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Spiky Strings and Spin Chains

ND 0805.4387 [hep-th]
ND + M. Losi 0812.1714 "

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AdS/CFT

⇒ strong/weak coupling duality

planar $\mathcal{N}=4$ SYM $\equiv$ free strings on $AdS_5 \times S^5$

$\lambda = g^2 N$

$\lambda \ll 1$

perturbative gauge theory

$\lambda \gg 1$

semiclassical strings

Q] How do gauge theory "partons" emerge from string?

Plan

- $S \to \infty$ limit $\nearrow$ gauge theory $\lambda \ll 1$; $\searrow$ string theory $\lambda \gg 1$

- **Main Result**

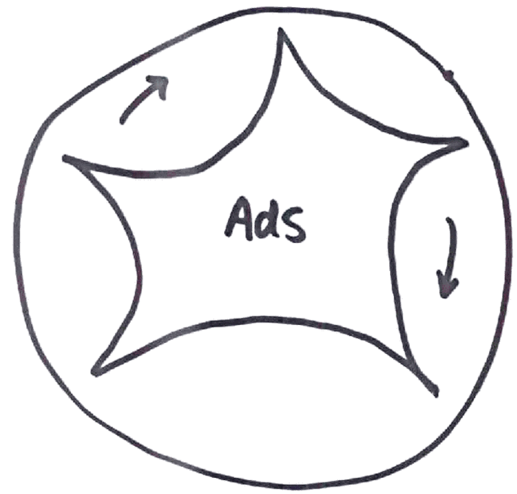
Precise agreement of spectra unexpected

- **Interpretation**

Spiky string $\longleftrightarrow$ Spin Chain

Spiky strings

Kruczenski (Alday + Maldacena)



Partons $\longleftrightarrow$ Spikes ?

This talk

Exact analysis of generic (semi-) classical solutions in large spin limit ......

$\mathcal{N}=4$ SUSY Yang-Mills $\lambda \ll 1$

$sl(2)$ sector,

$$\hat{O} = \mathrm{Tr}_N \left[\mathcal{D}_+^{s_1} Z \, \mathcal{D}_+^{s_2} Z \ldots\ldots \mathcal{D}_+^{s_J} Z \right]$$

- Twist J ($= U(1)_R$ charge)
- Spin $S = \sum s_i$
- Dimension $\Delta = (S+J) + O(\lambda)$

Will study limit,

$S \to \infty$, J fixed

Twist 2

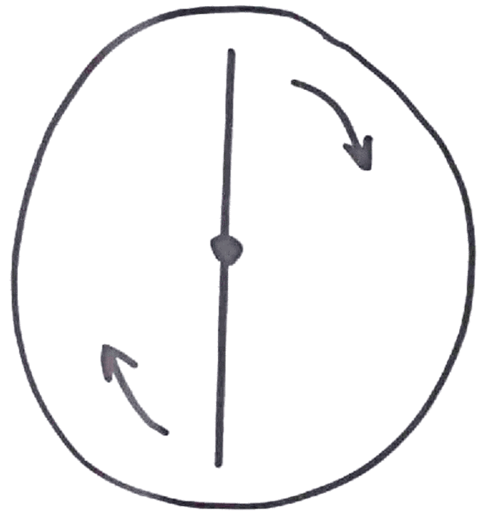
$$\Delta - S - J \approx 2\Gamma(\lambda) \log S + \ldots\ldots$$

$$\Gamma(\lambda) = \lambda/4\pi^2 + O(\lambda^2)$$

Strings on $AdS_3 \subset AdS_5 \times S^5$

$\lambda \gg 1$

Gubser et al.



$$\Delta - S \approx 2\Gamma(\lambda) \log S + \ldots\ldots$$

semiclassical string,

$$\Gamma(\lambda) = \frac{\sqrt{\lambda}}{2\pi} + O(\lambda^0)$$

Twist J   Belitsky et al
Beisert + Staudacher

Spin chain description,

$$\hat{O} = \mathrm{Tr}_N\left[\mathcal{D}_+^{S_1} Z\, \mathcal{D}_+^{S_2} Z \ldots\ldots \mathcal{D}_+^{S_J} Z\right]$$

Classical "spin" at each site,

$$\mathcal{L}_j \in sl(2,\mathbb{R}) \qquad j = 1, 2, \ldots, J$$

$$\left\{\mathcal{L}_i^A, \mathcal{L}_j^B\right\}_{P.B.} = 2i\,\epsilon^{ABC}\,\delta_{ij}\,\mathcal{L}_j^C$$

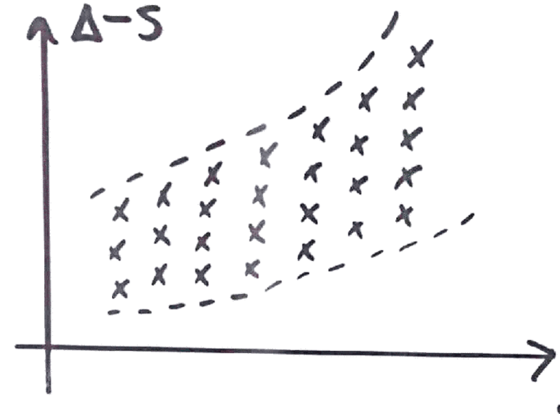
Hamiltonian,

$$\Delta - S = \frac{\lambda}{8\pi^2}\sum_{j=1}^{J} \log\left(|\vec{L}_j + \vec{L}_{j+1}|^2\right)$$

.... is integrable

$\Rightarrow$ exact solution

action/angle variables + WKB



$$2\Gamma \log S \leq \Delta - S \leq J\Gamma \log S$$

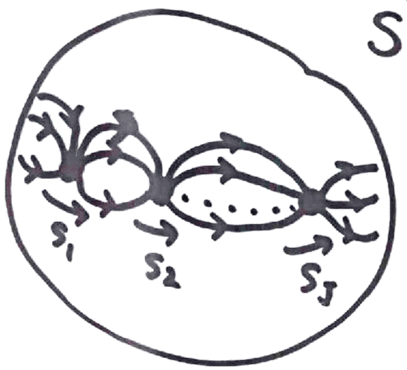
- Solution determined by spectral curve

$$\Sigma_{spin}: \quad t + 1/t = \left(2 + \frac{q_2}{x^2} + \ldots\ldots + \frac{q_J}{x^J}\right)$$

genus J−2

$$\hat{O} = \mathrm{Tr}_N\left[\mathcal{D}_+^{S_1} Z\, \mathcal{D}_+^{S_2} Z \ldots\ldots \mathcal{D}_+^{S_J} Z\right]$$

state ↔ operator

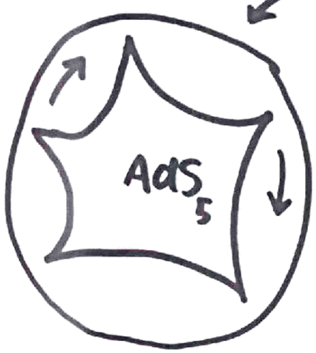


S^3

J "partons"

dual picture??

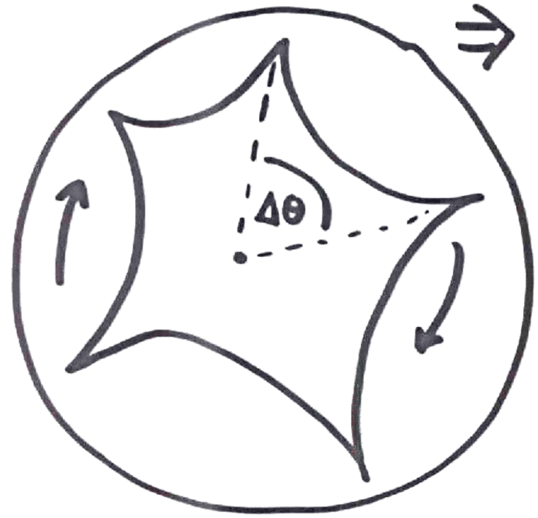
$\partial(AdS_5) = S^3 \times \mathbb{R}$



Kruczenski solution

k spikes

$\Rightarrow \Delta\theta = \frac{2\pi}{k}$



$$\Delta - S \approx k\,\Gamma(\lambda) \log S$$

- $k = J$??
- Moduli ??

Semiclassical Strings $\lambda \gg 1$

- $AdS_3 \times S^1$
- static, conformal gauge $\quad \sigma \sim \sigma + 2\pi$

$\Rightarrow SL(2,\mathbb{R})$ PCM

$$S_\sigma = \frac{\sqrt{\lambda}}{4\pi}\int d^2\sigma \, \mathrm{tr}_2\left[j_+ j_- \right]$$

$$j_\pm = g^{-1}\partial_\pm g, \quad g(\sigma,\tau) \in SL(2,\mathbb{R}) \cong AdS$$

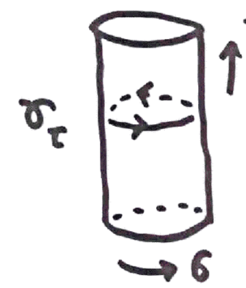
Lax pair,

$$\mathcal{L}\vec{\psi} = \left[\partial_\sigma + \frac{1}{2}\left(\frac{j_+}{1-x} - \frac{j_-}{1+x}\right)\right]\cdot\vec{\psi} =$$

$$\mathcal{M}\vec{\psi} = \left[\partial_\tau + \frac{1}{2}\left(\frac{j_+}{1-x} + \frac{j_-}{1+x}\right)\right]\cdot\vec{\psi} =$$

Eqn of Motion $\Leftrightarrow \quad [\mathcal{L},\mathcal{M}] = 0 \quad \forall x \in \mathbb{C}$

- Monodromy, $\begin{pmatrix}\mathcal{L}\\ \mathcal{M}\end{pmatrix} = d - J$



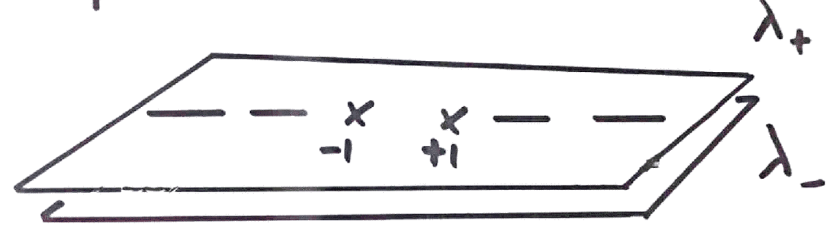
$$\Omega = P\exp\left[\oint_{\sigma_\tau} J(\sigma,\tau;x)\right]$$

$$\vec{\psi}(\sigma+2\pi) = \Omega\cdot\vec{\psi}(\sigma)$$

- $[\mathcal{L},\mathcal{M}] = 0 \Rightarrow$ eigenvalues $\lambda_\pm = e^{\pm i}$ of Ω are conserved $\forall x \in \mathbb{C}$

- Analyticity

$\lambda_\pm(x)$ live on double-cover of complex x-plane



"k-gap" solutions $\Leftrightarrow$ k cuts

Spectral Problem

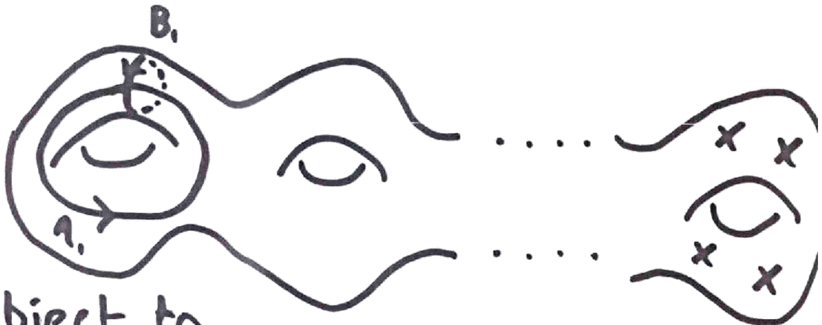
Kazakov et al

Find (Σ, dp)

hyperelliptic genus $k-1$

meromorphic

Σ



Subject to,

- normalisation,

mode numbers

$$\oint_{A_i} dp = 0 \quad , \quad \oint_{B_i} dp = 2\pi n_i$$

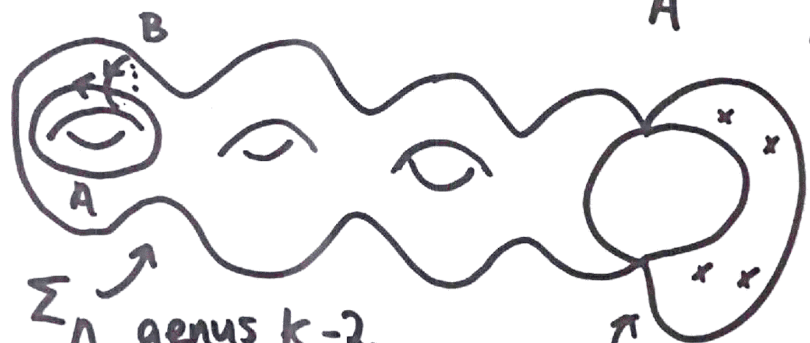
$i = 1, \ldots, k$

- Virasoro,

$$dp \longrightarrow \frac{\pi J}{(x \mp 1)^2} \quad , \quad x \to \pm 1^{\pm}$$

Large Spin Limit

$$\Sigma \to \Sigma_A \cup \Sigma_B$$



Σ_A genus $k-2$

Σ_B genus 0

- Solve period conditions,

$$\Sigma_A : \; t + 1/t = 2 + \frac{q_2}{x^2} + \ldots\ldots + \frac{q_k}{x^k}$$

$t = e^{ip_1(x)}$

- string energy,

$$\Delta - S \approx \frac{\sqrt{\lambda}}{2\pi}\left(k \log S + \log(q_k) + \ldots\right)$$

- WKB,

$$\oint_{A_i} x \frac{dt}{t} \in \mathbb{Z} \cdot \frac{1}{\sqrt{\lambda}}$$

Main Result

Semiclassical spectrum of k gap string $\xrightarrow{S\to\infty}$ Semiclassical spectrum of gauge theory spin chain, Length = k

up to single normalisation,

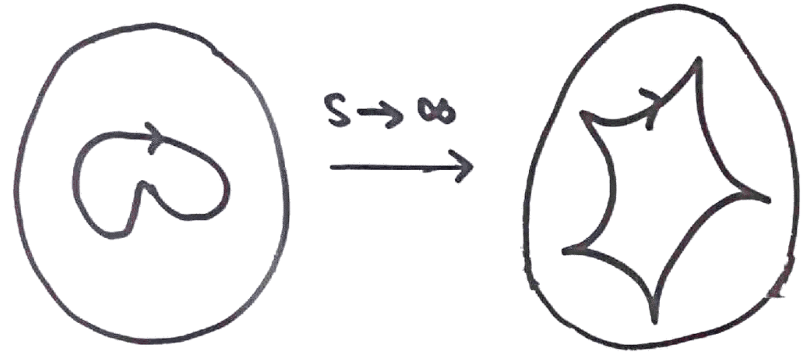
$$\sqrt{\lambda}/2\pi \longrightarrow \lambda/4\pi^2$$

Equality as integrable systems,

curve ✓
symplectic form ✓
Hamiltonian ✓

Interpretation

- Generic K-gap solution develops k spikes as $S\to\infty$



- Hamiltonian,

$$\Delta - S \approx \frac{\sqrt{\lambda}}{2\pi}\Big(\underbrace{k\log S}_{\text{static limit}} + \underbrace{\log(q_K)}_{\text{slow motion of spikes}} + \ldots\Big)$$

Connection to spin chain ND

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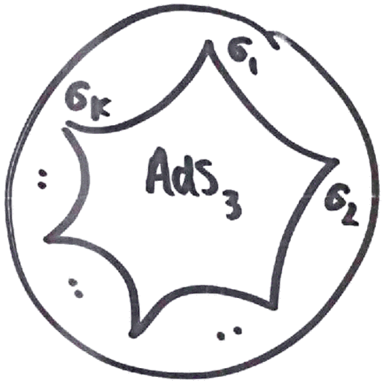
spikes at
$\sigma = \sigma_k$
$k = 1, \dots, K$



As $S \to \infty$, charge density localises at spikes,

$$j_\tau = g^{-1}(\partial_\tau g) \to \sum_{k=1}^{K} L_k \, \delta(\sigma - \sigma_k)$$

map,

$L_k \in sl(2, \mathbb{R})$

$$L_k^{\pm}, L_k^{0} \longleftrightarrow \mathcal{L}_k^{\pm}, \mathcal{L}_k^{0}$$

Limiting Solutions $S \to \infty$

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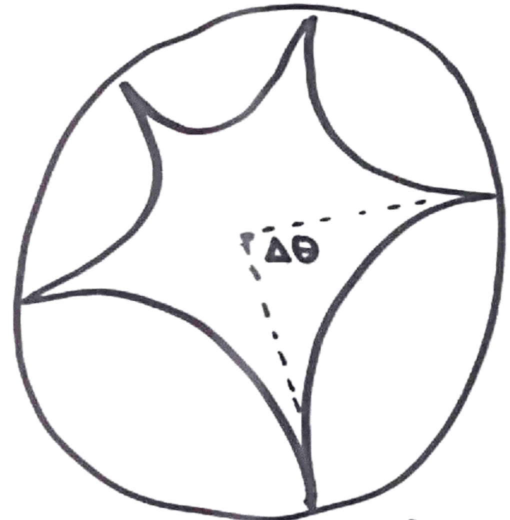

moduli $\rightleftharpoons$ Angular seperation

$$\Delta\theta_i = \theta_{i+1} - \theta_i$$

$$q_k \approx \sum_{1 \leq j_1 \dots \leq j_k \leq K} \prod_{l=1}^{k} \sin\left(\frac{\theta_{j_{l+1}} - \theta_{j_l}}{2}\right)$$

Comments/Outlook/Conclusions

- Unexplained agreement with gauge theory

  Wrapping corrections?

- Connection to gluon scattering and light-like Wilson loops?

- AdS pp-wave limit Krucsenski et al

DLCQ (integrable!) for Poincaré patch.