Superprotected n-point functions of local operators in $\mathcal{N} = 4$ supersymmetric YM

Nadav Drukker
Humboldt Universität zu Berlin

Kavli Institute for Theoretical Physics
May 12, 2009

N.D. and Jan Plefka, `arXiv:0812.3341, 0901.3653`

Introduction

$\mathcal{N} = 4$ SYM

Matter content: Gluons A_μ, 6 scalars Φ^I, 4 gluinos $\Psi^A_\alpha, \Psi^A_{\dot\alpha}$.

Most symmetric 4d gauge theory, symmetry $psu(2,2|4)$ including $so(4,2) \times so(6)$:

Conformal:	$P_\mu, K_\mu, M_{\mu\nu}, D\,,$	R-symmetry:	R_{IJ}
SUSY:	$Q^A_\alpha, \bar{Q}_{A\dot\alpha}\,,$	Superconformal:	$S^\alpha_A, \bar{S}^{A\dot\alpha}$

In total 32 supercharges.

Dual to string theory on $AdS_5 \times S^5$ (will not use today).

I've been involved in finding interesting families of operators with 16 or fewer SUSYs.

This gives enough freedom to be interesting but still under control.

Chiral primary operators

Simplest operators: $\mathrm{Tr}\,[\Phi_{\{I_1}\cdots\Phi_{I_J\}}]$ with $_{\{\dots\}}$ sym. tracless.

A nice basis is $Z_u(x)=u^I\Phi^I(x)$ and take $\mathcal{O}=\mathrm{Tr}\,[Z_u(x)^J]$.

SUSY: $\delta\Phi^I=\bar\Psi\gamma^5\rho^I(\epsilon_0+\gamma_\mu x^\mu\epsilon_1)$.

$$\delta Z(0)=0 \qquad\Rightarrow\qquad u^I\rho^I\epsilon_0=0 \qquad\Leftrightarrow\qquad u\cdot u=0$$

$\mathrm{Tr}\,Z^J$ preserves 24 supersymmetry generators.

$\left\langle\mathrm{Tr}\,Z_1^J(x_1)\,\mathrm{Tr}\,Z_2^J(x_2)\right\rangle\sim J\left(\frac{u_1^I\cdot u_2^I}{(x_1-x_2)^2}\right)^J\equiv J[12]^J$ is an exact statement and receives no quantum corrections.

$\mathrm{Tr}\,Z^J(x_2)$ preserves also 24 generators. It shares 16 preserved supercharges with the other one. Which generators are common depends on the positions x_1, x_2.

The three-point function:

$$\left\langle \operatorname{Tr} Z_1^{J_1}(x_1)\, \operatorname{Tr} Z_2^{J_2}(x_2)\, \operatorname{Tr} Z_3^{J_3}(x_3) \right\rangle \sim [12]^{J_1+J_2-J_3}\,[23]^{J_2+J_3-J_1}\,[13]^{J_3+J_1-J_2}$$

This is an exact statement and receives no quantum corrections. (Lee, Minwalla, Rangamani, Seiberg)

All three operators share at least 8 supercharges.

The 4-point function is very complicated

$$\left\langle \operatorname{Tr} Z_1^{k}(x_1)\, \operatorname{Tr} Z_2^{k}(x_2)\, \operatorname{Tr} Z_3^{k}(x_3)\, \operatorname{Tr} Z_4^{k}(x_4) \right\rangle = \text{tree} + \mathcal{R} \sum_{m+n+l=k-2} \mathcal{F}_{mnl}^{(k)}(s,t,\lambda)\, \mathcal{X}^m\, \mathcal{Y}^n\, \mathcal{Z}^l$$

where (Arutyunov, Dolan, Osborn, Penati, Santambrogio, Sokatchev,. . .)

$$\mathcal{X} = [12][34]\,, \qquad \mathcal{Y} = [13][24]\,, \qquad \mathcal{Z} = [14][23]\,.$$

The conformal invariant cross ratios are

$$s = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}\,, \qquad t = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}\,, \qquad x_{ij}^2 = (x_i - x_j)^2\,.$$

and

$$\mathcal{R} = s\,\mathcal{X}^2 + \mathcal{Y}^2 + t\,\mathcal{Z}^2 + (s-t-1)\,\mathcal{Y}\,\mathcal{Z} + (1-s-t)\,\mathcal{X}\,\mathcal{Z} + (t-s-1)\,\mathcal{X}\,\mathcal{Y}\,.$$

$\mathcal{F}^{(k)}_{mnl}(s,t,\lambda)$ is known to two loop order.

We would like to attribute the complexity of the 4-point function to the fact that four 1/2 BPS local operators will generically not share any supersymmetries.

The purpose of this talk is to answer the following questions:

The purpose of this talk is to answer the following questions:

1. Are there ways to choose four operators or more to preserve SUSY?

The purpose of this talk is to answer the following questions:

1. Are there ways to choose four operators or more to preserve SUSY?
2. Will the n-point function be simpler than the generic case?

The purpose of this talk is to answer the following questions:

1. Are there ways to choose four operators or more to preserve SUSY?
2. Will the n-point function be simpler than the generic case?
3. Can I write a paper on a subject other than Wilson loops???

The purpose of this talk is to answer the following questions:

1. Are there ways to choose four operators or more to preserve SUSY?
2. Will the n-point function be simpler than the generic case?
3. Can this be used to understand more general n-point functions?

Outline

- Introduction
- Trivial example
- Example I
  - four-point function
  - Symmetry
- Example II
- Side result: General one-loop insertion formula
- Some five and six-point functions
- Outlook

Trivial example

Take $u_i^I = (1, i, 0, 0, 0, 0)$, so $Z = \Phi^1 + i\Phi^2$.

$\mathrm{Tr}\, Z^J(0)$ preserves 8 super-Poincaré charges (Qs) and all 16 superconformal ones (the Ss). At a different position $\mathrm{Tr}\, Z^J(x)$ will no longer preserve the Ss, but it will still preserve the same eight Qs (as well as 16 other linear combinations of generators).

Any number of such operators will therefore share eight supercharges.

Indeed the n-point function is protected

$$\left\langle \mathrm{Tr}\, Z^{J_1}(x_1)\, \mathrm{Tr}\, Z^{J_2}(x_2) \cdots \mathrm{Tr}\, Z^{J_n}(x_n) \right\rangle = 0\,.$$

This is a rather trivial example....

Same is almost true with $\mathcal{N} = 1$ SUSY — chiral ring.

Example I

Central idea: Make $u^I = u^I(x)$ space-time dependent.

We will take the following combination of the six real scalar fields

de Medeiros, Hull, Spence, Figueroa-O'Farrill, hep-th/0111190

$$C = 2i\, x_\mu \Phi^\mu + i\,(1-(x)^2)\Phi^5 + (1+(x)^2)\Phi^6\,.$$

For example

$$C(0) \propto \Phi^6 + i\Phi^5\,, \quad C(\infty) \propto \Phi^6 - i\Phi^5\,, \quad C(1,0,0,0) \propto \Phi^6 + i\Phi^1\,.$$

A crucial property:

$$[12] = \langle C(x_1)\, C(x_2)\rangle \propto \frac{(1+x_1^2)(1+x_2^2) - (1-x_1^2)(1-x_2^2) - 4x_1^\mu x_2^\mu}{(x_1-x_2)^2}$$

This is a constant independent of x_1 and x_2!

four-point function

2 and 3-point functions of chiral operators are always given by free contractions. For operators $\mathrm{Tr}\, C^{J_i}$ it will depend on J_1, J_2 and J_3, but not on the coupling and not on the positions.

The 4-point functions has the tree-contractions and generally the interactions

$$\left\langle \mathrm{Tr}\, C^k(x_1)\, \mathrm{Tr}\, C^k(x_2)\, \mathrm{Tr}\, C^k(x_3)\, \mathrm{Tr}\, C^k(x_4) \right\rangle = \text{tree} + \mathcal{R} \sum_{m+n+l=k-2} \mathcal{F}^{(k)}_{mnl}(s,t)\, \mathcal{X}^m\, \mathcal{Y}^n\, \mathcal{Z}^l$$

s and t are arbitrary, but for our operators $\mathcal{X} = \mathcal{Y} = \mathcal{Z}$, so

$$\mathcal{R} = s\,\mathcal{X}^2 + \mathcal{Y}^2 + t\,\mathcal{Z}^2 + (s-t-1)\,\mathcal{Y}\,\mathcal{Z} + (1-s-t)\,\mathcal{X}\,\mathcal{Z} + (t-s-1)\,\mathcal{X}\,\mathcal{Y} = 0\,.$$

I will prove later that there are no one-loop corrections to any n-point function of these operators.

$$C = 2i\, x_\mu \Phi^\mu + i(1 - (x)^2)\Phi^5 + (1 + (x)^2)\Phi^6$$

$$C = \Phi^6 + i x_m \Phi^m \,, \qquad m = 1, \cdots, 5 \qquad x^2 = 1$$

Supersymmetry

Require $\delta C = 0$ for all x^μ $\quad\Rightarrow\quad (\gamma^{\mu\nu} + \rho^{\mu\nu})\epsilon_0 = 0, \quad \epsilon_1 = i\gamma^1\rho^{16}\epsilon_0.$

Break R-symmetry $su(4) \to su(2) \times su(2)$ such that $\mathbf{4} \to (\mathbf{2}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2})$

Two solutions: $\epsilon_{0\,a}^{+\alpha} = \delta_a^\alpha \epsilon_0^+$ and $\epsilon_{0\,\dot{a}}^{-\dot{\alpha}} = \delta_{\dot{a}}^{\dot{\alpha}} \epsilon_0^-$.

Writing the generators schematically as

$$\left(\begin{array}{c|c} P, K, M, D & Q \\ \hline S & R \end{array}\right)$$

The preserved supercharges are $\mathrm{tr}_{\,su(4)} Q$ and $\mathrm{tr}_{\,su(4)} S$.

Our operators are covariant under the twisted generators

$$\tilde{P}_\mu = P_\mu + T_{5\mu} + iT_{6\mu}\,, \qquad \tilde{M}_{\mu\nu} = M_{\mu\nu} + T_{\mu\nu}\,,$$

$$\tilde{D} = D + iT_{56}\,, \qquad \tilde{K}_\mu = K_\mu - T_{5\mu} + iT_{6\mu}\,.$$

with twisted dimension $\tilde{\Delta} = 0$.

Hence the two point function is a constant!

Special cases

- As for all $1/2$ BPS chiral primaries, two of our operators will share 16 supercharges.

Special cases

- As for all $1/2$ BPS chiral primaries, two of our operators will share 16 supercharges.
- Three operators (or any number of operators along a line/circle) will share 8 supercharges.

Special cases

- As for all $1/2$ BPS chiral primaries, two of our operators will share 16 supercharges.
- Three operators (or any number of operators along a line/circle) will share 8 supercharges.
- Four operators (or any number of operators along a plane/sphere) will share 4 supercharges.

Special cases

- As for all 1/2 BPS chiral primaries, two of our operators will share 16 supercharges.
- Three operators (or any number of operators along a line/circle) will share 8 supercharges.
- Four operators (or any number of operators along a plane/sphere) will share 4 supercharges.
- The most generic configuration will share 2 supercharges.

Special cases

- As for all $1/2$ BPS chiral primaries, two of our operators will share 16 supercharges.
- Three operators (or any number of operators along a line/circle) will share 8 supercharges.
- Four operators (or any number of operators along a plane/sphere) will share 4 supercharges.
- The most generic configuration will share 2 supercharges.
- They all transform covariantly under the "twisted symmetries". Including 15 bosonic generators of $so(5,1)$, and 15 fermionic ones. Together they make the algebra $q(4)$.

As part of the algebra we can generate one copy of $so(5,1)$ from the fermionic generators

$$\tilde{P}_\mu = \{\mathcal{Q}^\pm\,, \tilde{Q}^\pm_\mu\}\,, \qquad \tilde{D} = \{\mathcal{Q}^\pm\,, \tilde{Q}^\pm_D\}\,,$$

Using the Ward identity

$$\frac{\partial}{\partial\, x_1^\mu}\Big\langle \text{Tr}\; C^{J_1}(x_1)\;\cdots\;\text{Tr}\; C^{J_n}(x_n)\Big\rangle$$
$$= \mathcal{Q}^+\Big\langle J_1\,\text{Tr}\,\Big[\big\{Q^+_\mu, C\big\}C^{J_1-1}(x_1)\Big]\;\cdots\;\text{Tr}\; C^{J_n}(x_n)\Big\rangle = 0\,.$$

So it is position-independent.

Can also write $\mathcal{S} = \{\mathcal{Q}^\pm\,, \Psi^\pm\}$, do

$$\frac{\partial}{\partial\, g^2_{YM}}\Big\langle \text{Tr}\; C^{J_1}(x_1)\;\text{Tr}\;\cdots\;\text{Tr}\; C^{J_n}(x_n)\Big\rangle_{\text{pert}}$$
$$\propto \mathcal{Q}^+\Big\langle \Psi^+\;\text{Tr}\; C^{J_1}(x_1)\;\cdots\;\text{Tr}\; C^{J_n}(x_n)\Big\rangle_{\text{pert}} = 0\,.$$

So there are no perturbative correction to the free field-thoery answer.

Example II

Using the complex coordinates $w = x_1 + ix_2$ and $\bar{w} = x_1 - ix_2$ in the plane

$$Z = i(1 - \bar{w}^2)\Phi^1 + (1 + \bar{w}^2)\Phi^2 - 2i\bar{w}\Phi^3 \,.$$

This is not the same as C restricted to the plane!

Generically four supercharges are preserved.

Now the two-point function is

$$[12] = \langle Z(w_1)\, Z(w_2)\rangle \propto \frac{(\bar{w}_1 - \bar{w}_2)^2}{|w_1 - w_2|^2} = \frac{\bar{w}_1 - \bar{w}_2}{w_1 - w_2}$$

the same as a (matrix) 2d conformal field of weight $(\frac{1}{2}, -\frac{1}{2})$.

Symmetry

There is a natural $SL(2,\mathbb{C}) \simeq SL(2,\mathbb{R}) \times \bar{SL}(2,\mathbb{R})$ and $su(2)_{123} \times su(2)_{456}$.

Z has weight $\frac{1}{2}$ under $SL(2,\mathbb{R})$.

Z has weight $-\frac{1}{2}$ under $\bar{SL}(2,\mathbb{R}) + su(2)_{123}$.

The commutators of supercharges gives $\bar{SL}(2,\mathbb{R}) + \frac{1}{2} su(2)_{123}$.

Under these generators Z has dimension zero, but is not a primary.

Topological theory?

four-point function

Now we write the complex cross ratio

$$s = \mu\bar{\mu}\,, \qquad t = (1-\mu)(1-\bar{\mu})\,, \qquad \mu = \frac{w_{12}w_{34}}{w_{13}w_{24}}.$$

Then

$$\mathcal{R} = \Big(\mu\,(\mathcal{X}-\mathcal{Z}) + \mathcal{Z} - \mathcal{Y}\Big)\Big(\bar{\mu}\,(\mathcal{X}-\mathcal{Z}) + \mathcal{Z} - \mathcal{Y}\Big).$$

Now using

$$\frac{\mathcal{X}}{\mathcal{Y}} = \frac{\bar{\mu}}{\mu}\,, \qquad \frac{\mathcal{Z}}{\mathcal{Y}} = \frac{1-\bar{\mu}}{1-\mu}\,.$$

We find

$$\mu\,(\mathcal{X}-\mathcal{Z}) + \mathcal{Z} - \mathcal{Y} = 0\,,$$

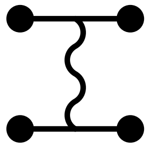
so again there are no quantum corrections to the four-point functions.

General one-loop insertion formula

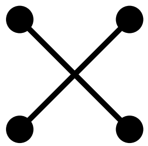
The following discussion applies to correlation functions of all Chiral primary operators, not only "superprotected".

All one-loop graphs involve the basic interaction vertices

$$= -\lambda\,(u_1\cdot u_2)\,(Y_{112}+Y_{122})$$



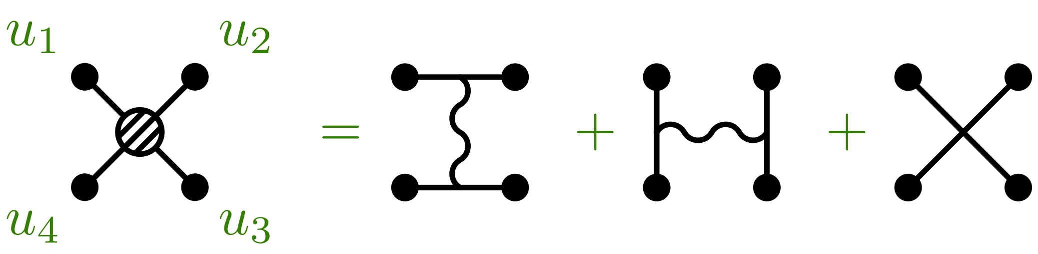
$$= \frac{\lambda}{2}\,(u_1\cdot u_2)\,(u_3\cdot u_4)\,I_{12}\,I_{34}\,F_{12,34}$$



$$= \frac{\lambda}{2}\Big[2\,(u_2\cdot u_3)\,(u_1\cdot u_4) - (u_2\cdot u_4)\,(u_1\cdot u_3) - (u_1\cdot u_2)\,(u_3\cdot u_4)\Big]\,X_{1234}$$

I is a free propagator, Y is a scalar triangle integral X is a box integral, and F is a derivative of the H-integral, which can be written as a sum of X and Y terms.

Summing over all possible insertions of these interactions, they all cancel unless the points 1234 are distinct and all the complicated terms in F cancel against corner interaction, so we are left with an effective four-point interaction

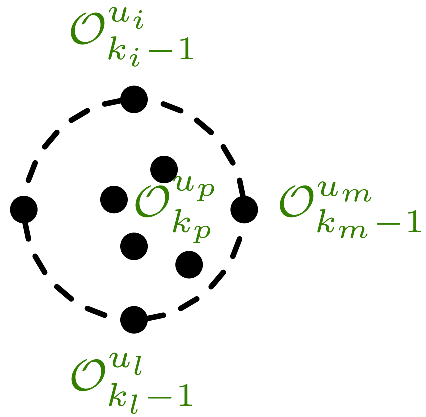
$D_{1234} \equiv$  $-$ corners

$$= \frac{\lambda}{2}\,\frac{X_{1234}}{I_{13}I_{24}}\big(2\,[13][24] + (s-1-t)[14][23] + (t-1-s)[12][34]\big)$$

So the general one loop graph is the sum over choices of four points

$$\left\langle \mathcal{O}^{u_1}_{J_1}\cdots\mathcal{O}^{u_n}_{J_n}\right\rangle_{\text{1-loop}} = \sum_{i,j,k,l} J_i J_j J_k J_l\, D_{ijkl}$$
$$\left\langle \mathcal{O}^{u_i}_{J_i-1}\mathcal{O}^{u_j}_{J_j-1}\mathcal{O}^{u_k}_{J_k-1}\mathcal{O}^{u_l}_{J_l-1}\,\middle|\,\mathcal{O}^{u_1}_{J_1}\cdots\mathcal{O}^{u_n}_{J_n}\right\rangle_{\text{tree, disc}}$$

$$\left\langle \mathcal{O}^{u_i}_{k_i-1}\mathcal{O}^{u_j}_{k_j-1}\mathcal{O}^{u_l}_{k_l-1}\mathcal{O}^{u_m}_{k_m-1}\right| \prod_{p\neq i,j,l,m}\mathcal{O}^{u_p}_{k_p}\Big\rangle_{\text{tree, disc}} =$$



Using this we calculated some four, five and six point functions at one loop.

$$\langle \mathcal{O}^{u_1}_2\mathcal{O}^{u_2}_2\mathcal{O}^{u_3}_2\mathcal{O}^{u_4}_2\rangle_{\text{1-loop}} = 16\Big(D_{1234}\,\langle \mathcal{O}^{u_1}_1\mathcal{O}^{u_2}_1\mathcal{O}^{u_3}_1\mathcal{O}^{u_4}_1\rangle_{\text{tree, disc}}$$
$$+\,D_{1324}\,\langle \mathcal{O}^{u_1}_1\mathcal{O}^{u_3}_1\mathcal{O}^{u_2}_1\mathcal{O}^{u_4}_1\rangle_{\text{tree, disc}} + D_{1243}\,\langle \mathcal{O}^{u_1}_1\mathcal{O}^{u_2}_1\mathcal{O}^{u_4}_1\mathcal{O}^{u_3}_1\rangle_{\text{tree, disc}}\Big).$$

For a given ordering there are two planar tree diagrams with a pair of contractions: $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$

$$\langle \mathcal{O}^{u_1}_2\mathcal{O}^{u_2}_2\mathcal{O}^{u_3}_2\mathcal{O}^{u_4}_2\rangle_{\text{1-loop}} = 16\Big(D_{1234}(\mathcal{X}+\mathcal{Z}) + D_{1243}(\mathcal{Y}+\mathcal{X}) + D_{1324}(\mathcal{Z}+\mathcal{Y})\Big)$$
$$= -\,16\Big(D_{1234}\mathcal{Y} + D_{1243}\mathcal{Z} + D_{1324}\mathcal{X}\Big) = -\frac{\lambda}{\pi^2}\,\Phi(s,t)\,\mathcal{R}.$$

Used $(D_{1234}+D_{1243}+D_{1324}) = 0$.
The result is minus the sum of all non-planar contractions.

Some 5-point functions

The 5-point function

$$\langle \mathcal{O}_2^{u_1} \mathcal{O}_2^{u_2} \mathcal{O}_2^{u_3} \mathcal{O}_3^{u_4} \mathcal{O}_3^{u_5} \rangle_{\text{1-loop}}$$

can be written as sum of terms like

$$\begin{aligned} & D_{1234} \langle \mathcal{O}_1^{u_1} \mathcal{O}_1^{u_2} \mathcal{O}_1^{u_3} \mathcal{O}_2^{u_4} \,|\, \mathcal{O}_3^{u_5} \rangle_{\text{tree, disc}} + 1 \leftrightarrow 2 + 2 \leftrightarrow 3 \\ & + D_{1245} \langle \mathcal{O}_1^{u_1} \mathcal{O}_1^{u_2} \mathcal{O}_2^{u_4} \mathcal{O}_2^{u_5} \,|\, \mathcal{O}_3^{u_3} \rangle_{\text{tree, disc}} + 1 \leftrightarrow 2 + 2 \leftrightarrow 4 \\ & + \cdots \end{aligned}$$

Using $(D_{1234} + D_{1243} + D_{1324}) = 0$ it can be written in terms of four and five-point functions of operators of dimension two times free-field contractions

$$\begin{aligned} & \frac{9}{4}[45] \langle \mathcal{O}_2^{u_1} \mathcal{O}_2^{u_2} \mathcal{O}_2^{u_3} \mathcal{O}_2^{u_4} \mathcal{O}_2^{u_5} \rangle_{\text{1-loop}} + \frac{9}{2}\,[41][15] \langle \mathcal{O}_2^{u_2} \mathcal{O}_2^{u_3} \mathcal{O}_2^{u_4} \mathcal{O}_2^{u_5} \rangle_{\text{1-loop}} \\ & \quad + \frac{9}{2}\,[42][25] \langle \mathcal{O}_2^{u_1} \mathcal{O}_2^{u_3} \mathcal{O}_2^{u_4} \mathcal{O}_2^{u_5} \rangle_{\text{1-loop}} + \frac{9}{2}\,[43][35] \langle \mathcal{O}_2^{u_1} \mathcal{O}_2^{u_2} \mathcal{O}_2^{u_4} \mathcal{O}_2^{u_5} \rangle_{\text{1-loop}} \,. \end{aligned}$$

Another example:

$$\langle \mathcal{O}_2^{u_1}\mathcal{O}_2^{u_2}\mathcal{O}_2^{u_3}\mathcal{O}_5^{u_4}\mathcal{O}_5^{u_5}\rangle = \frac{25}{4}[45]^3\langle \mathcal{O}_2^{u_1}\mathcal{O}_2^{u_2}\mathcal{O}_2^{u_3}\mathcal{O}_2^{u_4}\mathcal{O}_2^{u_5}\rangle$$

$$+\frac{75}{2}[35][35][45]^2\langle \mathcal{O}_2^{u_1}\mathcal{O}_2^{u_2}\mathcal{O}_2^{u_4}\mathcal{O}_2^{u_5}\rangle + \frac{75}{2}[25][25][45]^2\langle \mathcal{O}_2^{u_1}\mathcal{O}_2^{u_3}\mathcal{O}_2^{u_4}\mathcal{O}_2^{u_5}\rangle$$

$$+\frac{75}{2}[15][15][45]^2\langle \mathcal{O}_2^{u_2}\mathcal{O}_2^{u_3}\mathcal{O}_2^{u_4}\mathcal{O}_2^{u_5}\rangle$$

In all the examples we checked they can be written as the basic four and five-point functions times free contractions.

Would require better understanding of planar disc graphs to generalize.

Back to the “superprotected” operators

$$D_{1234} = \frac{\lambda}{2}\,\frac{X_{1234}}{I_{13}I_{24}}\Big(2\mathcal{Y} + (s-1-t)\mathcal{Z} + (t-1-s)\mathcal{X}\Big) \propto \frac{\partial\mathcal{R}}{\partial\mathcal{Y}}$$

$\frac{X_{1234}}{I_{13}I_{24}}$ is a transcendental function of the cross-ratios s and t, so cancelations should happen for each choice of four points independently.

Indeed for example I: $\mathcal{X} = \mathcal{Y} = \mathcal{Z}$ are constant, so $D_{ijkl} = 0$!

For example II this doesn't happen. We have to use the modular property

$$D_{1234} = -\frac{1}{\mu}\,D_{1324} = -\frac{1}{1-\mu}\,D_{1243}.$$

We checked five and six-point functions with total weight ≤ 16 and found that the sum of these triplets always vanished for example II!

Conclusions

- The answers are:

Conclusions

- The answers are:
  1. Yes: One can choose more than three operators that share SUSYs.

Conclusions

- The answers are:
  1. Yes: One can choose more than three operators that share SUSYs.
  2. Yes: They seem to receive no perturbative corrections.

Conclusions

- The answers are:
    1. Yes: One can choose more than three operators that share SUSYs.
    2. Yes: They seem to receive no perturbative corrections.
    3. Yes: I can write papers not mentioning Wilson loops.

Conclusions

- The answers are:
  1. Yes: One can choose more than three operators that share SUSYs.
  2. Yes: They seem to receive no perturbative corrections.
  3. Yes: I can write papers not mentioning Wilson loops.
- Possible to combine with BPS Wilson loop operators, surface operators, etc.

Conclusions

- The answers are:
  1. Yes: One can choose more than three operators that share SUSYs.
  2. Yes: They seem to receive no perturbative corrections.
  3. ~~Yes: I can write papers not mentioning Wilson loops.~~
- Possible to combine with BPS Wilson loop operators, surface operators, etc.

Outlook

- Many open questions:
  - Prove that there are no quantum corrections.
  - Understand the twisted symmetry and its multiplets.
  - Understand the topological theories.
  - Calculate instanton corrections.
  - Do the AdS calculation.

Outlook

- Many open questions:
  - Prove that there are no quantum corrections.
  - Understand the twisted symmetry and its multiplets.
  - Understand the topological theories.
  - Calculate instanton corrections.
  - Do the AdS calculation.
- Other examples: Any three $1/2$ BPS operators share eight supercharges. Should be possible to add more operators on the line.
  But on $\mathbb{R}^2$, $\mathbb{R}^3$?
  Use $\mathcal{R} = 0$?

Outlook

- Many open questions:
  - Prove that there are no quantum corrections.
  - Understand the twisted symmetry and its multiplets.
  - Understand the topological theories.
  - Calculate instanton corrections.
  - Do the AdS calculation.
- Other examples: Any three $1/2$ BPS operators share eight supercharges. Should be possible to add more operators on the line.
  But on $\mathbb{R}^2$, $\mathbb{R}^3$?
  Use $\mathcal{R} = 0$?
- Grand goal: Using these operators as the starting point to calculate n-point functions of other operators. BPS or not.
  Take operators deviating only by a small amount from those:
  Insert other fields into long operators...

The end