Brane Tilings, M2 Branes, CS Theories

Amihay Hanany

In collaboration with

Alberto Zaffaroni

Yang-Hui He

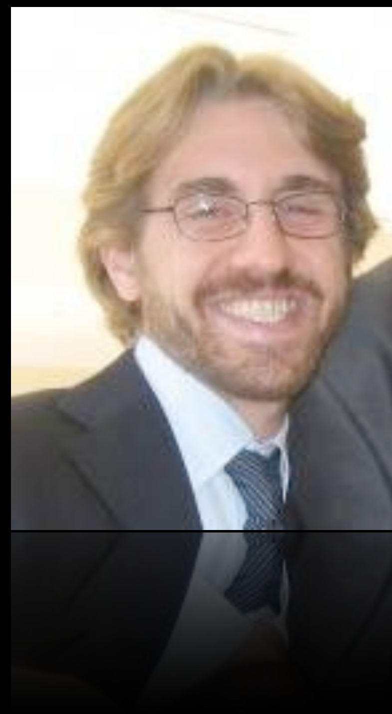
David Vegh

Sebastian Franco

Jaemo Park

Diego Rodriguez-Gomez

Noppadol Mekareeya



Giuseppe Torri

John Davey

Motivation

- Introduce a large class of SCFT's in 2+1d
- What is the world volume theory of a stack of N M2 branes in M theory?
- Understand Chern Simons (CS) theories better
- Algebraic Geometry - Quiver Gauge Theories

Motivation: AdS/CFT

- Long standing problem:
- What is the theory dual to AdS_4 x H^7
- H^7 Sasaki Einstein
- M2 probing CY4 - Cone over H^7

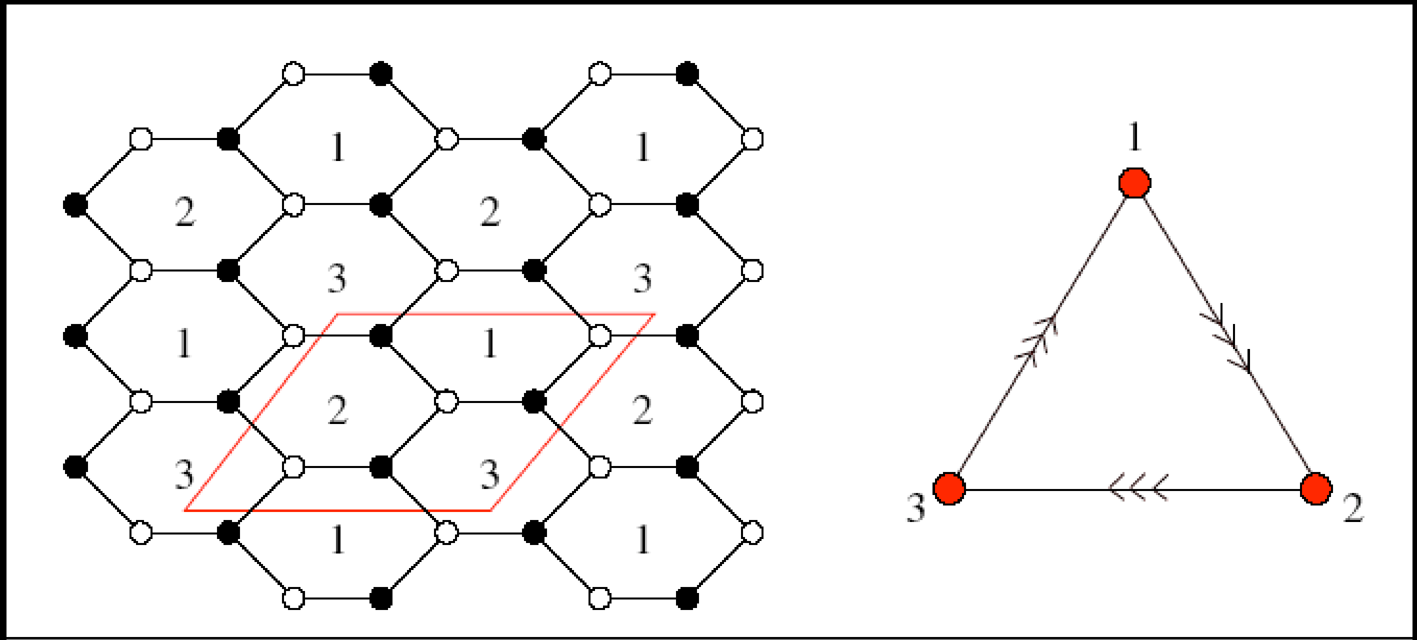
Recall in 3+1 dimensions

AdS/CFT

- Have a good understanding for the case of N D3 branes probing CY3
- $AdS_5 \times H^5$, H^5 Sasaki Einstein base of CY3
- Best description in terms of "Brane Tilings"

Brane Tilings Dictionary

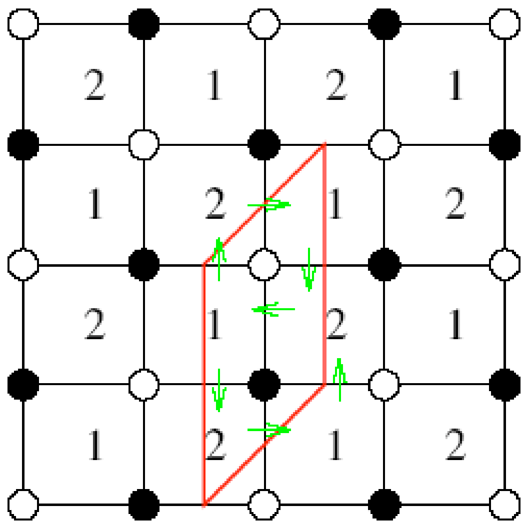
- Face (tile) - U(N) Gauge group
- Edge - A bi-fundamental chiral multiplet
- Node - Interaction term in W
- 2+1d: Each Face - integer CS level



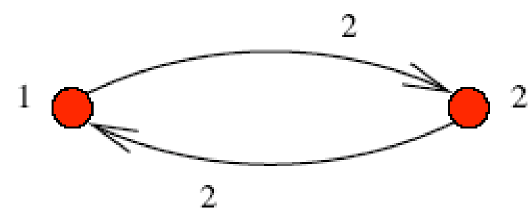
3 Hexagon tiling

Ex: Chessboard Tiling

$$CY_6 = conifold$$



brane tiling



quiver

$$W = X_{12}^{(1)} X_{21}^{(1)} X_{12}^{(2)} X_{21}^{(2)} - X_{12}^{(1)} X_{21}^{(2)} X_{12}^{(2)} X_{21}^{(1)}$$

Brane Tilings
Simple properties

- Arrows oriented in an alternating fashion around each face
- Bi-partite: arrows oriented (counter)clockwise around (black) white nodes
- black (white) nodes connected to white (black) nodes only

Brane Tilings Properties

- odd sided faces are forbidden - anomaly cancellation condition in 3+1d
- white (black) nodes with + (-) sign in W
- These rules define a unique Lagrangian in 3+1 & in 2+1 dimensions, 4 SUSY's

The 2+1d Lagrangian

$$
\begin{aligned}
- & \int d^4\theta \sum_{X_{ab}} X^{\dagger}_{ab} e^{-V_a} X_{ab} e^{V_b} \\
+ & \; i \int d^4\theta \sum_{a=1}^{G} k_a \int_0^1 dt V_a \bar{D}^{\alpha} (e^{tV_a} D_{\alpha} e^{-tV_a}) \\
+ & \int d^2\theta W(X_{ab}) + \text{c.c.}
\end{aligned}
$$

Choice of CS levels

$$\sum_{a=1}^{G} k_a = 0, \quad \mathrm{gcd}(\{k_a\}) = 1$$

$$C = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ k_1 & k_2 & k_3 & \dots & k_G \end{pmatrix} .$$

Vacuum Equations

$$\begin{aligned} \partial_{X_{ab}} W &= 0 \\ \mu_a(X) := \sum_{b=1}^{G} X_{ab} X_{ab}^{\dagger} - \sum_{c=1}^{G} X_{ca}^{\dagger} X_{ca} + [X_{aa}, X_{aa}^{\dagger}] &= 4 k_a \sigma_a \\ \sigma_a X_{ab} - X_{ab} \sigma_b &= 0 \end{aligned}$$

Fibration of a CY3 over a line

Forward Algorithm

INPUT 1: Quiver $\rightarrow d_{G\times E} \rightarrow (Q_D)_{(G-2)\times c} = \ker(C)_{(G-2)\times G} \cdot \tilde{Q}_{G\times c}$;

with $d_{G\times E} := \tilde{Q}_{G\times c} \cdot (P^T)_{c\times E}$

INPUT 2: CS Levels $\rightarrow C_{2\times G} \nearrow$

INPUT 3: Superpotential $\rightarrow P_{E\times c} \nearrow \rightarrow (Q_F)_{(c-G-2)\times c} = [\ker P]^t$;

$\downarrow$

$$(Q_t)_{(c-4)\times c} = \begin{pmatrix} (Q_D)_{(G-2)\times c} \\ (Q_F)_{(c-G-2)\times c} \end{pmatrix} \rightarrow$$

OUTPUT: $(G_t)_{4\times c} = [\mathrm{Ker}(Q_t)]^t$

Solving Vacuum Equations

- First set of equations - F terms - Master Space
- Second set - D terms - some vanish some not
- CS levels <> FI parameter
- Third set - a new ingredient in 2+1d

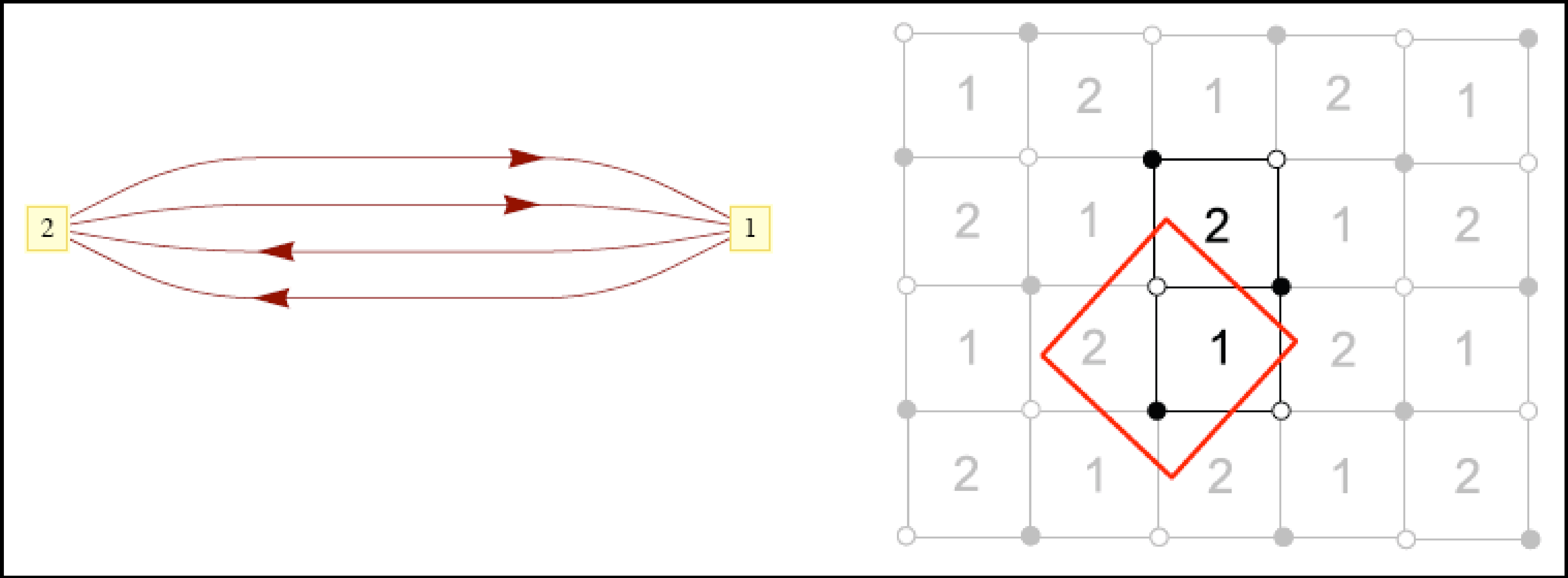
Some (new) Tools

- Master Space
- Lattice of generators - toric diagram
- Kasteleyn matrix
- Perfect Matchings

Master Space

- Solution to F term equations
- in 3+1d - combined baryonic & mesonic moduli space
- Toric, singular non-compact CY cone of dim G+2

Example: Chessboard Tiling; CS levels (1,-1)



$$W = \mathrm{Tr}(X^1_{12} X^1_{21} X^2_{12} X^2_{21} - X^1_{12} X^2_{21} X^2_{12} X^1_{21})$$

Chessboard tiling - $\mathbb{C}^4$

- For N=1 W =0, no F terms
- Master space is $\mathbb{C}^4$
- Third set of equations set σ's equal
- Second set of equations set value of σ
- CS levels 0 & in 3+1d: Conifold

CS levels on Edges

- Assign CS levels n's to edges such that
- d is the incidence matrix of the quiver

$$k_a = \sum_i d_{ai} n_i$$

chessboard fundamental domain

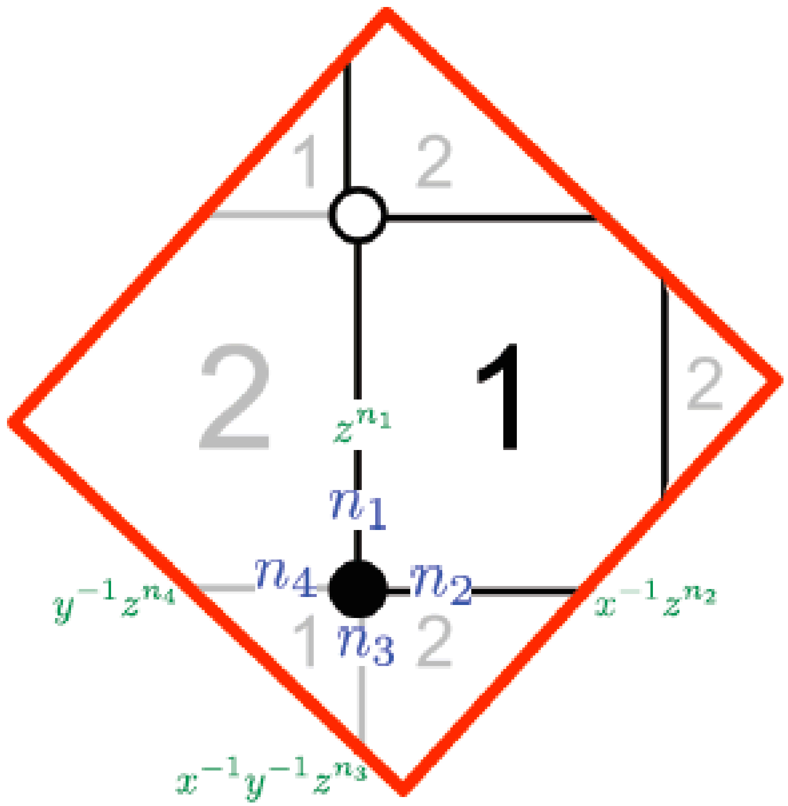
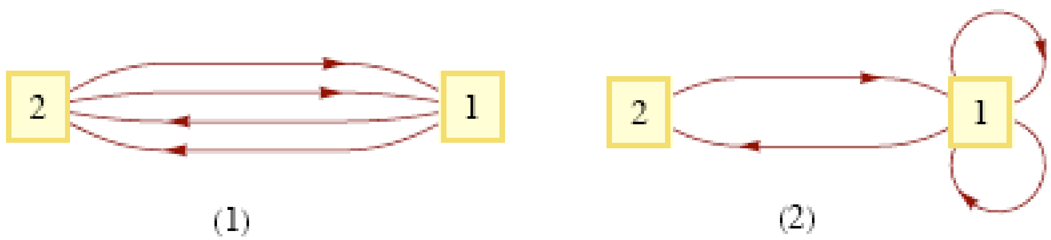


Figure 2: [Phase I of $\mathbb{C}^4$] The fundamental domain of the tiling for the $\mathscr{C}$ model: Assignments of the integers n_i to the edges are shown in blue and the weights for these edges are shown in green.

Useful in computing the toric diagram

4 fields in the quiver



$$W_{(1)} = \mathrm{Tr}(X^1_{12}X^1_{21}X^2_{12}X^2_{21} - X^1_{12}X^2_{21}X^2_{12}X^1_{21}); \qquad W_{(2)} = \mathrm{Tr}(X_{12}[\phi^1_2, \phi^2_2]X_{21})$$

Figure 1: The quivers with 4 fields and 2 nodes. There are 2 solutions and the 2-term superpotentials are also given. The moduli space in both cases is just the trivial CY 4-fold $\mathbb{C}^4$.

1 hexagon; 1 double edge, G=2

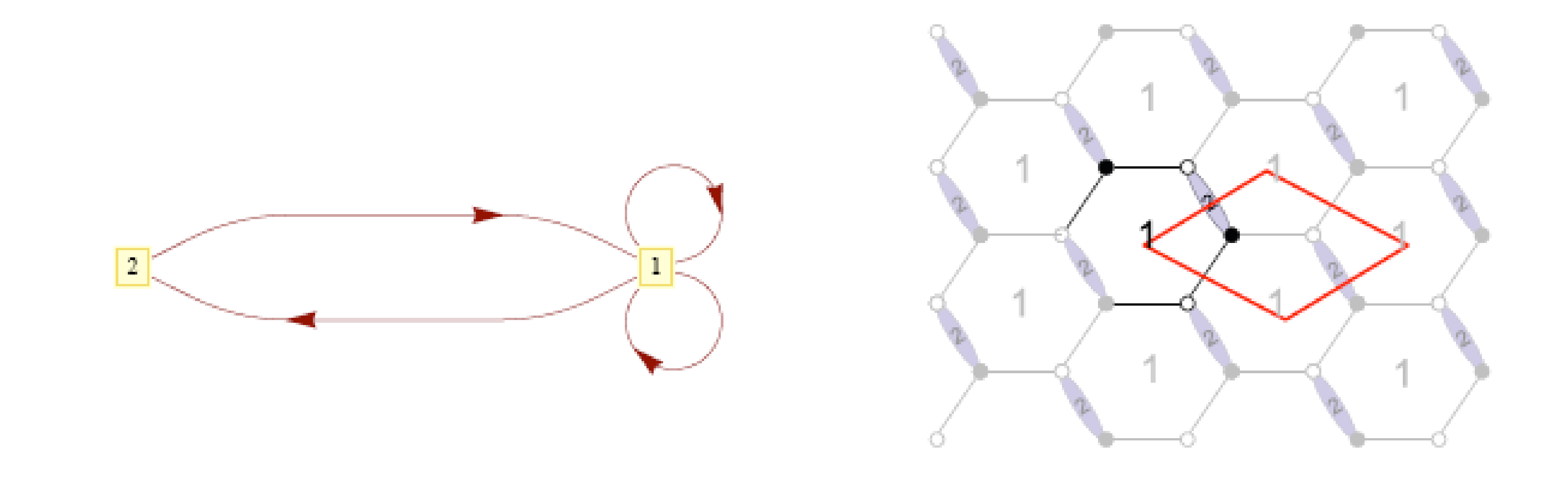
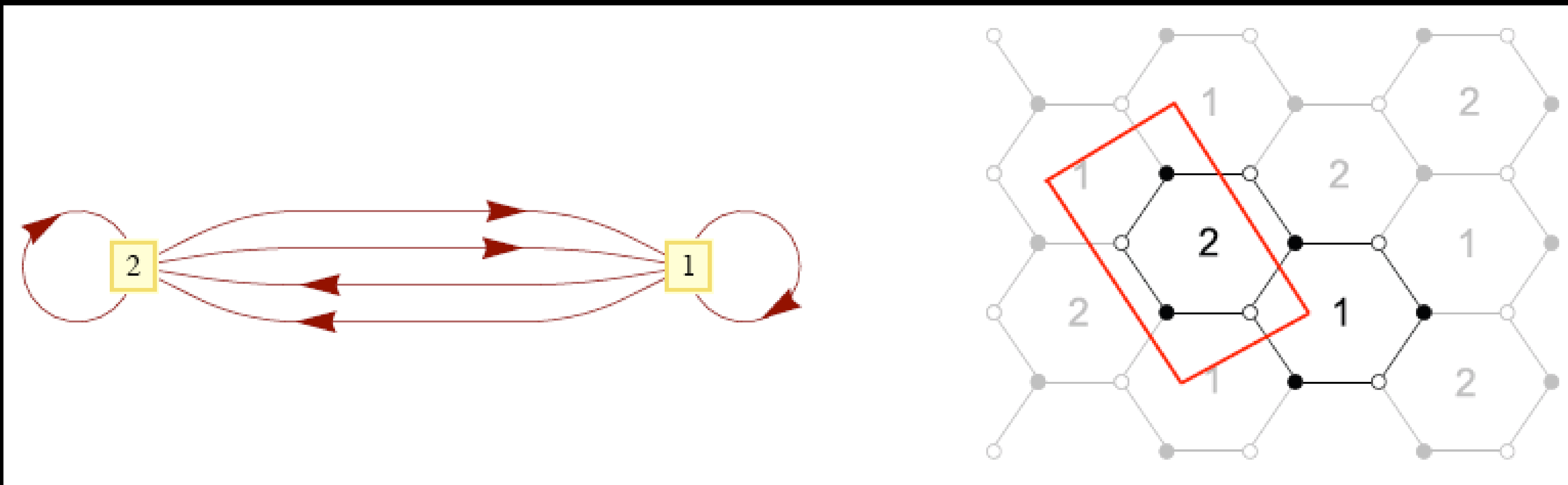


Figure 3: (i) Quiver diagram for phase 2 of the $\mathbb{C}^4$ theory. (ii) Tiling for phase 2 of the $\mathbb{C}^4$ theory.

Toric Duality

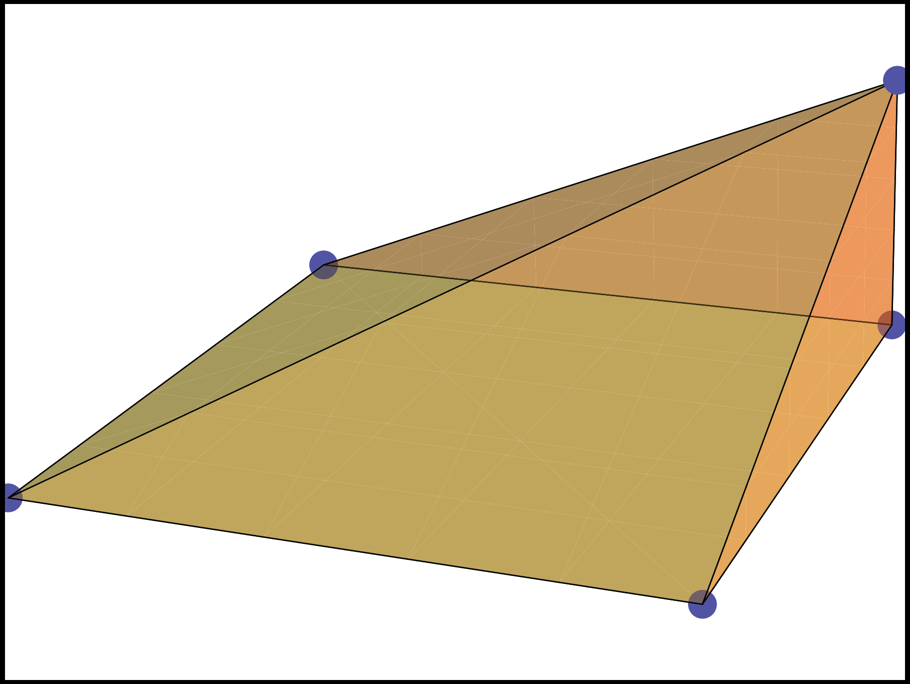
2 hexagon tiling; (1-,1)
Conifold ($\mathscr{C}$) x $\mathbb{C}$ $\mathbb{I}\mathbb{I}$



$$W = \phi_1(X_{12}^1 X_{21}^2 - X_{12}^2 X_{21}^1) + \phi_2(X_{21}^1 X_{12}^2 - X_{21}^2 X_{12}^1)$$

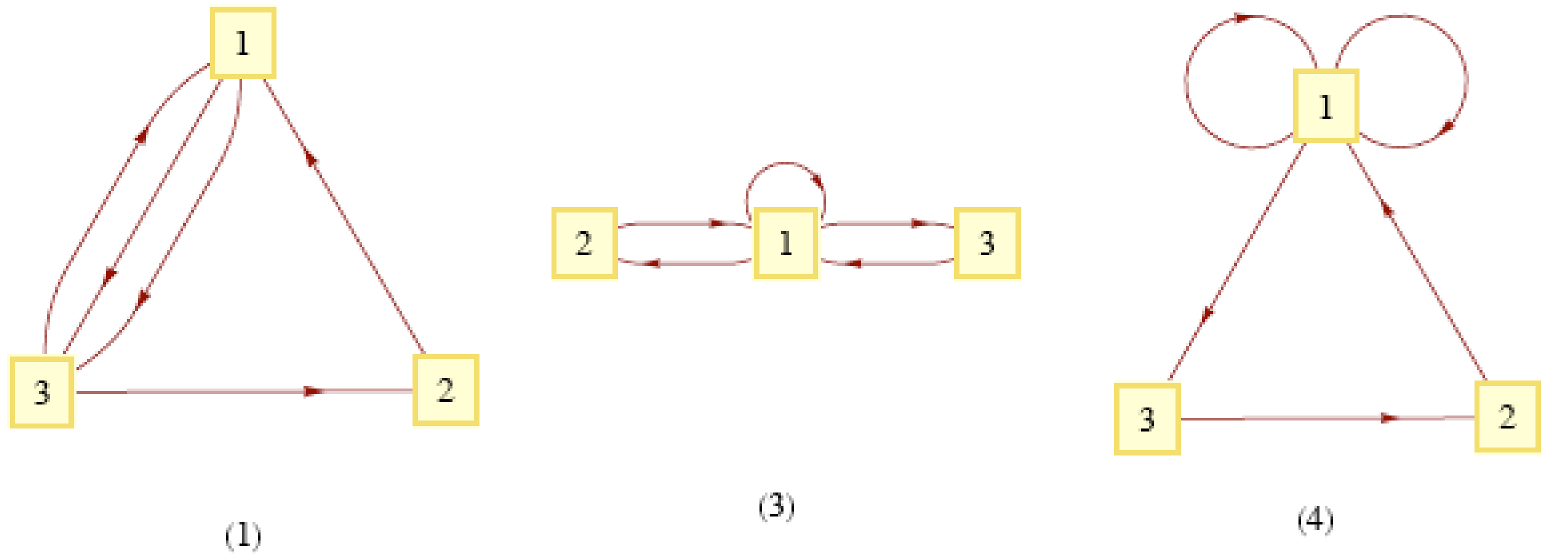
Ex: 2 hexagon tiling Conifold x $\mathbb{C}$ $\mathbb{II}$

- In 3+1d this is C^2/Z_2 x C
- Master space - 2+1d mesonic moduli space
- Non trivial scaling dimensions
- 1/2 for ϕ's, 3/4 for X's
- Non-trivial SCFT in the IR
- a test of AdS/CFT



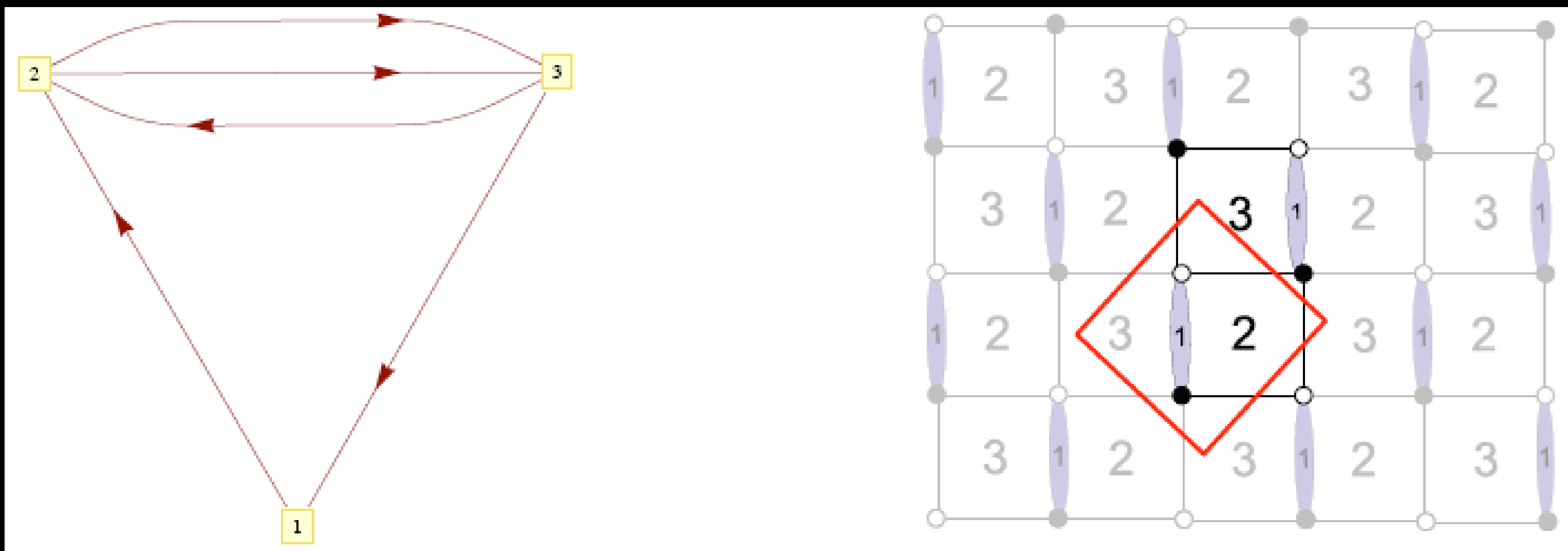
Toric Diagram $\mathcal{C} \times \mathbb{C}$

5 fields in the Quiver Master space - $\mathbb{C}^5$



$$W_{(1)} = \mathrm{Tr}(X_{21}X^1_{13}X_{31}X^2_{13}X_{32} - X_{21}X^2_{13}X_{31}X^1_{13}X_{32}) \; ;$$
$$W_{(3)} = \mathrm{Tr}(X_{21}\phi_1 X_{13}X_{31}X_{12} - X_{21}X_{13}X_{31}\phi_1 X_{12}) \; ;$$
$$W_{(4)} = \mathrm{Tr}(X_{21}[\phi^1_1, \phi^2_1]X_{13}X_{32}).$$

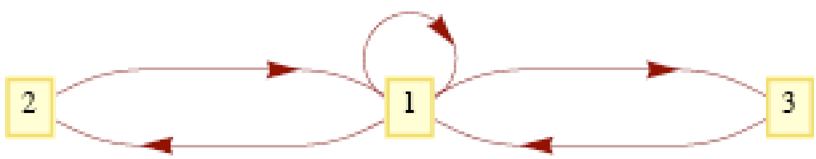
Chessboard tiling; 1 double edge; (1,-1,0)



Chessboard tiling; 1 double edge; (1,-1,0)

- mesonic moduli space is conifold x $\mathbb{C}$
- 1 dimensional baryonic moduli space
- Combined mesonic baryonic space - $\mathbb{C}^5$
- Scaling dimensions 1/2 for X_{12}, 3/8 other

Conifold x C
Phase III (0,1,-1); (-2,1,1)



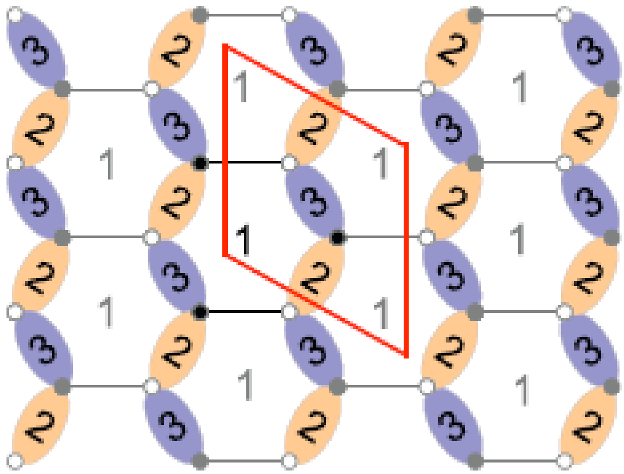


Figure 13: [Phase III of $\mathcal{C} \times \mathbb{C}$] (i) Quiver diagram of the $\mathscr{D}_2\mathscr{H}_1$ model. (ii) Tiling of the $\mathscr{D}_2\mathscr{H}_1$ model.

Global symmetry conifold x $\mathbb{C}$

- $SU(2) \times SU(2) \times U(1)_q \times U(1)_R \times U(1)_B$

Conifold x C
Table of charges

	$SU(2)_1$	$SU(2)_2$	$U(1)_q$	$U(1)_B$	$U(1)_R$	fugacity
p_1	1	0	1	1	3/8	$t^3 q b x_1$
p_2	-1	0	1	1	3/8	$t^3 q b / x_1$
p_3	0	1	1	-1	3/8	$t^3 q x_2 / b$
p_4	0	-1	1	-1	3/8	$t^3 q / (b x_2)$
p_5	0	0	-4	0	1/2	t^4 / q^4

Table 2: Charges under the global symmetry of the $\mathcal{C} \times \mathbb{C}$ theory. Here t is the fugacity associated with the $U(1)_R$ charges. The power of t counts R-charges in the unit of 1/8, q is the fugacity associated with the $U(1)_q$ charges, and x_1, x_2 are respectively the $SU(2)_1$, $SU(2)_2$ weights.

Perfect matchings	Generator of Phase I	Generator of Phase II	Generator of Phase III
$p_1 p_3$	$X_{13} X_{32}^1$	X_{12}^1	$X_{21} X_{12}$
$p_2 p_3$	$X_{13} X_{32}^2$	X_{21}^1	$X_{21} X_{13}$
$p_1 p_4$	$X_{23} X_{32}^1$	X_{12}^2	$X_{31} X_{12}$
$p_2 p_4$	$X_{23} X_{32}^2$	X_{21}^2	$X_{21} X_{13}$
p_5	X_{21}	$\phi_1 = \phi_2$	ϕ_1

Toric Duality
conifold x $\mathbb{C}$

- Three phases
- 2 tiles | 3 tiles | 3 tiles
- Master space: mesonic | mesonic baryonic | “
- mesonic generators: linear | bi-linear | “

Hilbert Series
conifold x $\mathbb{C}$

$$\frac{1}{(1-t_1 x_1 b)\left(1-\frac{t_1 x_2}{b}\right)\left(1-\frac{t_1 b}{x_1}\right)\left(1-\frac{t_1}{x_2 b}\right)(1-t_2)}$$

$$t_1 = t^3 q \qquad t_2 = t^4/q^4$$

Lattice of generators conifold x $\mathbb{C}$

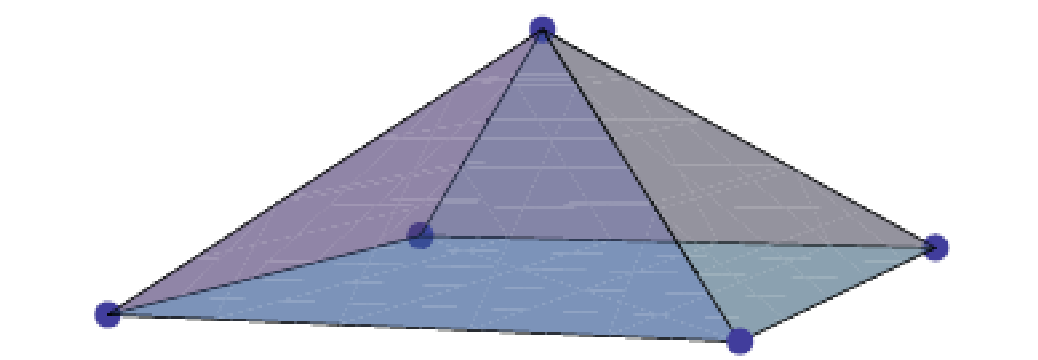


Figure 10: The lattice of generators of the $\mathcal{C} \times \mathbb{C}$ theory.

D3 theory

$$W = \mathrm{Tr}\left(X_{14}X_{42}X_{21}X_{12}X_{23}X_{31} - X_{14}X_{42}X_{23}X_{31}X_{12}X_{21}\right) \ . \qquad (5.1)$$

We choose the CS levels to be $(k_1, k_2, k_3, k_4) = (1, 1, -1, -1)$.

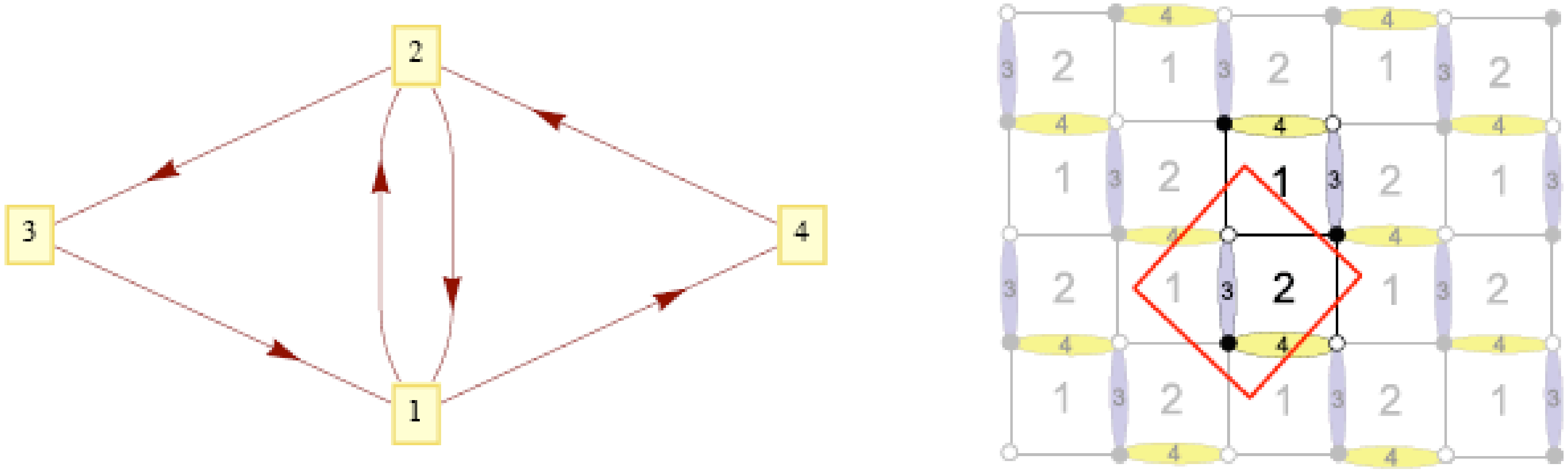


Figure 15: [Phase I of the D_3 theory] (i) Quiver diagram of the $\mathscr{D}_2\mathscr{C}$ model. (ii) Tiling of the $\mathscr{D}_2\mathscr{C}$ model.

D3 phase II

$$W = \mathrm{Tr}\left(X_{32}X_{23}X_{31}X_{13} - X_{23}X_{32}X_{21}X_{12} - \phi_1\left(X_{13}X_{31} - X_{12}X_{21}\right)\right) \ . \qquad (5.19)$$

We choose the CS levels to be $k_1 = 1,\ k_2 = -1,\ k_3 = 0$.

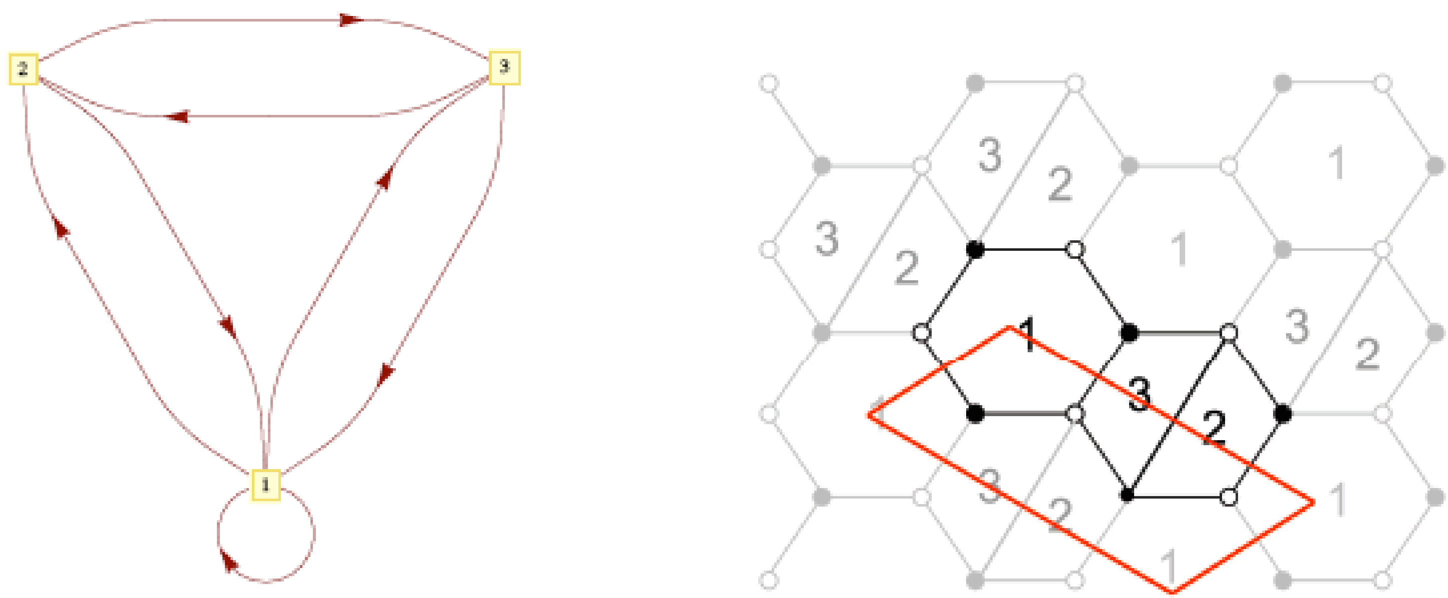


Figure 19: [Phase II of the D_3 theory] (i) Quiver diagram for the $\mathscr{H}_2\partial_1$ model. (ii) Tiling of the $\mathscr{H}_2\partial_1$ model.

D3 phase III

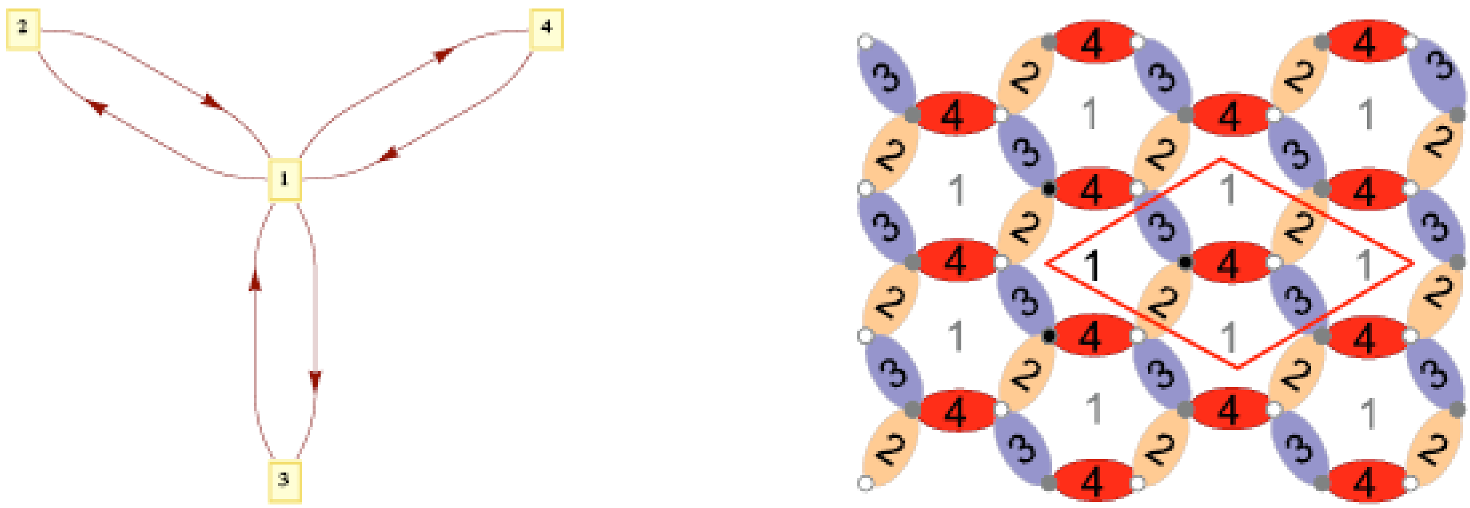


Figure 21: [Phase III of the D_3 theory] (i) Quiver diagram of the $\mathscr{D}_3\mathscr{H}_1$ model. (ii) Tiling of the $\mathscr{D}_3\mathscr{H}_1$ model.

$$W = \mathrm{Tr}\,(X_{13}X_{31}X_{14}X_{41}X_{12}X_{21} - X_{14}X_{41}X_{13}X_{31}X_{12}X_{21})$$

We choose the CS levels to be $(k_1, k_2, k_3, k_4) = (1, -1, 1, -1)$.

D3
Toric Diagram

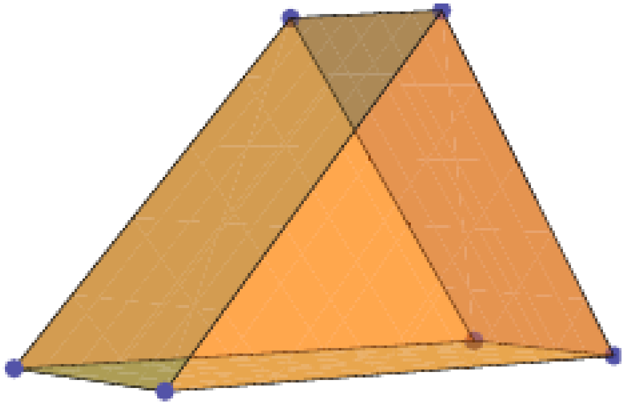


Figure 17: The toric diagram of the D_3 theory.

D3
Table of charges

	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$	$U(1)_{B_1}$	$U(1)_{B_2}$	fugacity
p_1	1	1	1	1/3	1	1	$tq_1q_2q_3b_1b_2$
p_2	−1	−1	0	1/3	0	1	$tb_2/(q_1q_2)$
p_3	1	0	−1	1/3	−1	0	$tq_1/(b_1q_3)$
p_4	−1	0	−1	1/3	1	0	$tb_1/(q_1q_3)$
p_5	−1	1	1	1/3	0	−1	$tq_3q_2/(q_1b_2)$
p_6	1	−1	0	1/3	−1	−1	$tq_1/(q_2b_1b_2)$

Perfect matchings	Generator of Phase I	Generator of Phase II	Generator of Phase III
p_1p_6	$X_{23}X_{14}$	X_{12}	$X_{41}X_{21}$
p_2p_5	$X_{42}X_{31}$	X_{21}	$X_{12}X_{14}$
p_3p_4	$X_{12}X_{21}$	ϕ_1	$X_{31}X_{13}$
$p_1p_3p_5$	$X_{23}X_{12}X_{31}$	$X_{23}X_{31}$	$X_{13}X_{41}X_{14}$
$p_2p_4p_6$	$X_{42}X_{21}X_{14}$	$X_{13}X_{32}$	$X_{31}X_{12}X_{21}$

D3
Lattice of Generators

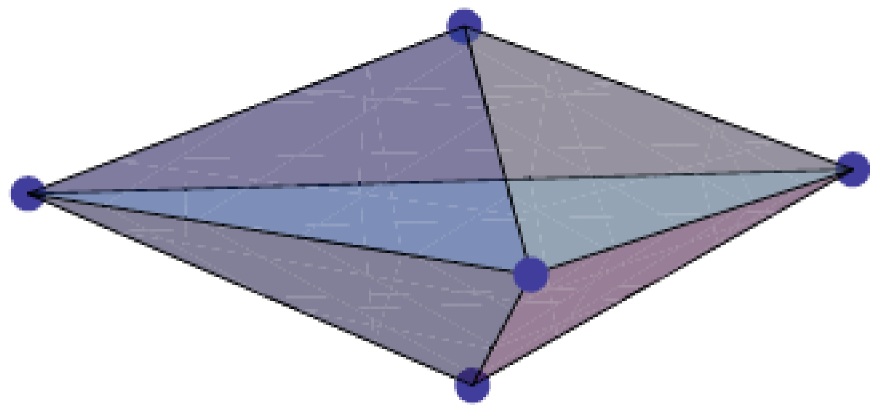


Figure 18: The lattice of generators of the D_3 theory.

Fano 3-folds

- 18 smooth toric Fano 3-folds
- translate toric data to brane tilings
- known for 10 cases, first 4:
- $\mathbb{P}^3$, $\mathbb{P}^2 \times \mathbb{P}^1$, $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$, $dP_1 \times \mathbb{P}^1$

M^{111}

CS (1,1,-2)

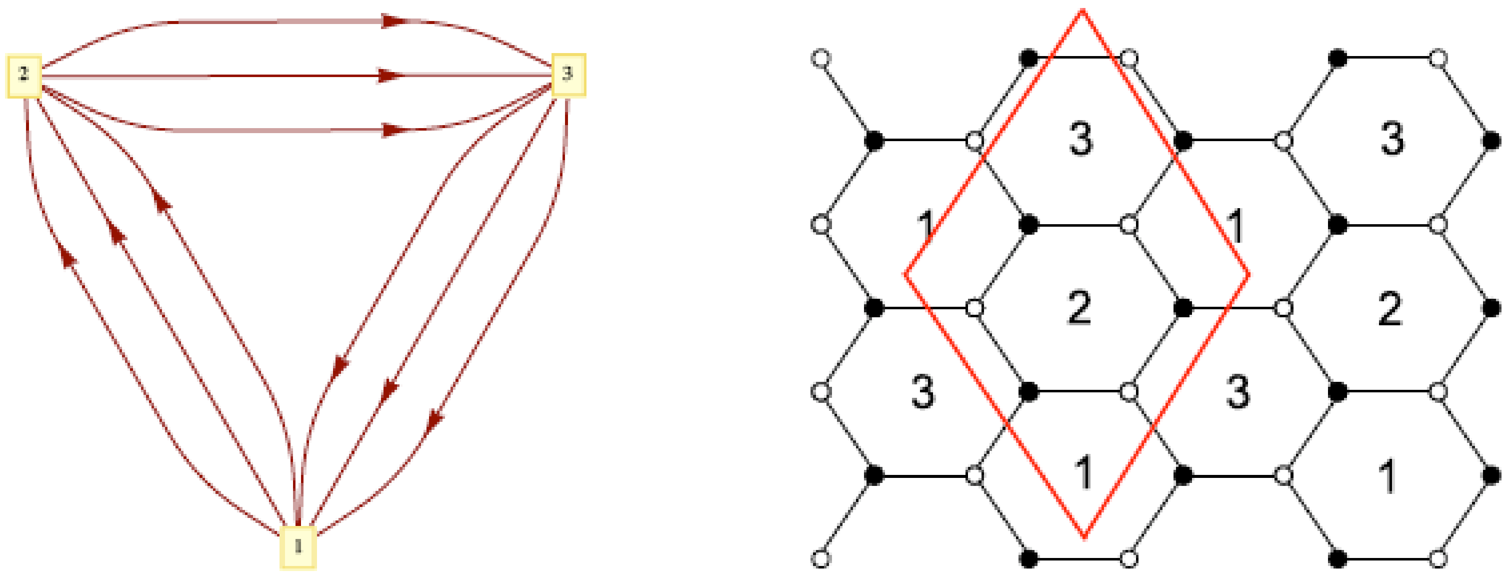


Figure 13: (i) Quiver diagram of the $M^{1,1,1}$ theory. (ii) Tiling of the $M^{1,1,1}$ theory.

M^{111}
Toric Diagram

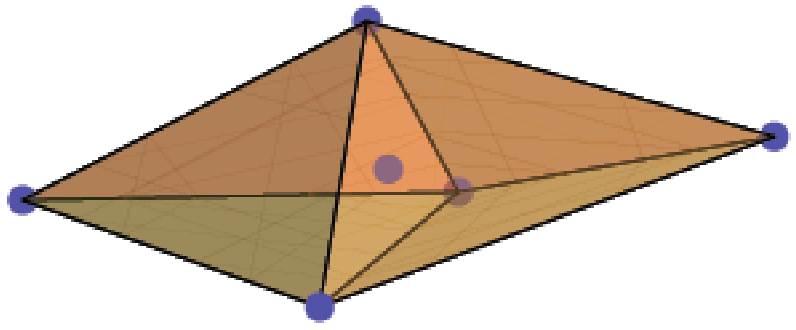


Figure 15: The toric diagram of the M^{111} theory.

M^{111}
Table of Charges

	$SU(3)$	$SU(2)$	$U(1)_R$	$U(1)_B$	fugacity
p_1	$(1,0)$	0	4/9	0	$t^4 y_1$
p_2	$(-1,1)$	0	4/9	0	$t^4 y_2/y_1$
p_3	$(0,-1)$	0	4/9	0	t^4/y_2
p_4	(0,0)	1	1/3	-1	$t^3 x/b$
p_5	(0,0)	-1	1/3	-1	$t^3/(xb)$
s_1	(0,0)	0	0	2	b^2

M¹¹¹ mesonic Hilbert Series

$$g^{\mathrm{mes}}(t,x,y_1,y_2;M^{1,1,1}) = \oint_{|b|=1} \frac{\mathrm{d}b}{2\pi i b}\; g^{\mathrm{Irr}\mathcal{F}^\flat}(t,x,y_1,y_2,b;M^{1,1,1})$$

$$= \frac{P_{M^{1,1,1}}(t,x,y_1,y_2)}{\left(1-\frac{t^{18}y_1^3}{x^2}\right)\left(1-t^{18}x^2y_1^3\right)\left(1-\frac{t^{18}x^2}{y_2^3}\right)\left(1-\frac{t^{18}}{x^2y_2^3}\right)\left(1-\frac{t^{18}y_2^2}{x^2y_1^3}\right)\left(1-\frac{t^{18}x^2y_2^3}{y_1^3}\right)}$$

$$= \sum_{j=0}^{\infty} [3j,0;2j]\, t^{18j} \qquad (2.25)$$

M^{111}

Lattice of generators

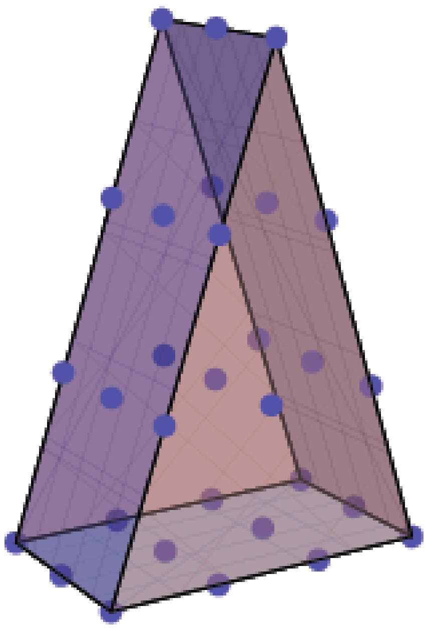


Figure 16: The lattice of generators of the $M^{1,1,1}$ theory

Q^{111}/Z_2
Phase I

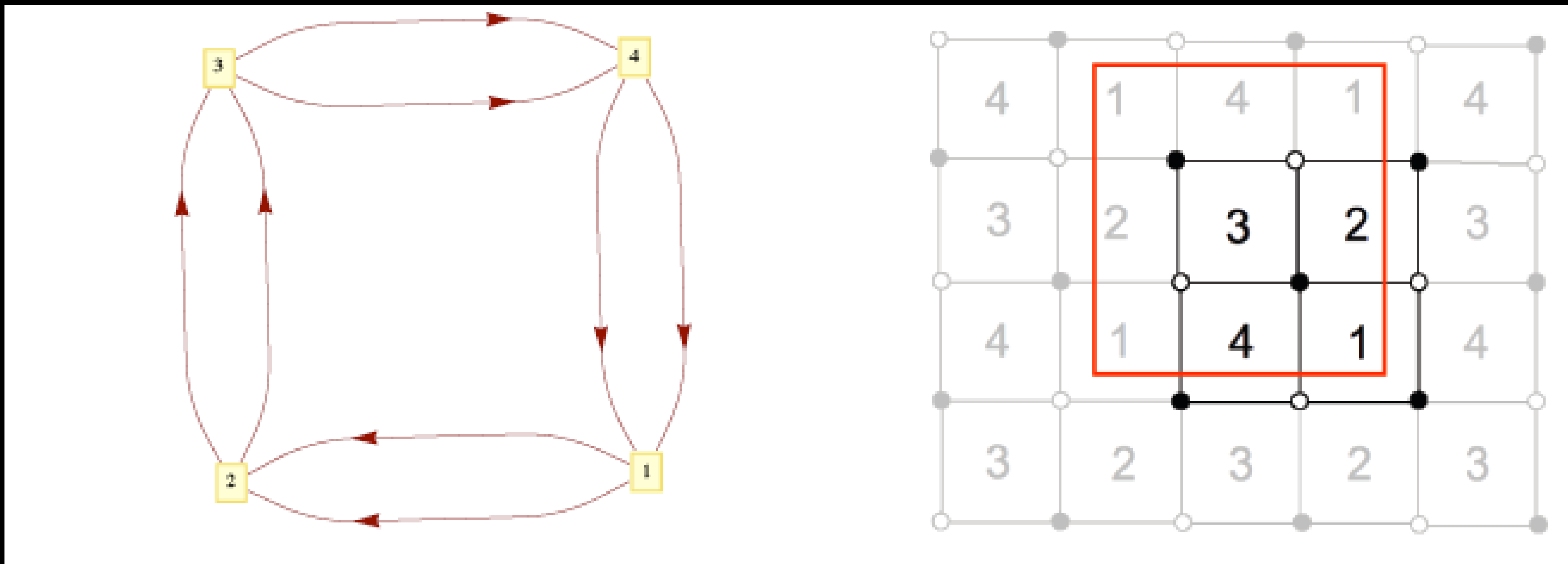


Figure 23: [Phase I of $Q^{1,1,1}/\mathbb{Z}_2$] (i) Quiver diagram for the $\mathscr{S}_4$ model. (ii) Tiling for the $\mathscr{S}_4$ model.

$$W = \epsilon_{ij}\epsilon_{pq}\,\mathrm{Tr}(X^i_{12}X^p_{23}X^j_{34}X^q_{41})$$

$$k_1 = -k_2 = -k_3 = k_4 = 1$$

Q^{111}/Z_2
Toric Diagram

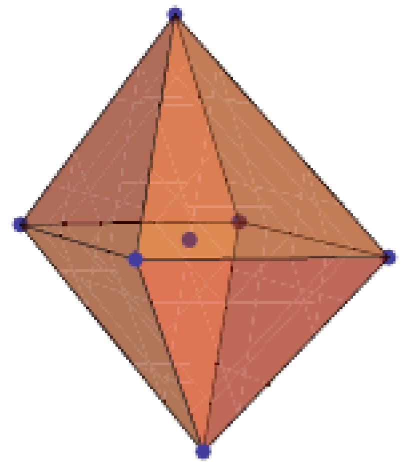


Figure 25: The toric diagram of the $Q^{1,1,1}/\mathbb{Z}_2$ theory.

Q^{111}/Z_2
Table of charges

	$SU(2)_1$	$SU(2)_2$	$SU(2)_3$	$U(1)_R$	$U(1)_{B_1}$	$U(1)_{B_2}$	fugacity
p_1	1	0	0	$1/3$	1	0	tb_1x_1
p_2	-1	0	0	$1/3$	1	0	tb_1/x_1
q_1	0	1	0	$1/3$	0	0	tx_2
q_2	0	-1	0	$1/3$	0	0	t/x_2
r_1	0	0	1	$1/3$	-1	-1	$tx_3/(b_1b_2)$
r_2	0	0	-1	$1/3$	-1	-1	$t/(x_3b_1b_2)$
s_1	0	0	0	0	0	2	b_2^2
s_2	0	0	0	0	0	0	1
s_3	0	0	0	0	0	0	1

Q^{111}/Z_2
Lattice of Generators

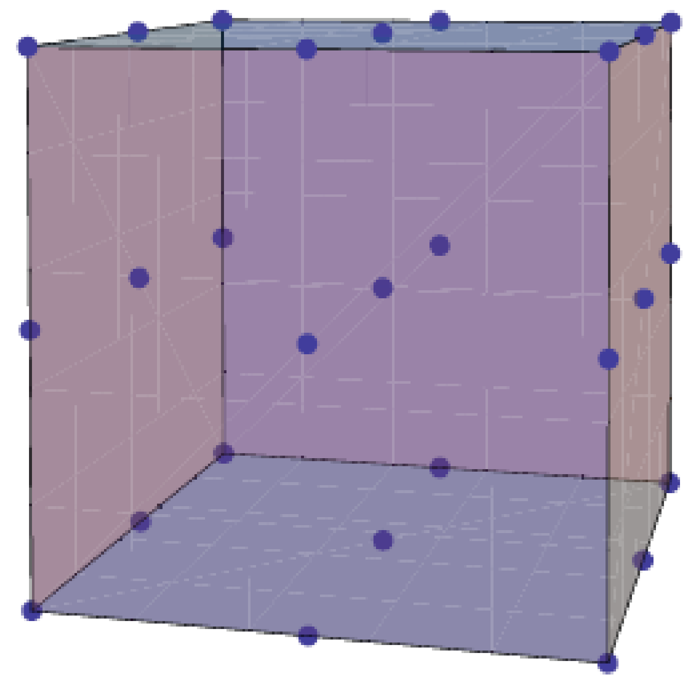


Figure 26: The lattice of generators of the $Q^{1,1,1}/\mathbb{Z}_2$ theory.

Fano 68 O(1,-1)$_{P^1 x P^1}$ (1,0,1,-2)

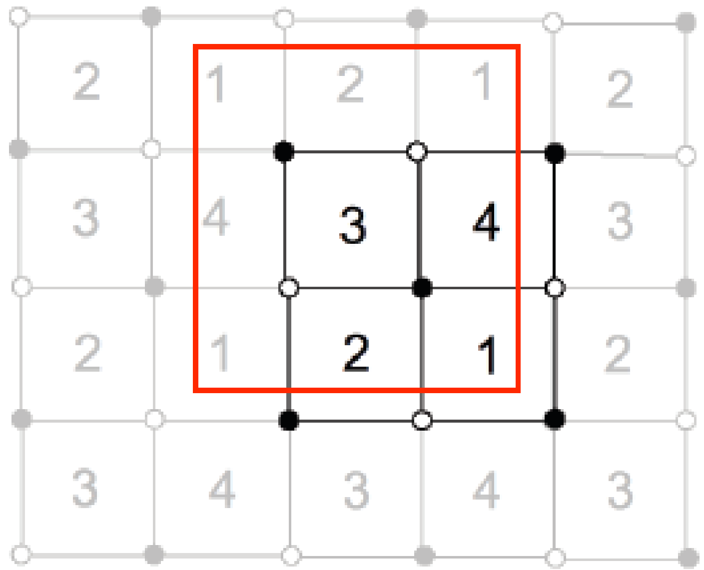


Figure 13: [Phase I of $\mathcal{C}_5$] (i) Quiver diagram of the $\mathcal{S}_4$ model. (ii) Tiling of the $\mathcal{S}_4$ model.

Fano 68 $O(1,-1)_{P^1 \times P^1}$ Table of charges

	$SU(2)_1$	$SU(2)_2$	$U(1)_q$	$U(1)_R$	$U(1)_{B_1}$	$U(1)_{B_2}$	fugacity
p_1	1	0	1	4/11	0	0	$t^4 x_1 q$
p_2	-1	0	1	4/11	0	0	$t^4 q/x_1$
q_1	0	1	-1	4/11	0	0	$t^4 x_2/q$
q_2	0	-1	-1	4/11	0	0	$t^4/(x_2 q)$
r_1	0	0	0	3/11	0	-1	t^3/b_2
r_2	0	0	0	3/11	0	-1	t^3/b_2
s_1	0	0	0	0	1	0	b_1
s_2	0	0	0	0	-1	2	b_2^2/b_1
s_3	0	0	0	0	0	0	1

Table 4: Charges of the perfect matchings under the global symmetry of the $\mathcal{C}_5$ theory. Here t is the fugacity of the R-charge, x_1, x_2 are the weights of the $SU(2)$ symmetry, q, b_1 and b_2 are, respectively, the charges under the mesonic abelian symmetry $U(1)$ and of the two baryonic $U(1)_{B_1}$ and $U(1)_{B_2}$.

dP2 bundle over P[1]
(1,-1,0,-1,1)

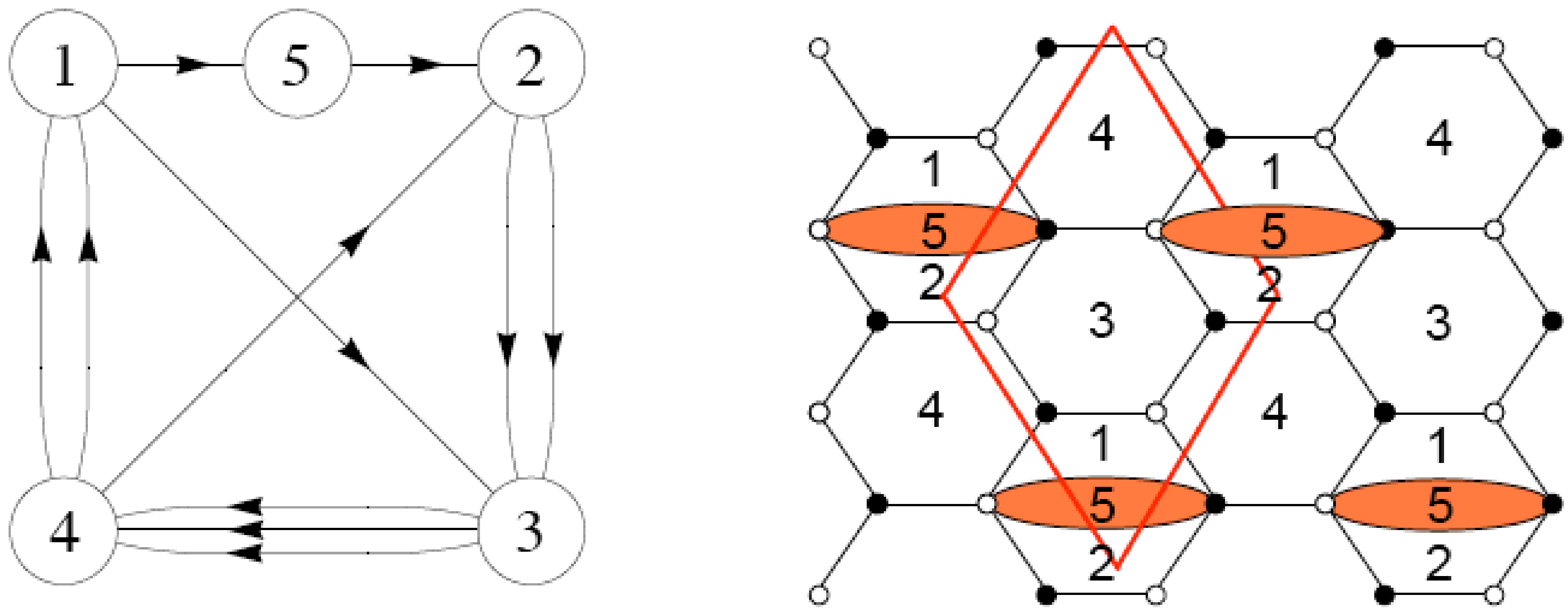


Figure 20: [$\mathcal{E}_1$] (i) Quiver diagram of the $\mathcal{H}_3\mathcal{D}_1\partial_1$ model. (ii) Tiling of the $\mathcal{H}_3\mathcal{D}_1\partial_1$ model.

dP2 x P^1

(1,1,-1,0,-1)

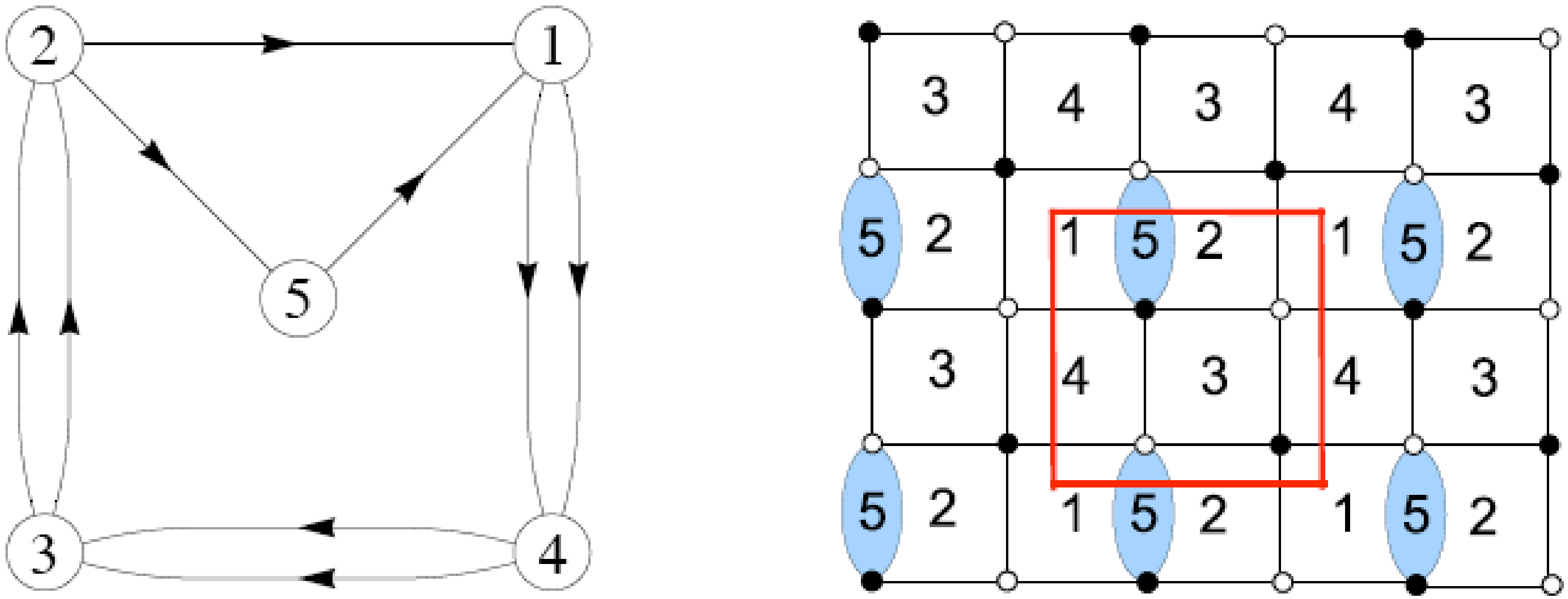


Figure 23: $[\mathcal{E}_3]$ (i) Quiver diagram of the $\mathcal{S}_4\mathcal{D}_1$ model. (ii) Tiling of the $\mathcal{S}_4\mathcal{D}_1$ model.

dP2 x P^1
Toric Diagram

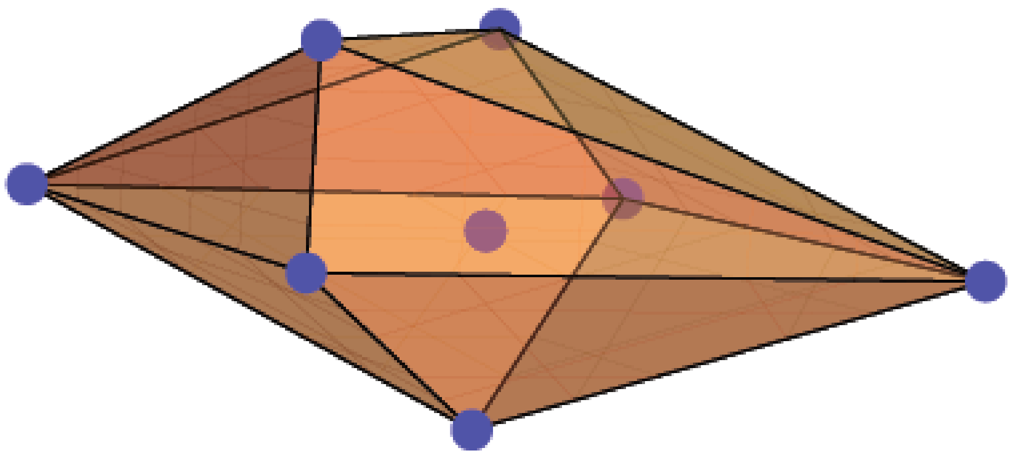


Figure 25: The toric diagram of the toric $\mathcal{E}_3$.

dP3 x P1
(0,0,0,0,1,-1)

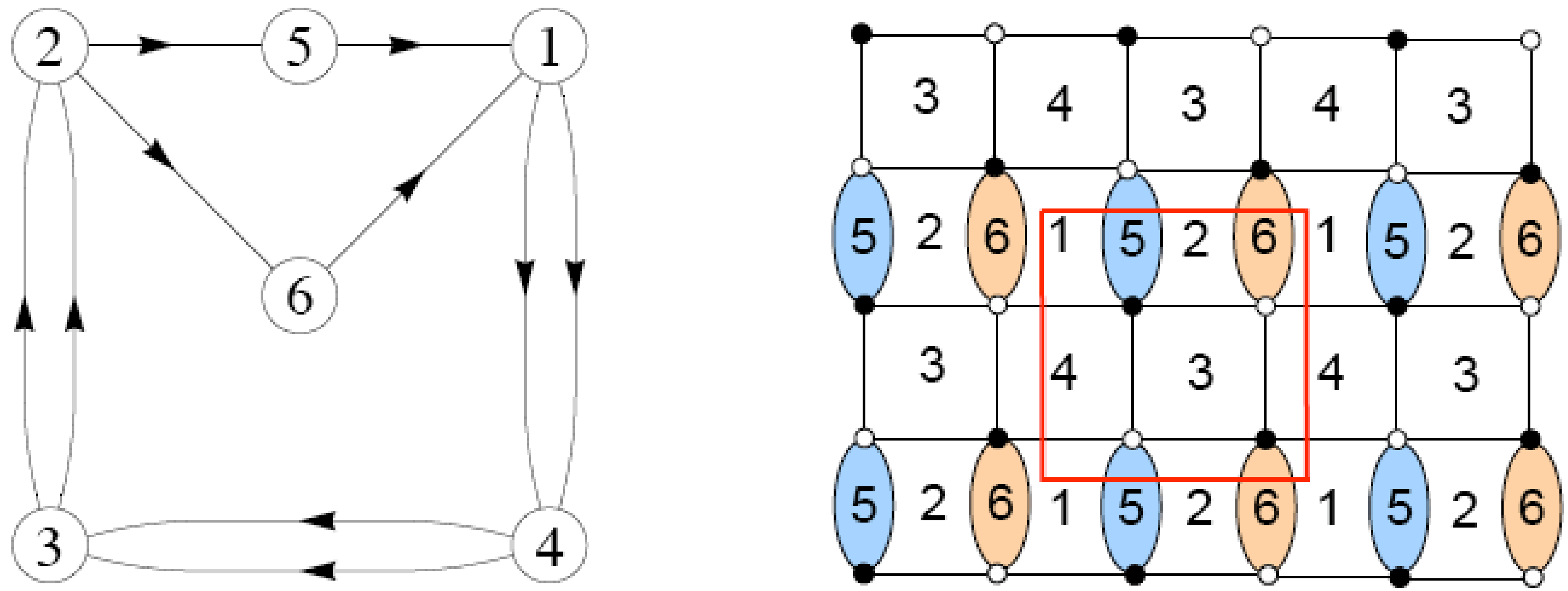


Figure 30: $[\mathcal{F}_1]$ (i) Quiver diagram of the $\mathcal{S}_4\mathcal{D}_2$ model. (ii) Tiling of the $\mathcal{S}_4\mathcal{D}_2$ model.

2+1d: current results

- Mesonic moduli space of vacua - CY4
- interacting SCFT's in the IR
- Non-trivial scaling dimensions
- Master space - partial baryonic & mesonic moduli space
- Hilbert Series

Summary

- All theories described are conjectured to live on the world volume of M2 branes probing the CY4 - mesonic moduli space
- Infinite families of SCFT's
- Count how many?
- Know for 2 terms in W and arbitrary G

Tools for study

- Mesonic moduli space
- Master space (including baryons)
- toric diagrams - lattice of generators
- toric duality

More technical tools

- Perfect matchings
- Kasteleyn matrix
- Hilbert Series
-

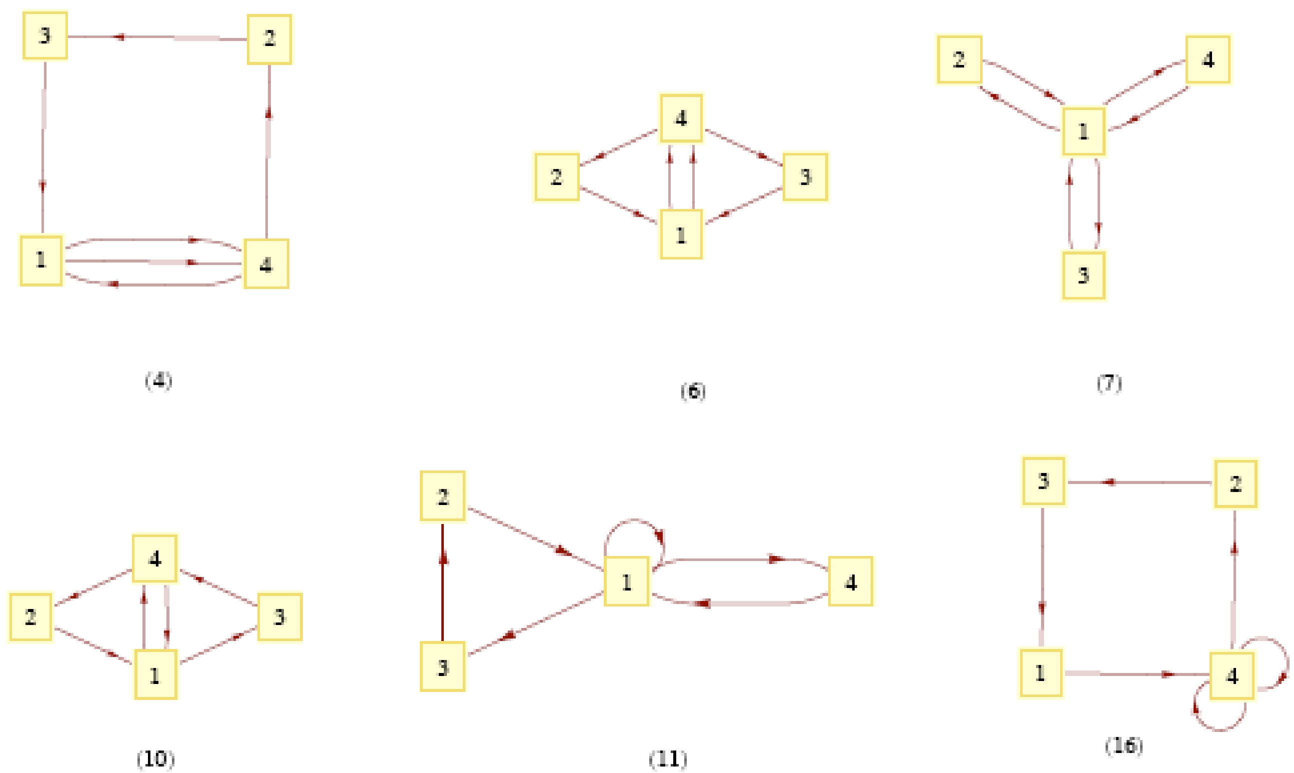
Classification of 2+1d theories?

- "order parameters"
- Number of gauge groups G
- Number of fields in the quiver E

Summary

- Infinitely many quivers
- Each represents a lattice of SCFT's in 2+1d
- A variety of scaling dimensions
- Toric Duality
- ...

6 fields in the Quiver



$$W_{(4)} = \mathrm{Tr}(X_{31}X^1_{14}X_{41}X^2_{14}X_{42}X_{23} - X_{31}X^2_{14}X_{41}X^1_{14}X_{42}X_{23}) \ ;$$
$$W_{(6)} = \mathrm{Tr}(X_{42}X_{21}(X^1_{14}X_{43}X_{31}X^2_{14} - X^2_{14}X_{43}X_{31}X^1_{14})) \ ;$$
$$W_{(7)} = \mathrm{Tr}(X_{12}X_{21}(X_{14}X_{41}X_{13}X_{31} - X_{13}X_{31}X_{14}X_{41})) \ ;$$
$$W_{(10)} = \mathrm{Tr}(X_{42}X_{21}X_{14}X_{41}X_{13}X_{34} - X_{42}X_{21}X_{13}X_{34}X_{41}X_{14}) \ ;$$
$$W_{(11)} = \mathrm{Tr}(X_{32}X_{21}\phi_1X_{14}X_{41}X_{13} - X_{32}X_{21}X_{14}X_{41}\phi_1X_{13}) \ ;$$
$$W_{(16)} = \mathrm{Tr}(X_{42}X_{23}X_{31}X_{14}[\phi^1_4, \phi^2_4])$$

G=2, E=4, Model I

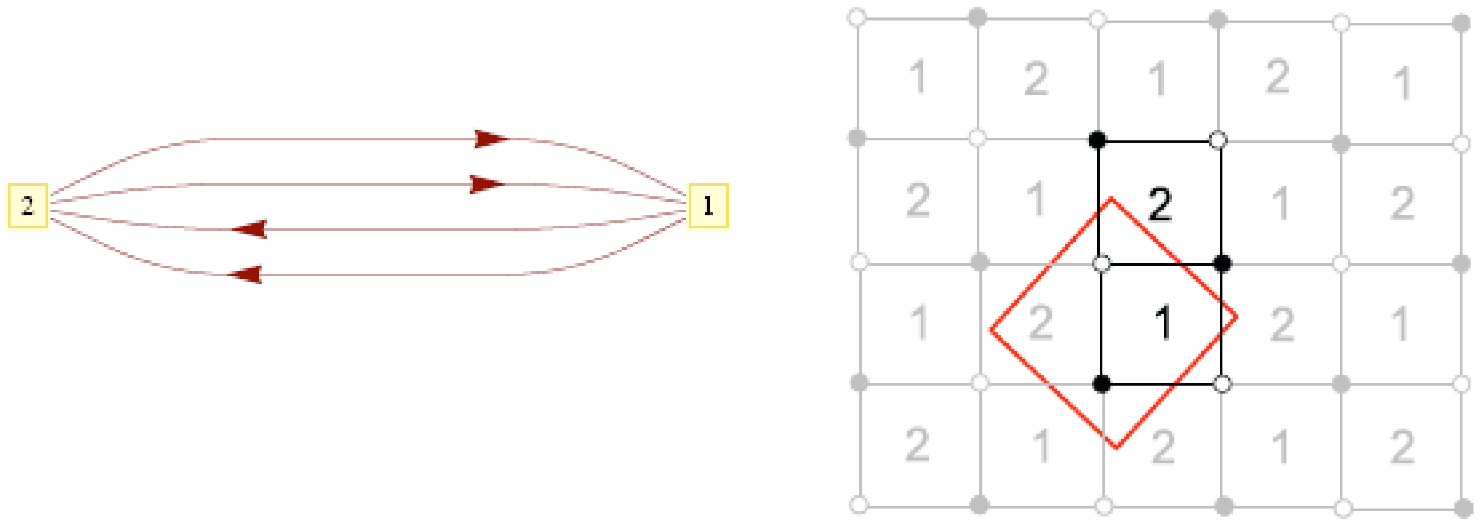


Figure 1: (i) Quiver diagram for the ABJM theory. (ii) Tiling for the ABJM theory.

G=4, E=6, Model IV

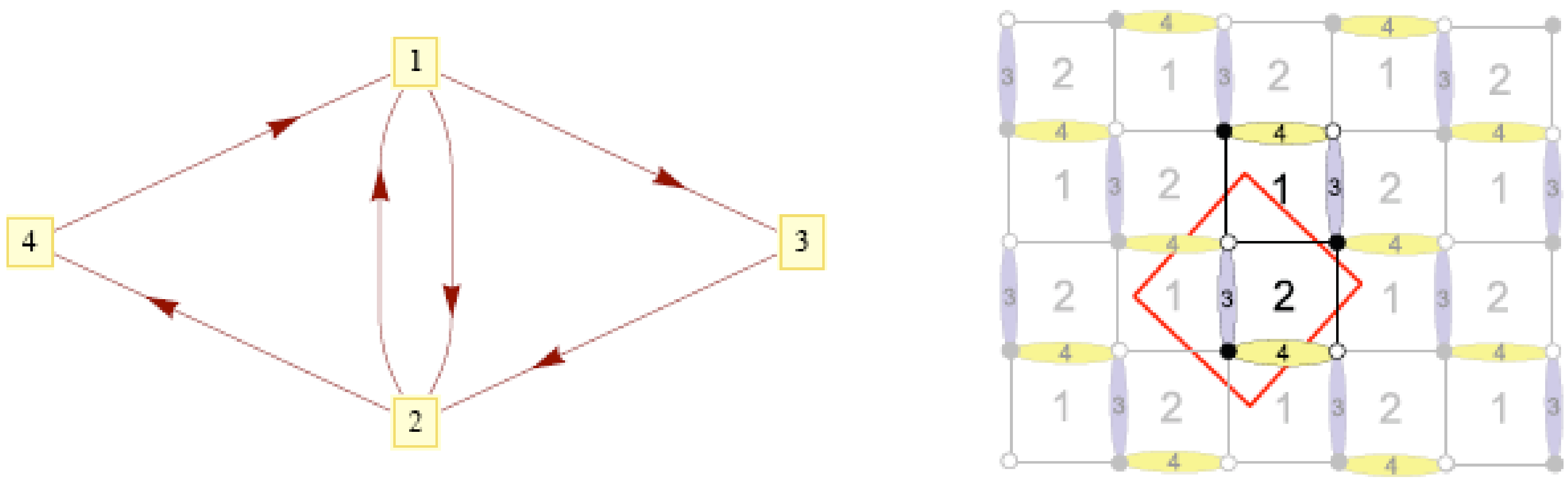


Figure 11: (i) Quiver diagram for phase 2 of the D_3 theory. (ii) Tiling for phase 2 of the D_3 theory.

Counting Quivers
1 Hexagon

$$f_1(t) = \frac{1}{(1-t)(1-t^2)(1-t^3)}$$

$$= 1 + t + 2t^2 + 3t^3 + \ldots$$

Counting Quivers Chessboard Tiling

$$\begin{aligned} f_2(t) &= \frac{1-t^6}{(1-t)(1-t^2)^2(1-t^3)(1-t^4)} \\ &= 1+t+3t^2+4t^3+8t^4+\dots \end{aligned}$$

3+1d: we know how to

- Compute the moduli space of vacua
- Spectrum of scaling dimensions
- Central charge and volume of SE manifold
- Master space - Baryonic & mesonic moduli space of vacua
- Hilbert Series - partition function to count the spectrum of the Chiral Ring

What is special in 2+1d?

- YM gauge coupling has dimension 1/2
- All IR theories are strongly coupled
- CS terms exactly marginal
- CS levels have dimension 0
- Integer coefficients
- Scale invariant

Simple observations in 2+1d CS theories

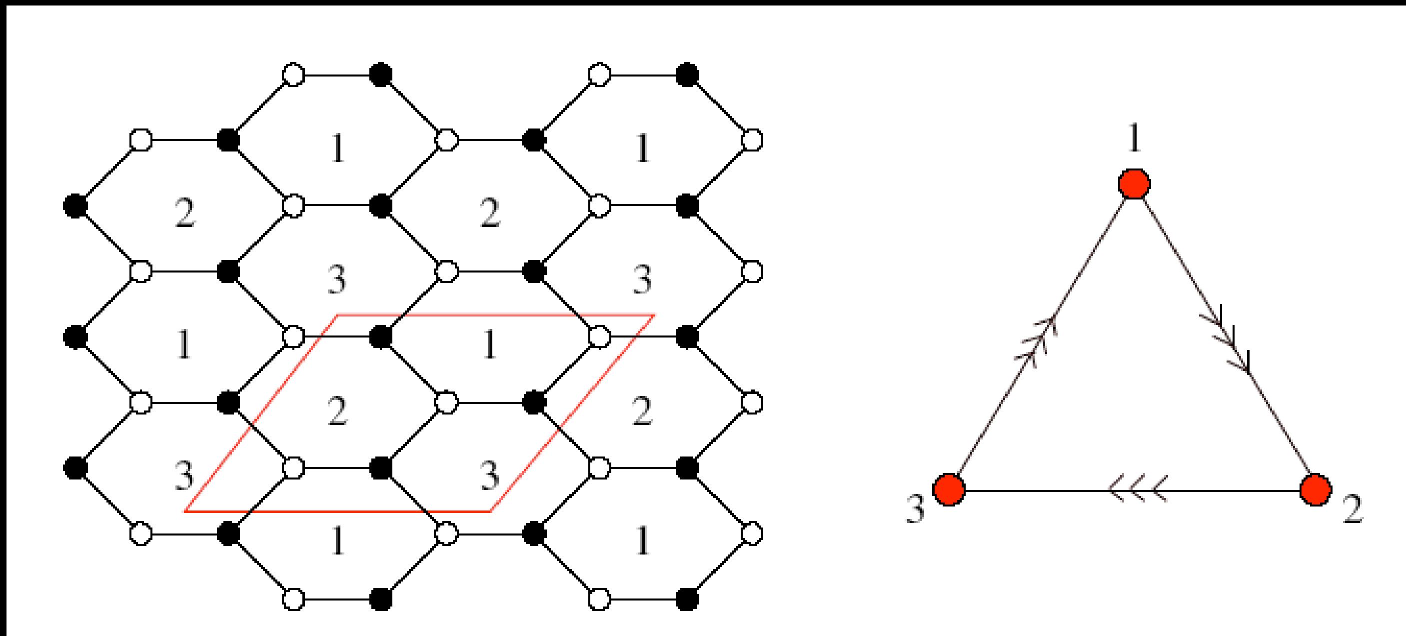
- No beta function for CS levels
- Finite renormalization - typically at 1 loop
- $\mathcal{N}$=2 supersymmetry (4 supercharges): no corrections
- Infinite family of SCFT's parametrized by CS terms

A lattice of SCFT's

- For one gauge group - a 1d lattice of SCFT's
- For a product of G gauge groups a G dimensional lattice of SCFT's
- If put c conditions on CS levels G-c dimensional sub - lattice of SCFT's

Nathan Broomhead

Dimer Models and Calabi-Yau Algebras



Periodic bipartite tiling

2+1d Lagrangians

- Given a 2d periodic, bipartite tiling with G tiles, add G CS levels, 1 for each tile.
- Largest known family of SCFT's in 2+1d!

Solving Vacuum Equations

- F terms - Master Space, G+2
- Third set of equations set σ's equal
- D terms - form linear combinations (LC)
- G equations, G-2 LC set to 0
- Divide by complexified gauge group
- Moduli space: toric singular CY4 cone