The Pomeron contribution to proton-proton scattering in AdS/QCD

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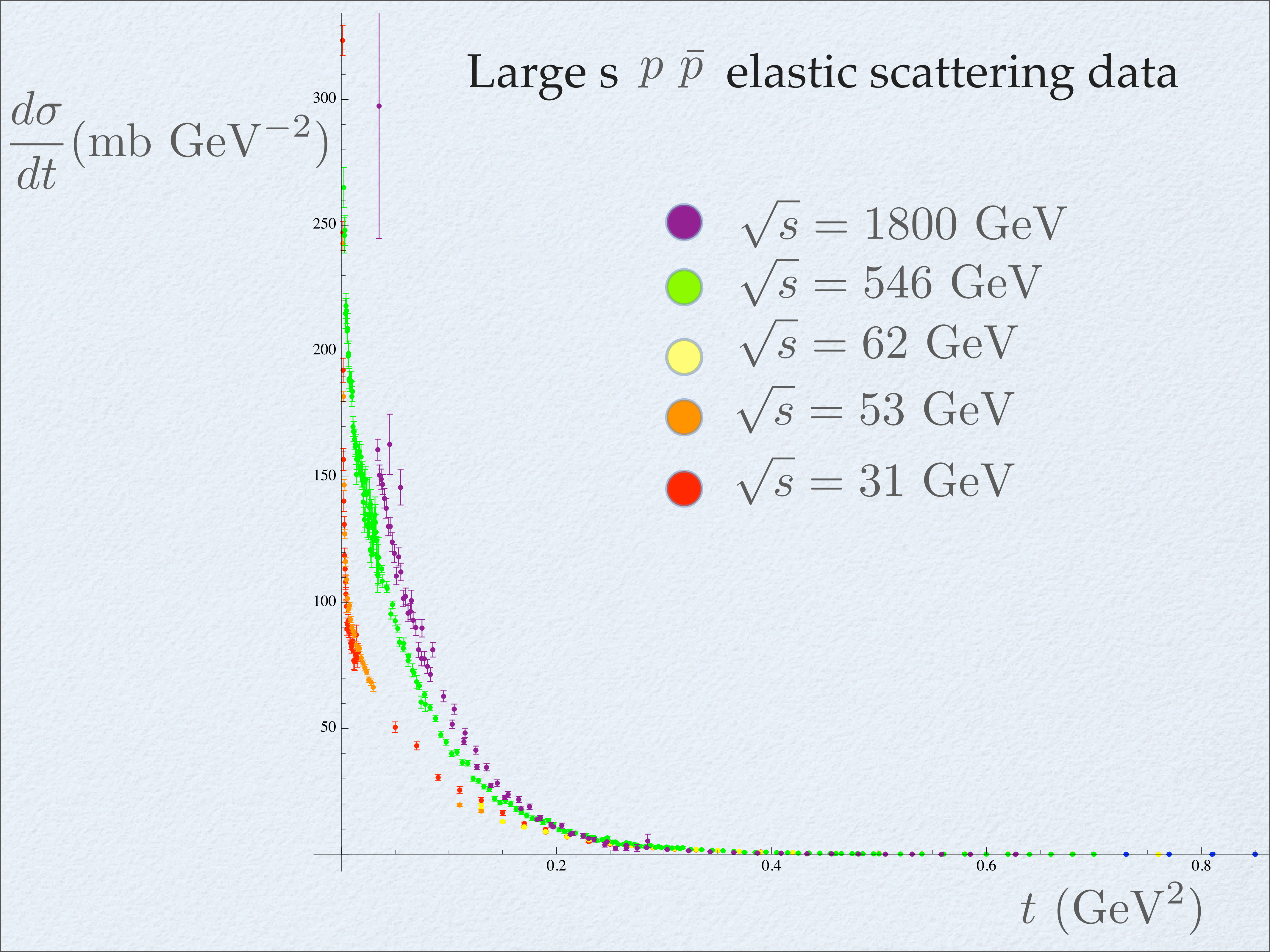
Outline

- The data and previous fits, summary of results
- QCD, string theory, and Regge theory
- The Pomeron
- Recipe for p-p elastic scattering in AdS/QCD
- Fitting to data
- Computation of parameters in a dual model
- A string theory prediction for the LHC


The Data

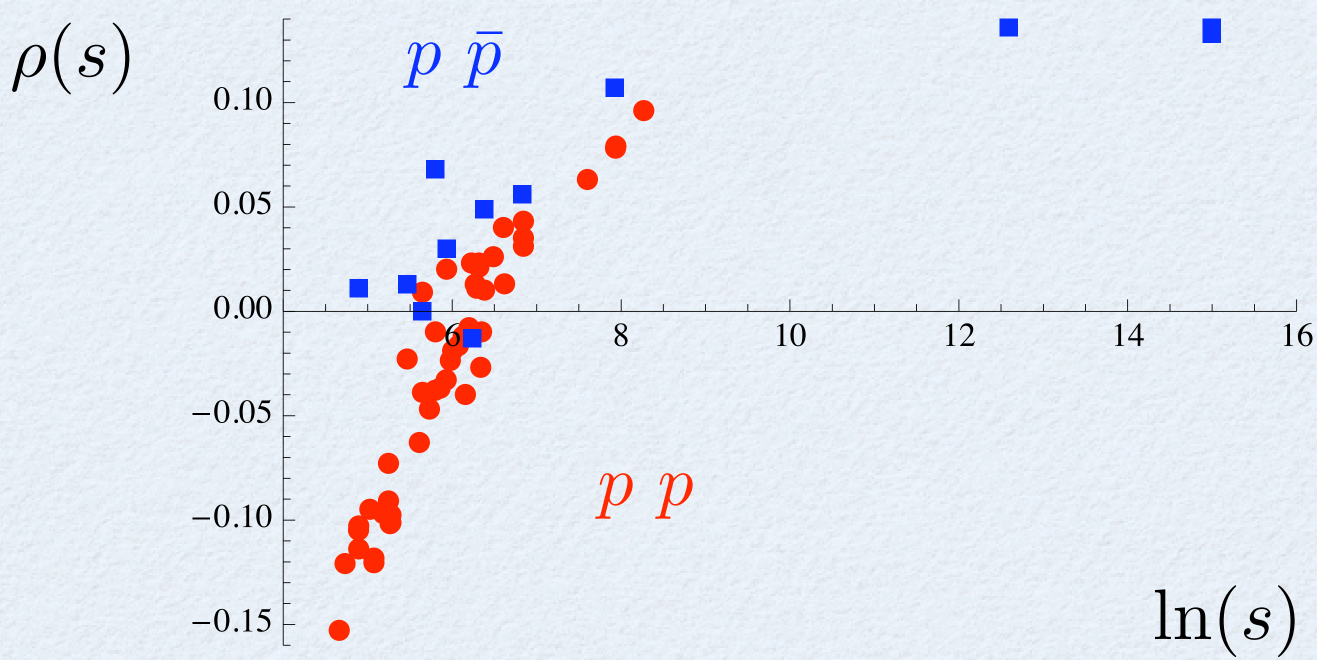


Large s $p\ \bar{p}$ elastic scattering data
$\frac{d\sigma}{dt}(\mathrm{mb\ GeV^{-2}})$
300
250
200
150
100
50
$\sqrt{s} = 1800$ GeV
$\sqrt{s} = 546$ GeV
$\sqrt{s} = 62$ GeV
$\sqrt{s} = 53$ GeV
$\sqrt{s} = 31$ GeV
0.2
0.4
0.6
0.8
$t\ (\mathrm{GeV}^2)$

A complete theory would tell us how to construct the invariant amplitudes $A^{pp}(s,t)$, $A^{p\bar{p}}(s,t)$ and we would then compute:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A(s,t)|^2 \quad \sigma_{tot} = \frac{1}{s}\Im A(s,0) \quad \rho(s) = \frac{\Re A(s,0)}{\Im A(s,0)}$$

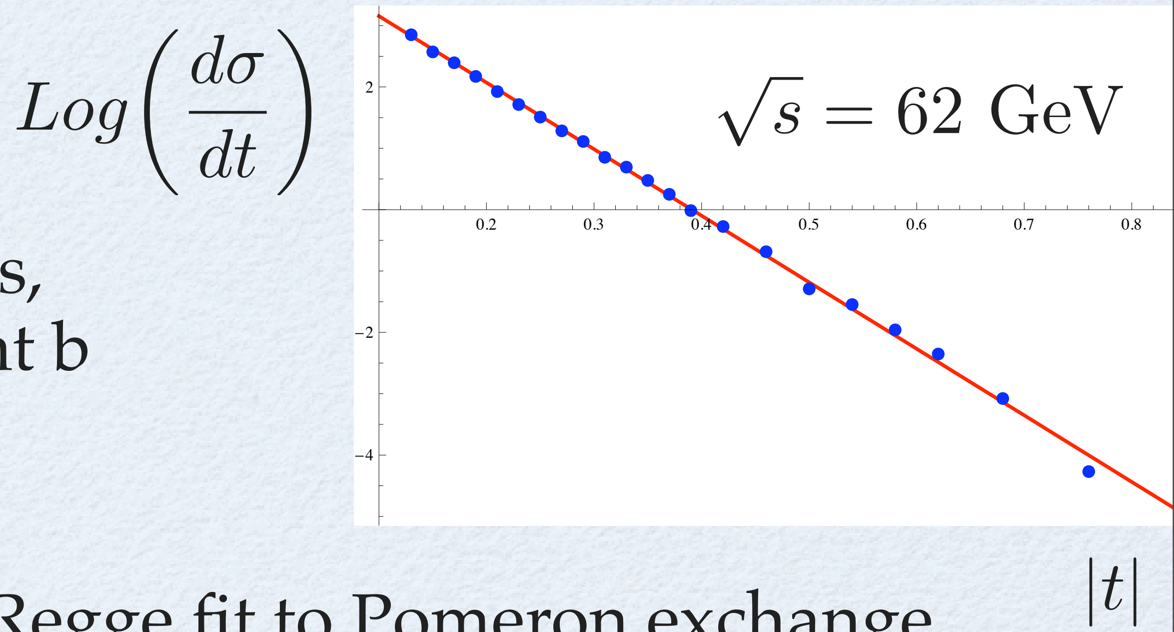
and compare to data, e.g.



The data is usually fit/modeled by one of 4 methods

- Experimental papers:

  $d\sigma/dt = ae^{bt}$ at fixed s,
  sometimes with different b
  for different ranges of t.



- Donnachie&Landshoff: Regge fit to Pomeron exchange using EM form factor $d\sigma/dt = C[F_1(t)]^4(\alpha' s)^{2\alpha(t)-2}$ with $\alpha(t) \simeq 1.08 + .25t$ plus Reggeons.

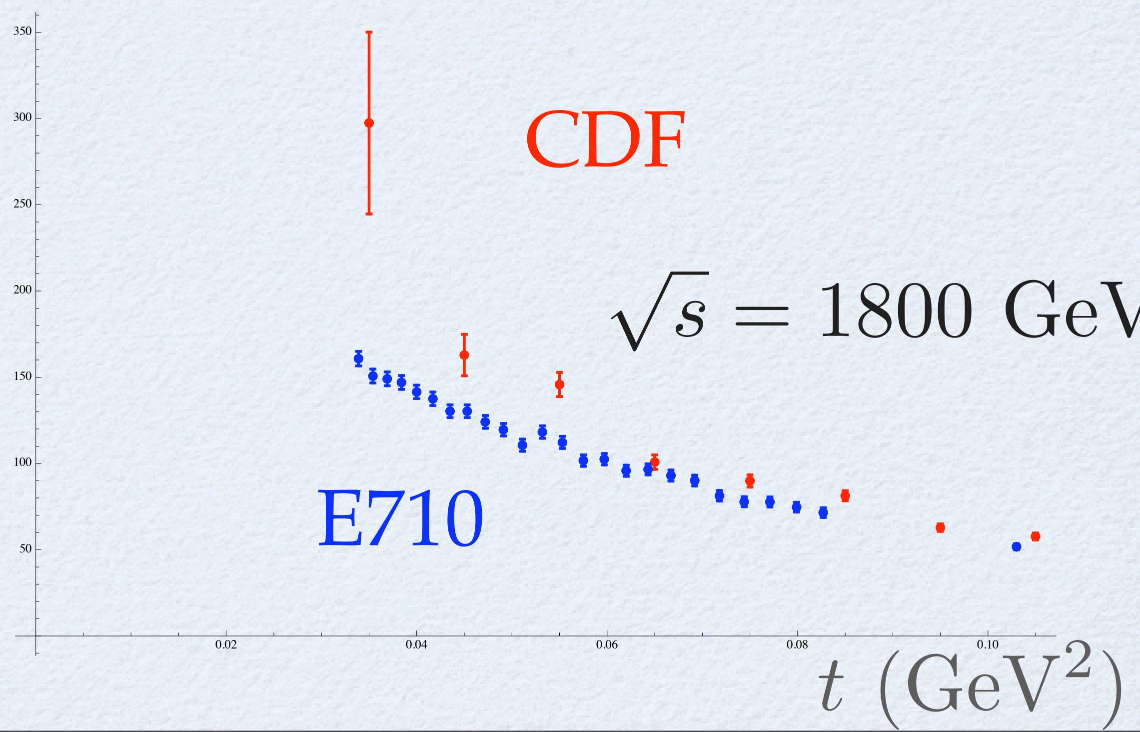
- PDG (Cudell et. al., Block and Halzen, Ishida and Igi): Fit to leading $\sigma_{tot} \simeq log^2(s)$ plus Reggeons.

- Khoze, Martin& Ryskin: Eikonal methods, multiple Pomeron exchange, triple Pomeron couplings...it is complicated.

It is difficult in any of these approaches to obtain a fit with a satisfactory χ^2/dof because of discrepancies between different data sets, issues of combining systematic errors with statistical errors and so on. In addition, the fits are mainly phenomenological with only general principles of analyticity and Regge theory to guide them.

In addition there is ``sieving'' of the data and sophisticated statistical ranking of models that is not easy for a novice to decipher.

Some of the data sets disagree:



Summary of our Results:

- We assume single Pomeron exchange, but *derive* a form factor related to matrix elements of the stress-tensor, and *compute* the prefactor from AdS/QCD.

- We compute parameters in the Sakai-Sugimoto model.

- We end up with a reasonable fit to large s data in the Regge regime. The computed parameters agree well with the best fit values.

QCD, String Theory, and Regge Theory

QCD and String Theory

Even before AdS/CFT there was a great deal of both experimental and theoretical evidence that QCD has a dual description in terms of string theory.

Theory: At large N_c QCD has an infinite tower of narrow resonances of arbitrarily high spin. The $1/N_c$ expansion is a topological expansion as in string theory.

Experiment: To a good approximation hadrons sit on linear Regge trajectories, $J = \alpha_0 + \alpha' M^2$, and many scattering processes exhibit Regge behavior at large s, fixed t.

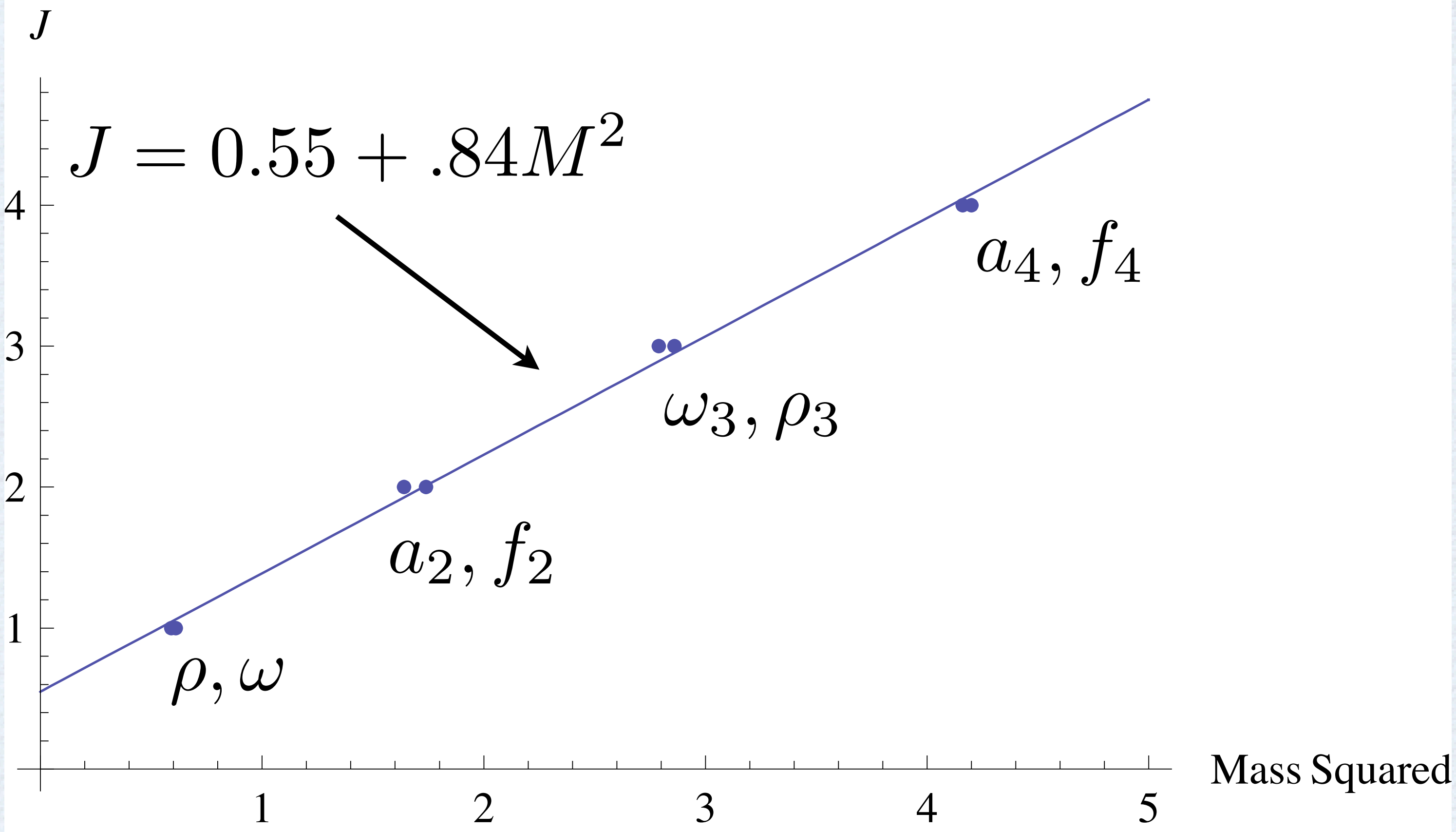
AdS/QCD: We now have dual models which seem to correctly describe parts of QCD. They have obvious flaws, but allow computations which were previously out of reach. We will study one more such calculation.

Regge Theory and QCD

Regge theory is the red-headed stepchild in the second marriage of string theory and QCD.

It is of course connected with analyticity in J and S-matrix theory. But, we can also think of it as a method to obtain the Regge behavior expected in a dual string theory in terms of a small number of parameters in a way that is consistent with general principles. There is a lot of successful phenomenology which should be updated with insight from AdS/QCD.

Tullio Regge



The leading meson Regge trajectories lie on straight lines at positive t and exhibit EXchange Degeneracy (EXD) with trajectories of both even and odd spin having roughly the same slope and intercept.

In 2-2 scattering at small $|t|$ we could try to exchange this tower of mesons, but exchange of spin J leads to amplitudes $\sim s^J$. Regge theory replaces this infinite sum of badly behaved amplitudes by a pole in the complex angular momentum plane at $J = \alpha(t)$.

The linearity of trajectories at positive t extends to small negative t with the same slope and intercept.

$$\frac{d\sigma}{dt} = \beta(t)(s)^{2\alpha(t)-2}$$

The linearity of the trajectory at $t < 0$ can be verified by using data to extract the effective trajectory from a plot of

$$Log(d\sigma/dt) \quad \text{vs.} \quad Log(s)$$

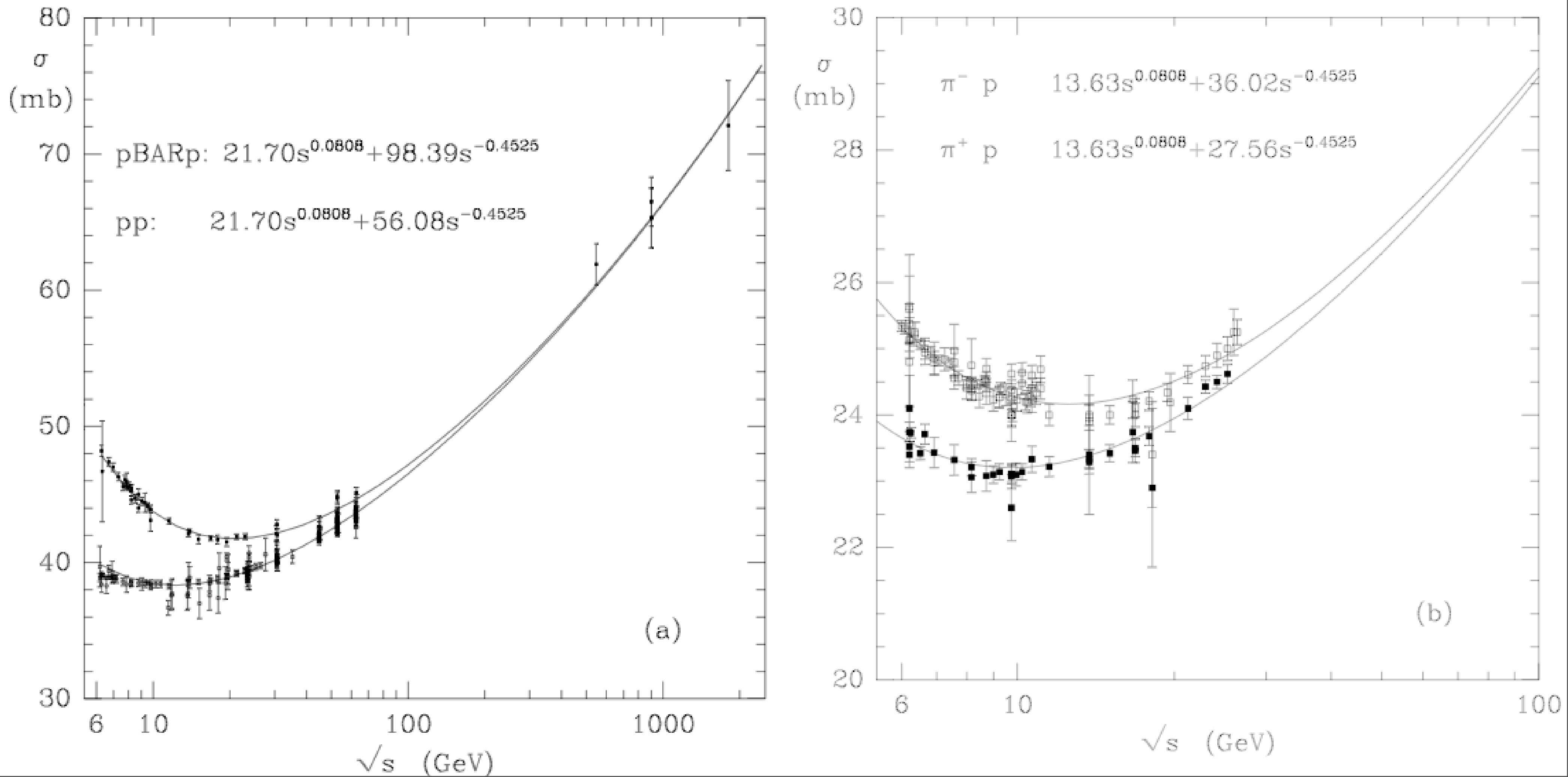
The Pomeron

The Pomeron

The Pomeron is a Regge trajectory introduced by Chew and Frautschi in 1961 to account for the then approximate constant behavior of total cross sections with increasing s. This was inconsistent with the known trajectories with $\alpha_{\mathcal{R}}(0) \simeq 0.55$ since total cross sections behave like $s^{\alpha(0)-1}$ and required a new trajectory with vacuum quantum numbers and an intercept $\alpha_{\mathcal{P}} \simeq 1$.

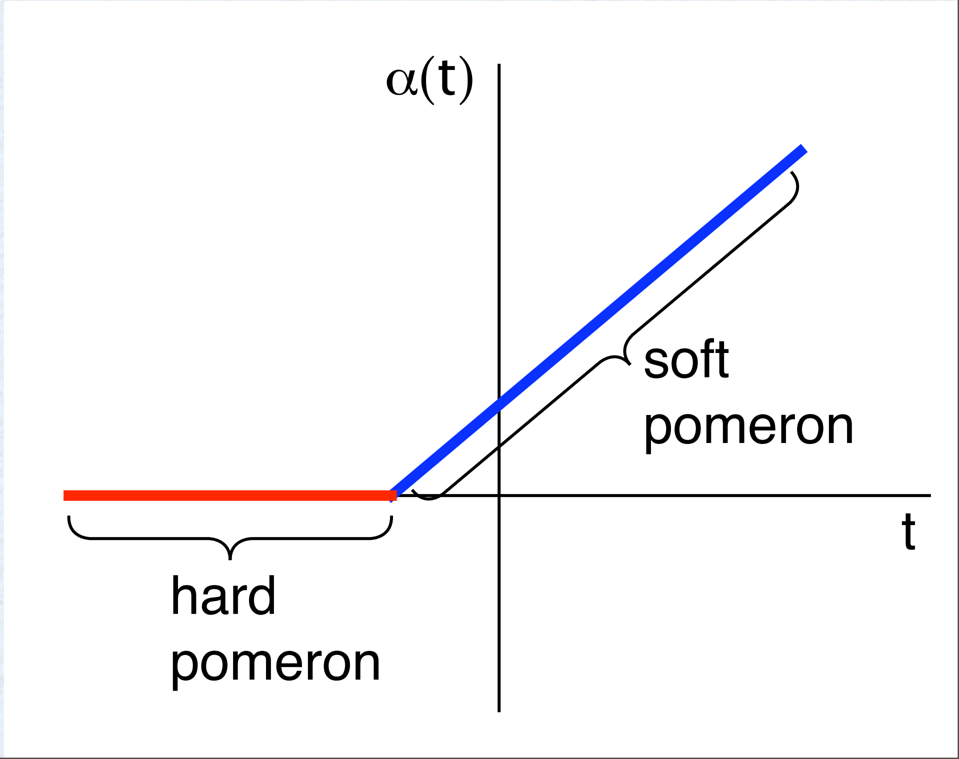
It is natural to identify this trajectory at positive t with glueball states and the Regge behavior at small negative t with closed string exchange in a string dual description of QCD. The lowest state on the leading trajectory is a 2^{++} glueball.

Total cross sections are well fit with two powers, one for Reggeon exchange with intercept $\alpha_{\mathcal{R}}(0) \simeq 0.55$ and one for Pomeron exchange with intercept $\alpha_{\mathcal{P}}(0) \simeq 1.08$ (Donnachie-Landshoff following earlier work of Collins, Gault and Martin).



There have been efforts (BPST) to use AdS/QCD ideas to connect the large negative t region of perturbative QCD/hard pomeron/BFKL to the positive t region of linear glueball trajectories. Unfortunately the most phenomenologically interesting region of small negative t is also the most model dependent. The following picture is not inconsistent with the analysis of BPST and we will exhibit some experimental evidence to support it.

Given a regime with a linear trajectory at small negative t we are led to a recipe for the Pomeron contribution to p-p scattering in the Regge limit.



Recipe for p-p elastic scattering in AdS/QCD

Recipe for p-p scattering in the Regge regime

1 dual model of QCD
4 protons treated as Skyrmions
1 spin 2 glueball extracted from a 5d graviton
1 closed string four point amplitude

Preheat two incoming protons to the desired c.o.m. energy. Compute the glueball wavefunction and its coupling to the proton stress tensor. Compute the stress tensor matrix elements and extract the dominant form factor in the Regge regime. Use this to calculate tree-level glueball exchange and extract a kinematic factor. Mix this into the dual amplitude, extract the Reggeized propagator and substitute this into the previously computed tree amplitude. Compare with data. Antiprotons may be substituted for protons according to taste.

In dual descriptions of QCD the 2^{++} glueball state arises as a mode of the 5D graviton. By definition, this perturbation couples to the 5D stress tensor. We assume (tensor meson dominance), and show explicitly in a specific model, that when reduced to 4D this leads dominantly to a coupling of the 2^{++} state to the proton stress-energy tensor. The p-p-glueball vertex then involves

$$\langle p', s' | T_{\mu\nu}(0) | p, s \rangle =$$

$$\bar{u}(p', s') \left[A(t) \gamma_{(\mu} P_{\nu)} + B(t) \frac{i P_{(\mu} \sigma_{\nu)\rho} q^\rho}{2 m_p} + C(t) \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{m_p} \right] u(p, s)$$

In the Regge limit of large s and fixed t the first form factor dominates. Tree-level exchange of the spin two glueball, but including this form factor at the vertices, leads to

$$\frac{d\sigma}{dt} = \frac{\lambda^4 A^4(t) s^2}{\pi(t - m_g^2)^2}$$

where λ governs the strength of the coupling of the glueball to the stress-energy tensor.

The form factor is model dependent, but in most models the form factors at small t are well fit by a simple dipole formula

$$A(t) = \frac{1}{(1-t/M_d^2)^2},$$

This formula cannot be correct at large s. It violates unitarity, and at large s and fixed t we expect the whole glueball trajectory to contribute to the differential cross section. We need Regge theory, or we can use string theory as a short cut.

Four point closed string amplitudes in the bosonic or superstring can be written in the general form

$$A(s,t) \simeq \frac{\Gamma[-\alpha(s)]\Gamma[-\alpha(t)]\Gamma[-\alpha(u)]}{\Gamma[-\alpha(t)-\alpha(s)]\Gamma[-\alpha(t)-\alpha(u)]\Gamma[-\alpha(u)-\alpha(s)]}K$$

where $\alpha(x)$ is a linear function of x and $K(1,2,3,4)$ is a polynomial in the momenta and polarization vectors/spinors of the initial and final states. This form is crossing symmetric.

We assume the same general form holds for closed string exchange in a curved space dual of QCD.

- Linearity: $\alpha(x) = a_0 + a' x$

- First pole is spin 2 glueball: $-a_o/a' = m_g^2$

- Mass shell: $\alpha(s) + \alpha(t) + \alpha(u) = a'(4m_p^2 - 3m_g^2) \equiv \chi$

- $K \sim s^2$ and residue of pole $\sim s^J$ identifies

  $J = \alpha_0 + \alpha' t$ with $\alpha_0 = 2 + 2a_0,\ \alpha' = 2a'$

With these identifications we then take the Regge limit of the resulting amplitude and use this to obtain a prescription for “Reggeizing the propagator”

$$\frac{1}{t-m_g^2} \rightarrow -\frac{a'\Gamma[-\chi]\Gamma[-\alpha(t)]}{\Gamma[\alpha(t)-\chi]} e^{-i\pi\alpha(t)} \left(a's\right)^{2\alpha(t)}$$

This agrees with the Feynman propagator at $\alpha(t) \sim 0$ and has an infinite sequence of poles at $\alpha(t) = n$ corresponding to exchange of massive spin $J = 2n + 2$ particles lying on a linear Regge trajectory.

We now combine this with our previous tree-level computation of spin two glueball exchange including the gravitational form factor.

We are then left with our final form for the Pomeron contribution to the differential cross-section for $p\,p$ or $p\,\bar{p}$ scattering in the Regge regime:

$$\frac{d\sigma}{dt} = \left(\frac{\lambda^4 A^4(t)}{\pi}\right)\underbrace{\left(\frac{\Gamma^2[-\chi]\Gamma^2[-\alpha(t)]}{\Gamma^2[\alpha(t)-\chi]}\right)(a's)^{4\alpha(t)+2}}_{\text{Regge}}$$

coupling form factor

In principle, with the correct dual and enough technical strength we would compute all four parameters, $a_0, a', \lambda, M_{\rm dip}$, and compare with data. At present the best we can do is to compute $\lambda, M_{\rm dip}$ in a specific dual theory, fit a_0, a' to data, and compare our fit with previous results.

Comparison to DL Model

Many papers cite a Pomeron exchange fit due to Donnachie and Landshoff:

$$\left(\frac{d\sigma}{dt}\right)_{DL} = \frac{\big(3\beta F_1(t)\big)^4}{4\pi}\left(\frac{s}{s_0}\right)^{2\alpha(t)-2}$$

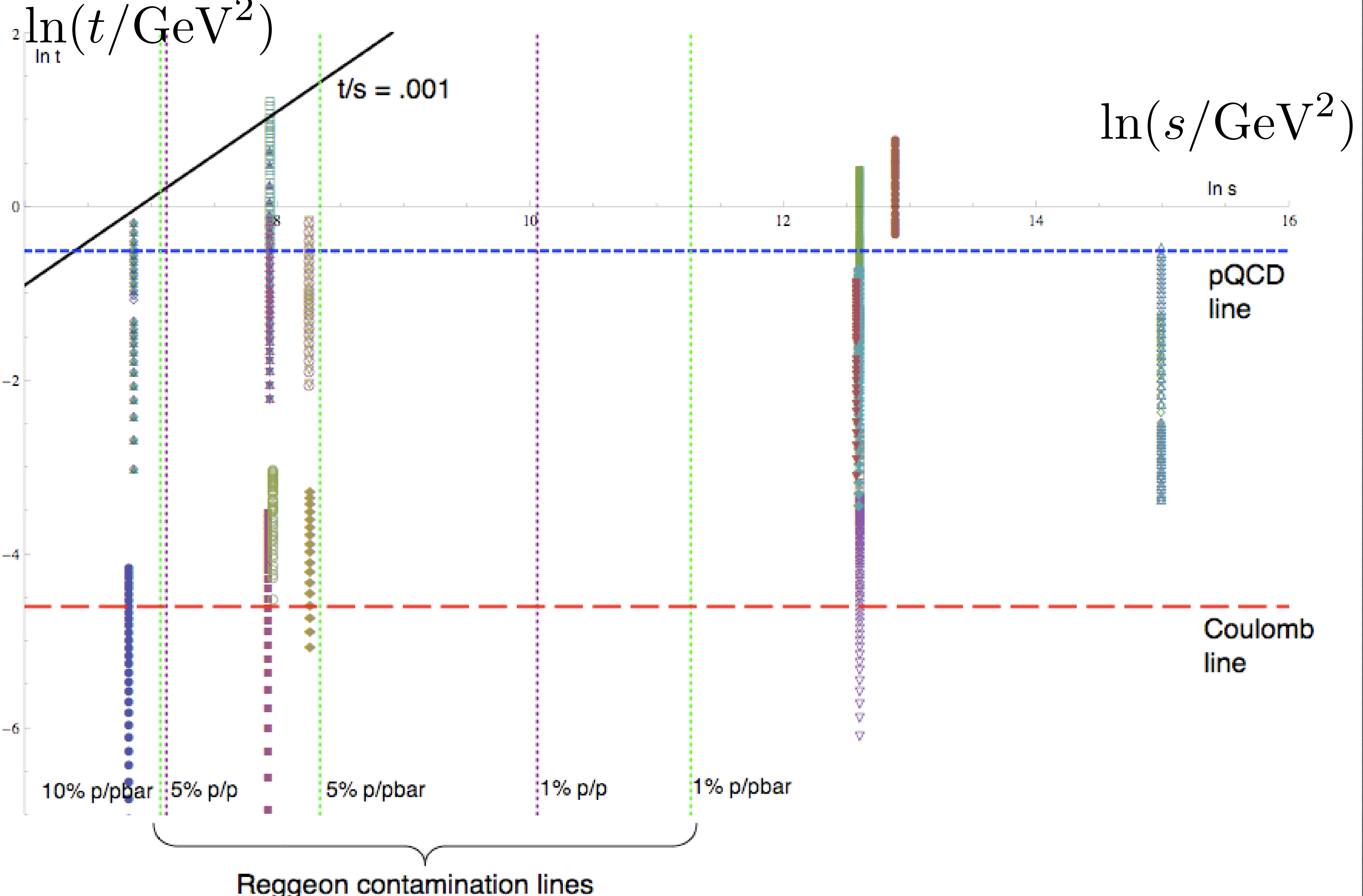
with $F_1(t)$ the electromagnetic form factor of the proton

$$F_1(t) = \frac{4m_p^2 - 2.79t}{4m_p^2 - t}\,\frac{1}{(1-t/0.71)^2}$$

We will fit the DL model to the same data set, varying β, α_0, α' rather than using their quoted values.

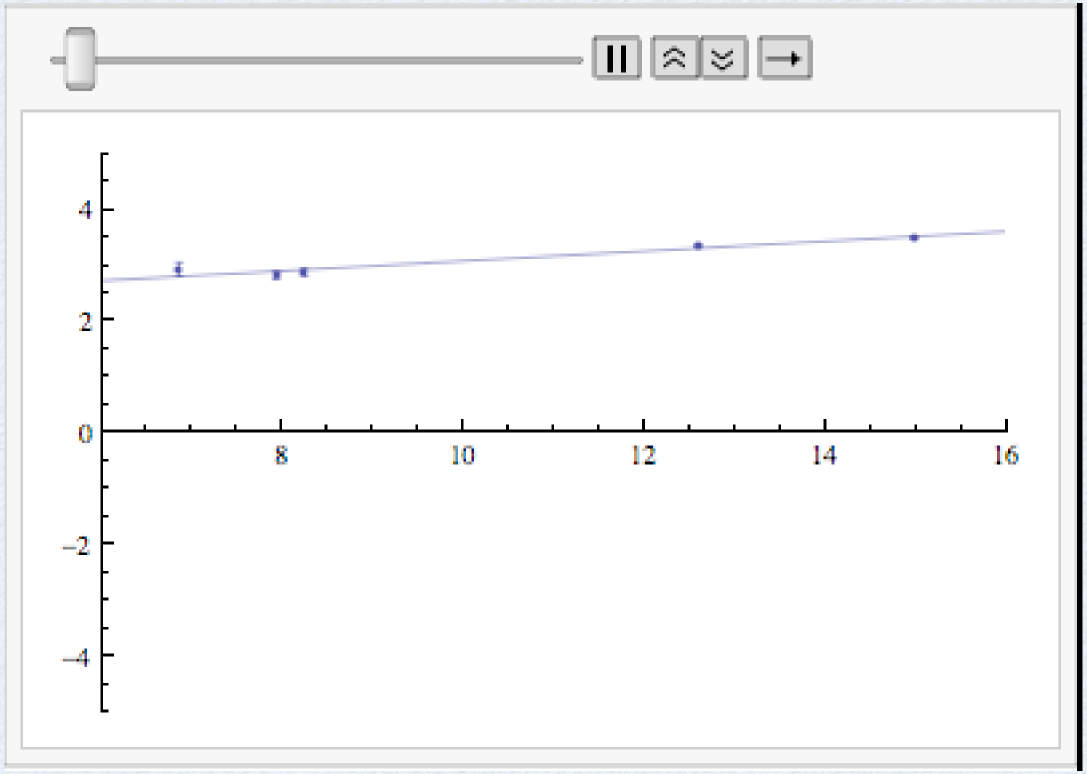
Fitting to Data

Parameter range for $p\,p$ and $p\,\bar{p}$ data, with cuts.



$$Log(\frac{d\sigma}{dt}) = Log(F(t)) + (4\alpha(t) + 2)Log(s)$$

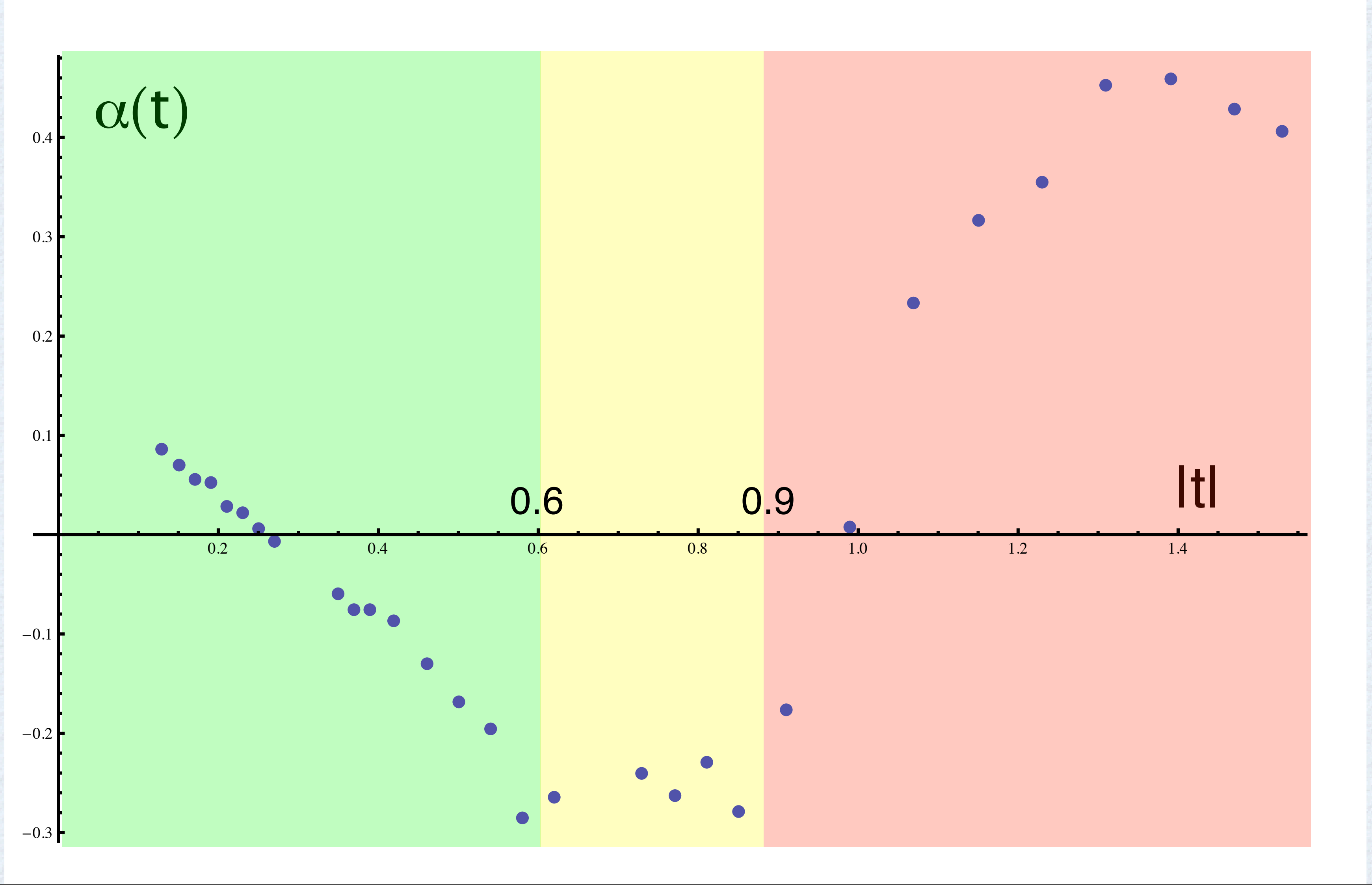
$$Log\left(\frac{d\sigma}{dt}\right)$$



Time is | t | !

$$Log(s/\mathrm{GeV}^2)$$

Extracting the trajectory from the data shows that it is quite linear for $|t| < 0.6$, has a flattish region, and then does something crazy. We will only try to fit data in the linear regime.



How well does our model fit the data? We don't want to try to model the full details of lower s data so we estimate the Reggeon contribution as a function of s and add this ``error'' in quadrature to the statistical errors and then do a chi squared fit to the resulting data. We find:

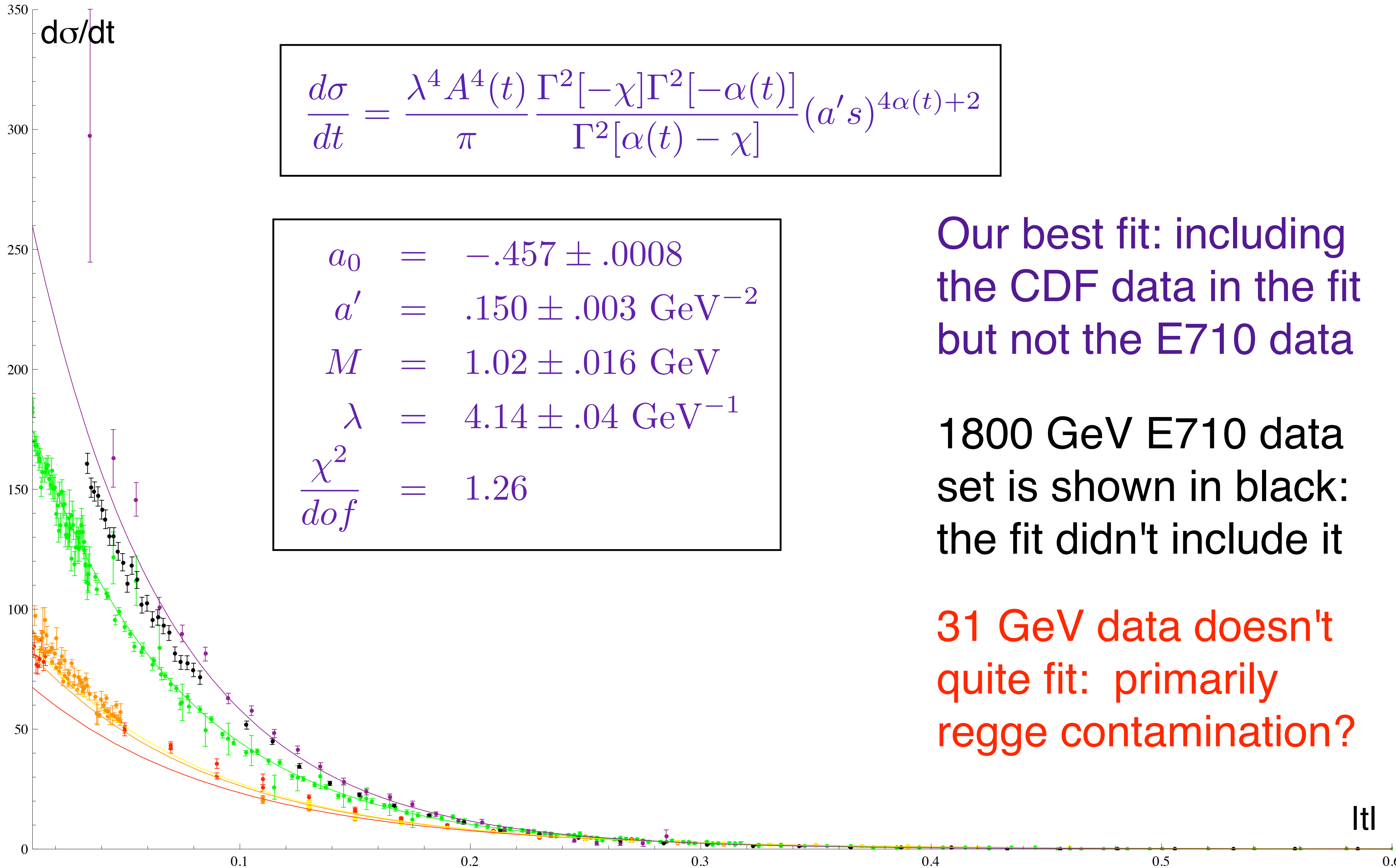
DHM fits

both data sets	just E710	just CDF
$\alpha_0 = 1.076 \pm .0016$	$\alpha_0 = 1.074 \pm .0016$	$\alpha_0 = 1.086 \pm .0016$
$\alpha' = .290 \pm .006 \text{ GeV}^{-2}$	$\alpha' = .286 \pm .006 \text{ GeV}^{-2}$	$\alpha' = .300 \pm .006 \text{ GeV}^{-2}$
$M = .983 \pm .016 \text{ GeV}$	$M = .970 \pm .016 \text{ GeV}$	$M = 1.02 \pm .016 \text{ GeV}$
$\lambda = 4.28 \pm .03 \text{ GeV}^{-1}$	$\lambda = 4.31 \pm .03 \text{ GeV}^{-1}$	$\lambda = 4.14 \pm .03 \text{ GeV}^{-1}$
$\frac{\chi^2}{d.o.f.} = 1.65$	$\frac{\chi^2}{d.o.f} = 1.41$	$\frac{\chi^2}{d.o.f.} = 1.26$

DL fits

both data sets	just E710	just CDF
$\alpha_0 = 1.076 \pm .0013$	$\alpha_0 = 1.075 \pm .0013$	$\alpha_0 = 1.082 \pm .0018$
$\alpha' = .289 \pm .003 \text{ GeV}^{-2}$	$\alpha' = .289 \pm .003 \text{ GeV}^{-2}$	$\alpha' = .289 \pm .003 \text{ GeV}^{-2}$
$\beta = 1.858 \pm .016 \text{ GeV}^{-1}$	$\beta = 1.877 \pm .016 \text{ GeV}^{-1}$	$\beta = 1.801 \pm .020 \text{ GeV}^{-1}$
$\frac{\chi^2}{d.o.f} = 1.97$	$\frac{\chi^2}{d.o.f.} = 1.66$	$\frac{\chi^2}{d.o.f} = 1.79$

Best fit to differential cross section



$$\frac{d\sigma}{dt} = \frac{\lambda^4 A^4(t)}{\pi} \frac{\Gamma^2[-\chi]\Gamma^2[-\alpha(t)]}{\Gamma^2[\alpha(t)-\chi]} (a' s)^{4\alpha(t)+2}$$

$$\begin{aligned} a_0 &= -.457 \pm .0008 \\ a' &= .150 \pm .003 \text{ GeV}^{-2} \\ M &= 1.02 \pm .016 \text{ GeV} \\ \lambda &= 4.14 \pm .04 \text{ GeV}^{-1} \\ \frac{\chi^2}{dof} &= 1.26 \end{aligned}$$

Our best fit: including the CDF data in the fit but not the E710 data

1800 GeV E710 data set is shown in black: the fit didn't include it

31 GeV data doesn't quite fit: primarily regge contamination?

Total cross section:

The total cross section is

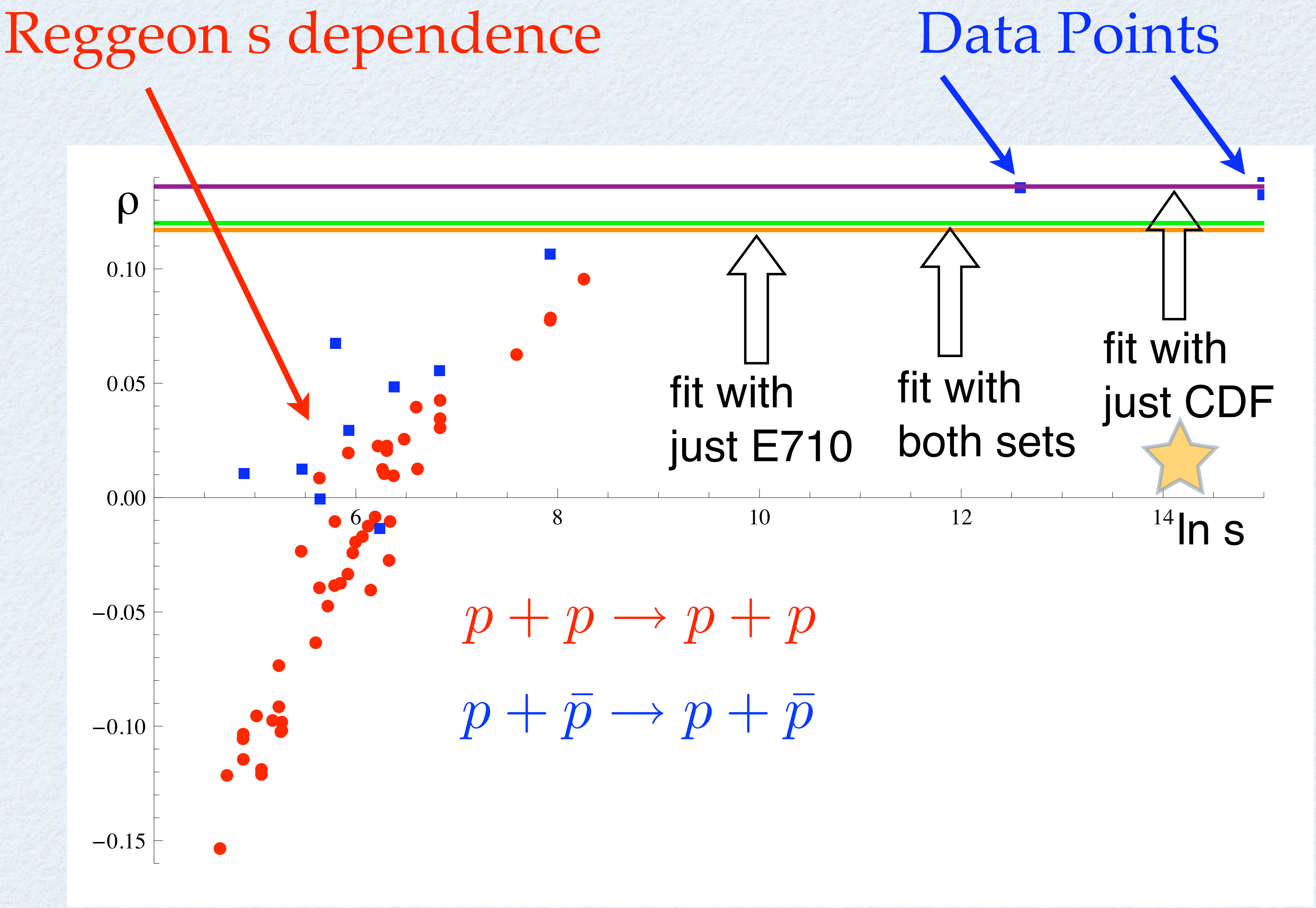
$$\sigma_{tot} = \frac{4\pi\lambda^2\Gamma[-\chi]}{\Gamma[1+a_0]\Gamma[a_0-\chi]}(a's)^{1+2a_0} \equiv Cs^b$$

The best fit values from fit to $d\sigma/dt$: $b = .085 \; C = 21.32$

Compare to fits to total cross section

Our fit to both 1800 sets	$b = .076, \; C = 23.73$
just E710	$b = .074, \; C = 24.43$
just CDF	$b = .086, \; C = 21.10$
DL fit	$b = .081, \; C = 21.70$

Prediction for $\rho = \frac{\Re A(s,t=0)}{\Im A(s,t=0)} \to -\cot a_0\pi = 0.136, \;\; s \to \infty$



Computing Parameters in the Sakai-Sugimoto Model

Quick Summary of Sakai-Sugimoto Model

N_c color D4-branes (replaced by SUGRA background)

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)\overbrace{d\tau^2}^{\text{period } \propto\, M_{KK}^{-1}}\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^\phi = g_s\left(\tfrac{U}{R}\right)^{3/4}, \quad F_4 = dC_3 = \tfrac{2\pi N_c}{V_4}\epsilon_4, \quad f(U) = 1 - \tfrac{U_{KK}^3}{U^3}$$

N_f flavor D8-branes have profile in (U, τ) (geometric realization of chiral symmetry breaking).

Fix M_{KK}, g_s in terms of m_ρ, f_π

Fields appearing in our analysis

Bulk

h_{MN}

10d graviton

$h_{\mu\nu}(x,U)$

other stuff

$h^{(n)}_{\mu\nu}(x)T_n(U)$

2^{++} glueball plus KK tower

Brane

A_M

9d gauge field

$A_\mu(x,U)$

$A_U(x,U)$

$A^{(n)}_\mu(x)\psi_n(U)$

$\varphi_0(x)\psi_0(U)$

KK tower of vector and axial-vector mesons

massless pions

look at this coupling

Quantities we can compute:

$\frac{a_0}{a'}$ (from glueball mass)

λ (from graviton-Skyrmion couplings)

M_d (from Skyrmion stress-tensor)

Do each in turn and compare to best fit values

Glueball Mass

Perturb around D4-brane background metric $h_{\mu\nu}(x)T(U)$ and solve eigenvalue eq's to compute glueball mass (Constable,Myers; Brower, Mathur, Tan):

$$\partial_U \left(U^4 f(U) \partial_U \left[(\frac{R}{U})^{3/2} T(U) \right] \right) = -m_g^2 \frac{R^{9/2}}{U^{1/2}} T(U)$$

Lowest eigenvalue gives mass of lowest 2^{++} glueball

$$m_g = 1.57 \ M_{KK} = 1.49 \ \text{GeV}$$

Fit value: $$m_g|_{\text{fit}} = \sqrt{\frac{-a_0}{a'}} = 1.75 \ \text{GeV}$$

$\Big\}$ both below value expected from lattice QCD

Graviton-Pion Coupling

A generalized Skyrme model arises naturally from the DBI action so we treat protons as Skyrmions

Decompose fields and substitute into DBI action:

$$A_\mu(x,U) = \mathcal{U}^{-1}(x)\partial_\mu\mathcal{U}(x)\psi_+(U) + \cdots$$

$$h_{\mu\nu}(x,U) = h_{\mu\nu}(x)T(U)$$

$$\mathcal{U}(x) = e^{-i\pi(x)/f_\pi}$$

$$S_{D8} \propto \int d^4x h_{\mu\nu}(x)\mathrm{Tr}\big(A_h(\mathcal{U}^{-1}\partial^\mu\mathcal{U})(\mathcal{U}^{-1}\partial^\nu\mathcal{U}) + B_h[\mathcal{U}^{-1}\partial^\mu\mathcal{U}, \mathcal{U}^{-1}\partial_\rho\mathcal{U}][\mathcal{U}^{-1}\partial^\nu\mathcal{U}, \mathcal{U}^{-1}\partial^\rho\mathcal{U}]\big)$$

$$= \lambda \int d^4x\ h_{\mu\nu}(x)T^{\mu\nu} + \text{ small corrections}$$

Glueball-proton coupling

Overlap integral from SS yields

$$\lambda = 0.39 f_\pi^{-1} = \boxed{4.18\ \mathrm{GeV}^{-1}}$$

Compare to fit value

$$\lambda|_{\mathrm{fit}} = \boxed{4.14 \pm 0.04\ \mathrm{GeV}^{-1}}$$

Dipole mass from the Skyrme model

In the Regge limit the first structure function dominates:

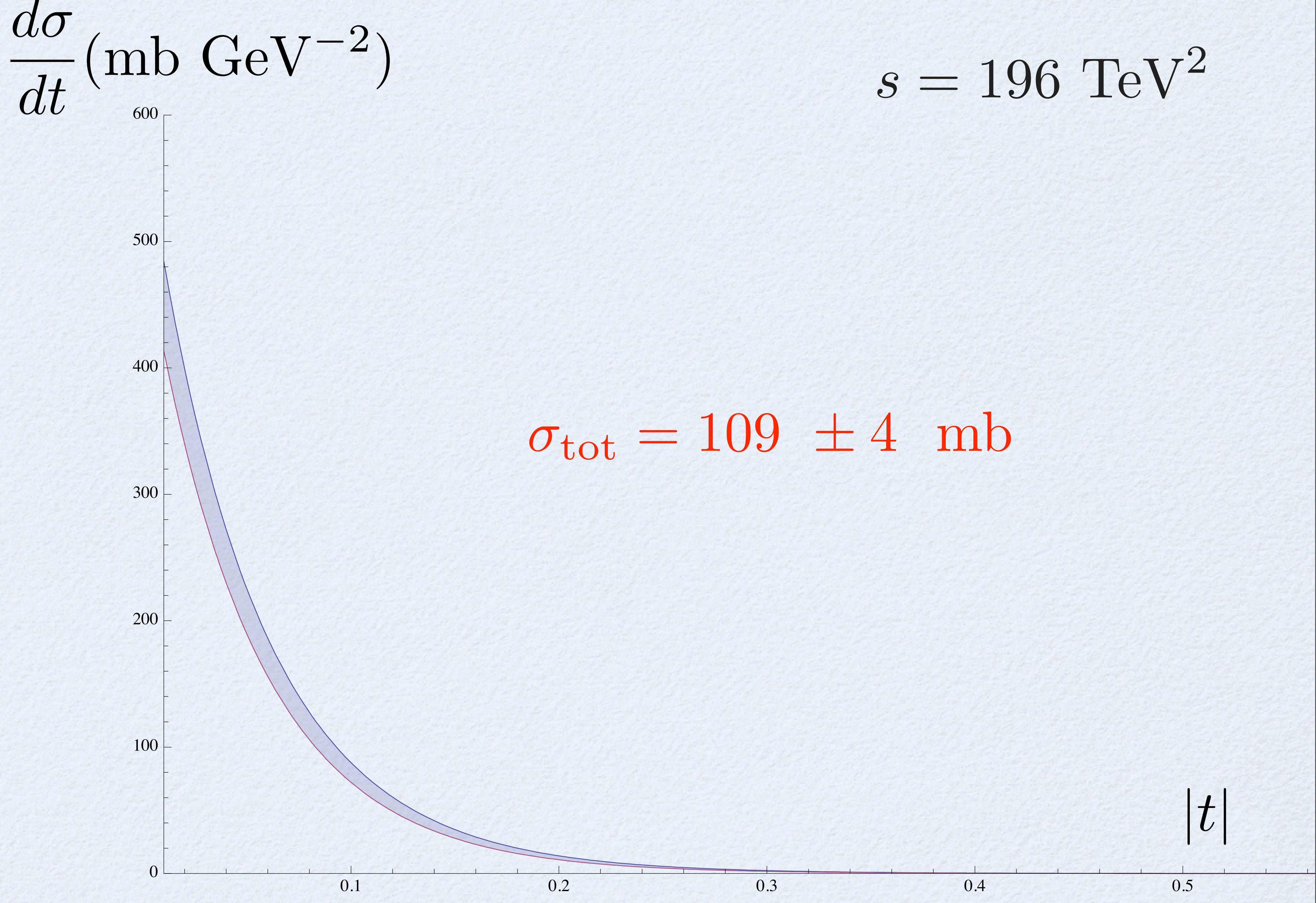
$$\langle p' s' | T_{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[A(t) \gamma_{(\mu} P_{\nu) + \dots} \right] u(p, s)$$

We did not compute $A(t)$ in the full SS model, but it has been computed in the Skyrme model (Cebula et.al.) and in a certain approximation in the soft-wall model (Abidin and Carlson). It is well approximated by a dipole form:

$$A(t) = \frac{1}{(1 - t/M_d^2)^2}, \qquad M_d = 1.17 \text{ GeV}$$

Fit value: $\quad M_d = 1.02 \pm 0.016$

Prediction for the LHC



$\frac{d\sigma}{dt}$ (mb GeV^{-2})
$s = 196$ TeV^2
$\sigma_{\rm tot} = 109 \pm 4$ mb
600
500
400
300
200
100
0
0.1
0.2
0.3
0.4
0.5
$|t|$

Conclusions Revisited

- Studied pp scattering using AdS/QCD to compute coupling and form factor.
- Found excellent agreement between fitted and computed parameters.
- Tests coupling of open-closed string sectors in a dual model of QCD.
- There are many interesting generalizations.