7th May 2009
@ KITP Santa Barbara

Nuclear Force from String Theory

Koji Hashimoto (RIKEN)

arXiv/0806.3122 Sakai, Sugimoto, KH

arXiv/0901.4449 Sakai, Sugimoto, KH

arXiv/0809.3141+ work in progress KH

1. From String Theory to Nuclear Physics

String theory attacking nuclear physics?

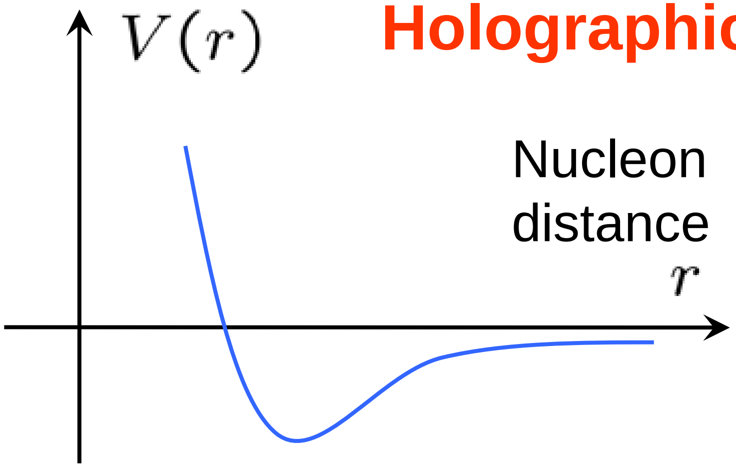
Most of applications of AdS/CFT to QCD, so far, are on hadron physics, not really nuclear physics!

Nuclear physics

Light nuclei : Can be studied by nuclear force

Heavy nuclei : Complicated quantum many body problem

Holographic QCD derivation of nuclear force ?



Long range : Pion exchange

0806.3122 [Sakai,Sugimoto,KH]

Cf. [Hong,Rho,Yee,Yi]

Short range : Strong repulsion

0901.4449 [Sakai,Sugimoto,KH]

String theory attacking nuclear physics?

Most of applications of AdS/CFT to QCD, so far, are on hadron physics, not really nuclear physics!

Nuclear physics

Light nuclei : Can be studied by nuclear force

Heavy nuclei : Complicated quantum many body problem

Nucleon number = large A

Nucleon = D-brane

Possible Gravity dual?

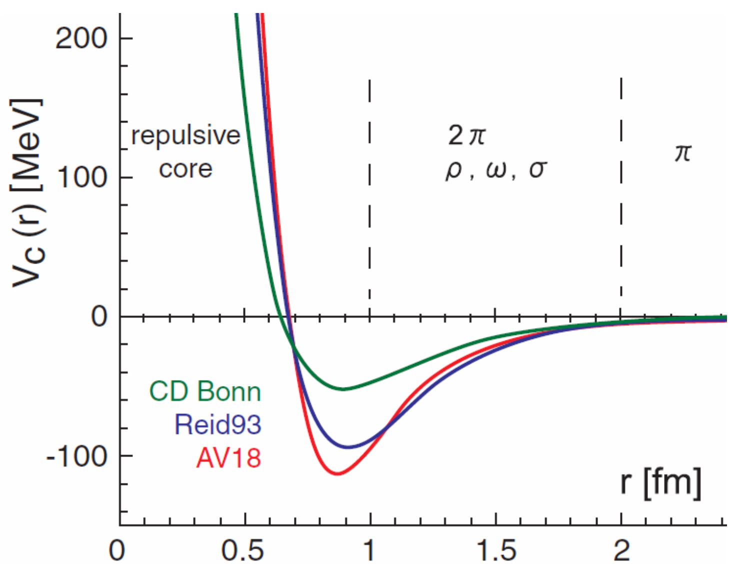
Heavy nuclei may have dual geometry description

arXiv/0809.3141+ work in progress KH

Repulsive core of nucleons

Nuclear force

- Short range :
  Strong repulsion
- Long range :
  Pion exchange

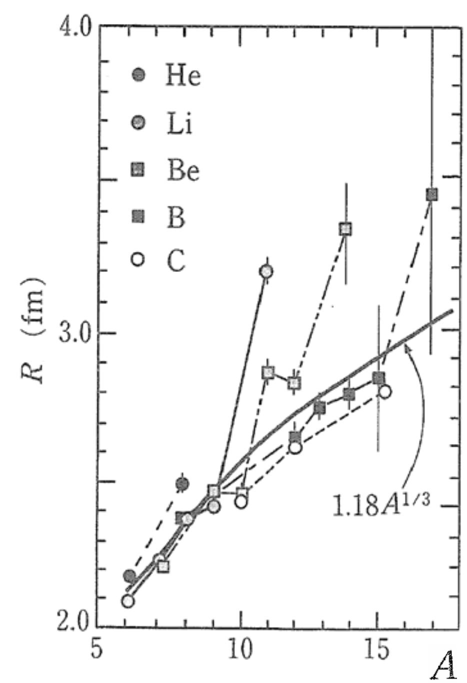


Taken from [Ishii, Aoki, Hatsuda(07)]

Experimental data

- Nucleon scattering
- Nuclear density saturation

$$R \sim 1.18 A^{1/3} [\mathrm{fm}]$$



Theoretical approaches for the repulsive core

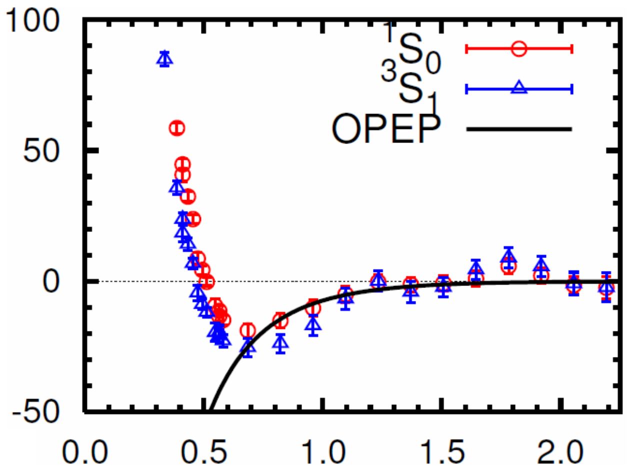
- Quark models   MIT bag model : 3 quarks inside a "bag"

  → Repulsive core due to Pauli principle [Oka,Yazaki(80)]

- Skrymions Baryon as a soliton of pion effective field theory

  → Repulsive core [Jackson,Jackson,Pasquier(85)]
  [VinhMau,Lacombe,Loiseau,Cottingham,Lisboa(85)] …

- Lattice QCD
  [Ishii, Aoki, Hatsuda(07)]



First-principle analytic derivation from strongly-coupled QCD?

We derive the repulsive core by holographic QCD

Our strategy

Skyrmi

Solit n pion ory

(4)

[Adkins

Witte]

[Skyrm (61)]

Hairy (dyo anton

CS(04)]

So in S ugimoto

mod MCS t

t al.(07)]

[Hong (07)]

iton quantization

Baryon spec

[Adki ,Nappi,

Witten(83)]

Chiral curre

[SSH(08)]

Charge r s, meson-b on couplings

[Jackson et.al

(85)]

o-soliton sect

[SSH(09)]

Nuclear Force at short ance

“Effective” theory

Only lower mesons

Many free parameters

Exact solution?

Analytic solution?

“Derived” from “QCD”

∞ # of mesons

Only 2 free parameters

Much more baryon states

in a unified manner

Exact solution

w/ complete moduli

Sakai-Sugimoto model

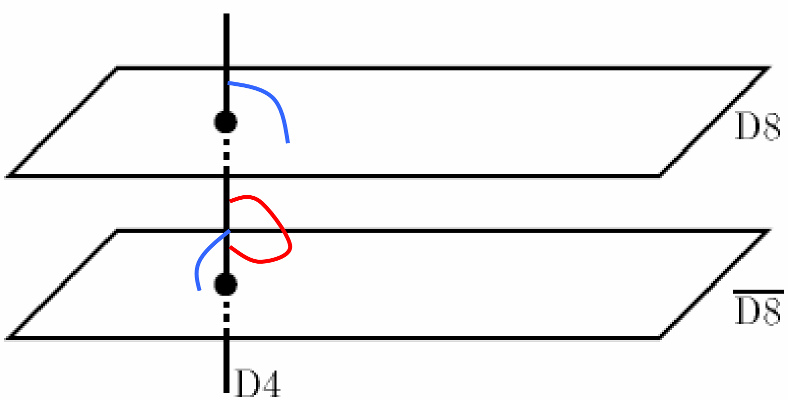
U(Nf) Yang-Mills-Chern-Simons in curved background

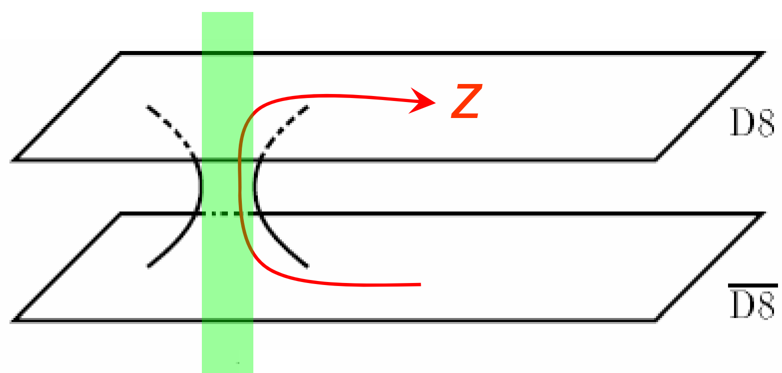
$$(M_{\mathrm{KK}}=1)$$

$$-\frac{\lambda N_c}{216\pi^3}\int d^4x dz \,\mathrm{tr}\left[\frac{1}{2}(1+z^2)^{-1/3}F_{\mu\nu}^2+(1+z^2)F_{\mu z}^2\right]$$
$$+\frac{N_c}{24\pi^2}\mathrm{tr}\int\left[AF^2-\frac{i}{2}A^3F-\frac{1}{10}A^5\right]$$

KK fluctuation analysis

A_μ → Vector meson tower spectrum
A_z → Massless pion





Plan

☑ 1. From String to Nuclear Physics — 6 slides

2. Baryons in SS model : Review — 3 slides

3. Nuclear Force at Long Range — 3 slides

[T.Sakai, S.Sugimoto and KH, 0806.3122]

3. Nuclear Force at Short Range — 3 slides

[T.Sakai, S.Sugimoto and KH, 0901.4449]

2. Baryons in SS model : Review

[Sakai,Sugimoto(04)]
[Hata,Sakai,Sugimoto,Yamato(07)]

Cf. [Hong, Rho, Yee, Yi (07)]

Baryons = YM instantons

Baryon = D4 wrapping S^4 [Witten(98), Gross,Ooguri(98)]

= Instanton in (x^1, x^2, x^3, z) of D8 wrapping S^4

[Sakai, Sugimoto(04)]

$$-\frac{\lambda N_c}{216\pi^3}\int d^4x dz \mathrm{tr}\left[\frac{1}{2}(1+z^2)^{-1/3}F_{\mu\nu}^2 + (1+z^2)F_{\mu z}^2\right] + \frac{N_c}{24\pi^2}\mathrm{tr}\int\left[A\boxed{F^2} - \frac{i}{2}A^3F - \frac{1}{10}A^5\right]$$

Instanton charge sources U(1)v

Quantization of instantons → Baryon spectrum

[Hata, Sakai, Sugimoto, Yamato (07)]
[Hong, Rho, Yee, Yi (07)]

Baryon solution : dyonic instanton

Small instanton localized at $z = 0$ → The background can be approximated by flat space

Solution : BPST instanton + electrostatic potential

$$A_M^{\rm cl} = -if(\xi) g \partial_M g^{-1} , \quad \widehat{A}_0^{\rm cl} = \frac{N_c}{8\pi^2 \kappa} \frac{1}{\xi^2} \left[1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2} \right] , \quad A_0 = \widehat{A}_M = 0$$

$$f(\xi) = \frac{\xi^2}{\xi^2 + \rho^2} , \quad g(x) = \frac{(z-Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi} , \quad \xi = \sqrt{(z-Z)^2 + |\vec{x} - \vec{X}|^2}$$

Inserting this back to the action leads to a potential

$$U(\rho, Z) = 8\pi^2 \kappa \left(1 + \frac{\rho^2}{6} + \frac{N_c^2}{5(8\pi^2 \kappa)^2} \frac{1}{\rho^2} + \frac{Z^2}{3} \right) \qquad \kappa = \frac{\lambda N_c}{216\pi^3}$$

U(1) Coulomb self-repulsion   Effect of curved space in SU(2)

→ Size is stabilized to be small, $\rho_{\rm cl}^2 = \frac{27\pi}{\lambda} \sqrt{\frac{6}{5}}, \; Z_{\rm cl} = 0$

Quantization of the instanton

Moduli space approximation : Moduli with small potentials

- Moduli : $(\vec{X}, Z)$, $y_I (I = 1,2,3,4, (y_I)^2 = \rho^2)$
- Lagrangian : $L = \frac{M_0}{2}(\dot{\vec{X}}^2 + \dot{Z}^2) + M_0\, \dot{y}_I^2 - U(\rho, Z)$ $M_0 = 8\pi^2\kappa$

y_I acted by $SO(4) \simeq (SU(2)_I \times SU(2)_J)/\mathbb{Z}_2$: isospin + spin

Z, ρ → Harmonic-like potential → Baryons labeled by n_ρ, n_z

Baryon states are given by wave function of the QM :

Proton : $|p\uparrow\rangle \propto R(\rho)\psi_Z(Z)(a_1 + i a_2)$ $a_I \equiv y_I/\rho$

$$R(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0}{\sqrt{6}}\rho^2}\ , \quad \psi_Z(Z) = e^{-\frac{M_0}{\sqrt{6}}Z^2}$$

3. Nuclear Force at Long Range

arXiv/0806.3122 (PTP)

T.Sakai, S.Sugimoto, KH

Static properties of baryons

Chiral currents

$J_L(r), J_R(r)$

→

Static properties

- Charge radius
- Magnetic moments
- Axial radius, coupling

Chiral symmetry = *U(Nf)* gauge symmetry at $z = \pm\infty$

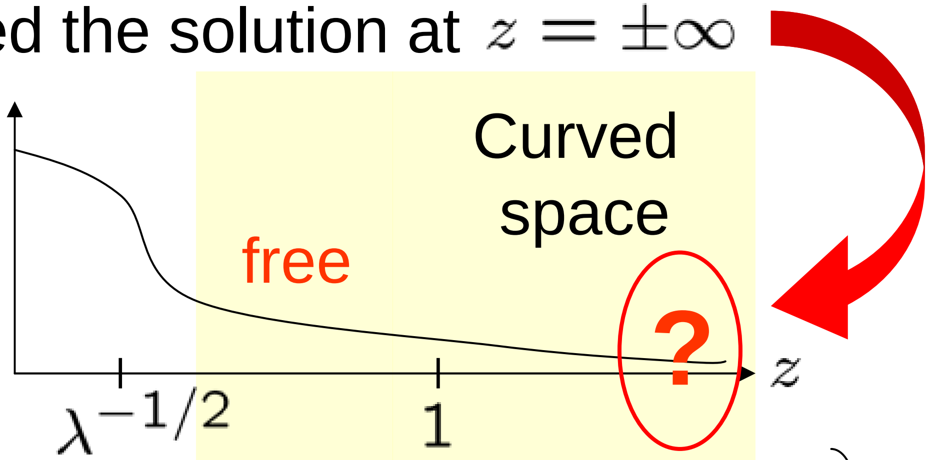
$$S\big|_{\mathcal{O}(\mathcal{A}_L,\mathcal{A}_R)} = -2\int d^4x\,\mathrm{tr}\left(\mathcal{A}_{L\mu}\mathcal{J}_L^{\mu} + \mathcal{A}_{R\mu}\mathcal{J}_R^{\mu}\right)$$

$$\left(\begin{array}{l} \mathcal{A}_\alpha(x^\mu, z) = \mathcal{A}_\alpha^{\mathrm{cl}}(x^\mu, z) + \delta\mathcal{A}_\alpha(x^\mu, z) \\ \delta\mathcal{A}_\mu(x^\mu, z\to+\infty) = \mathcal{A}_{L\mu}(x^\mu)\ ,\quad \delta\mathcal{A}_\mu(x^\mu, z\to-\infty) = \mathcal{A}_{R\mu}(x^\mu) \end{array}\right)$$

→ $$J_{L,R}(r) = \mp\frac{\lambda N_c}{216\pi^3}\left[(1+z^2)F_{\mu z}^{(\mathrm{sol})}\right]_{z=\pm\infty}$$

Evaluation of chiral currents

We need the solution at $z = \pm\infty$



Flat space solution
[HSSY]

Curved space
Green function

Connected
smoothly

Explicit expression of the chiral currents

Static properties of baryons

1) Baryon number current $J_B(r)$

$$1 = N_B = \int_0^\infty dr \rho_B(r), \quad \rho_B(r) \equiv 4\pi r^2 \langle J_B^0(r) \rangle$$

Isoscalar mean square radius $\langle r^2 \rangle_{I=0} = \int_0^\infty dr\, r^2 \rho_B(r)$

Our result : $\langle r^2 \rangle_{I=0}^{1/2} \simeq 0.742 \text{ fm}$

(Input : $M_{\text{KK}} = 949 \text{ MeV}$, $\kappa = 0.00745$)

2) Vector current $J_V^0(r)$

Isovector charge density turns out to be $\rho_{I=1}(r) = \rho_B(r)$

$$Q_{\text{em}} = I_3 + \frac{N_B}{2}$$

Charge radius $\langle r^2 \rangle_{\text{E,p}} = \langle r^2 \rangle_{I=0}$, $\langle r^2 \rangle_{\text{E,n}} = 0$

Our results for static properties

	Holographic QCD	Skyrmion	Experiment	Lattice QCD
$\langle r^2 \rangle_{E,\mathrm{p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.875\ \mathrm{fm})^2$	
$\langle r^2 \rangle_{E,\mathrm{n}}$	0	$-\infty$	$-0.116\ \mathrm{fm}^2$	
$\langle r^2 \rangle_A^{1/2}$	$0.537\ \mathrm{fm}$	—	$0.674\ \mathrm{fm}$	
μ_p	2.18	1.87	2.79	
μ_n	-1.34	-1.31	-1.91	
g_A	0.734	0.61	1.27	
$g_{\pi NN}$	7.46	8.9	13.2	
$g_{\rho NN}$	5.80	—	$4.2 \sim 6.5$	
$\mu_{\Delta^{++}}$	4.43	—	$3.7 - 7.5$	4.99
$\mu_{\Delta^{+}}$	2.32	—	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$	2.49
$\mu_{\Delta^{0}}$	0.204	—	—	0.06
$\mu_{\Delta^{-}}$	-1.91	—	—	-2.45

4. Nuclear Force at Short Range

arXiv/0901.4449

Sakai, Sugimoto, KH

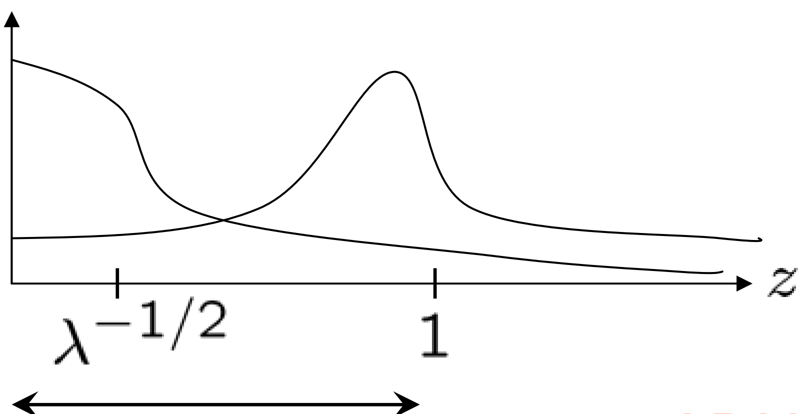
Nuclear force at short range

Two baryons = two instantons in curved spacetime

… Extremely difficult to solve analytically

If baryons are not far away from each other,

$$\mathcal{O}(\lambda^{-1/2}) < r < \mathcal{O}(1)$$



Flat space

YM 2-instanton in
flat space can be used

ADHM construction of 2 instantons
Full moduli dependence appears.
In total, 16 moduli = 8 x 2

Evaluation of interaction hamiltonian

Inserting the 2-instanton solution back to the action gives

$$H = H^{(1)} + H^{(2)} + \boxed{H_1(r)}$$

Interaction hamiltonian

$$H_1(r) = \frac{1}{r^2} f(Z_1, y_1^I, Z_2, y_2^I) \; + \mathcal{O}(1/r^3)$$

Nucleon-nucleon potential = VEV $\langle H_1(r) \rangle$
with 2-baryon asymptotic wavefunction $|p\uparrow\rangle_1 |p\uparrow\rangle_2$

$$\langle H_1(r) \rangle = V_C(r) + (3\hat{\mathbf{r}} \cdot \sigma_1 \hat{\mathbf{r}} \cdot \sigma_2 - \sigma_1 \cdot \sigma_2) V_T(r)$$

$$V_C(r) = \pi \left(\frac{27}{2} + \frac{32}{5} \vec{s}_1 \cdot \vec{s}_2 \, \vec{I}_1 \cdot \vec{I}_2 \right) \frac{N_c}{\lambda} \frac{1}{r^2}$$

$$V_T(r) = \frac{8\pi}{5} \vec{I}_1 \cdot \vec{I}_2 \frac{N_c}{\lambda} \frac{1}{r^2}$$

Strongly repulsive
with $1/r^2$ potential

Details of the interaction hamiltonian

ADHM construction enables computation of the hamiltonian

$$H_1 = H_1^{(U(1))} + H_1^{(SU(2))} - \frac{1}{2m_X}\nabla_1^2$$

$\boldsymbol{a}_i \equiv \boldsymbol{y}_i/\rho_i$

$a = 1/216\pi^3$

$$H_1^{(U(1))} = \frac{N_c}{8\pi^2 a}\frac{1}{|\boldsymbol{r}|^2}\left[\frac{1}{2} + \frac{2(\boldsymbol{a}_1\cdot\boldsymbol{a}_2)^2 - 1}{5}\left(\frac{\rho_2^2}{\rho_1^2} + \frac{\rho_1^2}{\rho_2^2}\right)\right]$$

"Coulomb" repulsion

$$H_1^{(SU(2))} = \frac{4\pi^2 a N_c}{3}\rho_1^2\rho_2^2\frac{r^a r^b}{|\boldsymbol{r}|^4}\,\mathrm{tr}(i\tau^a \boldsymbol{a}_2^{-1}\boldsymbol{a}_1)\,\mathrm{tr}(i\tau^b \boldsymbol{a}_2^{-1}\boldsymbol{a}_1)$$

Curved space

$$\nabla_1^2 = -\frac{1}{|\boldsymbol{r}|^2}\left[\frac{\rho_2^2}{2}\left(\frac{1}{\rho_1^3}\frac{\partial}{\partial\rho_1}\left(\rho_1^3\frac{\partial}{\partial\rho_1}\right) - \frac{4}{\rho_1^2}I_1^a I_1^a\right) + \frac{\rho_1^2}{2}\left(\frac{1}{\rho_2^3}\frac{\partial}{\partial\rho_2}\left(\rho_2^3\frac{\partial}{\partial\rho_2}\right) - \frac{4}{\rho_2^2}I_2^a I_2^a\right)\right.$$
$$\left. + 4I_1^a I_2^a + \rho_1\rho_2\frac{\partial}{\partial\rho_1}\frac{\partial}{\partial\rho_2} + \rho_1\frac{\partial}{\partial\rho_1} + \rho_2\frac{\partial}{\partial\rho_2} - \left(y_2^I\frac{\partial}{\partial y_1^I}\right)^2 - \left(y_1^I\frac{\partial}{\partial y_2^I}\right)^2\right].$$

Curved metric on moduli space of 2 instantons

$U(1)_V$ "Coulomb" repulsion dominates the repulsion

This is the tower starting with omega meson

Possible Existence of a Heavy Neutral Meson*

YOICHIRO NAMBU

The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois

(Received April 25, 1957)

IN an attempt to account for the charge distributions of the proton and the neutron as indicated by the electron scattering experiments,[1] we would like to consider the possibility that there may be a heavy neutral meson which can contribute to the form factor of the nucleon. We assume that this meson, ρ^0, is a vector field with isotopic spin zero and a mass two to three times that of the ordinary pion, coupled strongly to the nucleon field. An isolated ρ^0 would decay through virtual nucleon pair formation according to the follow-

if a resonance should occur for such a system.

(3) ρ^0 would contribute a repulsive nuclear force of Wigner type and short range ($\lesssim 0.7\times10^{-13}$ cm), more or less similar to the phenomenological hard core.

(4) The anomalous moment of the nucleon[3] should

5. Summary

Summary

Nuclear force from string theory :

We compute Nuclear force at short distances

→ Reproduction of the repulsive core,
providing an analytic form of the nuclear force

[T.Sakai, S.Sugimoto and KH, 0901.4449]

We compute also static properties of baryons, including meson-baryon coupling. → Long range nuclear force

They nicely match exp. data

[T.Sakai, S.Sugimoto and KH, 0806.3122]