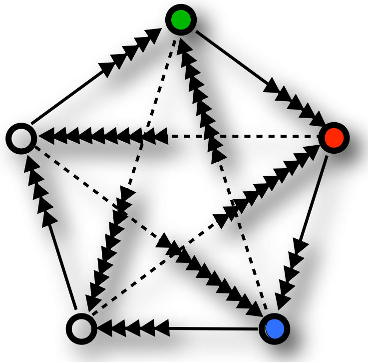
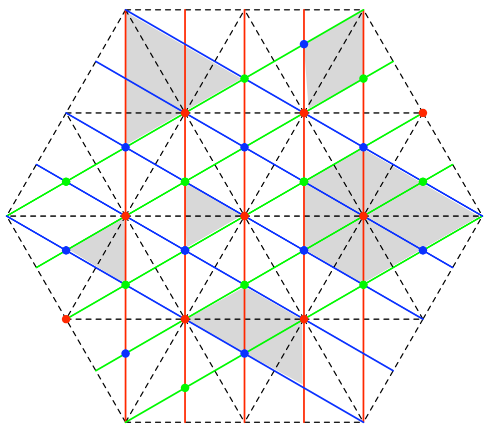
Matrix Factorizations, D-branes and Homological Mirror Symmetry

W.Lerche, KITP 03/2009



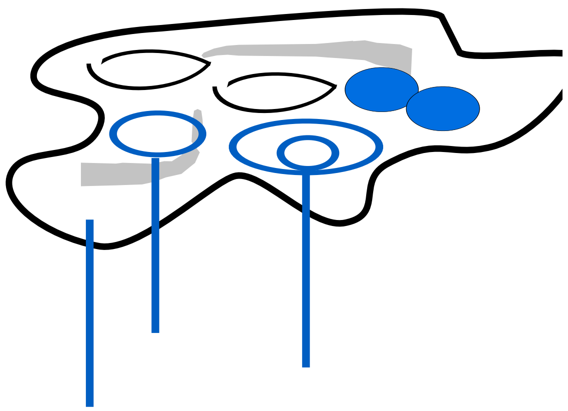
- Motivation, general remarks
- Mirror symmetry and D-branes
- Matrix factorizations and LG models
- Toy example: eff. superpotential for intersecting branes, applications

much more diverse instantons than for closed strings (world-sheet and D-brane instantons)



Part I Motivation: D-brane worlds

Typical brane + flux configuration on a Calabi-Yau space



closed string (bulk) moduli t

open string (brane location + bundle) moduli u

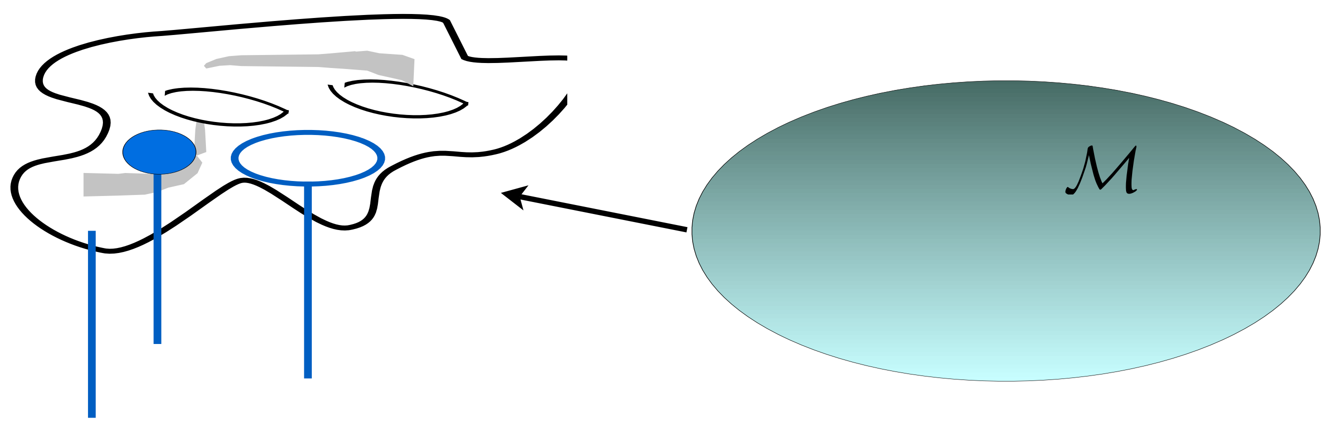
3+1 dim world volume with effective N=1 SUSY theory

What are the exact effective superpotential, the vacuum states, gauge couplings, etc ?

$$\mathcal{W}_{\text{eff}}(\Phi, t, u) = ?$$

Quantum geometry of D-branes

Classical geometry ("branes wrapping p-cycles", gauge bundle configurations on top of them) makes sense only at weak coupling/large radius!

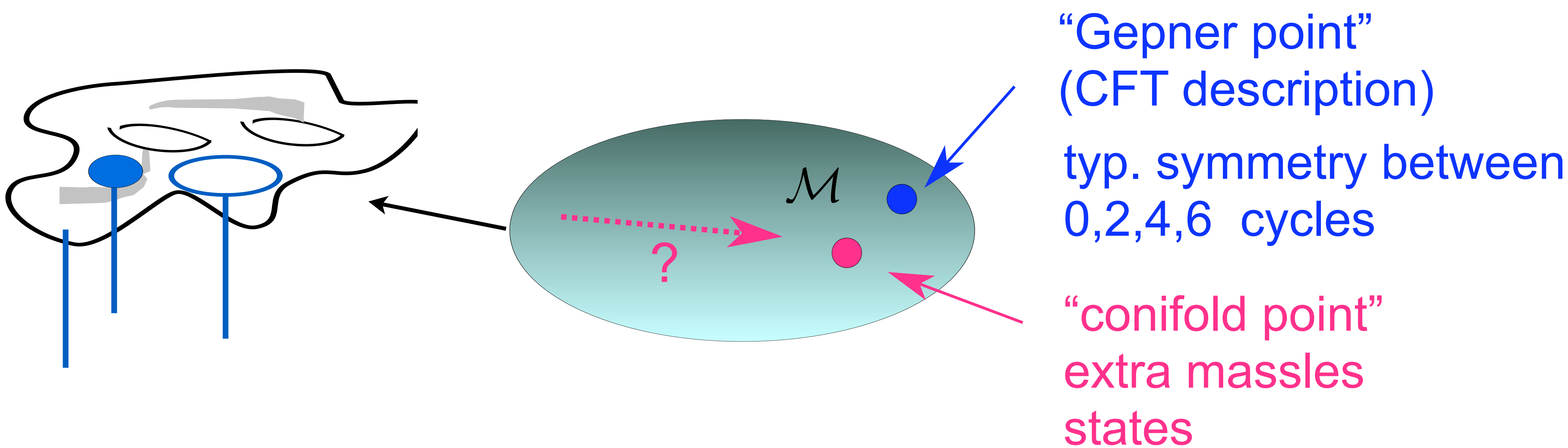


Classical geometry: cycles, gauge (“bundle”) configurations on them

In fact, practically all of string phenomenology deals with the boundary of the moduli space (weak coupling, large radius)

Quantum geometry of D-branes

Classical geometry ("branes wrapping p-cycles", gauge bundle configurations on top of them) makes sense only at weak coupling/large radius!



Classical geometry:
cycles, gauge (“bundle”) configurations on them

Quantum corrected geometry:
(instanton) corrections wipe out notions of classical geometry

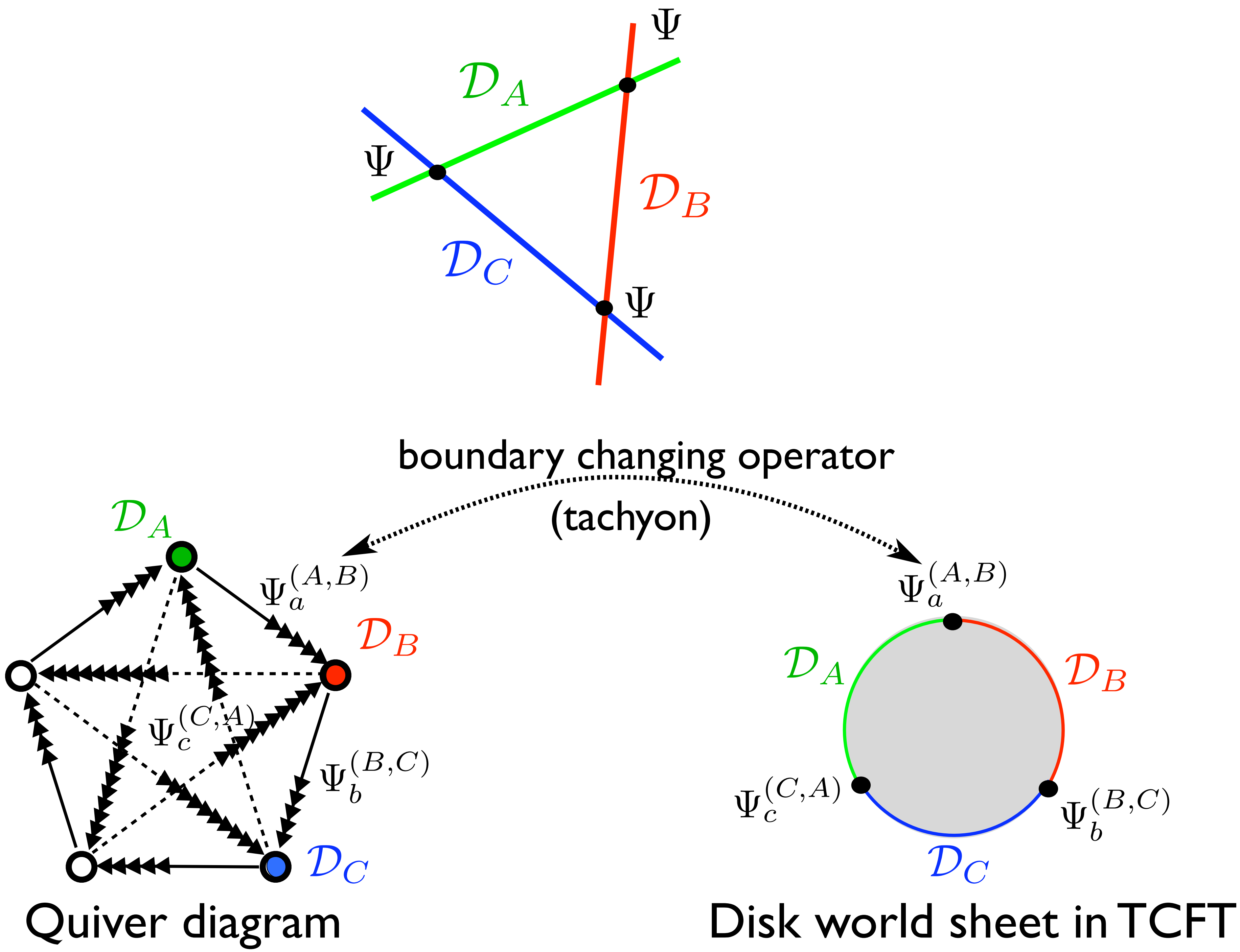
Quantum geometry of D-branes

Classical geometry ("branes wrapping p-cycles", gauge bundle configurations on top of them) makes sense only at weak coupling/large radius!

▷ Need to develop formalism capable of describing the physics of general D-brane configurations (here: topological B-type D-branes), incl their continuous deformation families over the moduli space

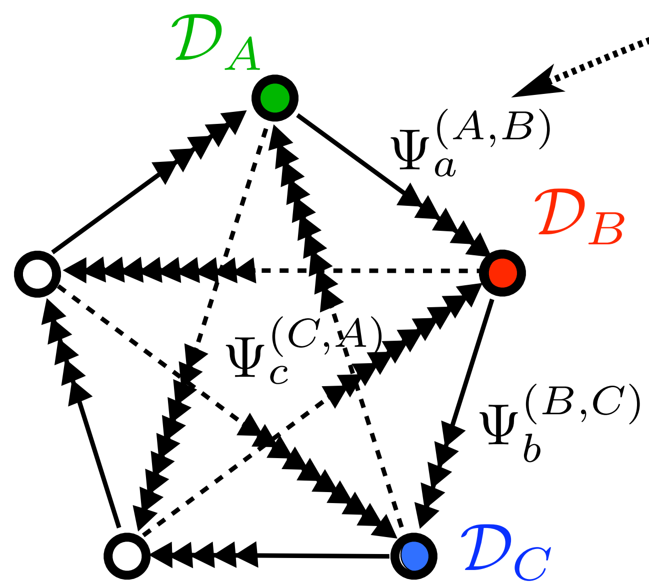
....well developed techniques (mirror symmetry) mostly for non-generic (non-compact, non-intersecting, integrable) brane configurations branes only !

Intersecting branes: eff. potential for quivers

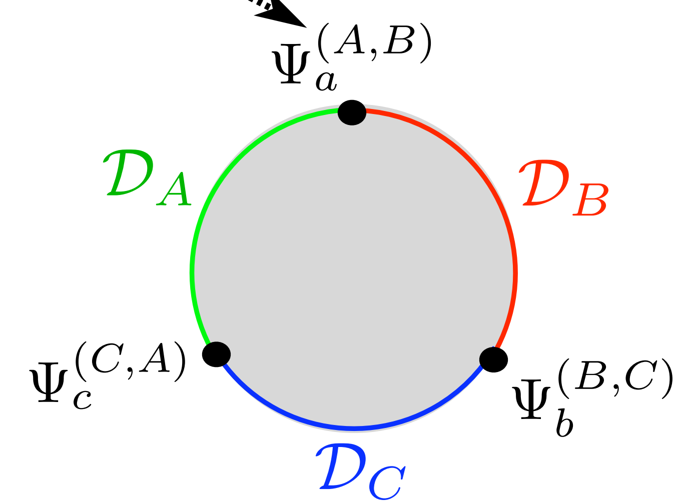


Intersecting branes: eff. potential for quivers

boundary changing operator
(tachyon)



Quiver diagram



Disk world sheet in TCFT

Superpotential ~ closed paths in quiver

$$\mathcal{W}_{eff}(T,u,t) = T_a T_b T_c \underbrace{\langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,A)} \rangle}_{C_{abc}(t,u)} + T_a T_b T_c T_d \underbrace{\langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,D)} \Psi_d^{(D,A)} \rangle}_{C_{abcd}(t,u)} + \ldots$$

tachyons

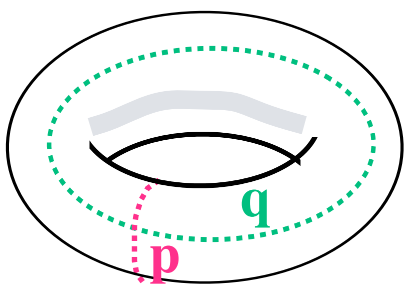
closed and open string moduli $\sim \mathrm{const} + \mathcal{O}(e^{-t}, e^{-u})$

instanton corrections... how to compute?

D-branes: homological mirror symmetry

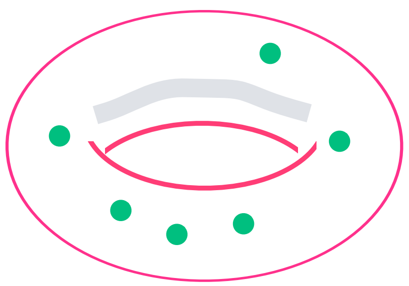
- Mirror symmetry acts between full categories descr. A- and B-branes!

A-Model



mirror symmetry

B-Model



D1 branes on (p,q) cycles

...“Fukaya category” of lagrangian cycles

$(N_2,N_0) = (r,c_1)$ of “gauge bundle”

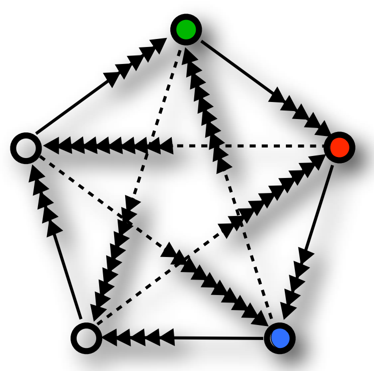
...“derived category” of coherent sheaves

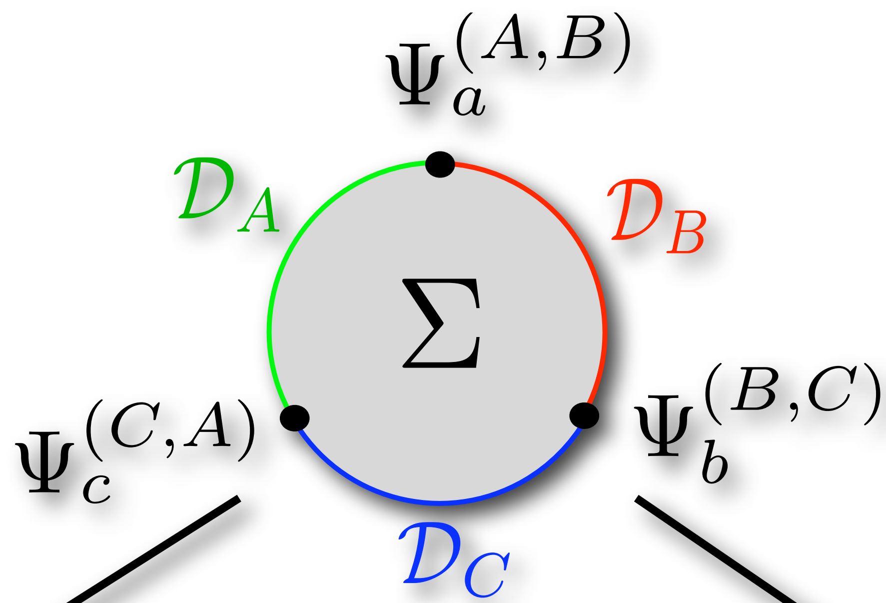
- There is much more to this than just quantum numbers (K-theory)

  Homological mirror symmetry also preserves the higher A_∞ products (open string correlators)

Mirror symmetry and open string TFT correlators

Disk amplitude for intersecting branes $C_{abc}(\tau; u_A, u_B, u_C) = \langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,A)} \rangle$



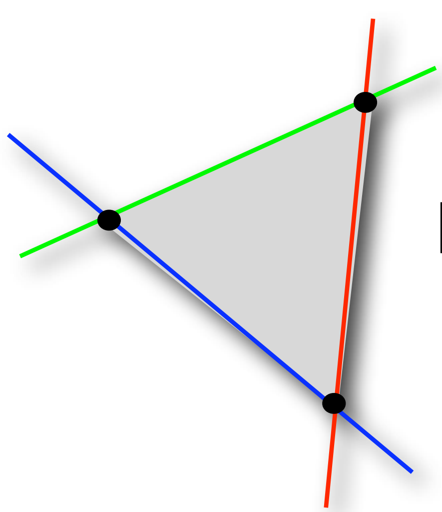


A-Model

localizes on holomorphic maps:
world-sheet instantons $\Sigma \longrightarrow T_2$

B-Model

(Landau-Ginzburg model)
localizes on constant maps:
classical



Fukaya products ←·····→ mirror symmetry ←·····→ Massey products

$C_{abc} \sim e^{-S_{\text{Inst}}}$

$S_{\text{Inst.}} \sim \text{Area}$

$\lambda_m(\Psi^{\otimes m}) \;=\; \Psi_{a_0} C^{a_0}_{a_1 \ldots a_m}$

B-model correlators

- General structure of 3-pt fct in (holom) Chern-Simons theory:

$$C_{abc} \;=\; \langle\, \Psi_a \Psi_b \Psi_c \,\rangle \;=\; \int_X \mathrm{Tr}[\Psi_a \wedge \Psi_b \wedge \Psi_c] \wedge \Omega^{(3,0)}$$

Wedge product of sections

$$\Psi_a \;\equiv\; \Psi_a^{(A,B)} \;\in\; \mathrm{Ext}^1(X; \bar{\mathcal{V}}^A, \mathcal{V}^B)$$

- Branes ~ vector bundles:
  sections are collections of vectors = matrices

- To perfom mirror map... need flat sections
  determined by some flatness equations

B-model correlators....

- For the bulk theory, the flatness equations coincide with the Picard-Fuchs equations for periods,
  arising from the variation of Hodge structures

- For open strings theory is not yet well developed
  (in general no flatness to begin with, obstructions)

- Physicist's pragmatic approach:
  obtain diffeqs from contact terms in Landau-Ginzburg formulation

Part II D-branes and Matrix Factorizations

Seek: Description of topol. B-type D-branes that captures the mathematical intricacies,
while allowing to do explicit computations

LG formulation of B-type branes in terms of matrix factorizations

Important: explicit moduli dependence of all quantities

Landau-Ginzburg description of B-type D-branes

- Consider bulk d=2 N=(2,2) LG model with superpotential:

$$\int_\Sigma d^2z d\theta^+ d\theta^- \, W_{LG}(x) + \mathrm{cc.}$$

B-type SUSY variations induce boundary ("Warner")-term:

$$\int_\Sigma d^2z d\theta^+ d\theta^- (\bar{Q}_+ + \bar{Q}_-) \, W_{LG} = \int_\Sigma d^2z d\theta^+ d\theta^- (\theta^+ \partial_+ + \theta^- \partial_-) W_{LG}$$

$$= \int_{\partial\Sigma} d\sigma d\theta \, W_{LG}$$

- Restore SUSY by adding boundary fermions $\Pi = (\pi + \theta^+ \ell)$

(... not quite chiral: $\bar{D}\,\Pi \; = \; E(x)|_{\partial\Sigma}$)

via a boundary potential: $\delta S \; = \; \int_{\partial\Sigma} d\sigma d\theta \, \Pi \, J(x)$

Condition for SUSY: $J(x) E(x) \; = \; W_{LG}(x)$

Matrix factorizations

- BRST operator: $Q(x) = \pi\, J(x) + \bar{\pi}\, E(x) = \begin{pmatrix} & J(x) \\ E(x) & \end{pmatrix}$

  thus SUSY condition implies a matrix factorization of W:

$$Q(x) \cdot Q(x) \;=\; W_{LG}(x)\, 1$$

  Total BRST operator $\mathcal{Q} \;=\; Q + Q_{bulk}$

  then squares to zero: $\mathcal{Q}^2 = 0$

- Generalization for n LG fields: need N=2^n boundary fermions, and

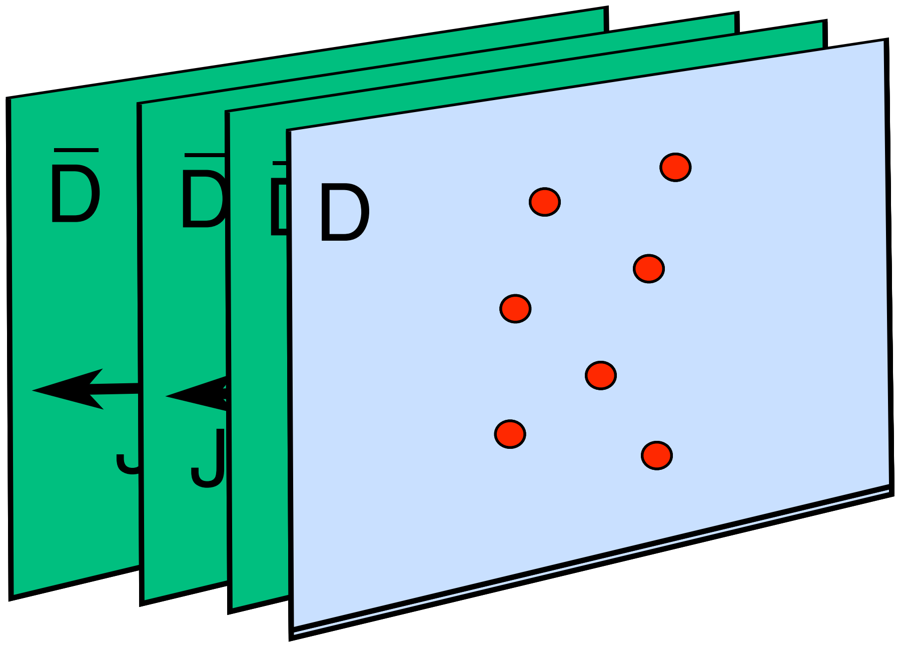
$$J_{N\times N} \cdot E_{N\times N} \;=\; E_{N\times N} \cdot J_{N\times N} = \; W_{LG}\, 1_{N\times N}$$

- B-type Susy D-branes:
  1:1 to all possible matrix factorizations of a given W=W(x,t)

Physical interpretation

- N... Chan-Paton labels of space-filling $D\bar{D}$ pairs

  Boundary potentials J,E form a tachyon profile that describes condensation to given B-type D-brane configuration in IR limit



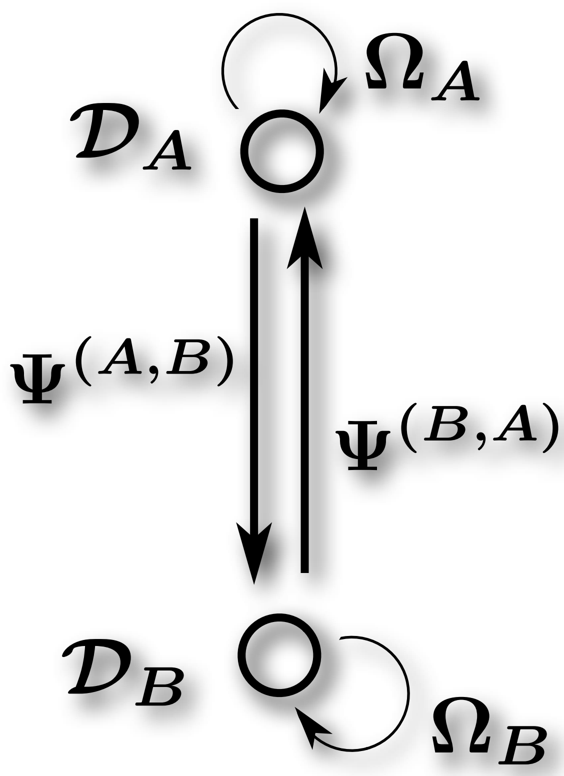
eg. $$J(x,u) = \prod^{n}(x-u_i)$$ c_1=n

- Geometrically: Maps J,E are sections of certain bundles

  Ker J, Ker E encode bundle data of branes: $(r,c_1,\ldots;u)$

Open string cohomology

- Physical open string spectrum is determined by the cohomology of the BRST operator:



$$[Q_A, \Omega_A] = 0\,, \quad \Omega_A \neq [Q_A, \Lambda]$$

boundary preserving
(square matrices)

$$Q_A \Psi^{(A,B)} - (-)^f \Psi^{(A,B)} Q_B = 0$$

boundary changing
(rectangular matrices)

$$[Q_B, \Omega_B] = 0\,, \quad \Omega_B \neq [Q_B, \Lambda]$$

- These are the ingredients of a nice category....

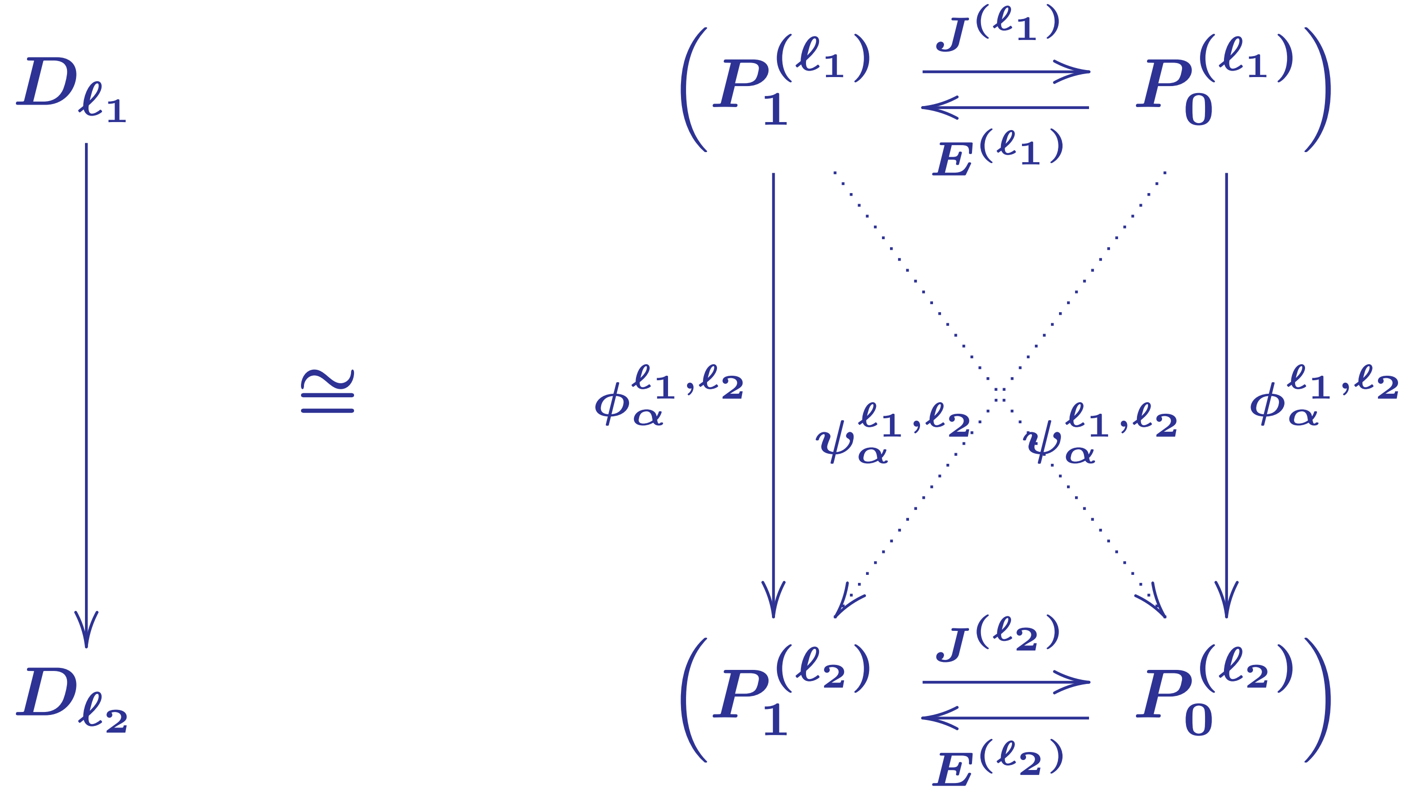
Kontsevich's category C_W

The LG model provides a concrete physical realization of a certain triangulated Z_2-graded category C_W

- objects: "complexes" (~composites of D$\overline{\text{D}}$ branes):

$$D_\ell \cong \left(P_1^{(\ell)} \underset{E^{(\ell)}}{\overset{J^{(\ell)}}{\rightleftarrows}} P_0^{(\ell)} \right), \qquad J^{(\ell)} E^{(\ell)} = W$$

- maps (boundary Q-cohomology):



Kontsevich's category C_W

The LG model provides a concrete physical realization of a certain triangulated Z_2-graded category C_W

Category of Matrix factorizations is isomorphic to D(Coh(M)), the derived category of coherent sheaves on M =
category of B-type D-branes! (O, HHP)

Part III: branes on the elliptic curve

- Simplest Calabi-Yau: the cubic torus

complex str modulus

$$T_2 : \quad W = \frac{1}{3}({x_1}^3 + {x_2}^3 + {x_3}^3) - a(\tau)\, x_1 x_2 x_3 = 0$$

- Mirror map:

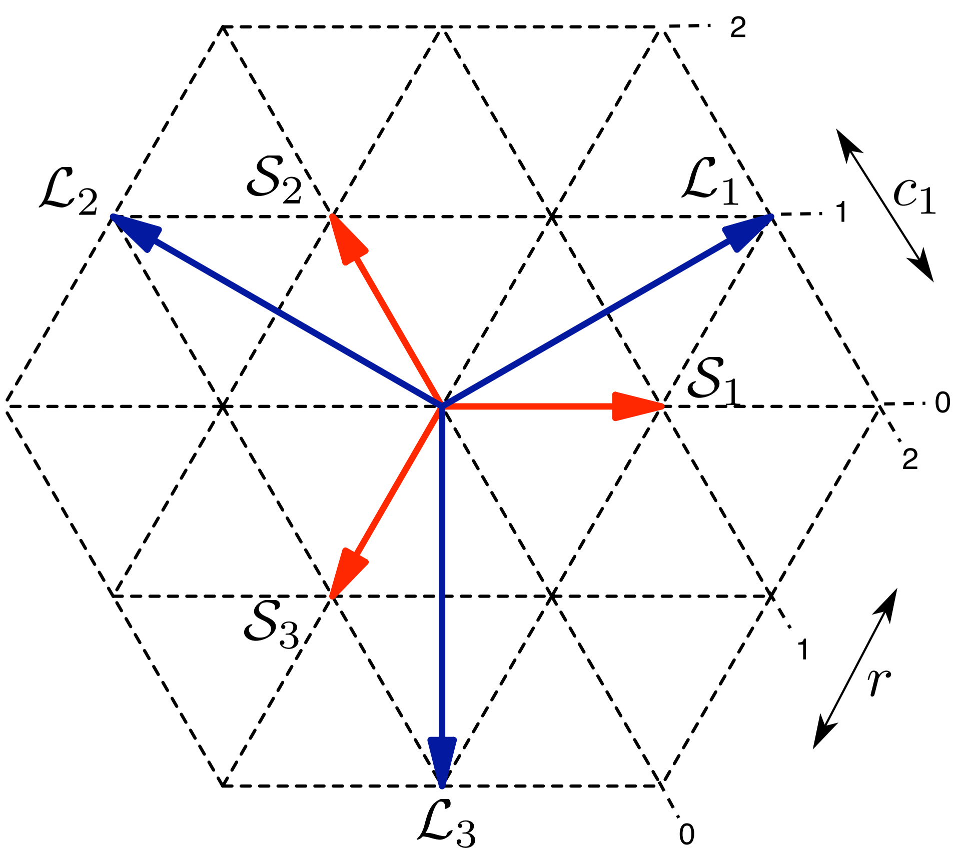
$$\frac{3\,a\,(a^3+8)}{\Delta} = J(\tau)^{1/3}\,, \qquad \Delta \equiv a^3 - 1$$

flat coo of complex structure moduli space =
Kahler parameter of mirror curve

Test example: branes on the elliptic curve

- B-type D-branes are composites of D2, D0 branes, characterized by $(N_2, N_0; u) \;=\; (\mathrm{rank}(V), c_1(V); u)$

... these are mirror to A-type D1-branes with wrapping numbers $(p, q) \;=\; (N_2, N_0)$



- We will consider the ``long-diagonal" branes with charges

$$(N_2, N_0)_{\mathcal{L}_A} = \{(-1,0), (-1,3), (2,-3)\}$$

3x3 matrix factorization

- Factorizations corresponding to the long diagonal branes L_i (i=1,2,3)

$$J_i = \begin{pmatrix} \alpha_1^{(i)} x_1 & \alpha_2^{(i)} x_3 & \alpha_3^{(i)} x_2 \\ \alpha_3^{(i)} x_3 & \alpha_1^{(i)} x_2 & \alpha_2^{(i)} x_1 \\ \alpha_2^{(i)} x_2 & \alpha_3^{(i)} x_1 & \alpha_1^{(i)} x_3 \end{pmatrix}$$

$$E_i = \begin{pmatrix} \frac{1}{\alpha_1^{(i)}} {x_1}^2 - \frac{\alpha_1^{(i)}}{\alpha_2^{(i)}\alpha_3^{(i)}} x_2 x_3 & \frac{1}{\alpha_3^{(i)}} {x_3}^2 - \frac{\alpha_3^{(i)}}{\alpha_1^{(i)}\alpha_2^{(i)}} x_1 x_2 & \frac{1}{\alpha_2^{(i)}} {x_2}^2 - \frac{\alpha_2^{(i)}}{\alpha_1^{(i)}\alpha_3^{(i)}} x_1 x_3 \\ \frac{1}{\alpha_2^{(i)}} {x_3}^2 - \frac{\alpha_2^{(i)}}{\alpha_1^{(i)}\alpha_3^{(i)}} x_1 x_2 & \frac{1}{\alpha_1^{(i)}} {x_2}^2 - \frac{\alpha_1^{(i)}}{\alpha_2^{(i)}\alpha_3^{(i)}} x_1 x_3 & \frac{1}{\alpha_3^{(i)}} {x_1}^2 - \frac{\alpha_3^{(i)}}{\alpha_1^{(i)}\alpha_2^{(i)}} x_2 x_3 \\ \frac{1}{\alpha_3^{(i)}} {x_2}^2 - \frac{\alpha_3^{(i)}}{\alpha_1^{(i)}\alpha_2^{(i)}} x_1 x_3 & \frac{1}{\alpha_2^{(i)}} {x_1}^2 - \frac{\alpha_2^{(i)}}{\alpha_1^{(i)}\alpha_3^{(i)}} x_2 x_3 & \frac{1}{\alpha_1^{(i)}} {x_3}^2 - \frac{\alpha_1^{(i)}}{\alpha_2^{(i)}\alpha_3^{(i)}} x_1 x_2 \end{pmatrix}$$

W, BHLW

These satisfy $J_i E_i = E_i J_i = W_{LG} \mathbf{1}$
if the parameters satisfy the cubic equation themselves:

$$W_{LG}(\alpha_i) \equiv {\alpha_1}^3 + {\alpha_2}^3 + {\alpha_3}^3 + a(\tau)\,\alpha_1\alpha_2\alpha_3 = 0$$

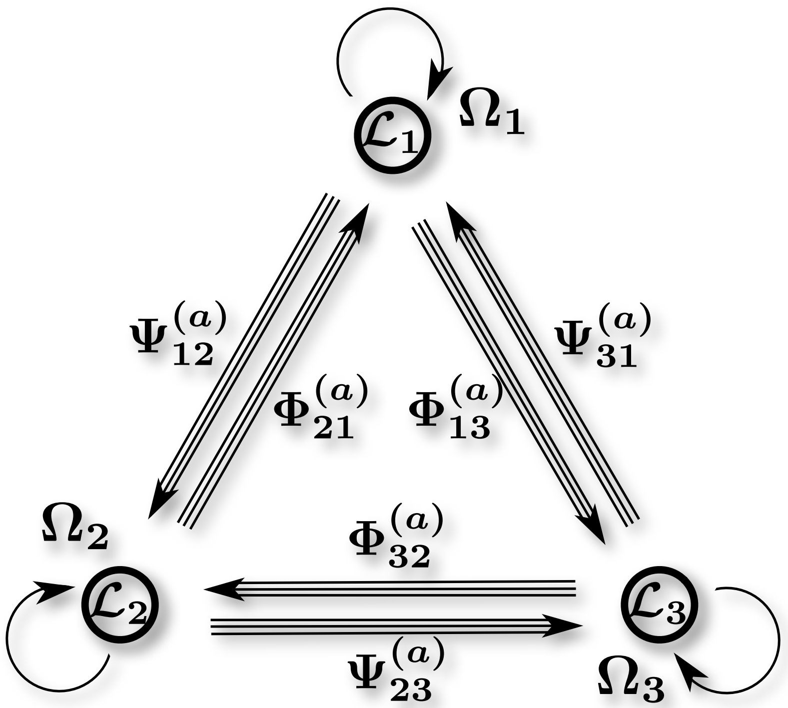
Thus the parameters parametrize the (jacobian) torus and can be represented by theta-sections:

$$\alpha_\ell^{(i)} \sim \Theta\left[\tfrac{1-\ell}{3} - \tfrac{1}{2} - \tfrac{1}{2} \,\middle|\, 3u_i, 3\tau\right]$$

u, τ...flat coordinates of open/closed moduli space (Kahler moduli in mirror A-model)

Open string BRST cohomology

Solving for the BRST cohomology yields explicit t,u-moduli dependent matrix valued maps, eg (a=1,2,3):



- marginal operators corr. to brane locations

$$\mathrm{Ext}^1(\mathcal{L}_A,\mathcal{L}_A):\quad \Omega_A = \partial_{u_A} Q(u_A)$$

- tachyon operators

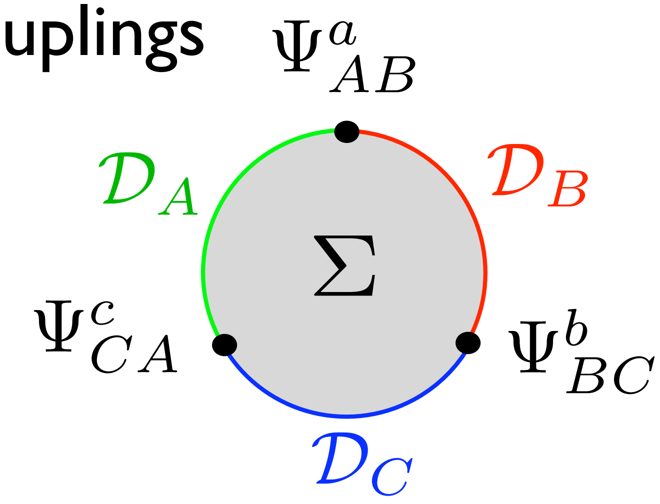
$$\mathrm{Ext}^1(\mathcal{L}_A,\mathcal{L}_B):\quad \Psi_{AB}^{(a)} = \begin{pmatrix} 0 & F_{AB}^{(a)} \\ G_{AB}^{(a)} & 0 \end{pmatrix}$$

with eg, $F_{12}^{(1)} = \begin{pmatrix} \zeta_1 & 0 & 0 \\ 0 & 0 & \zeta_2 \\ 0 & \zeta_3 & 0 \end{pmatrix}$ $G_{12}^{(1)} = \begin{pmatrix} \frac{\zeta_1}{\alpha_1^{(1)}\alpha_1^{(2)}}x_1 & \frac{\zeta_3}{\alpha_1^{(1)}\alpha_2^{(2)}}x_2 & \frac{\zeta_2}{\alpha_1^{(1)}\alpha_3^{(2)}}x_3 \\ \frac{\zeta_2}{\alpha_1^{(2)}\alpha_3^{(1)}}x_2 & \frac{\zeta_1}{\alpha_2^{(2)}\alpha_3^{(1)}}x_3 & \frac{\zeta_3}{\alpha_3^{(1)}\alpha_3^{(2)}}x_1 \\ \frac{\zeta_3}{\alpha_1^{(2)}\alpha_2^{(1)}}x_3 & \frac{\zeta_2}{\alpha_2^{(1)}\alpha_2^{(2)}}x_1 & \frac{\zeta_1}{\alpha_2^{(1)}\alpha_3^{(2)}}x_2 \end{pmatrix}$

and $\zeta_\ell \sim \Theta\left[\frac{1-\ell}{3} - \frac{1}{2} - \frac{1}{2} \,\middle|\, 3u_2 - 3u_1, 3\tau\right]$

Superpotential on brane intersection I

- Compute 3-point disk correlators = Yukawa couplings



$$\mathcal{W}_{eff} = T_a T_b T_c \, C_{abc}(\tau, u_i) + \ldots$$

$$C_{abc}(\tau, u_1, u_2, u_3) = \left\langle \Psi^a_{13}(u_1, u_3) \Psi^b_{32}(u_3, u_2) \Psi^c_{21}(u_2, u_1) \right\rangle$$

Use Kapustin-Li super-residue formula (from localization of path integral) for the matrix-valued, moduli-dependent operators:

$$= \frac{1}{2\pi i} \oint Str\Big[\big(\frac{dQ}{dW}\big)^{\otimes \wedge 3} \Psi^{(a)}_{13} \Psi^{(b)}_{32} \Psi^{(c)}_{21} \Big]$$

(A side remark)

- 90% of the actual work is not explained here

  The point is to determine proper "flat cohomology representatives" which includes their t-dependent normalization

- For the closed string, this is achieved by the Picard-Fuchs differential equations, which are based on the theory of Hodge variations ... the heart of mirror symmetry

  For open strings, a suitable non-commutative variant is not known

- Developed an approach based on contact terms that mimics Hodge theory for matrix valued operators, and leads to a matrix diffeq of the form:

$$\nabla_t \bar{\Psi}_a(t) \;=\; d\left(\frac{\phi\,\bar{\Psi}_a}{dW}\right)_+$$

$$\nabla_t \;\equiv\; \partial_t + U([\partial_t Q, *])$$

$$U \;\equiv\; \left(\frac{\{dQ,\, *\}}{dW}\right)_+$$

(propagator)

Superpotential on brane intersection II

- Solve differential eqn to determine t-dep of operators
- Insert in KL residue formula and make heavy use of theta-function identities such as the addition formula:

$$\theta_a[u_1] \cdot \theta_b[u_2] \;=\; \sum \theta_{a-b+c}[u_1 - u_2]\; \theta_{a+b+c}[u_1 + u_2]$$

(math: expresses product in Fukaya-category)

- Final result: theta functions

$$C_{111}(\tau,\xi) \;=\; e^{6\pi i \xi_1 \xi_2} q^{3{\xi_2}^2/2} \sum_m q^{3m^2/2} e^{6\pi i m \xi}$$

$$C_{123}(\tau,\xi) \;=\; e^{6\pi i \xi_1 \xi_2} q^{3{\xi_2}^2/2} \sum_m q^{3(m+1/3)^2/2} e^{6\pi i (m+1/3)\xi}$$

$$C_{132}(\tau,\xi) \;=\; e^{6\pi i \xi_1 \xi_2} q^{3{\xi_2}^2/2} \sum_m q^{3(m-1/3)^2/2} e^{6\pi i (m-1/3)\xi}$$

$$(\xi \equiv u_1 + u_2 + u_3 = \xi_1 + \tau \xi_2)$$

What's the interpretation of the q-series?

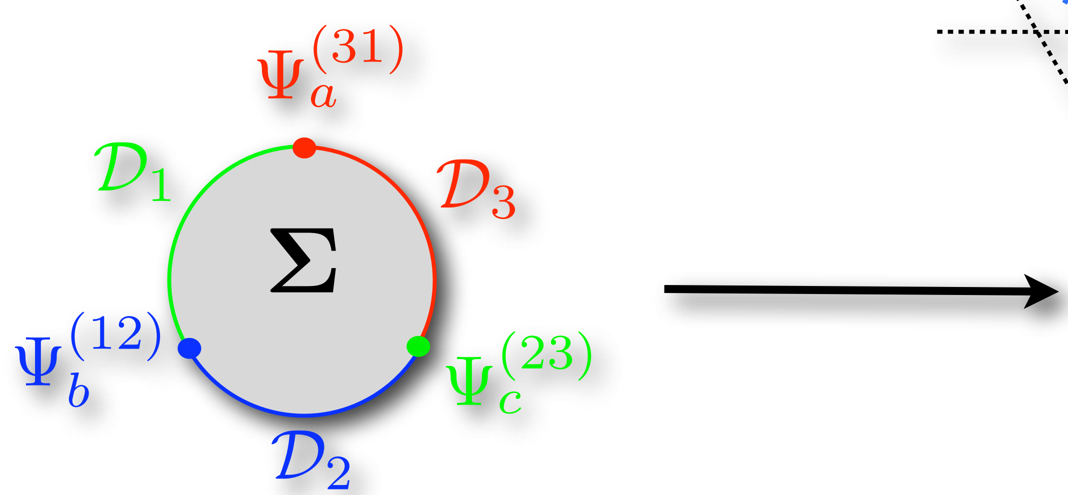
The topological A-Model: instantons

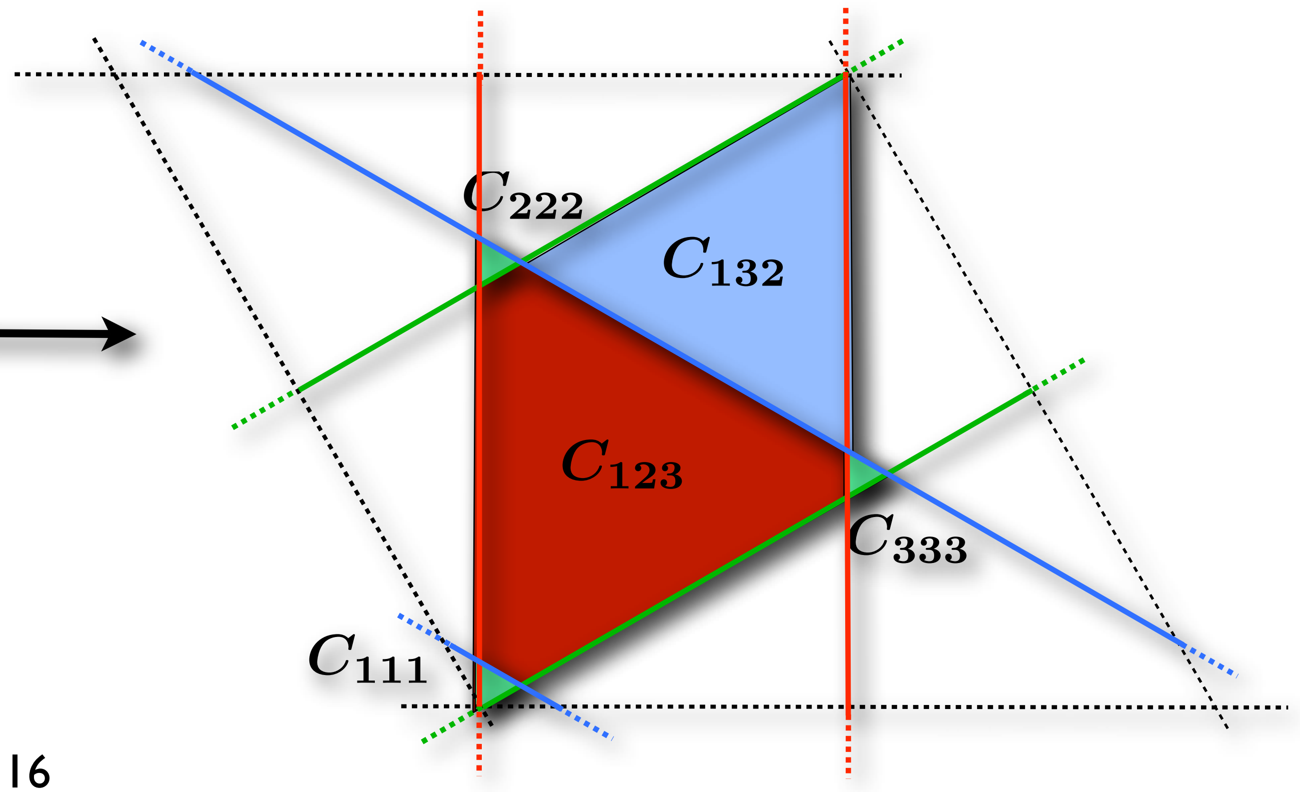
- Interpretation of q-series: In A model mirror language, these are contributions from triangular disk instantons whose world-sheets are bounded by the three D1-branes:

$$C_{abc} \sim e^{-S_{\rm inst}} \sim q^{\Delta_{abc}} + ...$$

(the u-dependence corresponds to position and Wilson line moduli)

Count maps: $\Sigma \to T_2$





Complete effective potential (long diag branes)

$$\mathcal{W}_{\text{eff}}(\tau, u, T) = \sum_{N=1}^{6} T^{(a_1)} \ldots T^{(a_N)} C^{(N)}_{a_1,\ldots,a_N}(\tau, u_1, \ldots, u_N)$$

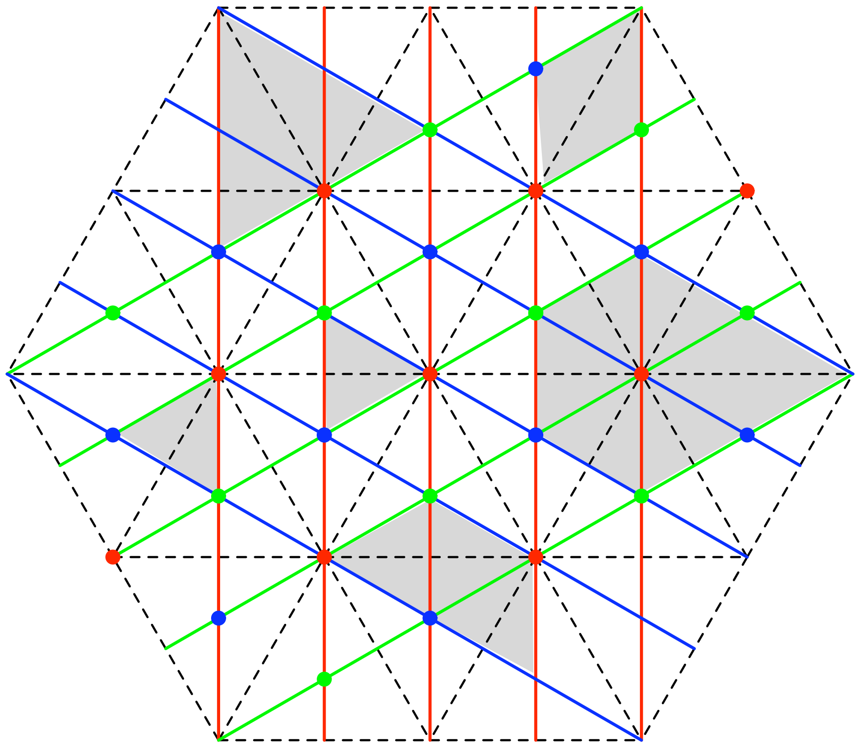
↑ tachyons ↑ moduli

- B-model: difficult to compute higher N-point Massey products with N>3!

  For (flat) elliptic curve, A-model is simpler....

- Generically, N-point functions get contributions from N-gonal instantons

  General structure:
  indefinite theta-functions summing over all lattice translates, positive areas



$$\sum_{m,n}{}' q^{mn} \equiv \left(\sum_{m,n>0} - \sum_{m,n<0} \right) q^{mn}$$

Polygons and instantons

N=4: trapezoids

$$\mathcal{T}_{ab\bar{c}\bar{d}}(\tau,u_i) \;=\; \delta^{(3)}_{a+b,\bar{c}+\bar{d}}\,\Theta_{trap}\begin{bmatrix}[b-\bar{c}]_3\\ [\bar{d}-\bar{c}+3/2]_3\end{bmatrix}(3\tau|3(u_1+u_2+u_4),3(u_1-u_3))$$

$$\Theta_{trap}\begin{bmatrix}a\\ b\end{bmatrix}(3\tau|3u,3v) \;=\; \sum_{m,n}{}' q^{\frac{1}{6}(a+3n)(a+3n+2(b+3m))}e^{2\pi i\left((a+3n)(u-1/6)+(b+3m)v\right)}$$

N=4: parallelograms

$$\mathcal{P}_{a\bar{b}c\bar{d}}(\tau,u_i) \;=\; \delta^{(3)}_{a+c,\bar{b}+\bar{d}}\,\Theta_{para}\begin{bmatrix}[c-\bar{b}]_3\\ [\bar{d}-c]_3\end{bmatrix}(3\tau|3(u_1-u_3),3(u_4-u_2))$$

$$\Theta_{para}\begin{bmatrix}a\\ b\end{bmatrix}(3\tau|3u,3v) \equiv \sum_{m,n}{}' q^{\frac{1}{3}(a+3n)(b+3m)}e^{2\pi i\left((b+3m)u+(a+3n)v\right)}$$

N=5: pentagons

$$\wp_{a\bar{b}\bar{c}\bar{d}\bar{e}}(\tau,u_i) = \delta^{(3)}_{a,\bar{b}+\bar{c}+\bar{d}+\bar{e}}\;\Theta_{penta}\begin{bmatrix}[-b-c-d]_3\\ [e+c+d]_3\\ [c-d+\frac{3}{2}]_3\end{bmatrix}(3\tau|3(u_5-u_2),3(u_1-u_4),3(u_3+u_2+u_4))$$

$$\Theta_{penta}\begin{bmatrix}a\\ b\\ c\end{bmatrix}(3\tau|3u,3v,3w) \equiv \sum_{m,n,k}{}' q^{\frac{1}{3}(a_>+3(n+k))(b_>+3(m+k))-\frac{1}{6}(c+3k)^2}e^{2\pi i\left((a_>+3(n+k))u+(b_>+3(m+k))v+(c+3k)(w-1/6)\right)}$$

N=6: hexagons

$$\mathcal{H}_{\bar{a}\bar{b}\bar{c}\bar{d}\bar{e}\bar{f}}(\tau,u_i) = \delta^{(3)}_{0,\bar{a}+\bar{b}+\bar{c}+\bar{d}+\bar{e}+\bar{f}}\Theta_{hexa}\begin{bmatrix}[-b-c-d]_3\\ [c+d+e]_3\\ [c-d+\frac{3}{2}]_3\\ [a-f+\frac{3}{2}]_3\end{bmatrix}(3\tau|3(u_5-u_2),3(u_1-u_4),3(u_3+u_2+u_4),3(-u_6-u_1-u_5))$$

$$\Theta_{hexa}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}(3\tau|3u,3v,3w,3z) \equiv \sum_{m,n,k,l}{}' q^{\frac{1}{3}(a+3n)(b+3m)-\frac{1}{6}(c+3k)^2-\frac{1}{6}(d+3l)^2}e^{2\pi i\left((a+3n)u+(b+3m)v+(c+3k)(w-1/6)+(d+3l)(z+1/6)\right)}$$

$$\sum_{m,n,k,l}{}' = \sum_{m,n\geq 0}^{\infty}\;\sum_{k\geq 0}^{<k_{max}}\;\sum_{l\geq 0}^{<l_{max}} - \sum_{m,n\leq -1}^{-\infty}\;\sum_{k\leq -1}^{>k_{min}}\;\sum_{l\leq -1}^{>l_{min}}$$

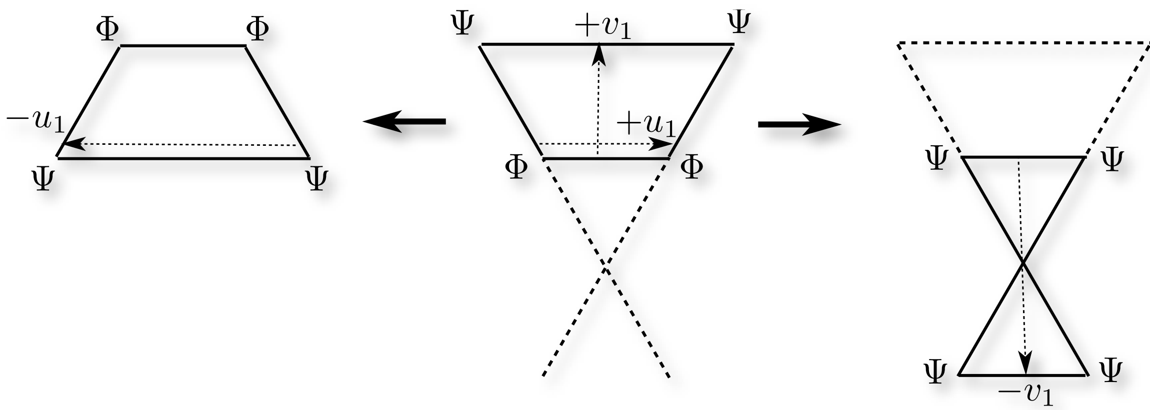
Global properties of open string moduli space

- Indefinite theta-fcts: singularities due to colliding branes

  eg., rewrite trapezoidal function in terms of Appel function:

$$\Theta_{trap}\left[\begin{array}{c} a \\ b \end{array}\right](3\tau|3u,3v) \;=\; e^{2\pi i v b}\sum_{n\in Z}\frac{q^{\frac{1}{6}(a+3n)(a+2b+3n)}e^{2\pi i(a+3n)(u-1/6)}}{1-q^{a+3n}e^{6\pi i v}}$$

- analytic continuation



Area becomes negative: resum instantons in terms of different geometry

“instanton flop”

Global properties of open string moduli space

- Indefinite theta-fcts: singularities due to colliding branes

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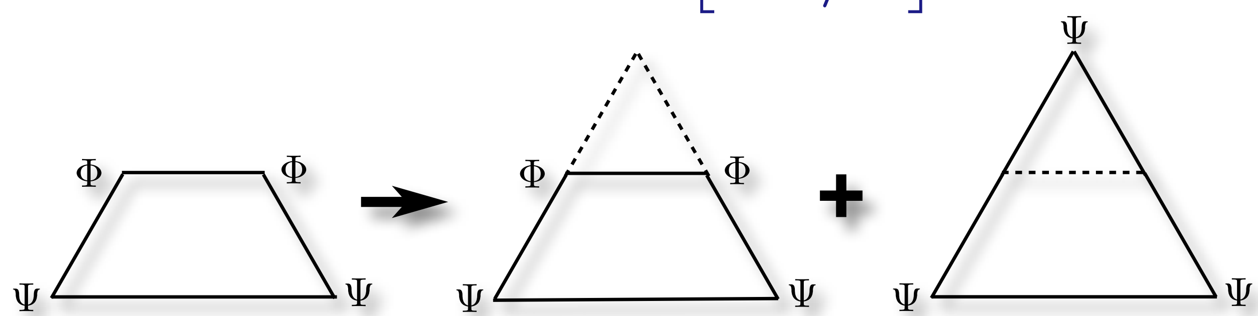
- monodromy

$$\Theta_{trap}\left[\begin{array}{c} a \\ b \end{array}\right](3\tau\,|\,3(u\pm\tau),\,3v) \;=\; e^{\mp 6\pi i v}\Theta_{trap}\left[\begin{array}{c} a \\ b \end{array}\right](3\tau\,|\,3u,\,3v)$$

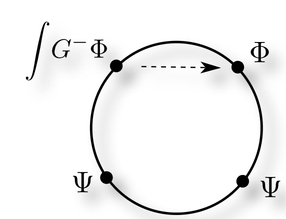
$$\mp\, e^{-2\pi i(u-\frac{1}{6})(b-\frac{3}{2}\pm\frac{3}{2})}e^{2\pi i v(b-\frac{3}{2}\mp\frac{3}{2})}q^{-\frac{1}{6}(b-\frac{3}{2}\pm\frac{3}{2})^2}\Theta\left[\begin{array}{c} a+b \\ -3/2 \end{array}\right](3\tau\,|\,3u)$$

induces "homotopy transformation",
modular anomaly of eff action
(compensate by non-lin field redef)



$$\mathcal{T}_{ab\bar{c}\bar{d}} \;\longrightarrow\; \mathcal{T}_{ab\bar{c}\bar{d}} + f_{\bar{c}\bar{d}}{}^{e}\Delta_{abe}$$

contact term



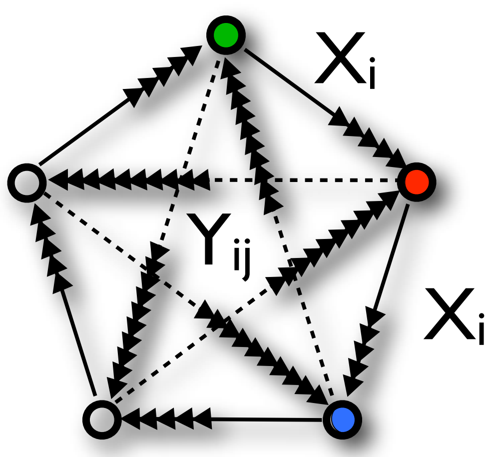
Summary and Outlook

- math: Cat of matrix factorizations ⟷ D(Coh(M))
  phyz: Boundary LG theory ⟷ Open string B-type top. CFT

- Represent all quantities in a quiver diagram (objects and maps) by explicit moduli-dependent, matrix-valued operators

- Combined with mirror symmetry this allows to explicitly compute instanton-corrected superpotentials (in particular, for intersecting brane configs). Requires solving a flatness DEQ.

- Generalization to M = CY 3-folds, eg. for quintic is cumbersome:



$$\mathcal{W}_{eff} = C_{XXY}(t)\,\mathrm{Tr}XXY + C_{XXYXXY}(t)\,\mathrm{Tr}(XXY)^2 + \ldots$$

t... interpolates between Gepner-point (BCFT) and large radius

... new results in enumerative geometry