Black Hole Complementarity from AdS_3/CFT_2

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Outline

Wavepackets

- Given an S-matrix, can build quasi-local observables by convoluting with on-shell wavepackets
- Example: 2d string theory
  - Polchinski (1994) studied the map between matrix observables and on-shell bulk tachyon observables
  - Found bulk causality rendered the continuous families of possible models inconsistent
  - (2003) 0A and 0B theories argued to be nonperturbatively consistent
- Will follow same strategy for AdS/CFT
  - convolute CFT correlators with on-shell wavepackets to build quasi-local bulk observables
  - Hamilton, Kabat, Lifschytz, Lowe
  - use AdS/CFT to study quantum gravity

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Problem

- At first sight, looks like observables are spread out over proper lengths $\sim R$
  - Bulk-to-boundary map, Gubser, Klebanov, Polyakov; Witten
  - $$\phi(p,r) \propto K(|p|r)e^{-ip\cdot x}\phi_0(p)$$
  - Natural infrared scale $= R$

Resolution

- Hamilton, Kabat, Lifschytz, D.L. : develop a coordinate space bulk-to-boundary map for Lorentzian AdS
  - can collapse the convolution to points spacelike separated from the bulk point (see also Rehren)
  - can re-express as a convolution over a disc by analytically continuing boundary coordinates
    - restrict to Δ =integer to avoid complications with branch cuts
  - size of disc simply determined by radial position of bulk observable
  - can impose UV and IR cutoffs and retain well-localized bulk observables

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Smearing functions-pure AdS_3

- Poincare coordinates

$$ds^2 = \frac{R^2}{Z^2}\left(-dT^2 + dX^2 + dZ^2\right) ,$$

- Express on-shell bulk field operator ϕ in terms of boundary conformal primary operator ϕ_0

$$\phi(T,X,Z) = \frac{\Delta - 1}{\pi}\int_{T'^2+Y'^2<Z^2} dT'dY' \left(\frac{Z^2 - T'^2 - Y'^2}{Z}\right)^{\Delta-2} \times \phi_0(T+T', X+iY')$$

$$\langle\phi_0(T,X)\phi_0(0,0)\rangle_{CFT} = \frac{1}{2\pi R}\frac{1}{(X^2-T^2)^{\Delta}}$$

Reproduces bulk correlators

- Using some nontrivial hypergeometric function integral identities, can reproduce the bulk Wightman function

$$G(x,x') = \frac{\sigma^{\Delta-2}}{4\pi R} \frac{\left(1-\sqrt{1-\sigma^{-2}}\right)^{\Delta-1}}{\sqrt{1-\sigma^{-2}}}.$$

- σ is the AdS invariant distance function, e.g.

$$\sigma(x,x') = \frac{-\delta T^2 + \delta X^2 + Z_1^2 + Z_2^2}{2 Z_1 Z_2}$$

- $i\varepsilon$ prescription needed: $T \to T - i\varepsilon$

Bulk causality

- Bulk causality has a simple geometric interpretation in terms of the intersections of discs on the boundary
- Landau singularity analysis to study $\langle[\phi(x),\phi(x')]\rangle$
- Nonvanishing pieces come from endpoint singularities of integrals, when $\varepsilon \longrightarrow 0$ pinches the contour of integration at the endpoint
- Upshot: nonzero commutator when edges of discs intersect
- Reproduces the condition of bulk timelike separations

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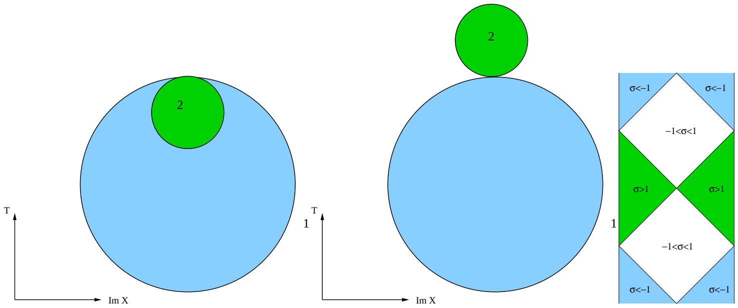
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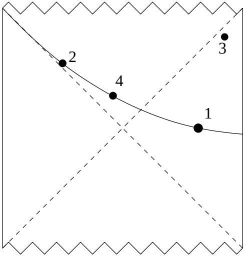
Smearing functions- BTZ

- Switching to Rindler coordinates allows us to easily generalize to BTZ

$$ds^2 = \frac{R^2}{r^2 - r_+^2} dr^2 - \frac{r^2 - r_+^2}{R^2} dt^2 + r^2 d\phi^2 ,$$

$$\sigma(t, r, \phi | t + x, r', \phi + iy) = \frac{rr'}{r_+^2} \left[\cos \frac{r_+ y}{R} \mp \left(\frac{r_+^2}{r^2} - 1 \right)^{1/2} \sinh \frac{r_+ x}{R^2} \right] ,$$

- Pure AdS, $\phi \in \mathbb{R}$, for BTZ $\phi \sim \phi + 2\pi$ and r_+ is the position of the horizon



- For points in the right wedge, mapping same as for pure AdS

$$\phi(t,r,\phi) = \frac{(\Delta-1)2^{\Delta-2}}{\pi R^3} \int_{disc} dx\, dy \lim_{r'\to\infty} \left(\sigma/r'\right)^{\Delta-2} \phi_0^R(t+x, \phi+iy)$$

- For points inside the horizon, get contributions from left and right boundaries

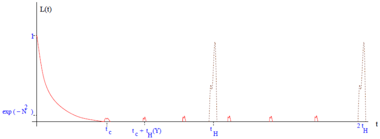
$$\begin{aligned}\phi(t,r,\phi) &= \frac{(\Delta-1)2^{\Delta-2}}{\pi R^3}\left[\int_{\sigma>0} dx\,dy \lim_{r'\to\infty}(\sigma/r')^{\Delta-2}\right. \\ &\times \phi_0^R(t+x,\phi+iy) \\ &+ \int_{\sigma<0} dx\,dy \lim_{r'\to\infty}(-\sigma/r')^{\Delta-2}(-1)^{\Delta} \\ &\times \phi_0^L(t+x,\phi+iy)\end{aligned}$$

- Can eliminate left contribution all together, using a complex time integral, as in real-time thermal field theory

$$\phi_0^L(t,\phi) = \phi_0^R(t+i\pi R^2/r_+,\phi)$$

Finite N Catastrophe

- We would have hoped that finite N effects would give some small correction to these results



- Boundary correlator initially shows thermal decay $e^{-T_H t}$, but starts to oscillate for $t > t_c$ (Barbon, Rabinovici; Kleban, Porrati, Rabadan)

$$e^{-\Gamma t_c} \sim e^{-S_{bh}} \Rightarrow t_c = \frac{2\pi^2 cR}{3}$$

Finite N Catastrophe

- If we use the exact boundary correlator with the above bulk-boundary map, correlators for points inside the BTZ horizon will diverge!

Cutoff Observables for finite N

- Proceed as follows:
  - impose IR cutoff for $t > t_c$ (actually on radius of disc that appears in bulk-boundary map)
  - will regard this as a success if it reproduces semiclassical correlators to reasonable accuracy when expected
  - can then use the difference between cutoff and semiclassical correlators as an estimate of finite N effects
- Note quasi-local bulk observables in quantum gravity do not have arbitrary accuracy
- Best you can do is try to optimize: trade off number of observables versus accuracy

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Reconstructing effective field theory in the bulk

- Bulk correlators will receive corrections perturbative in $1/N$ and nonperturbative in $1/N$
- Expect the perturbative corrections to be incorporated into renormalization of local terms in bulk effective action (c.f. Aharony and Komargodski; Giddings et al.; Penedones, Polchinski et al.)
- In general, terms nonperturbative in $1/N$ are not expected to look local around e.g. the BTZ metric
  - could try to interpret as local around some other saddle point
  - however we will just declare the effective action around the fixed BTZ metric invalid if these terms become important
  - quantity most sensitive to these terms is the commutator of bulk fields
  - a sure sign that a local effective action has broken down is that commutators become substantial at spacelike separations

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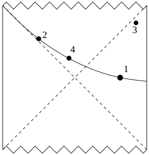
Breakdown of semiclassical approximation



- For points like 1 and 3, IR cutoff irrelevant for

$$r > r_c = r_+ + 2r_+ e^{-4\pi^2 c r_+/3R}$$

or proper distance from the horizon

$$ds = Re^{-2\pi c r_+/3R}$$

- For large central charge c, this is smaller than the Planck scale. With interactions, expect minimum distance from the horizon that can be probed is ℓ_{Planck}.

Black hole complementarity

- Therefore see bulk causality for $r > r_c$ even at finite N
  - determined by analytic structure of boundary correlator + cutoff bulk-boundary map
- For $r_{+<} r < r_c$ see a surface that behaves as a stretched horizon
- For a point inside $r < r_c$ (or inside the horizon) see commutator nonvanishing at bulk spacelike separations, provided $\delta t \gtrsim t_c$

$$\langle [\phi(1), \phi(3)] \rangle \sim e^{-S_{bh}}$$

- Of same order of magnitude as effects due to topology change from Euclidean path integral approach (Maldacena)

- Unitarity implies that these $e^{-S_{bh}}$ effects can add coherently when we pick particular correlators with of order $e^{S_{bh}}$ operators
- Local effective field theory will have order 1 errors when we compute such correlators
- Conclude local effective field theory works well around a local patch up to errors of order $e^{-S_{bh}}$ for small numbers of local operators
- Cutoff bulk operators give precise predictions for when effective field theory is valid
  - local general covariance is respected, away from regions of large curvature
  - breakdown in the infrared (for large t separations)
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Small black holes

- Problem: low temperature thermal ensemble not dominated by small black holes (Birmingham, Sachs, Solodukhin)
- Suppose we can find a way to turn on potentials to modify the ensemble to fix this
- Assume boundary correlator exhibits a thermal decay similar to above

$$\langle \phi_0(t)\phi_0(0)\rangle \sim e^{-T_H t}$$

- The same timescale t_c appears

$$t_c \sim S_{bh}/T_H \sim 1/T_H^3$$

in 4d

- Coincides with information retention time (Page; Susskind).
  - Works for general dimensions (Lowe, Thorlacius)

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- If the bulk to boundary map can be generalized to a slowly changing geometry, expect the same results to go through for commutators of operators
- Expect the same conclusions for effective field theory to carry over
- Matches expectations from information theory constraints on effective field theory (Lowe, Thorlacius)

Summary

- Found geometric picture of bulk causality entirely from CFT viewpoint
- Defined a family of quasi-local bulk operators at finite N
- Effective action that emerges matches expectations from black hole complementarity

- Outlook
  - General Δ?
  - Include perturbative interactions? Generalize to 3pt and 4pt functions?

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