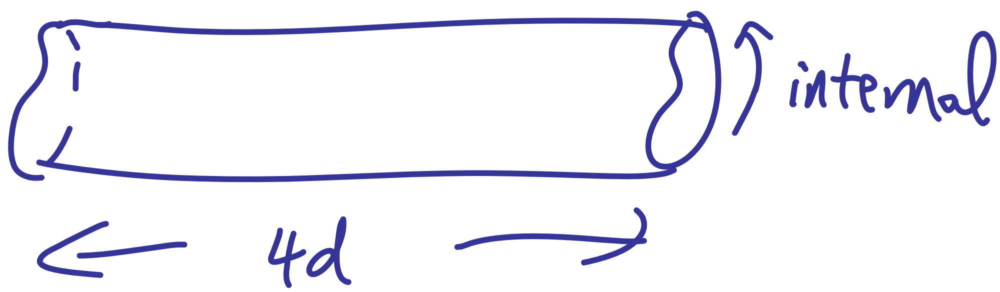
Toward 4d quantum gravity in string theory

Work in Progress with Joe Polchinski

We'd like a non-perturbative formulation of 4d physics:



with a hierarchy of energy scales

$$m_{\left\{\substack{KK \\ \text{string} \\ D}\right.} \gg \frac{1}{L_{4d}}$$

AdS/CFT (and previously, BFSS matrix theory) formulate some backgrounds non-perturbatively, but did not (yet) get down to 4d:

BFSS: 11d $\leftrightarrow$ N D0-brane Q.M.

4d (max susy) $\leftrightarrow$ $\underbrace{\text{D7-branes}}$ on T^7

↳ codim 2 → log potential → $c_{N\leq 24}$

AdS/CFT: ① $\underbrace{AdS_2 \times S^2}_{\text{want } L_{AdS}\to\infty}$ $\times$ $\underbrace{CY}_{\text{small ✓}}$

↳ IR divergences in AdS_2

AdS/CFT ②

$$AdS_4 \times \underbrace{\begin{cases} S^7 & (M) \\ S^7/\mathbb{Z}_k & \\ \mathbb{CP}^3 & (\mathrm{IIA}) \end{cases}}_{L_{internal} \sim L_{AdS}}$$

No hierarchy of scales in Freund-Rubin compactifications.

Basic reason: In 11/10 d Einstein equations

$$\underbrace{R_{MN} - \frac{1}{2} R G_{MN}}_{\text{Internal + 4d}} = 8\pi G \underbrace{T_{MN}}_{\text{flux}}$$

all three contributions are of the same order in the solution

For future reference, let us reproduce this in the language of the 4d effective potential energy:

$$S = \int \frac{d^{10}x\sqrt{G}}{\alpha'^4}\left(\frac{\mathcal{R}}{g_s^2} + F_p^2 + \cdots\right)$$

$\rightarrow$ 4d potential energy

$$U_4 = \frac{1}{\alpha'^2}\left\{-\int \frac{d^6x\sqrt{G_6}\,\mathcal{R}^{(6)}}{\alpha'^2 g_s^2} + \int \frac{d^6x\sqrt{G_6}}{\alpha'^2} F_p^2 + \cdots\right\}$$

$$\sim \frac{R^6}{\alpha'^2 g_s^2}\left(-\frac{1}{R^2} + \frac{g_s^2 Q^2}{R^{2p}} + \cdots\right)$$

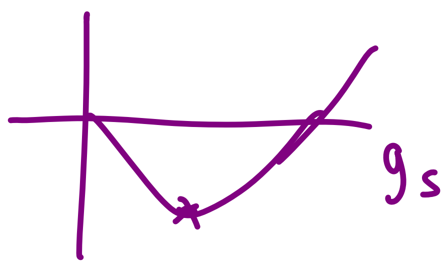
where $R \equiv$ size in string units.

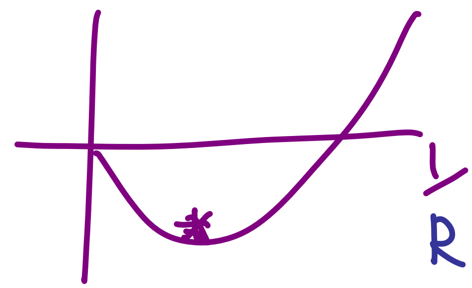
$$U_4 \sim M_p^4\left(\frac{g_s^2}{R^6}\right)\left(-\underbrace{\frac{1}{R^2}}_{U_{\mathcal{R}}} + \underbrace{g_s^2\frac{Q^2}{R^{2p}}}_{U_{F_p}} + \cdots\right)$$

in Einstein frame

e.g. IIA on $\mathbb{CP}^3$ w/ F_6 & F_2:

$$\frac{U_4}{M_p^4} \sim -\frac{g_s^2}{R^8} + \frac{g_s^4 Q_2^2}{R^{10}} + \frac{g_s^4 Q_6^2}{R^{18}}$$





Equivalently

$$U_4 \sim M_p^2\Big(\underbrace{-\frac{1}{R^2\alpha'} + \frac{g_s^2 Q^2}{R^{2p}\alpha'}}_{\text{“}\Lambda \sim \frac{1}{R_{AdS}^2\alpha'}} + \cdots\Big)$$

i.e. $\Lambda_{min} \sim \Lambda_{gR}$ in Freund-Rubin

- n.b. CFT_3 dual to IIA/$\mathbb{CP}^3$ is
  or M thy / S^7/Γ
  - IR limit of D2, D6, KK
  - strongly coupled CS

Schwarz, BL, ABJM, et seq.

In fact there is a large class of 3d CFTs obtained via RG flow from gauge theory with flavors

Appelquist/Heinz HET
Sachdev.. CMT

$$\frac{1}{g^2} = \frac{1}{g_0^2} + \frac{\Delta N}{E} + \cdots$$

classical 1-loop screening

(In 3d $N=4$, $\Delta N = N_f - 2N_c$ and this is exact)

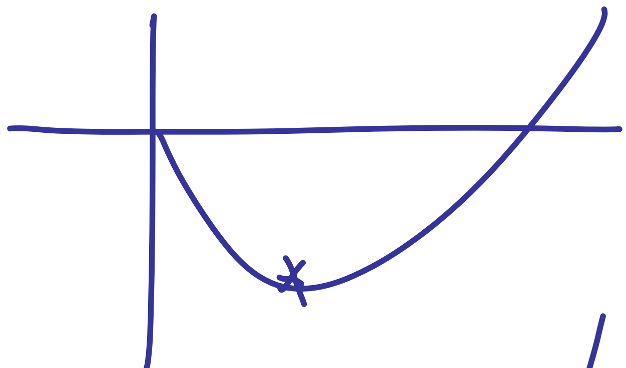
→ dimensionless coupling has fixed point

$$\frac{1}{\bar{g}^2} = \left.\frac{E}{g^2}\right|_{E\to 0} = \Delta N + \cdots$$

controlled at large N_f independently of SUSY.

e.g. D2-D6, D2-D6-orbifold ; Hanany-Witten ... examples : still Freund-Rubin (or stringy)

Similarly, compactification on S^5 w/ F_5 $\rightarrow$ $AdS_5 \times S^5$



fixed line

$\Updownarrow$

$N=4$ SYM

$L_{AdS} \sim L_{S^5}$

In general, the negative term(s) in the potential are key

e.g. $\left\{ \begin{array}{l} \text{positive curvature} \\ \text{O-planes} \\ -3|W|^2 \end{array} \right.$

Coming from the other direction, we can construct

$$(A)dS_4 \times X_{small}$$

in an apparently large number of ways B^2S DRS BP MSS GKP+KKLT ...

suggesting a rich set of dual CFT_3 s.

- Not a priori realized as near-horizon limit of brane system
- Can read off interesting properties:

$$N_{d.o.f.} \sim L^2_{(A)dS} M_p^2 \leq N^b$$

b ← betti #

N ← flux #

ES AAB

Plan: Start from known, Freund-Rubin dual pair:

$$\underbrace{AdS \times S}_{} \leftrightarrow QFT$$

(brane construction)

add ingredients which cancel or nearly cancel U_R $\leftrightarrow$ additional field content, couplings of QFT

$$\overbrace{\text{stabilize the moduli} \to AdS_4}^{} \leftrightarrow \overbrace{CFT_3}^{}$$

7-branes contribute to $\mathcal{U}$ naively as

$$\mathcal{U}_7 \sim M_p^4 \left(\frac{g_s^2}{R^6}\right)^2 \cdot \left(\underbrace{\tau_7}_{\text{tension in string units}} \cdot R^4 \right)$$

compare to curvature energy

$$\mathcal{U}_{\mathcal{R}} \sim M_p^4 \left(\frac{g_s^2}{R^6}\right)^2 \cdot R^6 \cdot \left(-\frac{1}{R^2 g_s^2} \right)$$

$\Rightarrow$ for $\tau_7 \sim \frac{1}{g_s^2}$, i.e. (p,q) 7Bs,

they compete.

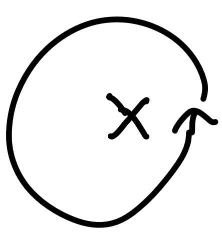
cf Aharony Fayazzuddin Maldacena

Of course 7Bs, being codimension 2, have large IR back reaction...

The interplay between curvature & 7-brane energy is accurately captured using the techniques of F-theory: Vafa '96

$$\begin{array}{ccc} T^2 & \to & X \\ & & \downarrow \\ & & B \end{array}$$

$$\tau_{T^2} = C_0 + \frac{i}{g_s} \quad \text{in IIB}$$



D7-brane: $\tau(u) \sim \frac{1}{2\pi i} \log u$

$\tau \to \tau + 1$ monodromy

(p,q) 7-brane $\tau \to \dfrac{(1+pq)\tau - p^2}{q^2\tau + (1-pq)}$

Plan: Start from known, Freund-Rubin dual pair:

$AdS \times S \leftrightarrow$ QFT (brane construction)

add 7-branes which cancel or nearly cancel $U_{\mathcal{R}}$ $\leftrightarrow$ additional flavors* couplings of QFT

stabilize the moduli $\to AdS_4$ $\leftrightarrow$ CFT_3

* in general, both electric & magnetic cf Douglas/Shenker, Argyres/Douglas, Argyres-Plesser-Seiberg-Witten

D3, D7 and Electric/Magnetic Matter

- 4d $\mathcal{N}=2$ $SU(2)$ SYM w/ N_f hypermultiplets

$[u$ (Coulomb branch)

Seiberg-Witten solution

monopole $\otimes$ dyon $\otimes$

$\otimes \leftarrow$ quark

AD/APSW : can <u>change</u> mass matrix M such that mutually nonlocal matter is light.

monopole $\otimes$ dyon + quark $\otimes$

- In brane constructions (Sen, Banks Douglas Seiberg, ...)

  $u \leftrightarrow$ D3 position

  $\otimes \leftrightarrow$ 7B position

The T^2 varying over B
can be described as

Vafa
Morrison-Vafa
Kachru Intriligator
Morrison Vafa
...

$$y^2 - x^3 - x f(u) z^4 - g(u) z^6 = 0$$

coordinates on B

i.e. as a degree 6 hypersurface
in $WP^2(2,3,1)$.

For a Kähler base B, one can
formulate the T^2 fibration $T^2 \to X \downarrow B$
as a hypersurface in $B \times WP^2(2,3,1)$,
and as the target space of a
(2,2) gauged linear σ-model

Witten

7-branes live at the locus

$$\Delta = 27 g^2 + 4 f^3 = 0$$

Let us start with IIB on

$$Y_5 \times S^1 \quad \text{with} \quad \int_Y F_5 = N_c$$

where Y_5 is an S^1 (Hopf) fibration over a Kähler base

(7-branes will be extended along fiber)

$$\begin{array}{ccc} S^1_f & \rightarrow & Y \\ & & \downarrow \\ & & B \end{array}$$

First study 7-branes on B

Examples: $Y = S^5$, $B = \mathbb{CP}^2$ or $\mathbb{WP}^2$

Topologically $S^2 \times S^3$

$$\left\{ \begin{array}{ll} Y = T^{1,1} & B = \mathbb{CP}^1 \times \mathbb{CP}^1 \\ Y = Y^{pq} & B = \ldots \\ Y = L^{abc} & B = \ldots \end{array} \right.$$

Klebanov/Tseytlin/Strassler

Gauntlett Martelli Sparks Waldram

Cvetic Lu Pope

$$\begin{array}{ccc} S^1_f & \to & Y \\ & & \downarrow \\ & & B \end{array}$$

size R_f

size R

$\cong$ B with "metric flux" $F_{met} = J_B$

(useful description for small fiber)

Two classes of candidate examples: $Y \times S^1_\perp \times (4d)$

① 7-branes nearly cancel $U_R^{(B)}$

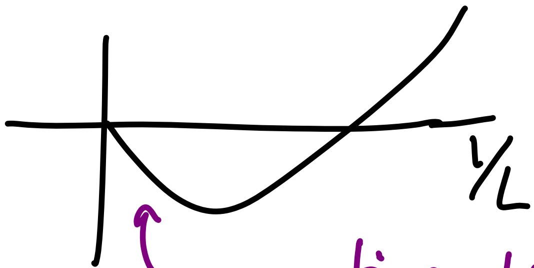
or $\frac{Q_3^2}{R_\perp^2 R^4}$

$$U\Big|_{g_s \sim 1} \sim M_p^4 \left(\frac{1}{R^4 R_6 R_f}\right)\left(\frac{R_f^2}{R^4} - \frac{\varepsilon}{R^2} + \frac{N_c^2}{R^8 R_f^2} + \frac{Q_1^2}{R_\perp^2}\right)$$

enforce with e.g. E_n 7-branes

↳ stable minimum with $R_f \ll R \ll R_{AdS}$

② $U_R + U_7 = 0$ (F-theory on CY)



negative term from O-planes

$\rightarrow$ again $R_{AdS} \gg R \gg R_f$

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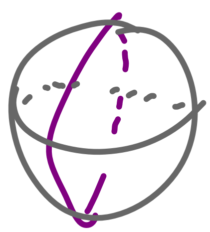
* Are 7-brane moduli tachyonic?

On S^5



allowed tachyon for $R \sim R_{AdS}$
but what about $R \ll R_{AdS}$?

Note that 7-brane moduli

$$y^2 - x^3 - x\,f(u)\,z^4 - g(u)\,z^6 = 0$$

are flat directions in the CY
and nearly flat for $U_R \propto \frac{\varepsilon}{R^2}$

To be specific, consider

$$Y = S^5 \quad \text{(topologically)}$$

Start from the $\mathbb{CP}^2$ model:

(2,2) chiral multiplets u_1, u_2, u_3

$U(1)$ Gauge symmetry

$$(u_1, u_2, u_3) \cong e^{2\pi i \alpha}(u_1, u_2, u_3)$$

$$\Rightarrow D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 - R^2\right)^2$$

$\hookrightarrow D = 0$ alone gives S^5

$\rightarrow \alpha$ parameterizes S^1_{fiber}

$$S^1_f \to S^5 \downarrow \mathbb{CP}^2$$

$$ds^2_{S^5} = d\sigma^2_{\mathbb{CP}^2} + R_f^2 (d\alpha + A)^2$$

$$dA = J$$

Gibbons, Pope

To add the 7-branes, want a T^2 fibration over $B = \mathbb{CP}^2$

Gauged Linear σ-model becomes

		u_1	u_2	u_3	x	y	z	P
T^2	$U(1)$	0	0	0	2	3	1	-6
$\times$ $\mathbb{CP}^2$	$U(1)$	1	1	1	q_x	$\frac{3}{2}q_x$	0	$-3q_x$

$$S_W = \int d^2\sigma\, d^2\theta\; P\left(y^2 - x^3 - f(u)\,x z^4 - g(u)\,z^6\right)$$

Now, $\sum_{\text{fields } I} q_I = 0$ is the Calabi-Yau condition (ensuring anomaly-free $U(1)\times U(1)$ R-symmetries appropriate to (2,2) SCFT) Witten

The running of R^2 in

$$D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 + q_x |x|^2 + \frac{3}{2} q_x |y|^2 - |p|^2 - R^2\right)^2$$

is $\mu \dfrac{\partial R^2}{\partial \mu} \sim \sum_I q_I$

Now $\sum q_I = 3 - \frac{1}{2} q_x$

and the degree of $G = y^2 - x^3 - f x z^4 - g z^6$

is $\deg_G = \deg_g = 3 q_x = 18 - 6 \sum q_I$

Fully canceling curvature energy

means $\sum q_I = 0 \Rightarrow \deg_g = 18$

$\Rightarrow \deg \Delta = 36 \Rightarrow$ 36 7-branes

(This agrees with naive result from $U_7 \sim \cdots (\Upsilon_7 \times \mathrm{vol})$ ✓)

- The 7Bs are extended along $U(1)$ fiber
- Similarly, on $\mathbb{CP}^1 \times \mathbb{CP}^1$, 48 7Bs cancels $\mathcal{U}_R$.

To stabilize, need negative term(s) in the potential.

→ introduce orientifold (mutually SUSY)

Altogether, take
$$\begin{array}{ccc} S^1_f \to S^5 & \times & S^1_\perp \\ \downarrow & & \\ \mathbb{CP}^2 & & \end{array}$$

with also worldvolume flux q on $S^1_f \times S^1_\perp$

(+ branes which compensate tadpole ...)

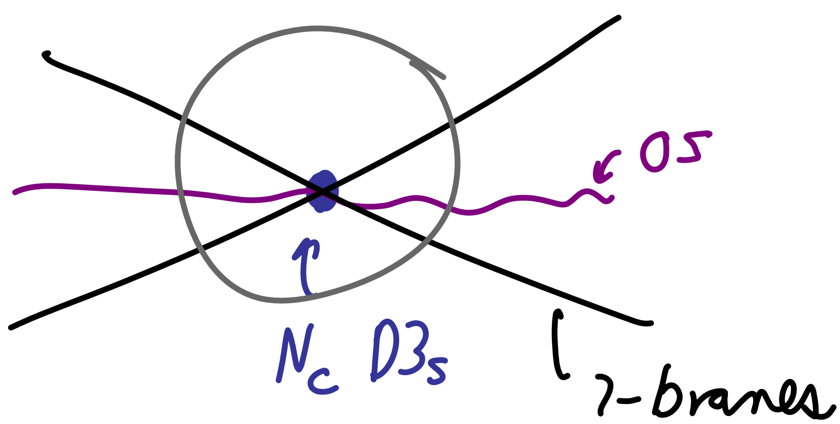
$$\mathcal{U} \sim \frac{M_p^4}{R_f R_\perp R^4}\left(\frac{R_f^2}{R^4} - \frac{1}{R_f R_\perp R^2} + \frac{N_c^2}{R^8 R_f^2} + \frac{q^2}{R_\perp^2 R_f^2 R^2}\right)$$

→ minimum, with hierarchy

$$R \sim (g^2 N_c)^{1/4} \qquad R_f \sim \frac{R}{g}$$

$$R_\perp \sim \frac{g^3}{R} \qquad R_{AdS} \sim g R$$

QFT side?



A complicated generalization of the D3 + D7 brane constructions for Seiberg/Witten, Argyres/Douglas, A Plessen, SW

Now generalize to cases where we do <u>not</u> fully cancel the curvature energy. Consider

T^2 fibration over $B = \underline{\underline{W}}\mathbb{P}^2$

Gauged Linear σ-model

		u_1	u_2	u_3	x	y	z	P
T^2	$U(1)$	0	0	0	2	3	1	-6
	$\times$							
$\underline{\underline{W}}\mathbb{P}^2$	$U(1)$	$\underline{\underline{w}}_1$	$\underline{\underline{w}}_2$	$\underline{\underline{w}}_3$	q_x	$\frac{3}{2}q_x$	0	$-3q_x$

$$S_W = \int d^2\sigma\, d^2\theta\; P\left(y^2 - x^3 - f(u)\,x z^4 - g(u)\,z^6\right)$$

Again $\beta_{R^2,\text{full}} \sim \sum q = \sum w - \frac{1}{2} q_x$ in the full system including the 7Bs.

Again $\beta_{R^2,\text{full}} \sim \sum q = \sum w - \frac{1}{2} q_x$ in the full system including the 7Bs.

For $\mathbb{WP}^2$ alone,

$$\beta_{R^2, \mathbb{WP}^2} = \sum w$$

$\Rightarrow$ If $\sum w - \frac{1}{2} q_x << \sum w$

then we almost cancel the curvature.

$$\rightarrow \mathcal{U}_{R,\text{full}} \sim M_p^4 \left(\frac{g_s^2}{\text{Vol}}\right)^2 \cdot \text{Vol.} \left(-\frac{\varepsilon}{R^2}\right)$$

with $\boxed{\varepsilon \sim \frac{\sum w - \frac{1}{2} q_x}{\sum w}}$

(using the NLSM result $\beta \sim R_{MN}$)

* What about the singularities of $WP^2(w_1, w_2, w_3)$?

$$(u_1, u_2, u_3) \cong (\lambda^{w_1} u_1, \lambda^{w_2} u_2, \lambda^{w_3} u_3)$$

$\rightarrow$ for $\lambda = e^{\frac{2\pi i}{w_1}} \in U(1)$

$(u_1, 0, 0)$ is a (generally non-SUSY) $\mathbb{C}^2/\mathbb{Z}_{w_3}$ orbifold fixed point.

By itself*, this has twisted tachyons which condense, smoothing it out

Adams
Polchinski
ES ...
.. Morrison ..

* Don't Panic! • Must include 7-branes

can restore SUSY

• and S^1_{fiber}: Removes $U(1)$ projection entirely

From WP^2 we get candidate examples of ...

① 7-branes nearly cancel $U_R^{(B)}$

$$U\Big|_{g_s \sim 1} \sim M_p^4 \left(\frac{1}{R^4 R_6 R_f}\right)\left(\frac{R_f^2}{R^4} - \frac{\varepsilon}{R^2} + \frac{N_c^2}{R^8 R_f^2} + \frac{Q_1^2}{R_\perp^2}\right)$$

enforce with e.g. E_n 7-branes

↳ stable minimum with $R_f \ll R \ll R_{AdS} \sim R_\perp^* / Q_1$

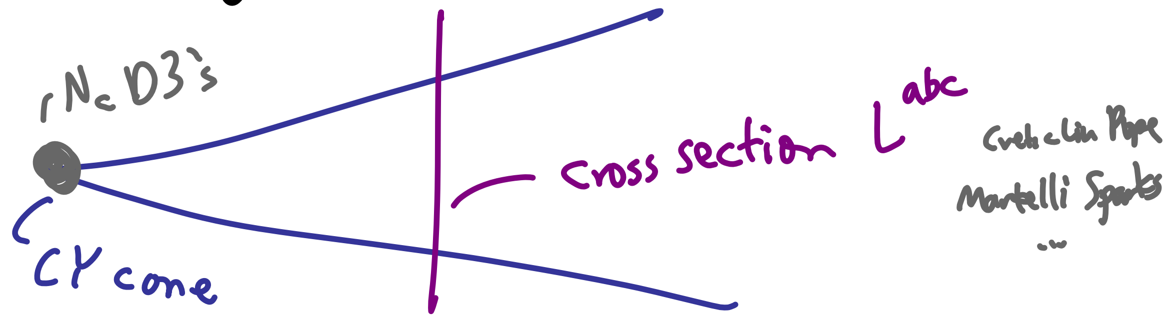
... as long as all other directions are stable (at most allowed tachyons)

*OK maybe F is for Fire ...

... but if replace $\frac{Q_1^2}{R_\perp^2}$ with $\frac{Q_3^2}{R_\perp^2 R^4}$, full hierarchy

→ $S^2 \times S^3$ examples

More general candidate examples:



- Start from a Calabi-Yau cone, given by GLSM with charges (Σ charges $=0$)

$$(c,\ a+b-c,\ -a,\ -b)$$

- Mod out by an additional (2,2) $U(1)$ to pick out 4d base in $S^1_f \to L$, $L \to B$

- Add 7-branes as before, imposing

$$\beta_{R^2_B,\ \text{with 7Bs}} \ll \beta_{R^2_B\ \text{alone}}$$

- Undo a (1,1) $U(1)$ to get S^1_{fiber}

- On the full cone, this gives our brane construction for the QFT

• On Y^{pq}

$S^1_f \to Y^{pq}$ (pq units of metric flux)

$\downarrow$

$B_4 \cong S^2 \times S^2$

$$ds^2 = ds^2_{B_4} + R_f^2 (d\chi + A)^2 \qquad dA \sim J_4$$

The naive 7-brane potential energy for 7-branes wrapped on SUSY cycles

$$\Sigma_{1,2} \sim S^3/\mathbb{Z}_p \qquad n_3 < n_4$$

$$\Sigma_{\{3,4\}} \sim S^3/\mathbb{Z}_{p\pm q} \qquad n_1 + n_2 + n_3 + n_4 = 40$$

$\rightarrow \varepsilon \sim \frac{q}{p}$ which $\ll 1$ for $p \gg q$

* still need to check against F-theory on B4

The string coupling

7-branes corresponding to mutually non-local flavors have $g_s \sim 1$

$\otimes$ e.g. $\tau \sim e^{\frac{i\pi}{3}}$ $f(u)=0$ branch

Dasgupta/Mukhi

In a (near-) SUSY background, the (approximate-) moduli are

- R^2 (GLSM D-term)
- polynomial coefficients in f, g

(other modes of the metric + dilaton have KK-scale masses $\sim \frac{1}{R\sqrt{\alpha'}}$.)

In the LSM, f + g are superpotential couplings $\Rightarrow$ don't run even at $\mathcal{O}(\varepsilon)$

$\Rightarrow$ expect $|m^2| \leq \mathcal{O}(\varepsilon^2)$

Sen Limit

In F theory, $\exists$ limit (Sen)

$$f = -3h^2 + \epsilon\eta \qquad \varepsilon \to 0$$
$$g = -2h^3 + \varepsilon h\eta - \varepsilon^2 \frac{\chi}{12}$$

for which $g_s \to 0$. i.e. all the (p,q) 7-branes boil down to O7-planes + D7-branes

Such examples, if they can also be stabilized (including g_s), would be purely electric on the QFT side.

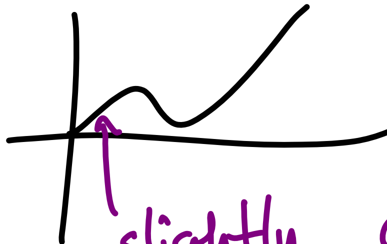
Remaining issues

- ~~SUSY~~ scale $\left(\ll \frac{1}{R}\right)$; (non-) perturbative stability in all directions
  Horowitz-Orgera-Polchinski instantons
- F-theory vs Naive Potential energy in general (agree for $\mathbb{CP}^1 \times \mathbb{CP}^1$, $\mathbb{CP}^2$)
- Analysis of QFT side/brane construction
- Sen limit examples

...

Future directions

- dS_4?



slightly over-cancel curvature energy

Now no tachyons are allowed.

- Giddings-Kachru-Polchinski + KKLT use 7-branes & F-theory on a positively curved base. Interpret as in above examples?
- Other applications (e.g. AdS/CMT?)

. . .