# Freedom and constraints in the landscape of magnetized/intersecting brane models

KITP workshop:
Fundamental Aspects of Superstring Theories

March 25, 2009

Washington (Wati) Taylor, MIT

| | | |
|---|---|---|
| | arXiv:0903.0386, | w/ V. Kumar |
| Based on: | hep-th/0606109, | w/ M. R. Douglas |
| | arXiv:090?.?, | w/ V. Rosenhaus |

# Outline

# 1. Philosophy

Can string theory make quantitative predictions for 4D low-E physics?

- A priori, unlikely q. gravity at $10^{19}$ GeV affects physics at 1 TeV
- Long hoped: low-energy physics computable from string theory (e.g. $e^-$ mass)
- Landscape: threatens predictability below Planck scale
- Issues:
  - Can't define theory completely
  - Probably understand only set of measure 0 of vacua ($G_2$, non-Kaehler, non-geometric,. . . )
  - Difficult to compute in known vacua (no simple world-sheet description w/R-R fields, etc.)
- Probabilities on landscape: difficult to interpret w/o understanding time-dependence & cosmology in ST

So how can we hope for quantitative predictions?

## Searching for stringy constraints

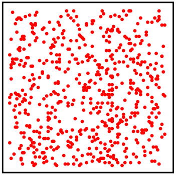
Given a large set $\mathcal{V}$ of vacua
(e.g. II flux vacua on CY, IBM on toroidal orientifolds, . . . )

And observables $\mathcal{O}$
(e.g. gauge group, matter content, couplings, . . . )

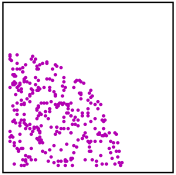
How are observables distributed?

Extreme cases

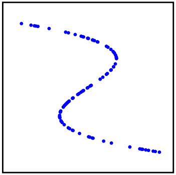
A: "Anything goes" (perhaps B: "Bounded"), C: "Constrained"



A) Anything goes



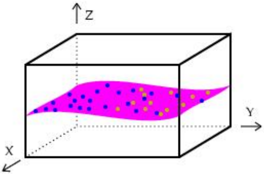
B) Bounded



C) Constrained

If there are constraints on low-energy physics

These constraints should be visible in every corner of the landscape




Common constraint on $\mathcal{O}$ in disparate sets of vacua $\mathcal{V}_1, \mathcal{V}_2$
$\Rightarrow$ hints at global constraint

Motivates looking at classes of vacua, looking for:

- Mathematical characterization of classes of vacua
- Range of low-energy field theories available in classes

— Complementary to standard approach of looking for realistic model

— Secondary motivation: find algorithms for constructing classes of models with specific properties like $H \subset G$ for $H = SU(2) \times SU(3)$

Landscape/Swampland problem for predictive phenomenology:

What range of EFT's arise in allowed (metastable) string vacua?

Possibility A: "Anything goes"

Perhaps virtually any variation on standard model below 100 TeV can be realized in string theory
(change masses, couplings, matter content, extensions, . . . )

— Predictivity difficult, maybe impossible in practice

— Progress requires real dynamics, measure etc.

Possibility C: Low-Energy Constraints

Perhaps not all low-E field theories allowed from ST

— May give constraints, (*e.g.* 3/19 of SM parameters)

Focus on theories w/SUSY —why?

– If SUSY at intermediate scale:

- If no constraints from ST on SUSY theory, low-energy constraints derivable from SUSY FT and SUSY breaking mechanism
- If constraints from ST on SUSY theory:
  $\Rightarrow$ constraints on broken SUSY theory at sub TeV scales

– If SUSY breaking at Planck scale: $\Rightarrow$ predictions very tough

So: look for constraints arising from ST
on SUSY field theory + gravity in 4 and higher dimensions

For today, focus on gauge group $G$, matter content
(representations, # generations)

Don't worry about moduli, cosmology

Question: does ST make definite predictions anywhere in landscape?

# 2. Magnetized Brane Models

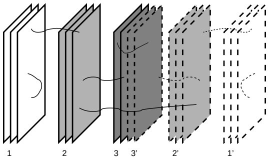
Magnetized brane models on K3 [V. Kumar/WT arXiv:0903.0386]

Result: Clean mathematical characterization of class of vacua:
6D $\mathcal{N} = 1$ SUSY models w/ rank 16 gauge group

Setup: Type I/heterotic SO(32) (Spin(32)/$\mathbb{Z}_2$) on K3

16 D9's (+ orientifold images) w/fluxes $\sim$ T-dual of IBM

"Stacks" of $N_a$ branes
w/ flux $f_a$




[Green/Schwarz/West, Honecker]

$$F = \begin{pmatrix} \left.\begin{matrix} f_1 & & \\ & \ddots & \\ & & f_1 \end{matrix}\right\} N_1 & & 0 \\ & \left.\begin{matrix} f_2 & & \\ & \ddots & \\ & & f_2 \end{matrix}\right\} N_2 & \\ 0 & & \ddots \end{pmatrix}$$

## Constraints on fluxes

- Fluxes $f_a$ integrally quantized

$$f_a \in H^2(K3, \mathbb{Z}) = \Gamma^{3,19} = U \oplus U \oplus U \oplus E_8(-1) \oplus E_8(-1) \quad \left[U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]$$

- $f_a$ generate even integral lattice $\Lambda \subset \Gamma^{3,19}$

Tadpole constraint : $\frac{1}{8\pi^2} \int_S \mathrm{Tr}\, \mathcal{F} \wedge \mathcal{F} = -\frac{1}{2} \int_S p_1(R) \Rightarrow \sum_a N_a f_a \cdot f_a = -24$

SUSY constraints : $\int f^a \wedge \Omega = 0 \quad \int f^a \wedge \bar{\Omega} = 0 \quad \int f^a \wedge J = 0$

$\Lambda$ negative definite $\Rightarrow \exists\, \Omega, J$ giving SUSY (pos. def. 3-plane in $\Gamma^{3,19}$)

**Constraints for SUSY vacuum:** Define matrix $m_{ab} = f_a \cdot f_b$

$$\sum_a N_a m_{aa} = -24, \qquad m_{ab} \text{ negative semidefinite}$$

## Parameters $N_a, m_{ab} \Rightarrow$ gauge group, matter content in 6D

**Gauge group:**

$$G = U(N_1) \times U(N_2) \times \cdots \times U(N_K) \times SO(32 - 2\sum_a N_a)$$

[Technical points: 1) Some U(1)'s anomalous $\to$ massive,
2) $G$ may be enhanced when $J \cdot f = 0$, $f^2 = -2$]

**Matter content:** depends only on $N_a$, $m_{ab} = f_a \cdot f_b$

| Rep. (+ c.c.) | # hypermultiplets |
|---|---|
| Adjoint $U(N_a)$ | 1 |
| $(N_a, \bar{N}_b)$ | $(-2 - (f_a - f_b)^2)$ |
| $(N_a, N_b)$ | $(-2 - (f_a + f_b)^2)$ |
| Antisym. $U(N_a)$ | $(-2 - 4f_a^2)$ |
| $(N_a, 2M)$ | $(-2 - f_a^2)$ |
| Neutral | 20 |

**Anomalies:** $F^4, R^4$ cancel (*e.g.* $n_H - n_V + 29n_T = 273$)

[Green/Schwarz/West, Intriligator, Honecker]

So for this class of models

$N_a, m_{ab}$ parameterize 6D physics

Tadpole + SUSY: $\sum_a N_a m_{aa} = -24, \quad m_{ab}$ neg. semidefinite

Question: Up to these constraints, what $N_a, m_{ab}$ possible?

Answer: **A**, "Anything goes"

Furthermore, realization of given $N_a, m_{ab} \sim$ unique up to duality
— Some discrete redundancy (generalize Banerjee/Sen dyon #'s)

Key: Nikulin results on lattice embeddings

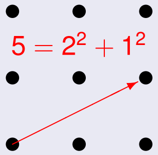
## Embedding theorems: vector (1D lattice) embeddings as example

Given lattice $\Lambda$ with $\langle x, y\rangle \in \mathbb{Z}$, desired norm $\nu$

Existence: $\exists x \in \Lambda : \langle x, x\rangle = \nu$?
Uniqueness: Is such an $x$ unique (up to $\Lambda$ automorphisms?)

**Positive definite**

*e.g.* Cartesian $\delta^a_b$




$\exists$ : not guaranteed
$(7 \neq a^2 + b^2)$

! : not guaranteed
$(8^2 + 1^2 = 7^2 + 4^2)$

**Even unimodular**

*e.g.* $E_8$ root lattice

$$E_8 = \{(x_1, \ldots, x_8) : \sum x_i \in 2\mathbb{Z}, x_i \in \mathbb{Z} \text{ or } \mathbb{Z} + 1/2 \ \forall i\}$$

$\exists$ : Guaranteed!

! : not guaranteed
(*e.g.* $\nu = 14 \Rightarrow 2x$'s w/ $\langle x, x\rangle = 14$)

**Even unimodular indefinite signature rank $\geq 4 + |$ sig. $|$**

*e.g.* $\Gamma^{2,2} = U \oplus U$,

$\exists$ : Guaranteed
$((1, x, 0, 0)^2 = 2x)$

! : Guaranteed unique
(Wall's theorem)
–!primitive $(x \neq nx')$
–! to automorphism

## Nikulin's theorem

Let $\mathcal{M}$ be an even lattice of signature $(t_+, t_-)$ and let $\mathcal{L}$ be an even, unimodular lattice of signature $(l_+, l_-)$. There exists a unique primitive embedding $\eta : \mathcal{M} \hookrightarrow \mathcal{L}$, provided the following conditions hold:

1. $l_+ - t_+ > 0$ and $l_- - t_- > 0$.
2. $l_+ + l_- - t_+ - t_- \geq 2 + l(A(\mathcal{M})_p)$ $\forall$ primes $p \neq 2$.
3. $l_+ + l_- - t_+ - t_- \geq l(A(\mathcal{M})_2)$;  if = then $A(\mathcal{M}) \cong \mathbb{Z}_2^3 \oplus A'$.

### Definitions:

$A(\mathcal{M}) = \mathcal{M}^*/\mathcal{M}$, $A(\mathcal{M})_p$ = subgroup of elements of order $p^k$ (any $k$)

$l(A(\mathcal{M})_p) = \#$ generators of $A(\mathcal{M})_p$

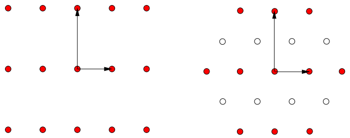
primitive: $\eta(\mathcal{M}) = \mathrm{Span}(\eta(\mathcal{M})) \cap \mathcal{L}$

Note: $l(A(\mathcal{M})_p) \leq t_+ + t_-$
so when $l_+, l_- = 3, 19$, $t_+ = 0$, $\Rightarrow$ always OK for $t_- \leq 10$

## Some consequences of Nikulin's theorem

(after some fiddling with special cases)

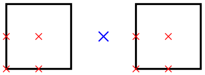
- For every $N_a, m_{ab}$ satisfying tadpole + SUSY, $m \Rightarrow \Lambda$ embeds into $\Gamma^{3,19}$, $m, \Lambda \rightarrow f \rightarrow \Omega, J$ ("Anything goes")
- $\Lambda \hookrightarrow \Gamma^{3,19}$ can be primitive in all cases except $(-2)^{12}$.
- Redundancy from overlattices w/ discrete embeddings (generalizes Banerjee & Sen dyon invariants)



Result: separates
Constraints (tadpole, SUSY, $N, m \rightarrow$ 6D physics)
Freedom (any $N, m : \sum_a N_a m_{aa} = -24, \quad m \leq 0$)

## Example of lattice embeddings

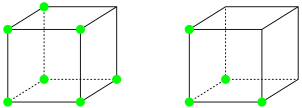
Kummer lattice $K \subset \Gamma^{3,19}$ from orbifold



Basis $E_i, i = 1, \ldots, 16, \{\sum_{i \in H} \frac{1}{2} E_i : H \text{ affine hyperplane in } \mathbb{Z}_2^4\}$

inner product $\langle E_i, E_j \rangle = -2\delta_{ij}$

can embed $(-2)^{11}$



can embed $(-2)^{12}$ but not in primitive fashion!

Application: find all models w/ $SU(3) \times SU(2) \subset G$

- Straightforward (ignoring possible enhancement of $G$)

- $N_1 = 3, N_2 = 2$

$$m = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix} = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

$$A < 0,\ C < 0,\ 3A + 2B \geq -24,\ AC - B^2 > 0$$

71 models. # "quarks" in $(3, 2) + (3, \bar{2})$ is $(-4 - 2m_{11} - 2m_{22}) \geq 4$

Point: just plug in values for desired quantities.

No extra info needed about rest of model
(unlike IBM toroidal orientifold models–next part)

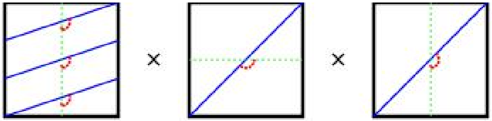
## Comments on K3 rank 16 models

- For *e.g.* $SO(32 - N) \times SU(N)$, construction gives all anomaly-allowed #'s of hypers in allowed rep's
- These are special points in moduli space.
  Turning on nonabelian fluxes $\rightarrow$ reduces rank, more general
- For complete analysis need to include enhancement from small instantons on shrinking cycles [Aspinwall/Morrison, Intriligator]

# 3. Intersecting Brane Models

## Intersecting Brane Models in IIA

Well known, simple models



$T^6/Z_2 \times Z_2$ orientifold well studied, $\Rightarrow \supset$ SM gauge group, 3 gens.
[Blumenhagen/Körs/Lüst(/Görlich/Ott), Ibáñez/Marchesano/Rabadan, Aldazabal/Franco/Ibáñez/Rabadan/Uranga, Cvetic/Shiu/Uranga, Cvetic/Li/Liu, Cremades/Ibanez/Marchesano, Kumar/Wells, March./Shiu]

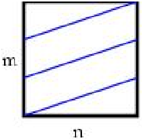
## Systematically studied:

Blumenhagen/Gmeiner/Honecker/Lüst/Weigand (Computer search)

Douglas/Taylor, Rosenhaus/Taylor (Complete analysis)

## IBM on $T^6/Z_2 \times Z_2$

Winding numbers on each torus




Tadpoles: $P = n_1 n_2 n_3, Q = -n_1 m_2 m_3, R = -m_1 n_2 m_3, S = -m_1 m_2 n_3$

Cancellation: $\sum_a P_a = \sum_a Q_a = \sum_a R_a = \sum_a S_a = 8$

SUSY conditions (when $P, Q, R, S > 0$):

$$\frac{1}{P} + \frac{j}{Q} + \frac{k}{R} + \frac{l}{S} = 0, \quad P + \frac{1}{j}Q + \frac{1}{k}R + \frac{1}{l}S > 0\,.$$

3 kinds of branes (up to $S_4$ symmetry):

**a**: $- + + +$, **b**: $+ + 0\ 0$, **c**: $+ 0\ 0\ 0$

moduli + **a** branes (negative tadpoles) make problem tricky.

Blumenhagen *et al.* [hep-th/]

Fix moduli $\vec{U} = (\hat{h}, \hat{j}, \hat{k}, \hat{l}) \in \mathbb{Z}^4$ $(j = \hat{j}/\hat{h}, k = \hat{k}/\hat{h}, l = \hat{l}/\hat{h})$,

Scan over $U = |\vec{U}|$, fixed $|\vec{U}| \rightarrow$ p.d. condition $\rightarrow$ finite solutions

- Scanned to $U = 12$, found $\sim 10^8$ models, complexity exponential
- Seemed found most models, decreasing tail
- $G$ components, # generations $\sim$ random

Douglas/Taylor [hep-th/0606109]

- Proved finite # total solutions, analytic bounds on $\mathfrak{a}$-brane combos

Rosenhaus/Taylor (to appear)

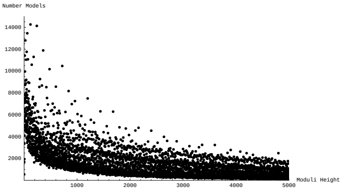
- Completed construction of all $\mathfrak{a}$-brane combinations (99,479)
- Allows construction of models w/ desired features
- Confirmed $\sim$ random distribution
- “Diversity in tail”

Example: consider $H = SU(3) \times SU(2) \times U(1) \subset G$ (distinct U(1))

— For each of $10^5$ a-brane combinations, add b's, c's

— Find all 3 + 2 + 1 combos ($\sim$ 13.4 M)

— Of these, for $\sim$ 12.9M moduli fixed, typical $U \sim 1000$




Explanation:

- More diverse 3 + 2 + 1 combos w/ large negative tadpole from a's
- Large negative tadpole $\rightarrow$ large moduli
- Fewer ways to complete models at large moduli w/ "extra" sector

Generations: # "quarks" $\sim \mathcal{O}(10)$, no suppression of 3 generations.

# Conclusions

## K3 Magnetized Brane Models

- Nikulin theorem $\Rightarrow$ simple math. characterization of model space
- Only constraints SUSY, tadpole, $N_a, m_{ab} \rightarrow$ 6D physics
- $\sum N_a m_{aa} = -24$, $m \leq 0 \Rightarrow$ exists lattice embedding
- Models unique up to dualities, discrete redundancy from overlattices
- Direct construction of models w/ desired properties

## IBM on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- Completed analysis of allowed negative tadpole (a-brane) combos
- Allows direct construction of models w/ desired properties
- “Diversity is in the tail” of this landscape slice

# Further Directions

- Extend analysis of 6D $N = 1$ SUSY models
  (nonabelian bundles, gauge enhancement,
  discrete $B$, bundles w/o v.s.)

- Apply K3 results, lattice embedding theorems in other contexts
  (less SUSY, lower dimensions, . . . )

- Find more general theoretical structure
  $\Rightarrow$ finite # IBM SUSY models

- Consider IBM/MBM on more general Calabi-Yau manifolds
  (proof of finite # solutions, . . . )