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Basics of String Field Theory

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"Fundamental Aspects
of Superstring theory"

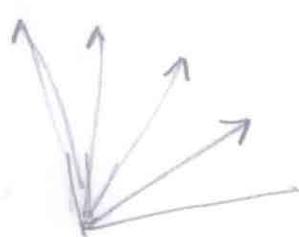
①

What is a string field?

String background CFT

$$\mathcal{H} = \underset{\text{CFT}}{(CFT)}_{\text{MATTER}} \otimes \underset{\text{CFT}}{(CFT)}_{\text{SUSY}}$$

Ψ string field is a vector in (some subspace)
 \mathcal{H} the state space \mathcal{H}_{CFT}
 with some linear con



Infinite dimensional space (primaries
 descendants etc)

$\mathcal{H}_{\text{CFT}} \simeq$ set of vertex operators

$S(\Psi) : \mathcal{H} \rightarrow \mathbb{R}$ a nonlinear
 function

Goes beyond perturbation

theory as Ψ need not be small

- Classical solutions
 and physical background independence

Achievements (pre 2000)

- ① Fundamental role of Q (BRST operator)

Gauge
fixed free
theory
Siegel (1984)



Gauge invariant
~~witten~~ Witten (1986)

$$Q\Psi = 0$$

physical states

$$\Psi \sim \Psi + QE$$

- ② New parameterizations of the moduli spaces of Riemann surfaces
[Numerical SFT]

- ③ Background independence for nearby CFT's.

Desaton Theorem.

- ④ Advances in BV quantization
BV Supergeometry.

Developments after 2000

2000 - 2003 Numerical evidence
that OSFT contains
classical solutions with
"large" Ψ . [Sen's tachyon cond]

Analytic developments in OSFT

12/05 → Tachyon Vacuum Analytic solution
(OSFT tachyon potential) M. Schnabl

01/07 → Marginal solutions. Rolling tachyon

Recent research developments:

Perturbation theory in Schnabl gauge
(linear b-gauges)

Boundary state in BCFT from
open string fields

The set of SFT's

Light-cone bosonic (1974 Kaku + Kikkawa)
Mandelstam

Covariant Open Bosonic (Witten 86)

SAFE!! Covariant Closed Bosonic (BZ 92)

Covariant open superstring (Berkovits 95)
NS

Heterotic NS (Okounkov BZ, Berkovits 04)

Light cone superstrings FT

NOT SO Vacuum string field theory

SAFE!! Cubic open superstring field theory (Witten, Thorn, Arefeva, Medvedev, Zubakov)

Covariantized light cone SFT's (Japanese group)

Missing

Natural type II ...

Open bosonic

"Q"

$$Q^2 = 0$$

$$S[\bar{\Phi}] = -\frac{1}{g_2} \left[\frac{1}{2} \langle \bar{\Phi}, Q \Phi \rangle + \frac{1}{3} \langle \bar{\Phi}, \Phi^* \bar{\Phi} \rangle \right]$$

$$|A\rangle \xrightarrow{\text{BPZ}} \langle A| \quad (\text{linear conjugation})$$

$$\langle A, B \rangle \equiv \langle A | B \rangle$$

$$\begin{cases} \langle A, B \rangle = (-)^{AB} \langle B, A \rangle \\ \langle QA, B \rangle = -(-1)^A \langle A, QB \rangle \end{cases}$$

signs $(-1)^A = (-)^{\epsilon(A)}$

$\epsilon(A) = 0 \pmod{2}$	A even
$\epsilon(A) = 1 \pmod{2}$	A odd

$$\begin{aligned} \langle \bar{\Phi}, Q \Phi \rangle &= (-1)^{\Phi(\bar{\Phi}+1)} \langle Q \bar{\Phi}, \Phi \rangle \\ &= \langle Q \bar{\Phi}, \Phi \rangle = -(-1)^{\bar{\Phi}} (\bar{\Phi}, Q \Phi) \\ \Rightarrow \boxed{\bar{\Phi} \text{ is odd}} \quad |\bar{\Phi}\rangle &= + \frac{g_1}{\overline{\text{odd}}} \underline{\text{even}} \end{aligned}$$

* : Associative

$$A * (B * C) = (A * B) * C$$

$$\langle A, B * C \rangle = \langle A * B, C \rangle$$

$$Q(A * B) = Q A * B + (-1)^A A * Q B$$

$$\epsilon(A * B) = \epsilon(A) + \epsilon(B)$$

$$\epsilon(QA) = 1 + \epsilon(A)$$

Two exercises:

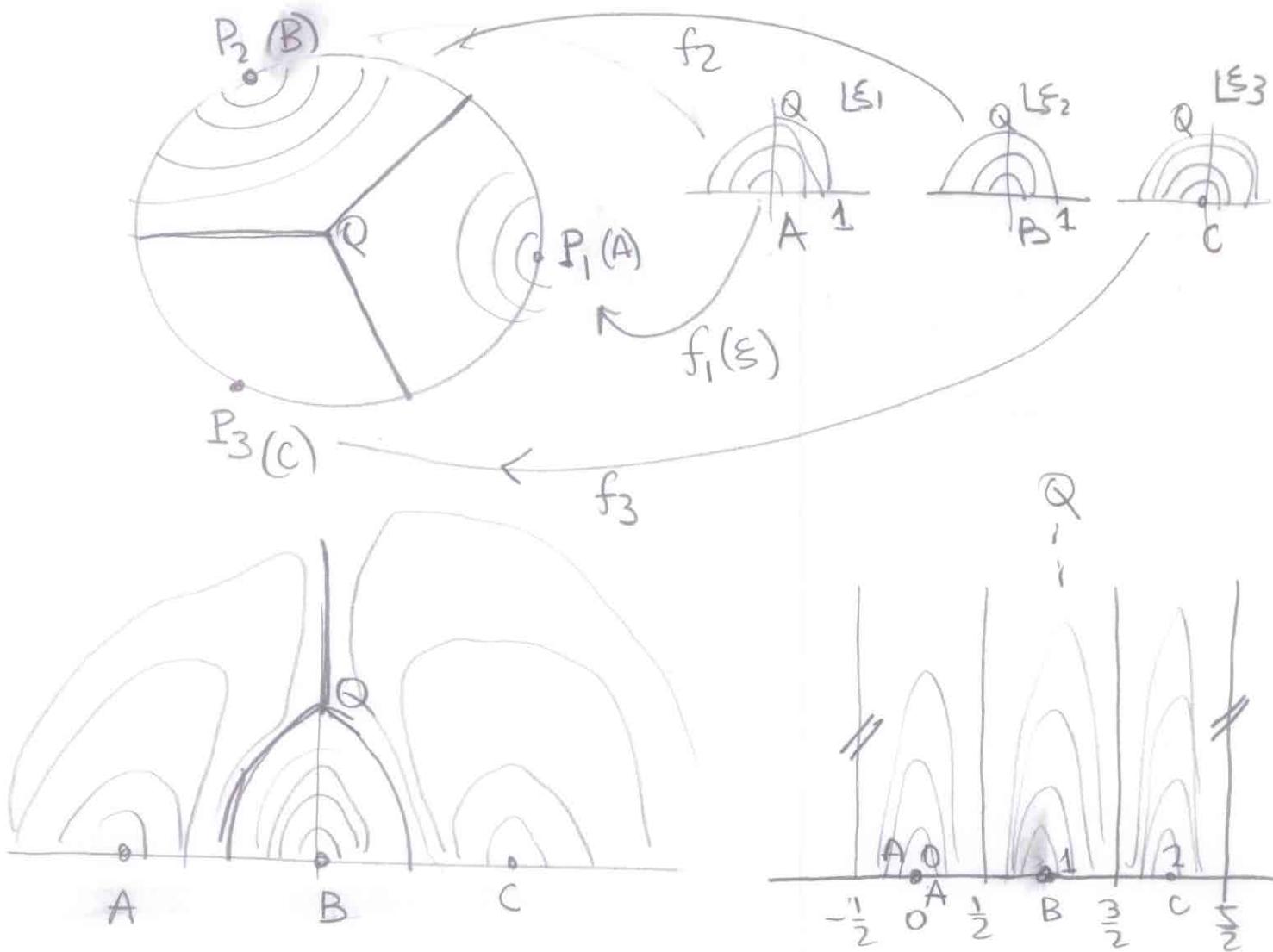
Show that $Q\Psi + \Psi^* \Psi = 0$ is the EOM

$$\delta\Phi = Q\Lambda + \Phi^*\Lambda - \Lambda^*\Phi$$

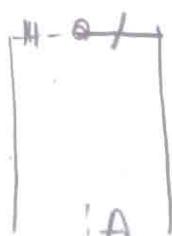
is a linearized gauge invariance

What is $*$?

$$\langle A, B^* C \rangle = \langle f_1 \circ A, f_2 \circ B, f_3 \circ C \rangle$$



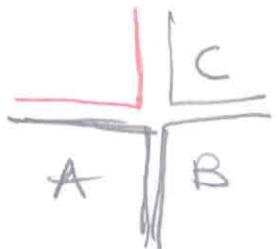
gauge invariance



String picture

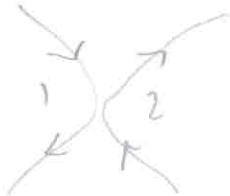
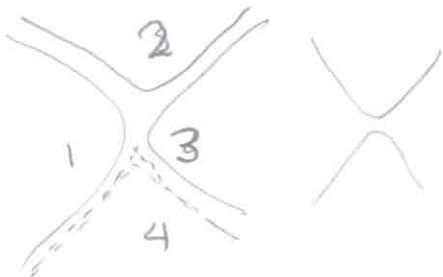


associative



Light cone

$$\Psi[\alpha, p^+, X^I(\sigma)]$$



Closed bosonic:

Is $Q|\Psi\rangle = 0$ the correct EOM?

Is $Q|\Psi\rangle = 0 \quad |\Psi\rangle \sim |\Psi\rangle + Q\Lambda$
correct cohomology?

No! subtle reasons but cannot write
a kinetic term: $\langle c \partial c \partial^2 c \bar{c} \bar{\partial} \bar{c} \bar{\partial}^2 \bar{c} \rangle$

gh#6
 $\langle \Psi Q \Psi \rangle$ will not work $c \bar{c} V_H$
 gh#2

No cohomology for $(L_0 - \bar{L}_0) \neq 0$

$$Q|\Psi\rangle = 0$$

$(L_0 - \bar{L}_0)|\Psi\rangle \neq 0$ since $L_0 - \bar{L}_0 = \{Q, b_0 - \bar{b}_0\}$
 $\rightarrow Q(b_0 - \bar{b}_0)|\Psi\rangle = 0$ means $b_0 - \bar{b}_0$ does
not kill $|\Psi\rangle$

but then $|\Psi\rangle = Q\left(\frac{b_0 - \bar{b}_0}{L_0 - \bar{L}_0} |\Psi\rangle\right)$

trivial

Impose both

$$\boxed{\begin{aligned} L_0 - \bar{L}_0 &= 0 \\ b_0 - \bar{b}_0 &= 0 \end{aligned} \quad \text{on } \mathcal{H}_{\text{CFT}}}$$

(9)

$$\langle A, B \rangle = \langle A | c_0^- | B \rangle \quad c_0^- = \frac{c_0 - \bar{c}_0}{2}$$

$$\{c_0^-, b_0^-\} = 1$$

$$S^{kin} = \langle \Psi | c_0^- Q | \Psi \rangle$$

$$b_0^- \Psi = 0$$

$$L_0^- \Psi = 0$$

Now $\delta \langle \Psi \rangle = Q(\Lambda) \quad b_0^- \Lambda = 0$

$$\delta \langle \Psi \rangle = \langle \Lambda | Q$$

$L_0^- \Lambda = 0$
otherwise
 Ψ goes out

$$\delta S^{kin} = \langle \Lambda | Q c_0^- Q | \Psi \rangle \quad |\Lambda \rangle = b_0^- | \Omega \rangle$$

$$= \langle \Omega | b_0^- Q c_0^- Q | \Psi \rangle$$

$$= - \langle \Omega | Q \underbrace{b_0^- c_0^-}_{1} Q | \Psi \rangle$$

$$= + \langle \Omega | Q c_0^- b_0^- Q | \Psi \rangle = 0$$

↑
still killed
by L_0^-

gauge invariance

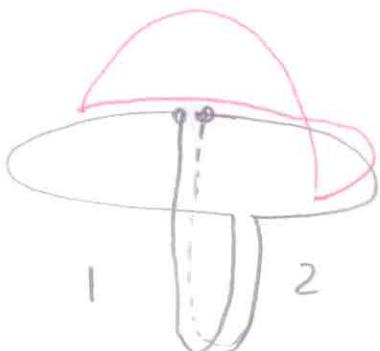
Cohomology problem for
closed strings is semirelative

$$\langle A, B \rangle = (-1)^{(A+1)(B+1)} \langle B, A \rangle$$

$$\langle QA, B \rangle = (-1)^A \langle A, QB \rangle$$

Product is (gr)commutative

$$[A, B] = (-1)^{AB} [B, A]$$



$$\epsilon[A, B] = \epsilon(A) + \epsilon(B) + 1$$

$$\theta([A, B]) = \theta(A) + \theta(B) - 1$$

$$S = -\frac{2}{\alpha' k^2} \left(\frac{1}{2} \langle \Psi, Q\Psi \rangle + \frac{1}{3!} \langle \Psi, [\Psi, [\Psi, \Psi]] \rangle + \dots \right)$$

Would be gauge invariant under

$$\delta \Psi = Q\Lambda + [\Psi, \Lambda] \quad \text{if}$$

$$[A, [B, C]] \pm [B, [C, A]] \pm [C, [A, B]] = 0$$

but it is not true ! RHS is $Q'(\dots)$

$$0 = Q[B_1, B_2, \dots, B_n] + \sum_{l=1}^n (-1)^{B_1 + \dots + B_{l-1}} [B_1, \dots, QB_l, \dots]$$

$$+ \sum_{\{i_e, j_e\}} \sigma(\{i_e\}, \{j_e\}) [B_{i_1}, \dots, B_{i_e}, [B_{j_1}, \dots, B_{j_e}]]$$

$$\text{To sum } Q[B_1, \dots, B_n] \rightarrow B_{i_1} \dots B_{i_e} Q[B_{j_1}, \dots, B_{j_e}]$$

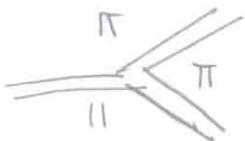
$$G([B_1, B_2, \dots, B_n]) = -1 - 2(n-2) + \sum_i \theta(B_i)$$

Then set

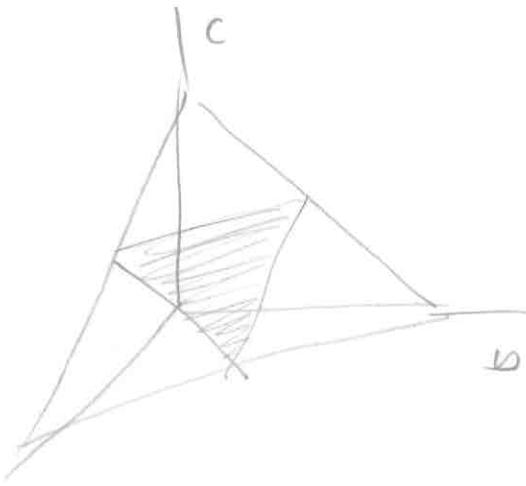
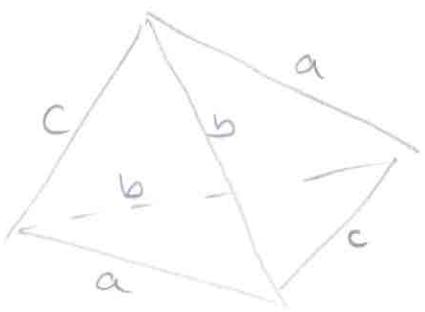
$$S = -\frac{2}{d' \kappa^2} \left(\frac{1}{2} \langle \Psi, Q\Psi \rangle + \sum_{n=3}^{\infty} \frac{1}{n!} \langle \Psi, [\Psi^{n-1}] \rangle \right)$$

$$\delta |\Psi\rangle = Q\Lambda + \sum_{n=1}^{\infty} \frac{1}{n!} [\Psi^n, \Lambda]$$

The moduli



really "means" concretely



conformal structures

$$c+b+c+b > 2\pi$$

$$c+b > \pi$$

$$a < \pi$$

$$b < \pi$$

$$c < \pi$$

N -point function gives n -faced polyhedra

All nontrivial closed paths

have length $> 2\pi$

Each face has length 2π

NS open Superstrings (wzw)

β, γ $\beta = \partial \xi e^{-\phi}$ $\rightarrow (\xi, n, \phi)$

grassmann even $\gamma = \eta e^\phi$

	dim	gh#	picture #
ξ	0	-1	+1
η	1	+1	-1
β	$\frac{3}{2}$	-1	0
γ	$-\frac{1}{2}$	+1	0
$e^{q\phi}$	$-\frac{1}{2}q(q+2)$	0	+q

dim $e^{q\phi} = -\frac{1}{2}q(q+2)$

$$[e^{-\phi}] = \frac{1}{2}$$

$$[e^\phi] = -\frac{3}{2}$$

Conventional approach

$$Q \hat{\Phi} = 0 \quad \text{with} \quad \hat{\Phi} = c e^{-\phi} V_M |0\rangle$$

$$\text{gh\# } \hat{\Phi} = 1$$

$$\text{picture\# } \hat{\Phi} = -1$$

Grassmann odd

$$c_1 e^{-\phi} \psi_{-\frac{1}{2}}^u |0\rangle$$

$$\downarrow \text{dim } \frac{1}{2} \quad \text{dim } \frac{1}{2}$$

$$\text{dim } -1$$

need total picture number -2

$$\langle \hat{\Phi} | Q \hat{\Phi} \rangle + \langle \hat{\Phi} | \hat{\Phi} \hat{\Phi} X \rangle$$

Try to get a string field Φ picture # 0 gh# 0

Φ	picture	gh#
	0	0

and
grassmann even

$$(X_0 = \int \frac{dz}{2\pi i z} X(z), \quad X(z) = \{Q, \xi(z)\})$$

Larue Hilbert space ξ_0 is included

$$\begin{array}{ll} Q & Q^2 = 0 \\ \eta_0 & \eta_0^2 = 0 \end{array} \quad \{Q, \eta_0\} = 0$$

Linearized field is

$$Q\eta_0 |\Phi\rangle = 0$$

Two gauge invariances

$$|\Phi\rangle \sim |\Phi\rangle + Q|\Lambda\rangle$$

$$|\Phi\rangle \sim |\Phi\rangle + \eta_0 |\Sigma\rangle$$

$$\{\eta_0, \xi_0\} = 1$$

$$\text{Gauge invariance } |\Phi\rangle \sim |\Phi\rangle + \eta_0 |\Sigma\rangle$$

allows us to choose $|\Phi\rangle$ such that

$$\xi_0 |\Phi\rangle = 0$$

(Proof: choose $|\Phi\rangle - \eta_0 \xi_0 |\Phi\rangle$)

If $|\Phi\rangle$ satisfies $\xi_0 |\Phi\rangle = 0$

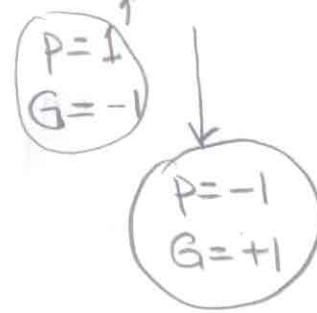
$$\text{then } |\Phi\rangle = \xi_0 |\hat{\Phi}\rangle$$

$$\text{Here } |\hat{\Phi}\rangle \sim |\Phi\rangle + \xi_0 |\chi\rangle$$

$$\text{so can choose } \eta_0 |\hat{\Phi}\rangle = 0$$

Summary: gauge where

$$|\tilde{\Phi}\rangle = \xi_0 |\hat{\Phi}\rangle, \text{ with } \eta_0 |\tilde{\Phi}\rangle = 0$$



good that is standard

Back to EOM

$$Q \eta_0 |\tilde{\Phi}\rangle = 0$$

$$Q \eta_0 \xi_0 |\hat{\Phi}\rangle = 0$$

$$Q |\hat{\Phi}\rangle = 0 \quad \checkmark$$

Nonlinear equation:

$$\eta_0 (\underbrace{e^{-\hat{\Phi}} Q e^{\hat{\Phi}}}_{\text{pure gauge solution of OSFT}}) = 0$$

pure gauge solution of OSFT $(\cdot e^{-\hat{\Lambda}} Q e^{\hat{\Lambda}})$
gh ≠ 0

gauge invariance

$$\delta e^{\tilde{\Phi}} = (Q \Lambda) e^{\tilde{\Phi}} + e^{\tilde{\Phi}} (\eta_0 \Omega)$$

Action: define $A_x = e^{-t\tilde{\Phi}} (x e^{t\tilde{\Phi}}) \times a$ denach

$$A_t = e^{-t\tilde{\Phi}} \partial_t e^{t\tilde{\Phi}}$$

$$S = -\frac{1}{2g^2} \int_0^1 dt \ll \partial_t (A_\eta A_Q) + A_t \{ A_Q, A_\eta \} \gg$$

Note the inner product

$$S = -\frac{1}{2g^2} \left[\langle\langle (\bar{e}^{-\phi} \eta e^\phi) (\bar{e}^{-\phi} Q e^\phi) \rangle\rangle + \int_0^t dt \langle\langle \bar{e}^{-t\phi} \partial_t e^{t\phi} \{ \bar{e}^{t\phi} Q \bar{e}^{+t\phi}, \bar{e}^{-t\phi} \eta e^{+t\phi} \} \rangle\rangle \right]$$

$$\langle S(z) c \partial c \partial^2 c(w) e^{-2\phi(y)} \rangle = 2$$

picture -1
ghost ≠ +2

$$\langle (Q\Phi)(\eta_0\Phi) \rangle$$

$gh = +2$

Heterotic Strings FT

<u>superstr.</u>	<u>bosonic open</u>
even Φ	Φ odd
$P=0$	$\Phi = -$
$G=0$	$G=1$

$$V \sim \Phi_{\text{super}} \otimes \Phi_{\text{bos}}$$

$P=0$
$G=1$
odd

$$(b_0 - \bar{b}_0) |V\rangle = (L_0 - \bar{L}_0) |V\rangle = 0$$

$$\langle A, B \rangle = \langle A | C_0^- | B \rangle$$

η is a derivation of all products

$$\langle g e^{-2\phi} c \partial c \partial^2 c \bar{c} \bar{\partial} \bar{c} \bar{\partial}^2 \bar{c} \rangle \neq 0$$

$G=5$
$P=-1$

for nonvanishing correlators

Now:

$$V = \underbrace{g c \bar{c} e^{-\phi} V_M |0\rangle}_{\text{usual}}$$

$$\eta Q V = 0 \text{ linearized EOM } \checkmark$$

$$S_2 = \frac{1}{2} \langle \eta V, QV \rangle$$

$$\downarrow \\ 1+1 + 1+1 + 1(c_0^-) = G=5$$

$$P=-1$$

(17)

$$S_3 = \frac{1}{3!} \left\langle \eta V, \underbrace{[V, QV]}_{gh=2} \right\rangle$$

$$gh[A, B] = gh[\bar{A}] + gh[\bar{B}] - 1$$

$$sh[A, B, C] = \begin{aligned} & gh(A) \\ & + sh(B) \\ & - 3 \end{aligned}$$

$$S_4 = \frac{1}{4!} \left(\left\langle \eta V, \underbrace{[V, \underbrace{QV}_{1}, \underbrace{QV}_{2}]}_{2} \right\rangle + \left\langle \eta V, [V, \underbrace{[V, QV]}_{2}] \right\rangle + \dots \right)$$

What about type II?

$$\Phi_{\text{super}} \otimes \bar{\Phi}_{\text{super}}$$

$$P = \bar{P} = 0$$

$$G = \bar{G} = 0$$

$$\left\langle g \bar{s} e^{-2\phi} \bar{e}^{-2\bar{\phi}} c \partial c \partial^2 c \bar{c} \bar{\partial} \bar{c} \partial^2 \bar{c} \right\rangle$$

$$G = 4$$

Gr even

$$P = -2$$

$$\left\langle F, Q \eta \bar{\eta} F \right\rangle \checkmark \quad \left\langle \eta \bar{\eta} F, \underbrace{QF}_{G=1} \right\rangle$$

\bar{e}_0^-

$$\left\langle \eta \bar{\eta} F, [F, \bar{F}] \right\rangle$$

"need 2 Q's"

$$[QF, QF] = 0$$

$$[F, [F, F]]$$

$$\langle \eta\eta F, [F, F, F, F] \rangle$$

need now 4Q's $\stackrel{-1}{-2}$ no good

$$\langle \eta\eta F, [F, F, F, F] \rangle$$

need now 6Q's $-1-4 = -5$
impossible ..

Analytic form for Helicistic SFT

EOM

$$\eta \bar{\Psi}_Q = 0$$

V_F

\hookrightarrow pure gauge solution
of bosonic CSFT.

Recall ..

$$\delta \Psi = \underset{gh\#2}{Q} \Lambda + \underset{gh\#1}{[\Psi, \Lambda]} + \frac{1}{2} [\Psi, \Psi, \Lambda]$$

$$\bar{\Psi}_Q = QV + \frac{1}{2} [V, QV]$$

$$+ \frac{1}{3!} ([V, QV, QV] + [V, [V, QV]])$$

+ ...

Ψ_Q = pure gauge field for $V(t)$ $\left[\begin{array}{l} V(0) = 0 \\ V(1) = V \end{array} \right]$

$$S = \frac{1}{\alpha'} \int_0^1 dt \langle \eta \Psi_t, \Psi_Q \rangle$$

$$\partial_t \Psi_Q = Q' \Psi_t$$

$$S = \frac{1}{\alpha} \left(\langle \bar{\Psi}_n, \bar{\Psi}_Q \rangle + k \int_0^t dt \langle \Psi_t, [\Psi_n, \Psi_Q]' \rangle \right) \quad (19)$$

for a linear homotopy

$$\nu(t) = t\nu$$