SANTA BARBARA
KITP, OCT 03

# HYPERBOLIC KAC-MOODY ALGEBRAS

# COSMOLOGICAL CHAOS

# AND A

# "SMALL TENSION EXPANSION" OF M THEORY

Thibault DAMOUR
I.H.E.S.

WORK WITH M. HENNEAUX, H. NICOLAI, ...

CG1

## FROM $E_7$ TO $E_{10}$

1978 CREMMER-JULIA-SCHERK $\mathcal{L}_{11}^{SUGRA} \sim R + (dA)^2 + A \wedge dA \wedge dA + \dots$

1978-79 CREMMER-JULIA: HIDDEN $E_7$ SYMMETRY OF $D=4$ REDUCED EOM
+ CONJECTURED $E_6$ IN $D=5$ AND $E_8$ IN $D=3$

1981 JULIA IN $D=2$: AFFINE EXTENSION $E_8^{\wedge} \equiv E_9$

1982 JULIA

"FINALLY, WE CAN GO TO 1 (TIME) DIMENSION: WE ARE NOW CONSIDERING THE SO-CALLED HOMOGENEOUS SPACE-TIMES. COULD IT BE THAT $E_{10}$ BE A SYMMETRY OF HOMOGENEOUS $N'=7$ SUPERGRAVITY?"

$E_{10} = E_8^{\wedge\wedge}$

"IT IS SOMETIMES CALLED HYPERBOLIC AND DOES NOT HAVE A SIMPLE INTERPRETATION LIKE SOME EXTENSION OF A LOOP GROUP. WE ARE IN A SITUATION WHERE PHYSICS COULD PROVIDE CONCRETE AND SIMPLE REALIZATIONS OF HIGHLY ABSTRACT MATHEMATICAL OBJECTS"

RECENTLY (TD, M. HENNEAUX, H. NICOLAI '02) SOME EVIDENCE FOR $E_{10}(\mathbb{R})$ IN A "SMALL-TENSION EXPANSION" OF $SUGRA_{11}$

NB1: HERE CONTINUOUS SYMMETRY $E_{10}(\mathbb{R})$: SOLUTION $\to$ SOLUTION'
NOT SOME DISCRETE VERSION $E_{10}(\mathbb{Z})$: STATE $\to$ PHYSICALLY EQUIVALENT STATE

NB2: OTHER STRUCTURES OF $E_{10}$ ALREADY SHOWED UP IN STRING/M THY

- SPECTRUM OF BPS STATES: ROOT LATTICE (Harvey, Moore 96)
- U-DUALITY OF M THY: WEYL GROUP (Obers, Pioline... 98; Banks, Fischler, Motl 99)
- GENERIC COSMOLOGICAL BEHAVIOUR: LOWEST APPROXIMATION OF COSMOLOGICAL BILLIARD = WEYL CHAMBER (TD, Henneaux '00)

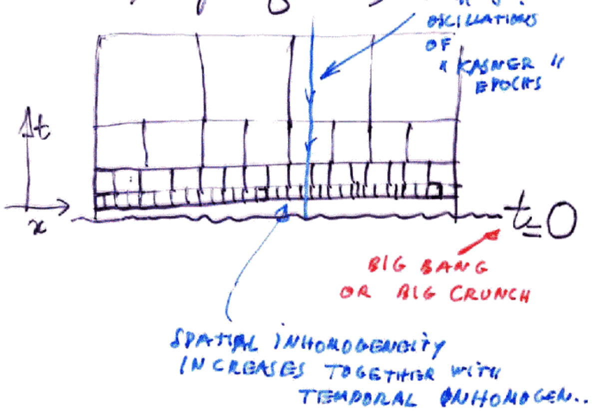
(1) DESCRIBE LEADING APPROXIMATION TO DYNAMICS OF COSMOLOGIC SOLU...

V1 F10

# CHAOS IN EINSTEIN'S COSMOLOGY

- Belinsky, Khalatnikov, Lifshitz 1969

INFINITELY OSCILLATORY BEHAVIOUR OF GENERIC COSMOLOGICAL SINGULARITY
FOR $\boxed{R_{\mu\nu}=0}$ IN $D=4$
"BKL" BEHAVIOUR




- SOME HOMOGENEOUS COSM. MODELS EXHIBIT THE BKL-TYPE CHAOS

Bianchi type IX, $ds^2 = -dt^2 + a^2(t)\,\omega_1^2 + b^2(t)\,\omega_2^2 + c^2(t)\,\omega_3^2$

$SO(3)$: $d\omega_1 = \omega_2 \wedge \omega_3$ etc..

$\Rightarrow$ ODE EQS FOR $a, b, c$

BKL '69
Misner '69

- Demaret, Henneaux, Spindel 1985

BKL oscillations DISAPPEAR in $D \geq 11$! $\Rightarrow$ MONOTONIC KASNER-LIKE POWER-LAW BEHAV.

- Belinsky, Khalatnikov 1973

[Theorem: Andersson, Rendall '00; Damour, Henneaux, Rendall, Weaver '02]

IF $\exists$ SCALAR FIELD $\left\{ \begin{array}{l} R_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi \\ \Box_g \varphi = 0 \end{array} \right.$ BKL oscillations DISAPPEAR in ANY $D$ $\Rightarrow$ MONOTONIC KASNER-LIKE

STRING THEORY: in $D \leq 10$ $\exists$ SCALAR: DILATON] EXPECT ...

V2 F11

# CHAOS IN GENERIC, INHOMOGENEOUS SUPERSTRING COSMOLOGY

Damour Henneaux hep-th/0003139

MASSLESS (BOSONIC) DEGREES OF FREEDOM OF SUPERSTRING TH'S

| | | NS-NS | | | R-R | | |
|---|---|---|---|---|---|---|---|
| $D=10$ | IIA | $g_{\mu\nu}$ | $\varphi$ | $B_{[\mu\nu]}$ | $A_\mu$ | $A_{[\mu\nu\lambda]}$ | |
| $D=10$ | IIB | $g_{\mu\nu}$ | $\varphi$ | $B_{[\mu\nu]}$ | $A$ | $A_{[\mu\nu]}$ | $A_{[\mu\nu\rho\sigma]}$ |
| $D=10$ | I | $g_{\mu\nu}$ | $\varphi$ | $B_{[\mu\nu]}$ | $A^a_\mu$ | | |
| $D=10$ | HET | $g_{\mu\nu}$ | $\varphi$ | $B_{[\mu\nu]}$ | $A^a_\mu$ | | |
| $D=11$ | M | $g_{\mu\nu}$ | | | | $A_{[\mu\nu\lambda]}$ | |

GENERAL MODEL

$$S = \int d^Dx \sqrt{g}\left[ R(g) - \partial_\mu\varphi\,\partial^\mu\varphi - \sum_p \frac{1}{2(p+1)!} e^{\lambda_p \varphi} (dA_p)^2 + \dots \right]$$

ANY $D$

EINSTEIN FRAME

NORMALIZED TO $-1$

SEVERAL $p$-FORMS $A_{\mu_1 \cdots \mu_p}$

DILATON COUPLING

INCLUDING OTHER COUPLINGS FOR LEADING APPROX.

BUT CRUCIAL FOR RECENT STUDY OF ...

V3 F12

# BKL-TYPE ANALYSIS OF $t \to 0$ SOLUTION

- EITHER IN FIELD EQS
- OR IN HAMILTONIAN CONSTRAINT.

BASIC PICTURE:

ONLY $(g_{\mu\nu}, \varphi)$

EPOCHS OF KASNER-LIKE FREE EVOLUTION

INTERRUPTED BY COLLISIONS ON THE "WALLS" DEFINED BY P-FORMS $p \geq 1$

## KASNER-LIKE FREE EVOLUTION

$$R_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi + \cdots$$

P-FORM CONT AT FIRST NEGLECTED

$$-g^{\mu\nu} \nabla_\mu \varphi = 0 + \cdots$$

$\boxed{d \equiv D-1 \text{ SPACE DIM}}$

$$g_{\mu\nu} dx^\mu dx^\nu \simeq -dt^2 + \sum_{i=1}^{d} t^{2p_i(x)} (\omega^i)^2$$

$\omega^i = e^i_j(x) dx^j$ d-bein

$$\varphi \simeq p_\varphi(x) \ln t + \psi(x)$$

(1) KEEP ONLY $\sim \frac{1}{t^2}$ TERMS IN FIELD EQ.

i.e. $\partial_t \gg \partial_x$

$$\boxed{p_\varphi^2 + \sum_{i=1}^{d} p_i^2 - \left(\sum_{i=1}^{d} p_i\right)^2 = 0 \qquad \sum_{i=1}^{d} p_i = 1}$$

CONSTRAINTS ON KASNER EXPONENTS $p_i, p_\varphi$

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## (2) "POTENTIAL WALLS" MODIFYING KASNER-LIKE BEHAVIOUR

HAMILTONIAN APPROACH:

GAUGE $N^i = 0$

$$S = \int dx^0 d^d x \left[\pi^{ij} \dot g_{ij} + \pi_\varphi \dot\varphi + \frac{1}{p!} \pi_A^{i_1 \cdots i_p} \dot A_{i_1 \cdots i_p} - N \mathcal{H} - N^i \mathcal{H}_i\right]$$

CONJUGATE MOMENTA — HAMILTONIAN CONSTRAINT

$$\mathcal{H} = \frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{1}{d-1}(\pi^i_i)^2\right) + \frac{1}{\sqrt{g}}\pi_\varphi^2 + \frac{1}{p!}\frac{1}{\sqrt{g}} \pi_A^{i_1 \cdots i_p} \pi_{A\, i_1 \cdots i_p} + \sqrt{g}\, U$$

KINETIC ENERGY TERMS — "$\vec{E}^2$" — POTENTIAL ENERGY

$$U = -{}^d R + g^{ij} \partial_i \varphi \partial_j \varphi + \frac{1}{(p+1)!} F^A_{i_1 \cdots i_{p+1}} F_A^{i_1 \cdots i_{p+1}}$$

GRAVITATIONAL POTENTIAL — "$\vec{B}^2$"

IWASAWA DECOMPOSITION OF METRIC (DHN)

$$\boxed{g_{ij}(t,x)\, dx^i dx^j = \sum_a e^{-2\beta^a(t,x)} \left(N^a_{\ i}(t,x)\, dx^i\right)^2}$$

DIAGONAL PART OF METRIC — UPPER TRIANGULAR $\begin{pmatrix} 1 & * & * \\ & 1 & * \\ 0 & & 1 \end{pmatrix}$

10 LEADING (WRT TIME DEPENDENCE) VARIABLES

$$\boxed{\begin{array}{l} \beta^\mu \;;\; \mu = 1, \cdots, 10 : \quad \beta^a \equiv -\ln g_{(a)(a)},\ a = 1, \cdots, d = 9 ; \quad \beta^{10} \equiv -\varphi \\ \text{in M-thy (SUGRA}_{11}) \quad \beta^\mu_M \equiv -\ln g_{(a)(a)} \quad a = 1, \cdots, d = 10 \end{array}}$$

GAUGE $N = \sqrt{g}$

$$\vec{E}^2 + \vec{B}^2 - R + \cdots$$

MAIN TERMS

LEADING DYNAMICS — KINETIC TERMS OF $\beta^\mu$

$$N\mathcal{H} = G^{\mu\nu} \pi_{\beta\mu} \pi_{\beta\nu} + V(\beta ; \text{OTHER VARIABLES})$$

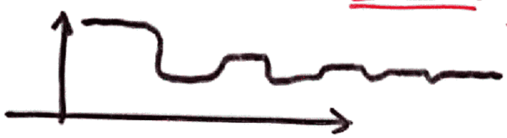
4.1

GENERAL RESULTS FROM

$$N\mathcal{H} = G^{\mu\nu}\pi_{\beta\mu}\pi_{\beta\nu} + \sum_A e^{-2w_A(\beta)} C_A[N^a_i, \pi^i_{N_a}, A_{q_1\ldots q_p}, \pi_A^{q_1\ldots q_p}]$$

AS $t \to 0$, i.e. $\sqrt{g} = e^{-\sum_\mu \beta^\mu} \to 0$

- ALL "OFF-DIAGONAL VARIABLES" $N^a_i$, $\pi^i_{N_a}$, $A_{a_1\ldots a_p}$, $\pi_A^{a_1\ldots a_p}$ FREEZE AS $t \to 0$



$-\ln t$

- INTERESTING DYNAMICS ONLY IN THE 10 DIAGONAL VARIABLES $\beta^\mu = -\ln g_{(\mu)(\mu)}$, $\mu = 1\ldots 10$

$-\ln t$

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TODA-LIKE MODEL FOR THE ASYMPTOTIC ($t \to 0$) DYNAMICS OF $\beta^\mu$ $[\beta^i = -\ln a_i,\ \beta^{10} = -\varphi$ OR $\beta^{10}_M = -\ln a_{10}]$, $\mu = 1, \ldots, 10$

$$\boxed{S = \int d\tau \left[ G_{\mu\nu} \frac{d\beta^\mu}{d\tau}\frac{d\beta^\nu}{d\tau} - V(\beta^\mu) \right]}$$

$$G^M_{\mu\nu}\, d\beta^\mu_M\, d\beta^\nu_M = \sum_{\mu=1}^{10} (d\beta^\mu_M)^2 - \left(\sum_{\mu=1}^{10} d\beta^\mu_M\right)^2$$

OR

$$G^S_{\mu\nu}\, d\beta^\mu_S\, d\beta^\nu_S = \sum_{i=1}^{9} (d\beta^i_S)^2 - (d\beta^0_S)^2$$

$$\beta^0_S = -\ln(\sqrt{g_S}\, e^{-2\Phi})$$

$$\boxed{V(\beta) \simeq \sum_A C_A(x)\, e^{-2 w_A(\beta)}}$$

SLOWLY VARYING IN TIME

WALL FORMS

+ HAMILTONIAN CONSTRAINTS $E = \dot\beta^2 + V = 0$

$\boxed{C_A^{(x)} > 0}$ FOR CRUCIAL WALLS

$$w_A(\beta) = \sum_{\mu=1}^{10} w_{A\mu}\beta^\mu$$

- NUMBER OF WALLS ~ 700 (MODEL-DEPENDENT WALLS)

E.G. IN M-THY:

GRAVITATIONAL WALLS → $g^M_{\alpha\beta\gamma} = 2\beta^\alpha + \sum_{\sigma \neq \alpha,\beta,\gamma} \beta^\sigma$ $\quad \alpha, \beta, \gamma = 1, \ldots, 10$

ELECTRIC 3-FORM WALLS → $e^{(3)}_{\alpha\beta\gamma} = \beta^\alpha + \beta^\beta + \beta^\gamma$ $\quad \alpha \neq \beta \neq \gamma$

MAGNETIC 3-FORM WALLS → $b^{(3)}_{\alpha_1\ldots\alpha_6} = \beta^{\alpha_1} + \beta^{\alpha_2} + \cdots + \beta^{\alpha_6}$ $\quad \alpha_1 \neq \cdots \neq \alpha_6$

- + SYMMETRY WALLS $w^S_{\alpha_1\alpha_2} = -\beta^{\alpha_1} + \beta^{\alpha_2}$ $\quad \alpha_1 < \alpha_2$

[OR QUOTIENTING BY "LARGE DIFFEOMORPHISMS" EXCHANGING $\beta^i$ ] $i = 1\ldots 10$

- GENERIC SOLUTION : E.G. M-THY CONTAINS $2 \times 128 = 256$ ARBITRARY FUNCTIONS OF $x^i$ APPEARING IN $x$-dependence OF 'KASNER-LIKE' SOLUTION

# TODA-MODEL FOR DOMINANT WALLS

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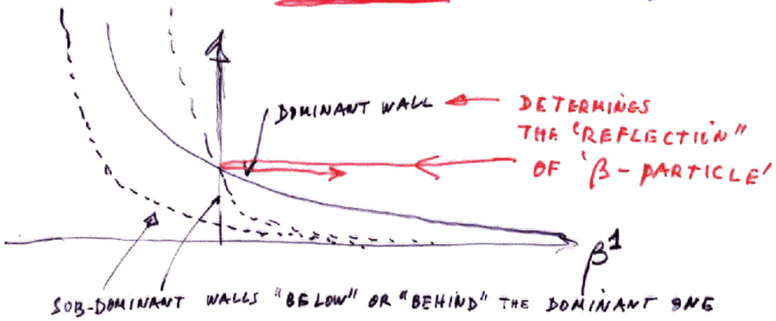


2 +1 BLOCKS OF EQUIVALENT TODA MODELS

$\mathcal{B}_2 = \{M, IIA, IIB\}$ ARE EQUIVALENT [ $M \leftrightarrow IIA$ EQUIV BECAUSE $KK_{red}$ ; $IIA \leftrightarrow IIB$ EQUIV: T-DUALITY ]

10 WALLS $\quad \beta^\mu = \beta^\mu_M$

$w^{[2]}_i(\beta) = -\beta^i + \beta^{i+1}$ $(i = 1 \dots 9)$ SYMMETRY WALLS

$w^{[2]}_{10}(\beta) = \beta^1 + \beta^2 + \beta^3$ ELECTRIC 3-FORM WALL

$\mathcal{B}_1 = \{I, HO, HE\}$ ARE EQUIVALENT [ $I \leftrightarrow$ HET EQUIV: S-DUALITY ]

10 WALLS $\quad \alpha^\mu \equiv (\beta^0_S, \beta^i_S)$ STRING-FRAME

$w^{[1]}_1(\alpha) = \alpha^1$ ELECTRIC 1-FORM WALL

$w^{[1]}_i(\alpha) = -\alpha^{i-1} + \alpha^i \quad i = 2, \dots, 9$ SYMMETRY

$w^{[1]}_{10} = \alpha^0 - \alpha^7 - \alpha^8 - \alpha^9$ MAGNETIC NS 2-FORM

$\mathcal{B}_0 = \{$CLOSED BOSONIC $D = 10\}$

10 WALLS

$w^{(0)}_1(\alpha) = \alpha^1 + \alpha^2$ ELECTRIC 2-FORM WALL

$w^{(0)}_i(\alpha) = -\alpha^{i-1} + \alpha^i \quad i = 2, \dots 9$ SYMMETRY

$w^{(0)}_{10}(\alpha) = \alpha^0 - \alpha^7 - \alpha^8 - \alpha^9$ MAGNETIC 2-FORM

# COXETER GROUP OF REFLECTIONS
# COXETER-DYNKIN DIAGRAMS

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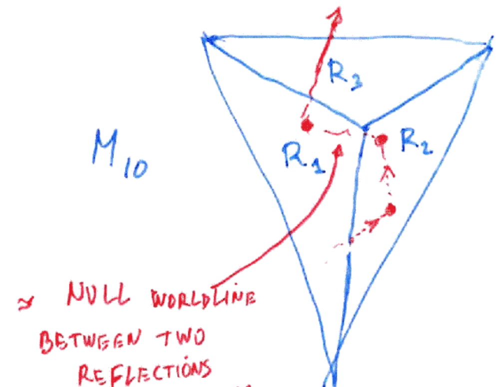
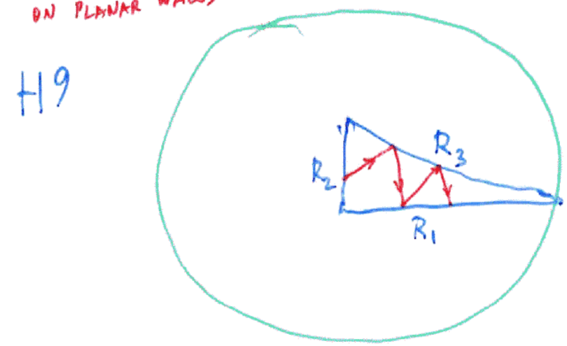



IN SHARP WALL APPROXIMATION DYNAMICS IS ENCODED IN A SEQUENCE OF REFLECTIONS IN HYPERPLANES

$\dots R_3 R_{10} \dots R_4 R_8 R_3 R_1 \dots$




QUESTIONS: IS THE GROUP GENERATED BY $R_1 \dots R_{10}$ DISCRETE?

IS IT A REMARKABLE GROUP?

→ COXETER-DYNKIN DIAGRAM: $R_i \leftrightarrow \overset{i}{\circ}$

$(R_i R_j)^{m_{ij}} = 1 \quad$ $m_{ij}=2$: $\circ \quad \circ$ (COMMUTE), $\quad m_{ij}=3$: $\circ\!-\!\circ$, $\quad m_{ij}=4$: $\circ\!=\!\circ$, $\dots$

$\theta_{ij} = \text{ANGLE}(w_i, w_j) = \frac{\pi}{m_{ij}} \qquad \theta_{ij} = \frac{\pi}{3} \qquad \theta_{ij} = \frac{\pi}{4}$

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# KAC-MOODY ALGEBRAS, DYNKIN DIAGRAMS

$SU(2)$ or $SL(2) = A_1$: $J_z$; $J_x + i J_y \equiv J_+$ RAISING OP.; $J_x - i J_y \equiv J_-$ LOWERING OP.

$$[J_z, J_+] = J_+$$
$$[J_z, J_-] = -J_-$$
$$[J_+, J_-] = 2 J_z$$

KAC-MOODY : $J_z \to h \in$ VectorSpace = CartanSubAlgebra. $[h_i, h_j] = 0$

RAISING OP.: $J_+ \to E_\alpha^{(s)}$ : $[h, E_\alpha^{(s)}] = \langle \alpha, h \rangle E_\alpha^{(s)}$

POSITIVE ROOT; ROOT; = LINEAR FORM in $h \in CSA$

LOWERING OP.: $J_- \to F_\alpha^{(s)} = E_{-\alpha}^{(s)}$ $[h, F_\alpha^{(s)}] = -\langle \alpha, h \rangle F_\alpha^{(s)}$

NEGATIVE ROOT

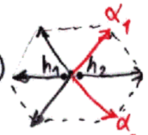
$$[E_\alpha^{(s)}, E_\beta^{(t)}] = N_{\alpha,\beta}^{s,t} \, E_{\alpha+\beta}$$

DECOMPOSITION IN SIMPLE ROOTS: $\alpha = \sum_{i=1}^{r} n^i \alpha_i$ $n^i \in \mathbb{N}$ FOR $>0$ ROOT

SIMPLE ROOTS

CARTAN MATRIX: $\boxed{a_{ij} = 2 \dfrac{(\alpha_i | \alpha_j)}{(\alpha_i | \alpha_i)}}$

ROOT DIAGRAM: DYNKIN DIAGRAM




ENCODES THE ANGLE BETWEEN $\alpha_1, \alpha_2$ $(\alpha_1 | \alpha_2) = -1$

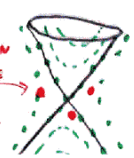
$a_{ij}$ +ve definite : Euclidean root system. e.g. $SU(3)$ Finite-dim Lie Alg. $r = 2$

$\det a_{ij} = 0$, but rank $a_{ij} = \dim - 1$ : Affine Kac-Moody Alg.

signature$(a_{ij}) = - + + \cdots +$ : Lorentzian Kac-Moody Alg.; e.g. $A_1^{\wedge\wedge} = A_1^{(H)} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -2 & 2 \end{pmatrix}$

HYPERBOLIC IF SUB-DYNKIN DIAGRAMS AFFINE OR EUCLIDEAN

3 SIMPLE ROOTS $\alpha^2 = 2$



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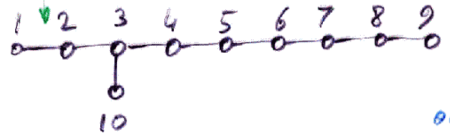
# DYNKIN DIAGRAMS OF SUPERSTRING COSMOLOG. BILLIARDS

means dihedral angle = 120°

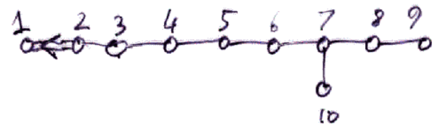
$B_2 = \{M, IIA, IIB\}$




$E_{10}$

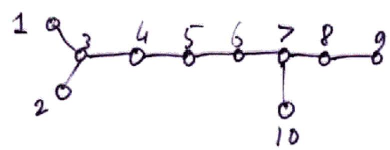
OK WITH EXPECTATION OF JULIA '82 FOR 1-D REDUCTION OF SUGRA

$B_3 = \{ I, HE, HO \}$




$BE_{10}$

$\Big[$ $B_0 = \{$closed bosonic $D=10\}$




$DE_{10}$ $\Big]$

C+N IDENTIFY: $\beta^\mu \leftrightarrow h = \sum_{\mu=1}^{10} \beta^\mu h_\mu$ CARTAN ELEMENT

LOG. SCALE FACTORS

$w_i \leftrightarrow$ SIMPLE ROOT OF $E_{10}$ OR $BE_{10}$

DOMINANT WALL FORMS

| | | |
|---|---|---|
| GROUP OF REFLECTIONS IN COSMO. BILLIARD | $\leftrightarrow$ | WEYL GROUP OF $E_{10}$ OR $BE_{10}$ |
| BILLIARD TABLE | $\leftrightarrow$ | WEYL CHAMBER |

NB: $E_{10}$, $BE_{10}$ ARE HYPERBOLIC KAC-MOODY ALGEBRAS.

signature$\left(\dfrac{a_{ij}}{d_i}\right) = - + + \cdots +$ ; INFINITE NUMBER OF ROOTS

INFINITE (BUT DISCRETE) WEYL GROUP

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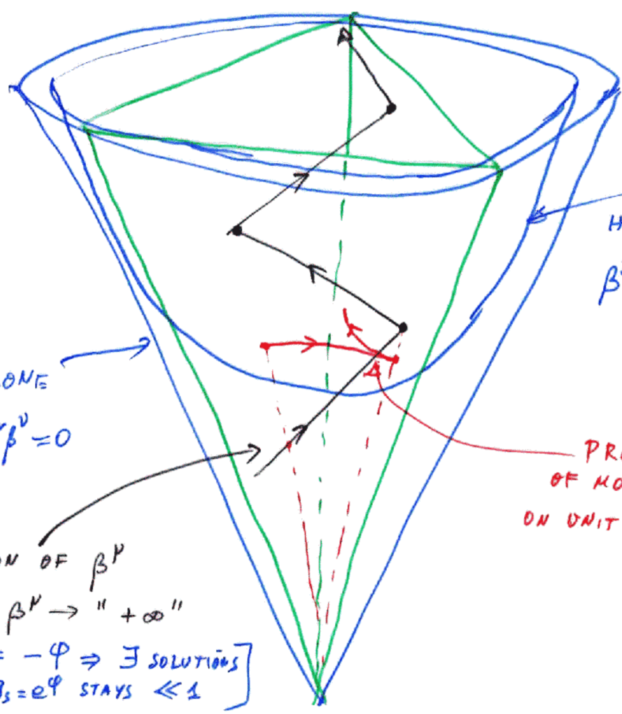
# PROJECTED COSMOLOGICAL BILLIARD



UNIT HYPERBOLOID $\beta^2 = -1$

LIGHT CONE $\beta^2 \equiv G_{\mu\nu}\beta^\mu\beta^\nu = 0$

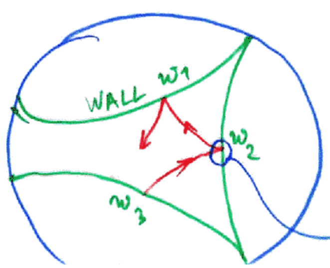
PROJECTION OF MOTION OF $\beta^\mu$ ON UNIT HYPERBOLOID

MOTION OF $\beta^\mu$ WITH $\beta^\mu \to$ " $+\infty$ "

[NB: AS $\beta^{10} = -\varphi \Rightarrow \exists$ SOLUTIONS WHERE $g_s = e^\varphi$ STAYS $\ll 1$]

PROJECTION MAP: $\underset{\text{LOGARITHMIC SCALE FACTORS}}{\beta^\mu} \longrightarrow \underset{\text{RATIOS OF LOG. SCALE FACTORS}}{\gamma^\mu \equiv \frac{\beta^\mu}{\sqrt{-\beta^2}}}$ : $\gamma^2 = -1$

UNIT HYPERBOLOID = LOBATCHEVSKII SPACE (HYPERBOLIC SPACE) : eg. $H_9$ FOR STRING/M, $H_2$ FOR D=4 GR




BILLIARD ON HYPERBOLIC SPACE

COLLISIONS BECOME SHARP WHEN $t \to 0$

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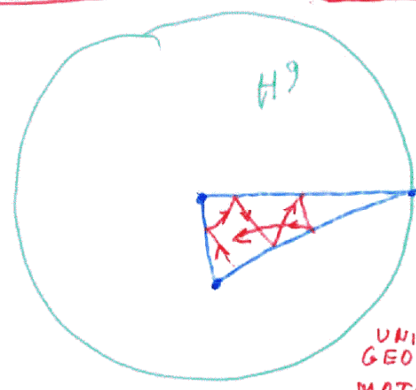
# CHAOTIC NATURE OF GENERIC COSMOLOGICAL SOLUTION

$\boxed{\text{SHARP-WALL APPROX}}$ PROJECTED ON $H^9$




$\frac{d \ln \rho}{dT} = \text{const} = 1$ as $\rho \to \infty$

new time

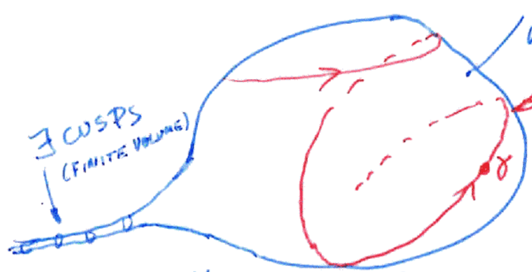
$$S = k \int dT \left[ -\left(\frac{d \ln \rho}{dT}\right)^2 + \left(\frac{d\vec{\gamma}}{dT}\right)^2 - V_T(\rho, \vec{\gamma}) \right]$$

UNIT-VELOCITY GEODESIC MOTION ON $H^9$

→ SHARP WALL AS $\rho \to \infty$

POINCARÉ THM + SELBERG'S LEMMA

⇒ USING SOME REFLECTIONS ⇒ ∃ HYPERBOLIC, FINITE-VOLUME SMOOTH MANIFOLD $\mathcal{M}$ WITHOUT BOUNDARY OBTAINED BY QUOTIENTING $H^9$ BY TORSION-FREE DISCRETE GROUP




GEODESIC MOTION ON $\mathcal{M}$

CURV $\mathcal{M} = -1$

→ THM OF HADAMARD-ANOSOV.

THE MOTION OF $\gamma$ IS ERGODIC, MIXING (ANOSOV FLOW) + POSITIVE ENTROPY, ∃ POSITIVE LYAPUNOV EXP...

QUANTUM BILLIARD: EXCEPTIONAL ARITHMETICAL CHAOS(?)

# KALUZA-KLEIN COSMOLOGY 12.1

PURE GRAVITY in $D \equiv d+1$: $S = \int d^D x \sqrt{g}\, R(g)$

GRAVITATIONAL WALL: $w_1(\beta) = 2\beta^1 + \sum_{i=2}^{d-2} \beta^i$ $(d \geq 4)$

$[w_1(\beta) = 2\beta^1$ in $d=3]$

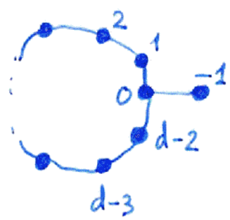
SYMMETRY WALLS: $w_i(\beta) = \beta^i - \beta^{i-1}$ $(i = 2, \ldots, n \equiv d-2)$

$w_0(\beta) = \beta^{d-1} - \beta^{d-2}$

$w_{-1}(\beta) = \beta^d - \beta^{d-1}$

DYNKIN DIAGRAM:

$AE_d \equiv A^{\wedge\wedge}_{d-2} \equiv A^{H}_{d-2}$




$d \geq 4$

$AE_3 \equiv A_1^{\wedge\wedge}$


$d = 3$

USUAL BKL CASE

$$A_{ij} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -2 & 2 \end{pmatrix}$$

## MORE GENERAL MODELS

Breitenlohner, Maison, Gibbons; Cremmer, Julia, Lü, Pope

Families of Lagrangians for $g_{\mu\nu}$ + other fields having classical groups $G$ as symmetry in $D=3$-reduction: $A_n, B_n, C_n, D_n, E_n, F_4, G_2$

GENERAL RESULT: ...

# CHAOS AND HYPERBOLIC KAC-MOODY ALGEBRAS 12.2

IN ANY DIMENSION THE COSMOLOGICAL OSCILLATIONS ARE DESCRIBED BY AN INDEFINITE KM ALGEBRA

SYMMETRIZABLE ALGEBRA: CARTAN-KILLING METRIC IS LORENTZIAN

"HYPERBOLIC" KM ≡ ANY SUBDIAGRAM IS FINITE OR AFFINE

NB: THE "GERM" OF $AE_d \equiv A^{\wedge\wedge}_{d-2}$ IS

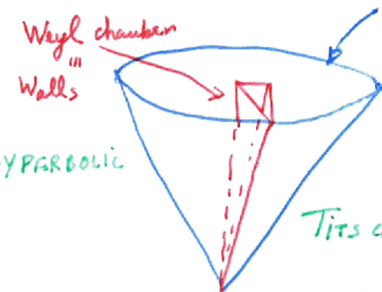
$A_{d-2}$ = Ehlers SYMMETRY GROUP OF THE COMPACTIFICATION OF $\int d^{d+1}x \sqrt{g}\, R(g)$ DOWN TO 3 DIMENSIONS

LINK CHAOS ↔ HYPERBOLICITY

$d \leq 9$

$D \leq 10$

Lorentz (future) cone $C^+$




HYPERBOLIC

$W^+ \subset C^+$

AND

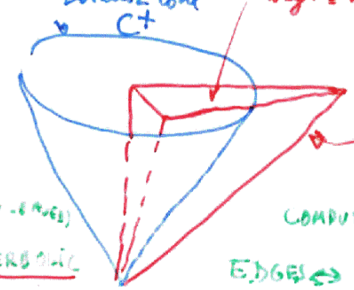
Tits cone $= \bigcup W_i(W^+) = C^+$

$d \geq 10$

$D \geq 11$

Lorentz cone $C^+$

Weyl = Wall = $W^+$



∃ spacelike edges

INDEFINITE (LORENTZIAN-ALGEBRA)

BUT NON HYPERBOLIC

COMPUTATION:

EDGES ↔ FUNDAMENTAL

E1

# $E_{10}$ AND A $\left\{\begin{matrix}\text{"SMALL TENSION"} \\ \text{BKL-LIKE}\end{matrix}\right\}$ EXPANSION OF M THEORY

UP TO NOW ONLY THE WEYL GROUP OF SOME HYPERBOLIC KM GROUP APPEARED, OR MORE PRECISELY: AT EACH SPATIAL POINT $x$ ASYMPTOTIC DYNAMICS OF SCALE FACTORS

$$ds^2 \simeq -N^2 d\tau^2 + \sum e^{-2\beta^a}(\theta^a)^2 \quad \text{GIVEN BY:}$$

$$\mathcal{L}_\tau = \underbrace{\sum_a (\dot\beta^a)^2 - \Big(\sum_a \dot\beta^a\Big)^2}_{G_{ab}\,\dot\beta^a\dot\beta^b} - \sum_i C_i(x)\, e^{-2\, w_i(\beta)}$$

Lorentz metric

LINEAR FORMS DESCRIBING THE SIMPLE ROOTS OF KM

BUT THE FULL KM GROUP HAS AN INFINITE # OF ROOTS:

ROOT SPACE

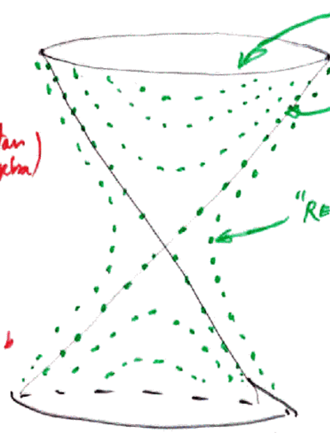
i.e. LINEAR FORMS ON $\beta^a$ (= Cartan subalgebra)

i.e. "covectors"

$\alpha(\beta) \equiv \alpha_a \beta^a$

DUAL METRIC

$(\alpha, \alpha') \equiv G^{ab}\alpha_a \alpha_b$



"IMAGINARY ROOTS" $\alpha^2 \leq 0$

"REAL ROOTS" $\alpha^2 \equiv (\alpha,\alpha) > 0$

SPACELIKE

"POSITIVE ROOTS" OF KM ALG:

$$\alpha = \sum_i n^i \alpha_i$$

"simple" roots : $\alpha_i(\beta) \equiv w_i(\beta)$

$i = 1, \cdots, r \equiv$ rank

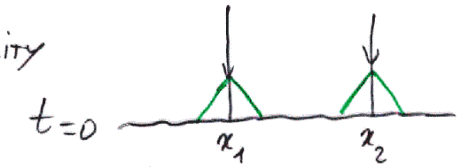
non-negative integers $n^i \in \mathbb{N}$

E2

# BKL AS A "SMALL TENSION EXPANSION"

NEAR SPACELIKE SINGULARITY

$\neq$ SPACE POINTS DECOUPLE




$\boxed{\text{"AS IF" } c \to 0}$ : $\ell_{\text{HORIZON}} \sim ct$ becomes smaller than $|x_2 - x_1|$

GRAVITATIONAL WAVES: $S = \frac{1}{2}\int d\hat t\, d^{10}x \left[\rho_b (\partial_t h_{ij})^2 - T_b (\partial_x h_{ij})^2\right]$

$\rho_b = \dfrac{c^2}{32\pi G}$

bulk tension $T_b = \dfrac{c^4}{32\pi G}$

$c = \sqrt{\dfrac{T_b}{\rho_b}}$

INTUITIVELY: ONE CAN THINK OF THE BKL-TYPE EXPANSION $\frac{1}{c}\frac{\partial h}{\partial t} \gg \frac{\partial h}{\partial x}$ AS A "$T_b \to 0$" LIMIT

SUGGESTS A POSSIBLE LINK WITH THE $T_s \sim M_s^2 \sim \frac{1}{\alpha'} \to 0$ LIMIT $[\alpha' \to \infty]$ CONSIDERED BY D. GROSS '88

NB: MOST OSCILLATIONS OCCUR WHEN $t/\ell_s = M_s t \ll 1$

THE $T_s \to 0$ LIMIT GIVES RISE TO AN INFINITE # OF RELATIONS BETWEEN STRING SCATTERING AMPLITUDES, SUGGESTING THE RESTORATION OF AN INFINITE SYMMETRY (Gross '88)

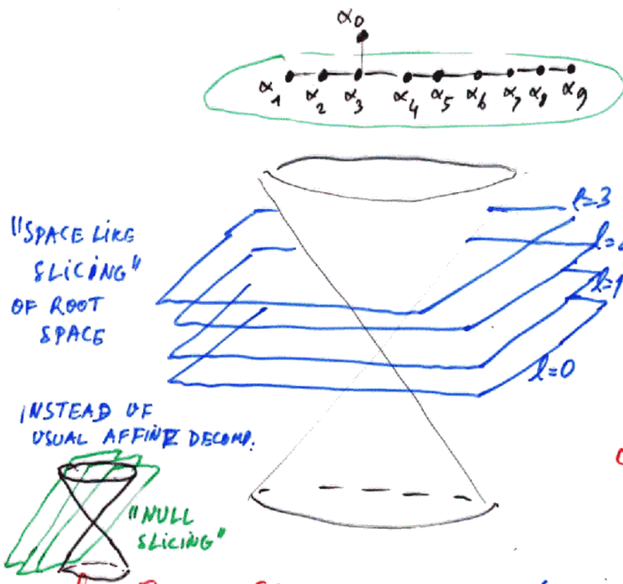
MOREOVER, Harvey, Moore '96 suggested $T_s \to 0$ reveals the role of the infinite tower of BPS states as a tower of "gauge bosons"

ANYWAY: We use an expansion linked to $t \to 0$ as a way a revealing a symmetry structure of the massless bosonic sector of string/M theory

E3

## NEW DECOMPOSITION OF $E_{10}$ : WRT $GL(10)$ SUBALGEBRA

$SL(10) = A_9$ SUBALGEBRA




ROOTS $\alpha = \ell\,\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j$

"LEVEL"

$ht(\alpha) = \ell + \sum_{j=1}^{9} m^j$

$\alpha_0 = \alpha_0^{\perp} + \alpha_0^{\parallel}$

TIME-LIKE $(\alpha_0^{\perp})^2 < 0$ ; IN $SL(10)$ HYPERPLANE $\{\alpha_i\}$

$\ell = 0$ : $GL(10)$ SUBALGEBRA : $[K^a{}_b, K^c{}_d] = K^a{}_d\,\delta^c_b - K^c{}_b\,\delta^a_d$

$\ell = 1$ : GENERATORS $E^{[a_1 a_2 a_3]}$, $F_{[a_1 a_2 a_3]}$ : $[K, E] = E$ ; $[E, F] = K$

$\ell = 2$ : $E^{[a_1 \cdots a_6]}$, $F_{[a_1 \cdots a_6]}$ :

$\ell = 3$ : $E^{a_0 | a_1 \cdots a_8}$, $F_{a_0 | a_1 \cdots a_8}$ :

$\ell = 4$ : $\oplus$

DETERMINATION OF COMMUTATORS : $[x, y] = z$

AND INVARIANT BILINEAR FORM : $(x \mid y)$

E4

## CONSTRUCTION OF A ONE-DIM. $E_{10}$-INVARIANT COSET MODEL

INFINITE DIMENSIONAL COSET SPACE $E_{10} / K(E_{10})$

MAXIMAL COMPACT SUBGROUP OF THE CANONICAL REAL FORM OF $E_{10}$

GENERIC $E_{10}$ GROUP ELEMENT

$$\mathcal{V} = e^{X} \quad X \in Lie(E_{10}) : X = X^i h_i + \sum_{\alpha \in \Delta^+} X^{\alpha,s} E^{(s)}_{\alpha} + \sum_{\alpha \in \Delta^+} Y^{\alpha,s} \underbrace{F^{(s)}_{\alpha}}_{E^{(s)}_{-\alpha}}$$

IWASAWA DECOMP. OF $E_{10}$ : $\mathcal{V} = K\,A\,N$

COMPACT ; ABELIAN ; "NULL"

CHEVALLEY INVOLUTION $\omega$ : $\omega(h_i) = -h_i$ ; $\omega(e_i) = -f_i$ ; $\omega(f_i) = -e_i$

DEFINE "TRANSPOSE" : $E^T_{\alpha,s} \equiv -\omega(E_{\alpha,s}) \propto F_{\alpha,s}$

$Lie\,K(E_{10})$ : SPANNED BY "ANTISYMMETRIC ELEMENTS": $E_{\alpha,s} - E^T_{\alpha,s}$

$E_{10}/K(E_{10})$ DEFINED BY
- GAUGE-FIXED $\mathcal{V} = A\,N = e^{X^i h_i + \sum_{\alpha \in \Delta^+} X^{\alpha,s} E_{\alpha,s}}$

  CARTAN ; ONLY RAISING
- PROJECTED DERIVATIVE: $v_{sym} \equiv \frac{1}{2}(v + v^T)$

$E_{10}$- INVARIANT ONE-DIMENSIONAL COSET MODEL:

$$\boxed{S_1^{(E_{10})} = \int \frac{dt}{n(t)} (v_{sym} \mid v_{sym}) = \int \frac{dt}{n(t)} \frac{1}{4} \left( \frac{d\mathcal{V}}{dt}\mathcal{V}^{-1} + \left(\frac{d\mathcal{V}}{dt}\mathcal{V}^{-1}\right)^T \Big| \frac{d\mathcal{V}}{dt}\mathcal{V}^{-1} + \left(\frac{d\mathcal{V}}{dt}\mathcal{V}^{-1}\right)^T \right)}$$

IMPOSES THE CONSTRAINT

VELOCITY $v = \frac{d\mathcal{V}}{dt}\mathcal{V}^{-1}$

PROJECTION $\perp K$

E5

## EXPLICIT PARAMETRIZATION OF $E_{10}/K(E_{10})$

$$\mathcal{V}(t) = e^{h^a{}_b(t) K^b{}_a} \, e^{\frac{1}{3!} A_{abc}(t) E^{abc} + \frac{1}{6!} A_{a_1\cdots a_6}(t) E^{a_1\cdots a_6} + \frac{1}{9!} A_{a_0|a_1\cdots a_8}(t) E^{a_0|a_1\cdots a_8} + \cdots}$$

"SL(10) field" $h^a{}_b(t)$; SL(10) generators; $\ell=1$ generators $A_{[abc]}(t)$; $\ell=2$ $A_{[a_1\cdots a_6]}(t)$; $\ell=3$ $A_{a_0|a_1\cdots a_8}(t)$

REMINISCENT OF SIMILAR ALGEBRAIC CONSTRUCTIONS OF Cremmer, Julia, Lu, Pope 98 and West '01

EXPLICIT FORM OF $E_{10}$-INVARIANT ACTION $S_1 = \int dt \, \mathcal{L}_1^{E_{10}}$

$$n \mathcal{L}_1^{E_{10}} = \frac{1}{4}\left(g^{ac}g^{bd} - g^{ab}g^{cd}\right) \dot{g}_{ab}\dot{g}_{cd} + \frac{1}{2}\frac{1}{3!} DA_{a_1a_2a_3} DA^{a_1a_2a_3}$$
$$+ \frac{1}{2}\frac{1}{6!} DA_{a_1\cdots a_6} DA^{a_1\cdots a_6} + \frac{1}{2}\frac{1}{9!} DA_{a_0|a_1\cdots a_8} DA^{a_0|a_1\cdots a_8} + \cdots$$

ALL INDICES RAISED BY $g^{ab}$

WHERE
$$g^{ab} = e^a{}_c e^b{}_c \quad \text{WITH} \quad e^a{}_b \equiv (\exp h)^a{}_b$$

AND
$$DA_{a_1\cdots a_3} = \dot{A}_{a_1a_2a_3}$$
$$DA_{a_1\cdots a_6} = \dot{A}_{a_1\cdots a_6} + 10 \, A_{[a_1\cdots a_3} \dot{A}_{a_4a_5a_6]}$$
$$DA_{a_1|a_2\cdots a_9} = \dot{A}_{a_1|a_2\cdots a_9} + 42 A_{\langle a_1\cdots a_3} \dot{A}_{a_4\cdots a_9\rangle} - 42 \dot{A}_{\langle a_1\cdots a_3} A_{a_4\cdots a_9\rangle}$$
$$+ 280 \, A_{\langle a_1\cdots a_3} A_{a_4\cdots a_6} \dot{A}_{a_7a_8a_9\rangle}$$

NB: ALL NUMERICAL COEFFICIENTS ARE UNIQUELY FIXED (MODULO FIELD REDEFINITIONS) BY THE STRUCTURE OF E

E6

## COMPARISON WITH CJS EQS OF MOTION

$$S_{11}^{(SUGRA)} = \int d^{11}x \sqrt{G}\left[\frac{1}{4}R(G) - \frac{1}{48}\mathcal{F}_{MNPQ}\mathcal{F}^{MNPQ} + \frac{2}{(12)^4}\varepsilon^{PQ\cdots Z}\mathcal{F}_{PQRS}\mathcal{F}_{TUVW}\mathcal{A}_{XYZ}\right]$$
$$\mathcal{F}_4 = d\mathcal{A}_3$$

EQS OF MOTION IN ZERO-SHIFT SLICING

$$ds^2 = -N^2 (dx^0)^2 + G_{ab}\theta^a\theta^b$$
$$\mathcal{F} = \frac{1}{3!}\mathcal{F}_{0abc}\, dx^0 \wedge \theta^a \wedge \theta^b \wedge \theta^c + \frac{1}{4!}\mathcal{F}_{abcd}\,\theta^a\wedge\theta^b\wedge\theta^c\wedge\theta^d$$

WITH TIME-INDEPENDENT SPATIAL ZEHNBEIN $\theta^a(x) = E^a{}_i(x)\, dx^i$

CHOOSE TIME-COORDINATE $x^0$ SO THAT $N = \sqrt{\det G_{ab}} \equiv \sqrt{G}$

NB: PROPER TIME $T = \int N\, dx^0 \neq x^0$ : $x^0 \to \infty$ AS $T \to 0$

$$\begin{cases} \partial_0\left(G^{ac}\partial_0 G_{cb}\right) = \frac{1}{6} G\mathcal{F}^{a\beta\gamma\delta}\mathcal{F}_{b\beta\gamma\delta} - \frac{1}{72} G\mathcal{F}^{\alpha\beta\gamma\delta}\mathcal{F}_{\alpha\beta\gamma\delta}\delta^a_b - 2G R^a{}_b(\Gamma, C) \\ \partial_0\left(G\mathcal{F}^{0abc}\right) = \frac{1}{144}\varepsilon^{abc a_1\cdots a_4 b_1\cdots b_4}\mathcal{F}_{0a_1\cdots a_4}\mathcal{F}_{b_1\cdots b_4} + \frac{3}{2} G\mathcal{F}^{de[ab}C^{c]}{}_{de} \\ \qquad - G\, C^e{}_{de}\mathcal{F}^{dabc} - \partial_d\left(G\mathcal{F}^{dabc}\right) \\ \partial_0 \mathcal{F}_{abcd} = 6\,\mathcal{F}_{0e[ab}C^e{}_{cd]} + 4\,\partial_{[a}\mathcal{F}_{0bcd]} \end{cases}$$

WHERE
$$2\,G_{ad}\Gamma^d{}_{bc} = C_{abc} + C_{bca} - C_{cab} + \partial_b G_{ca} + \partial_c G_{ab} - \partial_a G_{bc}$$

AND $C^a{}_{bc} = G^{ad} C_{dbc}$ ARE THE STRUCTURE COEFFICIENTS OF THE ZEHNBEIN: $d\theta^a = \frac{1}{2} C^a{}_{bc}\,\theta^b\wedge\theta^c$

E7

CORRESPONDENCE $SUGRA_{11} \longrightarrow E_{10}/K(E_{10})$

WHICH MAPS $\frac{\delta S_{11}^{SUGRA}}{\delta G_{\mu\nu}} = 0 = \frac{\delta S_{11}^{SUGRA}}{\delta A_{\mu\nu\lambda}} \longrightarrow 0 = \frac{\delta S^{E_{10}\ COSET}}{\delta h^a{}_b} = \frac{\delta S_1}{\delta A_{a_1 a_2 a_3}} = \frac{\delta S_1}{\delta A_{a_1 \cdots a_6}} = \cdots$

$E_{10}$ COSET ELEMENT UP TO $30^{TH}$ ORDER IN HEIGHT

$$\mathcal{V}(t) = e^{h^a{}_b(t) K^b{}_a}\, e^{\frac{1}{3!} A_{abc}(t) E^{abc} + \frac{1}{6!} A_{a_1\cdots a_6} E^{a_1\cdots a_6} + \frac{1}{9!} A_{a_0|a_1\cdots a_8}(t) E^{a_0|a_1\cdots a_8} + \cdots}$$

PARAMETRIZED BY: $h^a{}_b(t)$ $\quad A_{a_1 a_2 a_3}(t)$ $\quad A_{a_1\cdots a_6}(t)$ $\quad A_{a_0|a_1\cdots a_8}(t)$ $\quad \cdots$

SUGRA FIELD: $G_{\mu\nu}(x^0, \vec{x})$, $\mathcal{A}_{\mu\nu\lambda}(x^0, \vec{x})$

$$ds^2 = -N^2 (dx^0)^2 + G_{ab}\, \theta^a \theta^b$$

$$\mathcal{F} = d\mathcal{A} = \frac{1}{3!} \mathcal{F}_{0abc}\, dx^0 \wedge \theta^a \wedge \theta^b \wedge \theta^c + \frac{1}{4!} \mathcal{F}_{abcd}\, \theta^a \wedge \theta^b \wedge \theta^c \wedge \theta^d$$

MOVING FRAME: $\theta^a(x) = e^a{}_i(x)\, dx^i$

MAP: SUGRA $\rightarrow E_{10}/K(E_{10})$ : AT ANY FIXED SPATIAL $x$

$$t = x^0 = \int \frac{dT}{\sqrt{G}}$$

$$(e^{h(t)})^a{}_c\, (e^{h(t)})^b{}_c = G^{ab}(t, x)$$

$$\frac{dA_{a_1 a_2 a_3}}{dt} = DA_{a_1 a_2 a_3}(t) = \mathcal{F}_{0 a_1 a_2 a_3}(t, x)$$

$$\frac{d}{dt} A_6 + A_3 \dot{A}_3 \rightarrow DA^{a_1\cdots a_6}(t) = -\frac{1}{4!} \varepsilon^{a_1\cdots a_6 b_1\cdots b_4} \mathcal{F}_{b_1\cdots b_4}(t, x)$$

$$\dot{A}_9 + A_6 \dot{A}_3 + \cdots + A_3 A_3 \dot{A}_3 \rightarrow DA^{b|a_1\cdots a_8}(t) = +\frac{3}{2} \varepsilon^{a_1\cdots a_8 b_1 b_2} \left( C^b{}_{b_1 b_2}(x) + \frac{2}{9} \delta^b_{b_1} C^c{}_{b_2 c}(x) \right)$$

E8

THIS MAP BETWEEN

SUGRA geometric data at any given spatial point $x_0$

$$\left.\begin{array}{l} G_{ab}(t, x_0) \\ \mathcal{A}_{abc}(t, x_0) \\ \partial_a \mathcal{A}_{bcd}(t, x_0) \\ C^a{}_{bc}(t, x_0) \end{array}\right\} \longrightarrow \left\{\begin{array}{l} h_a{}^b(t) \\ A_{abc}(t) \\ A_{a_1\cdots a_6}(t) \\ A_{a_0|a_1\cdots a_8}(t) \end{array}\right.$$

PARAMETERS OF $E_{10}/K(E_{10})$ coset

TRANSFORMS

EOM $SUGRA_{11} \longrightarrow$ NULL GEODESIC OF COSET SPACE $E_{10}/K(E_{10})$


UP TO HEIGHT $< 30$ (INSTEAD OF $ht = 1$, BEFORE)

(AND LEVEL $\ell = 3$ INCLUDED)

HEIGHT EXPANSION $\sim$ EXPANSION IN DECREASING EXPONENTIALS $\sim$ EXPANSION IN SUCCESSIVE SPATIAL GRADIENTS

$ht(\alpha) = \sum m^i$ if $\alpha = \sum m^i \alpha_i$

$V(\beta) \sim \sum e^{-2\alpha(\beta)}$

- FOR $HT \leq 30$, PERFECT (AND REDUNDANT) MATCH OF ALL COEFFICIENTS
- $\ell = 1$ is SIMPLEST 1-DIM REDUCTION: $G_{ij}(t)$, $\mathcal{A}_{ijk}(t)$
  $\ell = 2$ AND $\ell = 3$ INCLUDE MORE AND MORE $x$-DEPENDENCE
- BEYOND $HT \geq 30$, ? MAP EXTENDED BY CONSIDERING HIGHER SPATIAL GRADIENTS OF $\mathcal{A}_{\mu\nu\lambda}(t,x)$, $G_{\mu\nu}(t,x)$ ?
- $\exists$ ENOUGH ROOM in $E_{10}/K(E_{10})$ FOR ALL SPATIAL GRADIENTS
  3 INFINITE TOWERS: $E_0^{b_1 b_2 b_3}$ $\quad E^{b_1\cdots b_6}$ $\quad E^{b_0|b_1\cdots b_8}$

14.2 CONC. 1

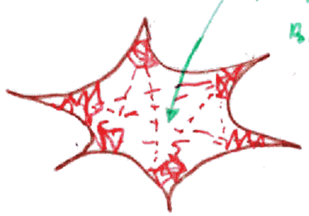
⇒ CHAOTIC QUANTUM BILLIARD SCENARIO OF VACUUM SELECTION IN STRING COSMOLOGY

(Damour, Henneaux 2001, à la Horne, Moore 1994)

∃ 6 STRING THEORIES


⇒ COMBINED QUANTUM BILLIARD, WITH CHAOTIC OSCILLATIONS WITHIN AND BETWEEN VARIOUS STRING MODELS

⇒ UNTIL THE UNIVERSE FINDS AN UNSTABLE (INFLATION?) PATCH OF PARAMETER SPACE WHICH CAN "BLOW UP" INTO OUR UNIVERSE



TUNNELING TRANSITIONS BETWEEN THE SEPARATE CHAOTIC BILLIARDS

↓

DEFINE (?) THE GENERIC INITIAL STATE OF THE UNIVERSE

15 CONC. 2

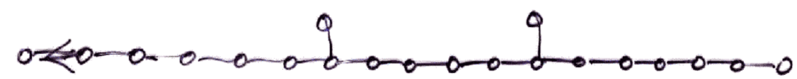
A "CHAOTIC QUANTUM BILLIARD" SCENARIO OF VACUUM SELECTION IN STRING THY

FOR FINITE-SPATIAL VOLUME UNIVERSES, THERE SHOULD EXIST A FINITE TRANSITION AMPLITUDE BETWEEN MODULI SPACES OF $\mathcal{B}_2$ AND $\mathcal{B}_1$

⇒ INITIAL STATE OF UNIVERSE MIGHT BE A COMBINED QUANTUM BILLIARD

? MAYBE A − − + + ⋯ + HYPER-HYPERBOLIC KM ALG.?

$F_{20}$ ?



(KAC + DAMOUR, HENNEAUX)

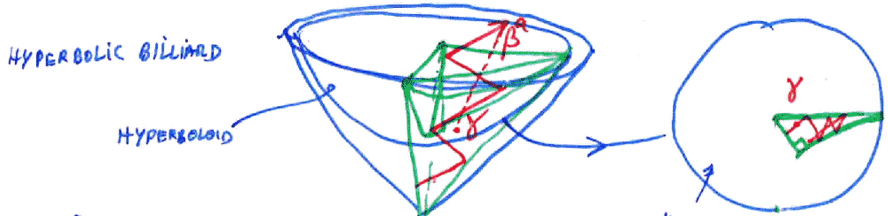
SIMILAR TO PROPOSAL OF HORNE, MOORE '94

BECAUSE OF FINITE VOLUME OF THIS (PROJECTED) BILLIARD

⇒ FINITE PROBABILITY OF EXPLORING THE REGIONS OF MODULI SPACE WHICH HAVE A CHANCE OF EVOLVING INTO OUR UNIVERSE.

# CONCLUSIONS

- UP TO NOW MOST STUDIES IN STRING COSMOLOGY HAVE LOOKED AT HIGHLY SPECIAL SOLUTIONS, ADMITTING A KASNER-LIKE (POWER-LAW) BEHAVIOUR AS $t \to 0$
- HOWEVER THE R.R. FORMS NECESSARILY IMPLY AN (GENERIC) ULTIMATE CHAOTIC BKL-LIKE BEHAVIOUR NEAR $t \simeq 0$
- ASYMPTOTIC BEHAVIOUR IS PROVEN TO BE: $\underbrace{\text{ANOSOV, ERGODIC, MIXING}}_{\text{MOST "CHAOTIC"}}$,... WHEN PROJECTED ON HYPERBOLIC SPACE




- POSSIBLE CHAOTIC QUANTUM BILLIARD SCENARIO OF VACUUM SELECTION IN STRING TH.
- HOWEVER THIS HYPERBOLIC BILLIARD MOTION HAS A LINK WITH SYMMETRY:
  - WEYL CHAMBER OF HYP. KAC-MOODY ALGEBRA
- RECENT RESULTS, SUGGEST THE FULL $E_{10}(\mathbb{R})$ KAC-MOODY GROUP MAY BE A HIDDEN INFINITE-DIMENSIONAL SYMMETRY OF SUGRA$_{11}$ EOM: TRANSFORMING SOLN $\to$ SOLN' ( SIMILAR TO "GEROCH GROUP = $A_1^\wedge$" OR "$E_9$" FOR D=2 REDUCTION)
- BKL-LIKE EXPANSION WOULD BE A WAY OF REVEALING THIS ENORMOUS HIDDEN SYMMETRY,
- CAN BE VIEWED AS "SMALL (BULK) TENSION" LIMIT $T_b = \frac{c^4}{\dots} \to 0$