Eternal Inflation

AND TUNNELING IN STRINGY LANDSCAPE

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1. Various models of inflation
2. Stochastic approach to inflation and tunneling
3. Eternal inflation
4. Wandering in stringy landscape: diffusion, tunneling, and all that
5. Can we populate the landscape jumping from Minkowski space?

Problems solved

Inflation:

1. Homogeneity
2. Isotropy
3. Flatness/entropy/mass
4. Horizon
5. Creation of large-scale structure
6. Monopoles, gravitinos...
Example 1.

Chaotic inflation

\[ V = \frac{m^2 \varphi^2}{2} \]

Inflation may begin in a domain of smallest possible size \( M_p^{-1} \sim 10^{-33} \text{cm} \) with a total mass \( 10^{-9} \text{ g} \) and total entropy \( O(1) \).

Successful inflation requires long accelerated expansion and exponentially large increase of entropy.

Eternal inflation provides good conditions for justification of anthropic principle.

The main idea which simplifies investigation of inflation: no-hair theorem for D5 space.

Inflation can start in a domain \( O(H^{-1}) \) and then this domain can grow indefinitely large. The processes inside the domain occur completely independently of initial conditions at the beginning of inflation, and of the processes at the boundary of inflationary domain.

In this sense inflation is anti-holographic.
Example 2.

**Neur Inflation**

\[ V \]

\[ \Phi \]

Inflation begins in a domain of size \( H^{-1} \sim \frac{M_p}{M_{pl}} \sim 10^6 M_p^{-1} \) with total mass \( M_p \cdot \frac{M_p}{M_{pl}} \sim 10^{12} M_p \) and with total entropy \( \frac{M_p}{M_{pl}} \sim 10^9 \).

This is not a real solution of flatness, horizon and entropy problems... unless one uses eternal inflation.

Historically, new inflation was introduced as a result of supercooling in the process of high temperature phase transition of the hot Big Bang. These ideas did not work, because to get out of thermal equilibrium, we need to go by 80-10 to the inflation conditions. We may still use the potential along the lines of chaotic inflation (i.e., assuming chaotic initial conditions).
Example 3.

Cyclic inflation

Inflation begins in a domain of initial size $10^{28}$ cm with total mass $10^{55}$ g and total entropy $10^{88}$

Flatness/entropy/mass problem unsolved

Scalar field fluctuations

$\delta \phi \sim \frac{H}{2\pi}$

$H^{-1}$

This process produces perturbations of density which lead to galaxy formation and CMB anisotropy
Diffusion equation describes the probability to find a field $\Phi$ at a given point $x,t$

By $\Phi$ we understand its component with wavelength $\geq H^{-1}$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \Phi} \left( \frac{H^3}{8\pi^2} \frac{\partial P}{\partial \Phi} + \frac{P}{3H} \frac{dV}{d\Phi} \right)$$

Stationary solutions (as in thermal equilibrium)

$$P \approx \exp \left( \frac{3M_p^4}{8V(\phi)} \right) \cdot \exp \left( -\frac{3M_p^4}{8V(\phi)} \right)$$

Hawking, Moss 82
Starobinsky 85

Solution normalization factor

$V \sim M_p^4$, the field can easily jump over Planckian barriers

This can easily happen during eternal chaotic inflation.
In new inflation it is more difficult.

$$S = \ln P = \frac{3M_p^4}{8V(\phi)}$$

Entropy

True for ANY number of fields

As in thermodynamics

$$P = \exp \left( -\frac{3M_p^4}{8V(\phi)} \frac{dV}{V(\phi)} \right)$$

Suppose $V(\phi) \sim V(\phi)$ ($\Delta V \ll V$)

$$P \approx O(1) \text{ for } \Delta V \sim \frac{V^2}{M_p^4}$$

A.L. 98
Suskind 2002

ds entropy without use of Euclidean methods
Supersymmetric $SU(5)$

$V$

$SU(5) \quad SU(4) \times U(1) \quad SU(3) \times SU(2) \times U(1)$

- no difference between weak, strong and electromagnetic interactions
- different interactions
- weak, strong and q.m. interactions

Spontaneous decompactification

Inflating $d=4$ and $d=6$ universes

Kandinsky Universe
Eternal Chaotic Inflation

\[ V = \frac{m^2 \phi^2}{2} \]

Produces fluctuations of all other fields with the amplitude that can be as large as \( \delta \phi \sim M_p \)

This process continues eternally \( \Rightarrow \) the universe "probes" all of its possible vacuum states

For \( \phi > \phi^* \sim M_p \sqrt{\frac{M_p}{m}} \)

Brownian motion more efficient than classical

In such case: During the time \( \Delta t = H^{-1} \) the size of an inflationary domain grows \( e^3 \sim 20 \) times, its volume grows \( e^3 \sim 20 \) times, and almost in a half of this volume (approximately 10 times greater than the volume of the original domain) the field \( \phi \) increases rather than decreases.

As a result, the total volume of the universe filled by a large field \( \phi \) \( (\phi > \phi^* \sim M_p \sqrt{\frac{M_p}{m}}) \) exponentially grows in time without end.
The tools to investigate the new theory:

\[ V(\varphi, \varphi_0, t) \rightarrow \text{the total volume of all parts of the universe which began their evolution at } \varphi = \varphi_0 \text{ and contain field } \varphi \text{ at the moment } t \]

It can be shown that in many theories

\[ V(\varphi, \varphi_0, t) \sim e^{-\frac{3M_p^2}{8V(\varphi_0)}} P(\varphi) x \int dH_{\text{max}} e^{x} \]

Stationary distribution!

THE UNIVERSE as it is
Is inflation future/past eternal?

1) Inflation along each particular geodesic IS NOT future eternal, it ends within finite time.

We call inflation future eternal iff there is no upper bound on the proper time of duration of inflation along ALL geodesics.

2) Inflation along each particular geodesic IS NOT past eternal (see talk by Guth).

However, there are no statements about an upper bound of the duration of inflation in the past along ALL geodesics.

In this sense, there is no information whether inflation is past-eternal or not.

Only in the thin-wall approx.
Coleman-De Luccia instanton

\[ \begin{align*}
\phi & \quad \text{false vacuum} \\
0.35 & \quad \text{bubble} \\
0.25 & \quad \text{new instanton} \\
0.15 & \quad \text{open universe}
\end{align*} \]

\[ \begin{align*}
\alpha & \quad \text{open universe} \\
25 & \quad \text{Euclidean}
\end{align*} \]

Making sense of CDL instantons

Note that \( \Phi_i \) does not correspond to the false vacuum.

So how these instantons can describe decay of the false vacuum?
Possible answers:
(Euclidean approach allows many interpretations)

1) CDL tunneling
   - $\Psi_i$ to $\Psi_f$
   - $dS$ space
   - Collapsing open universe
   - Expanding open universe

2) CDL instanton
   - $\Psi_i$ to $\Psi_f$
   - Expanding open universes
   - Two expanding open universes
   - (like in Hawking-Turok case but without singularities)

3) Deforming CDL at the North pole (at the initial point $\Psi_i$)

This procedure makes sense if the corresponding change of action is small,

$\Delta S < 1$
Estimate the price for bending CDL instantons:

$$\Delta S \sim \int \left( \frac{d\phi}{dt} \right)^2 a^3(t) dt$$

kinetic energy (other terms can be made small)

$$\sim \left( \frac{d\phi}{dt} \right)^2 H^3 \sin^3 Ht dt$$

$$\sim \left( \frac{d\phi}{dt} \right)^2 (aT)^4$$

$$> (d\phi)^4$$

$$\frac{d\phi}{dt} < 1$$

(sub Planckian)

Must be smaller than 1

$$\Delta \phi < 1$$

Super Planckian jumps from $$\phi_0$$ to $$\phi_i$$ are forbidden

Example

HM instanton

Legitimate if $$\phi_i - \phi_0 = \Delta \phi < 1$$

Does not work for typical inflationary potentials with $$\Delta \phi > 1$$
Deviating from Euclidean path

\[ T_0 = \frac{H_0}{2\pi} \sim \frac{\sqrt{V_0}}{M_p} \]

\[ E_1 = V_1 \cdot H_1^{-3} = \frac{M_p^3}{\sqrt{V_1}} \]

\[ P_1 \sim e^{\exp(-\frac{E_1}{T_0})} = e^{\exp(-\frac{M_p^4}{\sqrt{V_1} V_0})} \]

But in fact jump probably must occur in the whole original causal patch

\[ E_{10} = V_1 \cdot H_0^{-3} \]

\[ P_2 = e^{\exp(-\frac{M_p^4 V_1}{V_0^2})} \]

In either case, \( P > 0 \).

Is it possible than new universe will be formed from ours?

Is it possible to create a new universe?

100ky. In inflationary cosmology the initial weight of the universe may be as small as \( 10^{-57} \) q.

Is it a big deal?
Can we jump up from Minkowski space?

Creation of the universe in a laboratory due to quantum fluctuations A.L. G1

What do we need?

\[ V = \frac{1}{4} \varphi^4 \]

1) \( \varphi \) must be greater than \( M_p \)

2) \( \ell > H^{-1}, H^{-1} \)

If \( \varphi > M_p \), then the condition \( \ell > H^{-1} \)

is sufficient

\[ \langle \varphi^2 \rangle \sim \int \frac{d^4 k}{k^2} \sim H^2 \sim \frac{\lambda \varphi^4}{M_p^2} \]

\[ P \sim \exp \left( -\frac{\varphi^2}{2 \langle \varphi^2 \rangle} \right) = \exp \left( -\frac{M_p^2}{\lambda \varphi^2} \right) \]

For \( \varphi \sim M_p \Rightarrow P \sim \exp \left( -\frac{\lambda}{2} \right) \]

\[
\langle \varphi^2 \rangle \sim \frac{T^2 V(\varphi)^{1/2}}{2\pi^2 \varphi} \\
\]

\[
P_T \sim \exp \left( -\frac{\varphi^2}{2 \langle \varphi^2 \rangle} \right) \sim \exp \left( -c \frac{T^2 \varphi^2}{T \sqrt{V}} \right)
\]

\[
V \sim \lambda \varphi^4
\]

\[
P_T \sim \exp \left( -c \frac{\pi^2 M_p^2}{\sqrt{V} T} \right)
\]

But:

\[
T^4 \ll V(\varphi)
\]

\[
P_T < \exp \left( -c \frac{\pi^2}{\lambda^{3/4}} \right)
\]
Interpretation

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