Friction, dilatancy, boundary conditions, and constitutive relations in shear thickening suspensions

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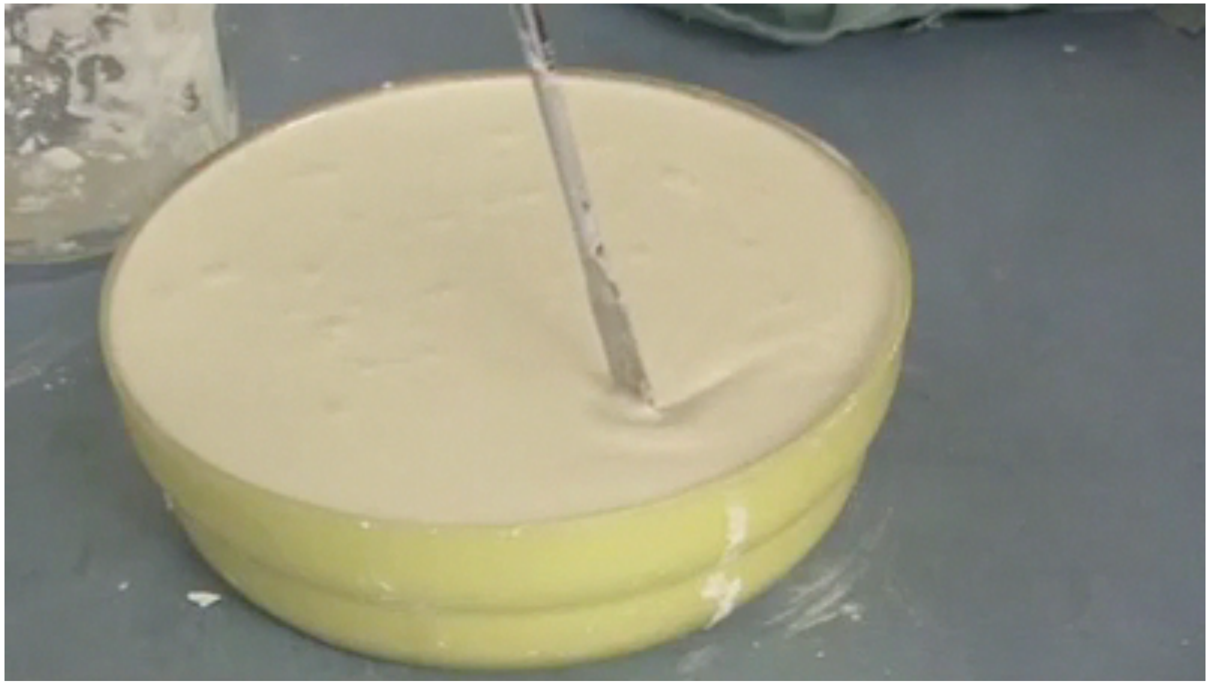
Discontinuous Shear Thickening (DST) fluids

suspension of cornstarch in water



Ben Allen

- What are *consequences* of friction-like stress scaling ($\tau=\mu\tau_N$) for constitutive relations, dilation, and the role of boundary conditions?
- What determines the strength of shear thickening (i.e. maximum stress scale)?

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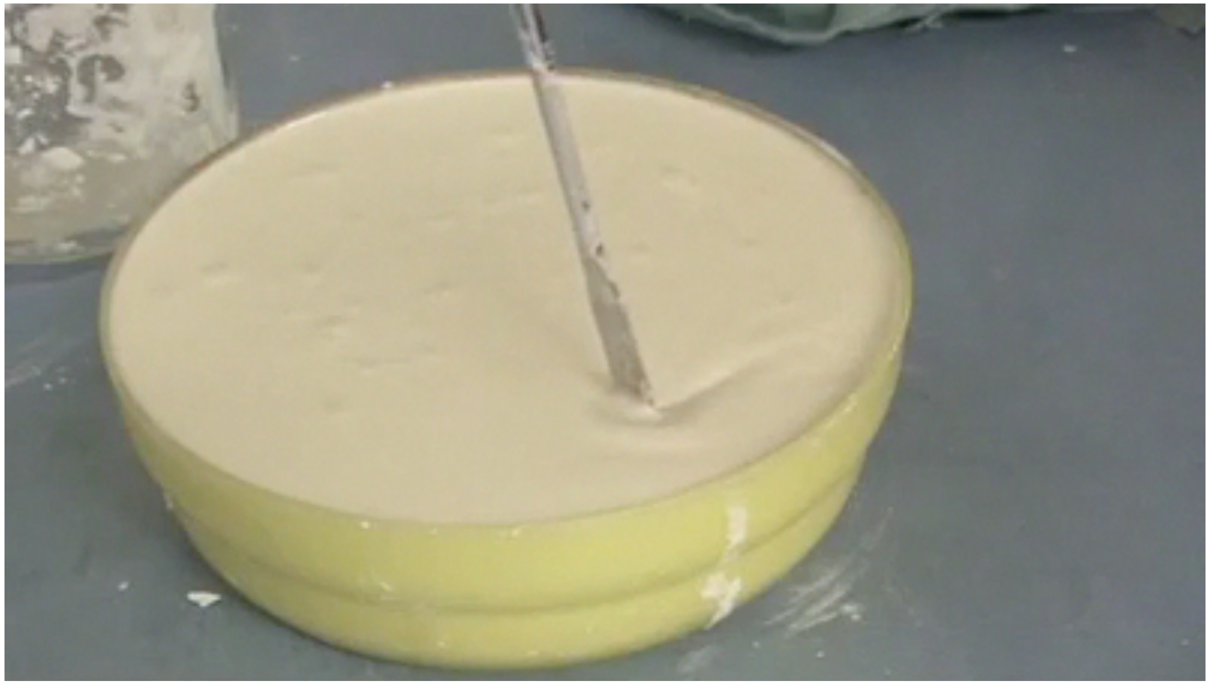

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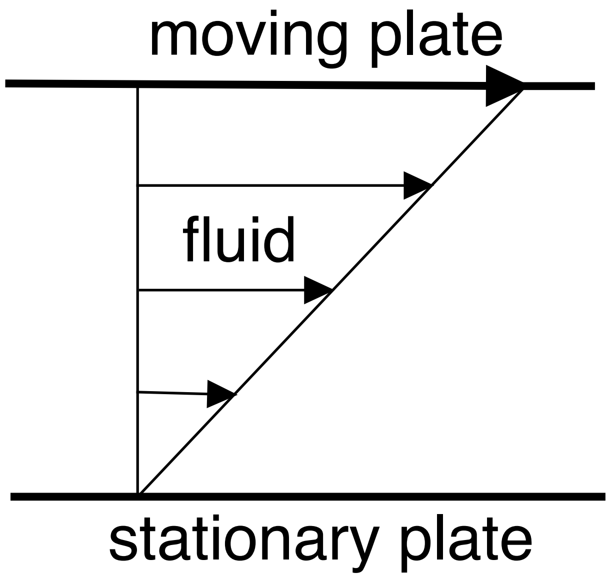
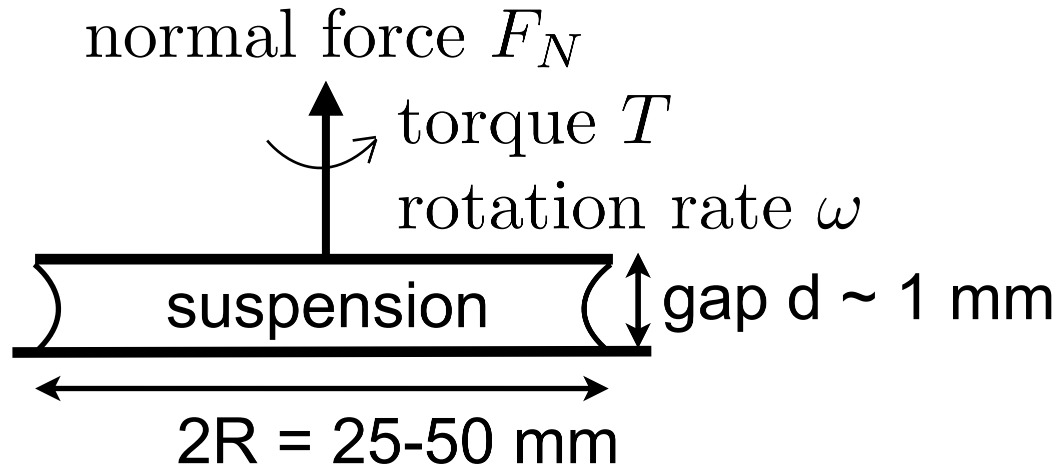
Steady-state rheology: viscosity is a measure of spatially averaged energy dissipation rate





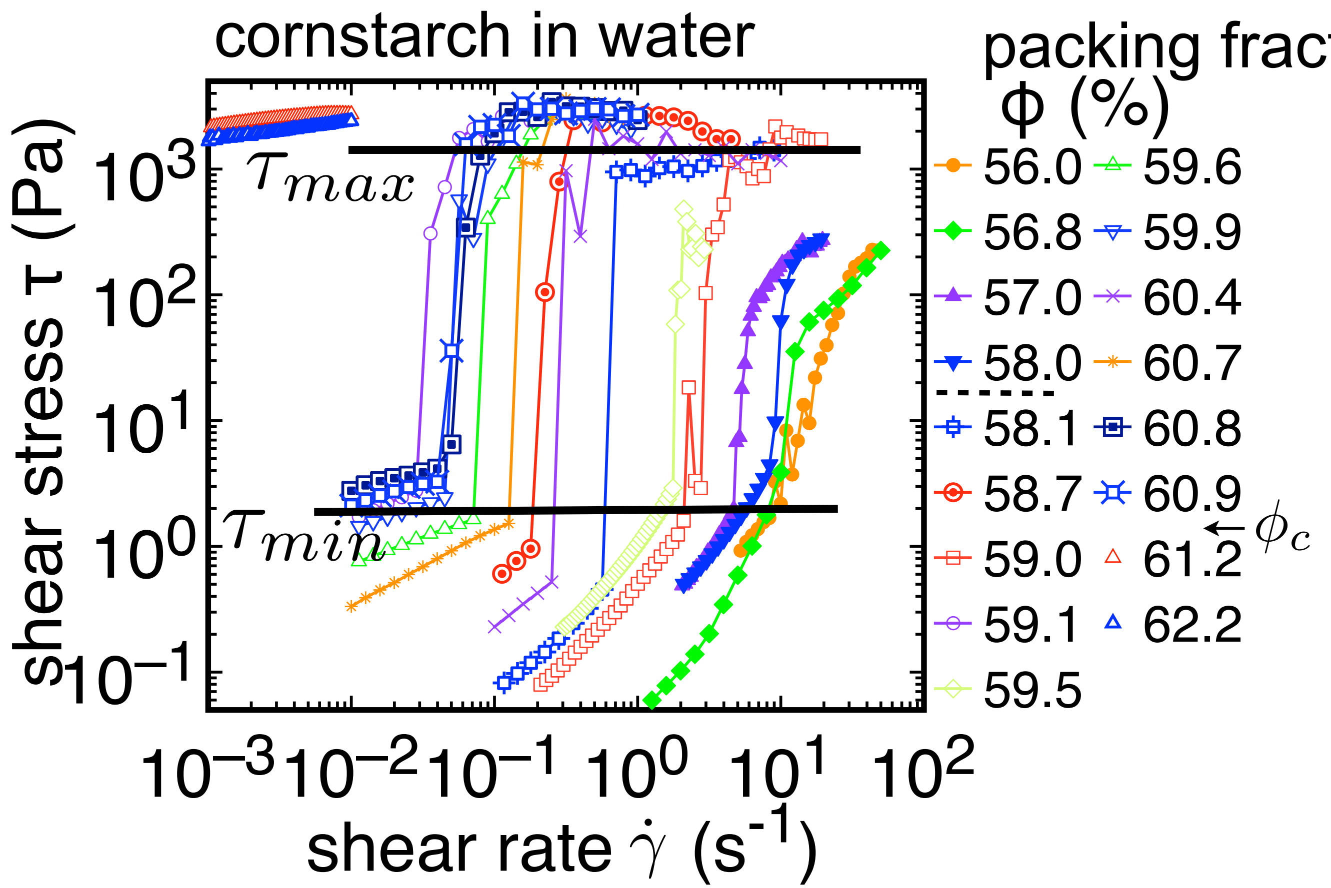
shear rate $\dot{\gamma} = \omega R/d$ (average velocity gradient)

shear stress $\tau = 2T/\pi R^3$ (average shear force/area)

normal stress $\tau_N = F_N/\pi R^2$ (average normal force/area)

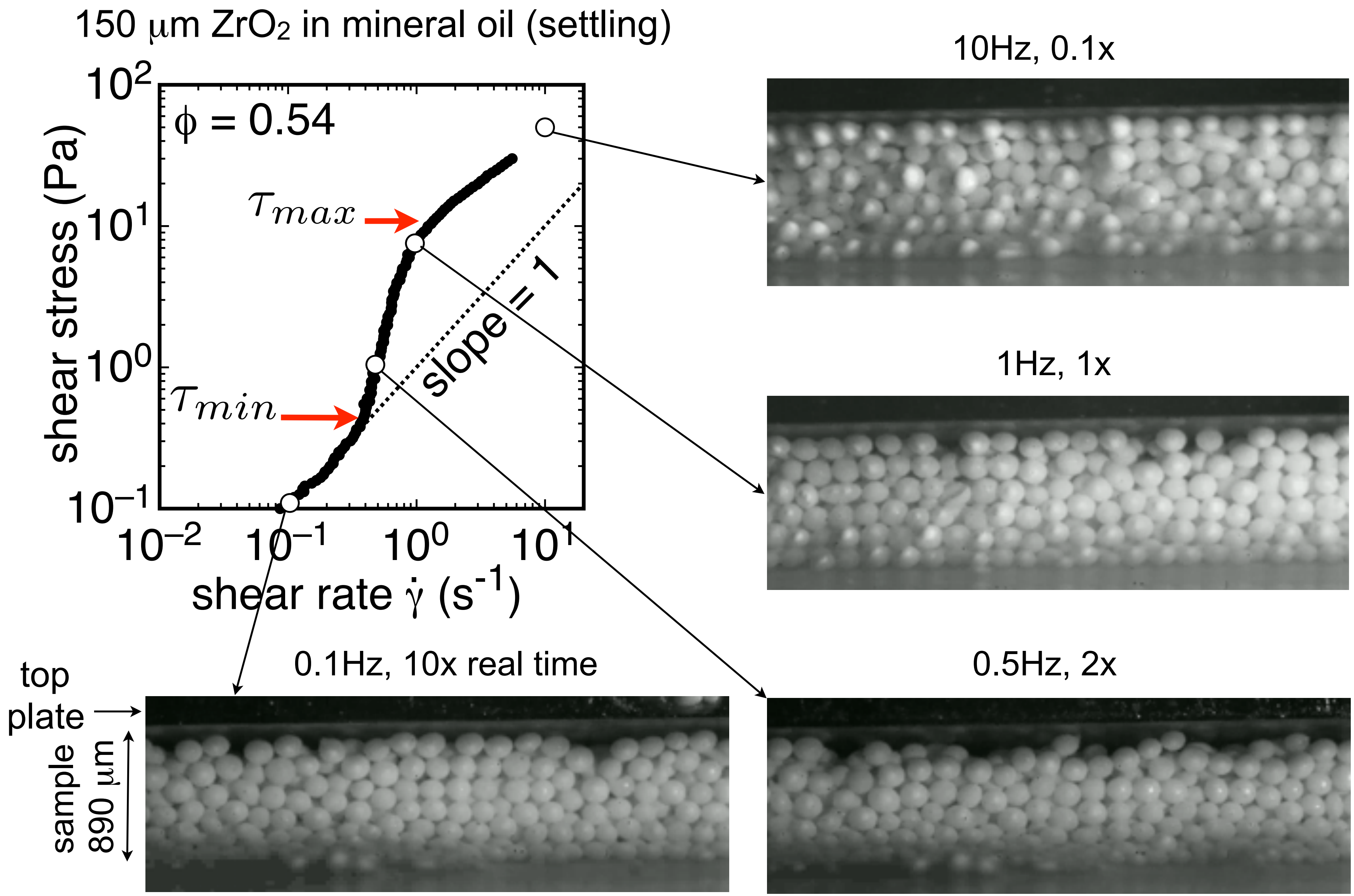
viscosity $\eta = \tau/\dot{\gamma}$

Discontinuous Shear Thickening viscosity curves



- "discontinuous" stress increase for $0.92\phi_c < \phi < \phi_c$ in rate-controlled measurements (ϕ_c has the same value as RLP, Brown & Jaeger PRL 2009)
- stress scales τ_{max} and τ_{min} bound the shear thickening regime -- what determines their scales?

Local constitutive relation can be obtained from shear profile of non-density matched suspensions



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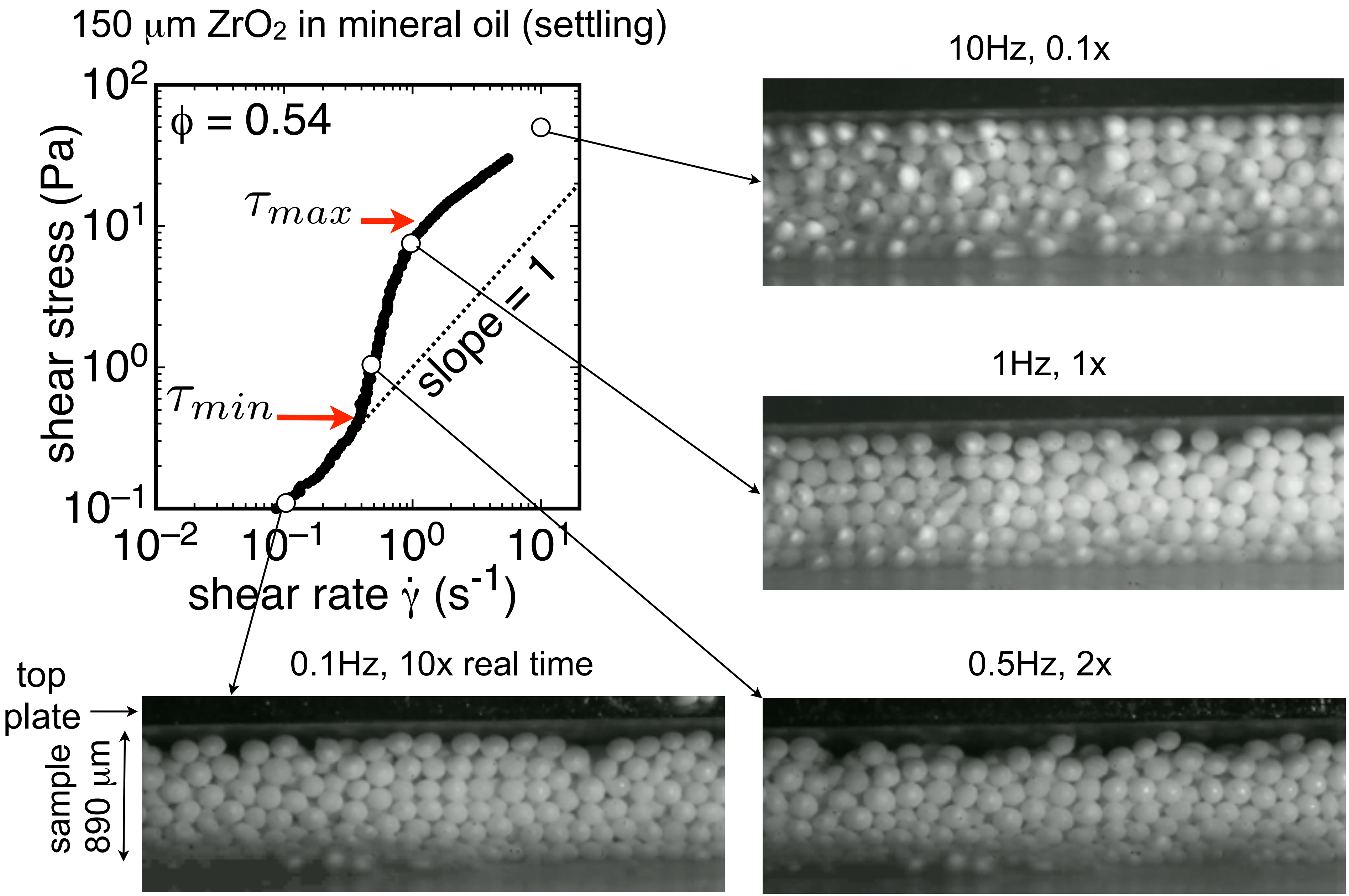
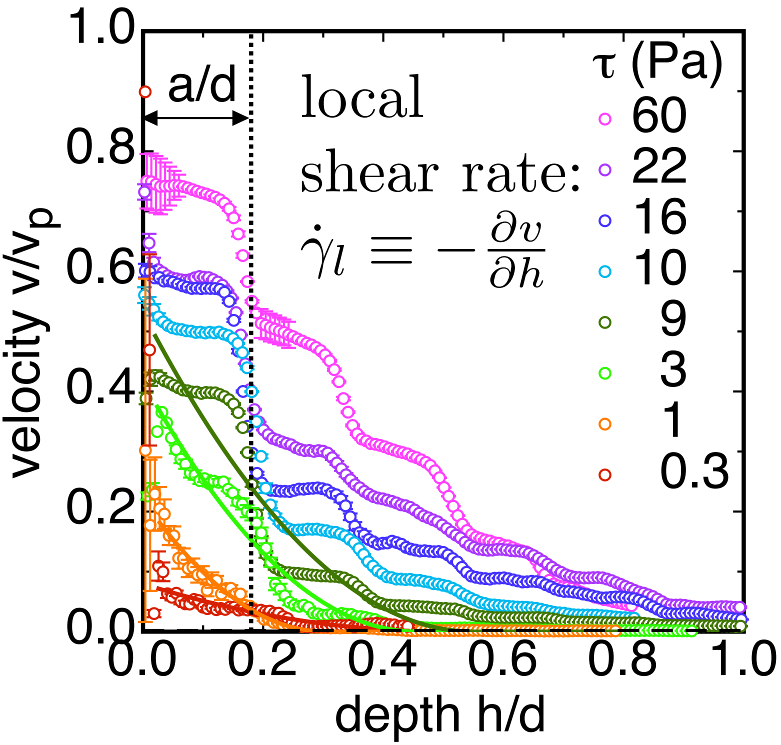


Shear thickening not dependent on local shear rate

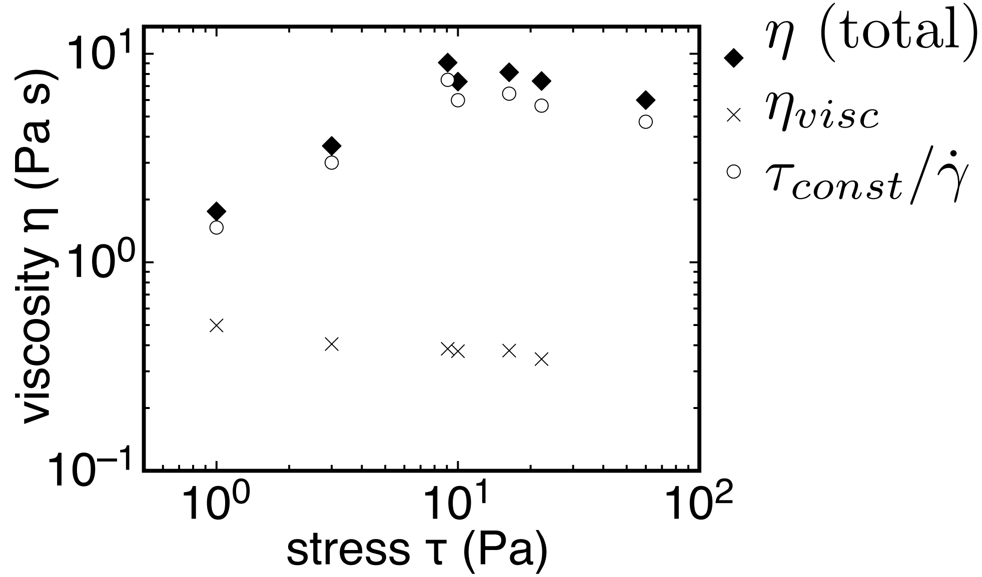
150 μm ZrO_2 in mineral oil (settling)



$$\tau(\dot{\gamma}_l, h) \approx \eta_{visc}\dot{\gamma}_l + \mu_g \Delta\rho g h + \tau_{const}$$

(viscous) (gravitational) (i.e. friction)

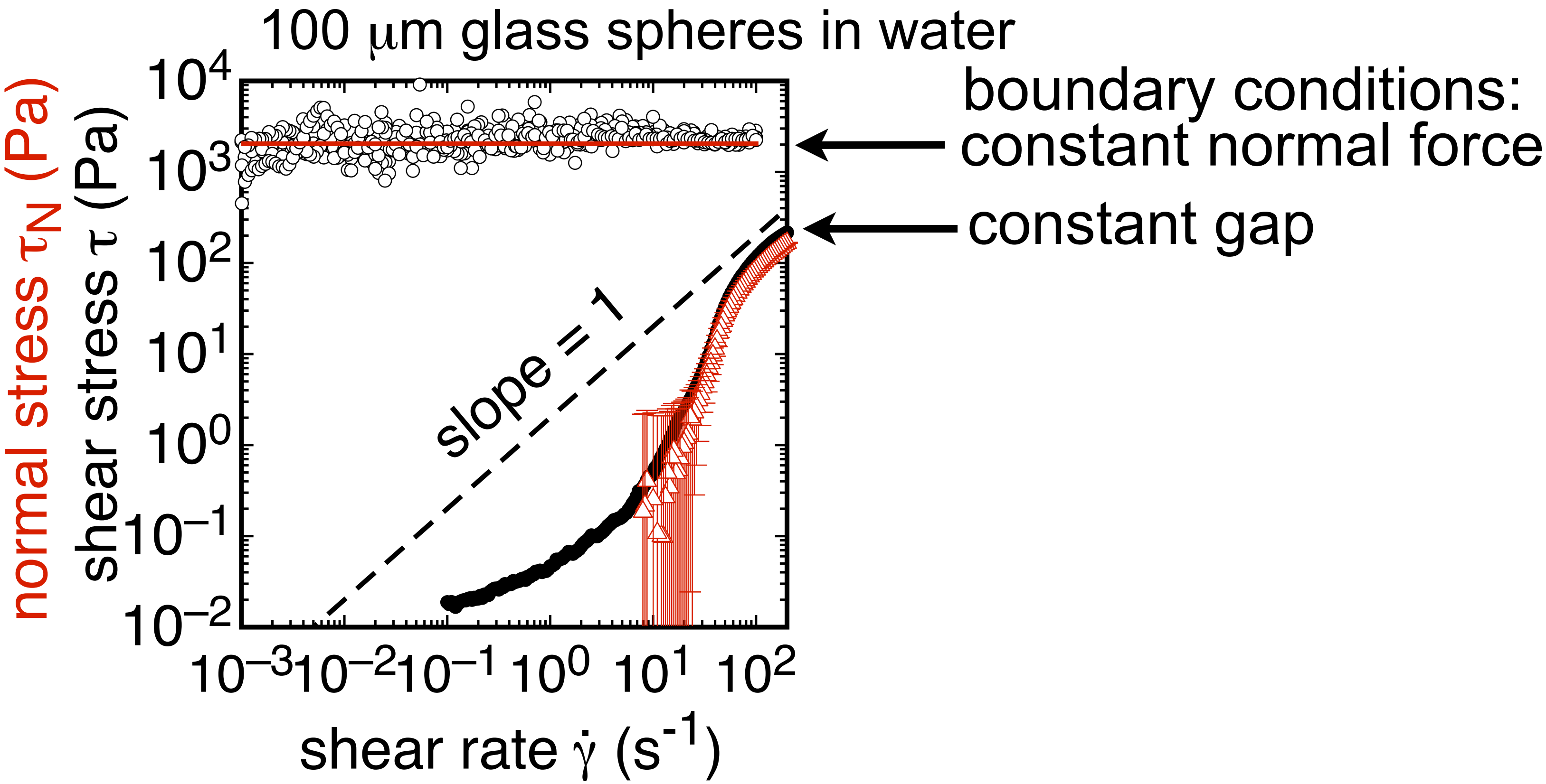
$$\frac{v}{v_p} = \frac{\tau_g}{2\tau_\nu}\left(\frac{h_c - h}{d}\right)^2$$



➡ majority of shear stress does is not dependent on local shear rate in shear thickening range and higher stress (Fall et al. PRL 2008, Brown & Jaeger J. Rheology 2012, Xu et al. EPL 2014, Overlez, Manneville, Colin)

Brown & Jaeger J. Rheology 2012

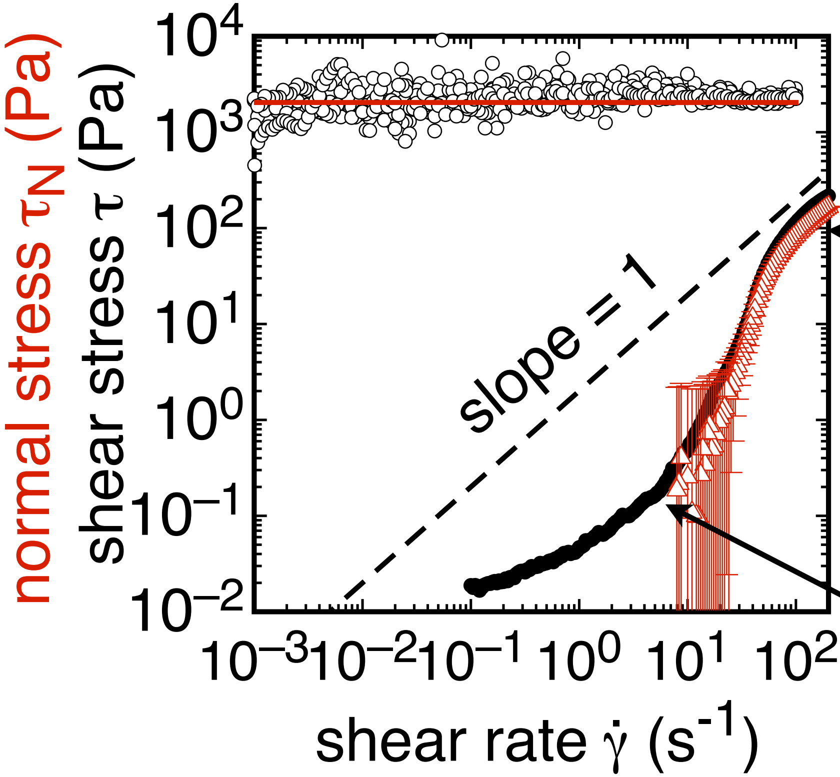
Shear stress comes from normal stress



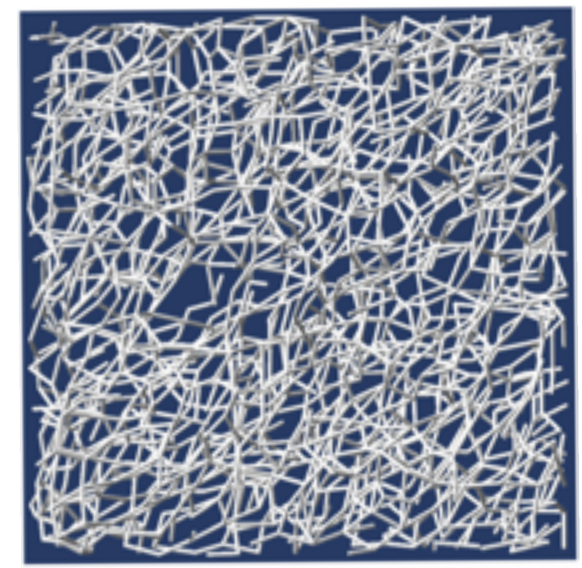
- existence of DST depends on boundary conditions
- $\tau_{const} = \mu_{eff}\tau_N$ with μ_{eff} ~1 -> effective friction (Lootens et al. PRL 2003, 2005, Heussinger PRE 2013, Seto et al. PRL 2013, & many others...)

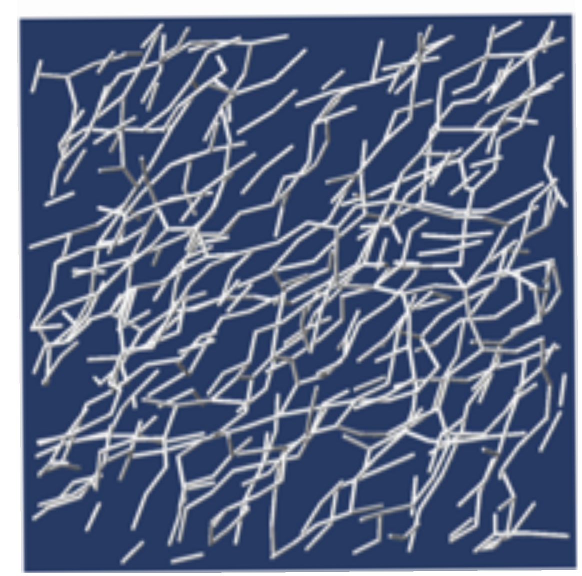
Brown & Jaeger J. Rheology 2012

Normal stress comes from force networks



force networks appear in DST regime (Seto et al. 2013, Brady, Cohen)



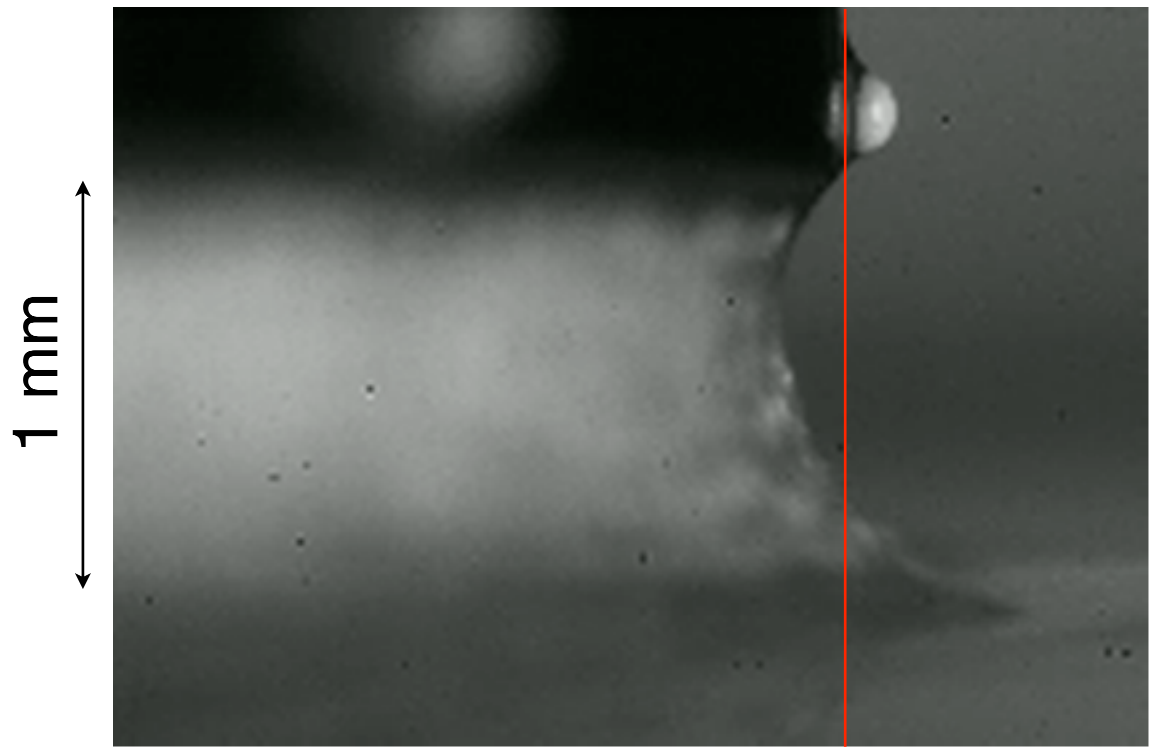


➡ load-bearing force networks
➡ positive normal stress
➡ tendency to expand (i.e. dilation)
➡ positive normal stress can remain in steady state only if dilation is frustrated by boundaries

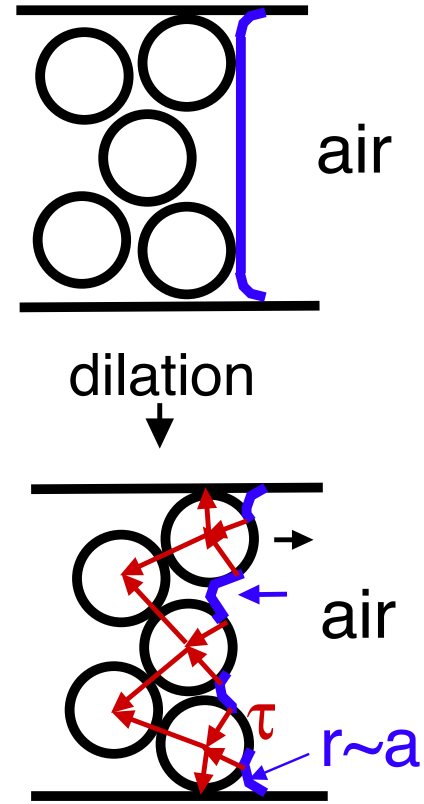
Dilation against liquid-air interface leads to confining stress from surface tension

150 μm ZrO_2 in mineral oil

side view (tangent to surface)
shear rate = 3 s^{-1}, playback at 0.33x



Brown & Jaeger J. Rheol. 2012



maximum confining stress:

$$\tau_{max} \approx \frac{\gamma}{r} \sim \frac{\gamma}{a}$$

$\gamma =$ surface tension

$a =$ particle diameter

Cates et al. J. Phys. Cond. Matt. 2005

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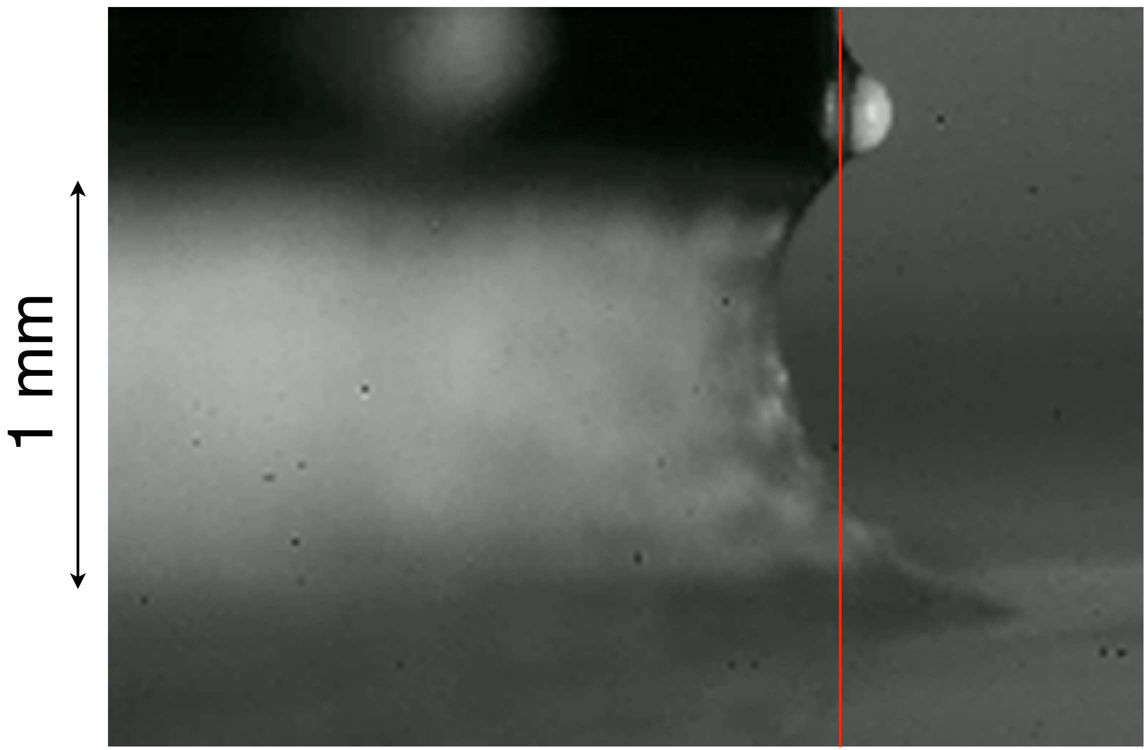

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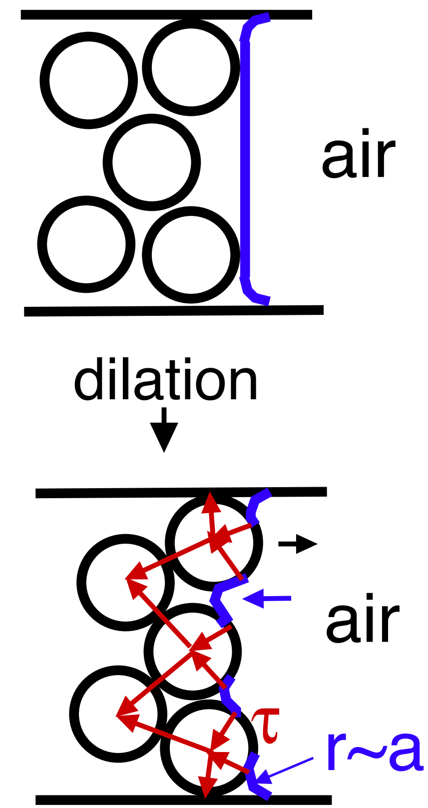

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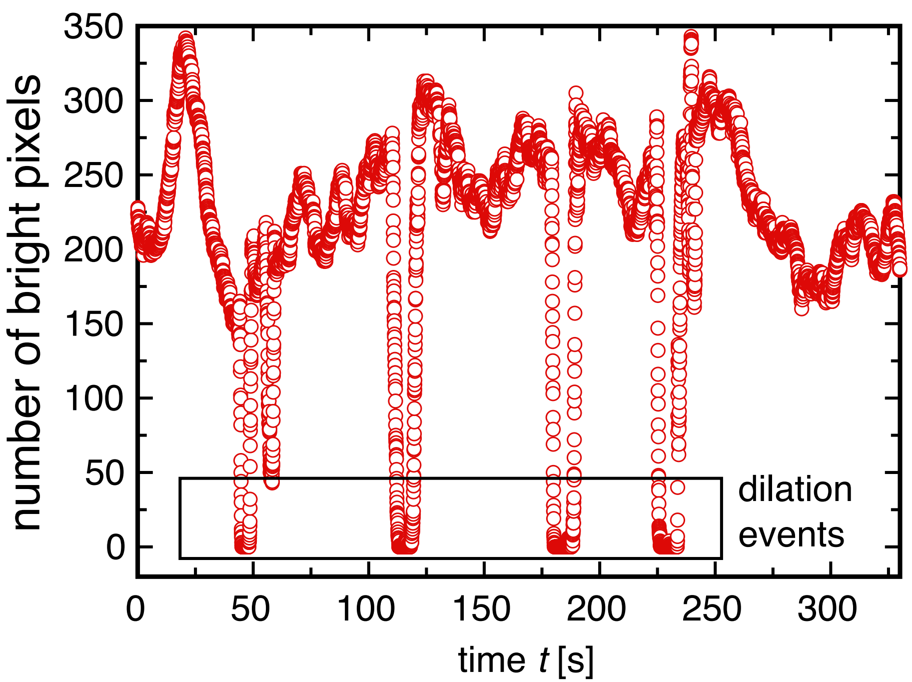
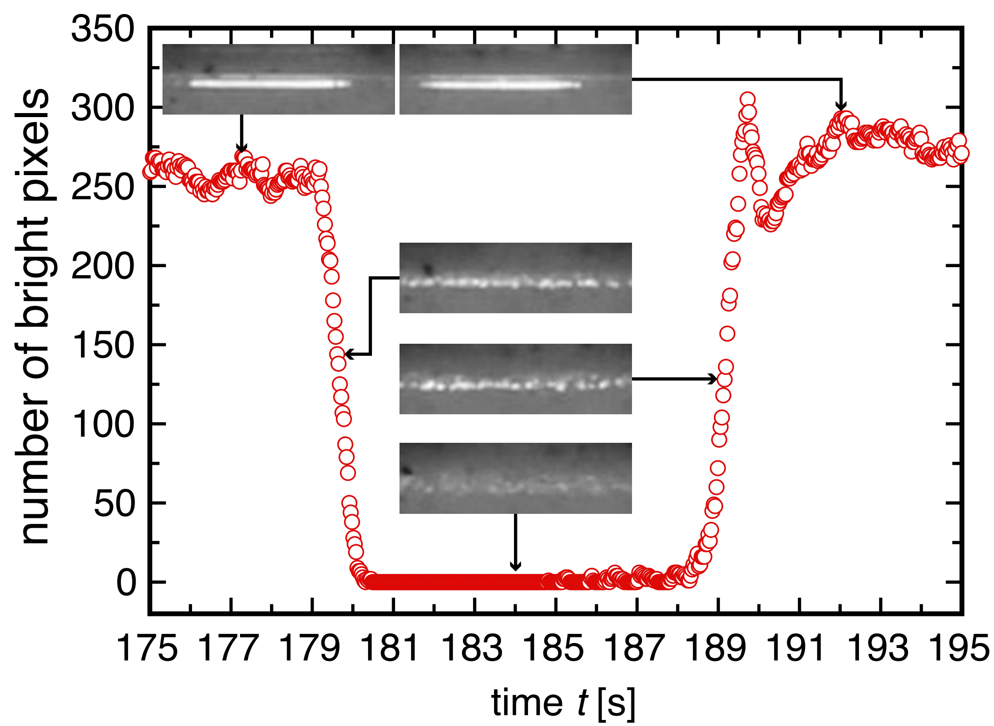
Dilation against liquid-air interface can be observed as a change in surface reflectivity

cornstarch in water, 200 Pa, real time





- events spatially localized and fluctuate in time (Blair)

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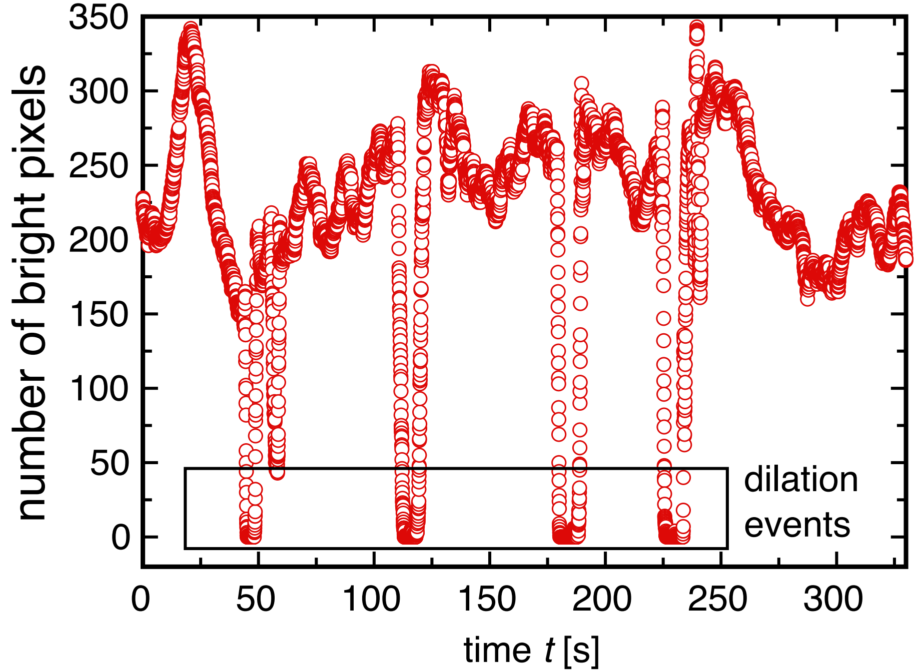

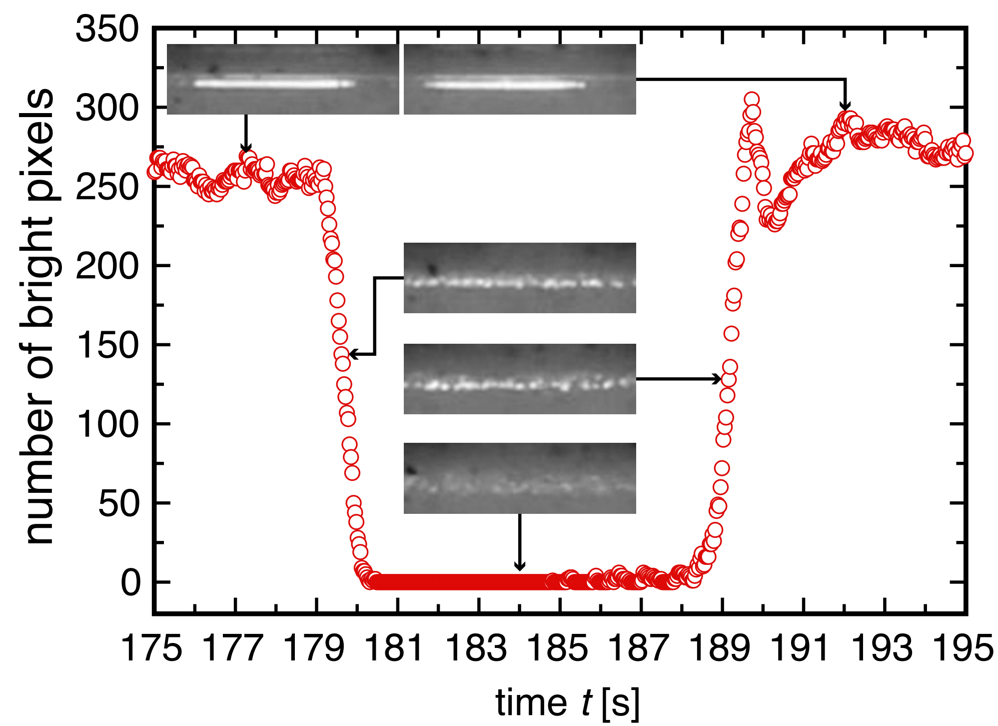

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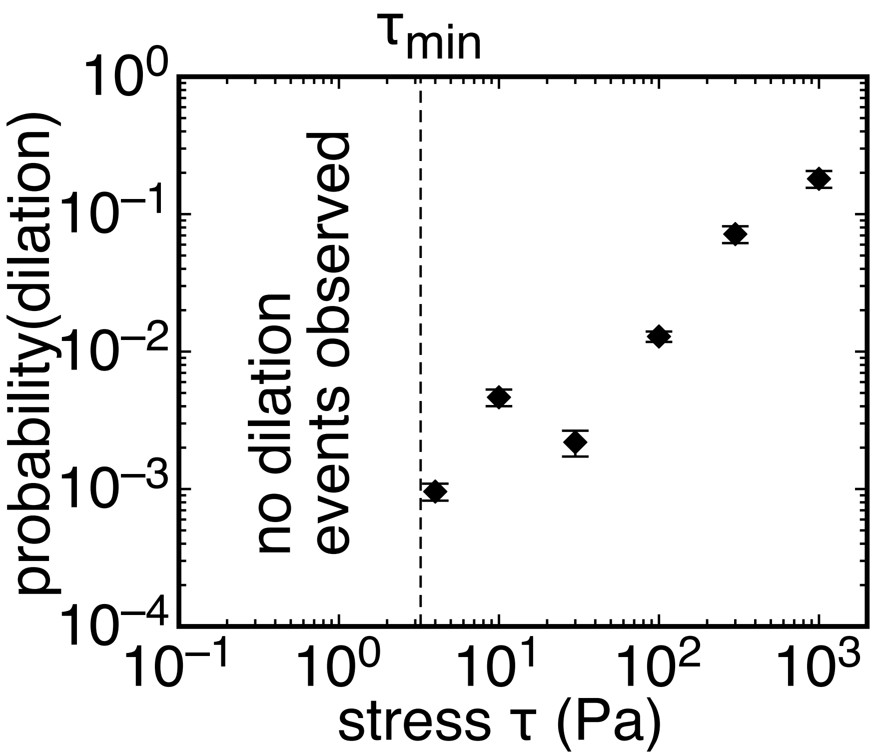
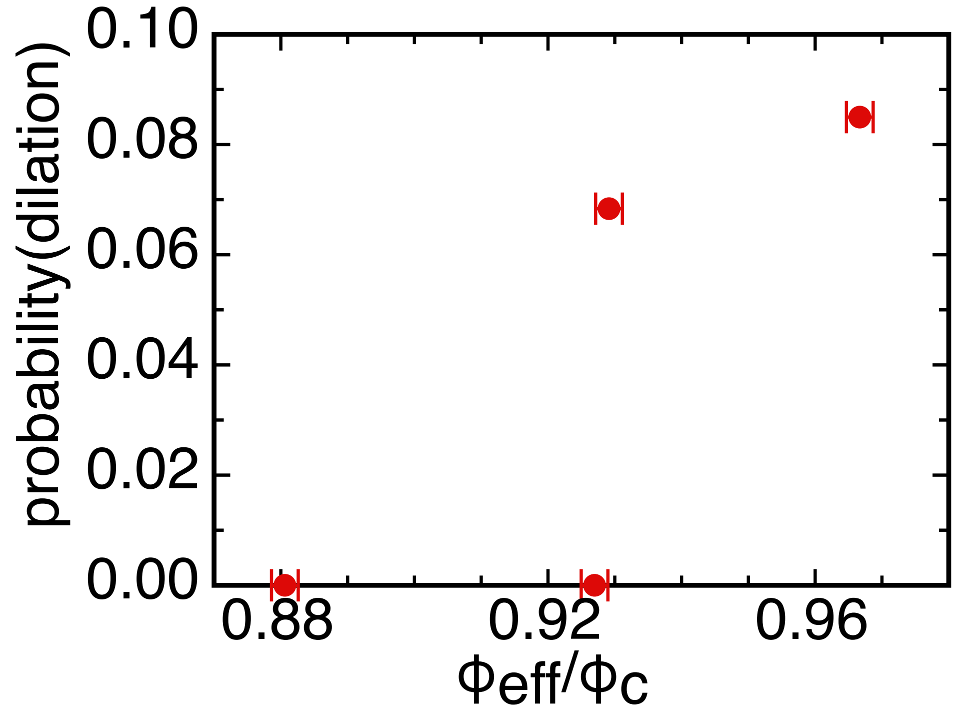
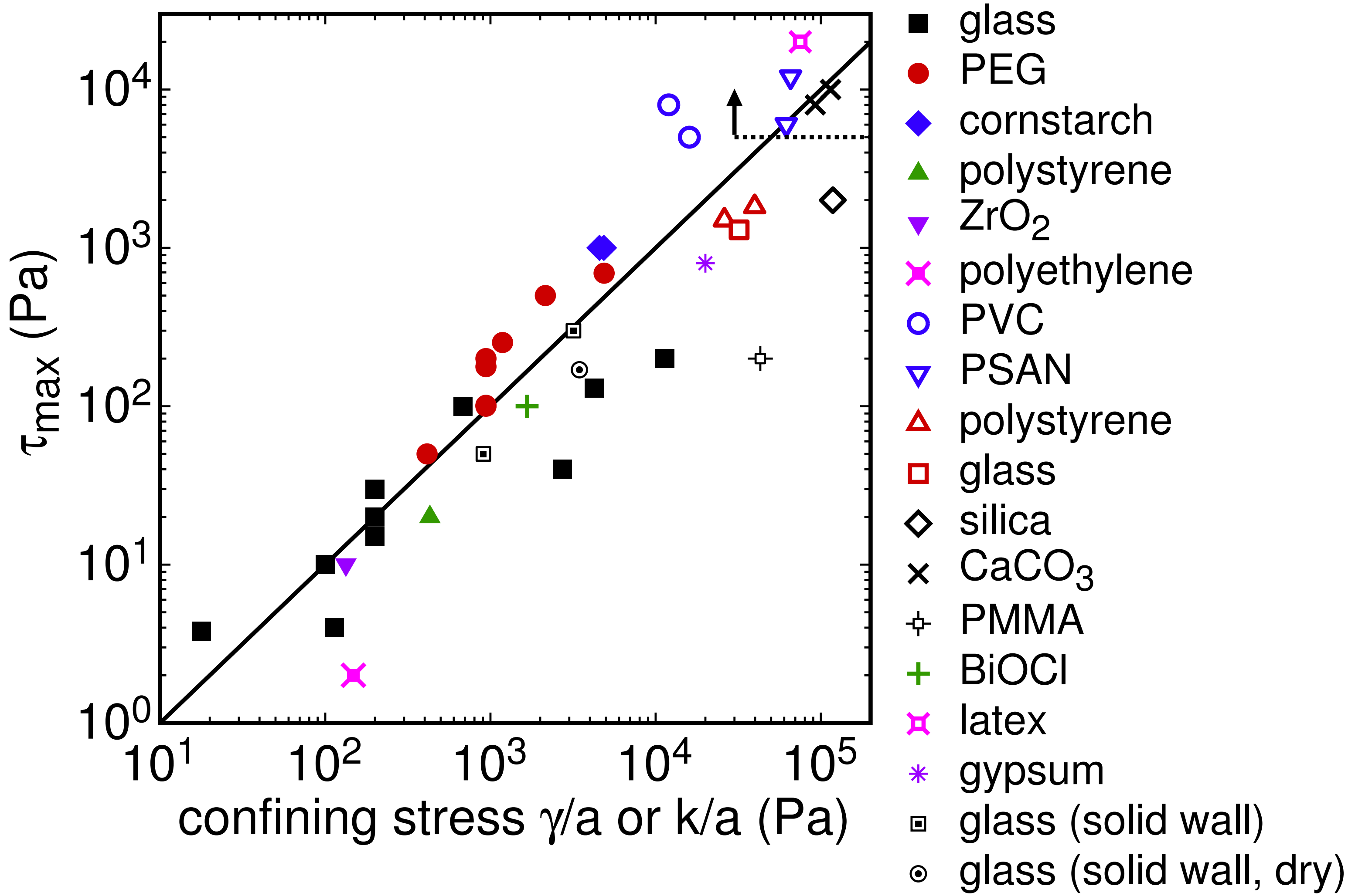
Dilation coincides with DST





- dilation observed above τ_{min} (onset of shear thickening) (Metzner & Whitlock 1958, Blair)
- dilation observed for $\phi > 0.92\phi_c$ (same ϕ-range as DST)

Maximum stress (τ_{max}) in DST regime limited by boundary stiffness



other cases:

- hard walls: wall stiffness k/a
- simulations with periodic BCs: particle stiffness k/a

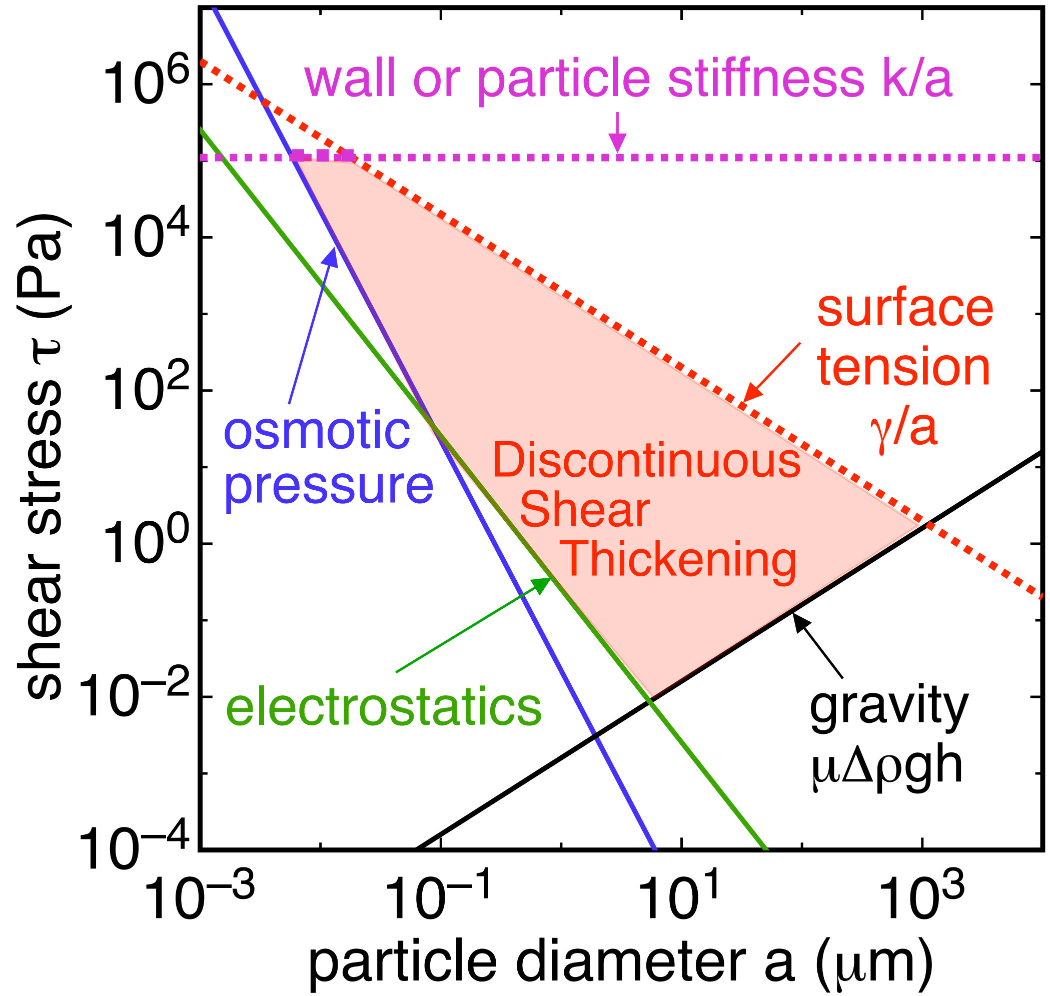
(Otsuki & Hayakawa PRE 2011

Brown & Jaeger, Reports on Progress in Physics, 2014

onset stress scale τ_{min}

various regimes depending on dominant force:

- electrostatic repulsion (Hoffman 1982, Maranzano & Wagner J. Rheol. 2001, Royer, ...)
- osmotic pressure in Brownian suspensions (Bergenholtz et al. 2002, Maranzano & Wagner J. Chem. Phys. 2002, Brady)
- gravity for settling particles (Brown & Jaeger J. Rheol. 2012)
- induced dipole-dipole attractions from applied fields (Brown et al. Nature: Materials 2010)



➡ generally: shear stress must exceed interparticle stresses that prevent pushing grains together and around each other to generate positive normal stress and dilation

Brown & Jaeger, Reports on Progress in Physics (2014)

Summary

$$\tau(\dot{\gamma}_l, h) \approx \eta_{visc}\dot{\gamma}_l + \tau_i + \mu_g \Delta\rho g h + \mu_{eff}\tau_N$$

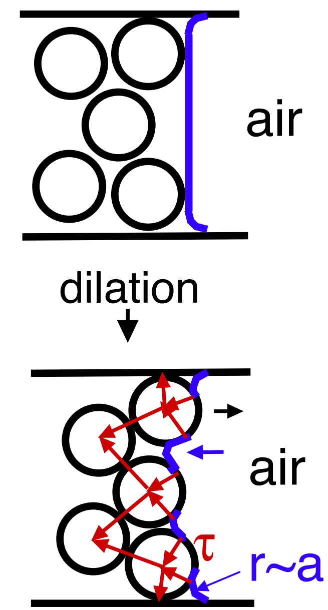
(viscous)

(gravitational)

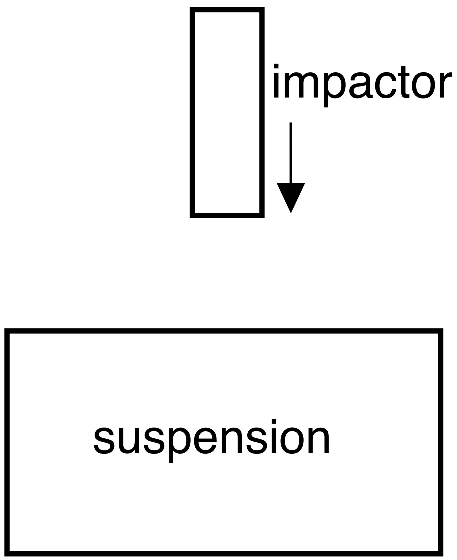
particle interactions that resist shear

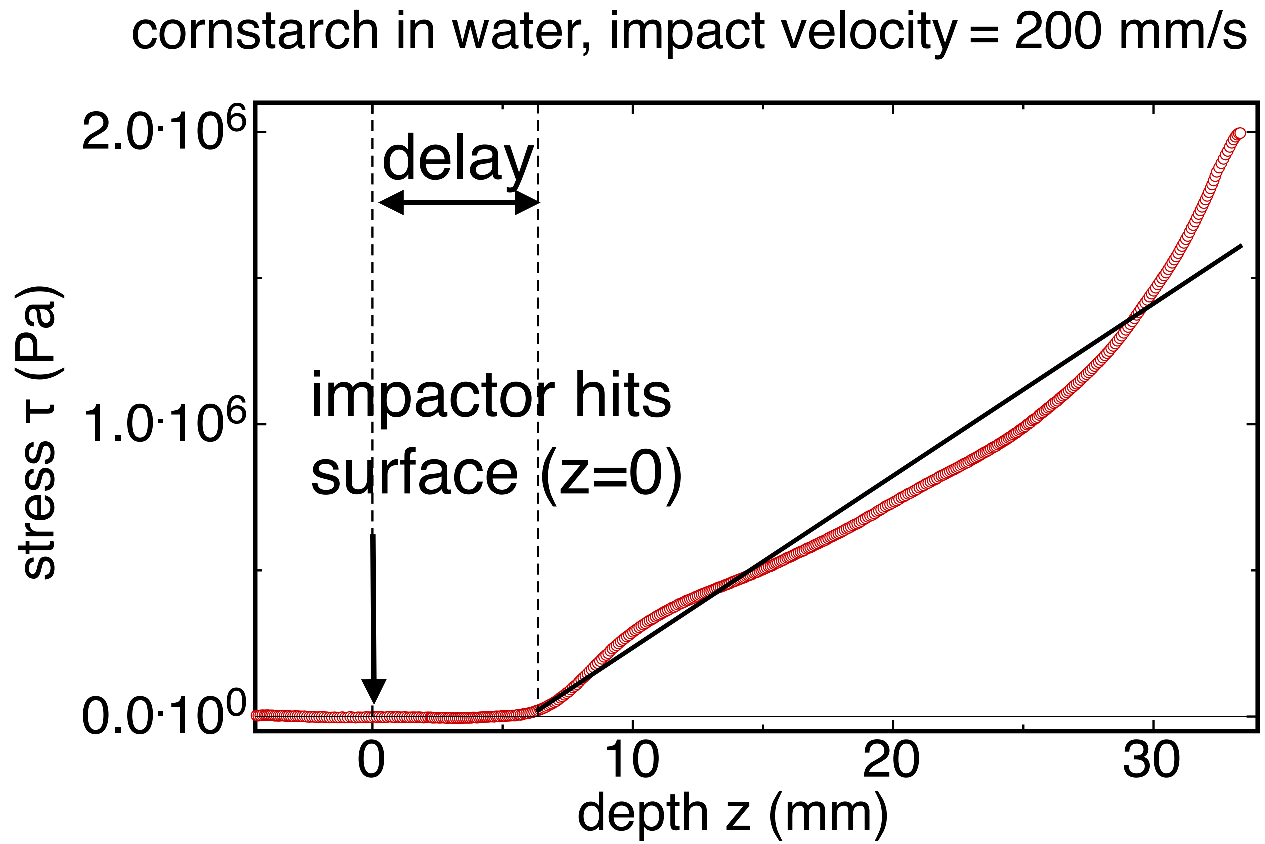
frictional: $\tau_N \sim \frac{k}{a}$
depends on boundary stiffness k in response to dilation

- shear stress depends mostly on normal stress rather than local shear rate
- system-spanning force networks lead to positive normal stress and dilation
- dilation against a boundary leads to confining stress k/a from boundary stiffness (usually from surface tension) that limits the strength of shear thickening (τ_{max})



Transient impact experiments





- stress more than enough to support a person's weight (~ 4×10^4 Pa) and much more than steady state shear (~10^3 Pa)
- **Open question:** what sets the scale of the stress (~10^6 Pa) here?

Maharjan et al arXiv:1407.0719