Suspensions of non-Brownian particles in a shear thickening matrix

Sarah Hormozi

OHIO UNIVERSITY

Ph.D. Student:

Yasaman Madraki
Department of Mechanical Engineering,
Ohio University, Athens, Ohio

- Shear thickening: Transition from CST to DST In prep for Phys. Rev. Letter. 2018

- Enhancing shear thickening, Phys. Rev. Fluids 2, 033301.

NSF
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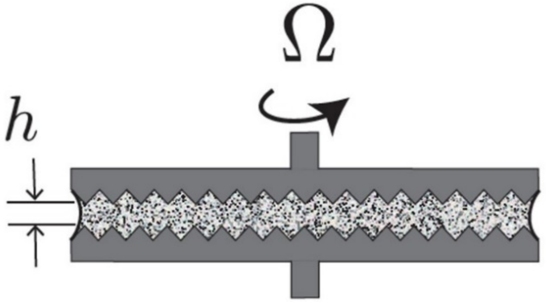
Intellectual Collaboration:

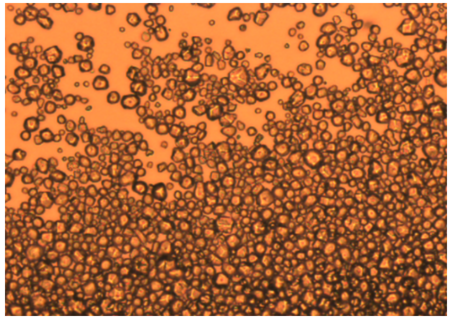
Guillaume Ovarlez
University of Bordeaux,
CNRS, Solvay, LOF, UMR, Pessac, France

Elizabeth Guazzelli
Aix- Marseille Univ, CNRS, IUSTI, Marseille, France

Olivier Pouliquen
Aix- Marseille Univ, CNRS, IUSTI, Marseille, France

Suspensions of cornstarch in water (Oobleck)



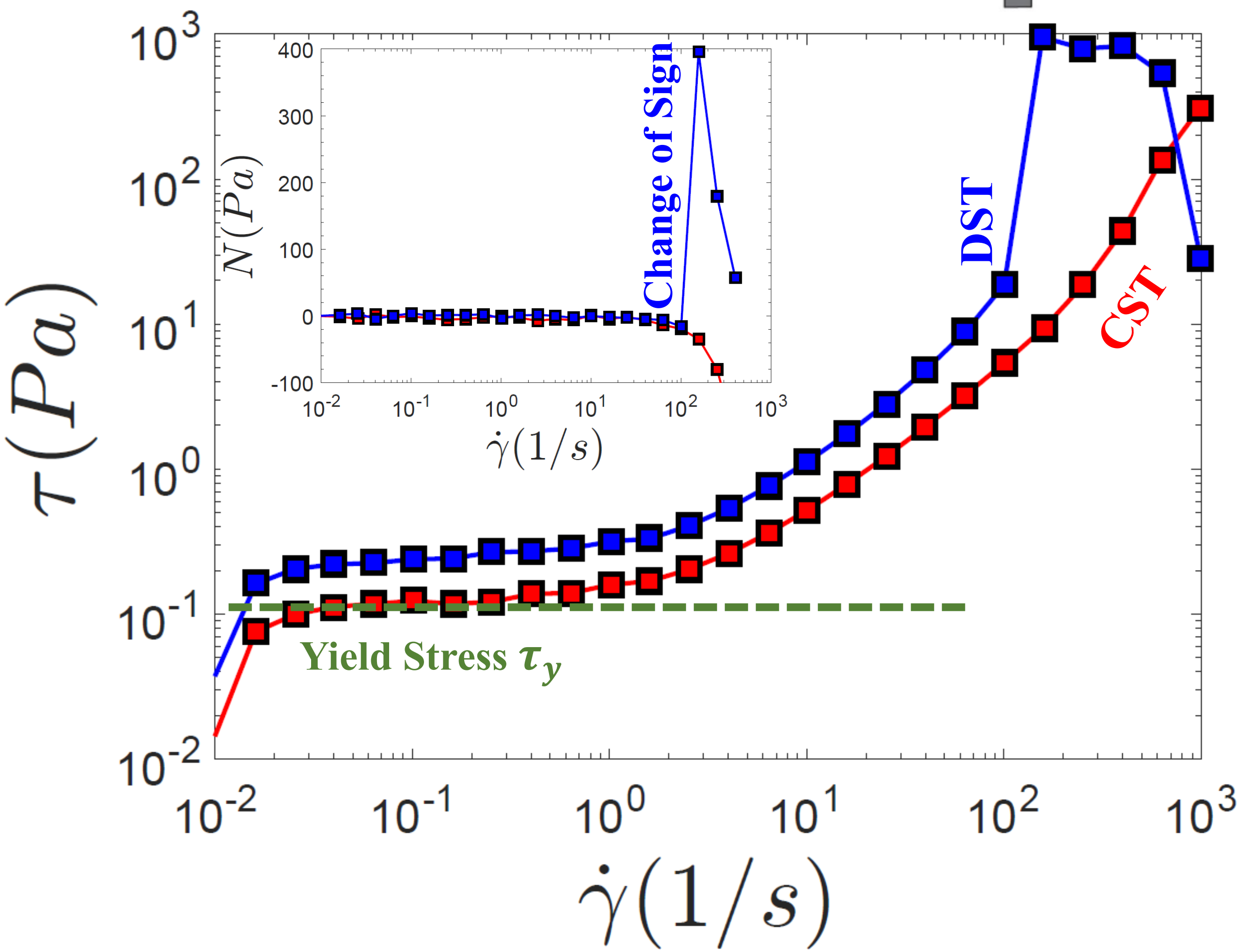


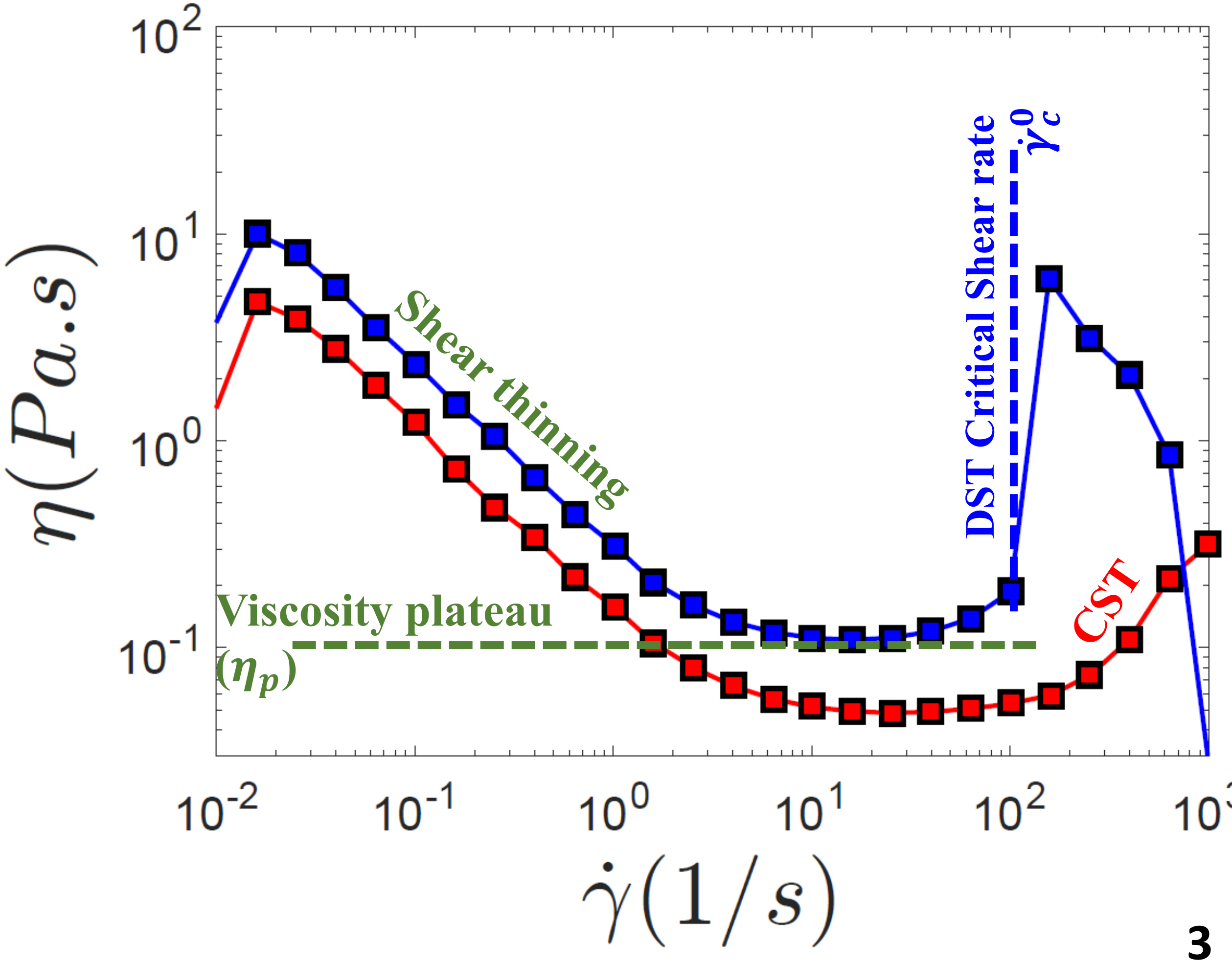
$5\mu m < d_{cs} < 20\mu m$

Cornstarch: $\rho = 1.68 \frac{gr}{cm^3}$

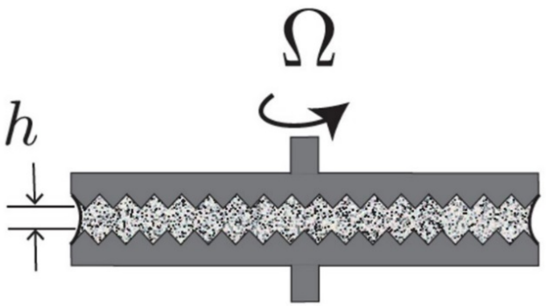
water+CsCl: $\rho \simeq 1.68\ gr/cm^3$

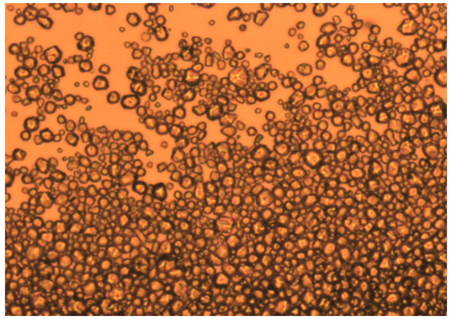
Micrograph of cornstarch suspension





Rheology: suspension of cornstarch in water





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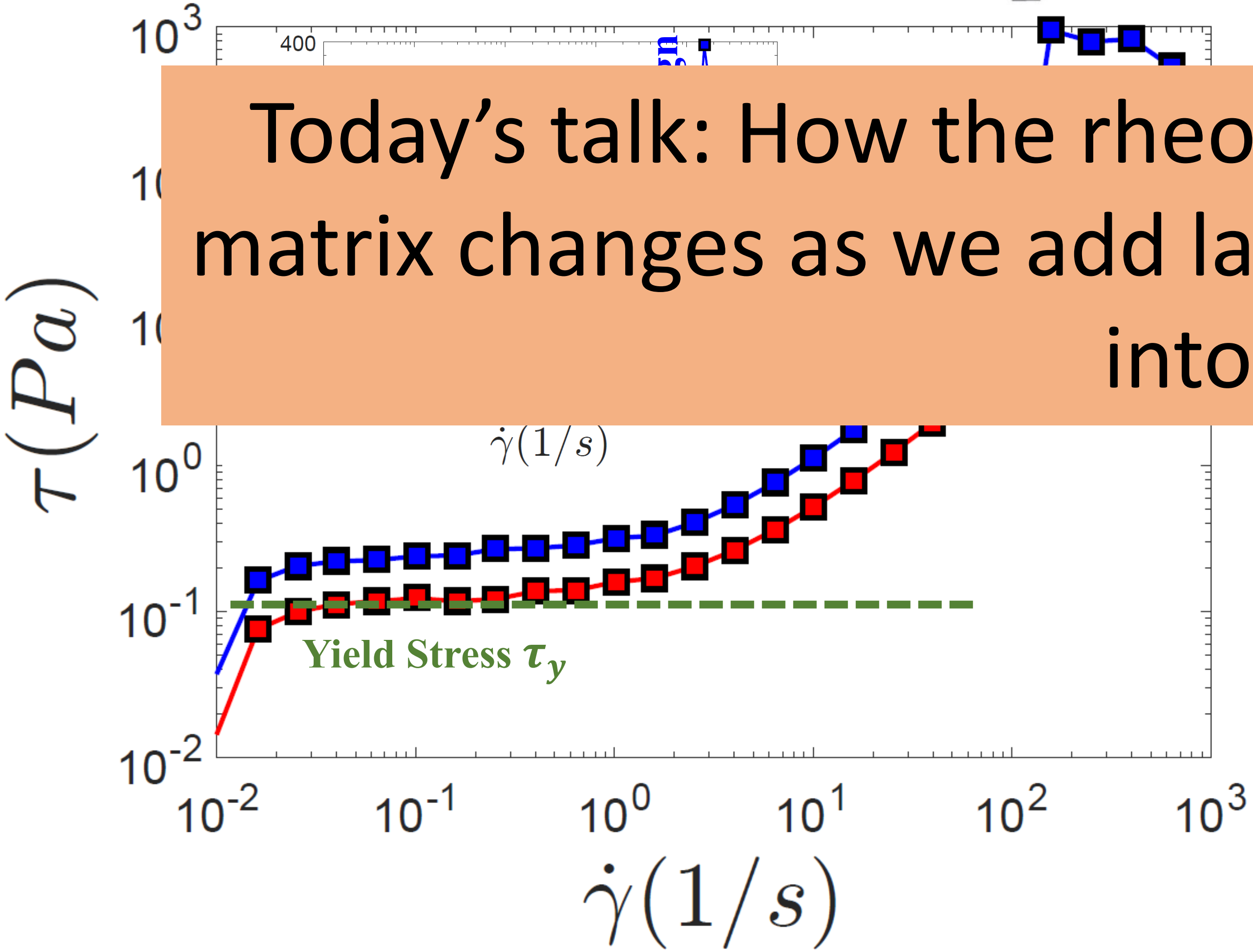
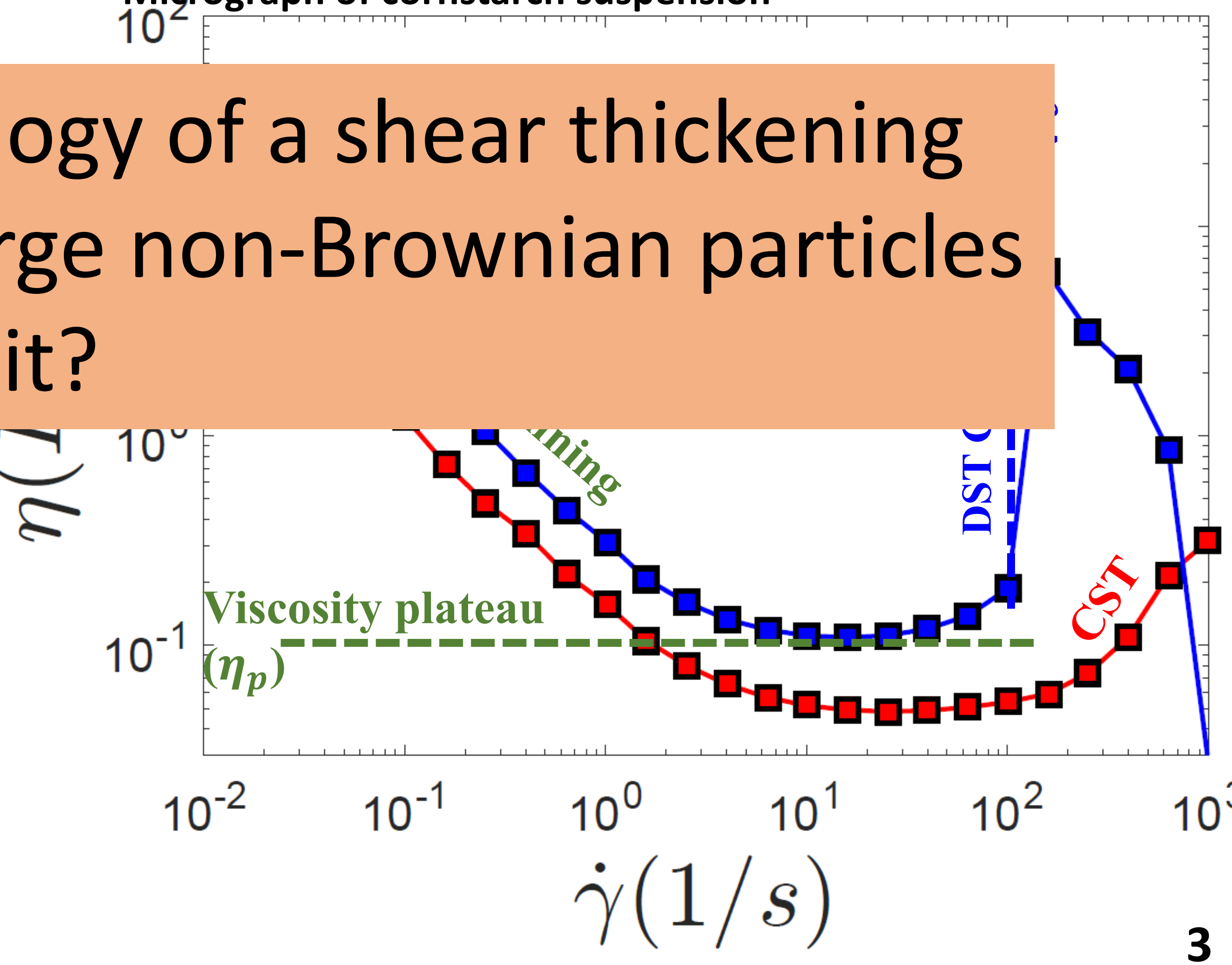


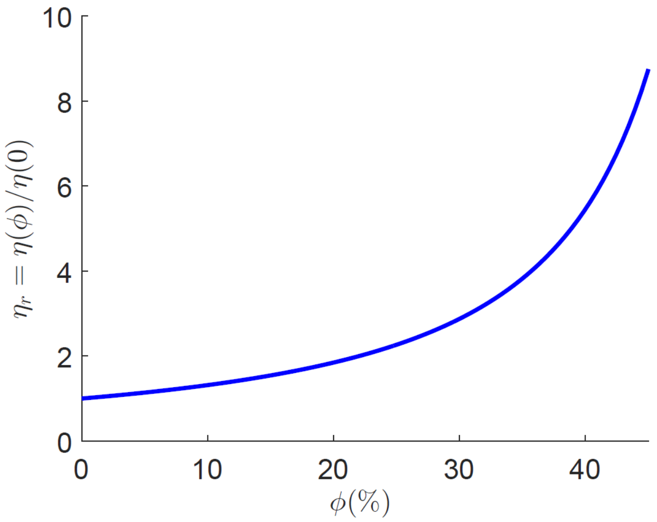


Today's talk: How the rheology of a shear thickening matrix changes as we add large non-Brownian particles into it?

Adding non-Brownian particles into Newtonian and generalized Newtonian fluids, Stokes limit

(Krieger & Dougherty, 1950)

Particles in a **Newtonian fluid**



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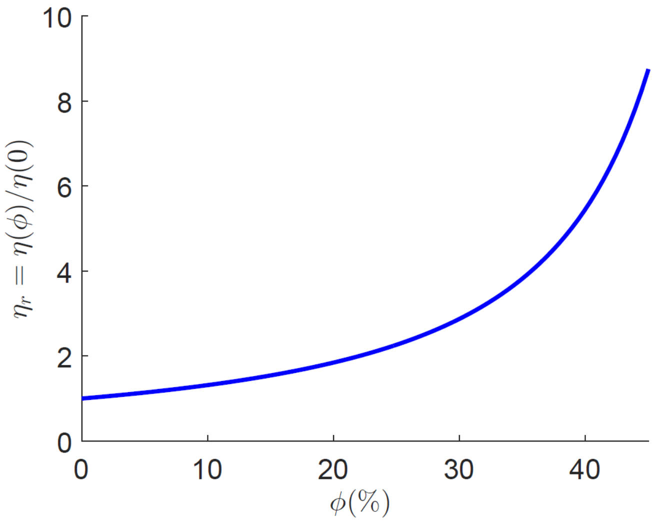

Particles in a **shear thinning** suspension

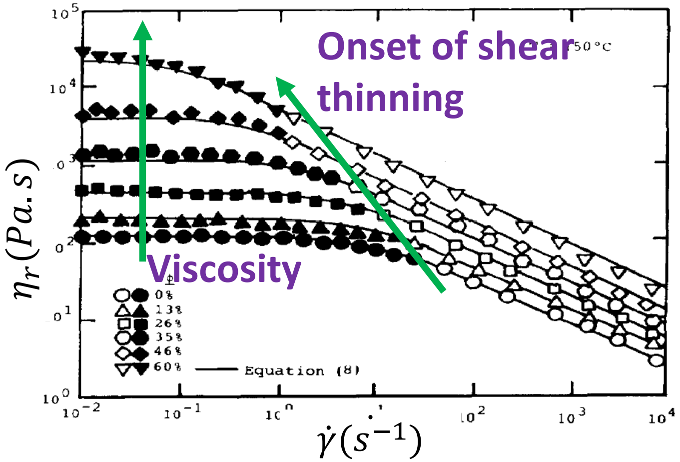

Fig. 4. Shear viscosity of glass spheres dispersed in the thermoplastic polymer at 150°C. ○● = 0%, △▲ = 13%, □■ = 26%, ⬡⬢ = 35%, ◇◆ = 46%, ▽▼ = 60%, — = Eq. (8).

(Polinski et al., *JOR* 1988; Liard, et al., 2014)

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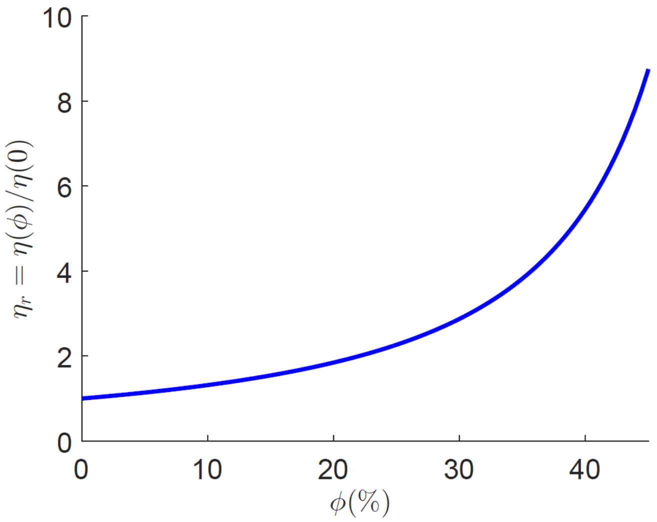

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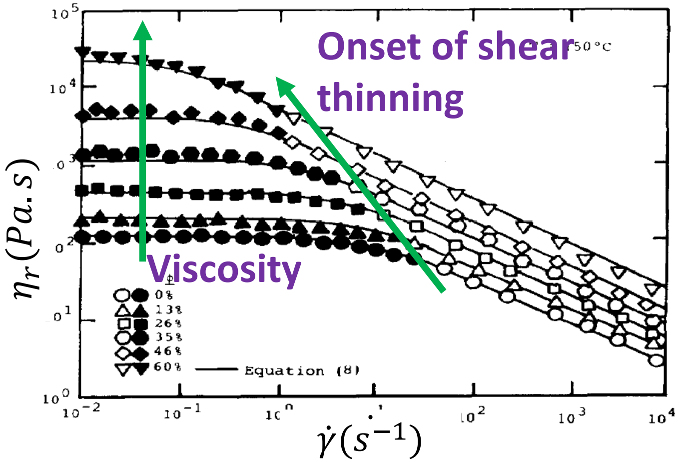

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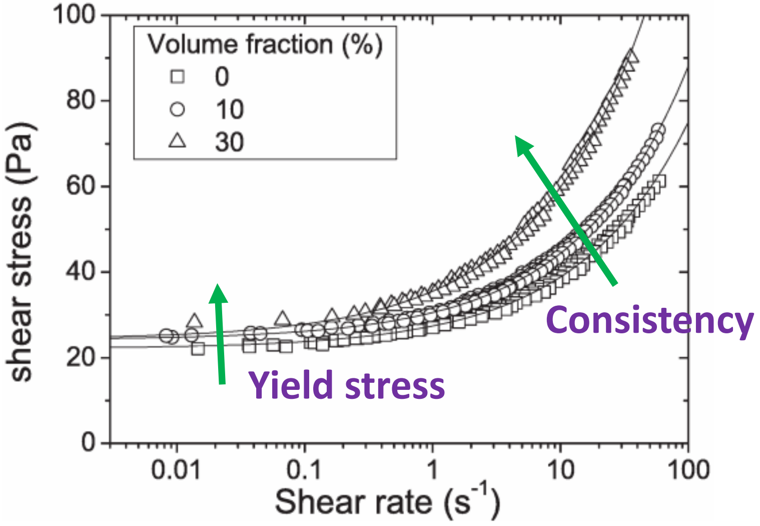
Particles in a **yield stress** suspension

(Dagois-Bohy, et al., 2015; Chateau, et al., 2008; Ovarlez, et al., 2015)



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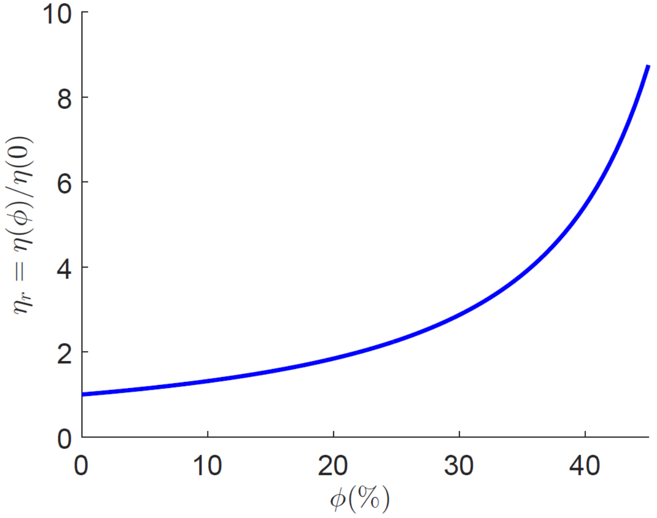

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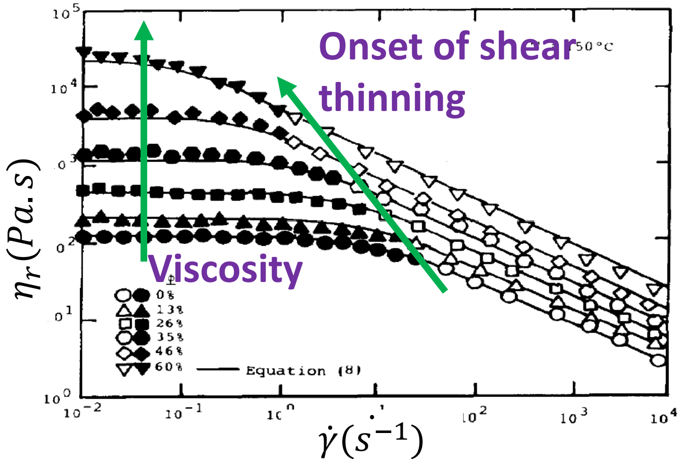

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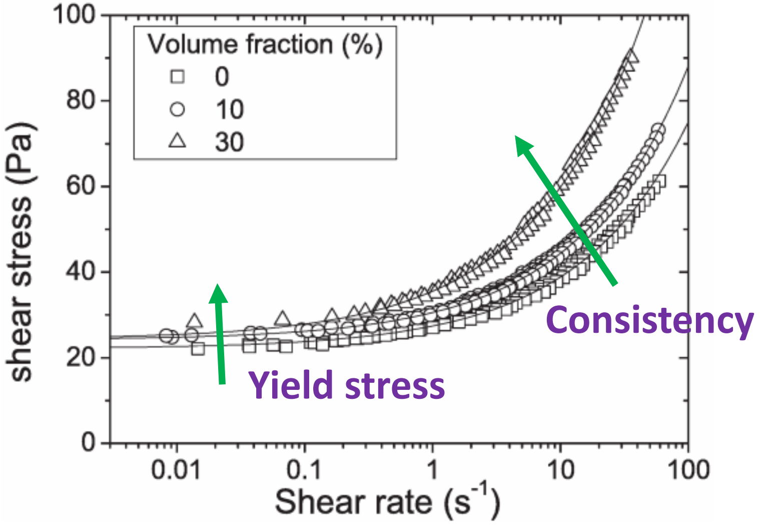

Particles in a **CST** suspension

(Cwalina & Wagner, 2016;
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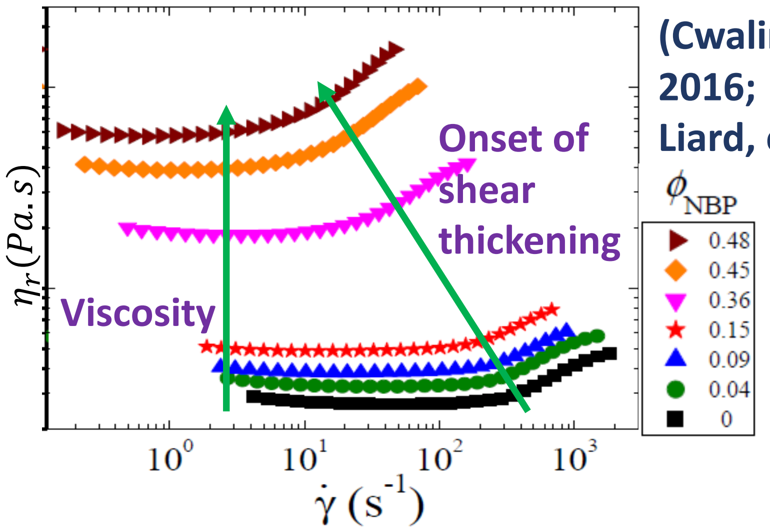
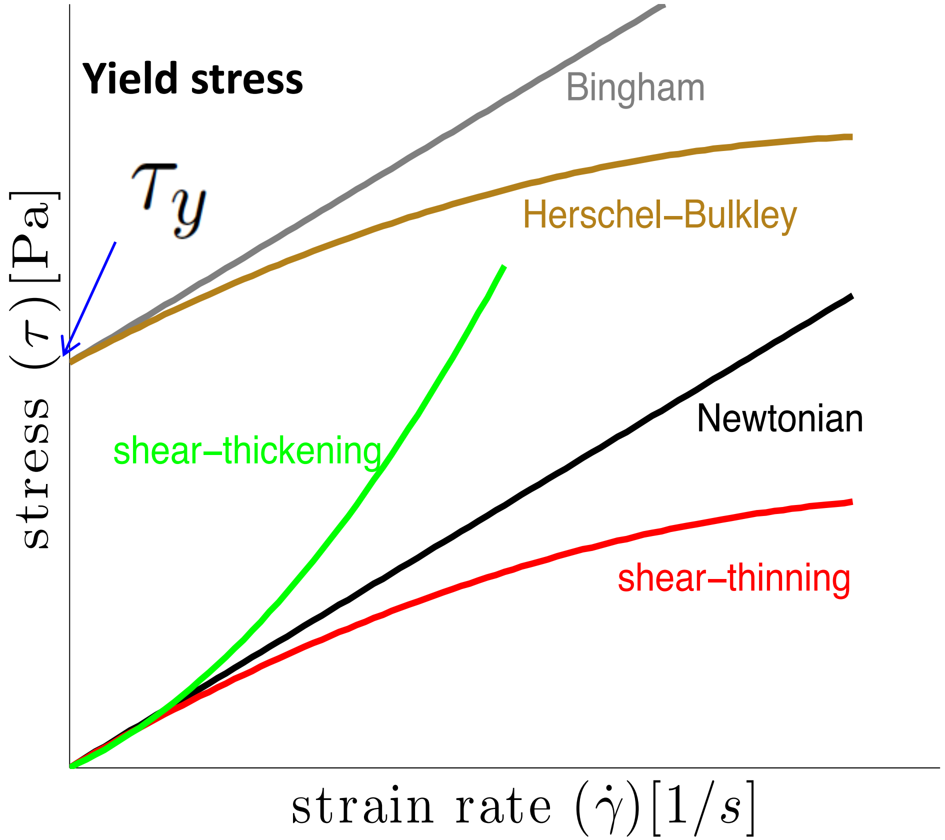


Explanation via a scaling argument

1. A generalized Newtonian fluid model

$$\begin{cases} \tau = k\dot{\gamma}^n & Power-law\ model \\ \tau = \tau_y + k\dot{\gamma}^n & Herschel-Bulkley\ model \end{cases}$$



$$\tau = \eta_{app}(\dot{\gamma})\dot{\gamma} \begin{cases} \eta_{app}(\dot{\gamma}) = k\dot{\gamma}^{n-1} & Power-law\ model \\ \eta_{app}(\dot{\gamma}) = \dfrac{\tau_y + k\dot{\gamma}^n}{\dot{\gamma}} & Herschel-Bulkley\ model \end{cases}$$

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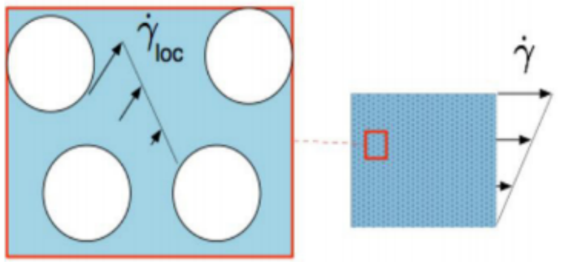

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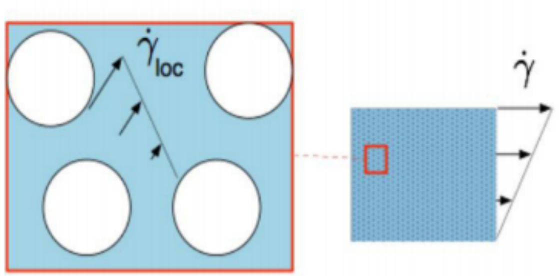

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4. Local shear rate mainly controlled by geometrical constraints, i.e. ϕ

$$\dot{\gamma}_{local} = \dot{\gamma}\mathcal{F}(\phi)$$

(Chateau, et al., JOR 2008; Lerner, et al., Phys Rev E 2012; Dagois-Bohy, et al., JFM 2015; Madraki et al., PRF 2017)

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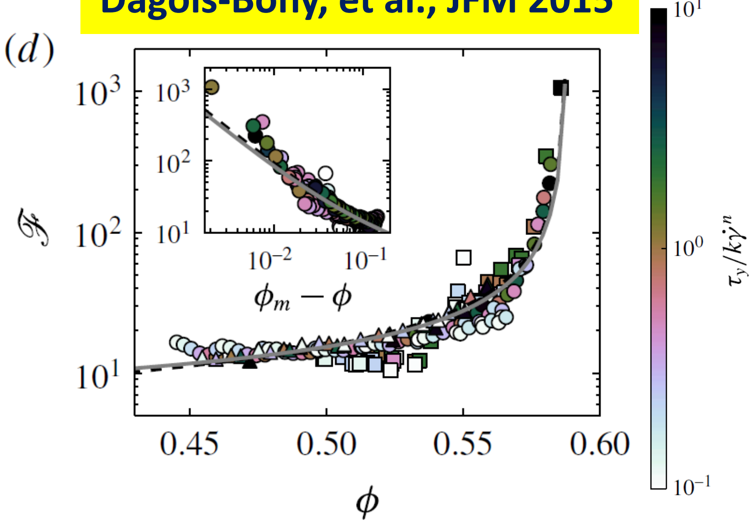

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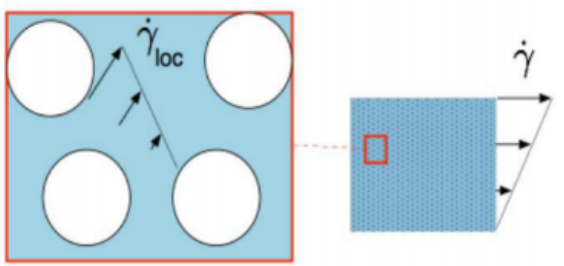

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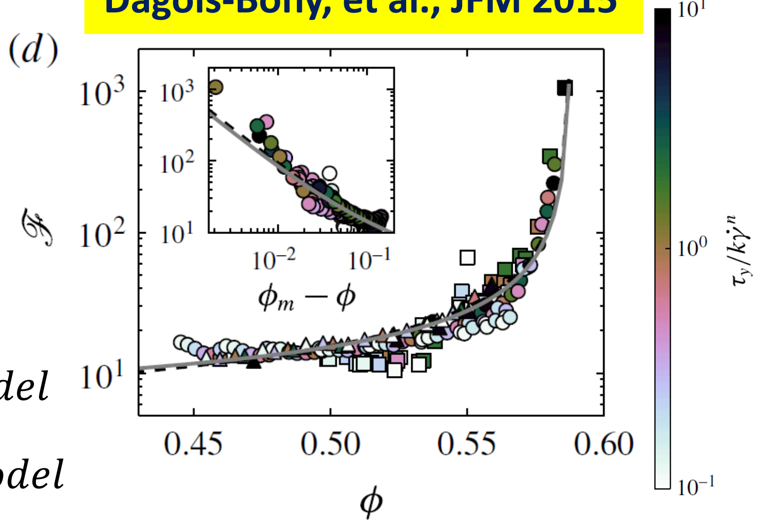
Constitutive laws for suspensions of particles in generalized Newtonian fluids:

$$\tau = \eta_{app}(\dot{\gamma})\dot{\gamma} \begin{cases} \tau = k(\dot{\gamma}\mathcal{F}(\phi))^{n-1}\dot{\gamma} = \kappa_s(\phi)\dot{\gamma}^n & Power-law\ model \\ \tau = \dfrac{\eta_r(\phi)\tau_y}{\mathcal{F}(\phi)} + k\mathcal{F}(\phi)^{n-1}\eta_r(\phi)\dot{\gamma}^n = \tau_{ys}(\phi) + \kappa_s(\phi)\dot{\gamma}^n & Herschel-Bulkley\ model \end{cases}$$

Dagois-Bohy, et al., JFM 2015

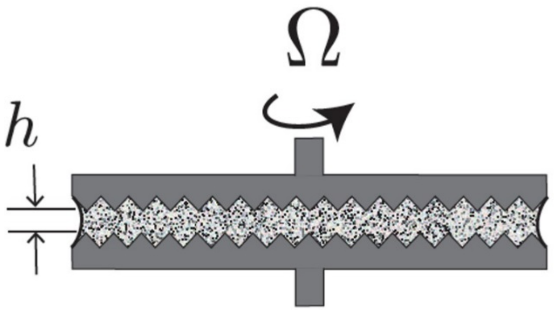


Adding large non-Brownian particles to an oobleck exhibiting DST!

Methods

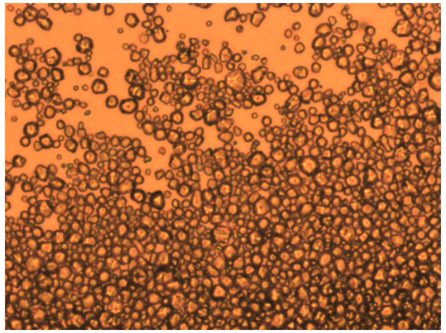
Experiment

- Serrated parallel plate was used to eliminate wall slip effect
- Shear ramp was imposed to obtain rheological results

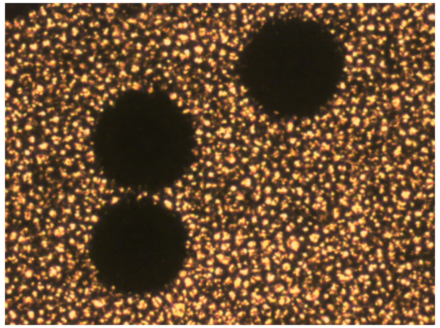


Materials

- Cornstarch: $\rho = 1.68\ gr/cm^3$
- Distilled water+CsCl: $\rho \simeq 1.68\ gr/cm^3$
- Silver coated PMMA*: $\rho = 1.34\ gr/cm^3$

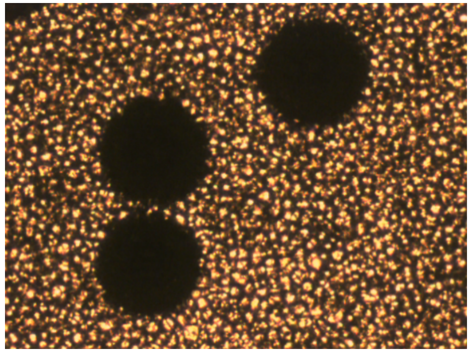


Micrograph of cornstarch suspension
$5\mu m < d_{cs} < 20\mu m$



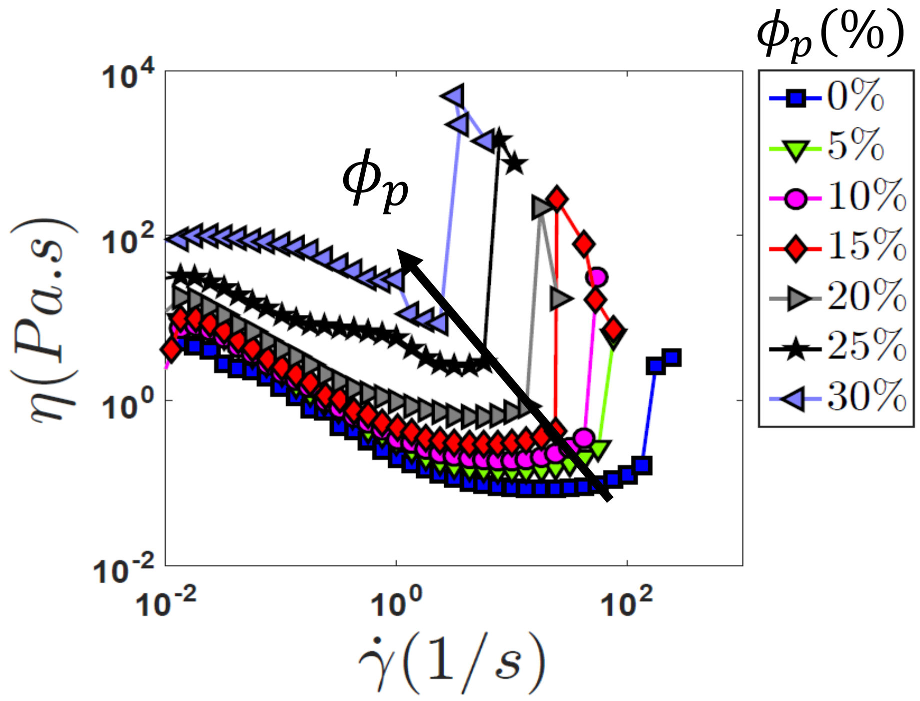
Micrograph of large particles (silver coated PMMA) in cornstarch suspension
$106\mu m < d_p < 125\mu m$

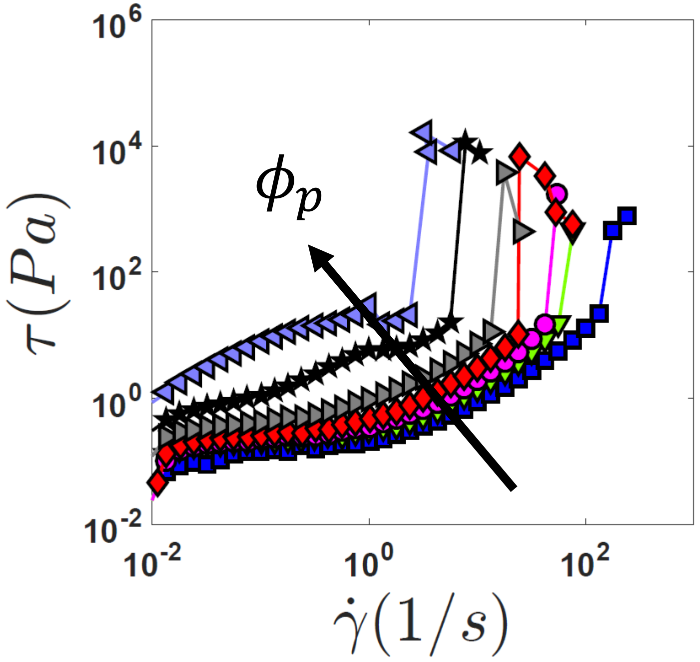
Rheology of particles in cornstarch suspensions

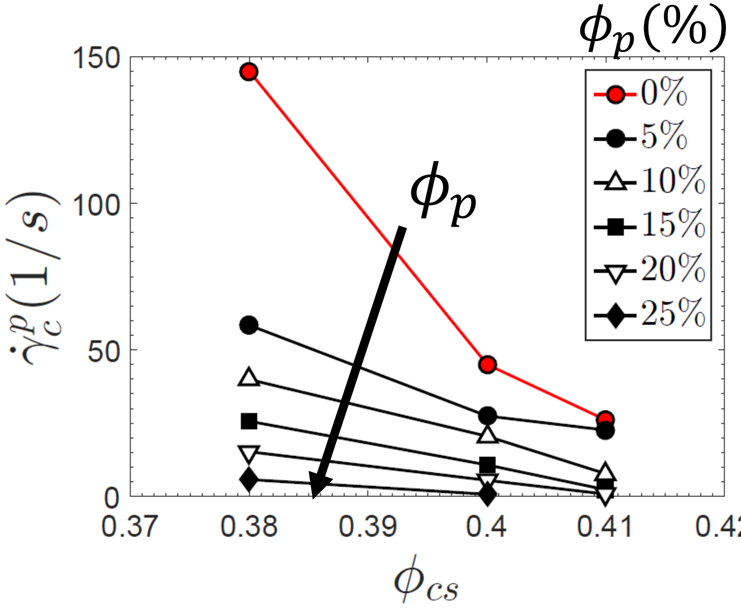


Adding large particles to a cornstarch suspension has dramatic effect on rheological behavior:

- DST transition
- Effective viscosity of the mixture



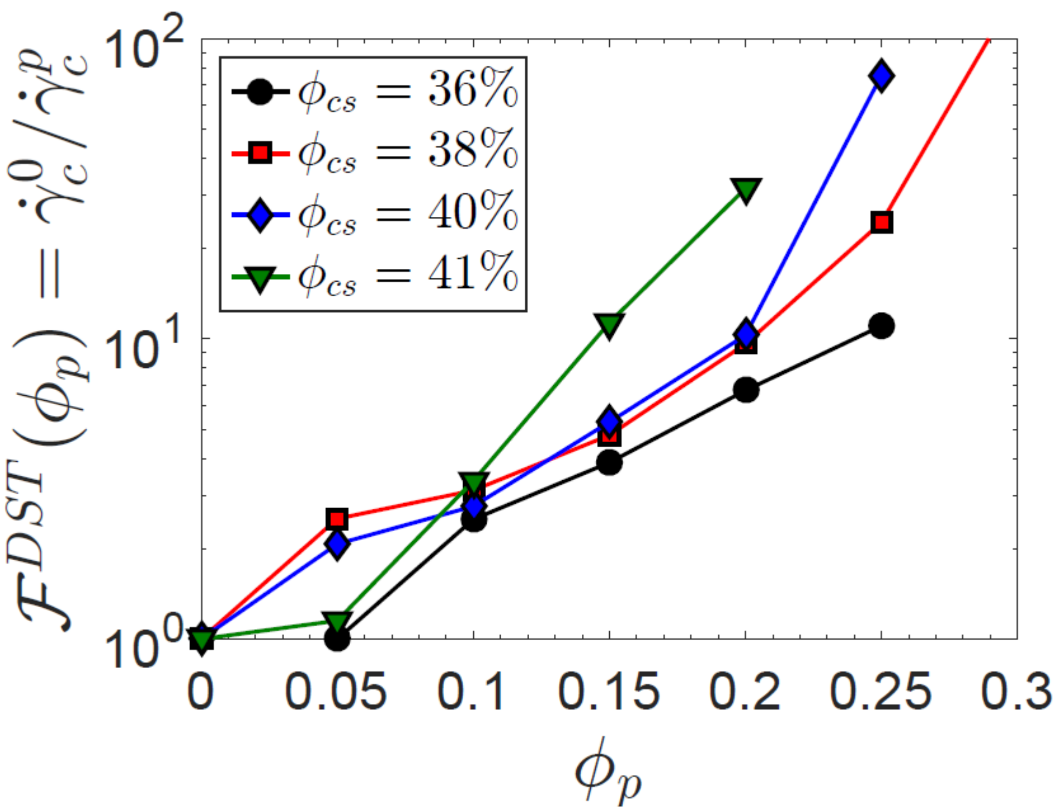


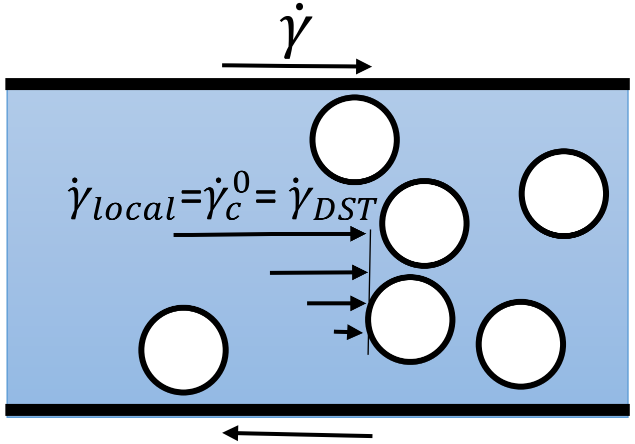


$$\phi_p = \frac{V_p}{V_p + V_{Cornstarch\ Suspension}}$$

$\dot{\gamma}_c^p$: The onset of DST for the mixture of large particles and cornstarch suspensions at different ϕ_p

Shift of critical shear rate at DST





- $\dot{\gamma}_{local} = \mathcal{F}(\phi_p)\dot{\gamma}$
- DST should be obtained when $\dot{\gamma}_{local}$ reaches the critical values for the pure cornstarch suspension, $\dot{\gamma}_c^0$ ($\dot{\gamma}_{local} = \dot{\gamma}_c^0$).
- $\mathcal{F}(\phi_p) = \frac{\dot{\gamma}_c^0}{\dot{\gamma}_c^p} = \frac{critical\ shear\ rate(\phi_p=0)}{critical\ shear\ rate(\phi_p)}$

Shift of rheological properties prior to DST

$$\eta_p(\phi_p)\dot{\gamma}^2 = (1-\phi_p)\eta_p(0)\dot{\gamma}_{\text{local}}^2$$

$$\eta_p(\phi_p) = \left(\mathcal{F}^{\eta_p}(\phi_p)\right)^2 \eta_p(0)(1-\phi_p)$$

$$\tau_y(\phi_p)\dot{\gamma} = (1-\phi_p)\tau_y(0)\dot{\gamma}_{\text{local}}$$

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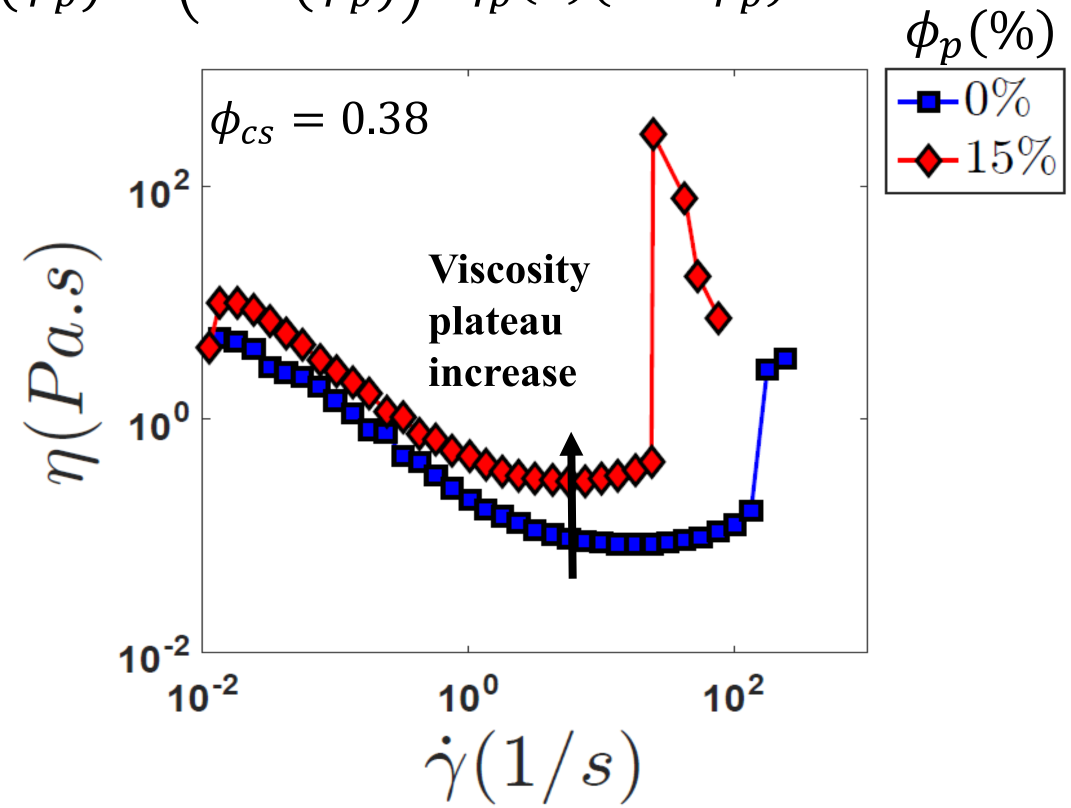
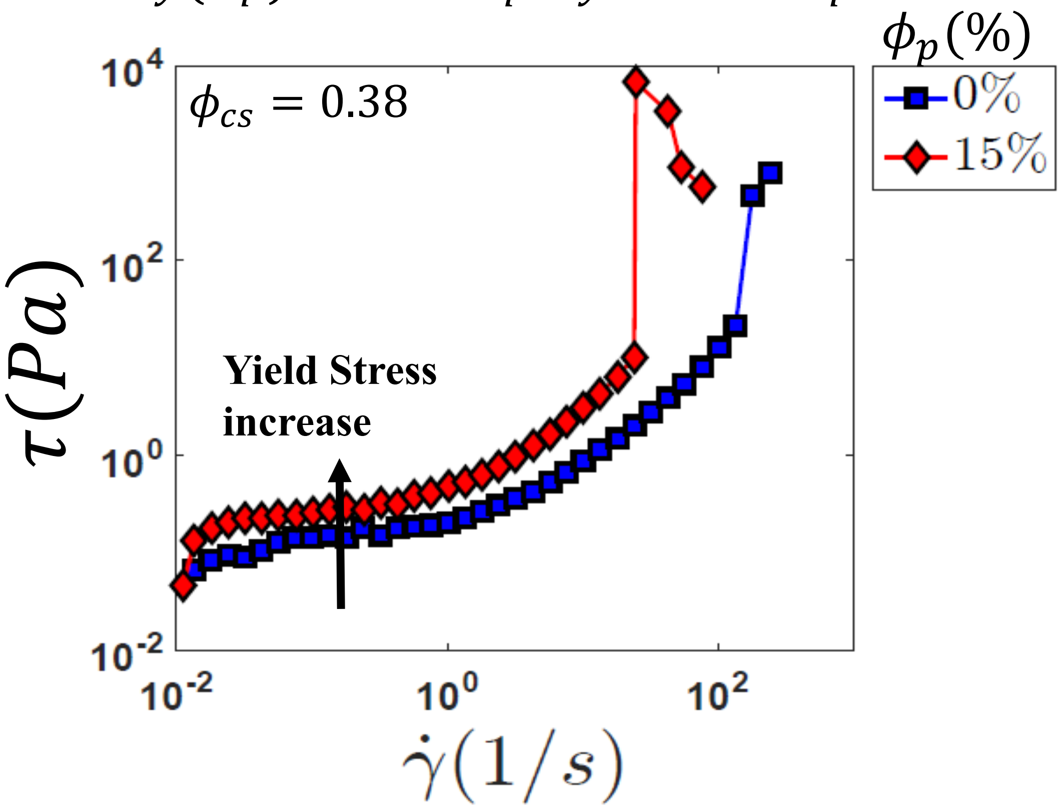
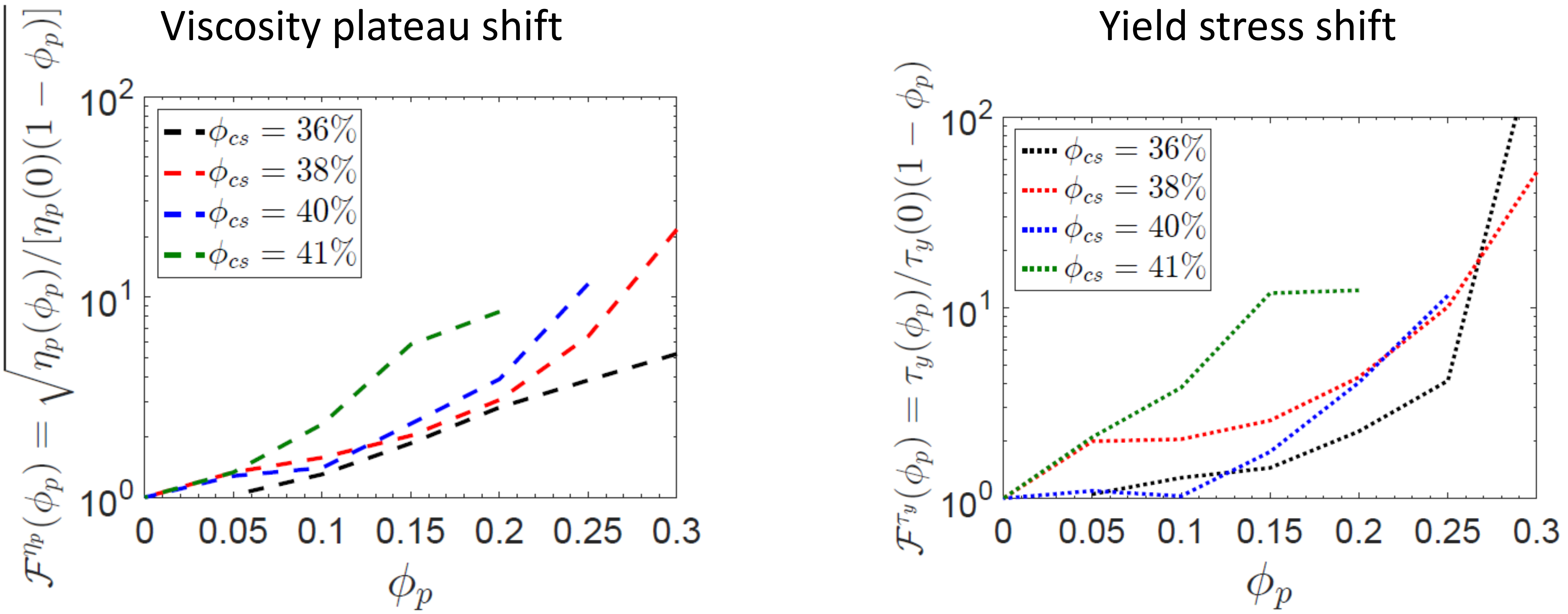




- Not only the DST transition is shifted by adding large spheres to cornstarch suspension, but also the **viscosity plateau** and **yield stress** are shifted.

Shift of rheological properties prior to DST

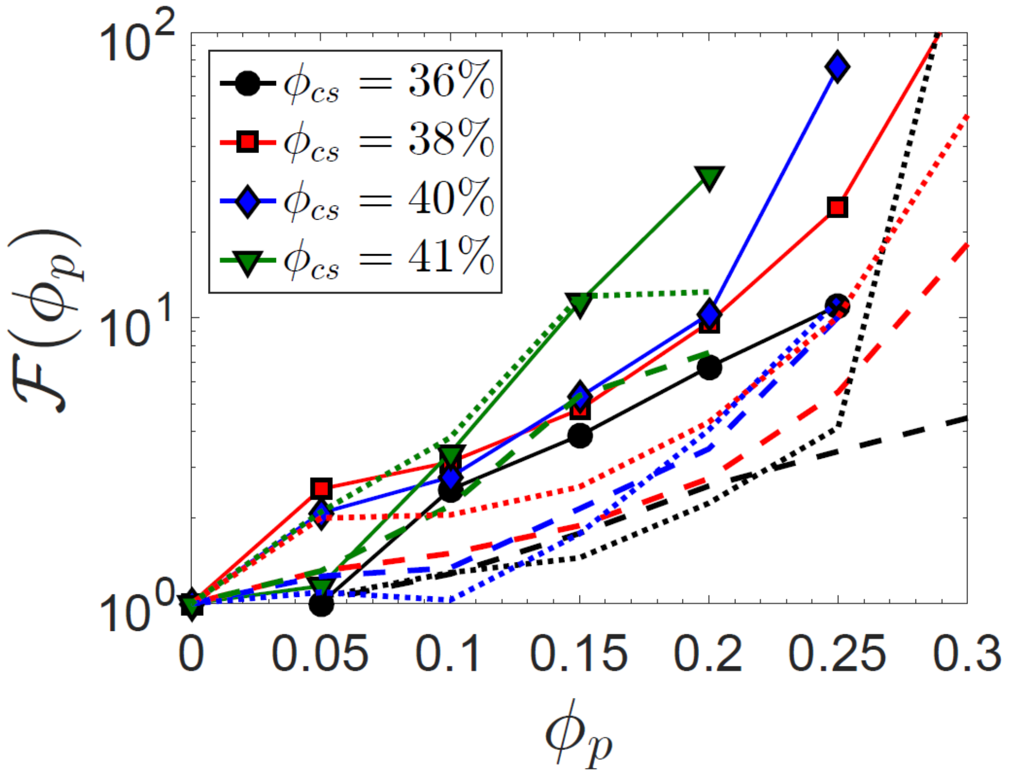


- Lever function can be alternatively estimated by measuring the shift in **viscosity plateau** and **yield stress**.

(Dagois-Bohy, et al., 2015; Chateau, et al., 2008; Ovarlez, et al., 2015)

Three values of $\mathcal{F}(\phi_p)$

- Predictions from the DST shift, $\mathcal{F}(\phi_p)$, are systematically larger than other two evaluations.



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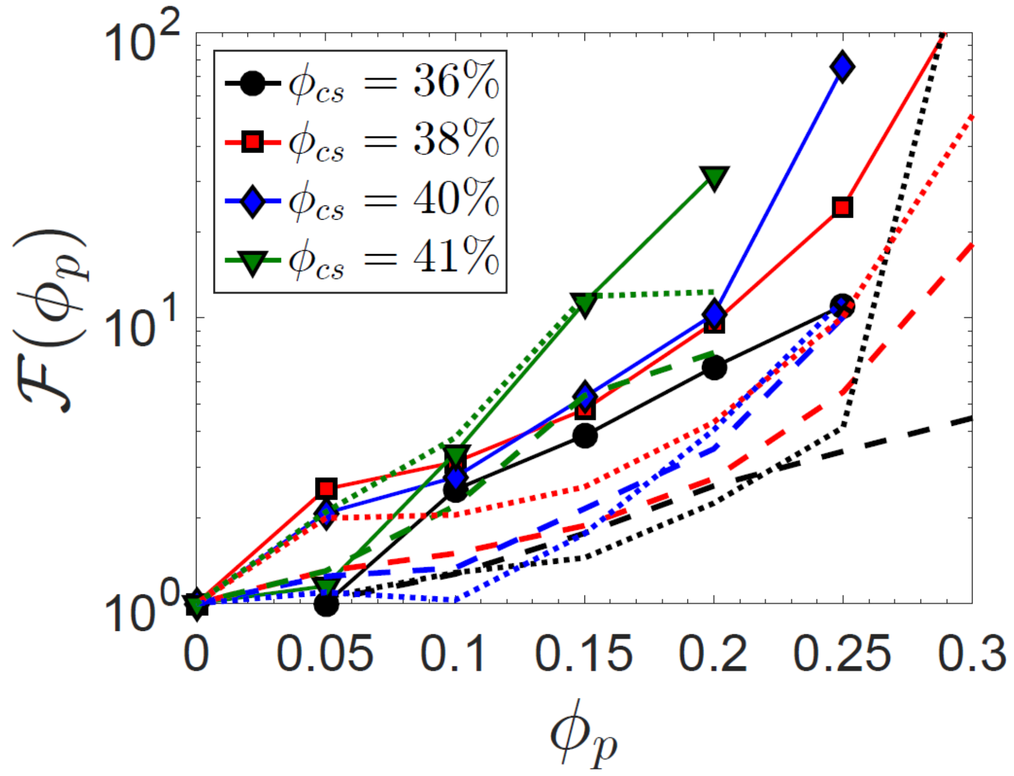
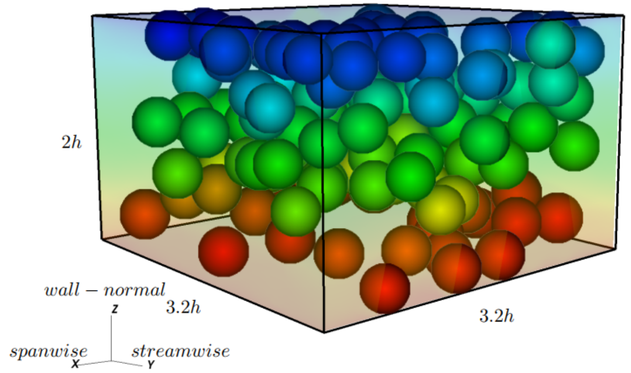


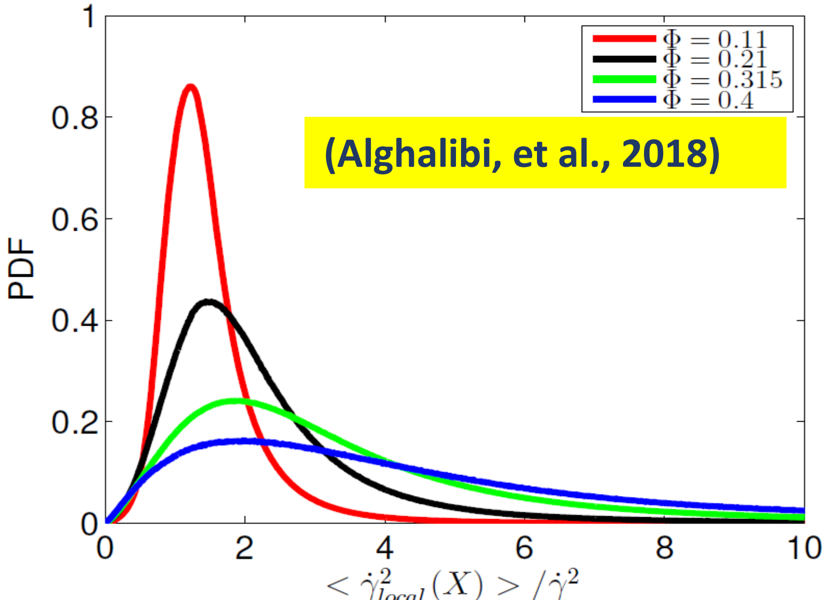
- Probability distribution function of local shear rate present increase of extreme values by adding larger particles to a background fluid. DST may be controlled by the extreme values of the local shear rate distribution.

(Liard, et al., 2014;

Alghalibi, et al., 2018;

Souzy, et al., 2017)





(Alghalibi, et al., 2018)

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Particles in a **shear thinning** suspension

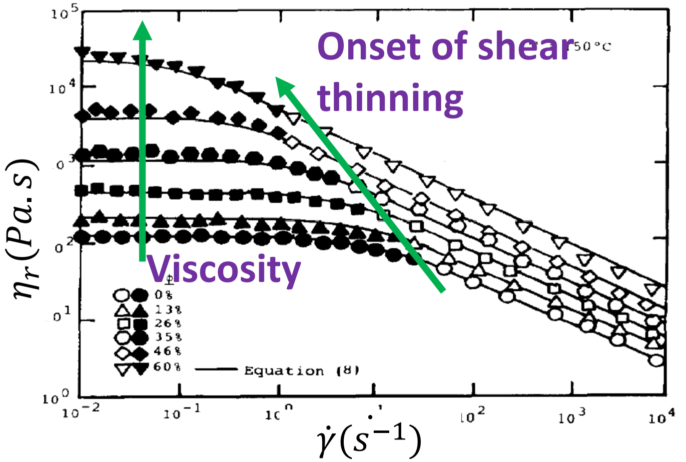

Fig. 4. Shear viscosity of glass spheres dispersed in the thermoplastic polymer at 150°C. ○● = 0%, △▲ = 13%, □■ = 26%, ⬡⬢ = 35%, ◇◆ = 46%, ▽▼ = 60%, — = Eq. (8).

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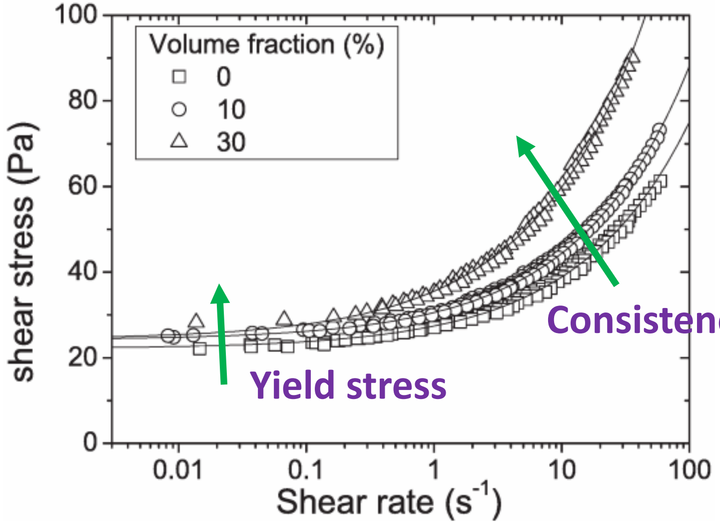
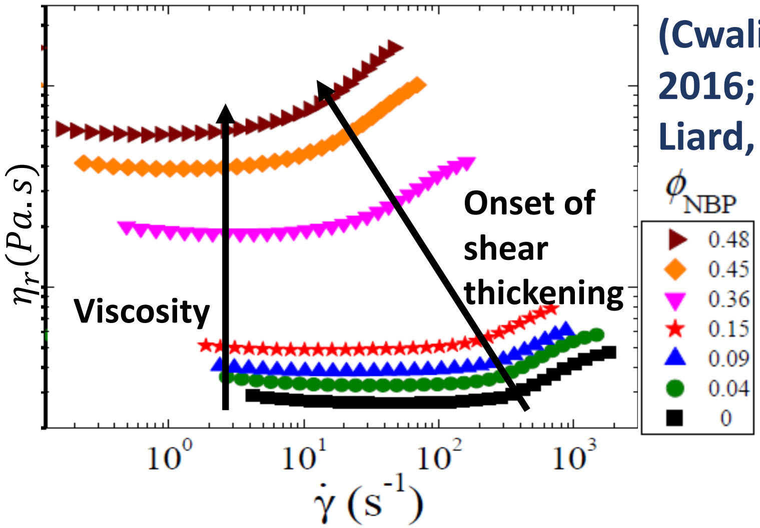


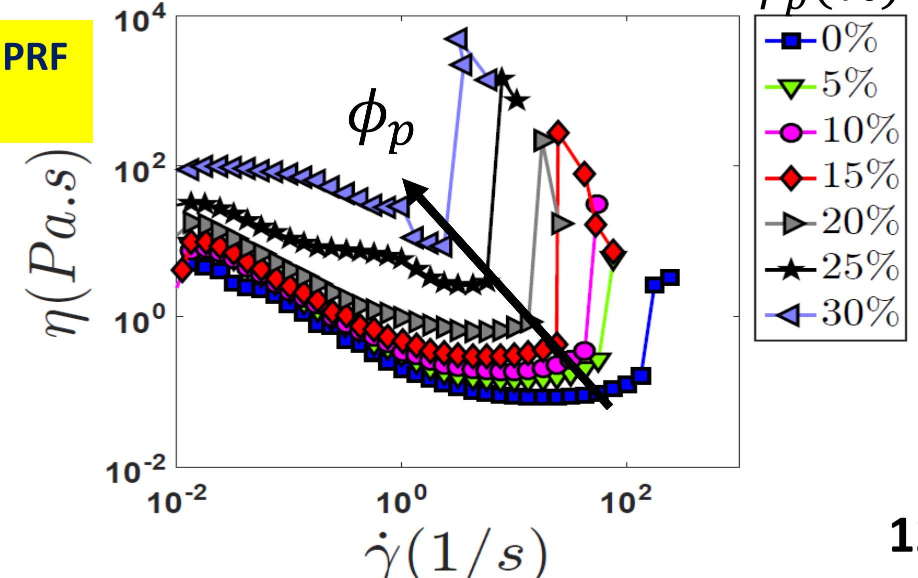
Particles in a **CST** suspension



(Cwalina & Wagner, 2016; Liard, et al., 2014)

Particles in a **DST** suspension

(Madraki et al., PRF 2017.)



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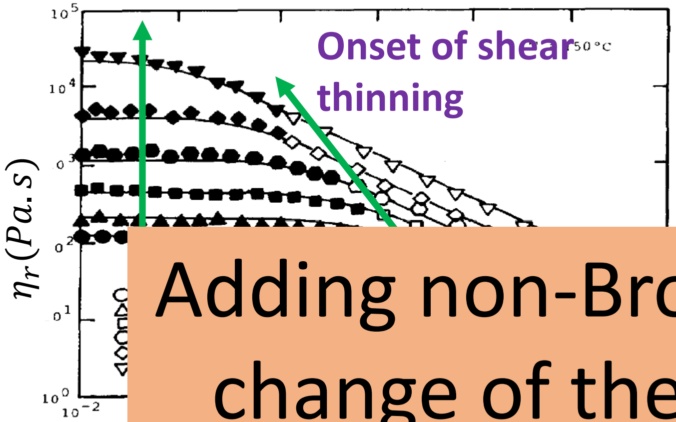

Fig. 4. She
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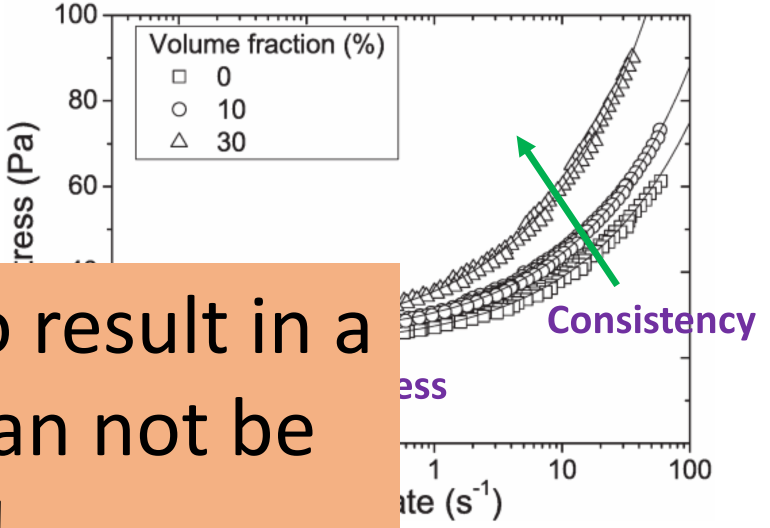
▽▼ = 60%, —

(Polinski et al., *JOR* 1988;
Liard, et al., 2014)

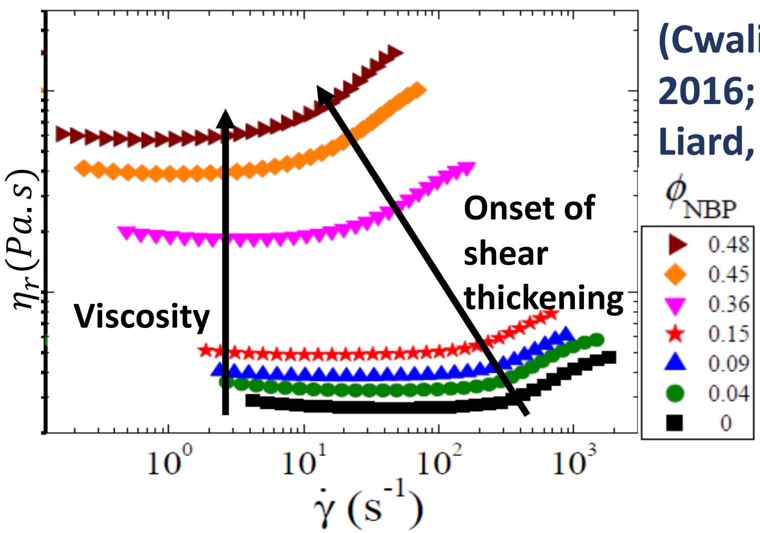
(Dagois-Bohy, et al., 2015;
Chateau, et al., 2008;
Ovarlez, et al., 2015)

Particles in a **yield stress** suspension



Adding non-Brownian particles may also result in a change of the Rheological behavior! Can not be explained via this framework!

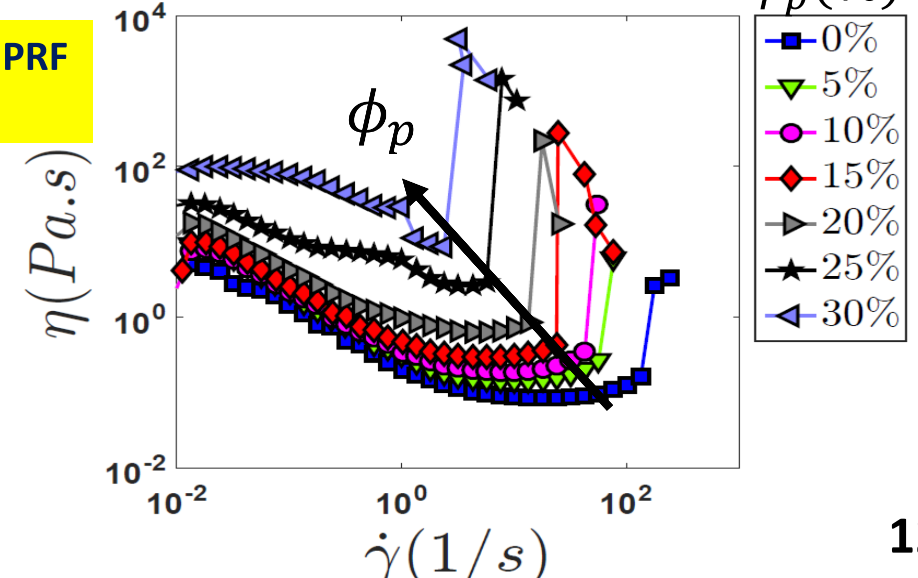
Particles in a **CST** suspension



(Cwalina & Wagner, 2016;
Liard, et al., 2014)

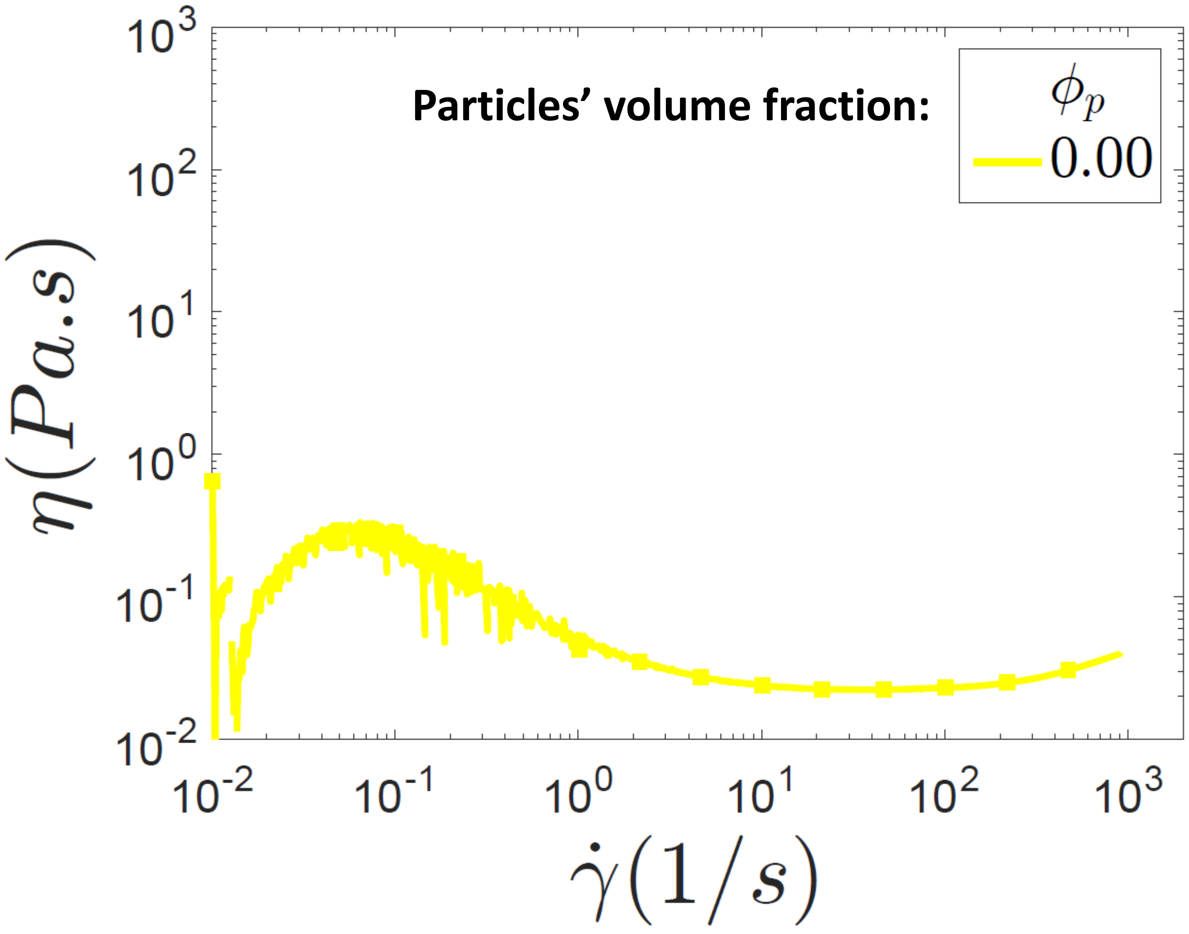
(Madraki et al., PRF 2017.)

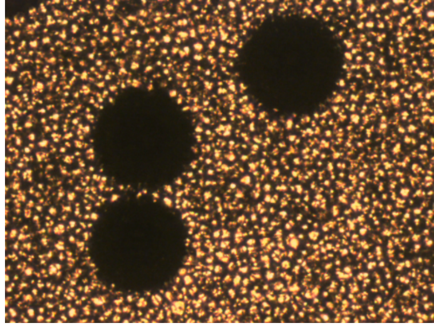
Particles in a **DST** suspension



CST to DST transition

Adding particles to a CST suspension, i.e. Cornstarch suspension $\phi_{cs} = 0.3$:





Silver coated PMMA in cornstarch suspension

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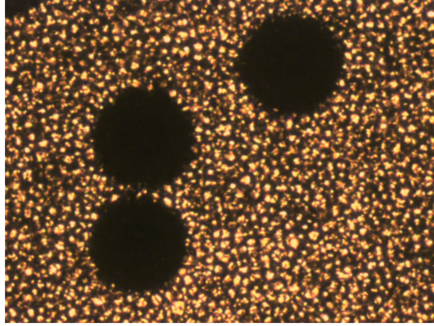
Adding particles to CST suspension, i.e. Cornstarch suspension $\phi_{cs} = 0.3$:

- **Low** particles' volume fraction **: increases the viscosity** and **expedites CST**
  (Cwalina & Wagner, 2015; Liard, et al., 2014)



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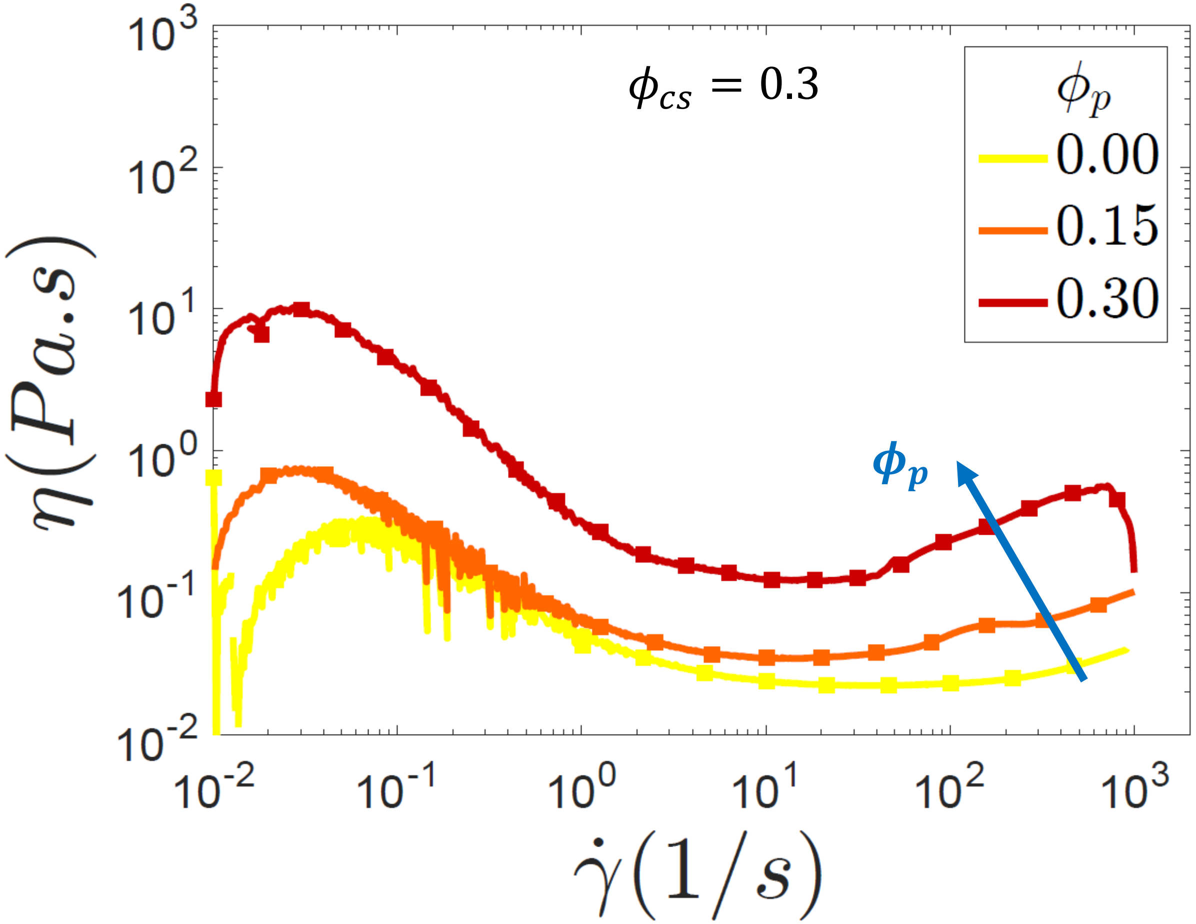

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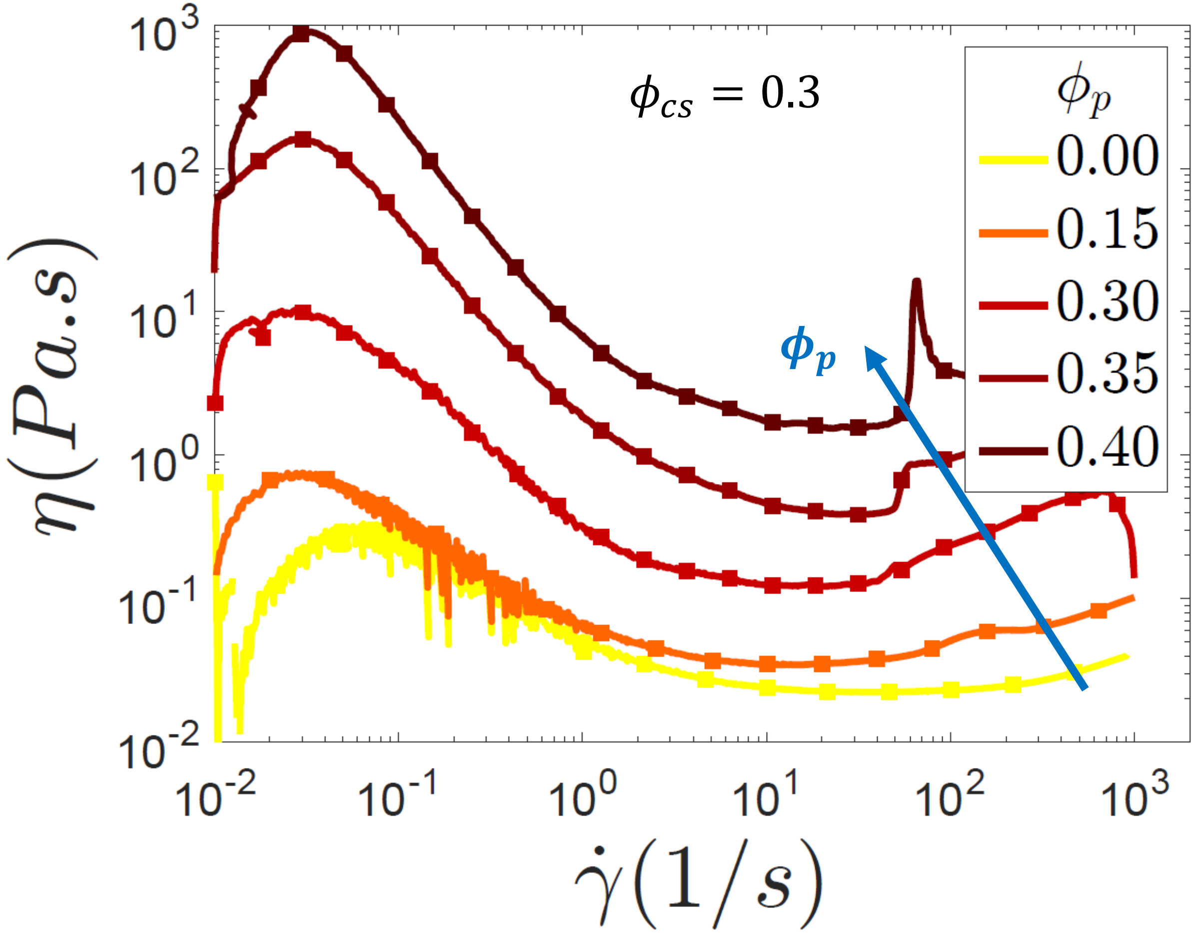
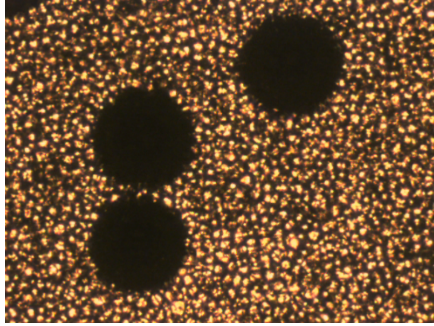
- **High** particles' volume fraction concentrations: **transitions the CST to DST**





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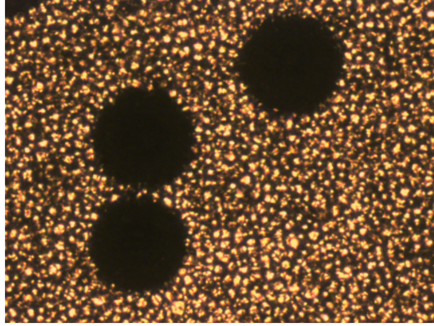

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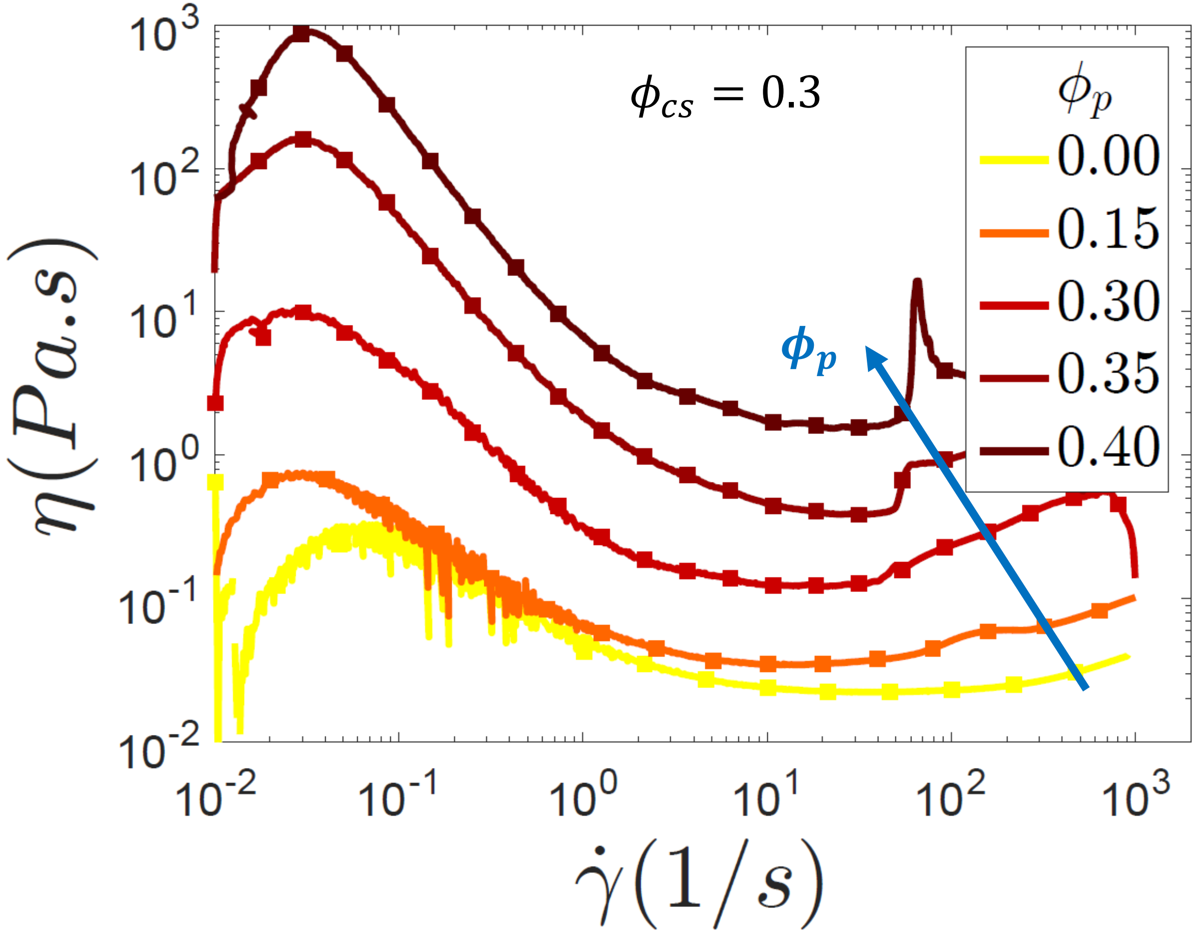
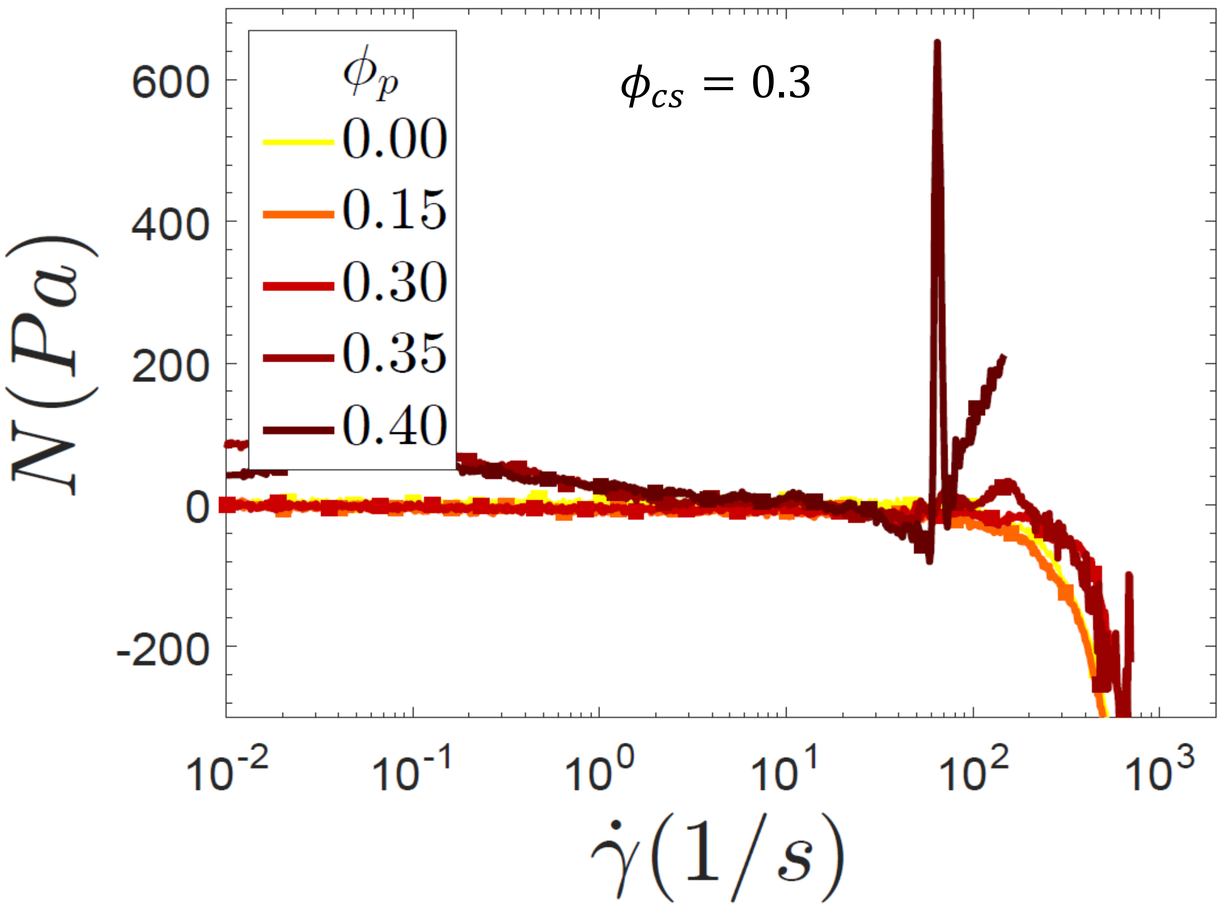
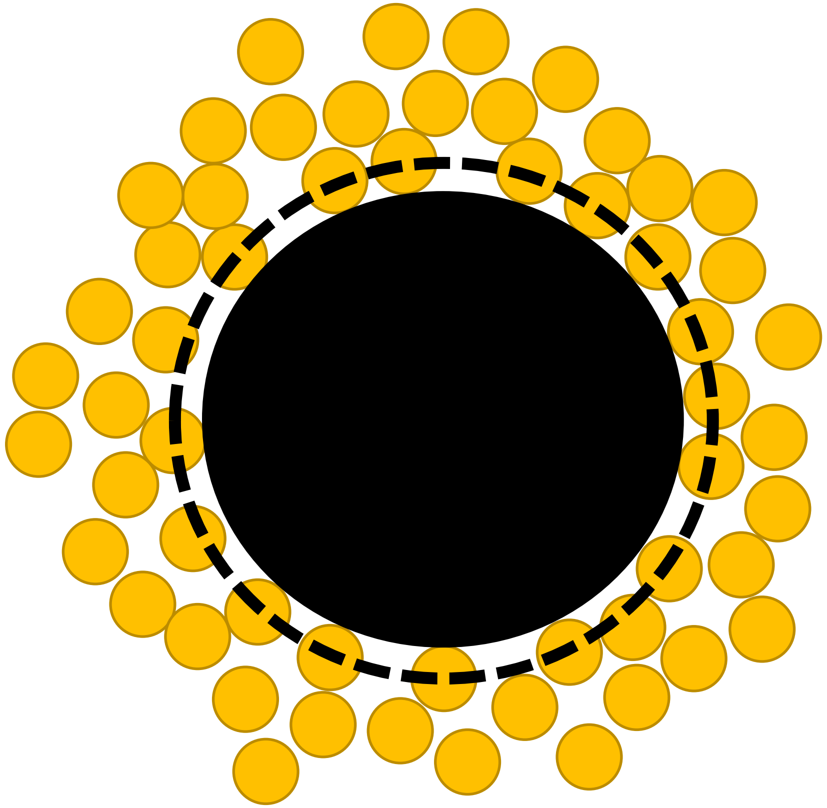




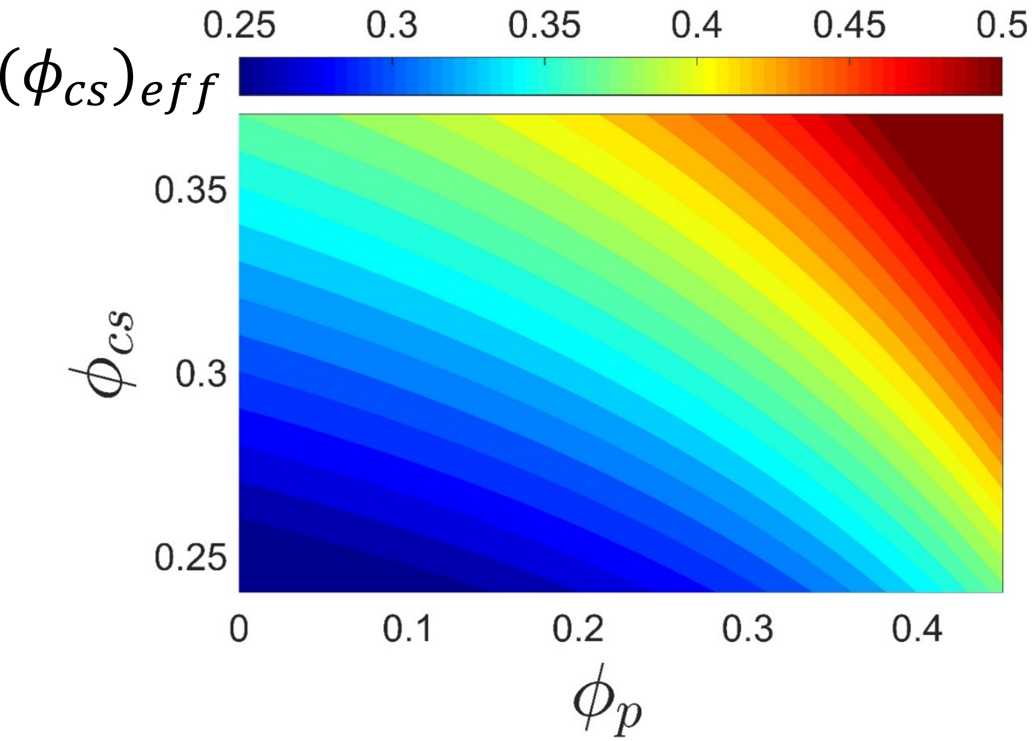
CST to DST transition: Excluded volume effect

- The center of cornstarch grains cannot come closer than their radius to the surface of a large particle.
- We must subtract the volume of this shell, mostly composed of water from the volume of water in the rest of the suspension.
- There is an increase of cornstarch concentration outside of these shells.
- local cornstarch volume fraction decreases at the large particle surface, compensated by increase elsewhere

The effective cornstarch volume fraction: $(\phi_{cs})_{eff} = f(\phi_{cs}, \phi_p, d_p)$



Excluded volume effect: Plotting the $(\phi_{cs})_{eff} = f(\phi_{cs}, \phi_p, d_p)$



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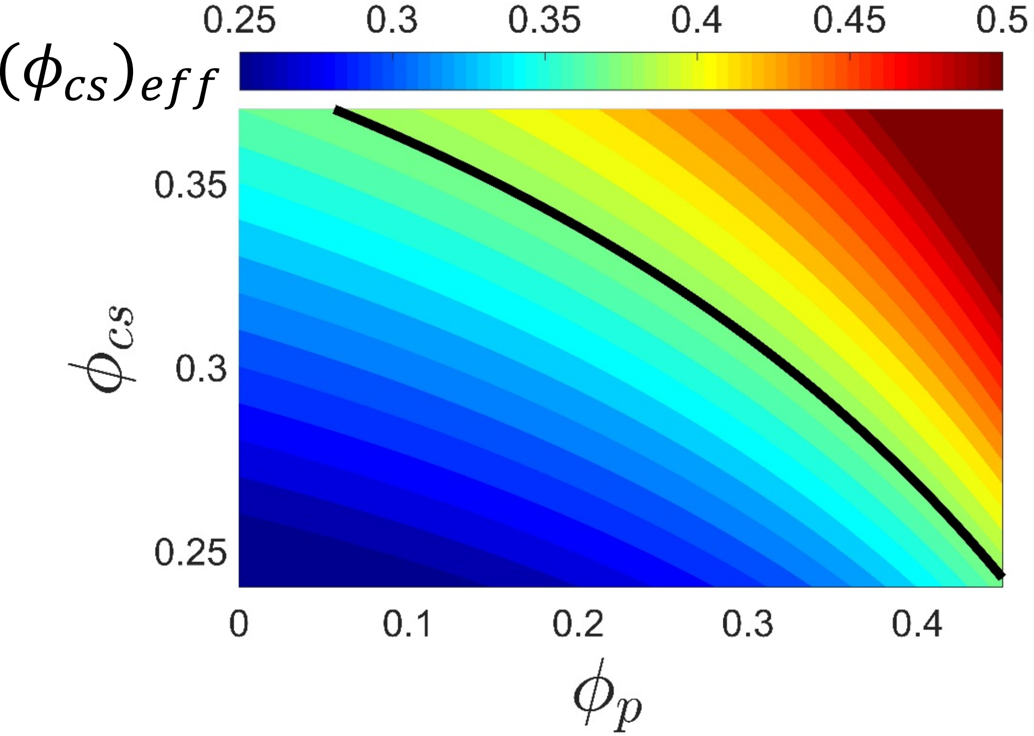

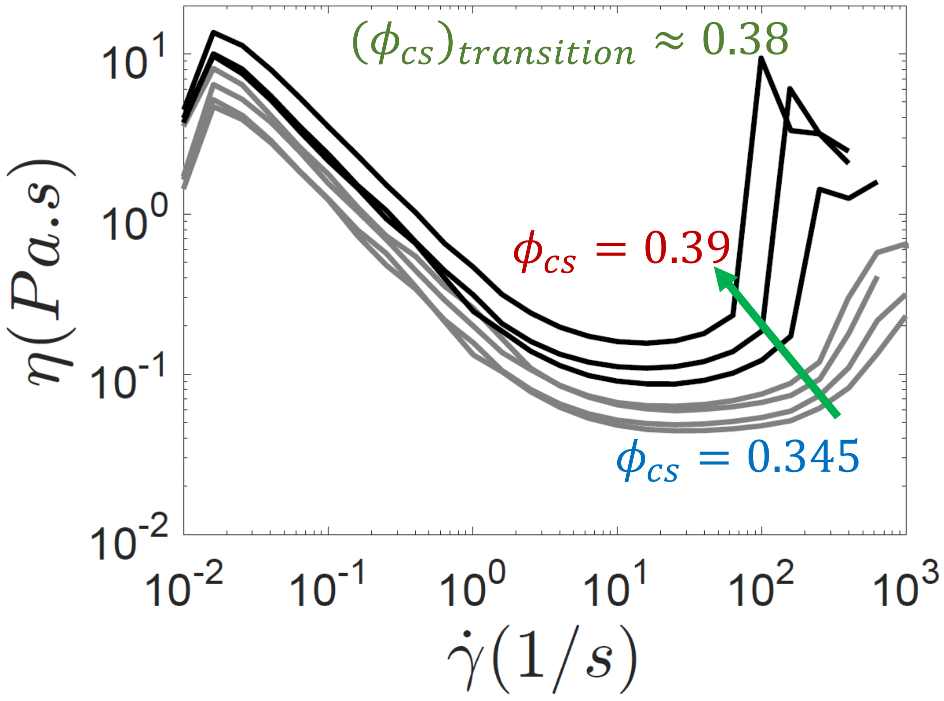

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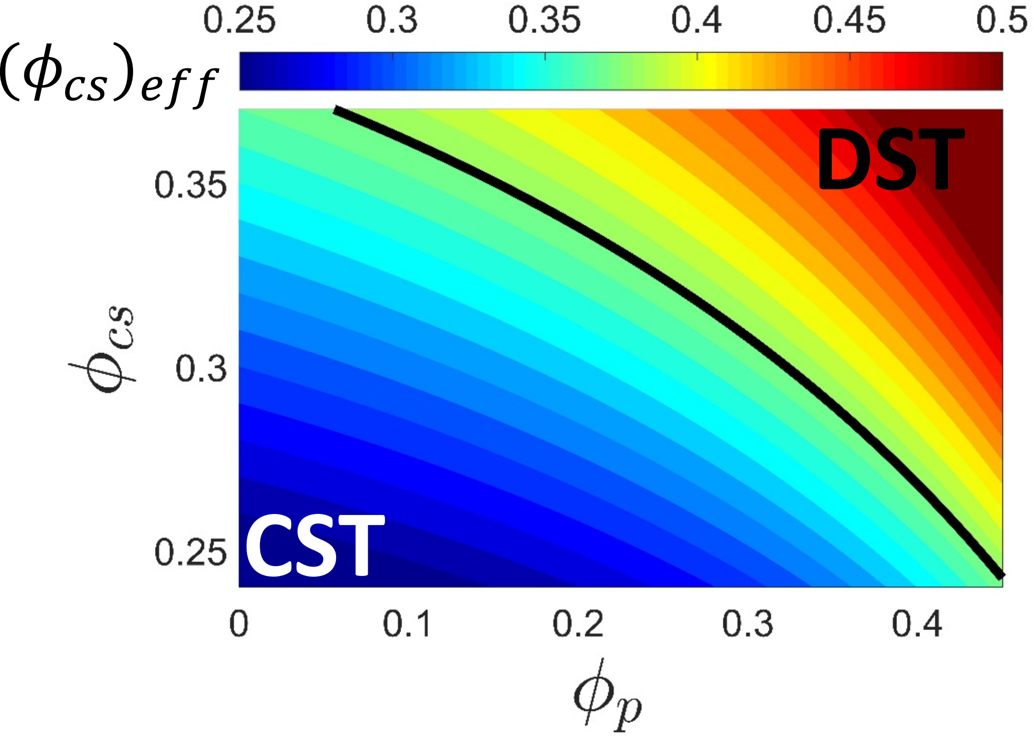

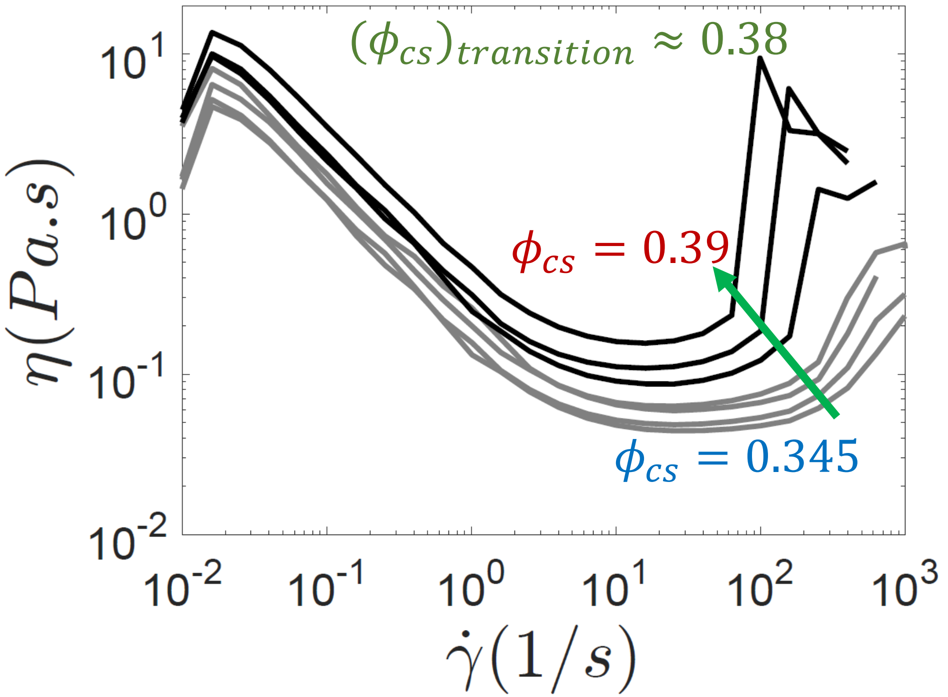

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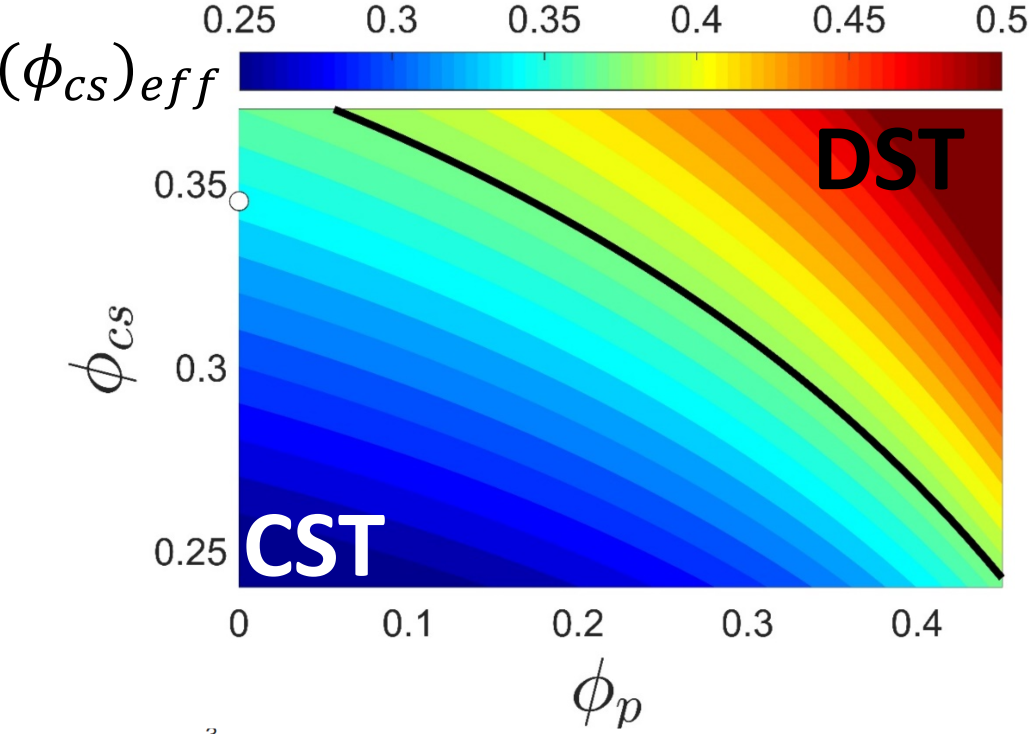
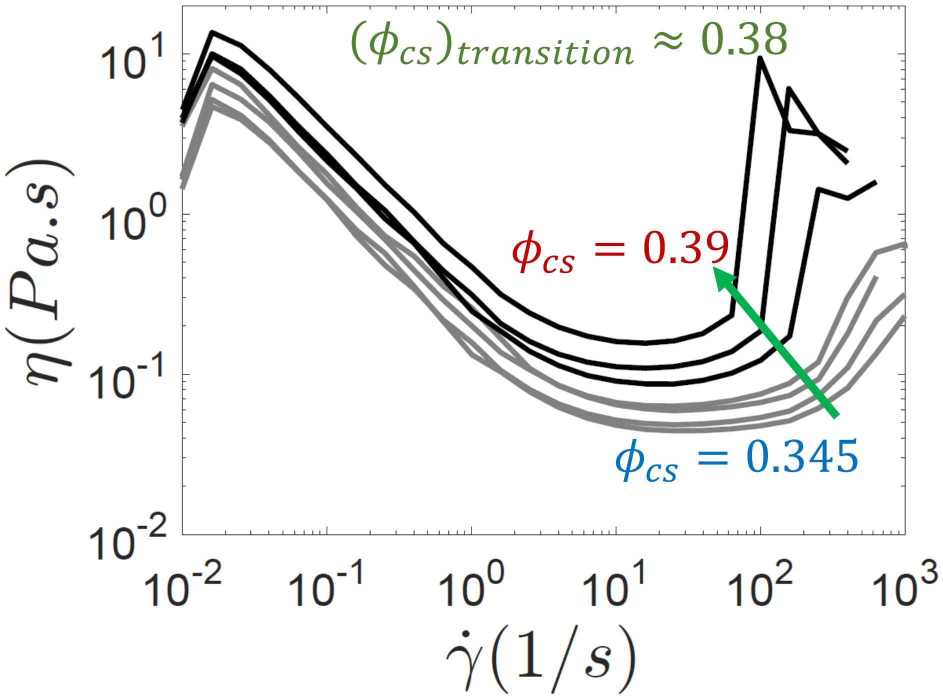
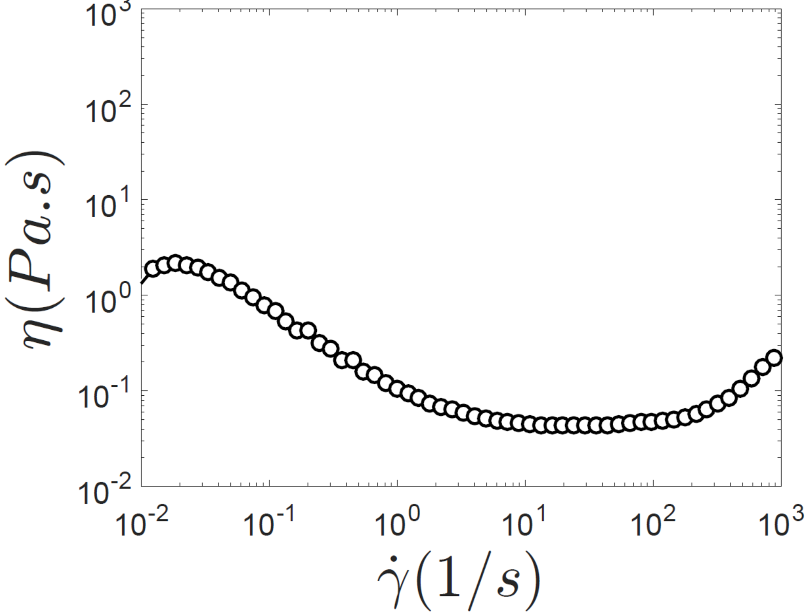
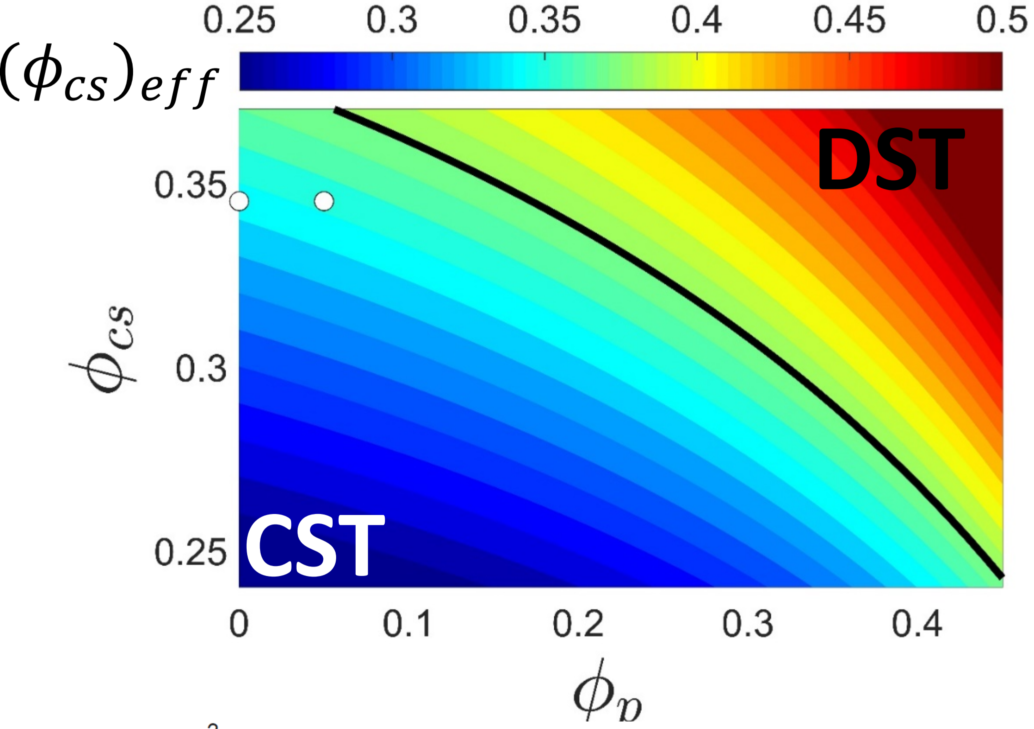


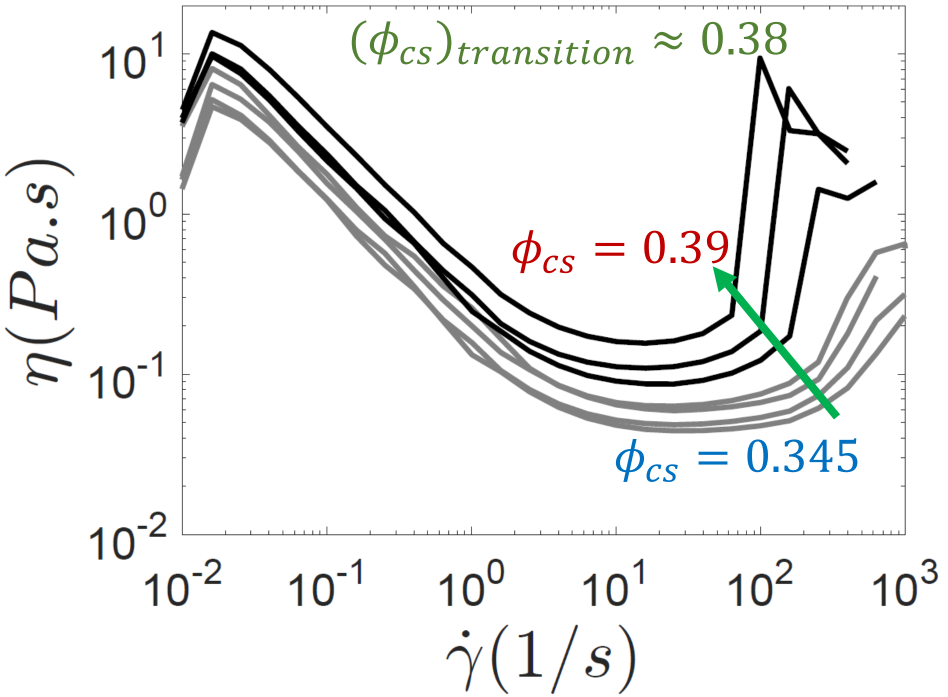


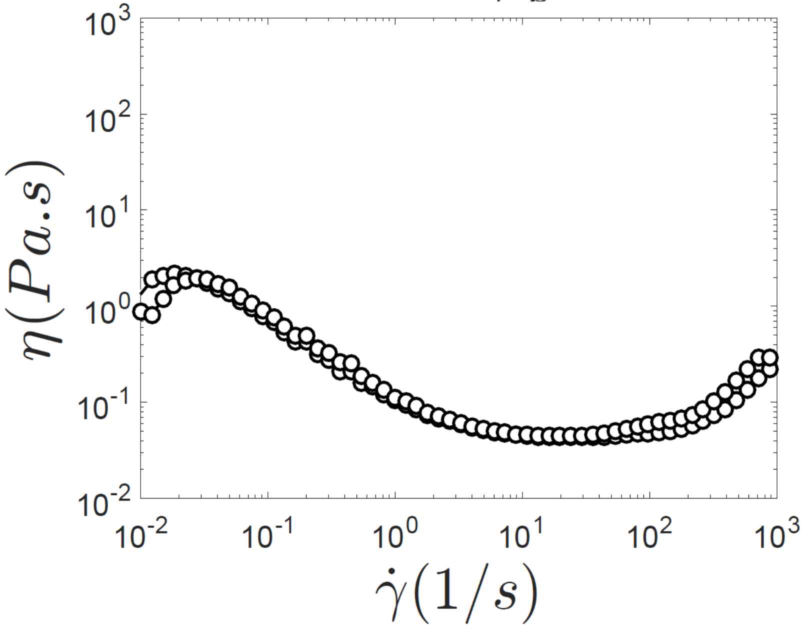


$\phi_{cs} = 34.5\%$
Silver Coated PMMA
$d_p = 106 - 125\ \mu m$

Excluded volume effect: Plotting the $(\phi_{cs})_{eff} = f(\phi_{cs}, \phi_p, d_p)$







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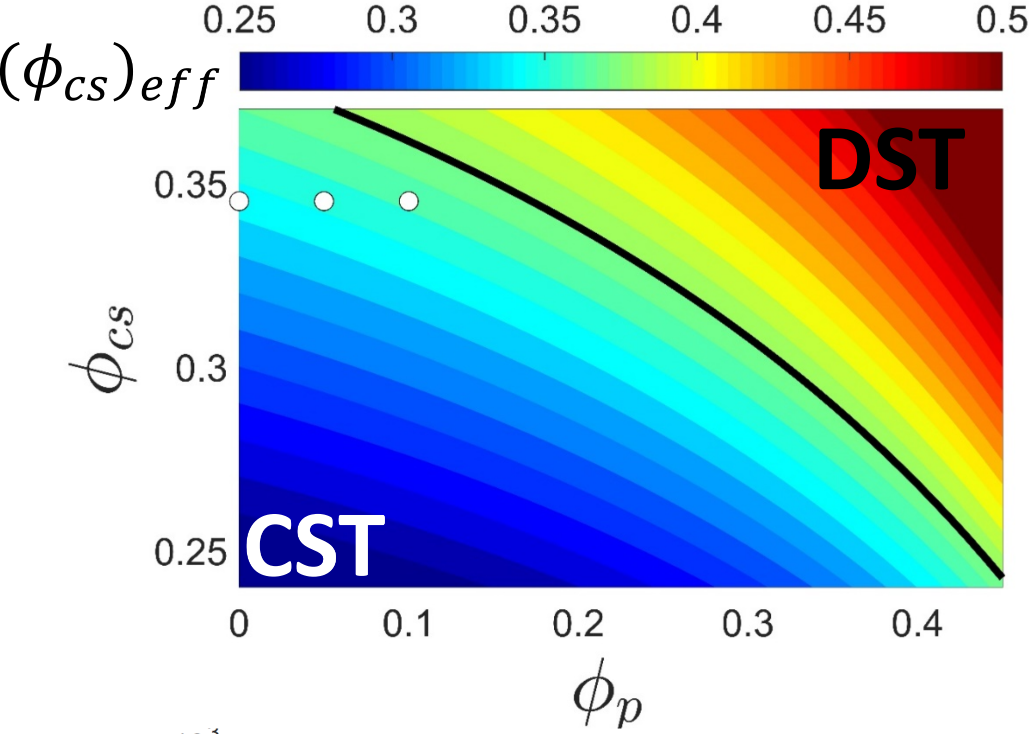

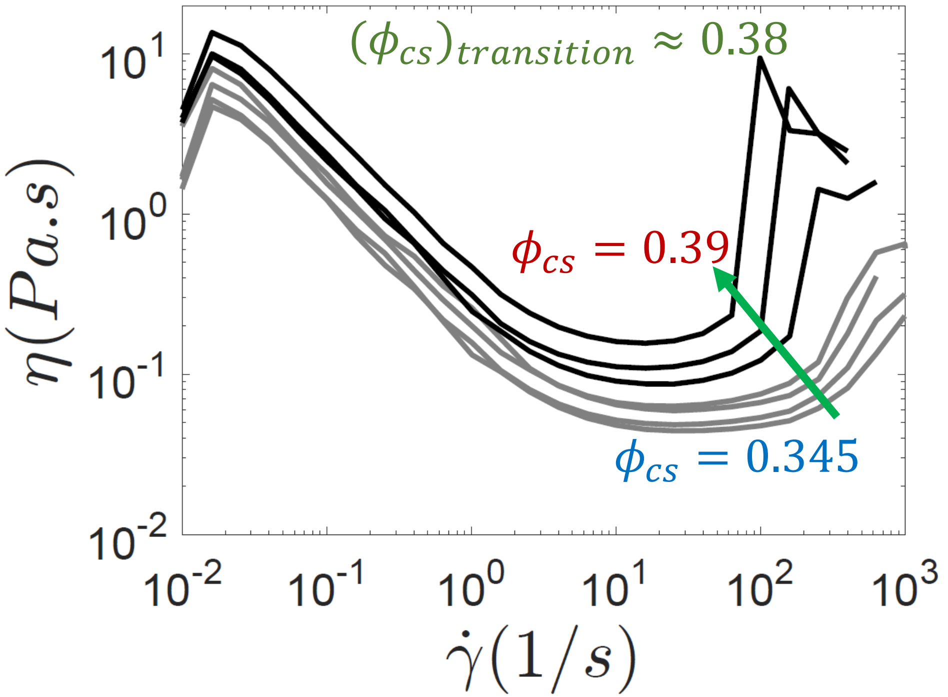

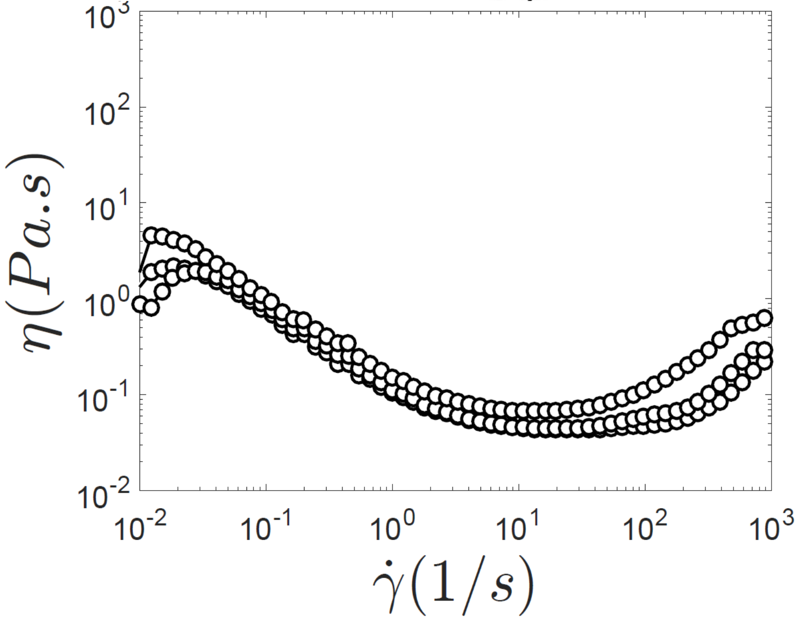

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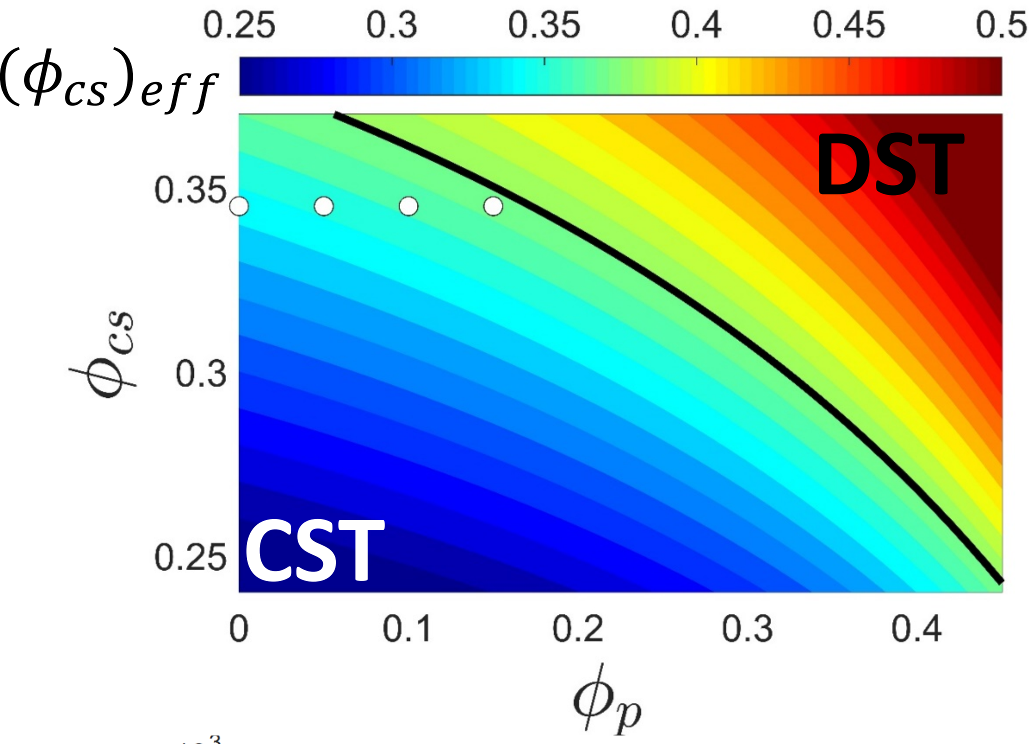

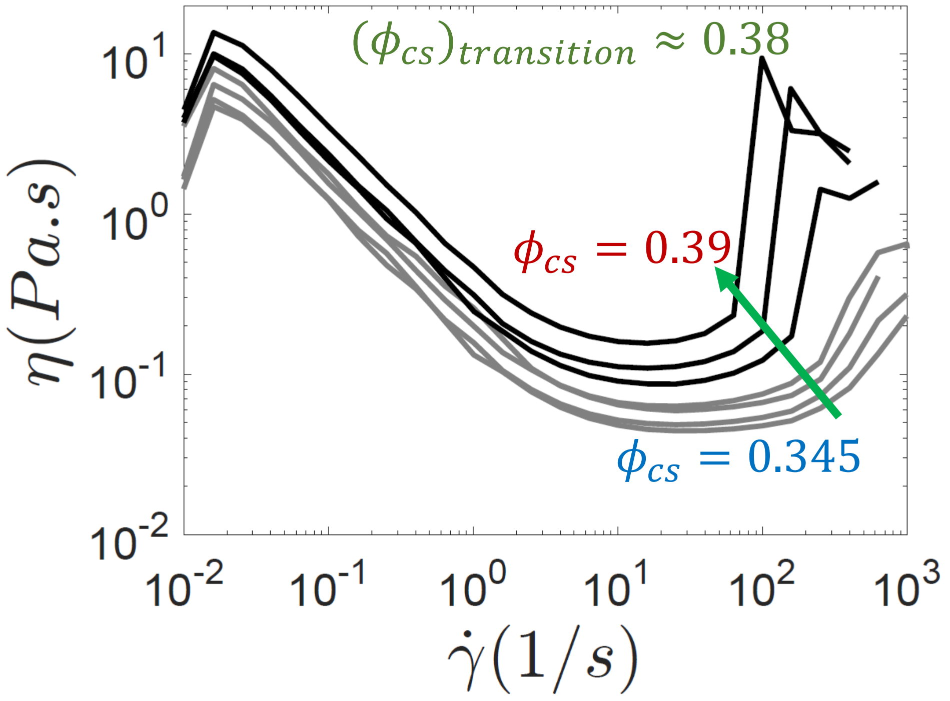

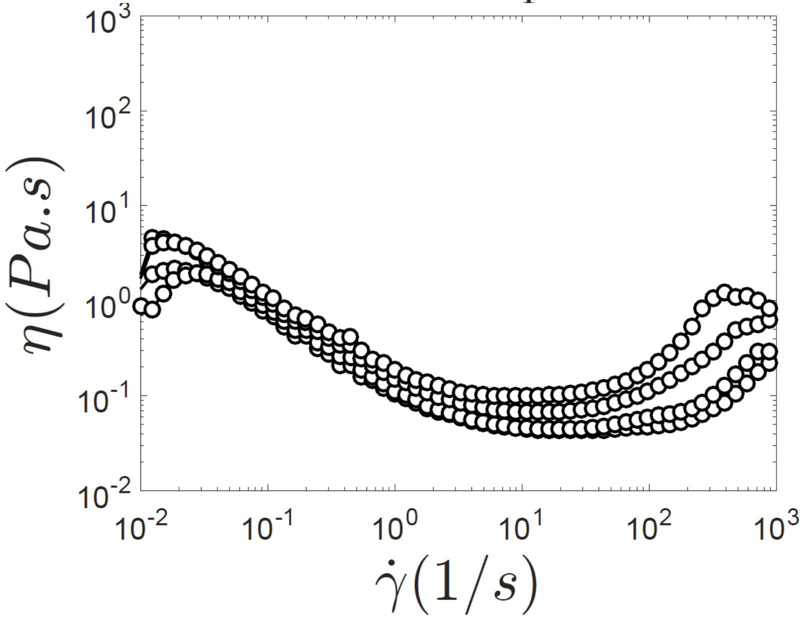

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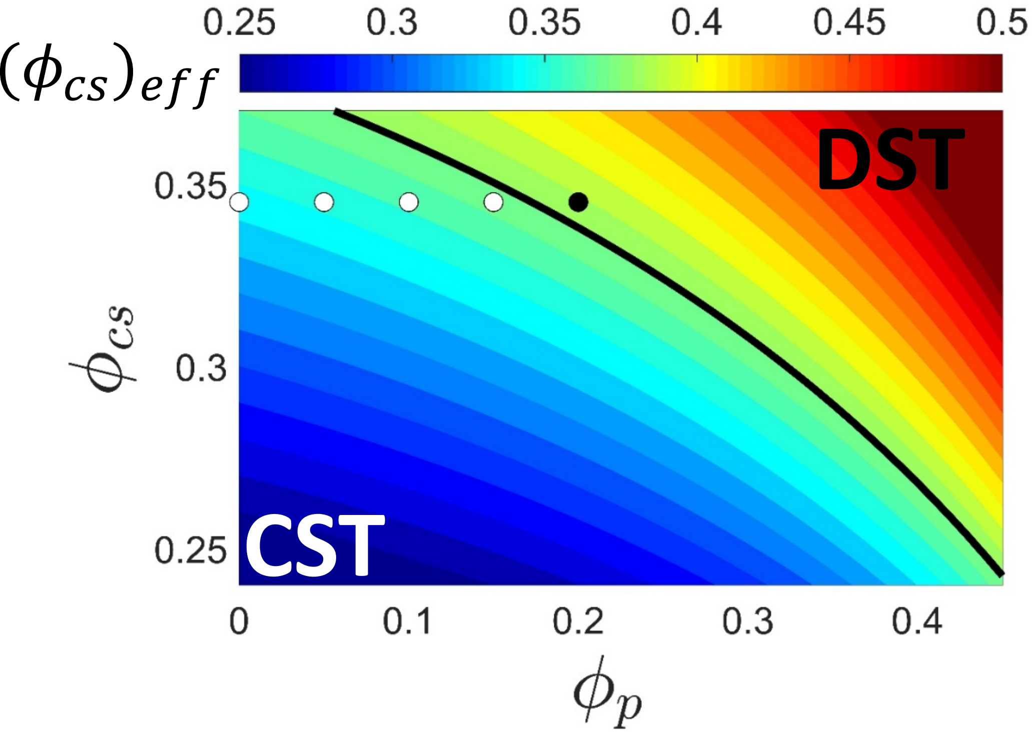

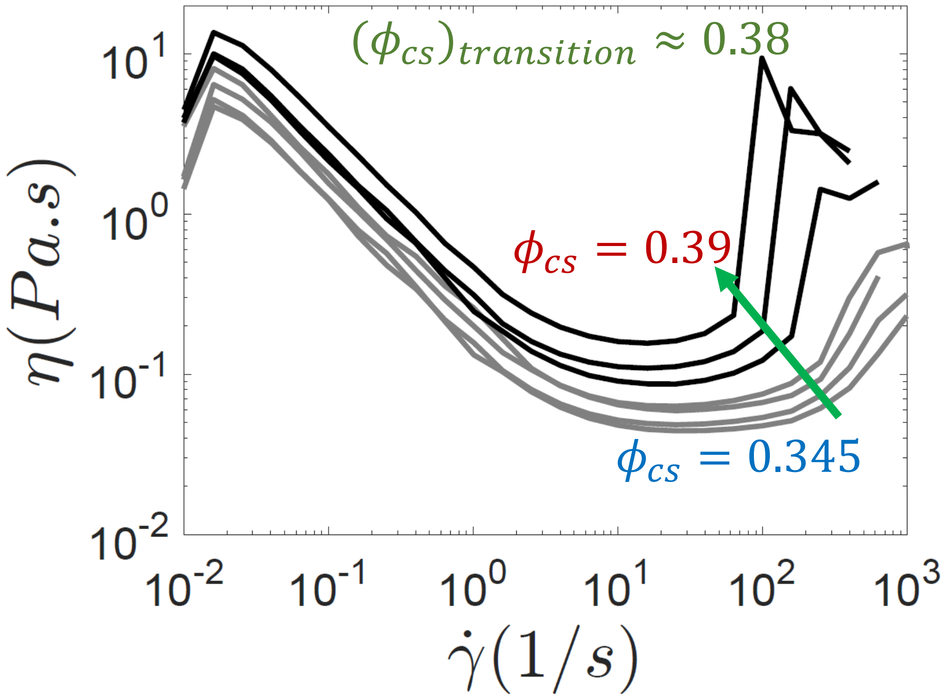

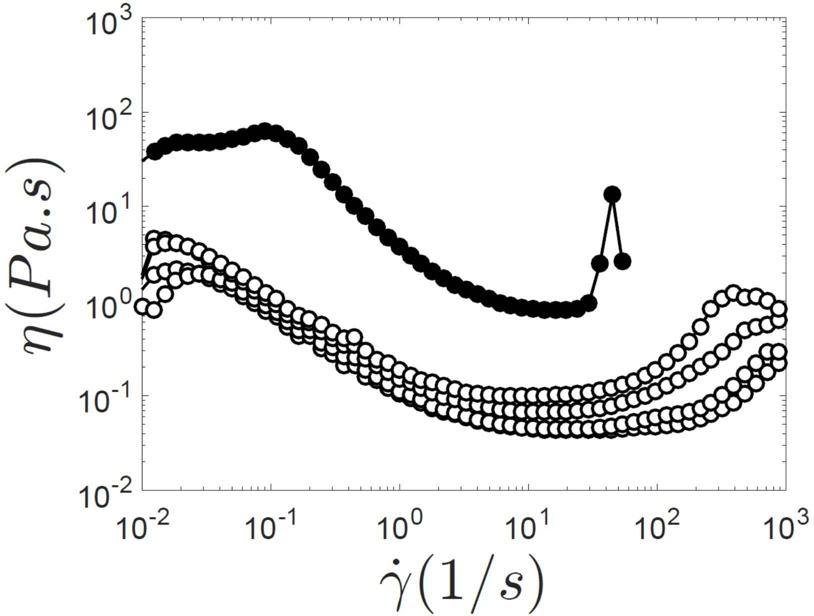

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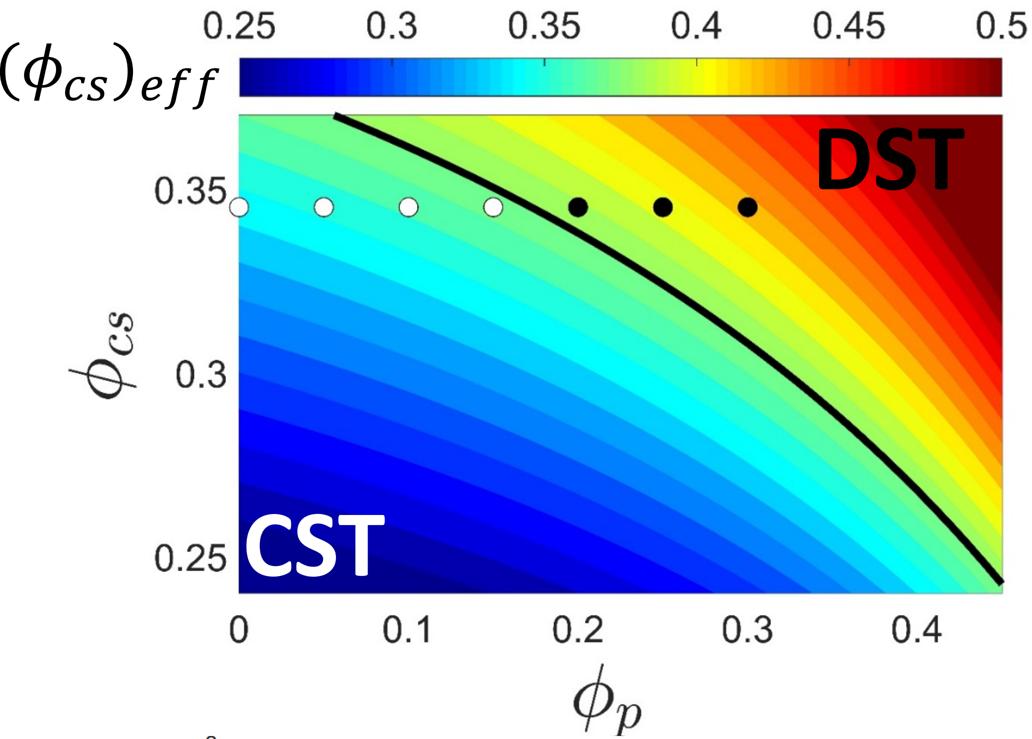

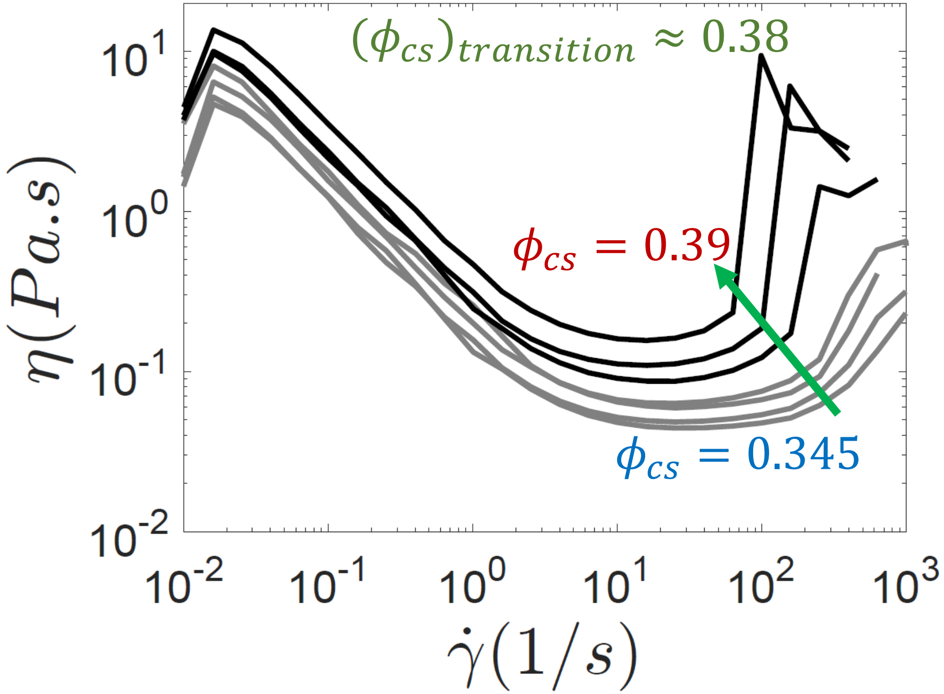

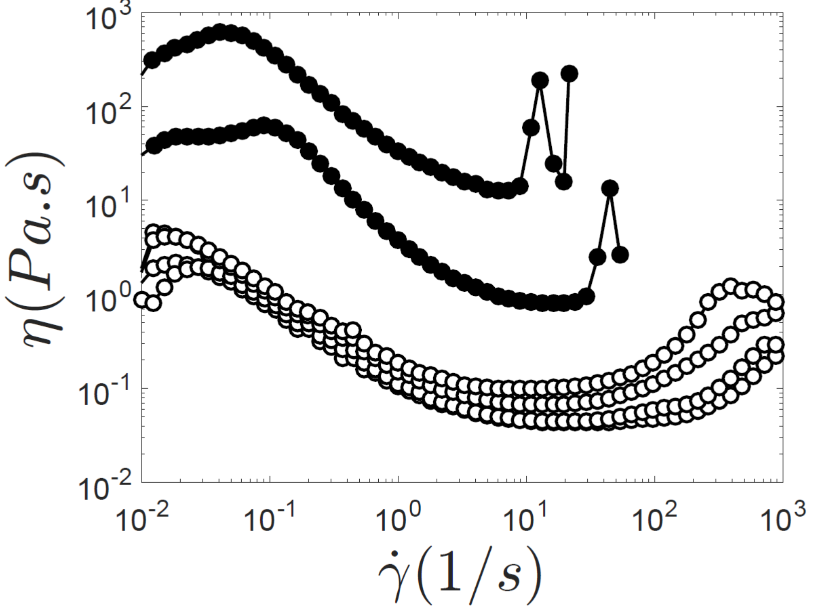

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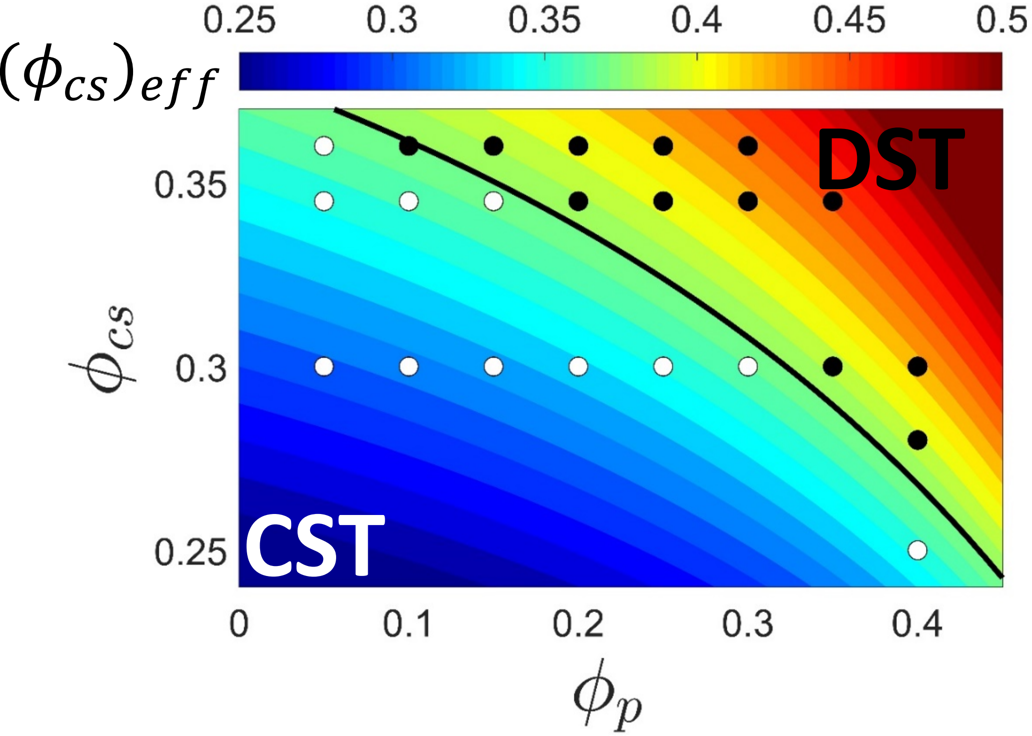

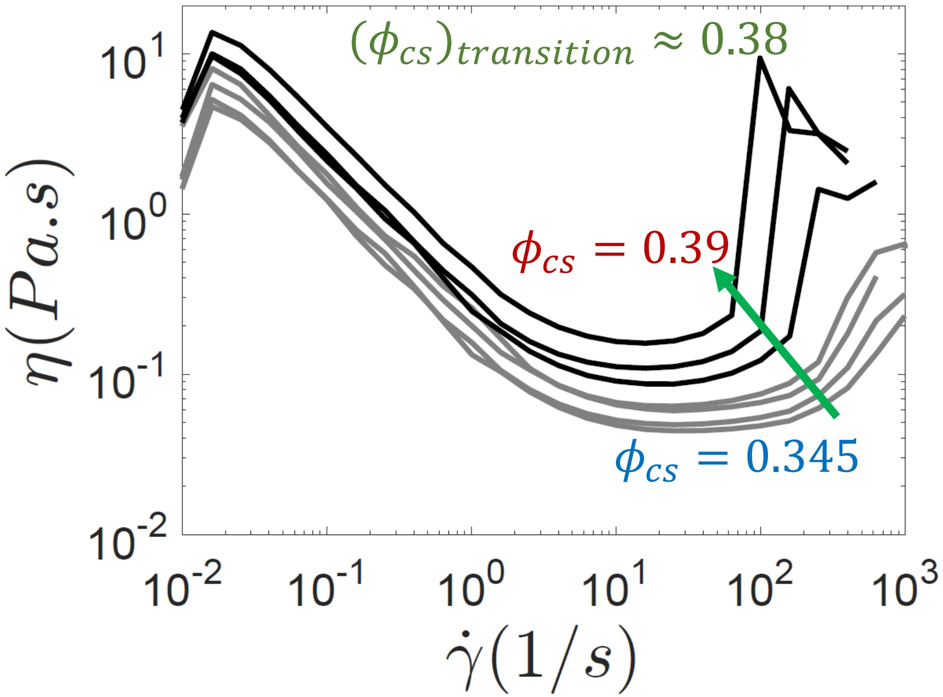

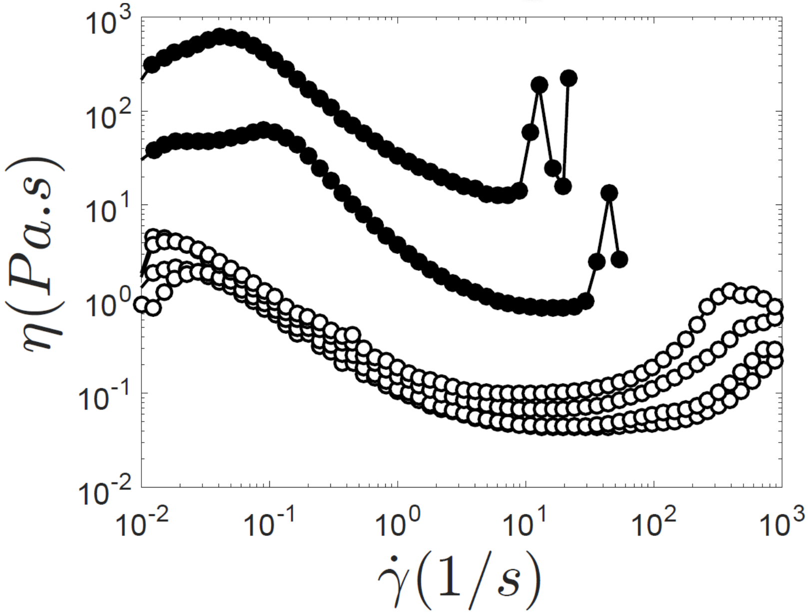

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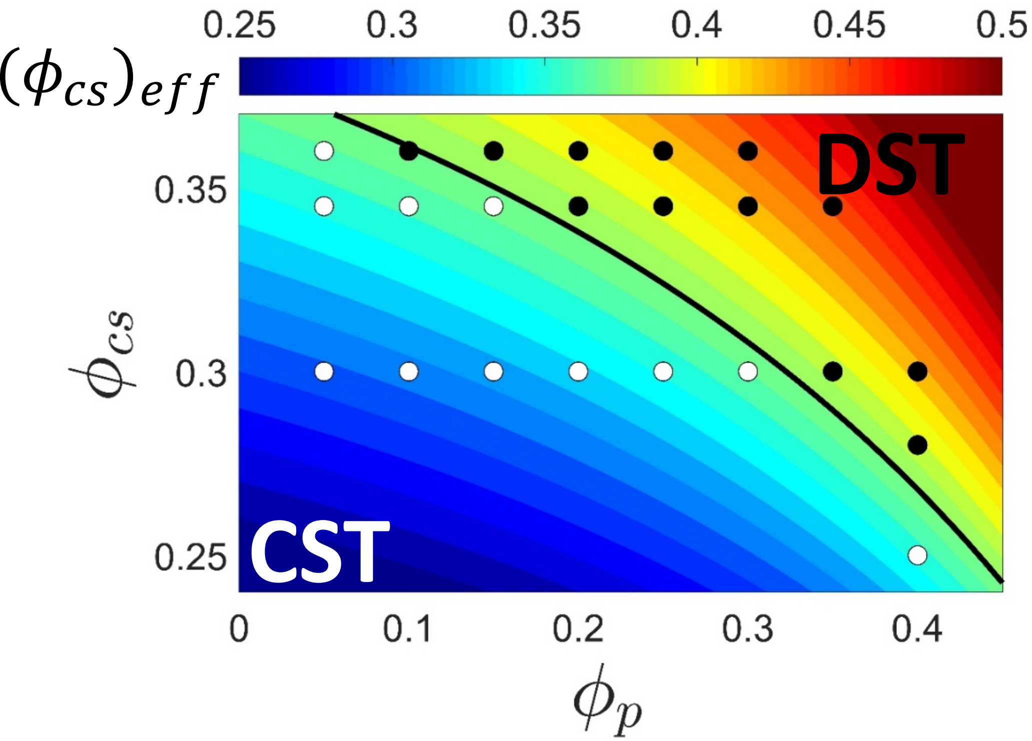

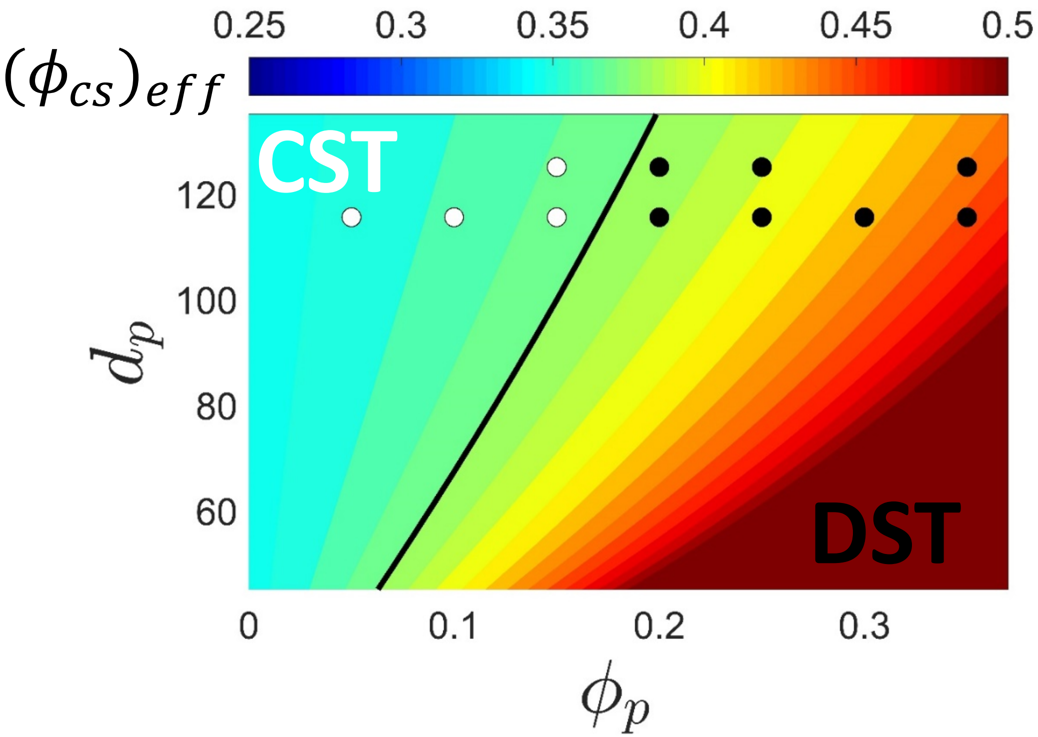

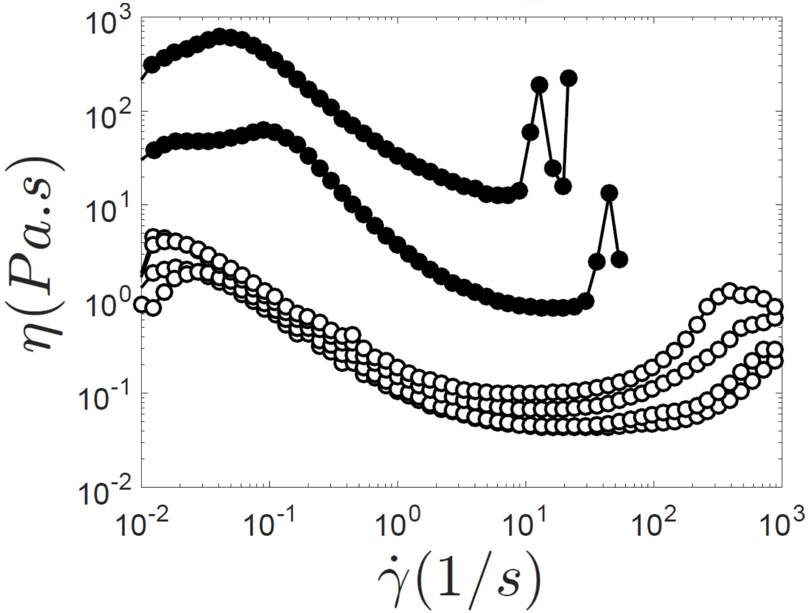

$\phi_{cs} = 34.5\%$
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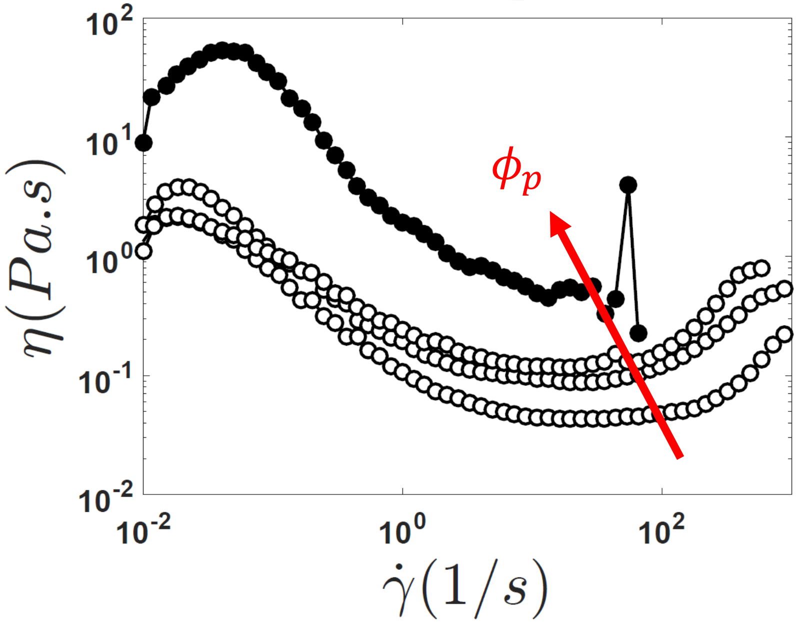
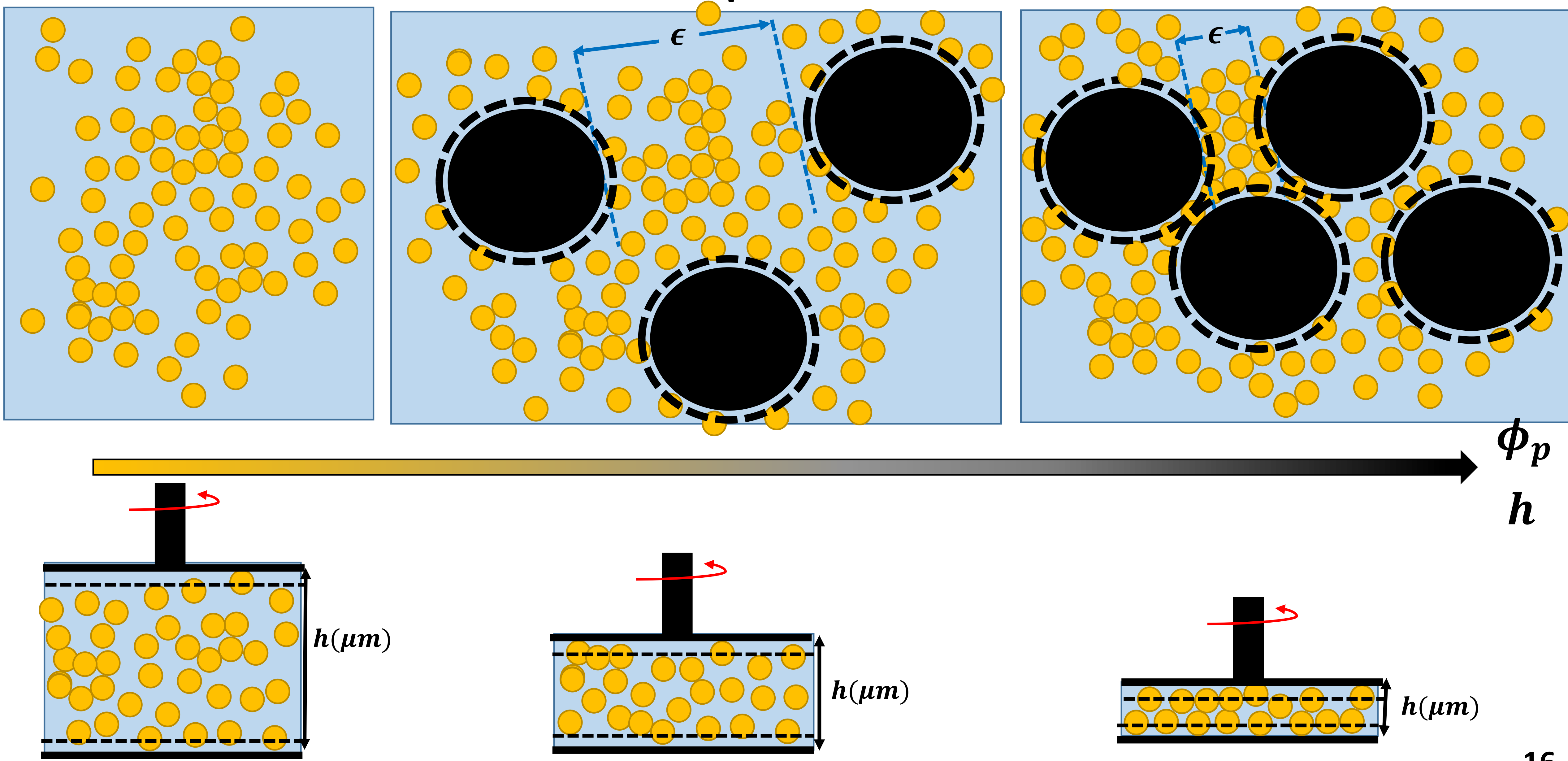


$\phi_{cs} = 34.5\%$
$d_p = 125 - 135\ \mu m$
Silver Coated PMMA

Simulating the excluded volume effect via rheometry of the pure cornstarch suspension



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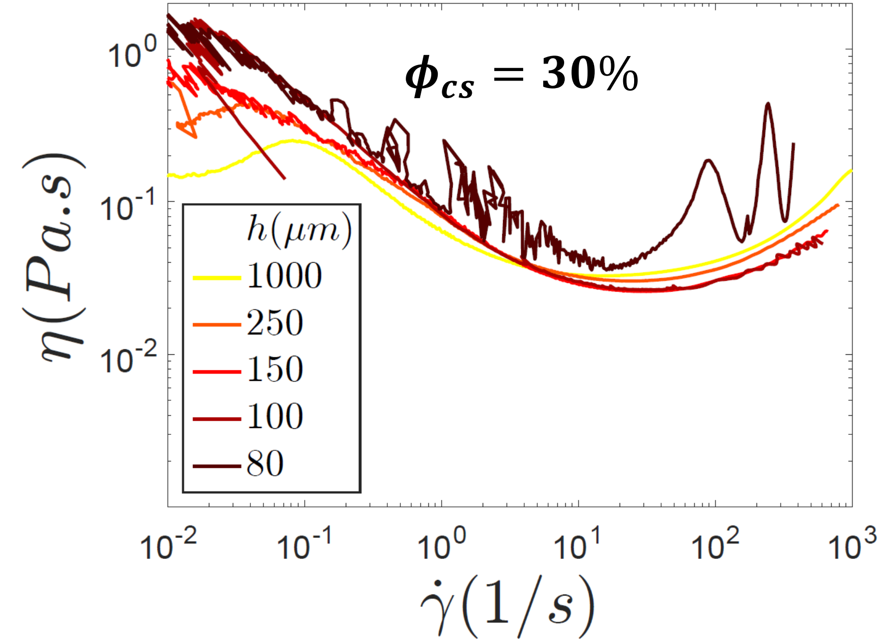
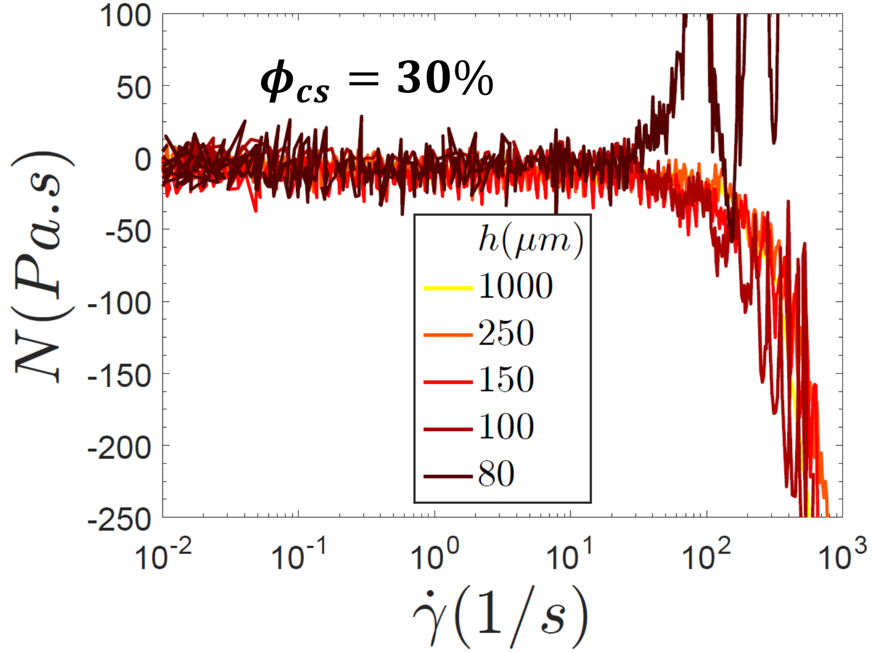
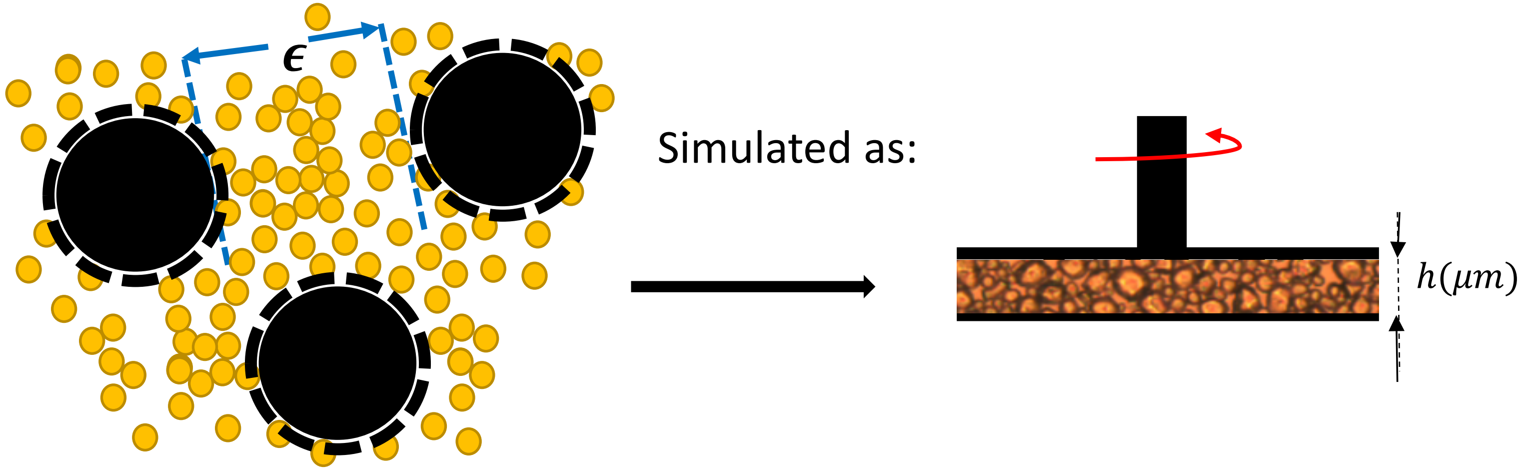
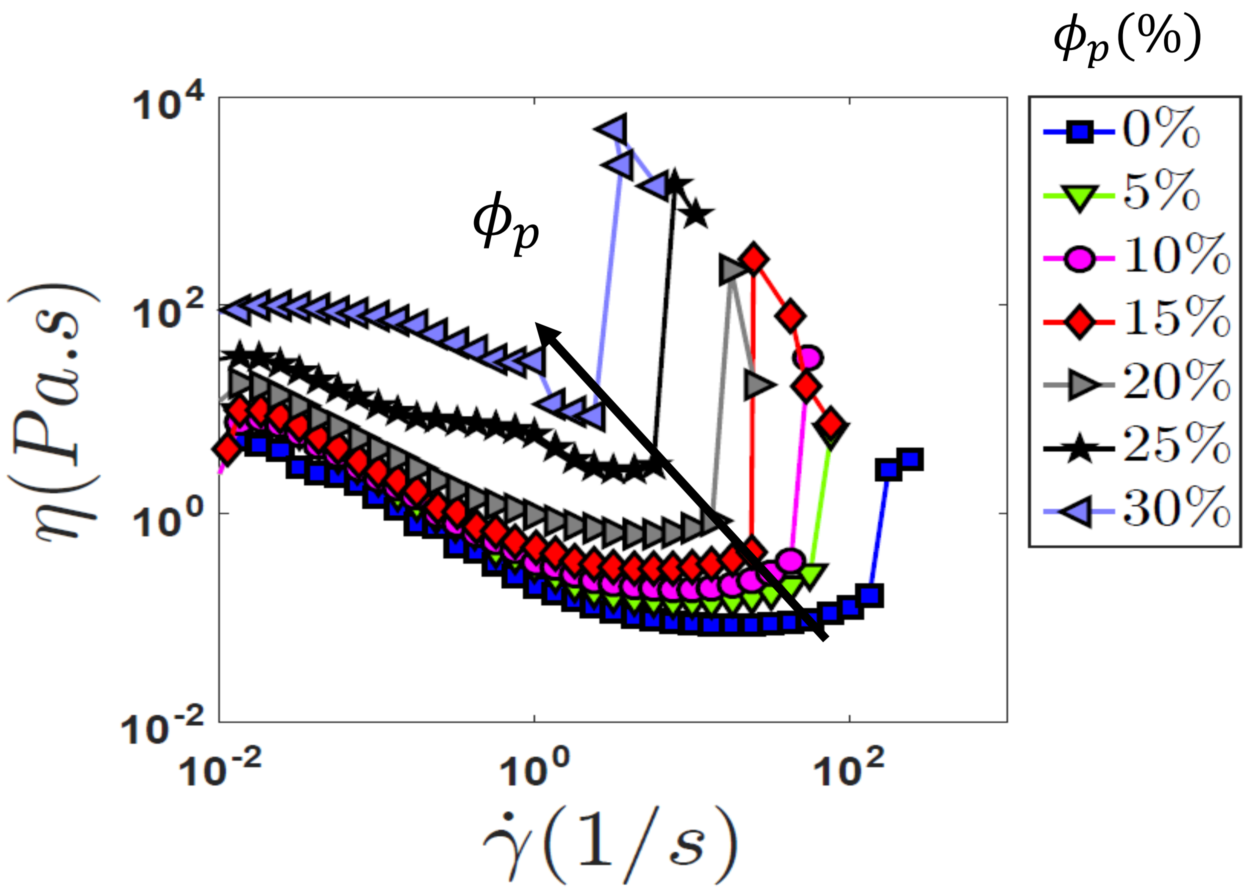


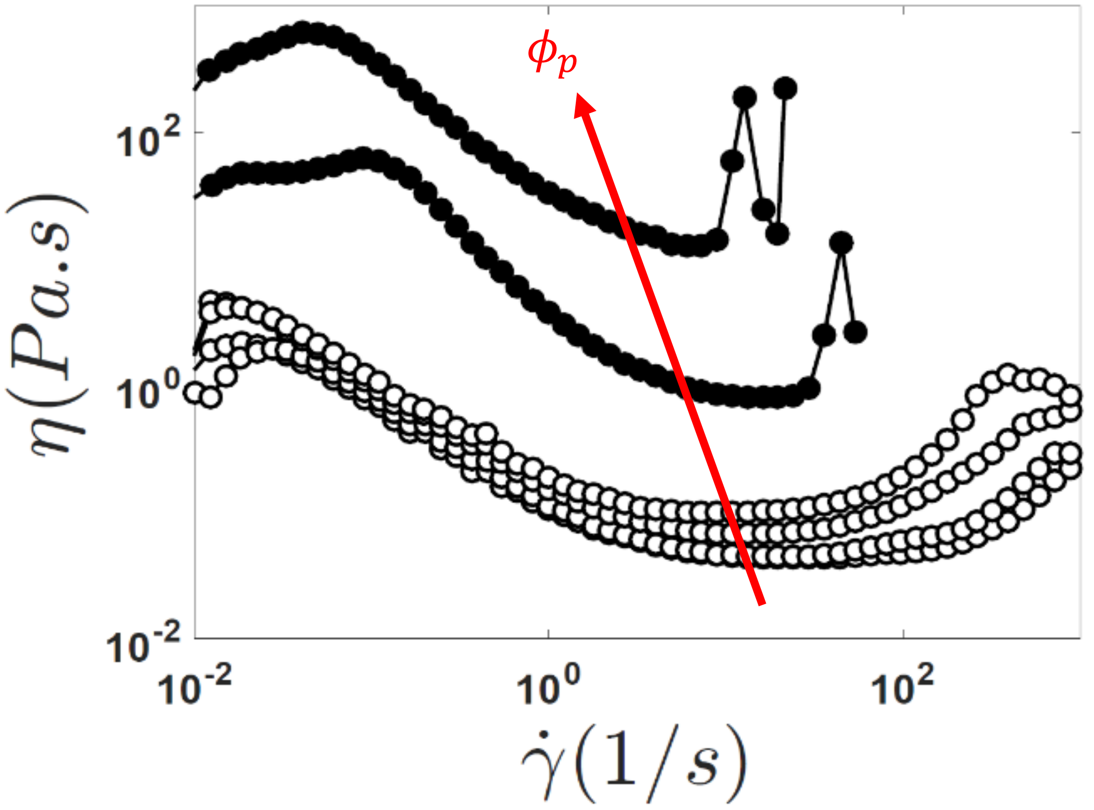




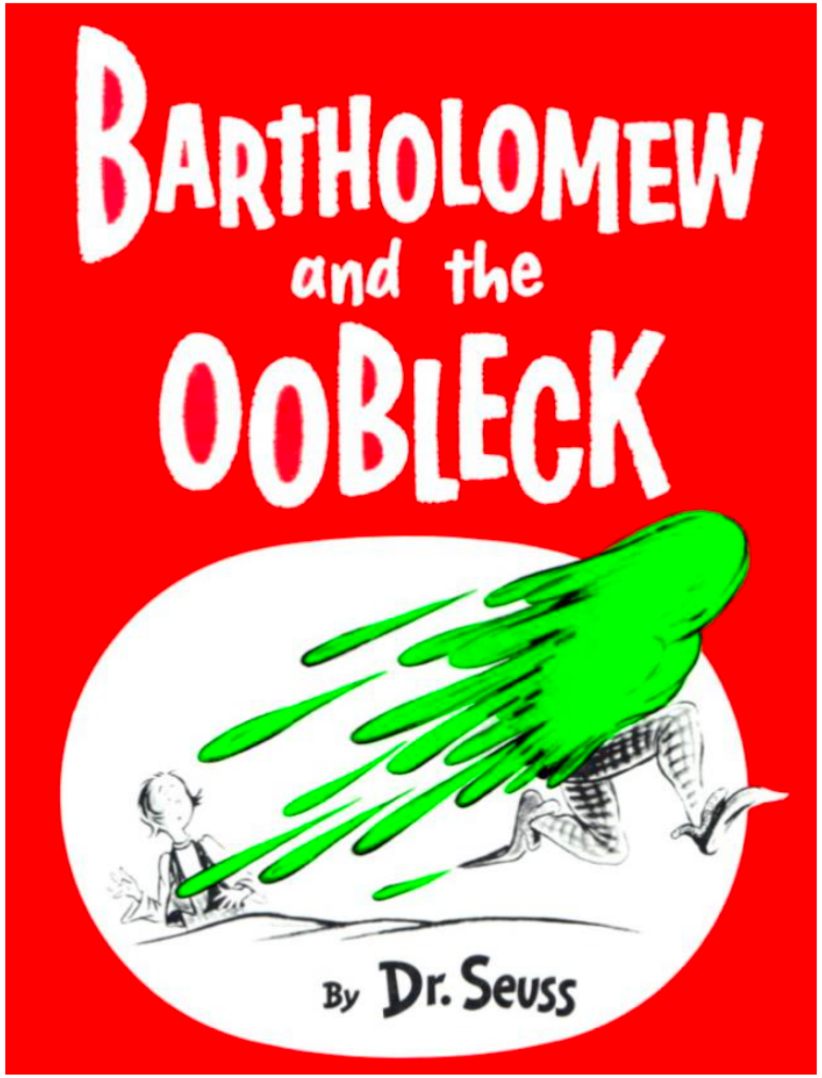
Summary

- By increasing large particles volume fraction in a DST suspension shear thickening enhances, specifically DST, can be explained via a scaling argument!
- By adding high volume fractions of large particles to a CST suspension, the suspension transitions from CST to DST, can be explained via an excluded volume effect.





Thank You!



Bartholomew and the Oobleck is a 1949 book by Dr. Seuss. It follows the adventures of a young boy named Bartholomew, who must rescue his kingdom from a sticky substance called Oobleck!