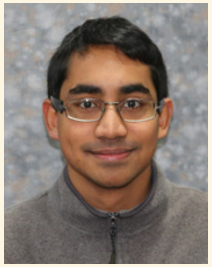
VORTICITY BANDING IN SHEAR THICKENING SUSPENSIONS

Romain Mari

Laboratoire Interdisciplinaire de Physique, CNRS-Université Grenoble-Alpes



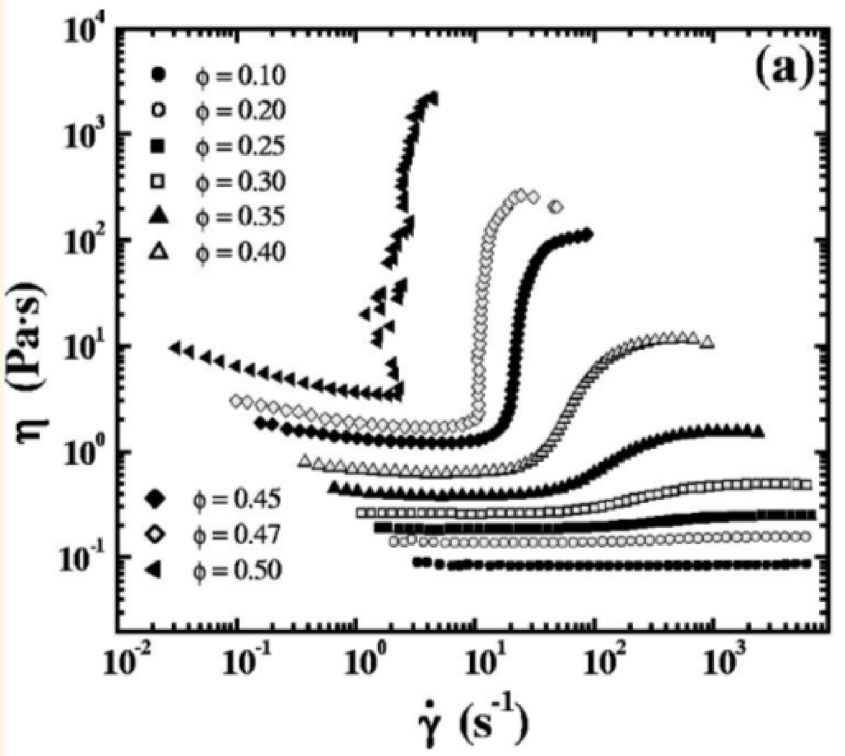
Rahul Chacko   Suzanne Fielding   Mike Cates

Dept. of Physics, Durham   DAMTP, Cambridge

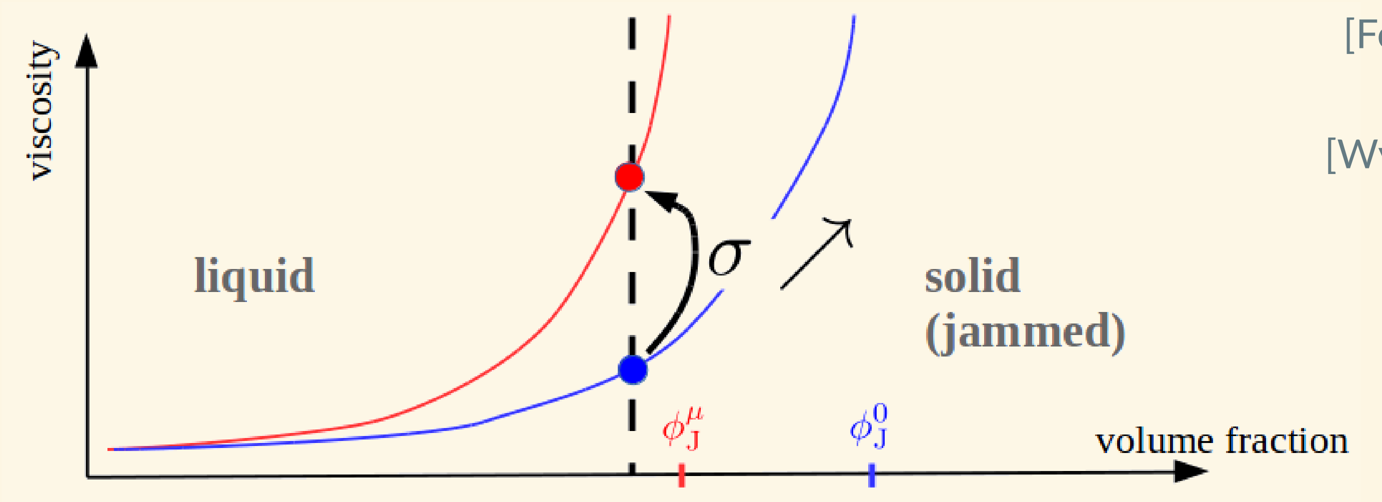
SHEAR THICKENING

[Egres & Wagner, JOR 2005]

~500nm calcium carbonate + polymer brush in PEG 200

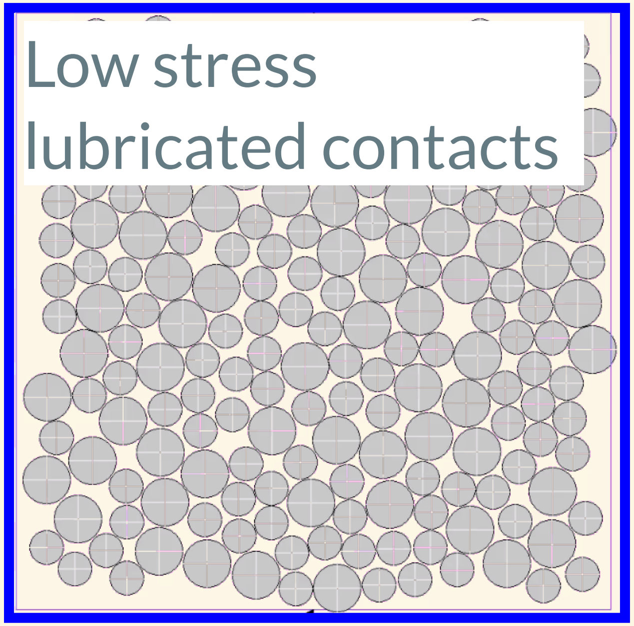


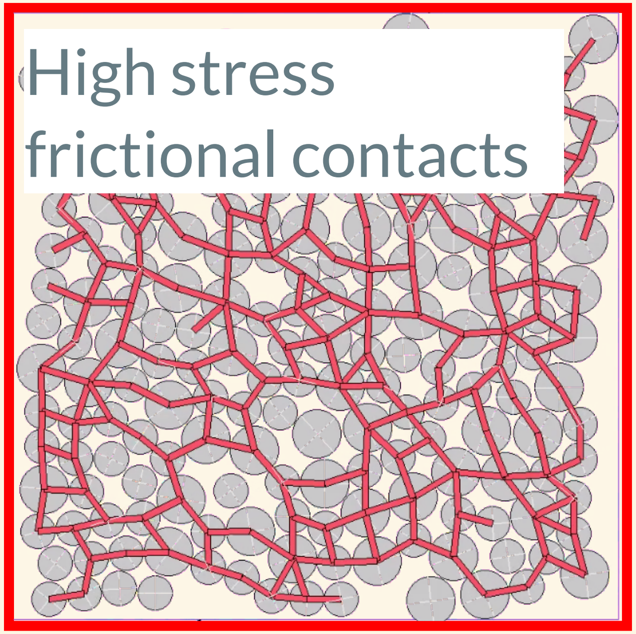
THICKENING SCENARIO



[Fernandez et al, PRL 2013]
[Seto et al, PRL 2013]
[Heussinger, PRE 2013]
[Wyart and Cates PRL 2013]
[Mari et al, JOR 2014]

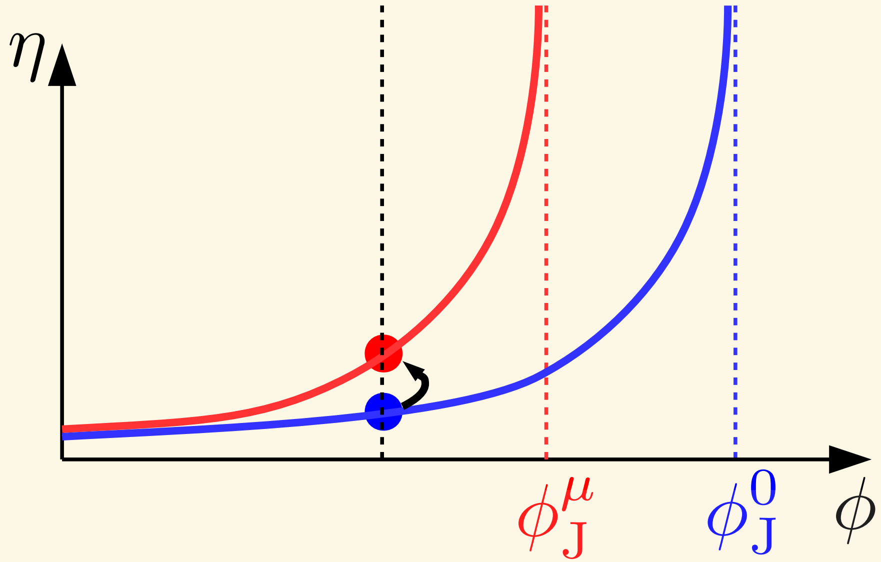
- Frictional contacts
- Repulsive force, stress scale $\sigma^* = \frac{\text{force}}{\text{radius}^2}$



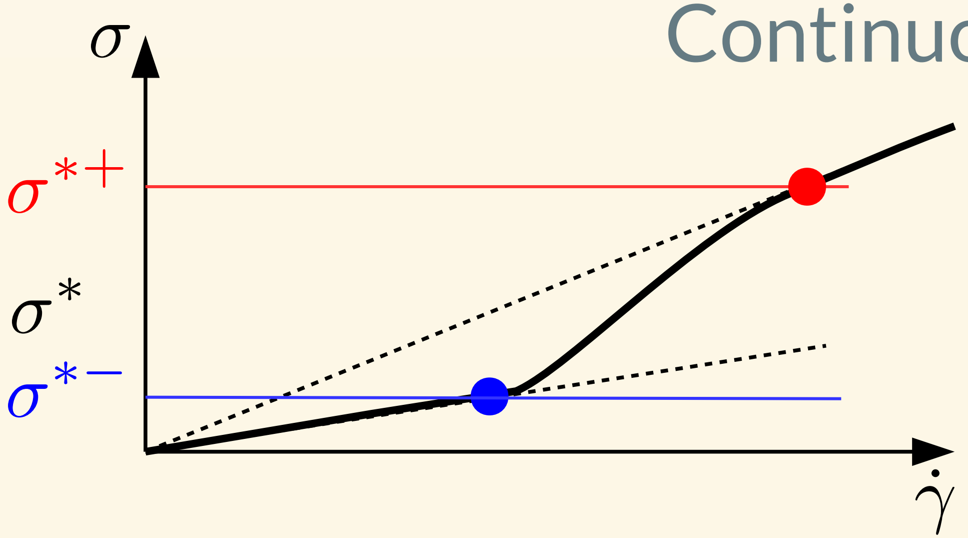


THICKENING SCENARIO

[Wyart & Cates, PRL 2014]

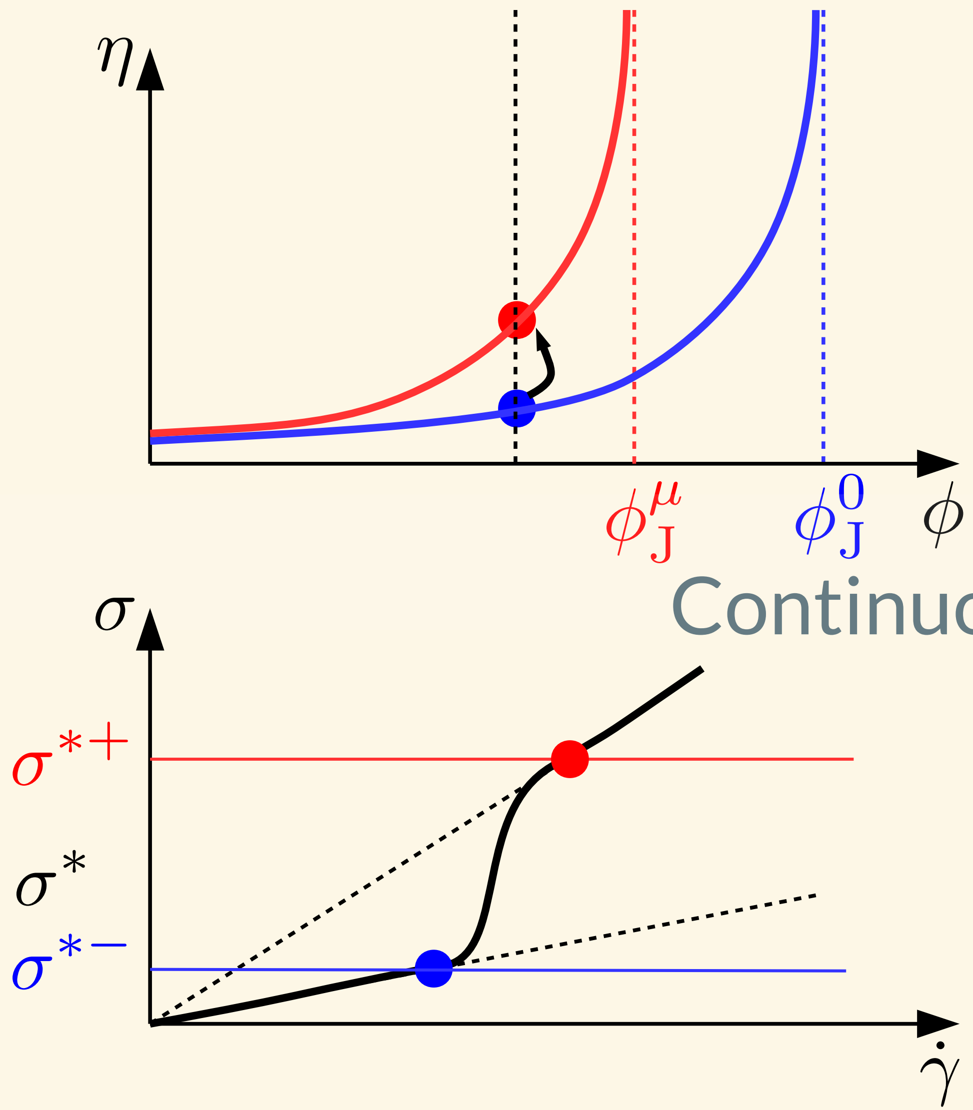


Continuous Shear Thickening



THICKENING SCENARIO

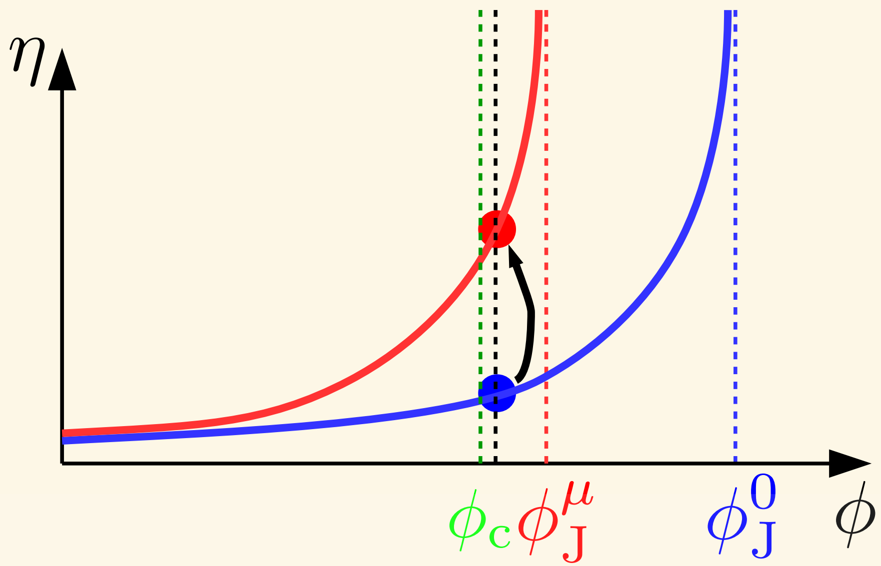
[Wyart & Cates, PRL 2014]



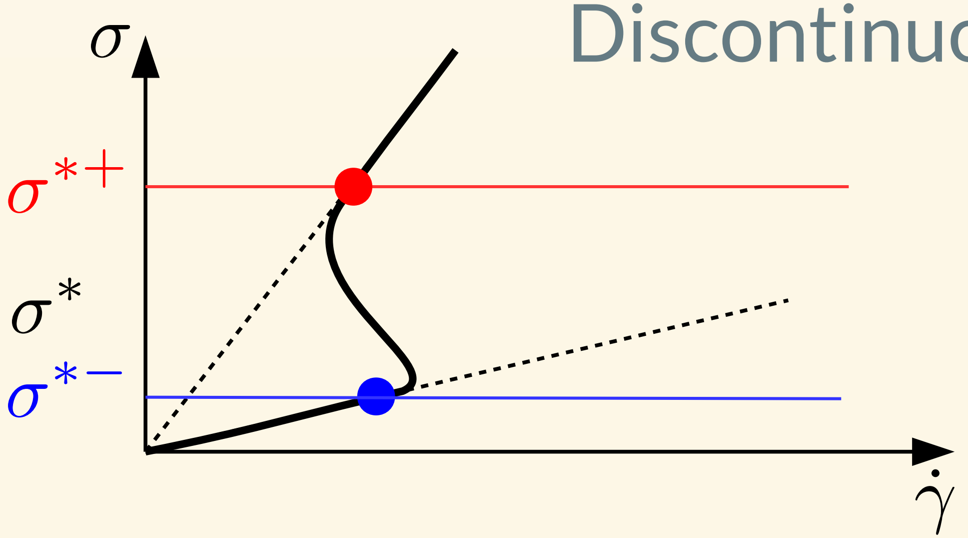
Continuous Shear Thickening

THICKENING SCENARIO

[Wyart & Cates, PRL 2014]

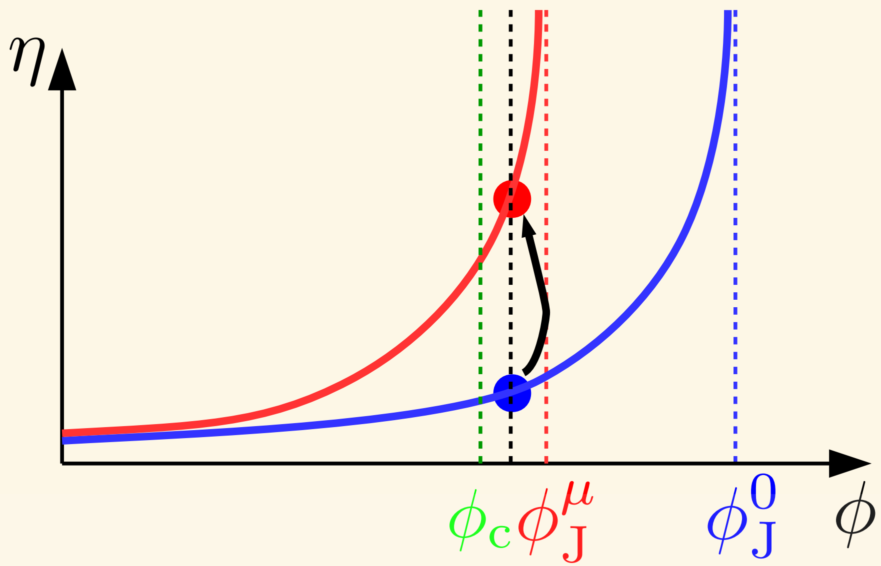


Discontinuous Shear Thickening

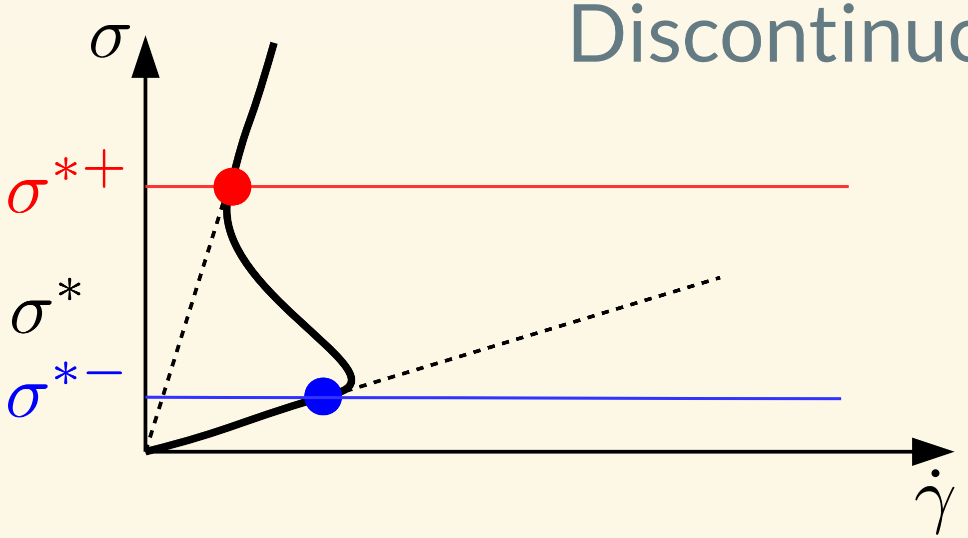


THICKENING SCENARIO

[Wyart & Cates, PRL 2014]

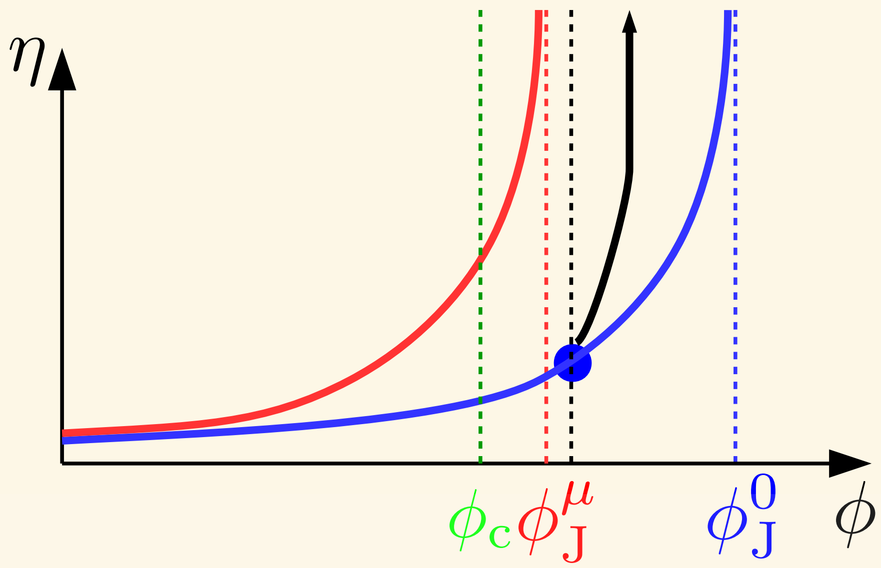


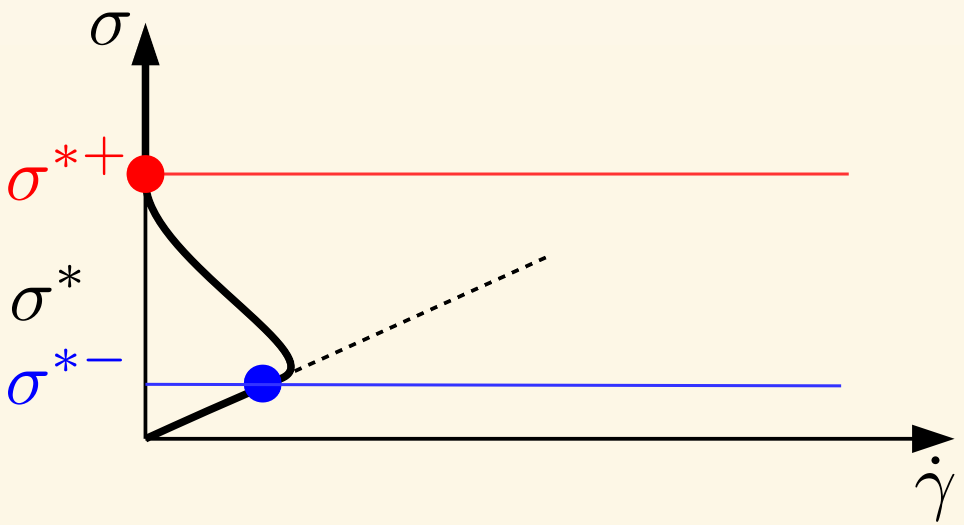
Discontinuous Shear Thickening



THICKENING SCENARIO

[Wyart & Cates, PRL 2014]

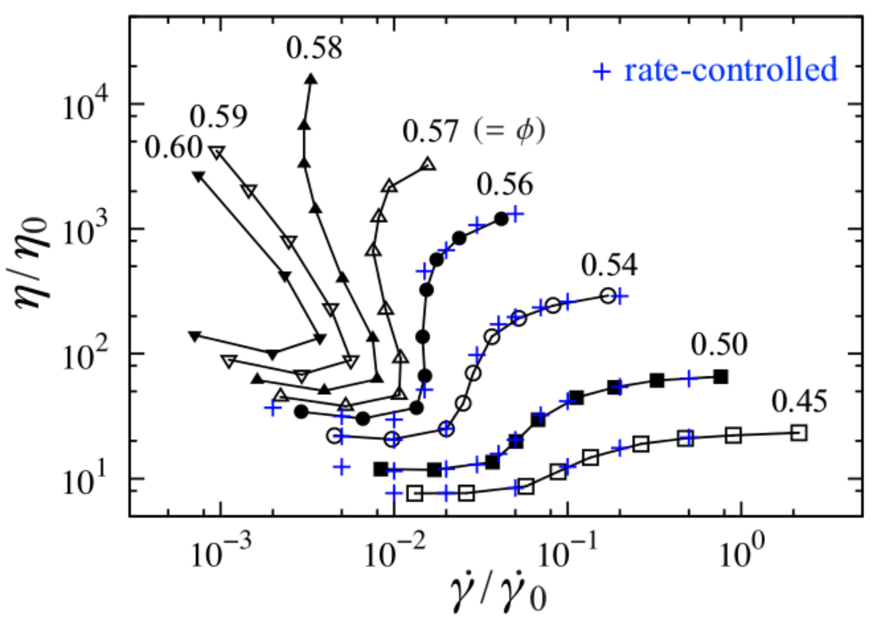




Shear Jamming

STRESS-CONTROLLED SIMULATIONS

[Mari, Seto, Morris & Denn, PRE 2015]



Non-monotonic flow curves:

- S-shaped (discontinuous thickening)
- Arches (shear jamming)

WYART-CATES MODEL

[Wyart & Cates, PRL 2014]

"Minimal constitutive model" with qualitative features of ST:

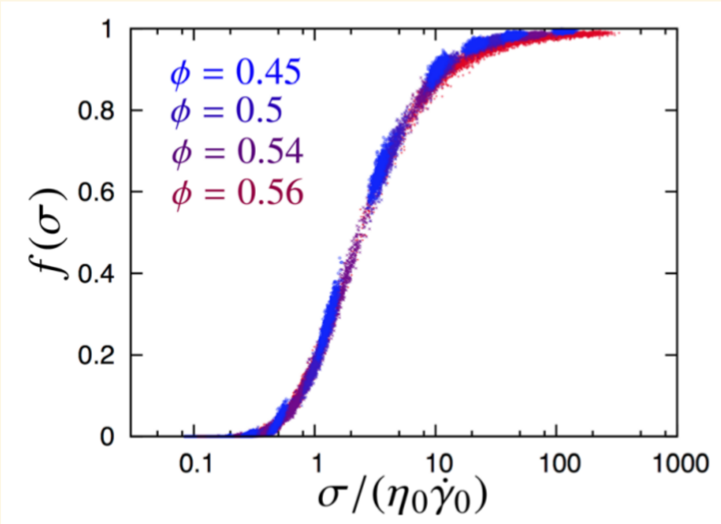
$$\sigma = \eta(\phi, f)\dot{\gamma}$$

$$\eta(\phi, f) = \eta_0(\phi_{\mathrm{J}}(f) - \phi)^{-2}$$
$$\phi_J(f) = f\phi_{\mathrm{J}}^{\mu} + (1-f)\phi_{\mathrm{J}}^{0}$$

$$f = f(\sigma)$$

In practice,
$f(\sigma) \approx \exp(-C\sigma^*/\sigma)$

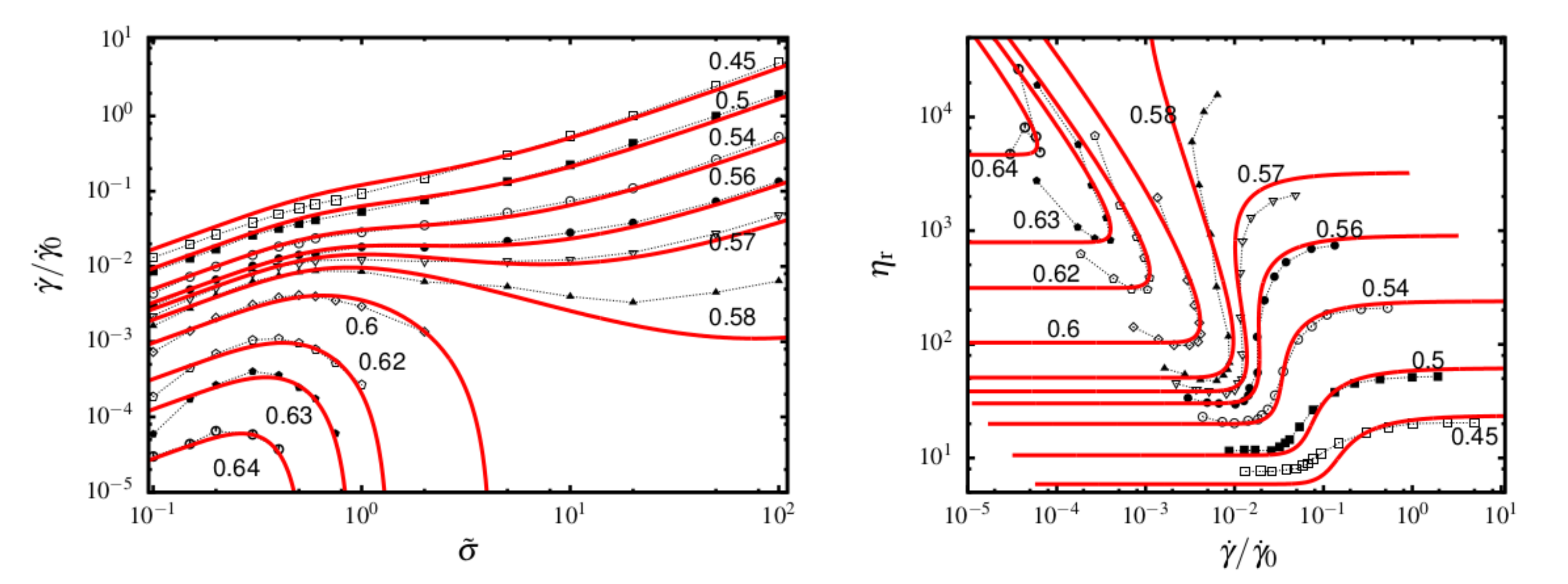
f: "fraction of frictional contacts"
$f = 0$: only lubricated contacts
$f = 1$: only frictional contacts



WYART-CATES MODEL

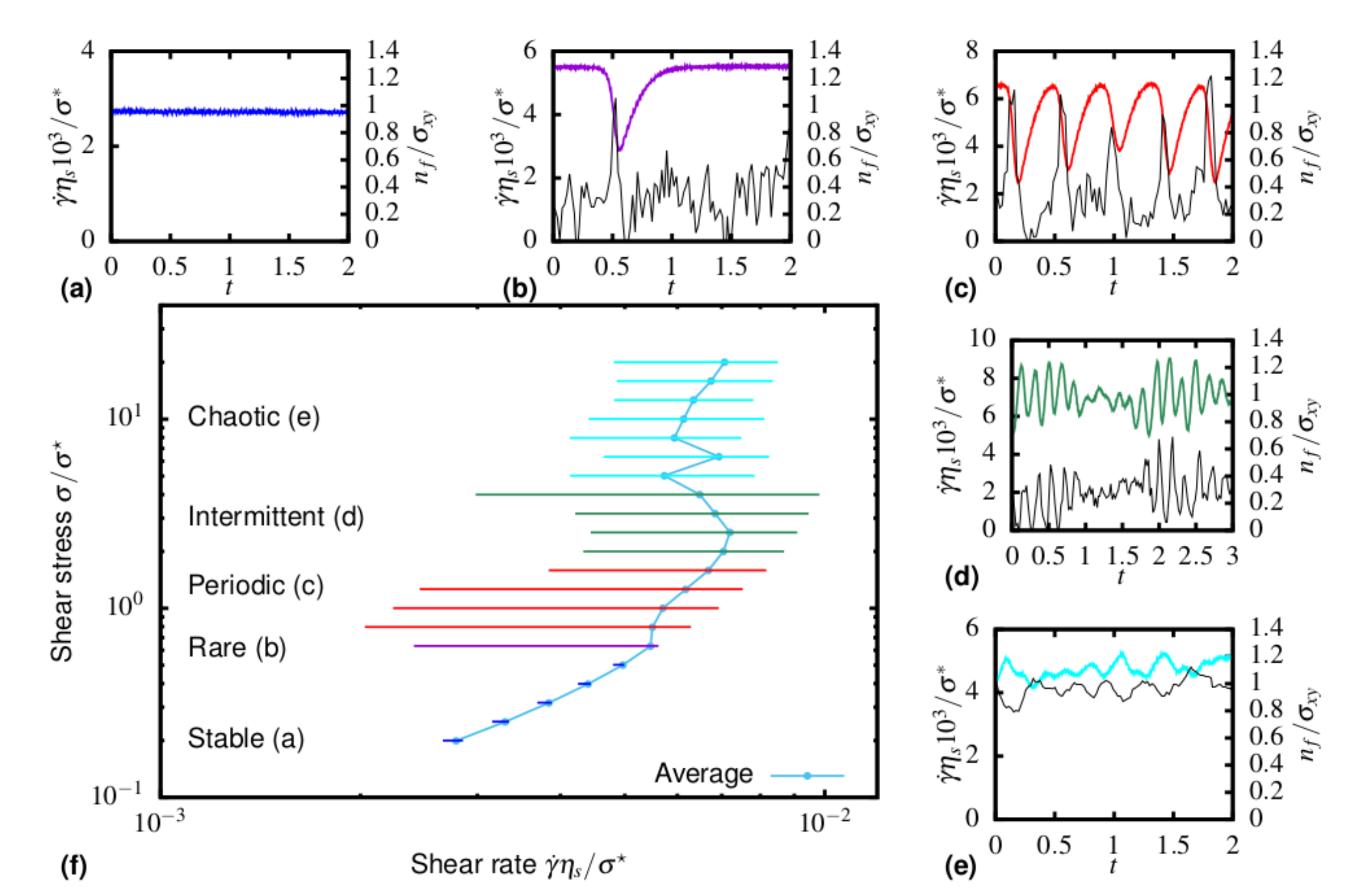
[Singh, Mari, Morri & Denn, JoR 2018]

Comparison with simulations

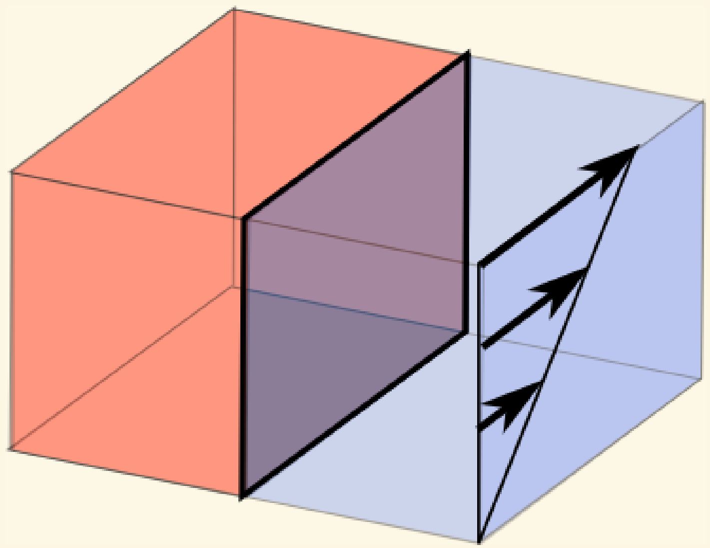


FLOW INSTABILITIES

[Hermes et al, 2015]



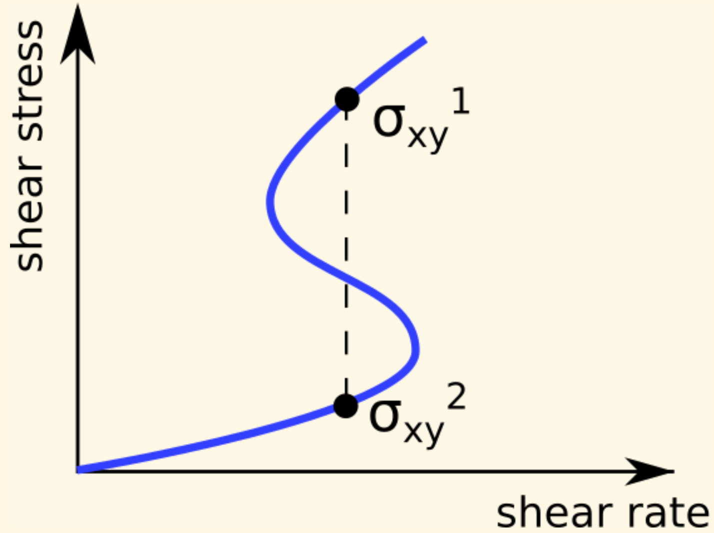
STEADY VORTICITY BANDING



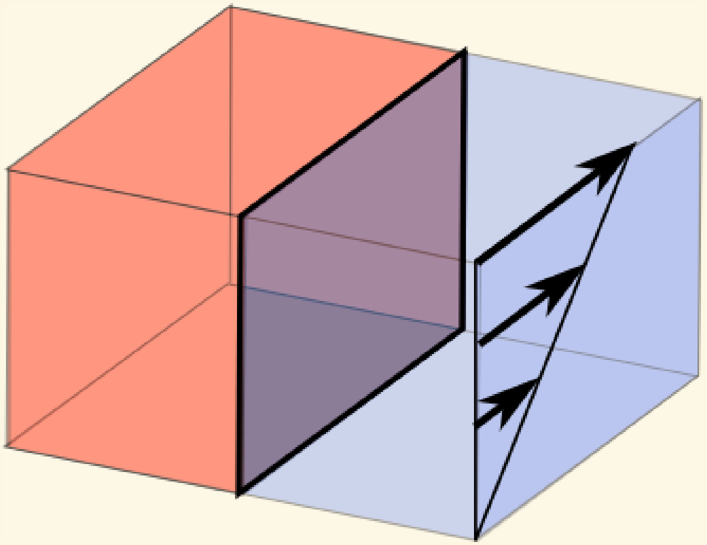
At the interface:

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}$$

$$\dot{\gamma}^{(1)} = \dot{\gamma}^{(2)}$$



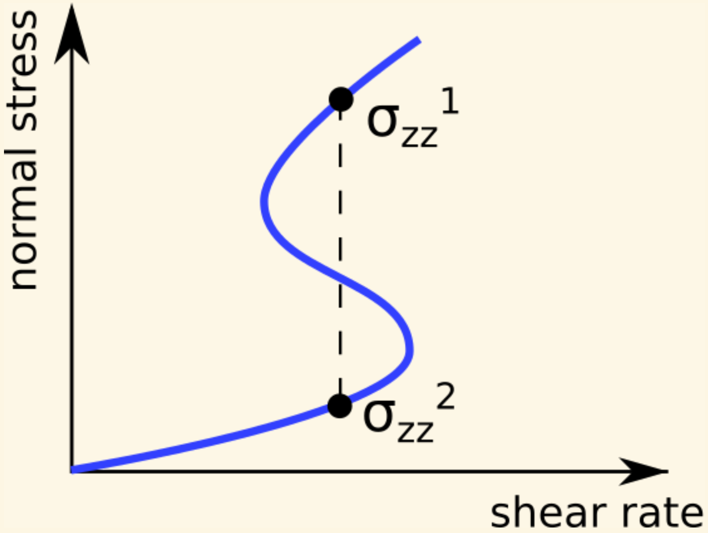
STEADY VORTICITY BANDING



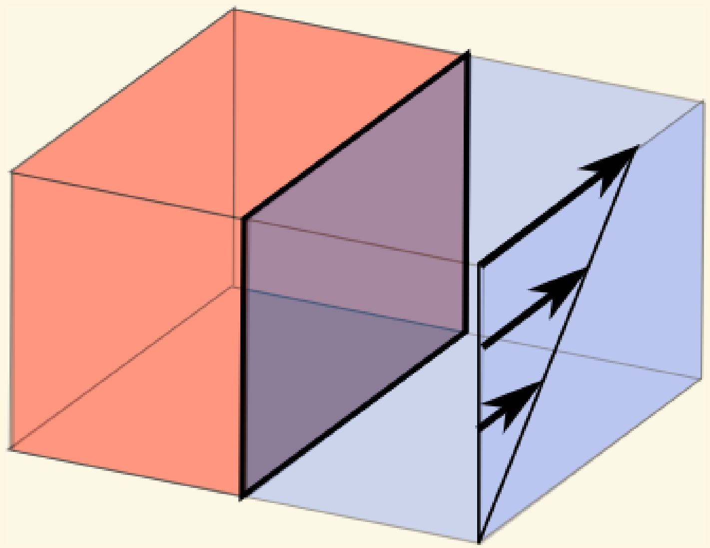
At the interface:

$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}$$

$$\dot{\gamma}^{(1)} = \dot{\gamma}^{(2)}$$



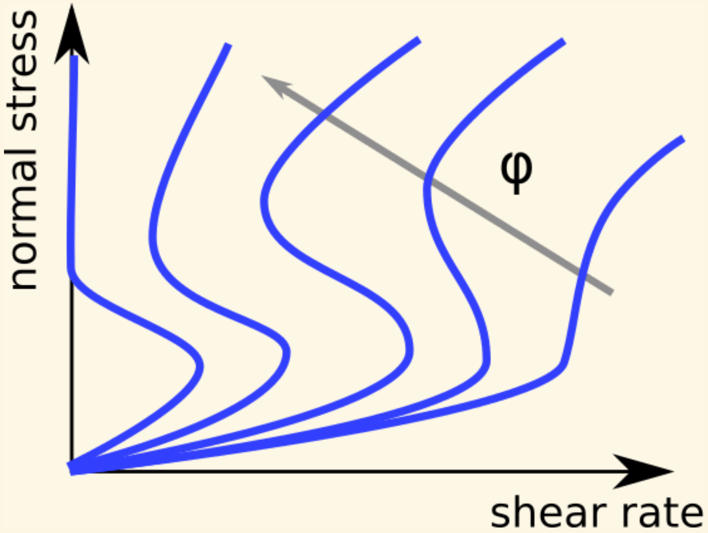
STEADY VORTICITY BANDING



At the interface:

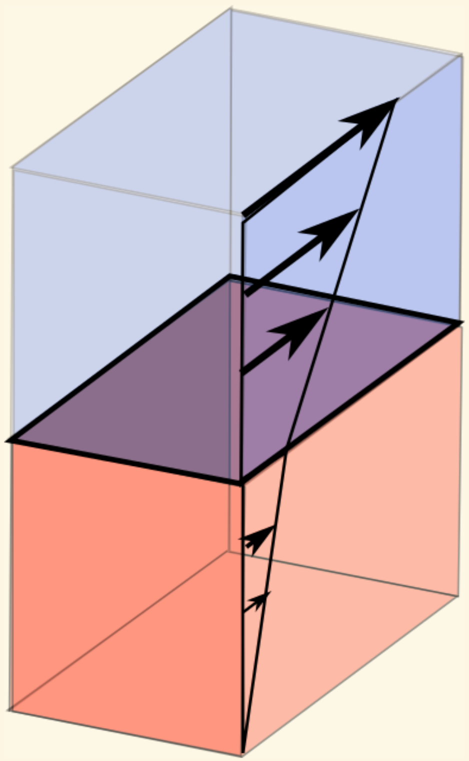
$$\sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}$$

$$\dot{\gamma}^{(1)} = \dot{\gamma}^{(2)}$$



Impossible!

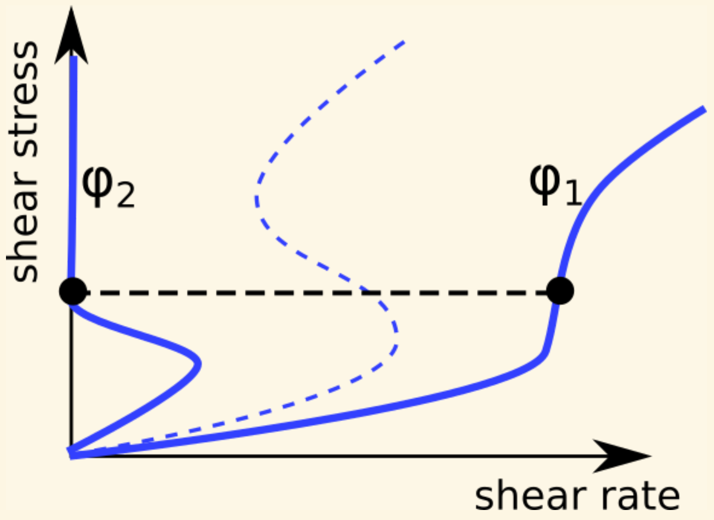
STEADY GRADIENT BANDING



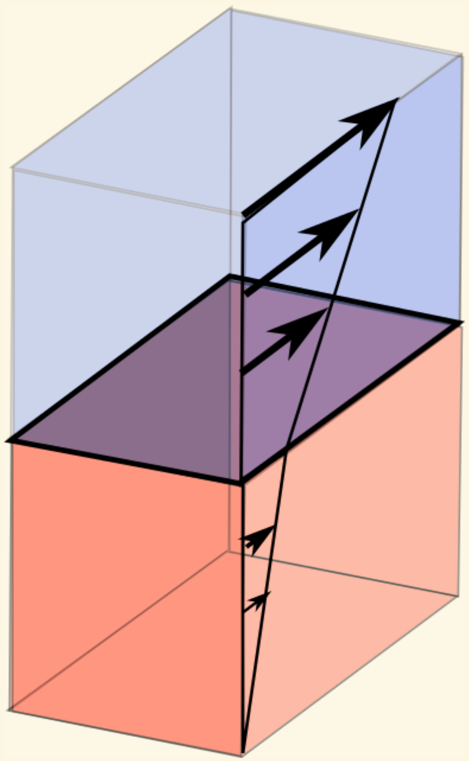
At the interface:

$$\sigma_{xy}^{(1)} = \sigma_{xy}^{(2)}$$

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}$$



STEADY GRADIENT BANDING

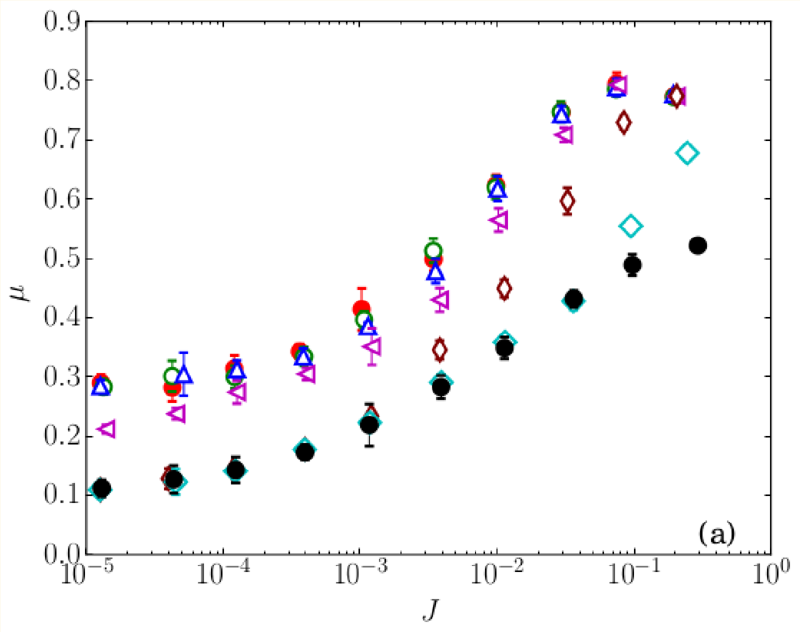


Impossible!

At the interface:

$$\sigma_{xy}^{(1)} = \sigma_{xy}^{(2)}$$
$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}$$

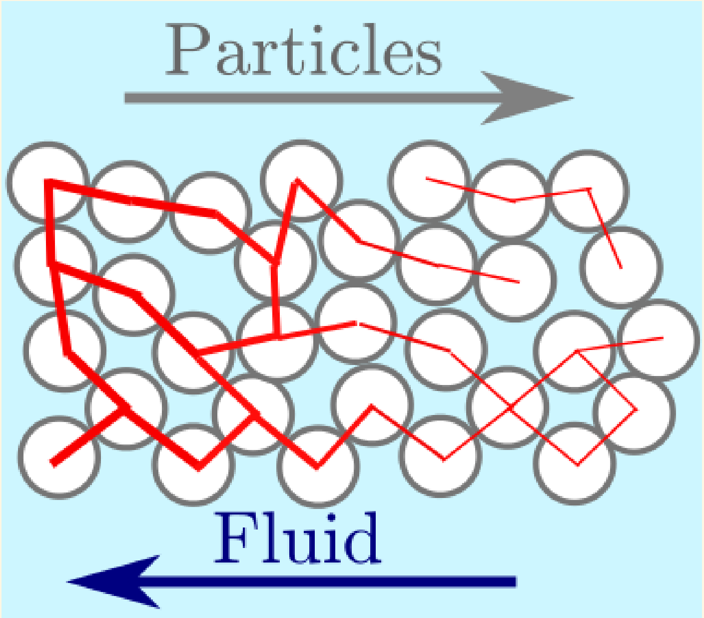
[Dong & Trulsson, Phys. Rev. Fluids 2017]



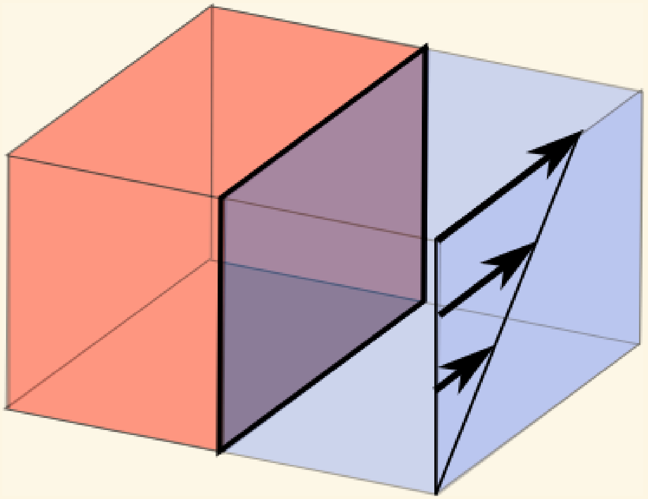
BANDING AND PARTICLE MIGRATION

Suspension balance model [Nott & Brady, JFM 1994]:

$$\nabla \cdot \Sigma^{\mathrm{p}} = \phi R(\phi)(\mathbf{v}^{\mathrm{p}} - \mathbf{v}^{\mathrm{p+f}})$$



VORTICITY INSTABILITY MODEL



Reducing the problem to 1d

$\Sigma \to \sigma_{zz} \equiv \sigma$

$\mathbf{v} \to v_z \equiv v$

Conservation relations

Mass conservation:

$\partial_t \phi + \partial_z(\phi v) = 0$

Momentum conservation:

$\partial_z \sigma = -R\phi v$

Stress control:

$L_z^{-1} \int dz\, \sigma = \bar{\sigma}$

Constitutive model:

Wyart-Cates + linear response:

$\sigma = \eta(\phi, f)\dot{\gamma}$

$\eta(\phi, f) = \eta_0(\phi_J(f) - \phi)^{-2}$

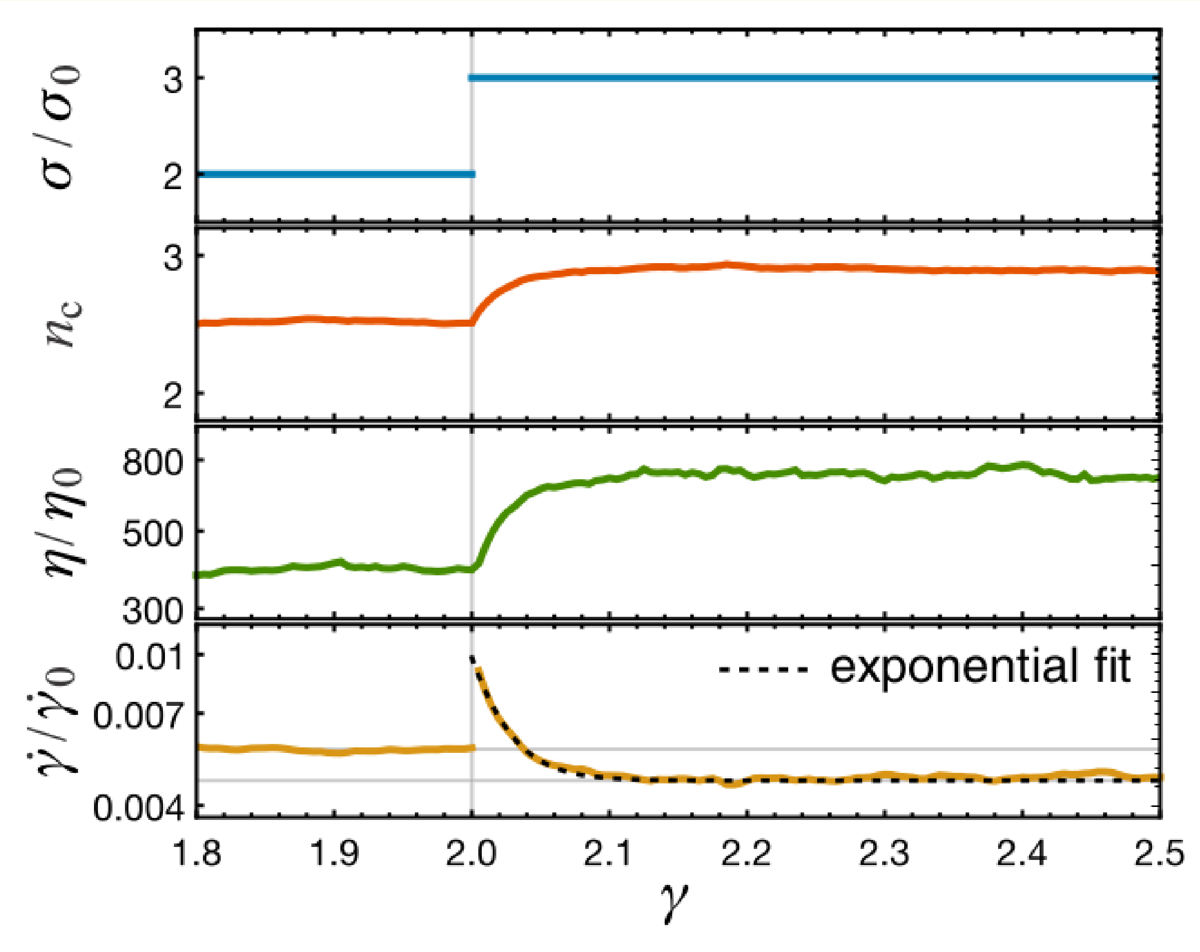
$\phi_J(f) = f\phi_J^{\mu} + (1-f)\phi_J^0$

$\partial_t f = -\dot{\gamma}\gamma_0^{-1}\left[f - f^*(\sigma)\right]$

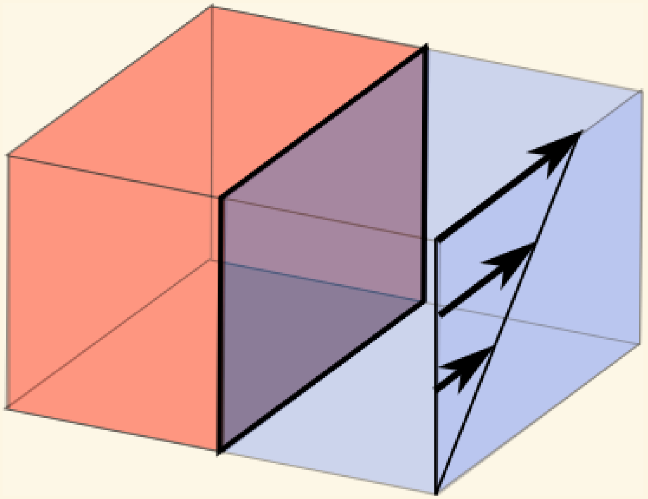
$f^*(\sigma) = \exp(-\sigma^*/\sigma)$

LINEAR RESPONSE OF THE MICROSTRUCTURE

[Mari, Seto, Morris & Denn, PRE 2015]



VORTICITY INSTABILITY MODEL



Reducing the problem to 1d

$$\Sigma \to \sigma_{zz} \equiv \sigma$$

$$\mathbf{v} \to v_z \equiv v$$

Conservation relations

Mass conservation:

$$\partial_t \phi + \partial_z(\phi v) = 0$$

Momentum conservation:

$$\partial_z \sigma = -R\phi v$$

Stress control:

$$L_z^{-1} \int dz\, \sigma = \bar{\sigma}$$

Constitutive model:

Wyart-Cates + linear response:

$$\sigma = \eta(\phi, f)\dot{\gamma}$$

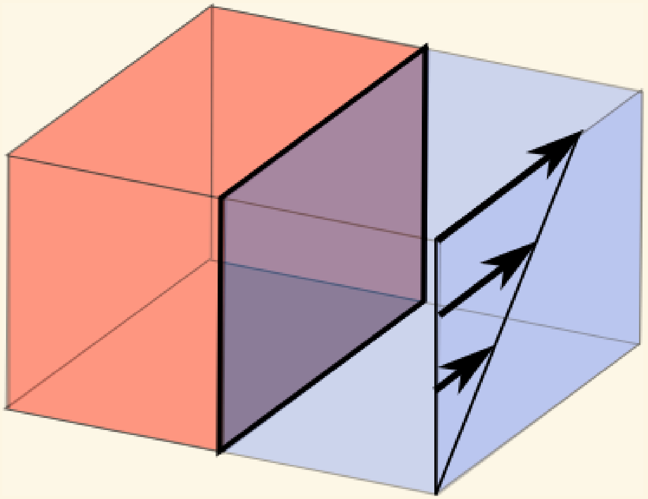
$$\eta(\phi, f) = \eta_0 (\phi_\mathrm{J}(f) - \phi)^{-2}$$

$$\phi_J(f) = f\phi_\mathrm{J}^{\mu} + (1-f)\phi_\mathrm{J}^{0}$$

$$\partial_t f = -\dot{\gamma}\gamma_0^{-1}\left[f - f^*(\sigma)\right]$$

$$f^*(\sigma) = \exp(-\sigma^*/\sigma)$$

VORTICITY INSTABILITY MODEL



Reducing the problem to 1d

$$\Sigma \to \sigma_{zz} \equiv \sigma$$
$$\mathbf{v} \to v_z \equiv v$$

Coupled dynamical equations

$$\partial_t \phi = R^{-1} \partial_z^2 \sigma$$
$$\partial_t f = -\dot{\gamma}\gamma_0^{-1} \left[f - f^*(\sigma) \right]$$

Coupling through:

$$\sigma = \eta(\phi, f)\dot{\gamma}$$
$$\eta(\phi, f) = \eta_0 (\phi_\mathrm{J}(f) - \phi)^{-2}$$
$$\phi_J(f) = f\phi_\mathrm{J}^\mu + (1-f)\phi_\mathrm{J}^0$$
$$f^*(\sigma) = \exp(-\sigma^*/\sigma)$$
$$L_z^{-1} \int dz\, \sigma = \bar{\sigma}$$

VORTICITY INSTABILITY MODEL

Non-dimensionalize with units:

- Time η_0/σ^*
- Length L_z
- Stress σ^*

Change of variables:

- $t \to s = \gamma_0^{-1} t$
- $R \to \alpha = \gamma_0^{-1} R$

Coupled dynamical equations

$$\partial_s \phi = \alpha^{-1} \partial_z^2 \sigma$$
$$\partial_s f = -\dot{\gamma}\left[f - f^*(\sigma)\right]$$

Coupling through:

$$\sigma = \eta(\phi, f)\dot{\gamma}$$
$$\eta(\phi, f) = (\phi_{\mathrm{J}}(f) - \phi)^{-2}$$
$$\phi_J(f) = f\phi_{\mathrm{J}}^{\mu} + (1 - f)\phi_{\mathrm{J}}^{0}$$
$$f^*(\sigma) = \exp(-1/\sigma)$$
$$\int dz\, \sigma = \bar{\sigma}$$

3 parameters: $\bar{\phi}, \bar{\sigma}, \alpha = L_z^2 R/(\gamma_0 \eta_0) \gg 1$

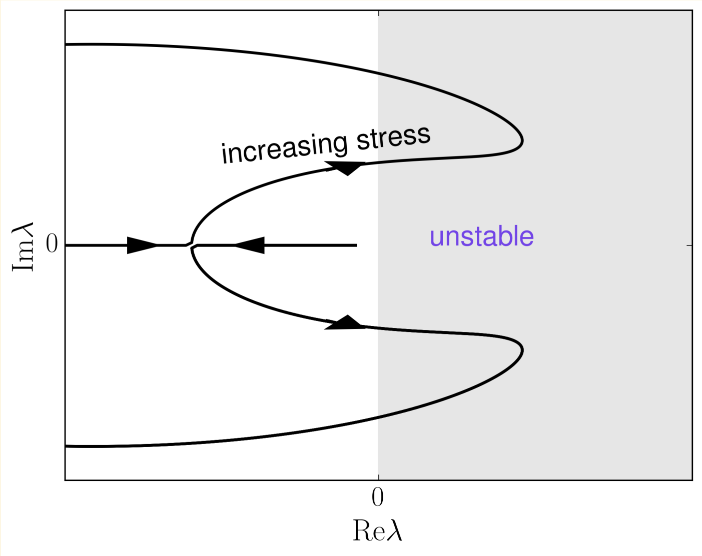
VORTICITY INSTABILITY MODEL

Linear stability analysis:
$$X = X_0 + \delta X e^{ikz+\lambda t}$$

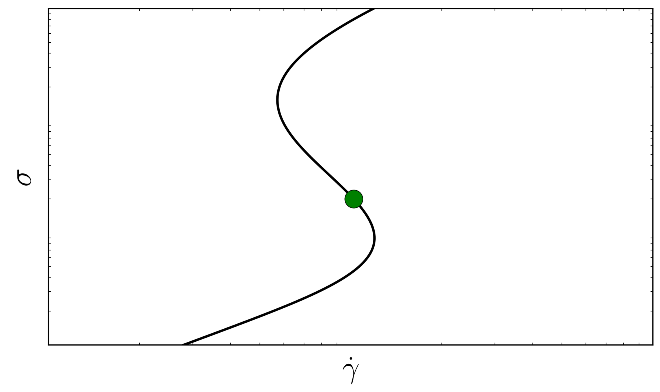
Unstable when $\eta \partial_\sigma \dot{\gamma} < -\frac{k^2}{\phi \alpha} \partial_\phi \eta$

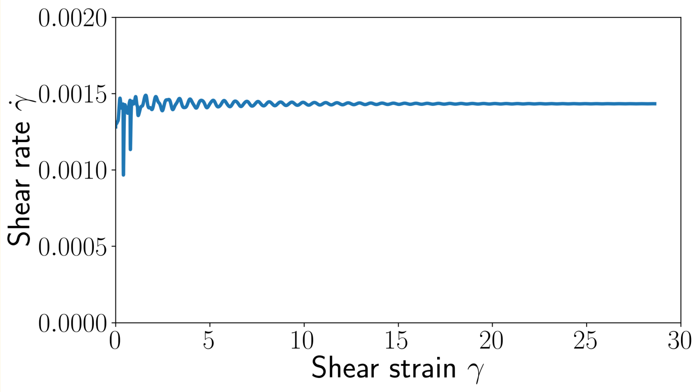
Hopf bifurcation, $\mathrm{Re}\lambda > 0$ and $\mathrm{Im}\lambda \neq 0$

Instability towards *traveling bands*

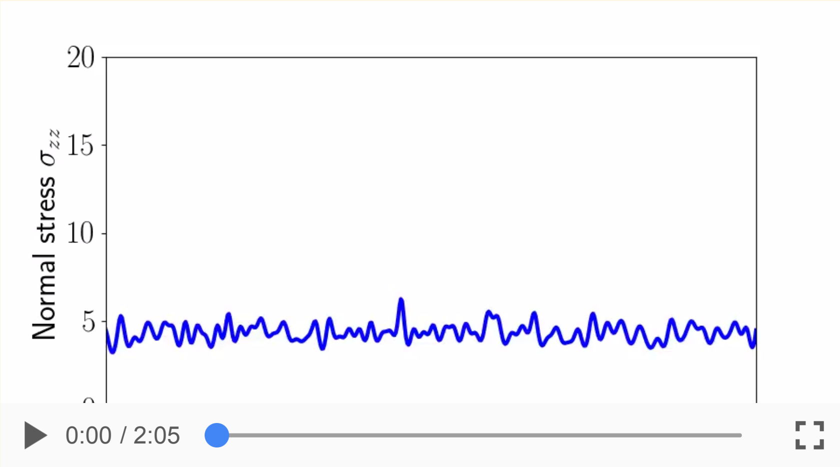


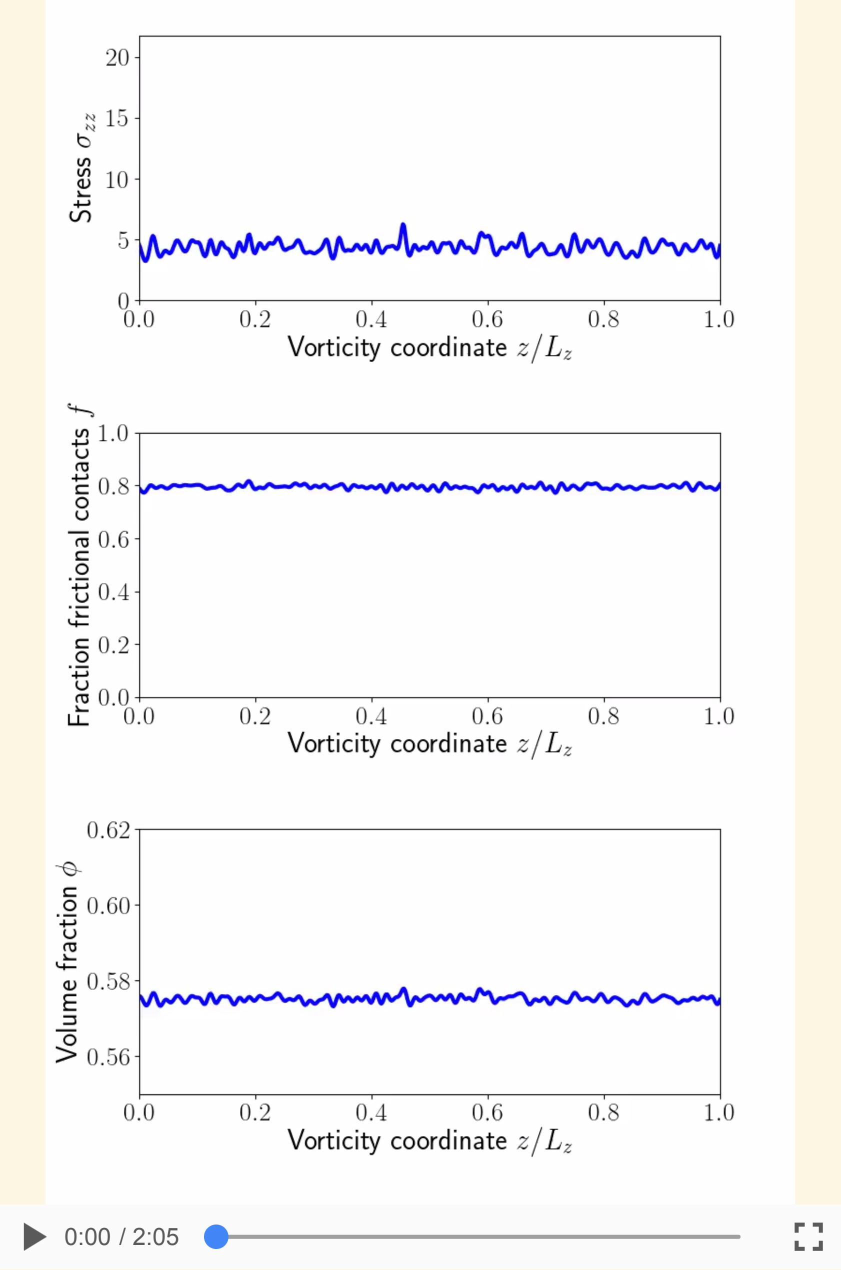
TRAVELING BANDS





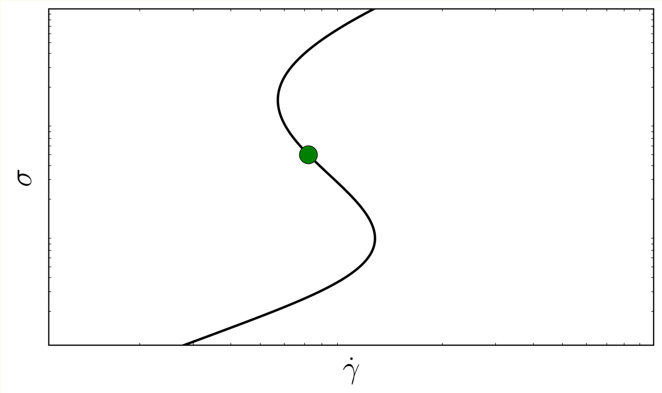
Stress field along vorticity

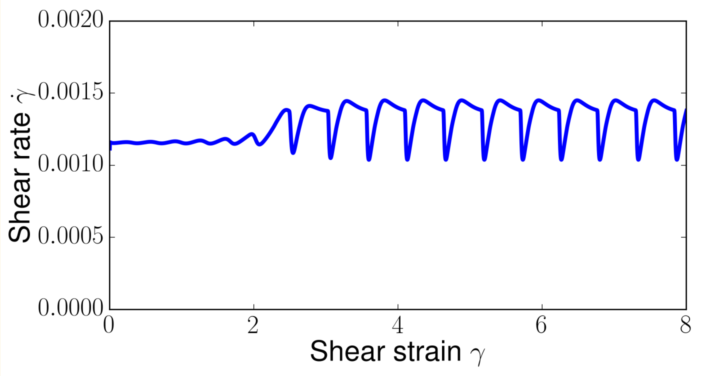




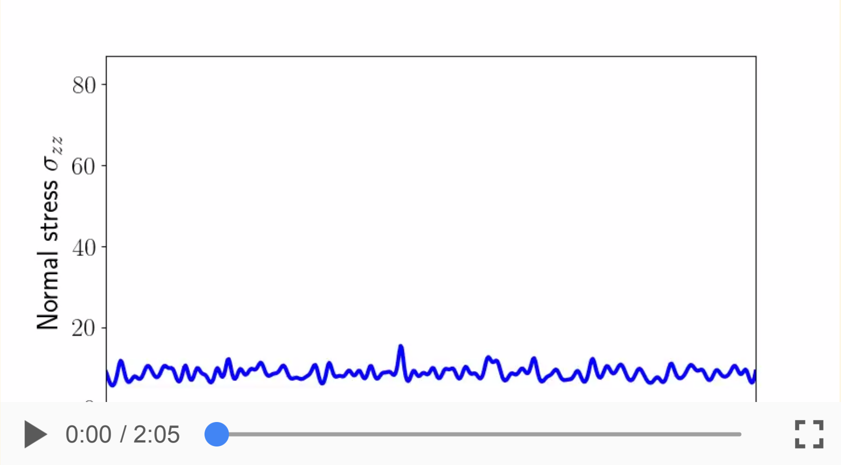
Stress σ_{zz}
Fraction frictional contacts f
Volume fraction ϕ
Vorticity coordinate z/L_z
Vorticity coordinate z/L_z
Vorticity coordinate z/L_z
0:00 / 2:05

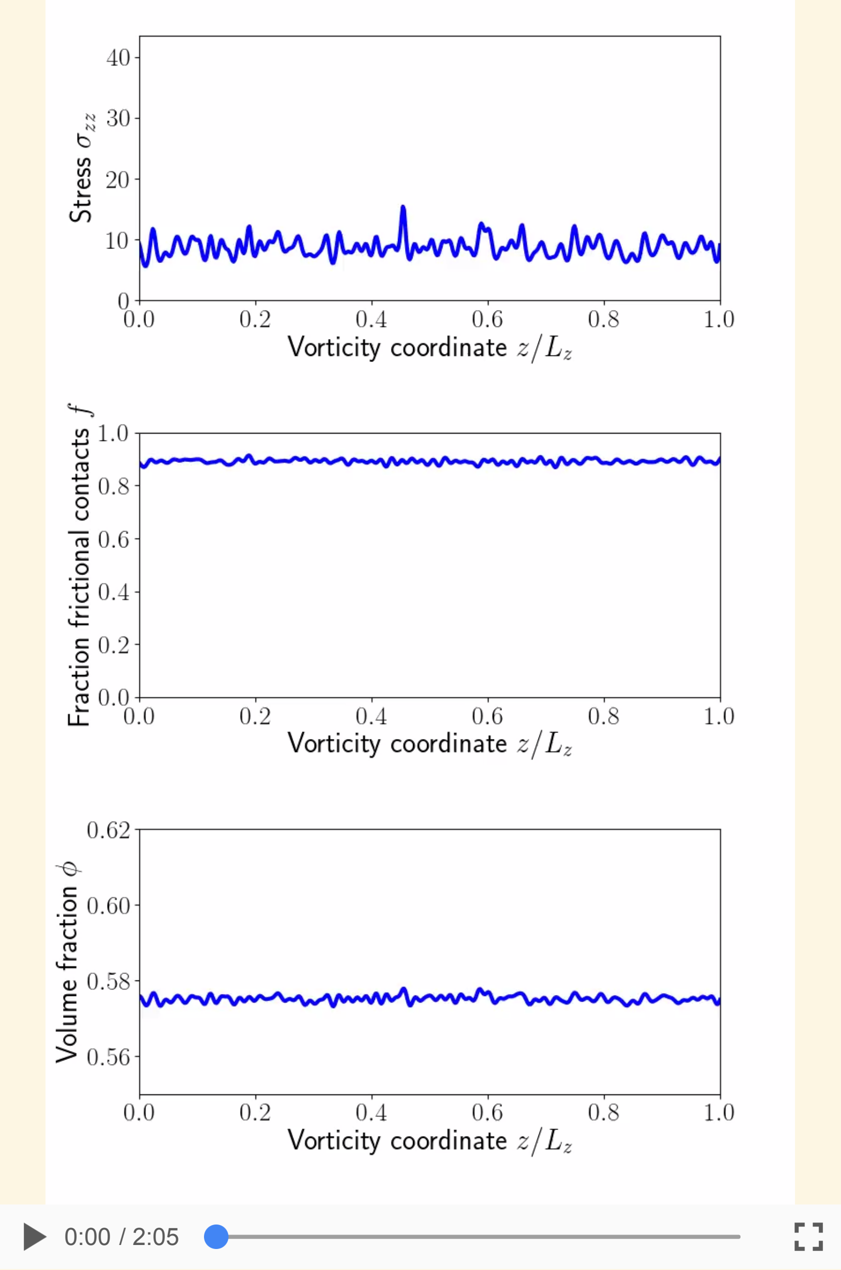
TRAVELING BANDS





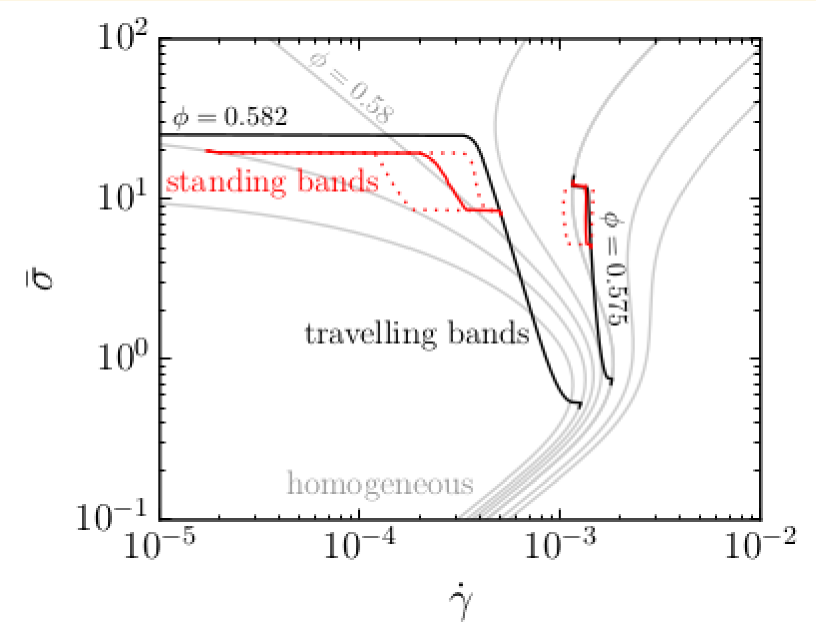
Stress field along vorticity





Stress σ_{zz}
Vorticity coordinate z/L_z
Fraction frictional contacts f
Vorticity coordinate z/L_z
Volume fraction ϕ
Vorticity coordinate z/L_z
0:00 / 2:05

COMPOSITE FLOW CURVES

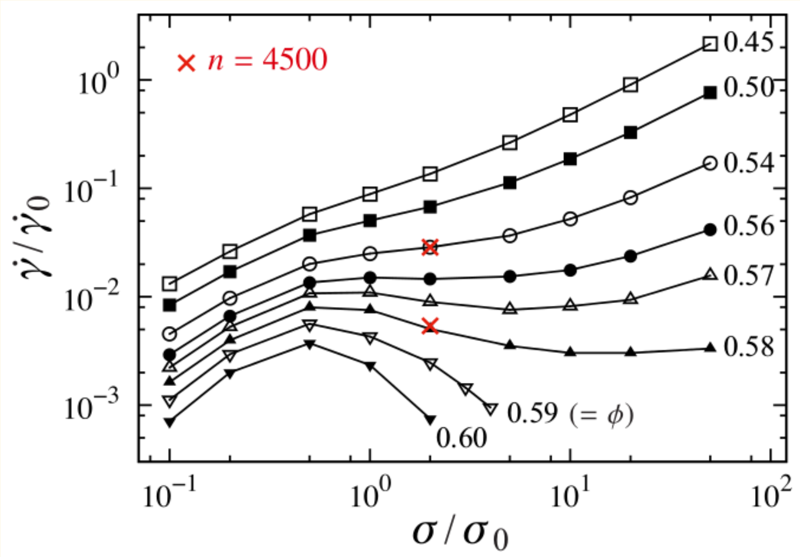


DEM + LUBRICATION SIMULATIONS

Instability for:

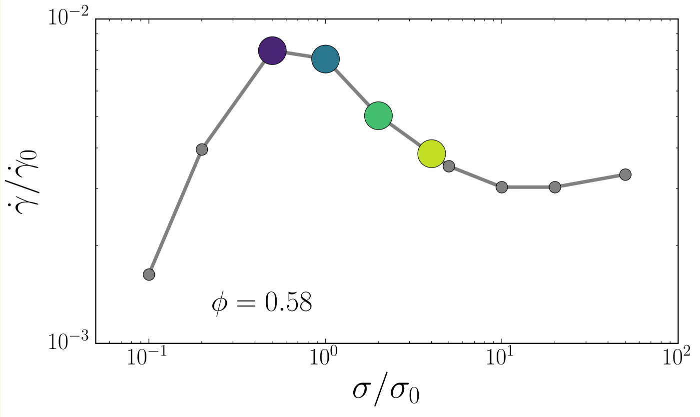
$$\eta\partial_\sigma\dot{\gamma} < -\frac{k^2\gamma_0}{\phi\alpha}\partial_\phi\eta$$

Need $L_z/a \gtrsim 100$



Simulations with very large aspect ratio in favor of the vorticity

DEM + LUBRICATION SIMULATIONS



$$\sigma/\sigma^* = 0.5$$

0:00 / 0:09

$$\sigma/\sigma^* = 1$$

0:00 / 0:15

$$\sigma/\sigma^* = 2$$

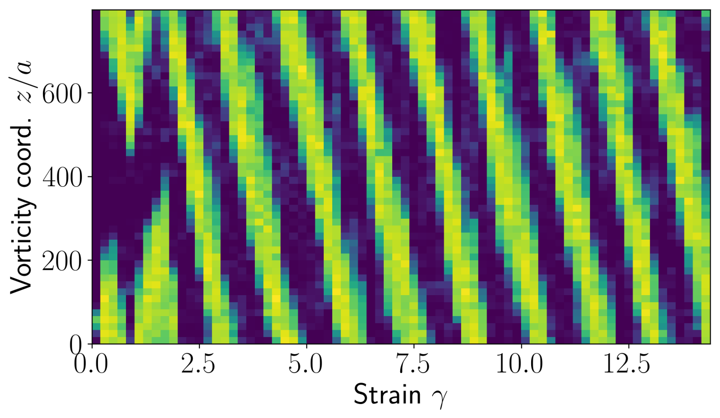
0:00 / 0:04

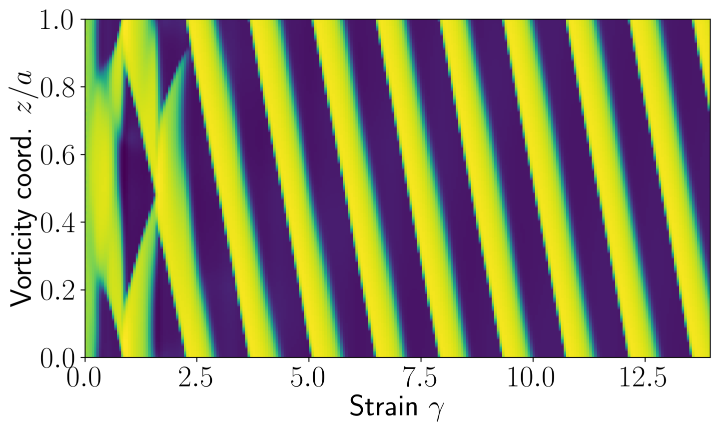
$$\sigma/\sigma^* = 4$$

0:00 / 0:01

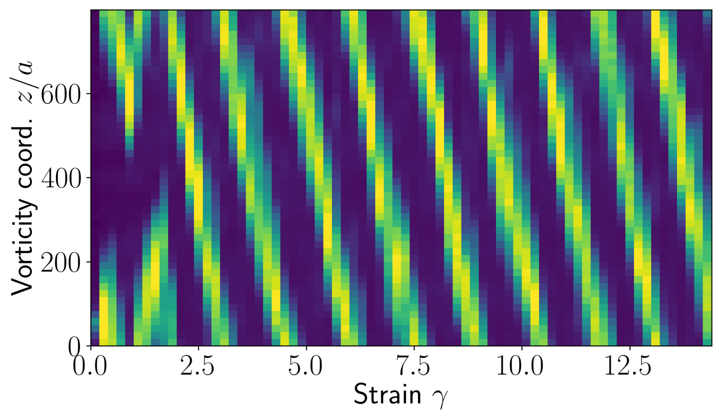
COMPARISON WITH MODEL

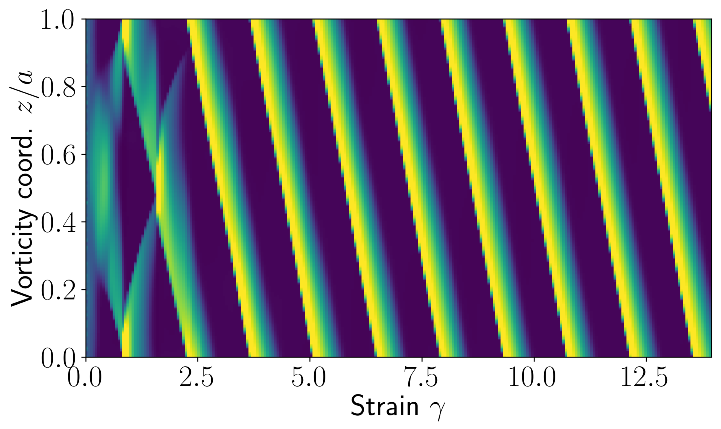
Friction field





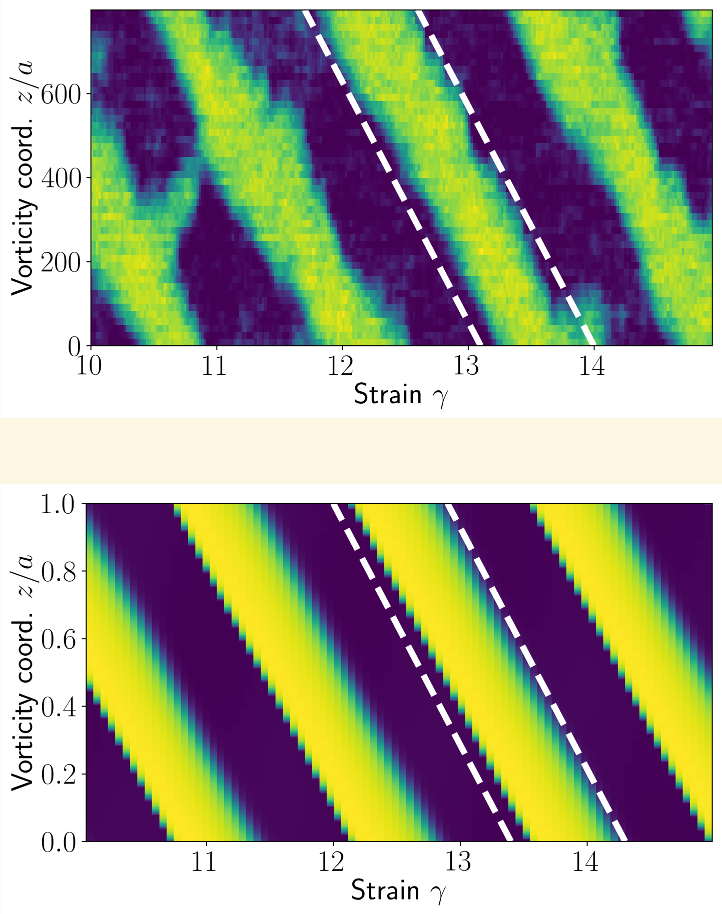
Stress field



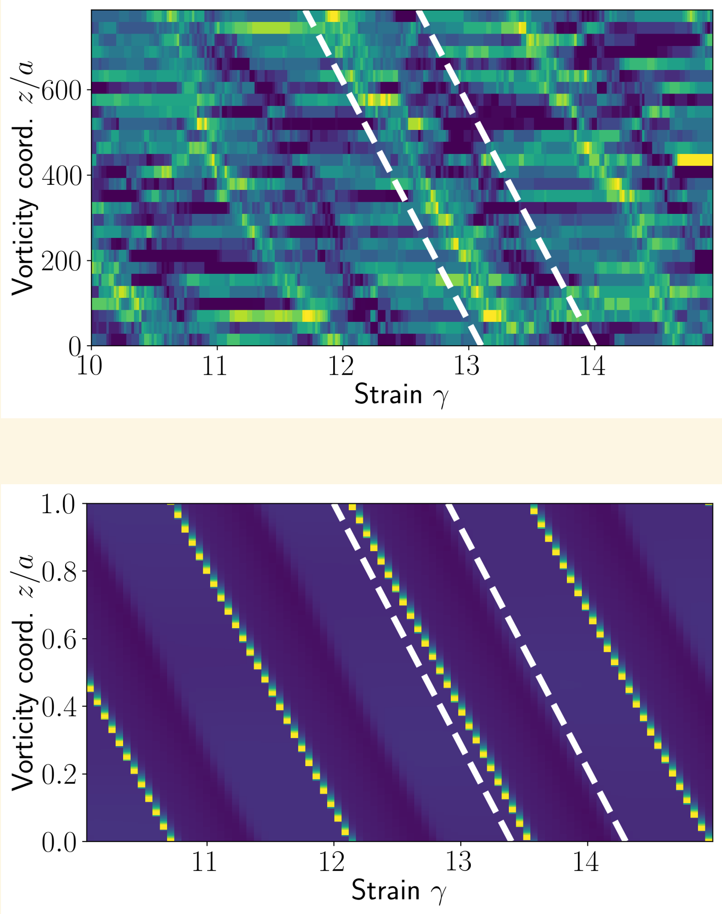


COMPARISON WITH MODEL

Friction field

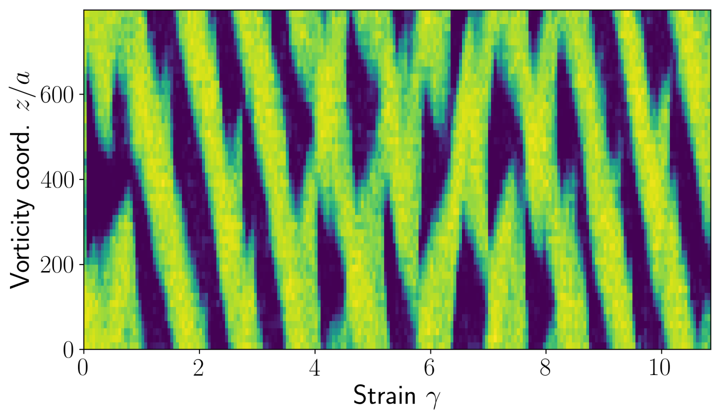


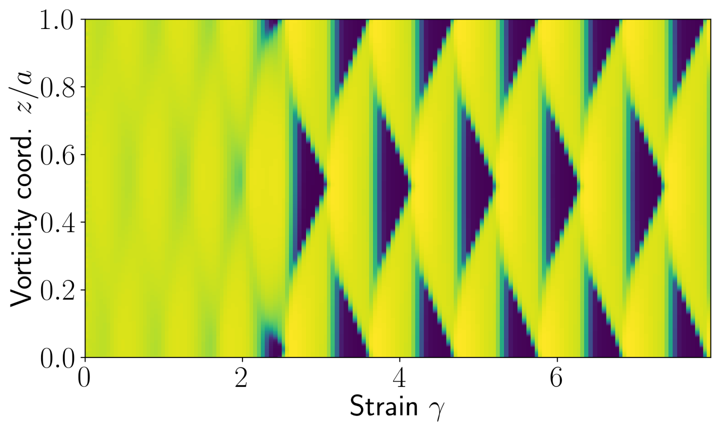
Volume fraction field



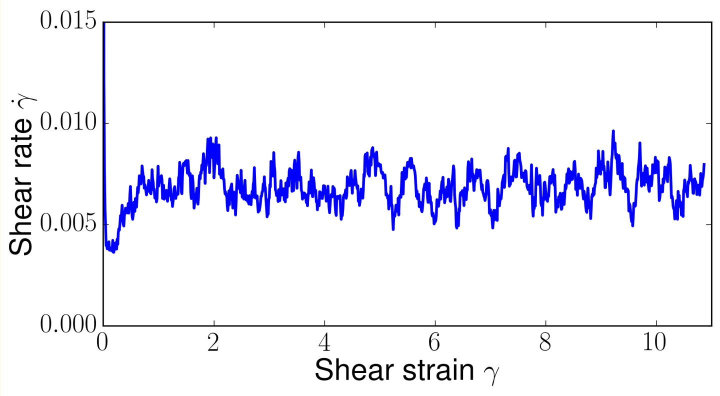
TWO-BAND STATE?

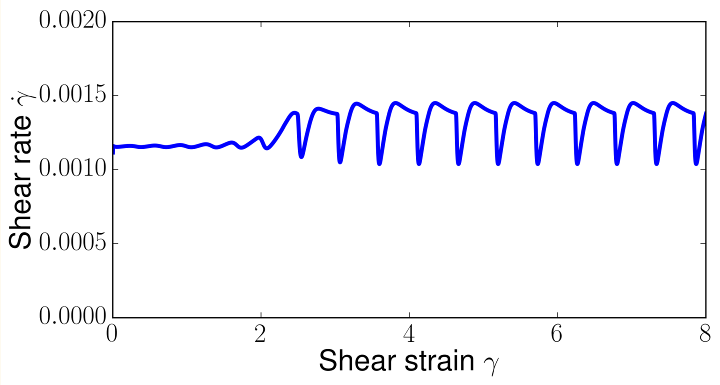
Friction field





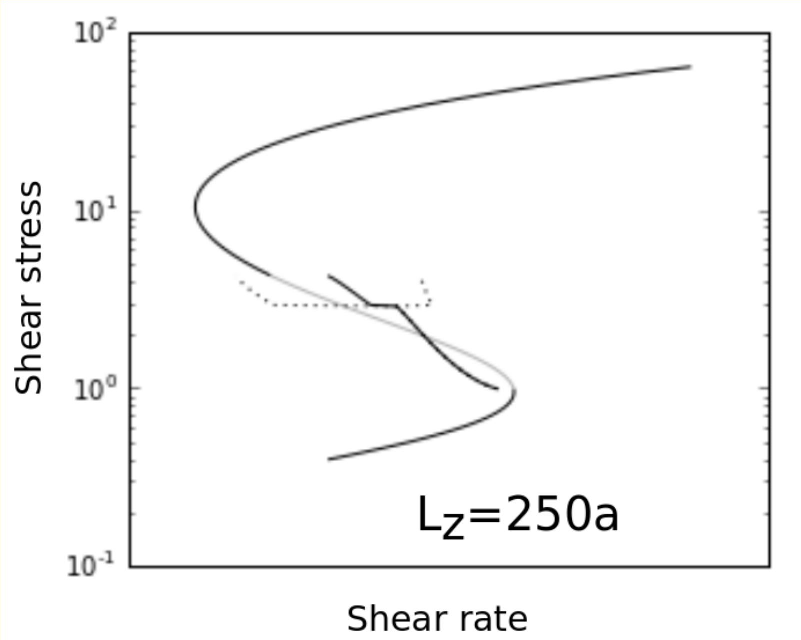
Strain rate

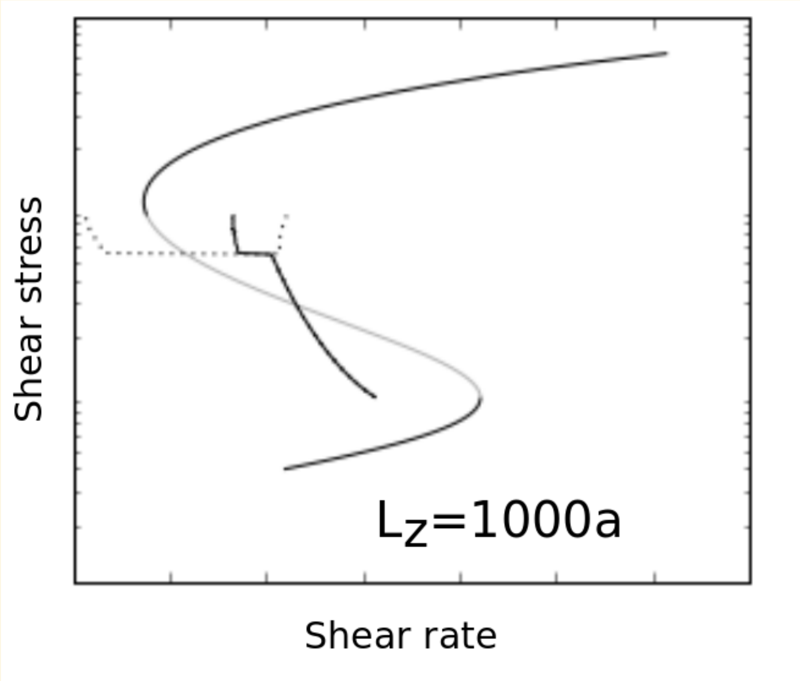




FLOW CURVES

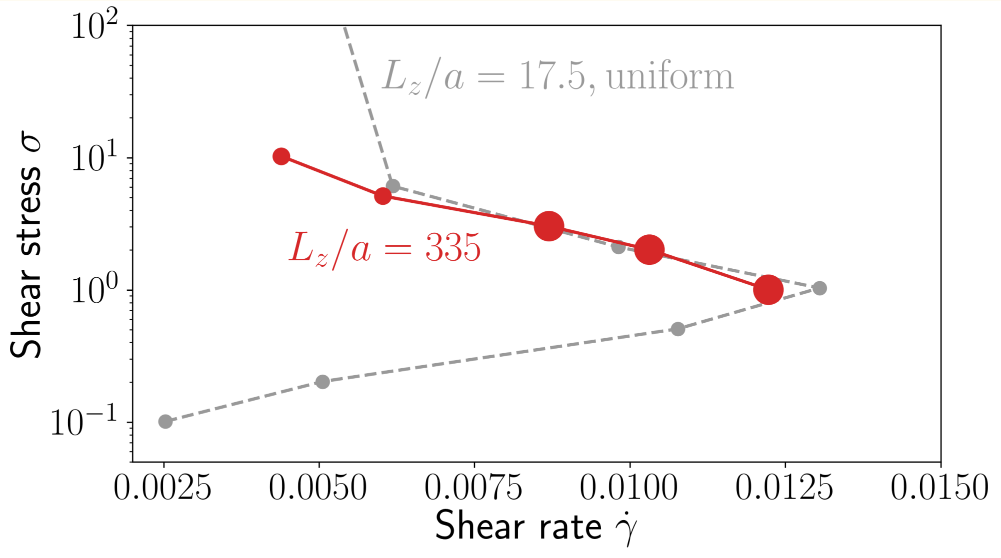
Model





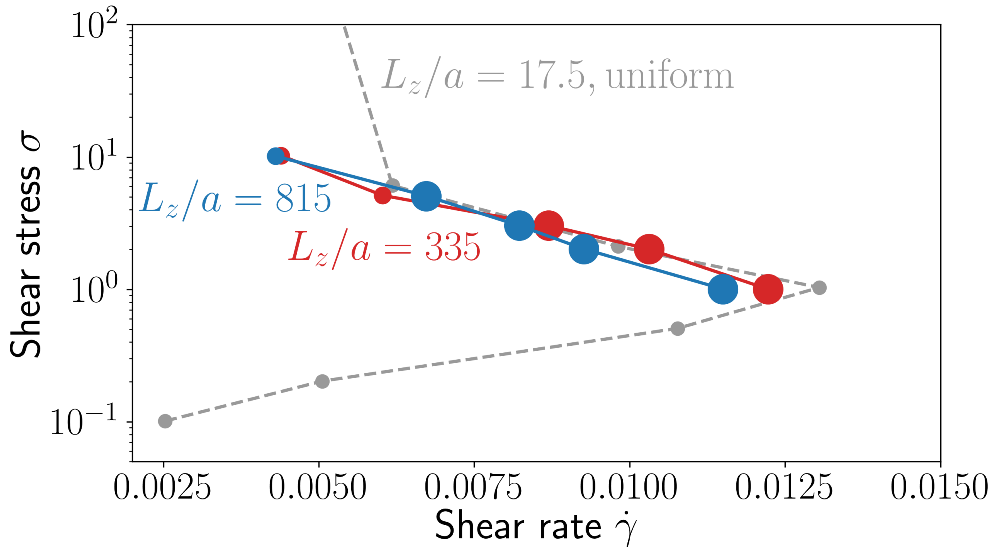
COMPARISON WITH MODEL

Simulations



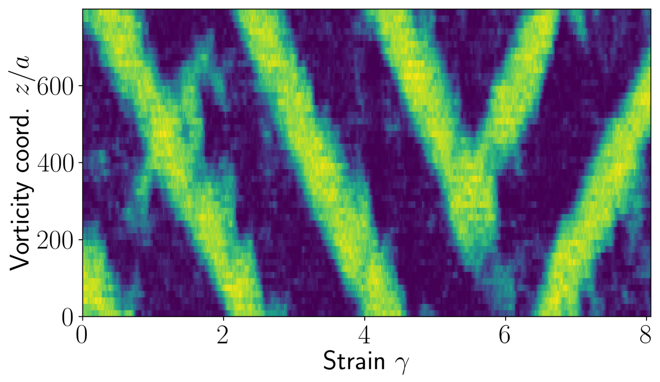
COMPARISON WITH MODEL

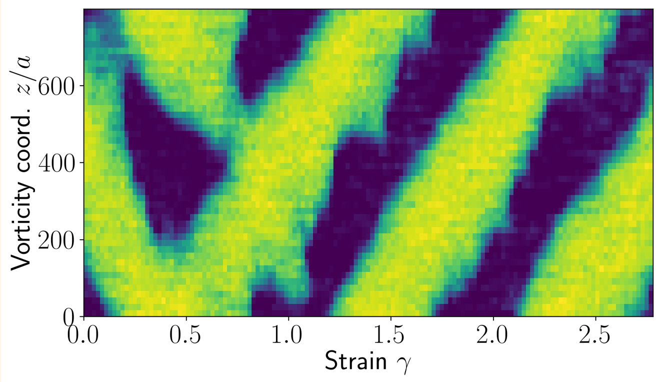
Simulations



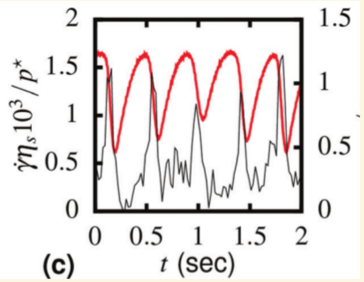
SOME OPEN QUESTIONS

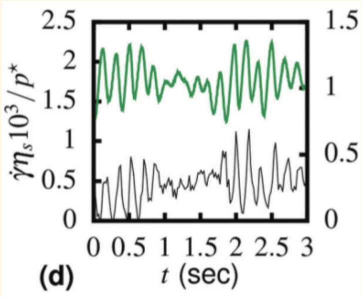
- Two-band state in simulations?
- Some simulation events not captured, role of "noise"?

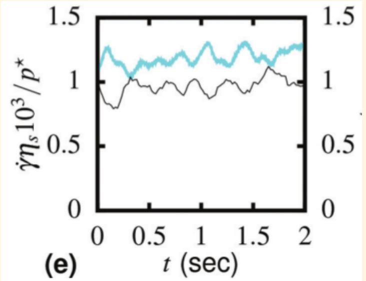




- Where are the rare, intermittent and chaotic regimes of experiments?







- Boundary conditions (free surfuce, imposed normal stress?)