Existing Models for Dense Suspensions

Hand, J. Fluid Mech. 13, 3 (1961)

e.g.,
Phan-Thien, J. Rheol. 39, 679 (1995)

Micro-Structure

$$\frac{d}{dt}\langle \mathbf{pp} \rangle - \mathbf{L}:\langle \mathbf{pp} \rangle - \langle \mathbf{pp} \rangle : \mathbf{L}^{T} = \dot{\gamma}\mathbf{I} - 2\mathbf{D}:\langle \mathbf{pppp} \rangle - 3\dot{\gamma}\langle \mathbf{pp} \rangle$$

where

$\mathbf{L} \equiv \nabla \mathbf{v}$, $2\mathbf{D} \equiv \mathbf{L} + \mathbf{L}^{T}$, "Non-Stokesian" $\dot{\gamma} \propto \sqrt{2\mathrm{tr}\mathbf{D}^{2}}$

Stress

$$\boldsymbol{\sigma} = \mu(\phi)\left(\mathbf{D}:\langle \mathbf{pppp} \rangle + \dot{\gamma}\langle \mathbf{pp} \rangle\right)$$

Closure

$$\mathbf{D}:\langle \mathbf{pppp} \rangle = \frac{1}{5}\left(6\langle \mathbf{pp} \rangle \cdot \mathbf{D} \cdot \langle \mathbf{pp} \rangle - \mathbf{D}:\langle \mathbf{pp} \rangle\langle \mathbf{pp} \rangle - 2\langle \mathbf{pp} \rangle^{2}:\mathbf{DI} + 2\langle \mathbf{pp} \rangle:\mathbf{DI}\right)$$

also,
Stickel, et al., J. Rheol. 50, 379 (2006)

and
Goddard, J. Fluid Mech. 568, 1 (2006)

Jenkins & La Ragione, J. Fluid Mech. 763, 218 (2015)

Pure Straining Trajectories

$$\frac{1}{s}\frac{ds}{d\gamma} = \frac{2}{3}F\left(\frac{1}{s} + 4\frac{\alpha}{\bar{s}}\right) + 4\frac{\alpha}{\bar{s}}\cos 2\theta$$

$$\left[\ln\left(\frac{1}{s}\right) + 3.84\right]\frac{d\theta}{d\gamma} = \left(4.77 - 3\alpha\frac{1}{\bar{s}}\right)\sin 2\theta - \frac{1}{2}\ln\left(\frac{1}{s}\right)S$$

$$S = -\frac{12\alpha}{(4\alpha - k + 1)}\frac{1/\bar{s}}{\ln(1/\bar{s}) - 0.96}\sin 2\theta$$

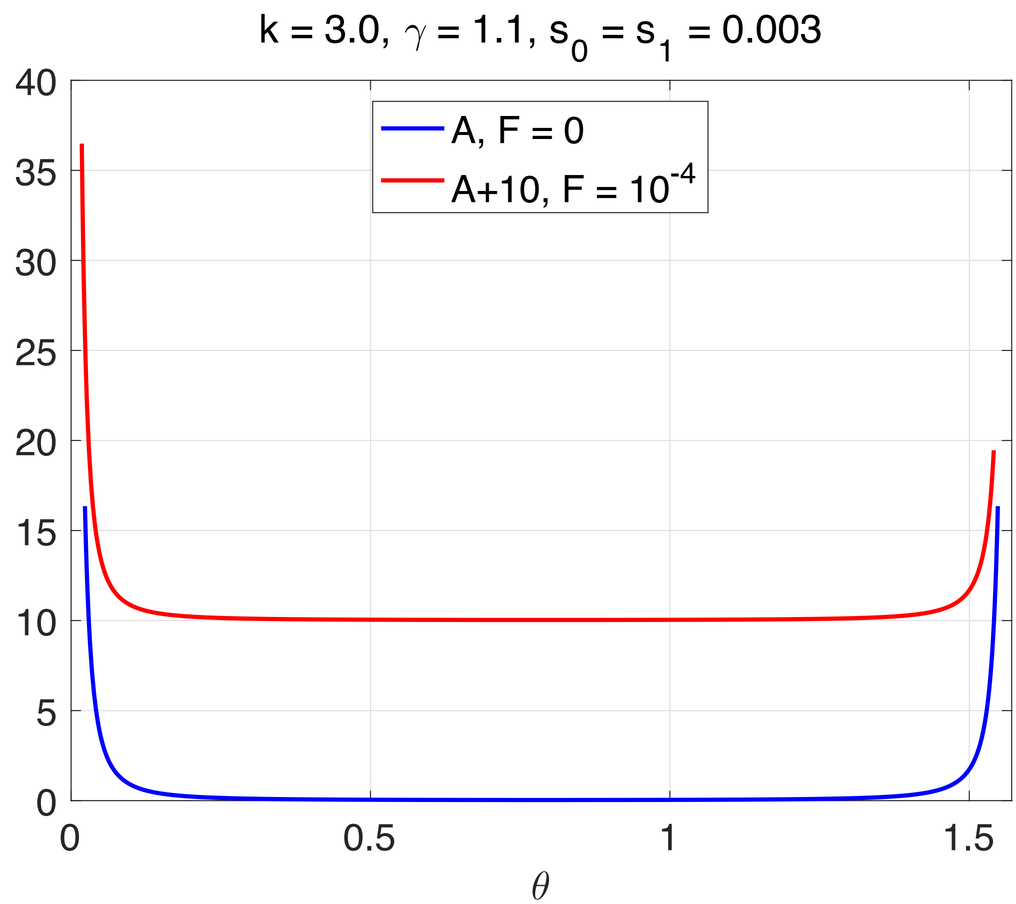
Particle Distribution

$$A(\theta)\frac{d\theta}{d\gamma} = \text{constant}$$

and

$$\int_0^{\pi/2} A(\theta)\,d\theta = \frac{k}{4}$$

Particle Distribution



Stress

$$t_{\alpha\beta} = \left[\nu \frac{\alpha}{\bar{s}} \int_0^{2\pi} A(\theta)\left(4F\hat{d}_\alpha \hat{d}_\beta - 6\hat{d}_\alpha \hat{d}_\beta \hat{d}_\gamma^{(BA)} \hat{d}_\mu^{(BA)}\right) d\theta \right.$$

$$+ \nu \left(9.54 - \frac{6}{\bar{s}}\right) \int_0^{2\pi} A(\theta) \hat{t}_\gamma^{(BA)} \hat{d}_\mu^{(BA)} \hat{t}_\alpha \hat{d}_\beta d\theta$$

$$\left. + \nu \frac{12\alpha}{(3\alpha - k + 1)} \frac{1/\bar{s}}{\ln(1/\bar{s})} \int_0^{2\pi} A(\theta) \ln\left(\frac{1}{s}\right) \hat{t}_\gamma^{(BA)} \hat{d}_\mu^{(BA)} \hat{t}_\alpha \hat{d}_\beta d\theta \right] \tilde{D}_{\gamma\mu}$$

with $\hat{t}_\alpha = \varepsilon_{\alpha\beta} \hat{d}_\beta$.