Shear banding in time-dependent flows of complex fluids

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Outline

Shear banding in time-dependent flow protocols

- shear startup
- step stress (creep)
- step strain
- large amplitude oscillatory shear

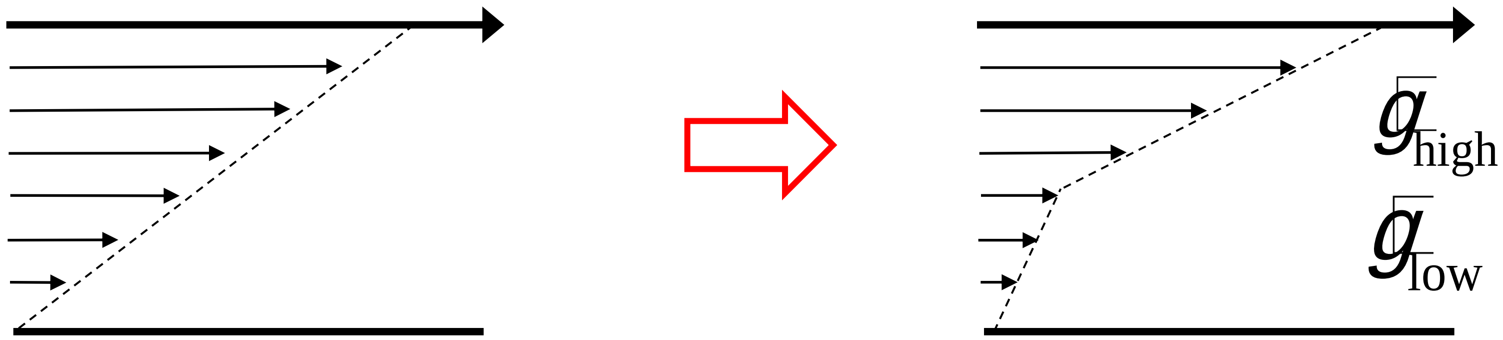
Aim: to provide fluid-universal criteria for onset of banding, covering

- polymeric fluids (polymer solutions/melts, wormy micelles)
- soft glassy materials (foams, emulsions, colloids, microgels)
- and everything else too…?

Warmup: criterion for formation of steady state shear bands

Initial state:
homogeneous shear flow

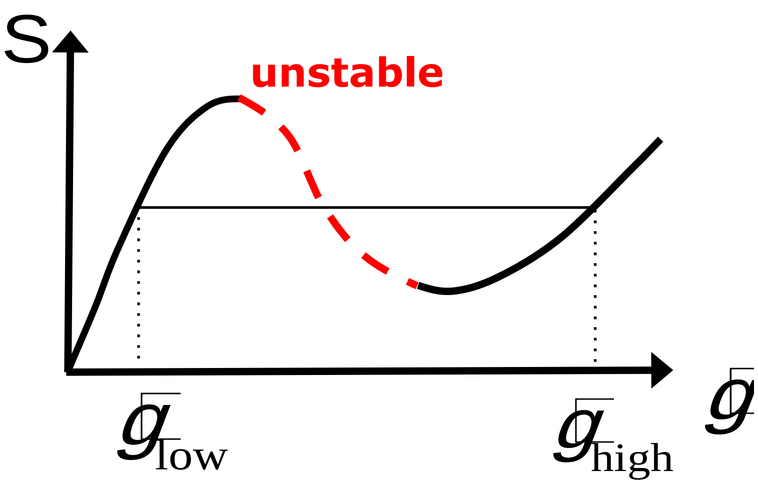
Steady state:
shear banded



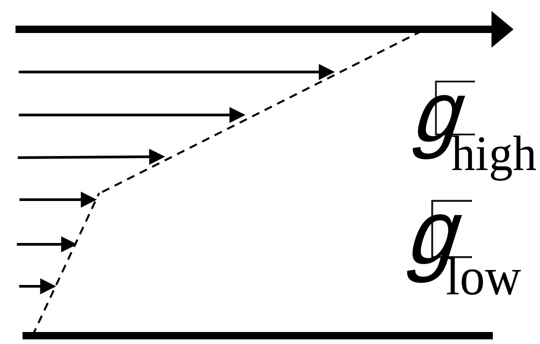
What is the criterion for this to happen?

Warmup: criterion for formation of steady state shear bands

Constitutive curve for homogeneous shear flow



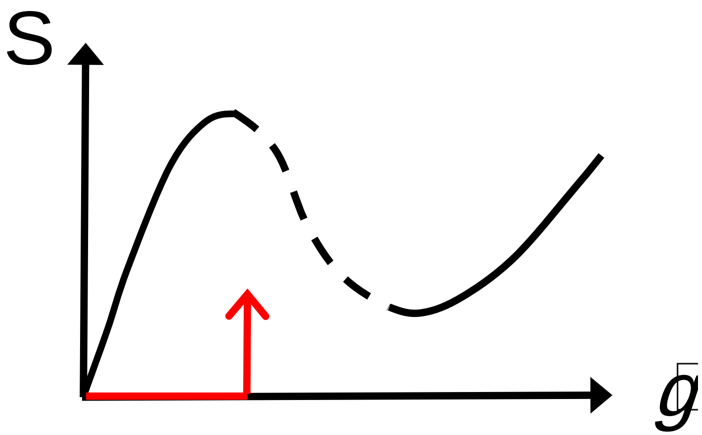
Steady state: shear banded



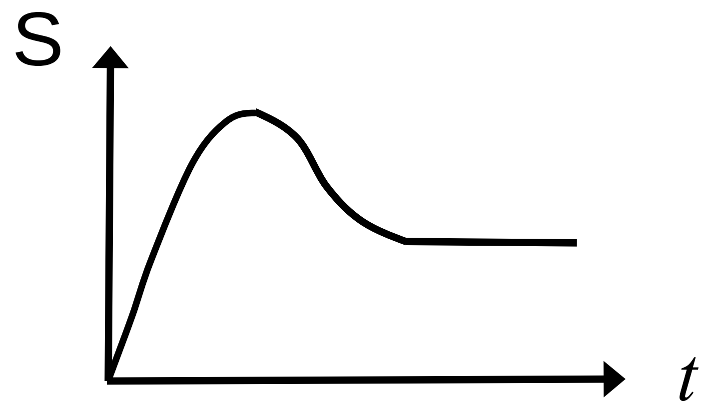
Criterion $dS/dg < 0$ is independent of fluid and model in question

Time-dependent flow protocols…

constitutive curve

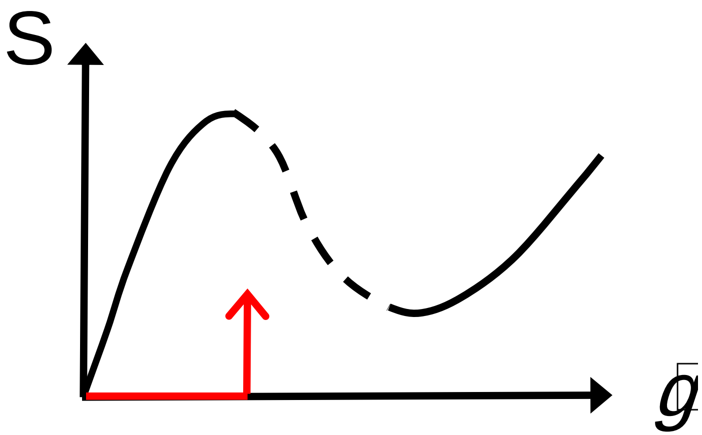


startup transient



Time-dependent flow protocols…

constitutive curve



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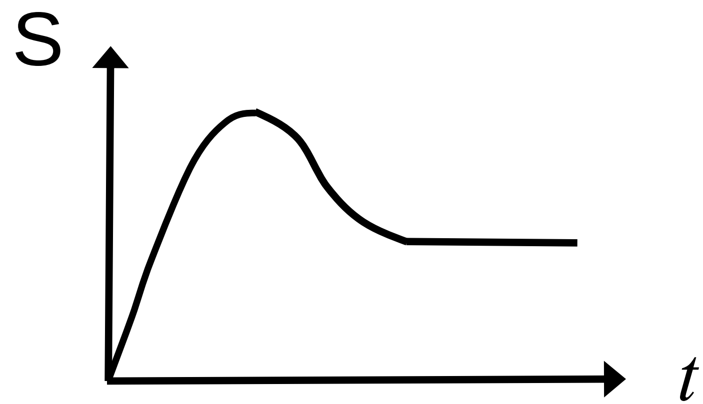
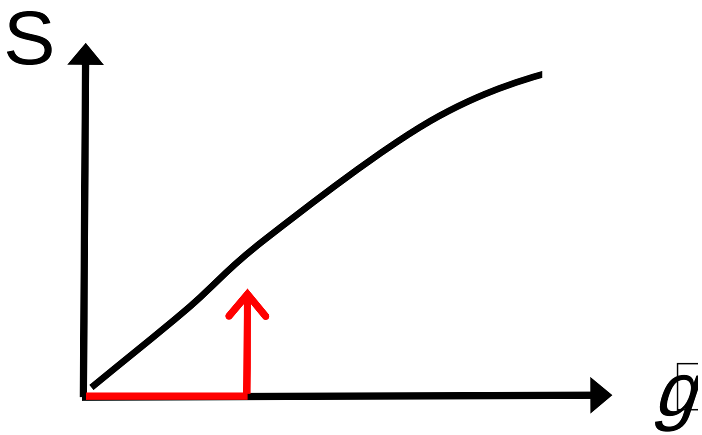


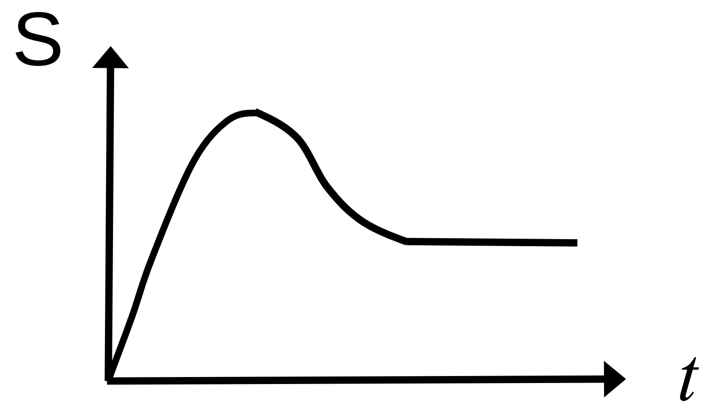
At what stage of progression to steady state do bands form?

Time-dependent flow protocols…

constitutive curve



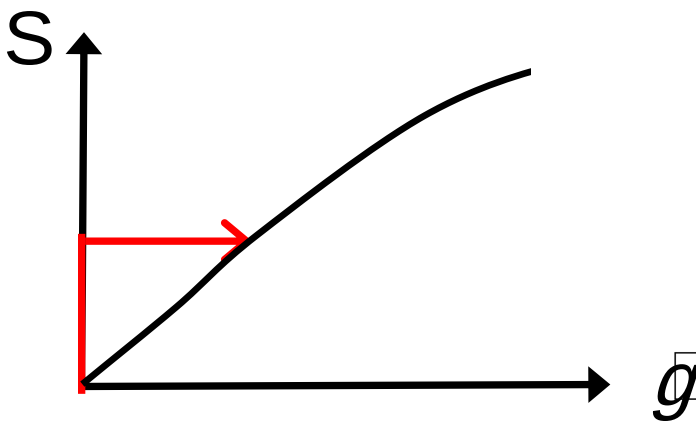
startup transient



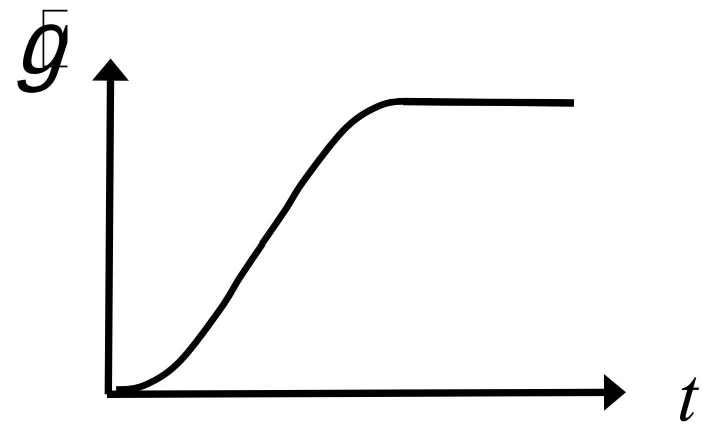
Can bands form transiently in shear startup, even if steady state unbanded?

Time-dependent flow protocols…

constitutive curve



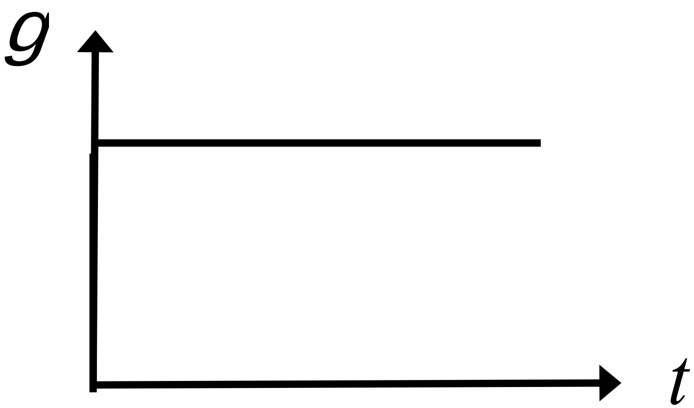
startup transient



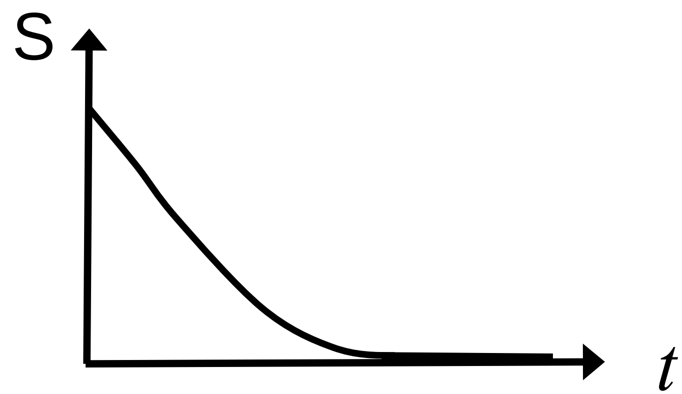
and in a step-stress (creep) experiment?

Time-dependent flow protocols…

Step strain



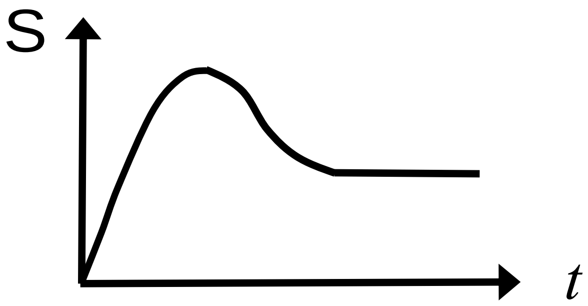
stress transient



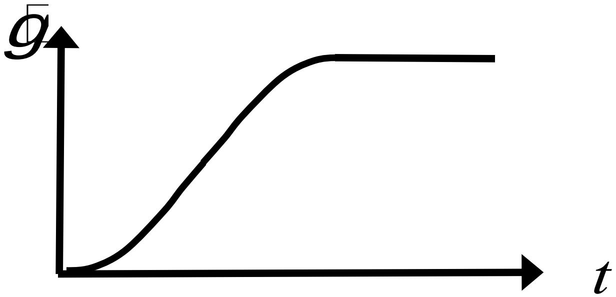
and in a step-strain experiment?

Aim here: derive <u>general</u> criteria for banding in time-dependent flows

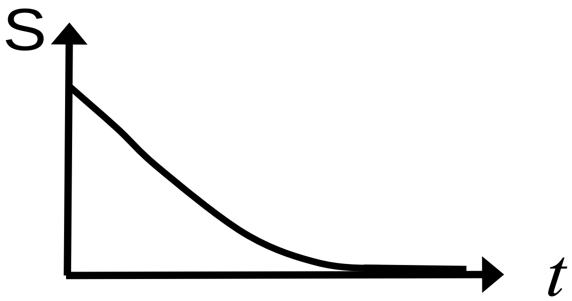
shear startup



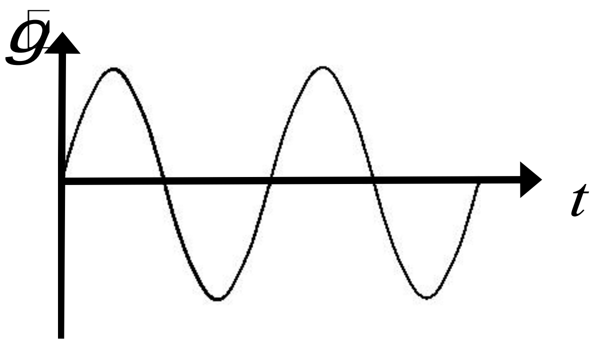
step stress



step strain



LAOS



that depend <u>only</u> on the shape of these rheological response functions

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shear startup protocol: the basic idea

polymeric fluids

soft glassy materials

general criterion

step stress

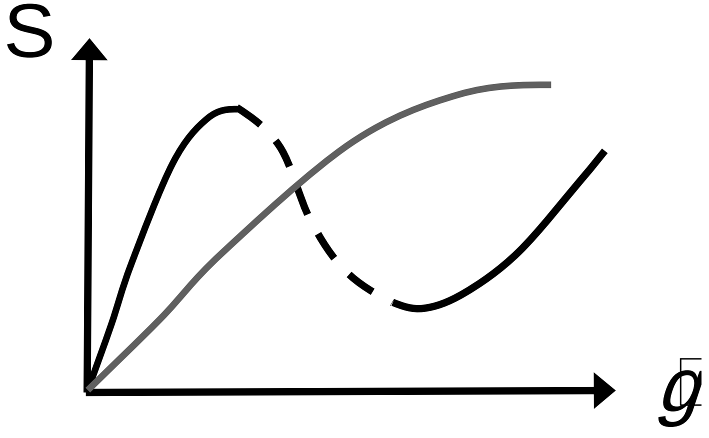
step strain

large amplitude oscillatory shear

Summary and outlook

Shear startup in complex fluids: the basic idea

steady state constitutive curve



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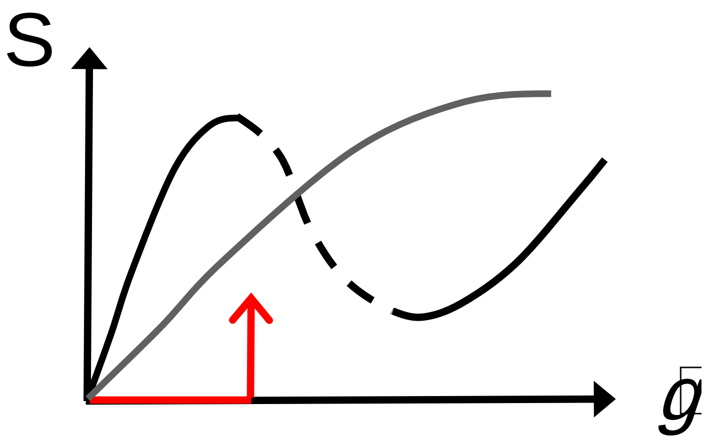
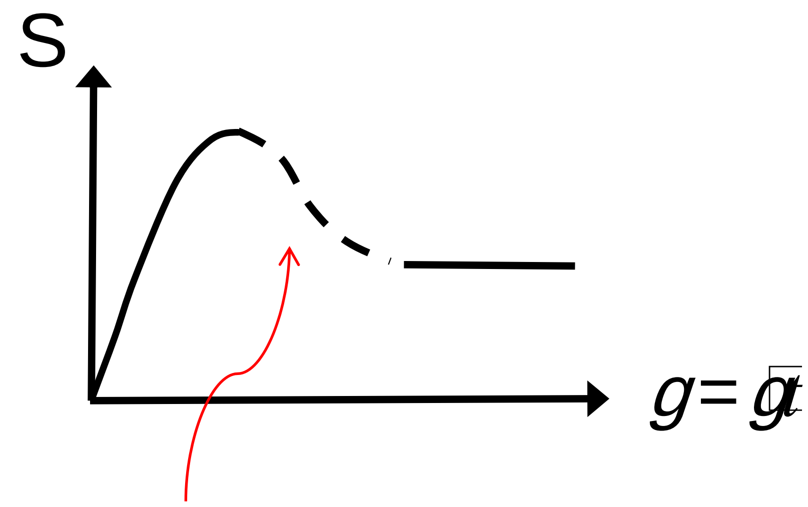


startup transient



stress overshoot commonly seen

[Marrucci + Grizzuti J. Rheol 1983]

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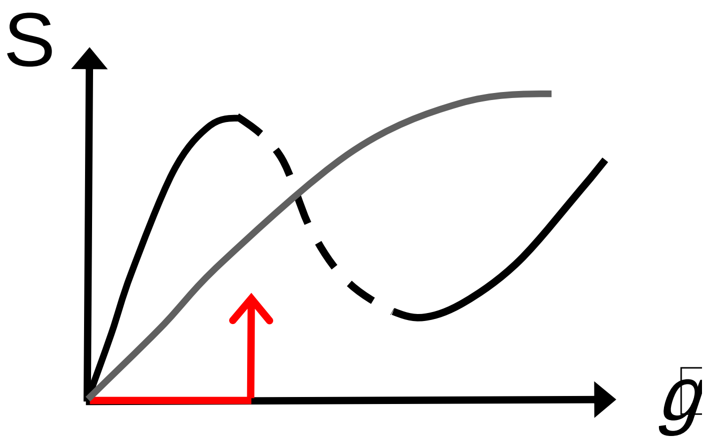
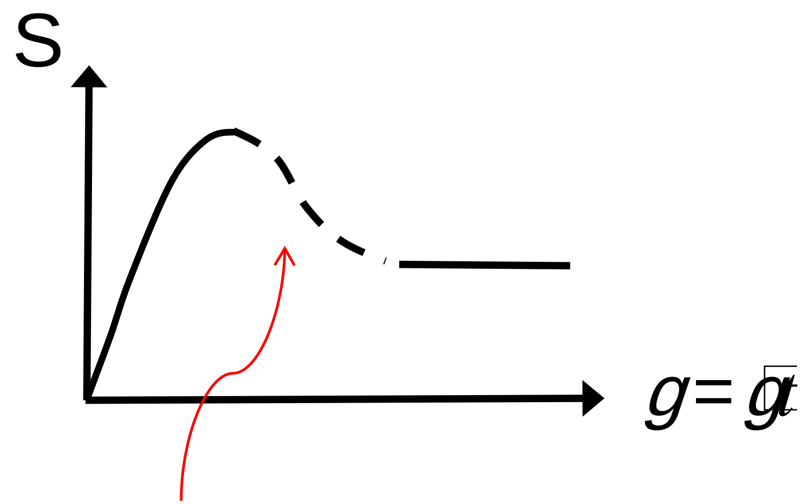


startup transient



stress overshoot commonly seen

associated with transient banding

[Marrucci + Grizzuti J. Rheol 1983]

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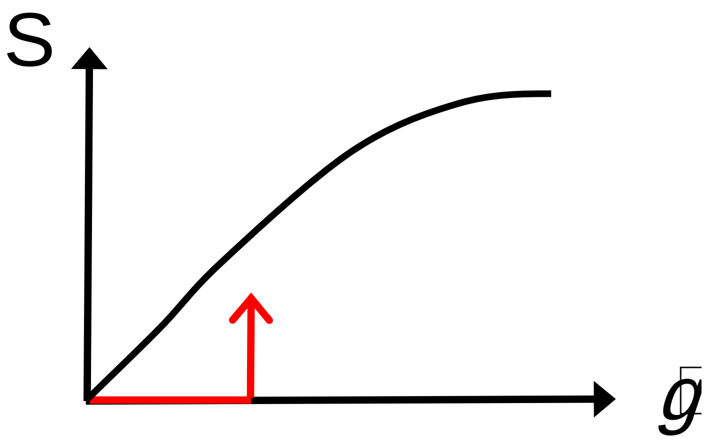
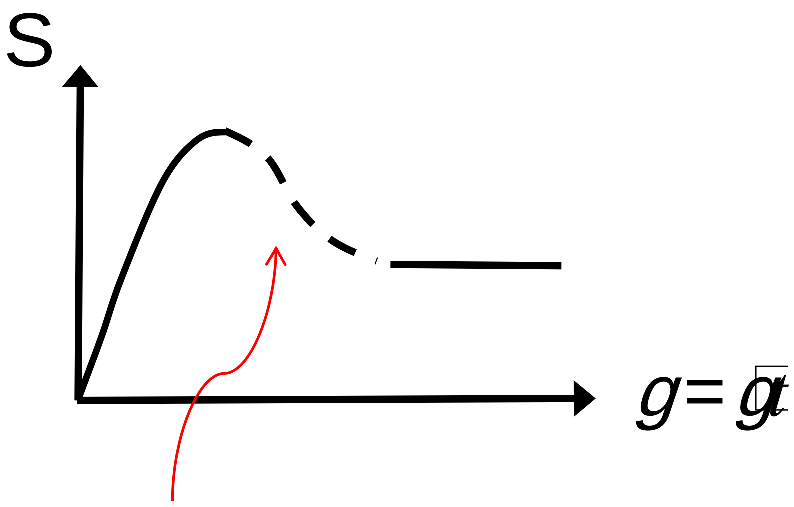


startup transient



stress overshoot commonly seen

associated with transient banding

even if steady state cc monotonic

[Marrucci + Grizzuti J. Rheol 1983]

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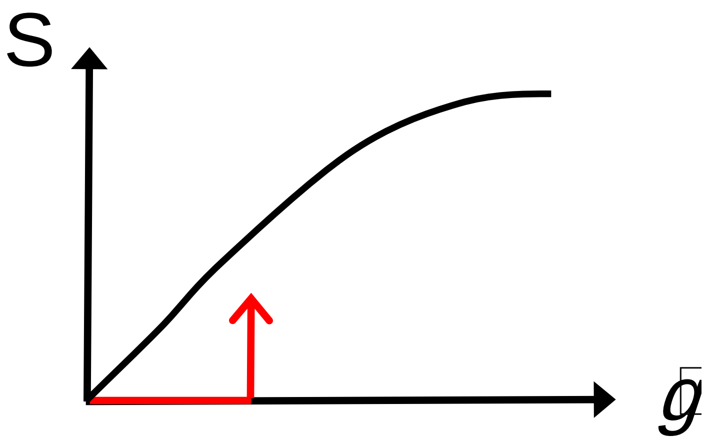
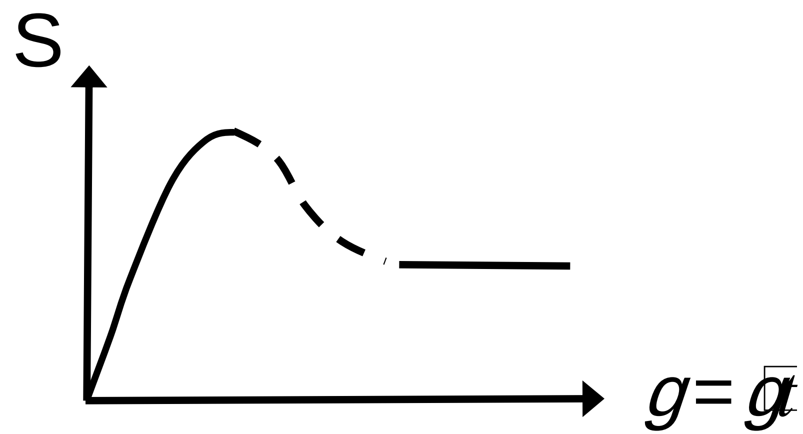
steady state constitutive curve



startup transient



Indeed: for rest of talk, constitutive curve will be monotonic

unless otherwise stated – no <u>steady state</u> banding !

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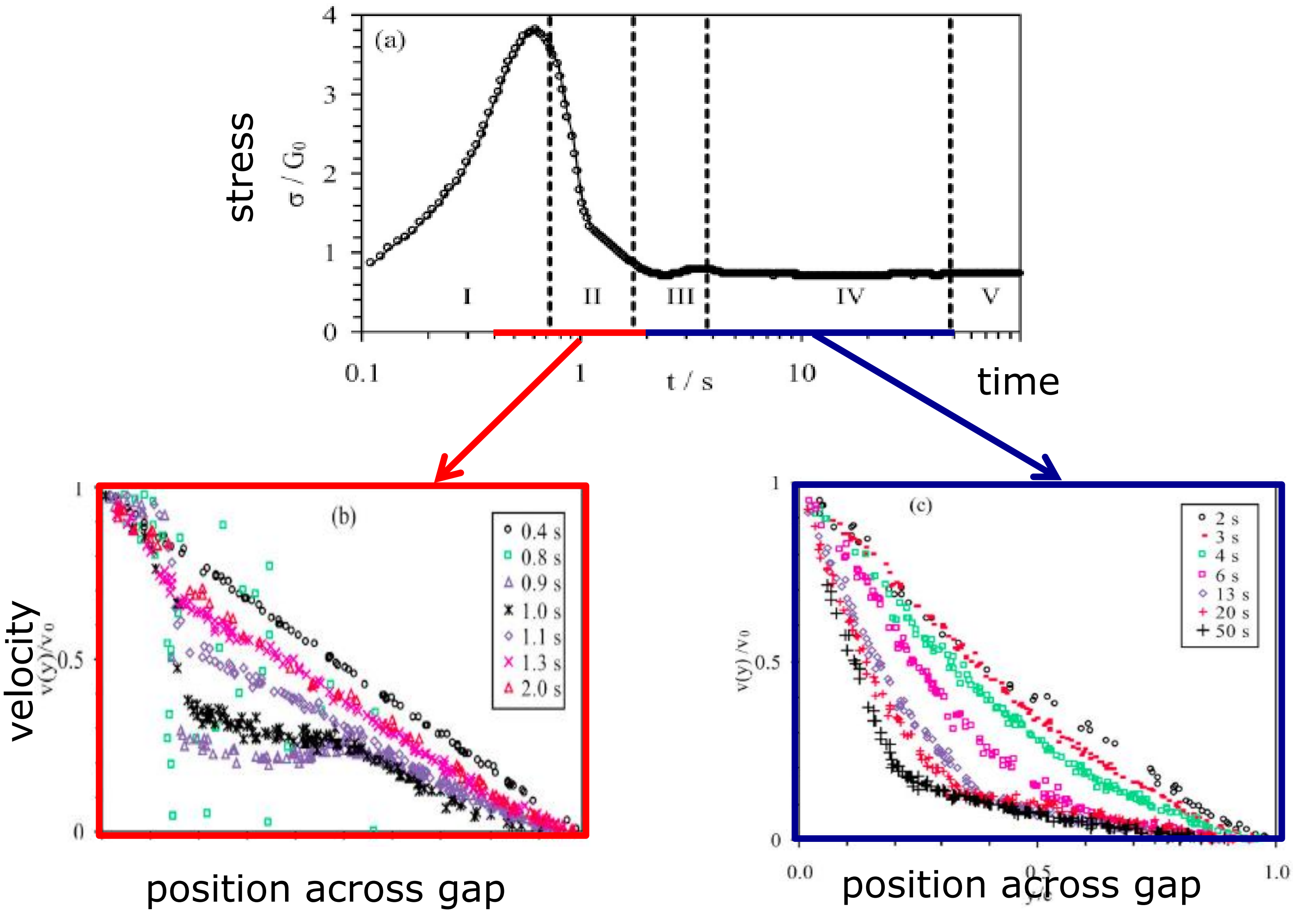
[Hu et al. J. Rheol 2008. See also S Q Wang et al. J. Rheol 2008,09; PRL 06; Macromol 08, 09]

Shear startup in polymeric fluids: experiments



[S Q Wang et al. Macromol 08; see also J. Rheol 2008,09; PRL 06; Macromol 09]

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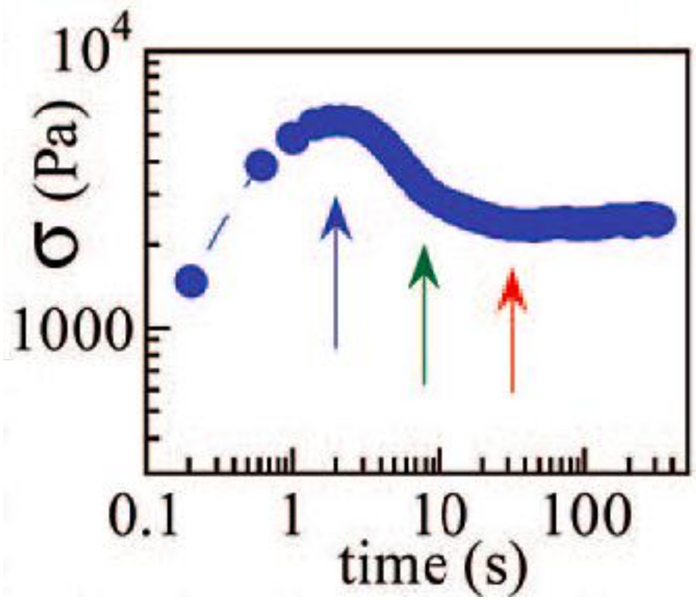
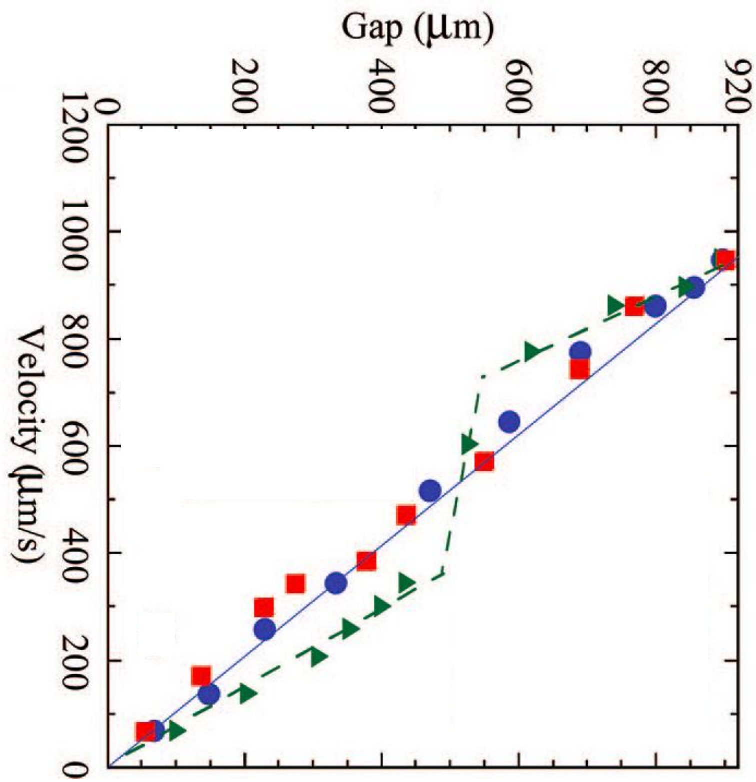




position across gap

[See also Zhou at al. J Rheol 08; and MD simulations of Cao and Likhtman PRL 2012]

Shear startup in polymeric fluids: simulation

Force balance in creeping flow

$$0 = \nabla\cdot\left(\Sigma + \underbrace{2\eta D}_{\text{solvent}} - pI\right)$$

Polymeric stress

$$\Sigma = G\left[\mathbf{W} - \mathbf{I}\right]$$

“Rolie-Poly” model for dynamics of polymer conformation tensor

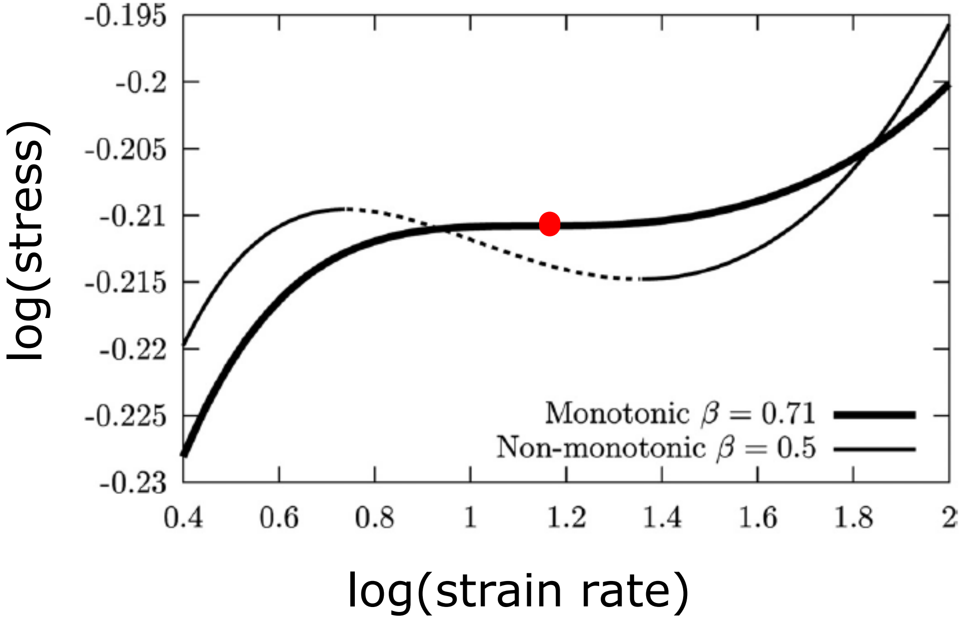
$$\overset{\nabla}{\mathbf{W}} = -\frac{1}{\tau}\left(\mathbf{W}-\mathbf{I}\right) - \frac{2}{\tau_R}\left(1-\sqrt{\frac{3}{T}}\right)\left[\mathbf{W} + \beta\left(\frac{T}{3}\right)^{\delta}\left(\mathbf{W}-\mathbf{I}\right)\right] + D\nabla^2\mathbf{W}$$

Use units in which G = 1, τ = 1

[Likhtman + Graham JNNFM 2003]

[Adams, FIelding, Olmsted J Rheol 2011]

Shear startup in polymeric fluids: simulation

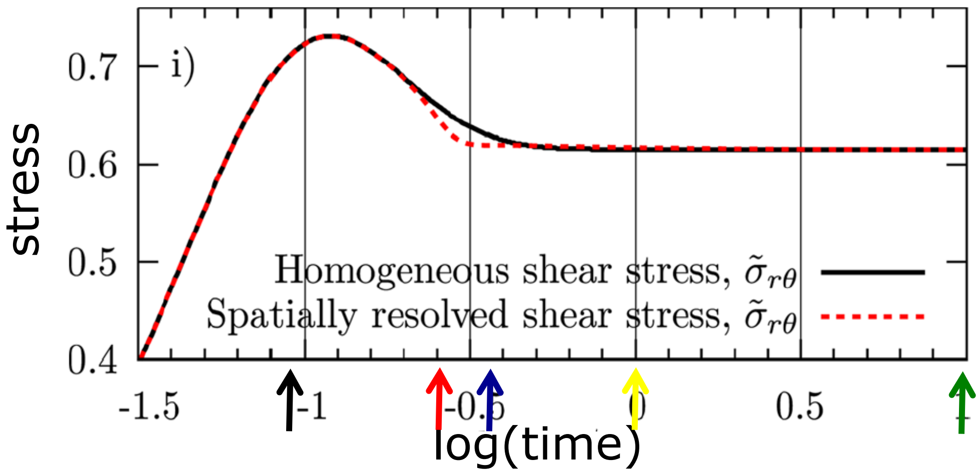


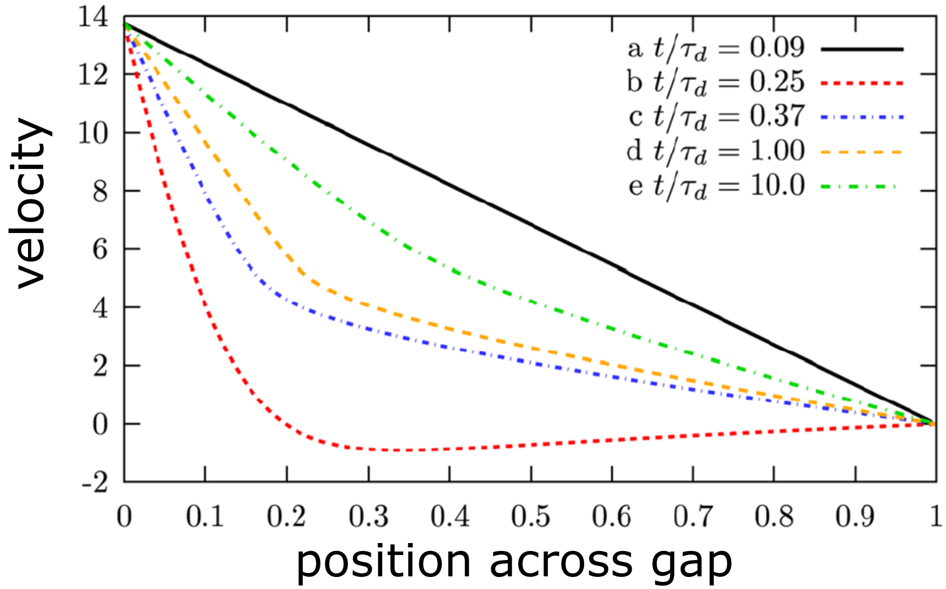
monotonic or not according to model parameters

Results on next slide for startup at shear rate ● on monotonic one

[Adams, FIelding, Olmsted J Rheol 2011]

Shear startup in polymeric fluids: simulation





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soft glassy materials

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Summary and outlook

Soft "glassy" materials

- Foams
- Dense emulsions
- Surfactant onion phases
- Gel bead suspensions
- Dense colloids (?)
- Gels (?)

Shared features:

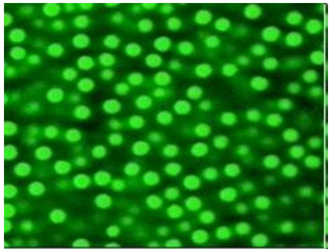
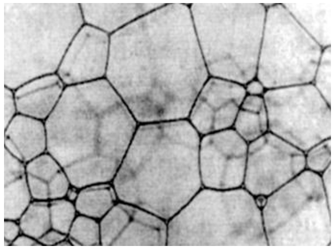
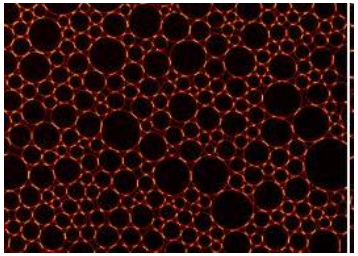
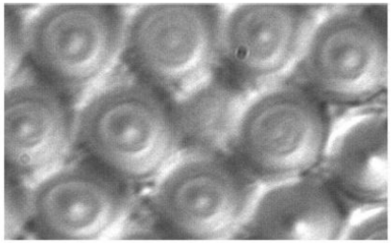
disorder

metastability

broken ergodicity

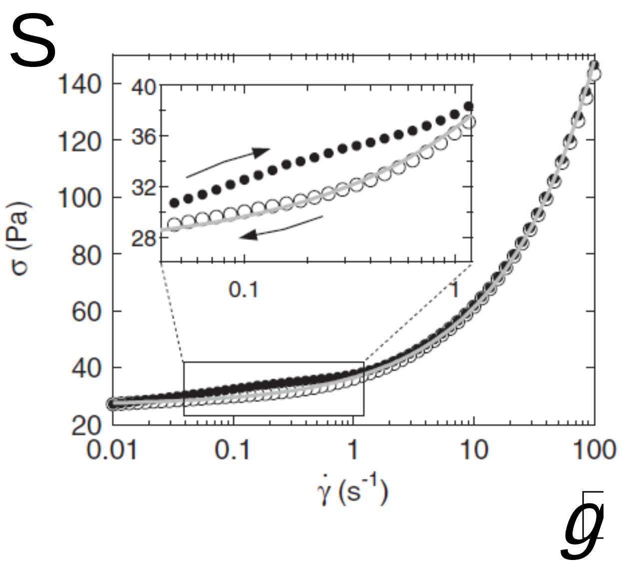
yield stress

ageing

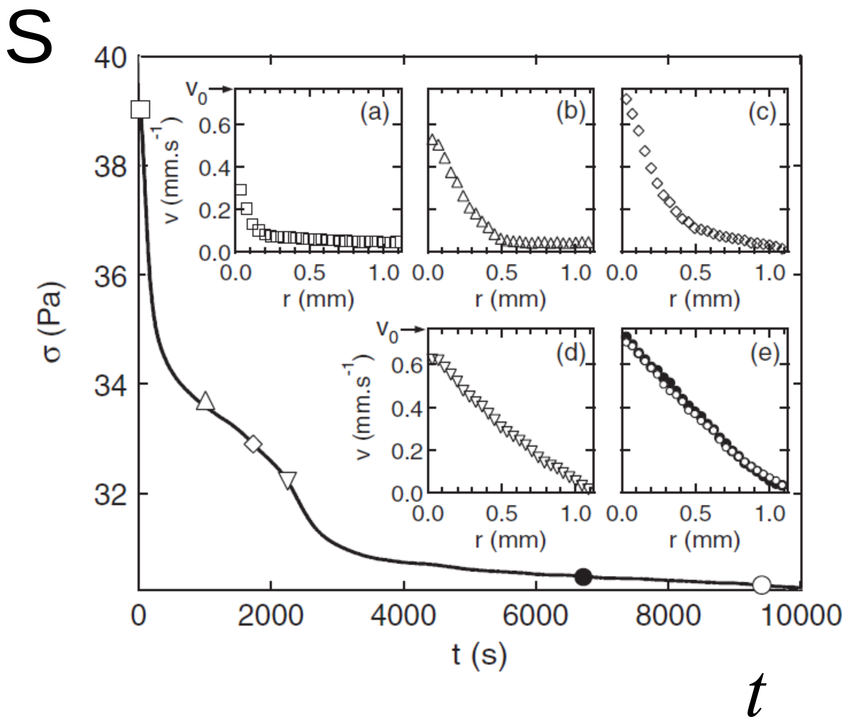





Shear startup in a soft glassy material: experiment

flow curve: yield stress



stress overshoot + transient bands



[Carbopol gel - Divoux et al. PRL 2010]

[Sollich et al PRL 97]

Soft glassy rheology (SGR) model

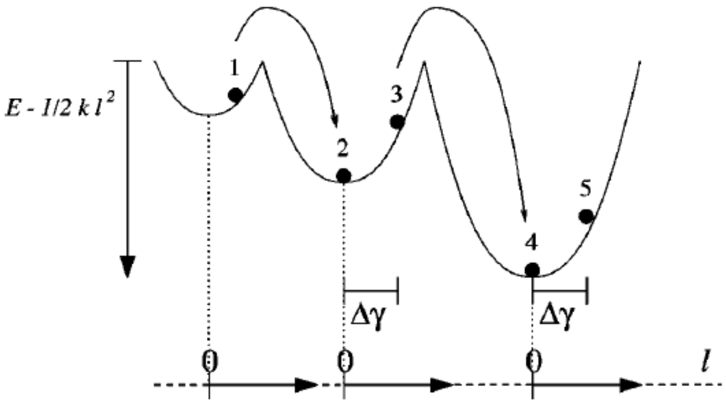
- Particles jump independently among traps at noise temperature x

- l = local strain

- $dl/dt = \dot{\gamma}(t)$ between hops

- Jump rate $\Gamma_0 \exp[-(E-k\, l^2/2)/x]$

- Trap depth distribution $\exp[-E]$
  $\Rightarrow$ glass transition at $x=1$



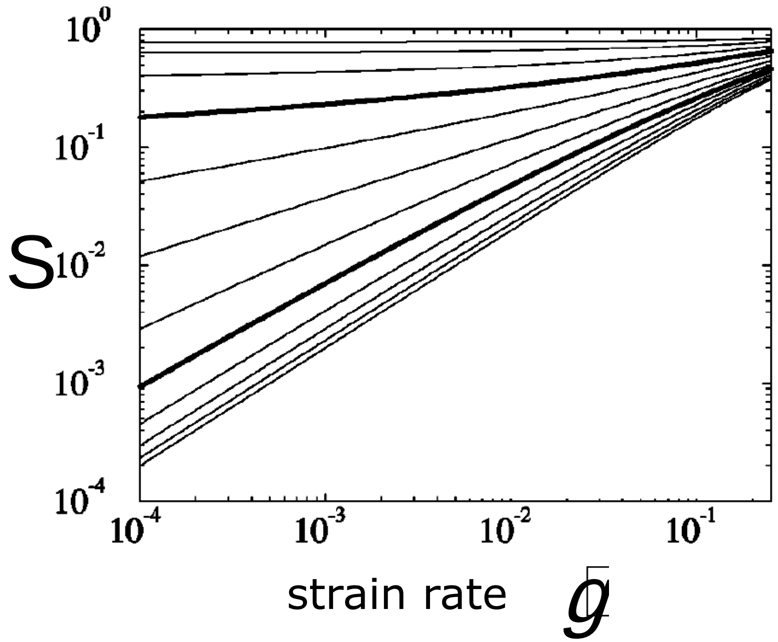
- Stress $\sigma = k\langle l\rangle$. Yield stress rises smoothly for $x < 1$

- Extended to account for spatial heterogeneity [Fielding et al. Soft Matter 2009]

See also: STZ (Langer, Falk et al) , MCT (Fuchs, Cates et al), fluidity (Coussot, Ajdari)

SGR model predictions

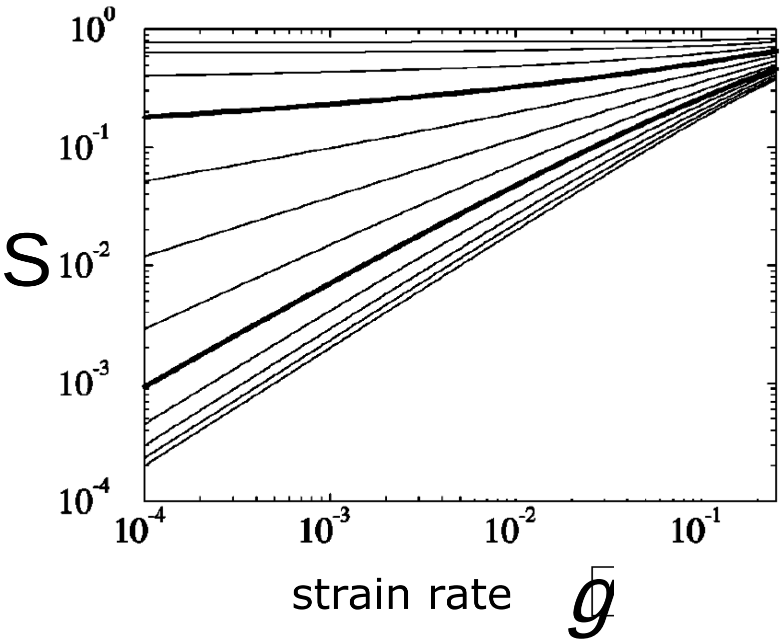
Flow curve: yield stress for x < 1



Monotonic - no steady state bands

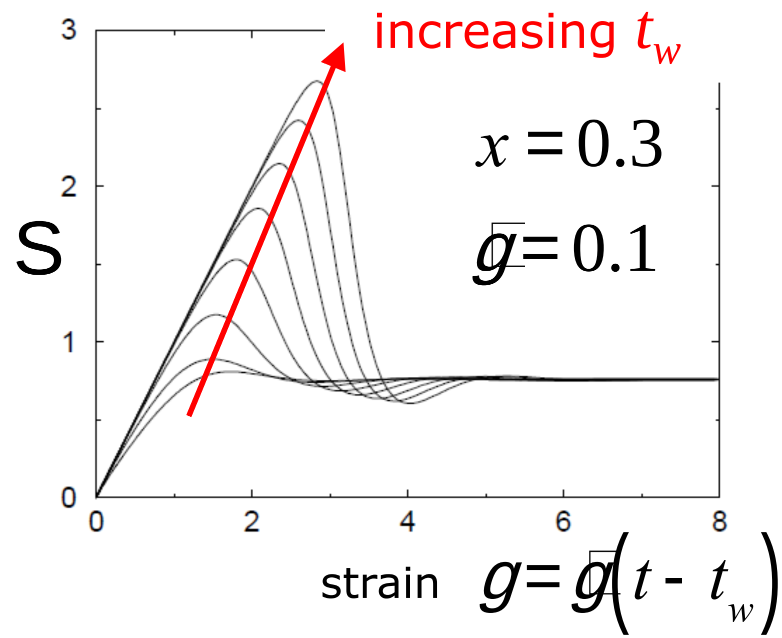
SGR model predictions

Flow curve: yield stress for x < 1



Monotonic - no steady state bands

Overshoot in shear startup

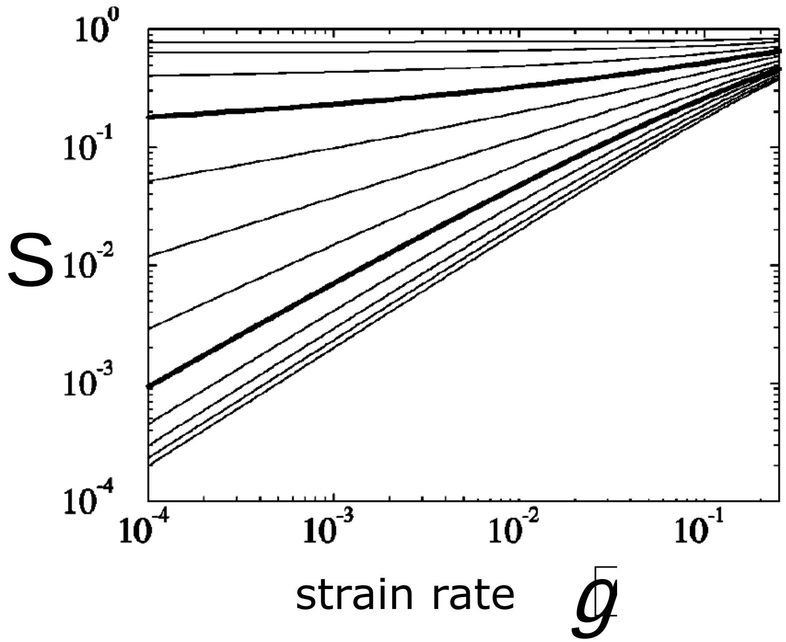


Prepare sample at time $t = 0$

and wait a time t_w before shearing

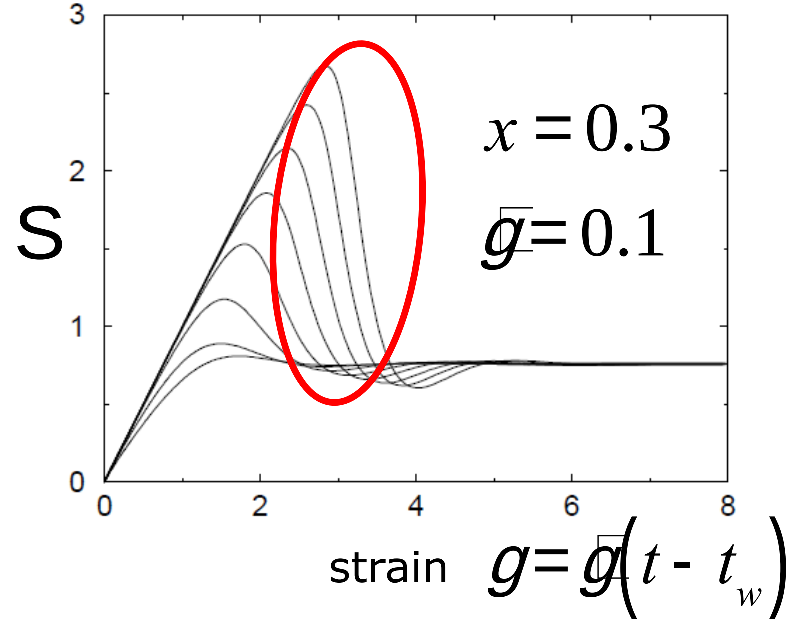
SGR model predictions

Flow curve: yield stress for x < 1



Monotonic - no steady state bands

Overshoot in shear startup



transient banding,

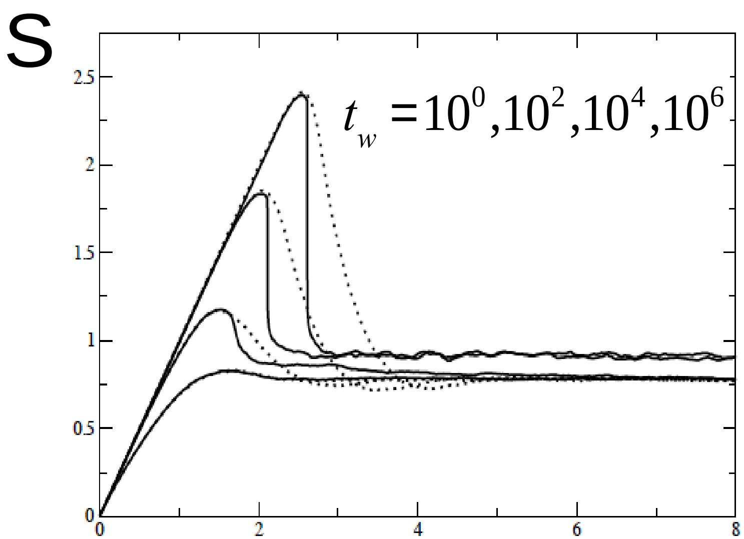
with strong age dependence?

[Moorcroft Cates Fielding PRL 11]

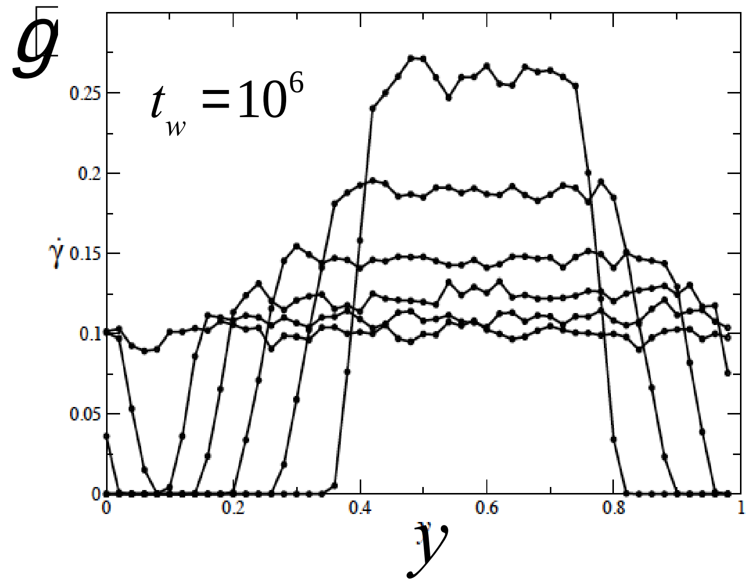
SGR model predictions

Overshoot in shear startup

“transient” bands



strain $\gamma = \dot{\gamma}(t - t_w)$



$\gamma = 38, 75, 125, 175, 225, 275$

persist longer than duration of realistic experiment….?

See also – binary Lennard Jones: Shi et al. PRL 2007; STZ Manning et al PRE 2007. 09

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polymeric fluids

soft glassy materials

general criterion

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large amplitude oscillatory shear

Summary and outlook

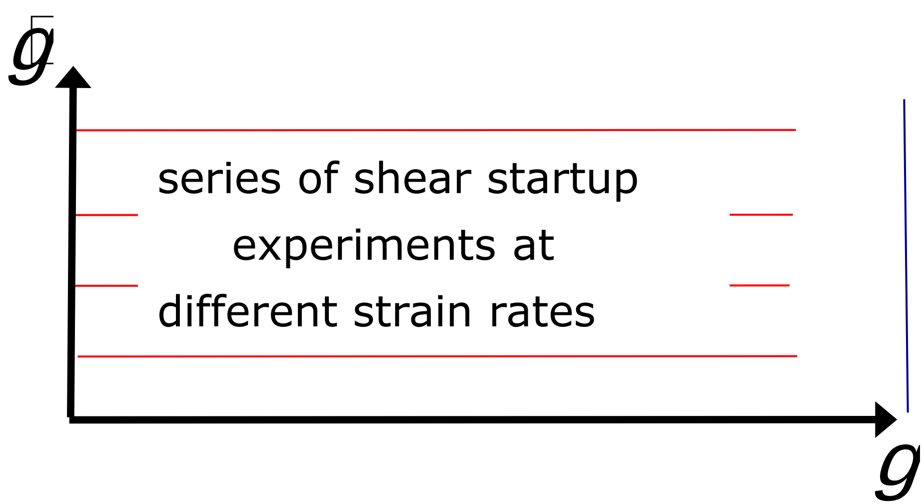
[Moorcroft + Fielding, PRL 2013]

General criterion

Criterion for banding in steady state known $d\Sigma / d\dot{\gamma} < 0$

Criterion for transient banding in startup associated with $d\Sigma / d\gamma < 0$

Consider series of startup runs at different $\dot{\gamma}$ each run to steady state $\gamma \rightarrow \infty$



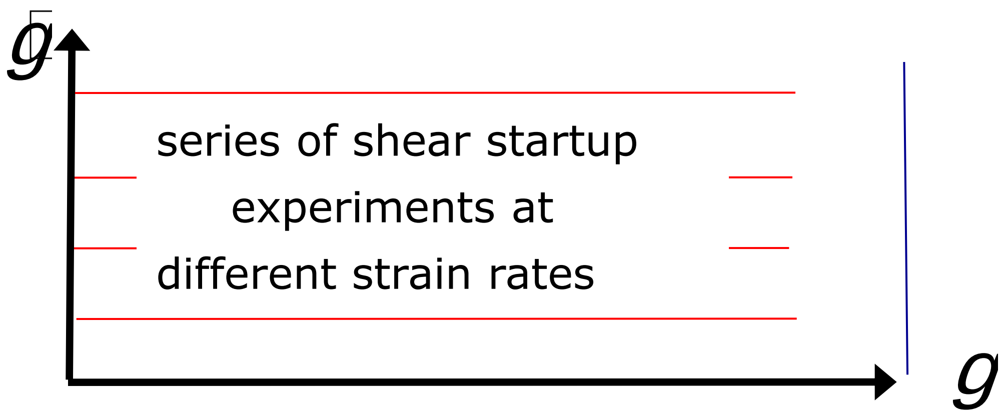
Can we arrive at a general condition on derivatives of $\Sigma(\gamma, \dot{\gamma})$?

[Moorcroft + Fielding, PRL 2013]

Highly generalised theoretical framework of shear rheology

$$\Sigma(t) = \sigma(y,t) + \eta\dot{\gamma}(y,t)$$ force balance in 1D

$$\partial_t \sigma = f(\sigma, \nu, \dot{\gamma}), \quad \partial_t \nu = g(\sigma, \nu, \dot{\gamma})$$ viscoelastic dynamics (rolie-poly, fluidity…)



Linear instability criterion for onset of shear banding

$$\Delta\partial_{\dot{\gamma}}\Sigma \; - \; T\partial_{\gamma}\Sigma + \dot{\gamma}\partial_{\gamma}^{2}\Sigma \; + \; \ldots\partial_{\gamma}\partial_{\dot{\gamma}}\Sigma \; < \; 0$$

"viscous" "elastic"

[Moorcroft + Fielding, PRL 2013]

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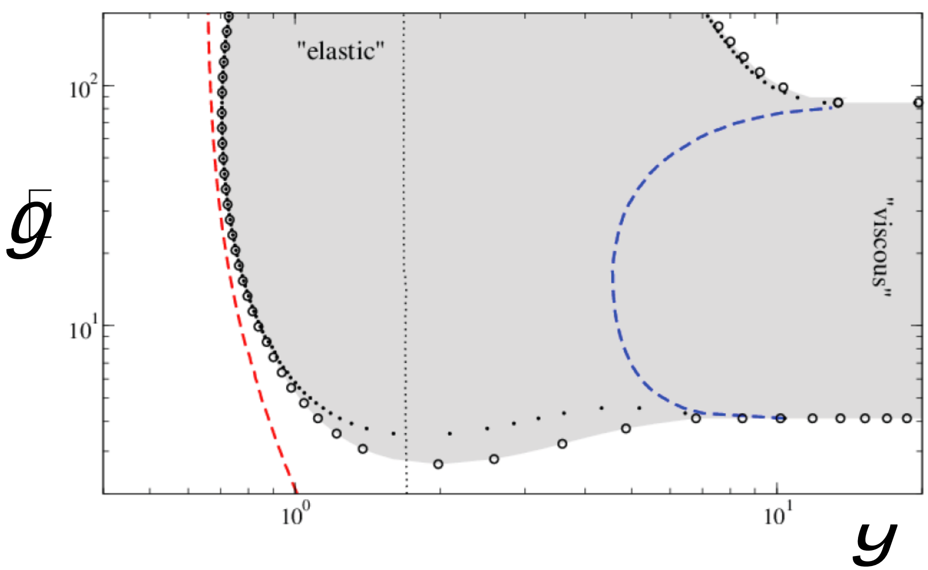

rolie-poly model

non-monotonic constitutive curve

$$D\partial_{\dot{\gamma}}\Sigma - T\partial_{\gamma}\Sigma + \dot{\gamma}\partial^2_{\gamma}\Sigma + \ldots\partial_{\gamma}\partial_{\dot{\gamma}}\Sigma < 0$$

"viscous" "elastic"

[Moorcroft + Fielding, PRL 2013]

Highly generalised theoretical framework of shear rheology

$$\Sigma(t) = \sigma(y,t) + \eta\dot{\gamma}(y,t)$$

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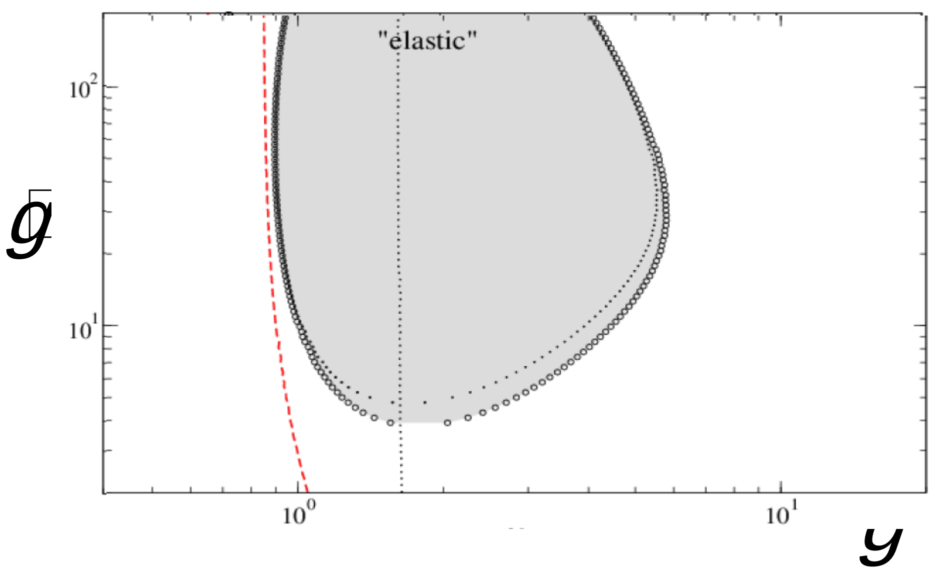
viscoelastic dynamics



rolie-poly model

monotonic constitutive curve

$$D\partial_{\dot{\gamma}}\Sigma - \left(T\partial_{\gamma}\Sigma + \dot{\gamma}\partial_{\gamma}^{2}\Sigma\right) + \ldots\partial_{\gamma}\partial_{\dot{\gamma}}\Sigma < 0$$

"viscous" "elastic"

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[Moorcroft + Fielding, PRL 2013]

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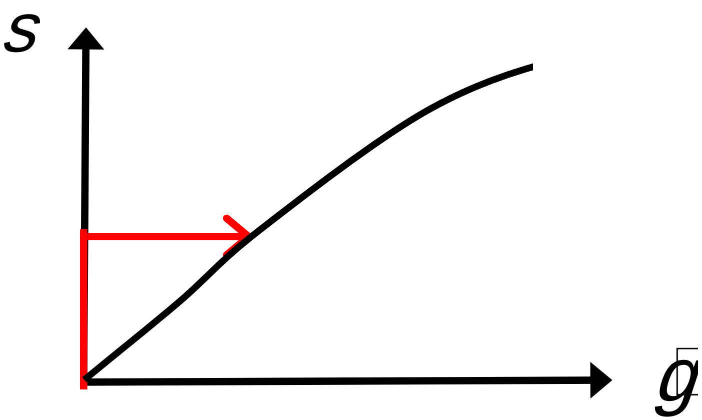
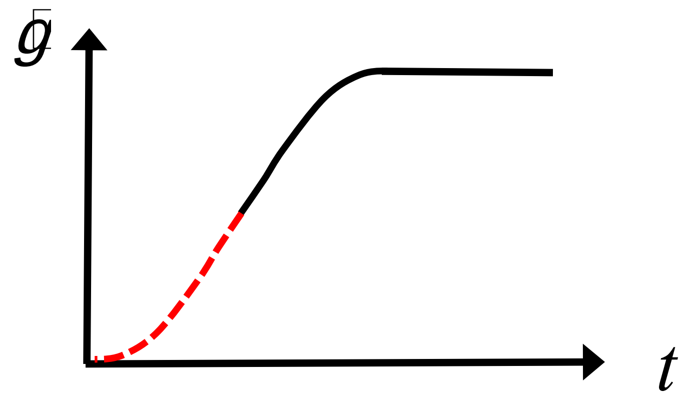




$$\Sigma(t) = \sigma(y,t) + \eta\dot{\gamma}(y,t)$$

$$\partial_t \sigma = f(\sigma, \nu, \mu, \ldots, \dot{\gamma}),$$

$$\partial_t \nu = \ldots, \partial_t \mu = \ldots$$

force balance in 1D

viscoelastic dynamics

highly general

(Rolie Poly, SGR)

Linear instability criterion

for onset of shear banding

$$\partial_t^2 \dot{\gamma} / \partial_t \dot{\gamma} > 0$$

Outline

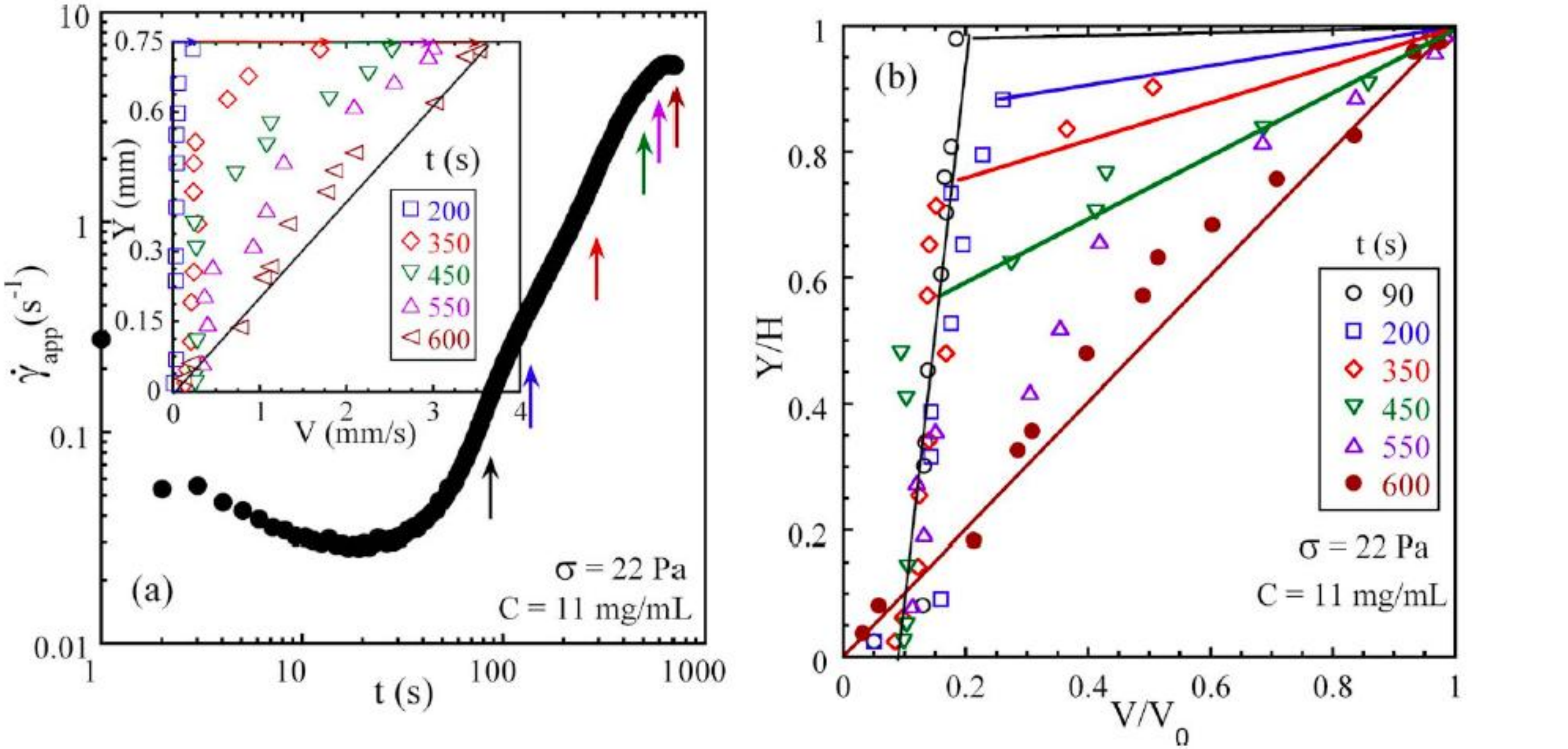
Shear banding in time-dependent flow protocols

- shear startup
- step stress protocol:
  - general criterion
  - polymeric fluids
  - soft glassy materials
- step strain
- large amplitude oscillatory shear

Summary and outlook

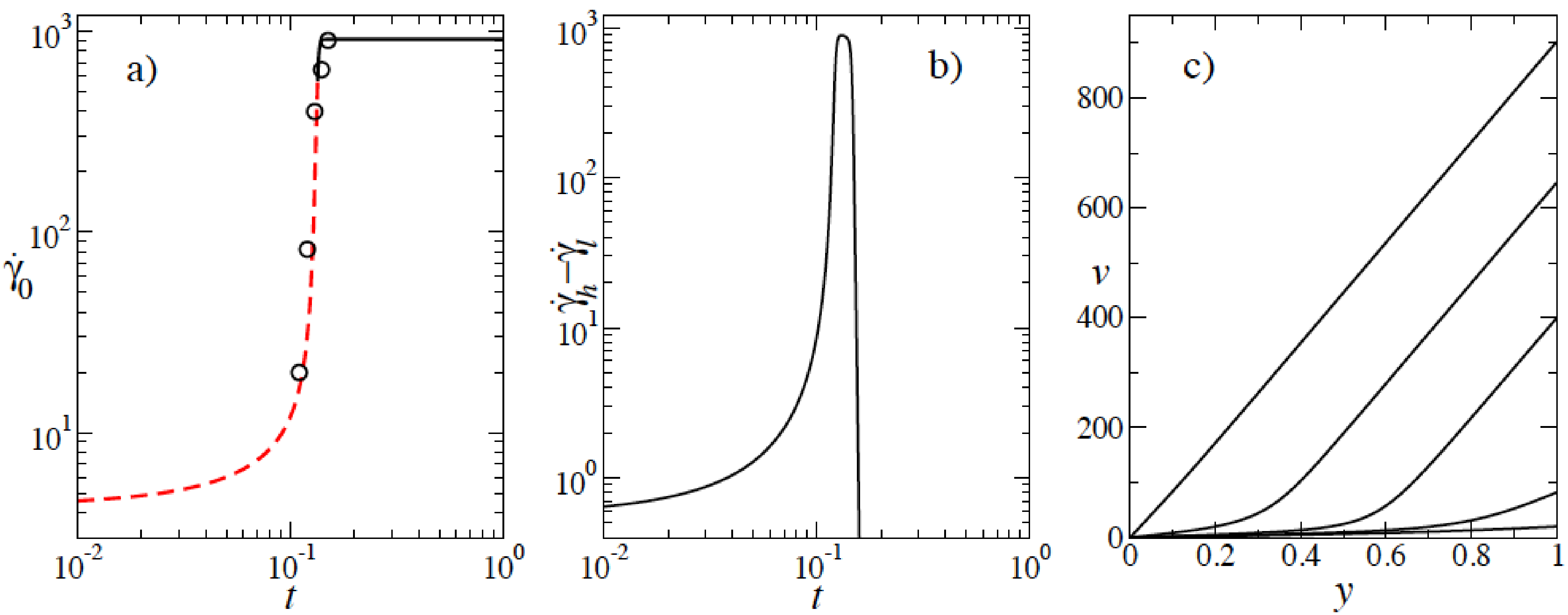
[Boukany and Wang J Rheol 2008]

Shear banding in step stress protocol: polymer experiments



Consistent with prediction $\partial_t^2 \bar{\dot{\gamma}} / \partial_t \bar{\dot{\gamma}} > 0$

Shear banding in step stress protocol: Rolie Poly model



Consistent with prediction $\partial_t^2 \dot{\gamma} / \partial_t \dot{\gamma} > 0$

Outline

Shear banding in time-dependent flow protocols

shear startup

step stress protocol: general criterion

polymeric fluids

soft glassy materials

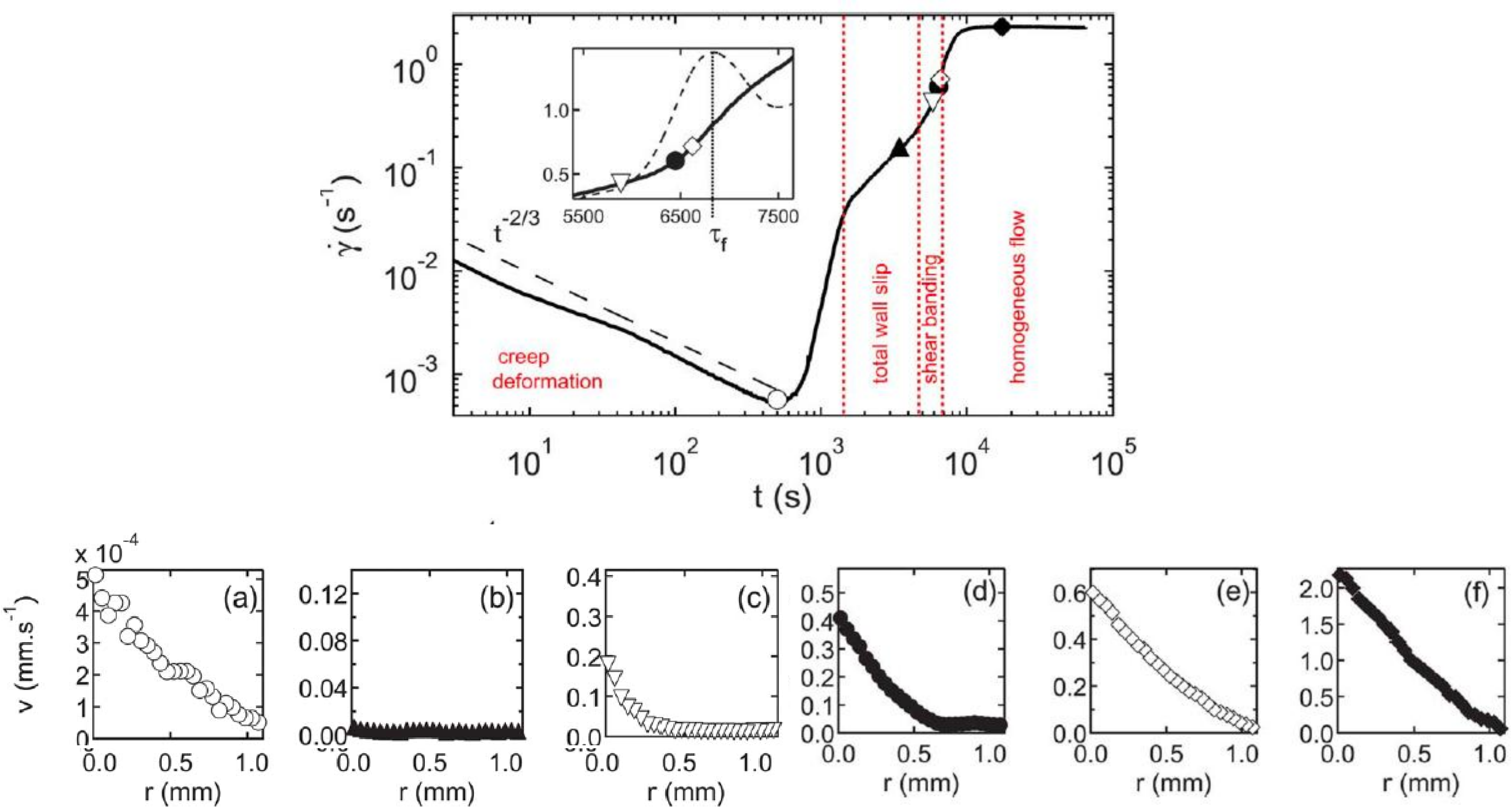
step strain

large amplitude oscillatory shear

Summary and outlook

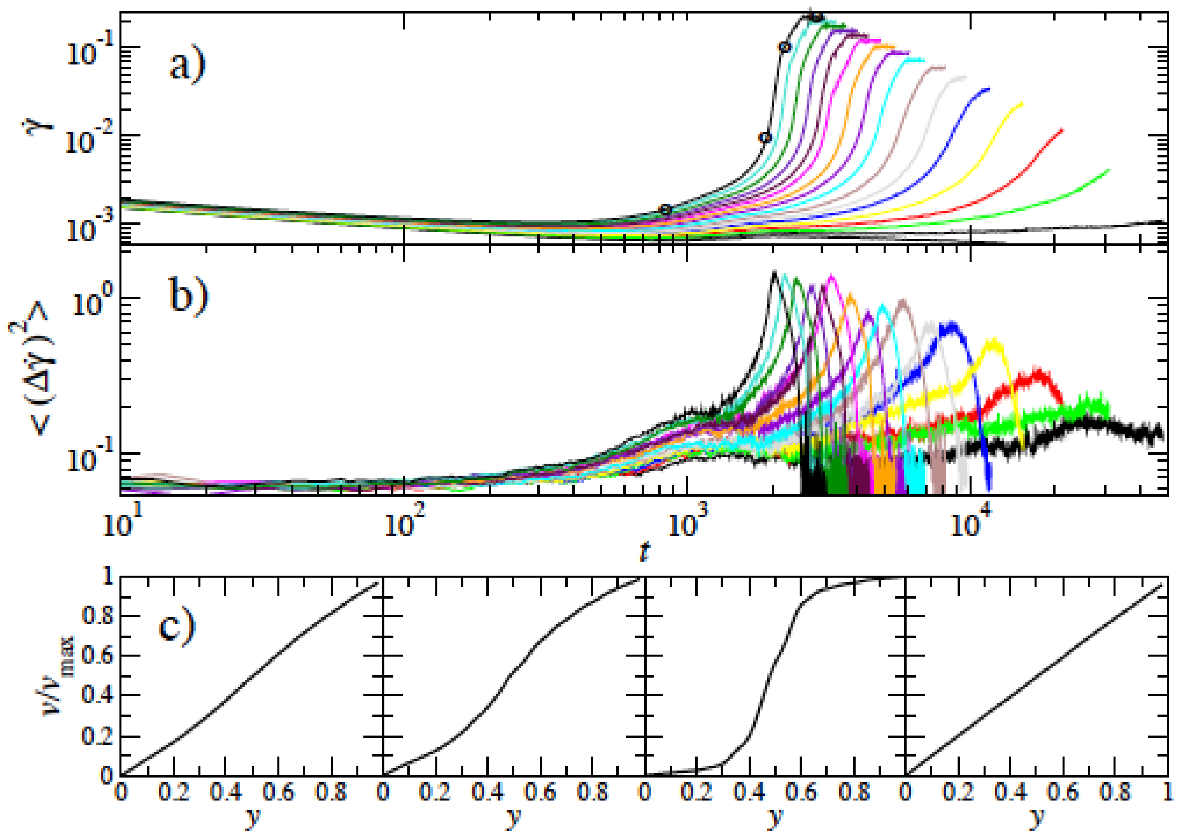
[Divoux, Barentin, Manneville, Soft Matter, 2011]

Shear banding in step stress (creep) protocol: soft glassy material



[Moorcroft + Fielding PRL 2013]

Shear banding in step stress (creep) protocol: SGR model



Consistent with prediction $\partial_t^2 \dot{\bar{\gamma}} / \partial_t \dot{\bar{\gamma}} > 0$

Outline

Shear banding in time-dependent flow protocols

- shear startup
- step stress
- step strain protocol: Moorcroft + SMF JoR 2014
- large amplitude oscillatory shear

Summary and outlook

Outline

Shear banding in time-dependent flow protocols

- shear startup
- step stress
- step strain protocol: Moorcroft + SMF JoR 2014
- large amplitude oscillatory shear (LAOS)

Summary and outlook

LAOS: protocol with a sustained time dependence

- Shear startup and step stress are only transiently time-dependent,

  so any associated shear banding is also only transient

- Consider now flows with a sustained time-dependence,

  (of which LAOS is a simple - and much-studied- example)…

  Question - Do we see sustained shear banding here,

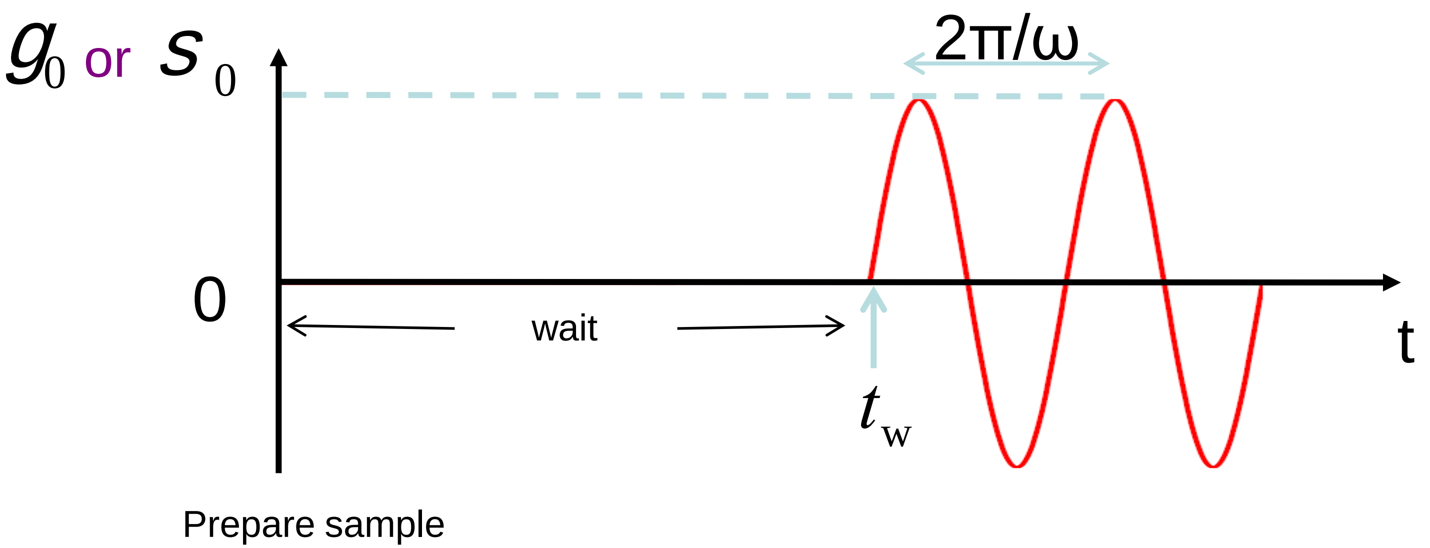
  arising simply due to the time-dependence of the flow,

  even in fluids that don't support bands in steady state?

LAOS: defining the protocol

Prepare sample at time t=0, then age it for waiting time t_w

Execute many cycles of either LAOStrain or LAOStress



After many cycles, response independent of cycle number and initial t_w

Any run prescribed fully by (γ_0, ω) or $(\dot{\gamma}_0, \omega)$ or (σ_0, ω)

LAOS: the basic intuition

- LAOStrain:

  a bit like a repeating series of forward and reverse startup runs

  expect banding associated with stress overshoot in each half cycle?

- LAOStress:

  a bit like a repeating series of positive and negative stress steps

  expect banding associated with yielding in each half cycle?

Outline

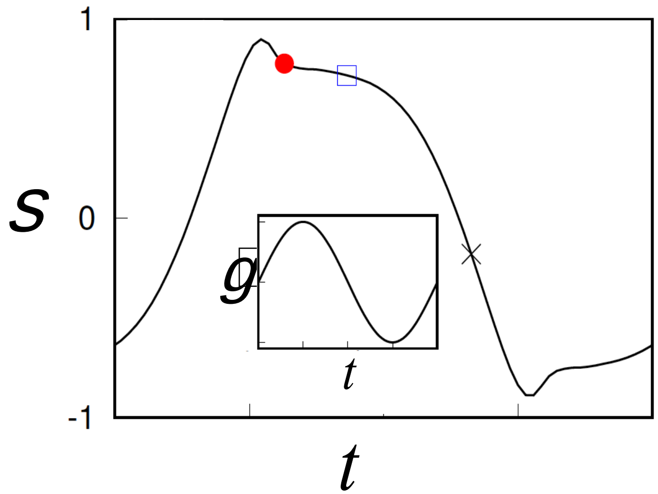
Shear banding in time-dependent flow protocols

- shear startup
- step stress
- step strain: Moorcroft + SMF JoR 2014
- large amplitude oscillatory shear
  - soft glassy materials
  - polymeric fluids

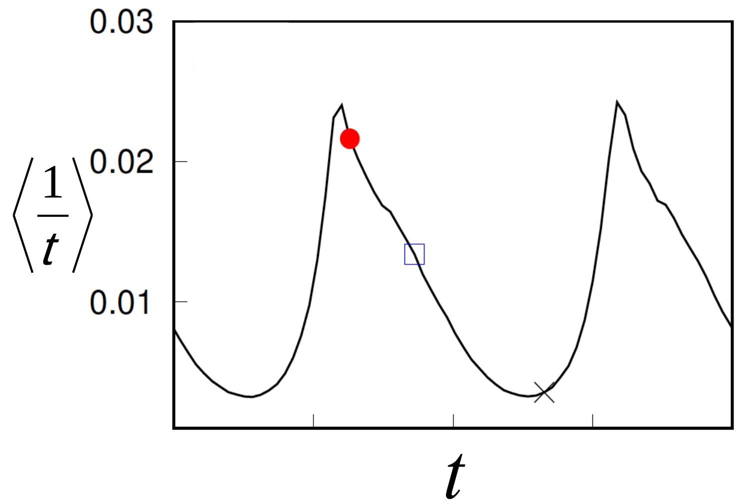
Summary and outlook

Large amplitude oscillatory shear strain (LAOStrain)

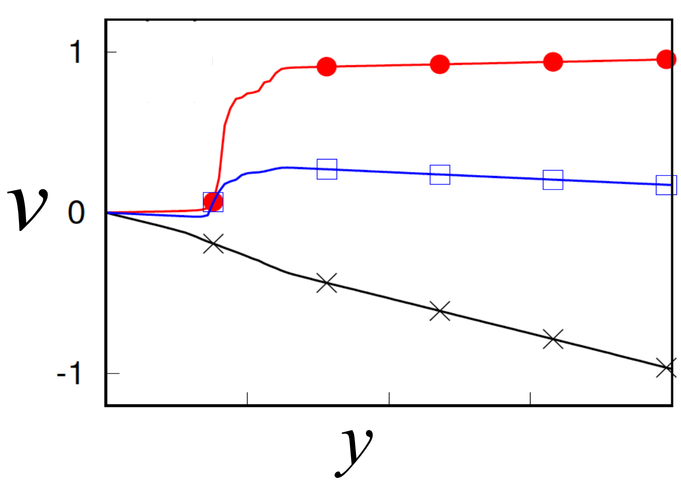
stress over cycle



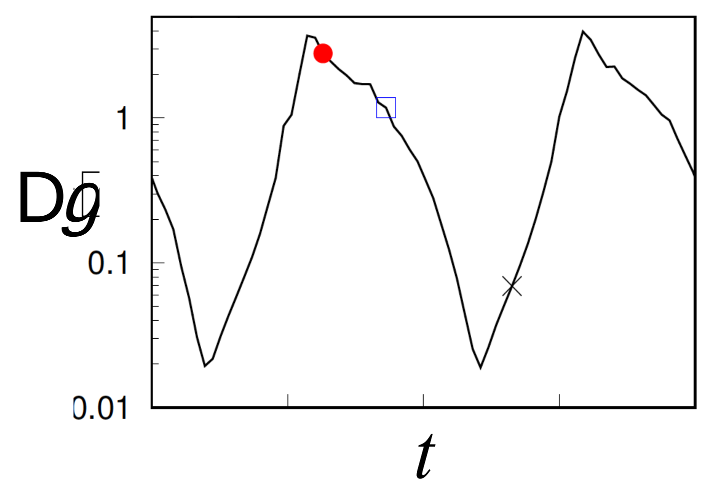
inverse `age' over cycle



snapshot banded profiles

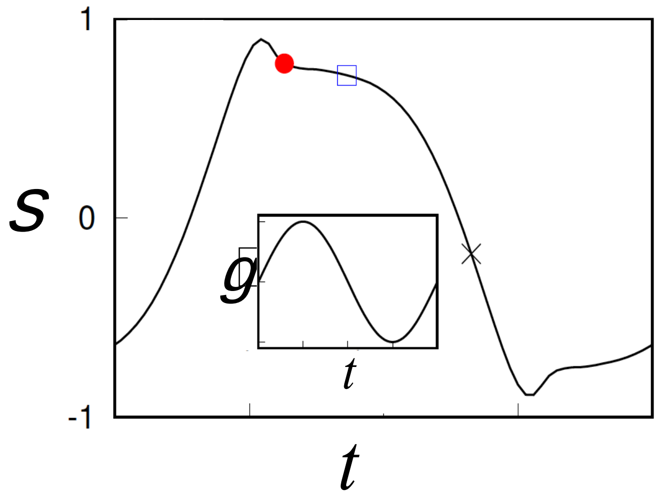


banding over cycle

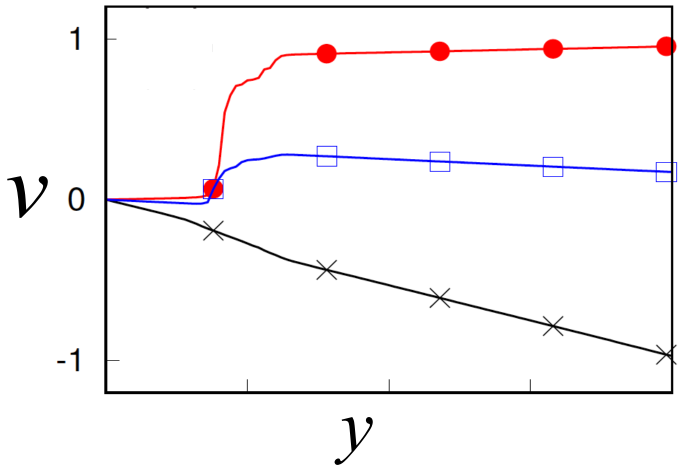


Large amplitude oscillatory shear strain (LAOStrain)

stress over cycle



snapshot banded profiles



<u>Confirms basic intuition:</u>

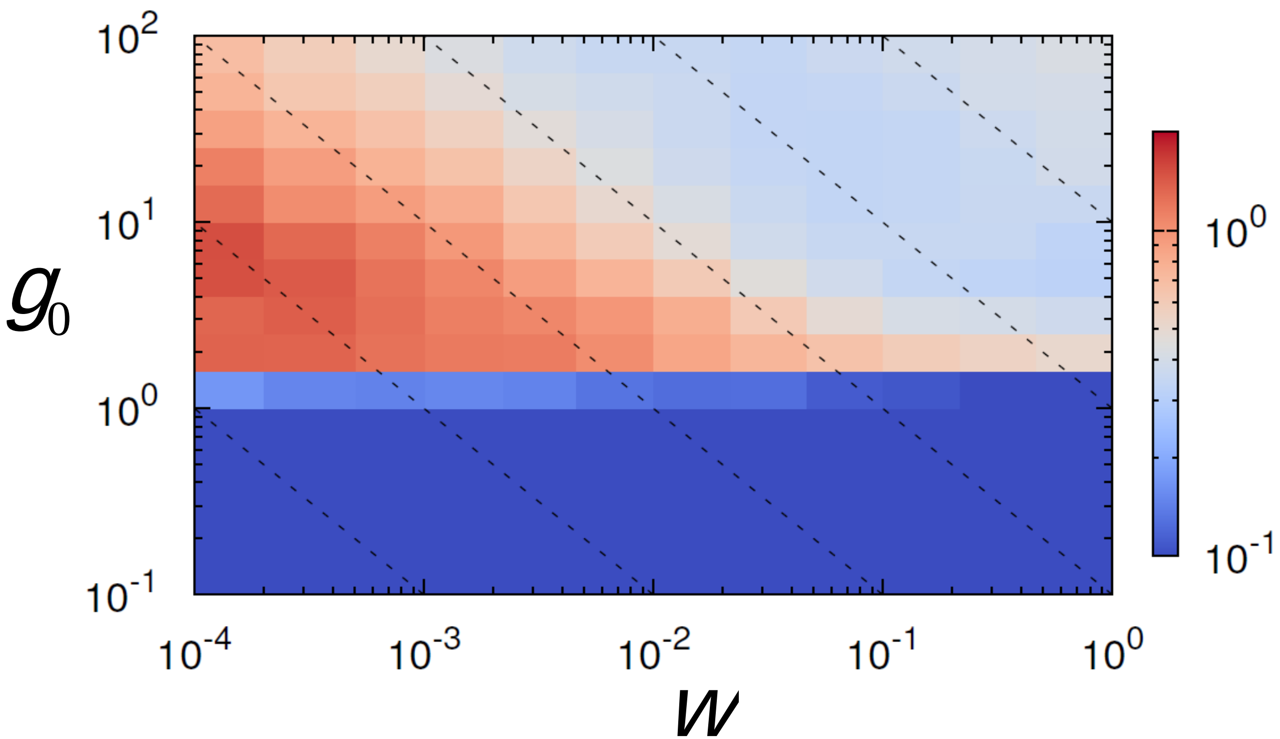
- see banding in LAOStrain
- associated with stress overshoot
- repeated competation over cycle

  between aging and `rejuvenation'

[Radhakrishnan + Fielding PRL 2017, J. Rheol 2018]

LAOStrain: dynamic phase diagram

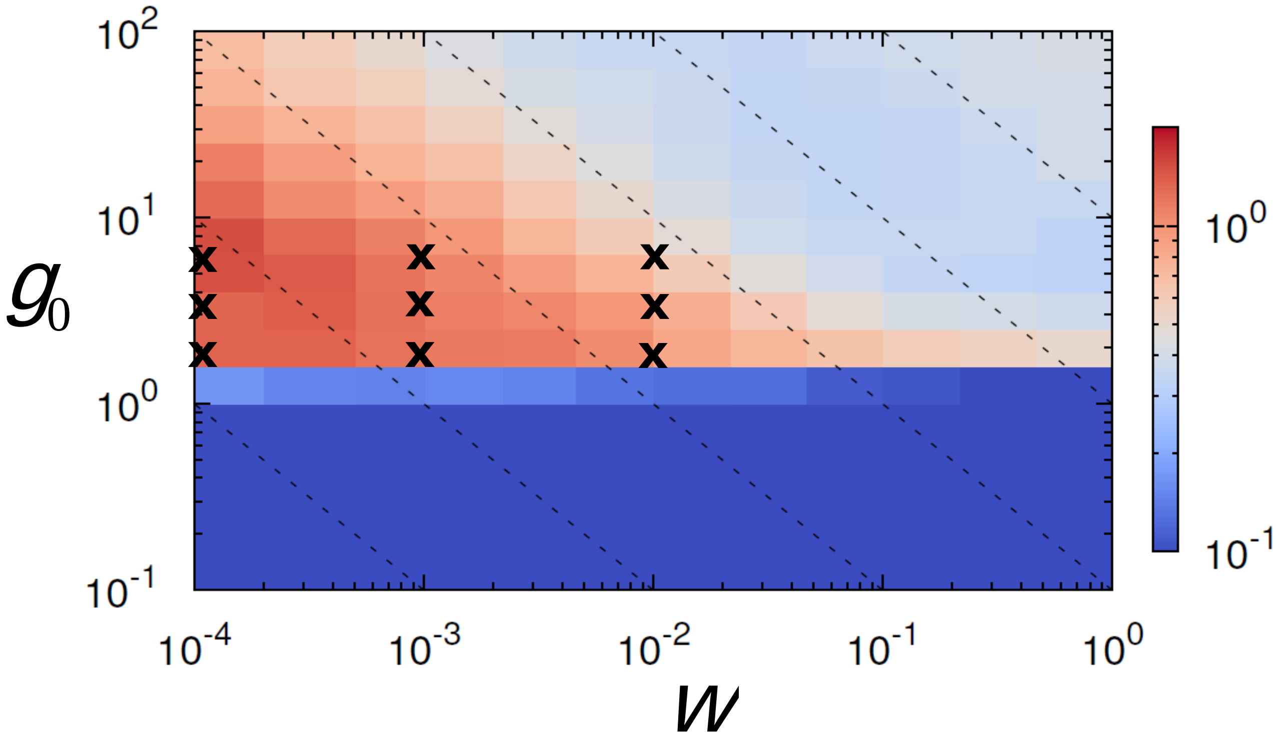
Cycle averaged degree of banding as a function of amplitude and frequency of imposed strain oscillation



[Radhakrishnan + Fielding PRL 2017, J. Rheol 2018]

LAOStrain: dynamic phase diagram

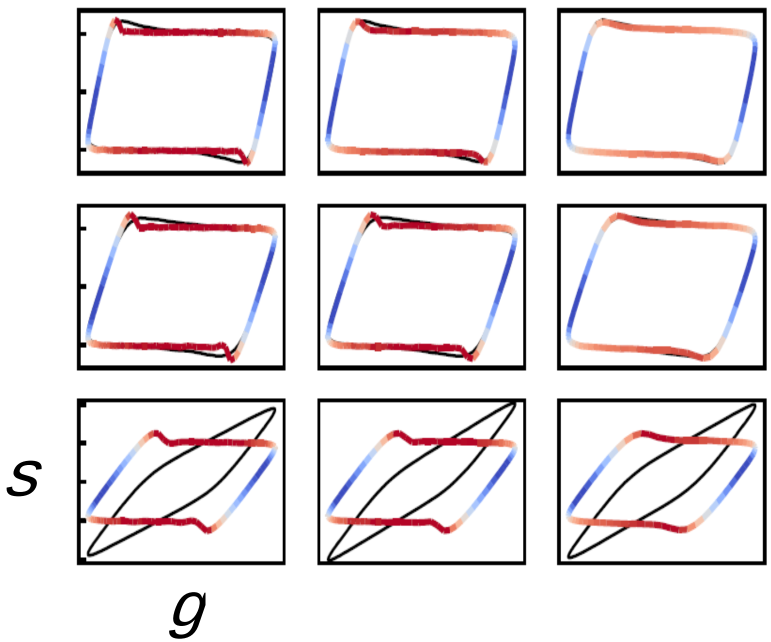
Cycle averaged degree of banding as a function of amplitude and frequency of imposed strain oscillation



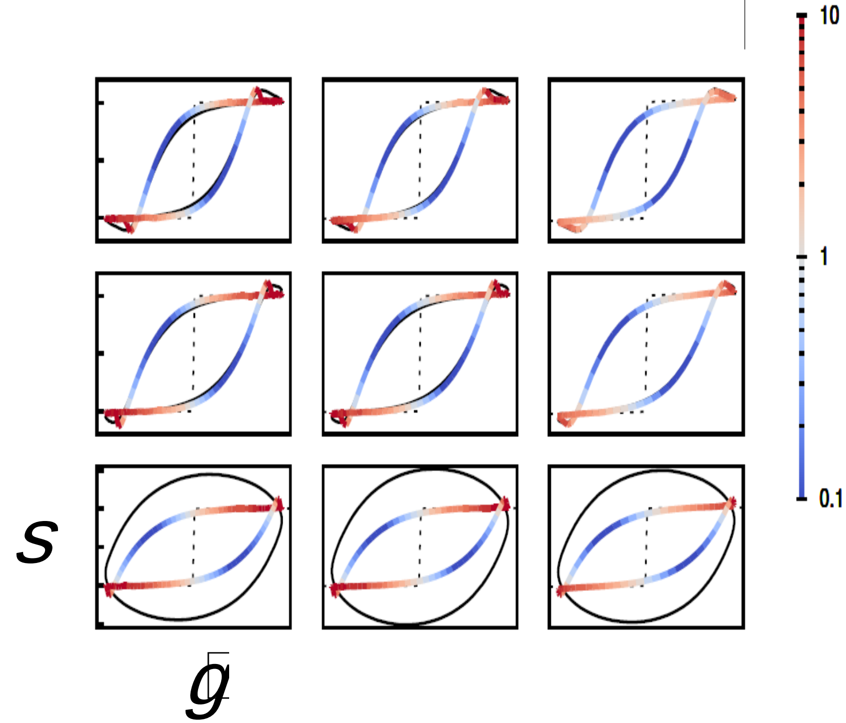
[Radhakrishnan + Fielding PRL 2017, J. Rheol 2018]

Pipkin diagrams: grid of Lissajous-Bowditch curves

Elastic: stress vs strain round cycle



Viscous: stress vs strain rate

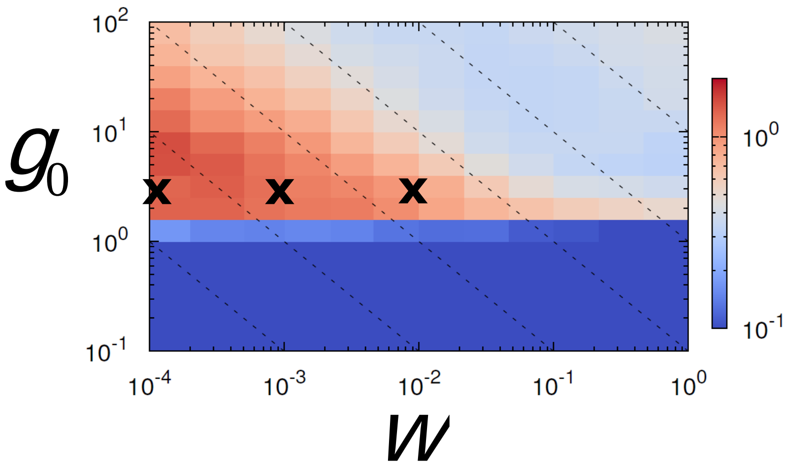


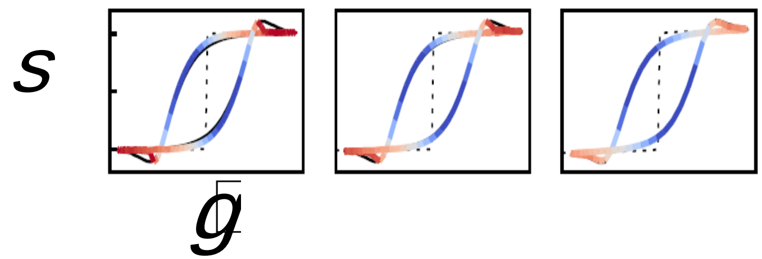
Colourscale: degree of banding round cycle

Important: shape of curve strongly changed by banding

[Radhakrishnan + Fielding PRL 2017, J. Rheol 2018]

Still see banding even in zero-frequency limit $w \to 0$

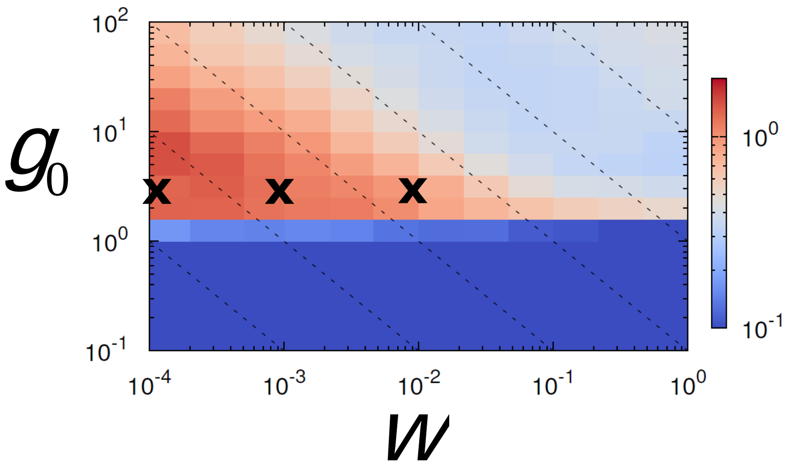


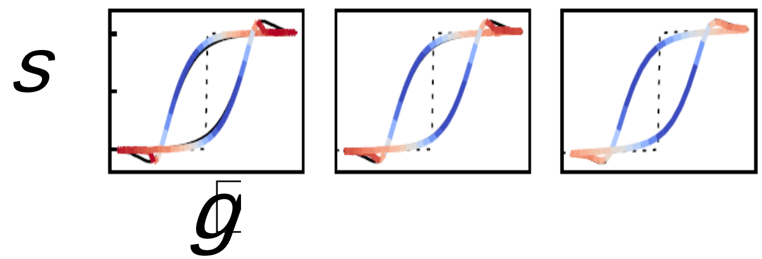


Puzzle: In zero frequency limit might a priori expect quasistatic sweeps up and down steady state flow curve with no banding (because flow curve is monotonic)

[Radhakrishnan + Fielding PRL 2017, J. Rheol 2018]

Still see banding even in zero-frequency limit $\omega \to 0$





Puzzle: In zero frequency limit might a priori expect quasistatic sweeps up and down steady state flow curve with no banding (because flow curve is monotonic)

Resolution: ageing glassy material has no fixed inverse relaxation time against which can set frequency to be small $\omega\tau \to 0$
Instead: repeating ageing and rejuvenation in each cycle

Same in: square/triangular/sawtooth strain and LAOStress

Outline

Shear banding in time-dependent flow protocols

shear startup

step stress

step strain: Moorcroft + SMF JoR 2014

large amplitude oscillatory shear (LAOS)

soft glassy materials

polymeric fluids

Summary and outlook

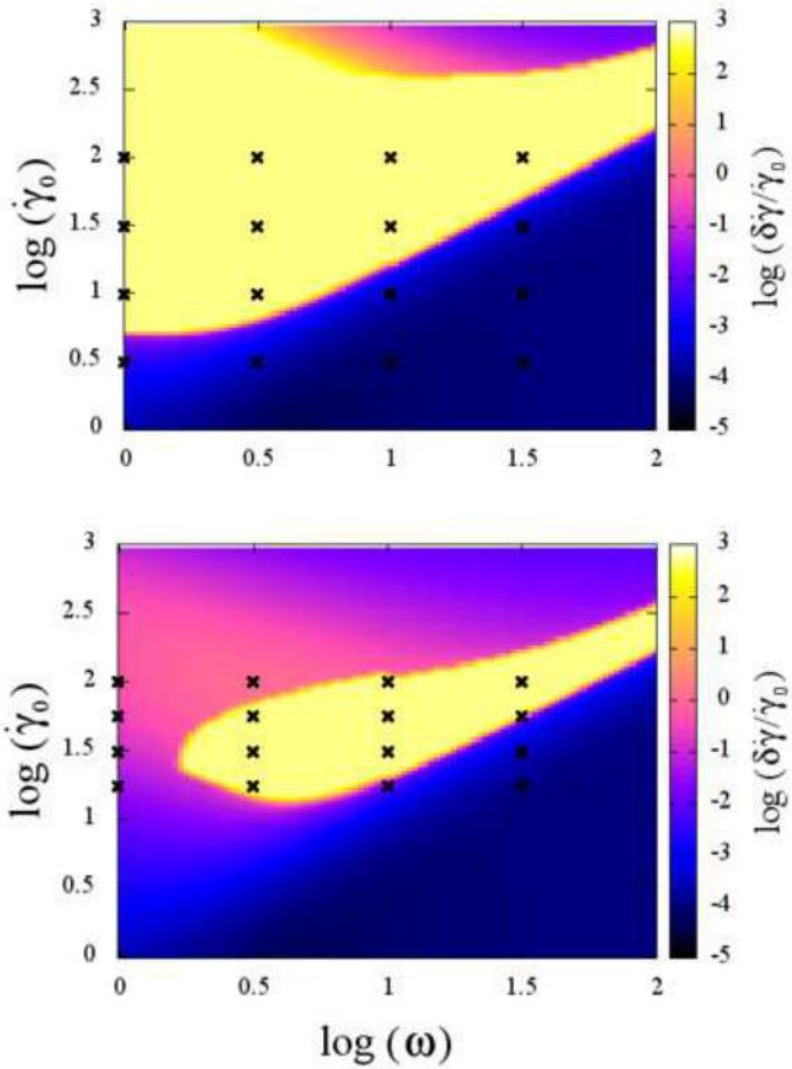
[Carter, Girkin, Fielding, JoR 2016; see also Adams + Olmsted PRL 2009]

Large amplitude oscillatory shear strain: Rolie-poly model

Low frequency
shear banding

high frequency
'elastic' banding

<u>Non-monotonic</u>
underlying
constitutive curve



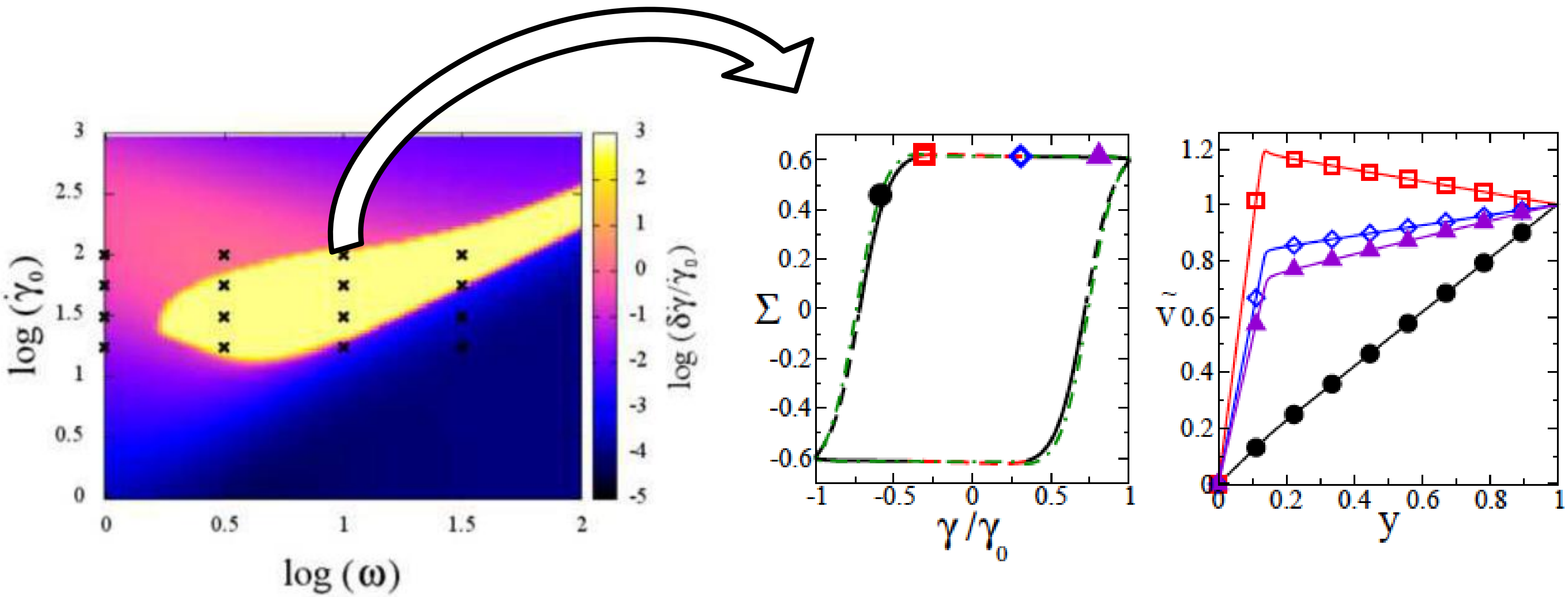
high frequency
'elastic' banding
only

<u>Monotonic</u>
underlying
constitutive curve

[Carter, Girkin, Fielding, JoR 2016; see also Adams + Olmsted PRL 2009]

Large amplitude oscillatory shear strain: Rolie-poly model

Monotonic underlying constitutive curve   high frequency ‘elastic’ banding



Summary - polymeric fluids, soft glasses (and all else?) predicted to band:

- Transiently in shear startup, associated with stress overshoot

- Transiently following imposition of step stress, as sample yields

  note: in (soft) glasses this 'transience' likely appears permanent and/or

  in (hard) glasses might 'break' the sample – game over anyway

- In a sustained way in time-periodic flow protocols

- Do complex fluids have a generic predisposition to form shear bands

  in flows with a strong enough time dependence ?

- Do glasses have a generic predisposition to form shear bands

  in flows of even arbitrarily slow time dependence ?

Durham University

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erc

A couple of review articles (SMF):

Summarising results in soft glasses:

Report on Progress in Physics, 2014

Summarising criteria in general:

Journal of Rheololgy, 2016