Shear-induced inhomogeneities in concentrated suspensions

Guillaume Ovarlez, LOF, CNRS-Solvay-Univ. Bordeaux

with contributions from

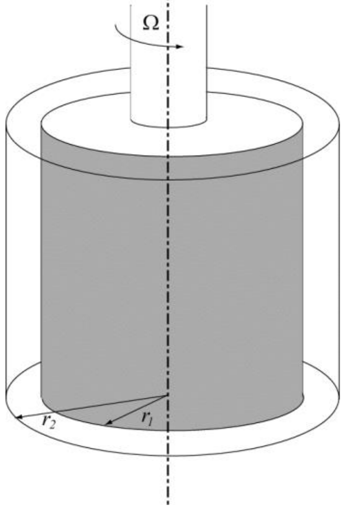
Elisabeth Guazzelli

Abdoulaye Fall, Daniel Bonn & Anaël Lemaître

Guillaume Chatté, Nicolas Lenoir & Annie Colin

From rheometry to physics…

Rheometry



Constitutive law:

$$\boxed{\tau(\dot{\gamma})}$$

nature

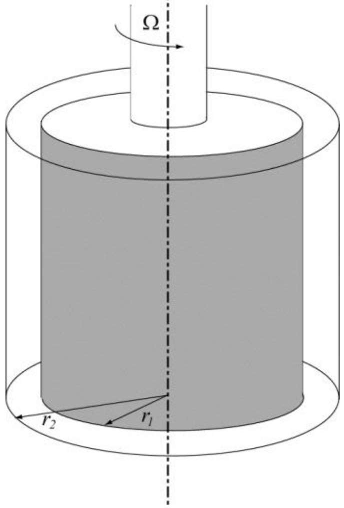
Science

AAAS

PHYSICS

From rheometry to physics...

Rheometry



The (boring?) rheologist question

Constitutive law:

$$\boxed{\tau(\dot{\gamma})}$$

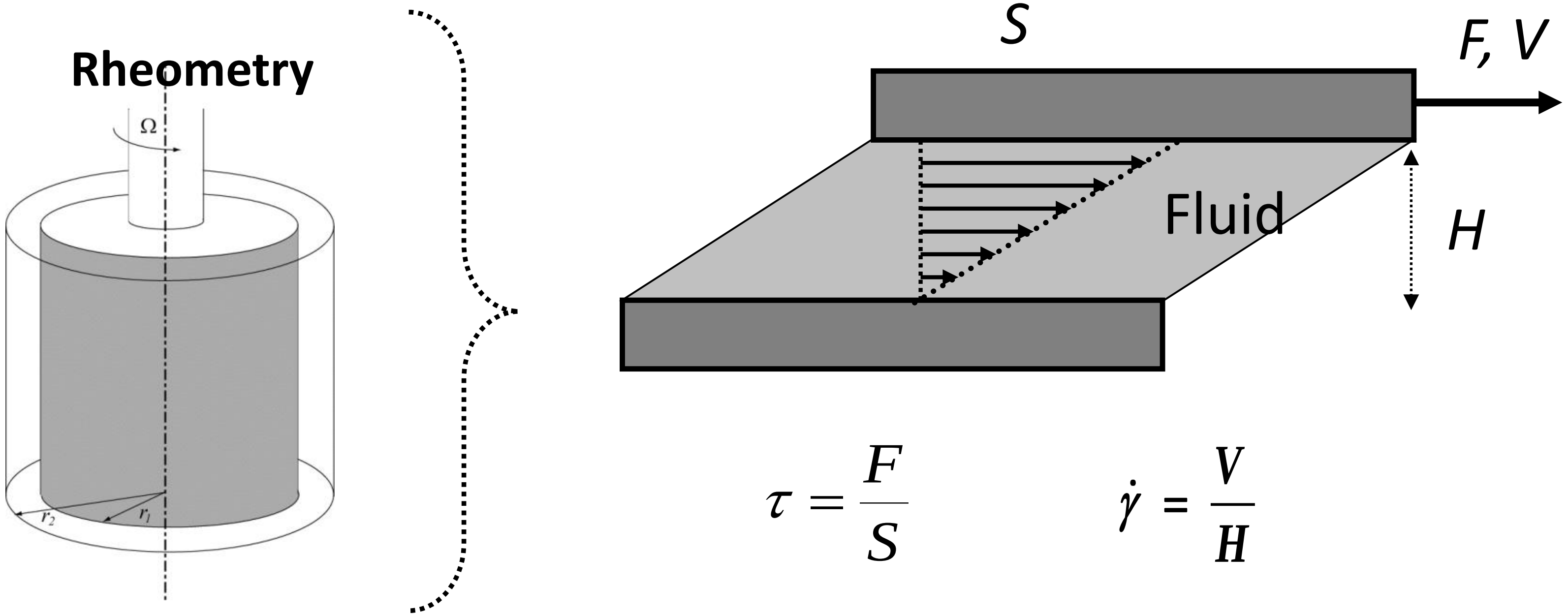
nature

Science

AAAS

PHYSICS

From rheometry to physics…



$$\tau = \frac{F}{S} \qquad \dot{\gamma} = \frac{V}{H}$$

***Macroscopic** measurement:*

F(V)

(or T(Ω))

The rheologist question →

Hypotheses :

- **homogeneous shear**
- **homogeneous material**

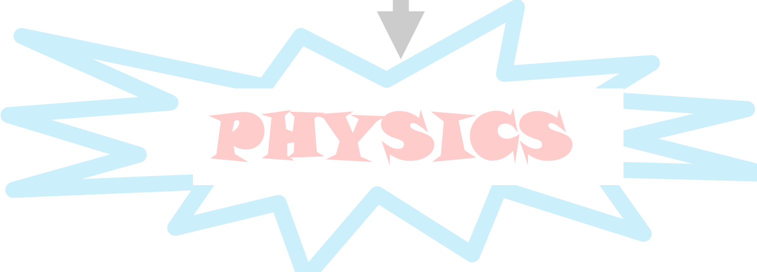
Constitutive law?????

$$\boxed{\tau(\dot{\gamma})}$$

nature

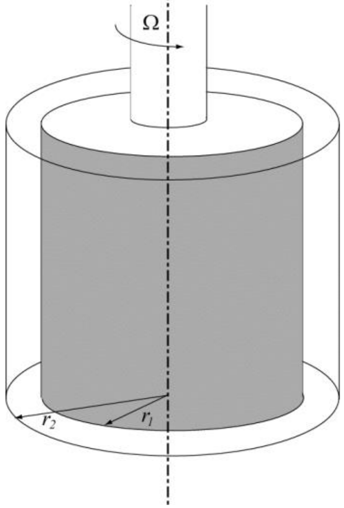
Science

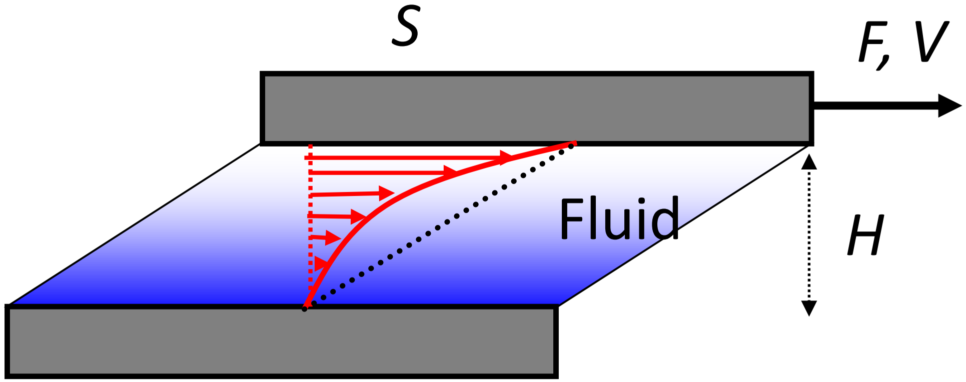
AAAS



From rheometry to physics…

Rheometry





$$\tau = \frac{F}{S}$$

$$\cancel{\dot{\gamma} = \frac{V}{H}}$$

The rheologist question

***Macroscopic** measurement:*

F(V)

(or T(Ω))

?

Hypotheses :

- ~~**homogeneous shear**~~
- ~~**homogeneous material**~~

***Constitutive** law:*

$$\tau(\dot{\gamma}, x)$$

Bad evaluation of shear rate

Bad knowledge of the material

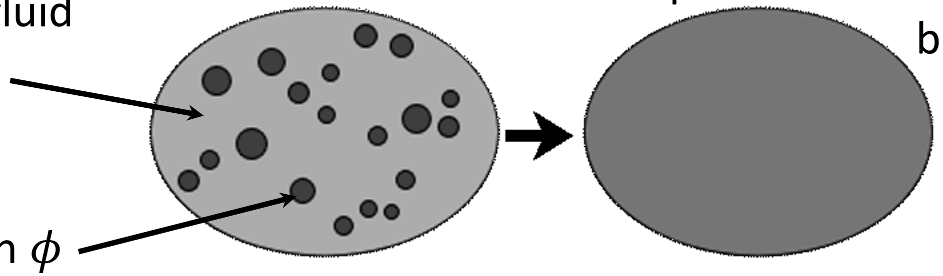
JOURNAL OF RHEOLOGY®

Shear-induced migration in viscous suspensions

Shear-induced migration in shear-thickening suspensions:
(1) Role of interparticle contacts

Shear-induced migration in shear-thickening suspensions
(2) Jamming

Macroscopic behavior of viscous nonBrownian suspensions



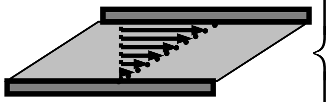
Newtonian fluid

$\tau = \eta_0 \dot{\gamma}$

volume fraction ϕ

Suspension : **linear** material

but **non-Newtonian**

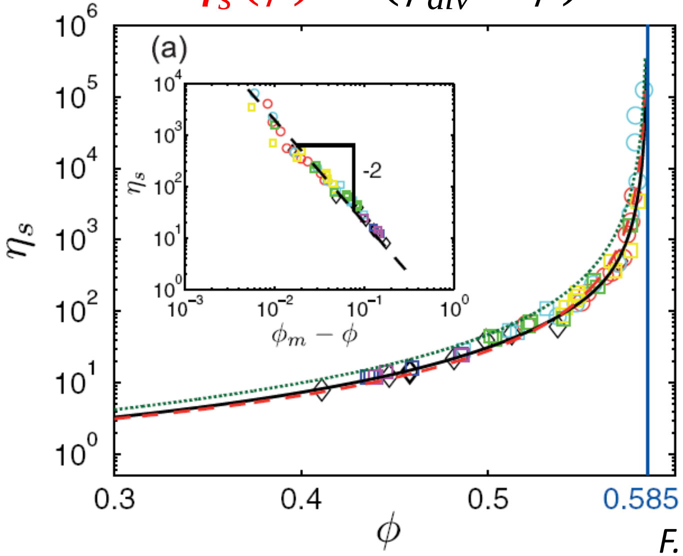


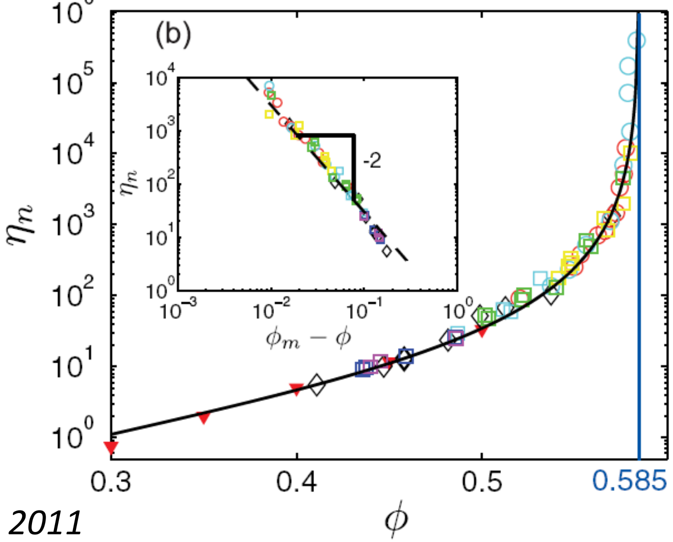
shear stress: $\tau = \eta_s(\phi)\eta_0\dot{\gamma}$

normal stresses differences: $N_1, N_2 \propto \eta_n(\phi)\eta_0\dot{\gamma}$

$\eta_s(\phi) \propto (\phi_{div} - \phi)^{-2}$

$\eta_n(\phi) \propto (\phi_{div} - \phi)^{-2}$



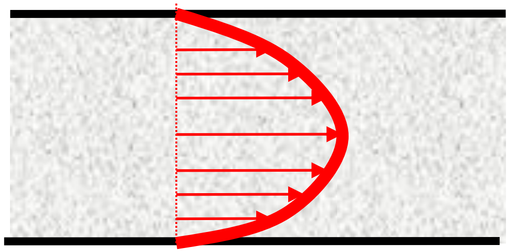


F. Boyer et al., PRL 2011

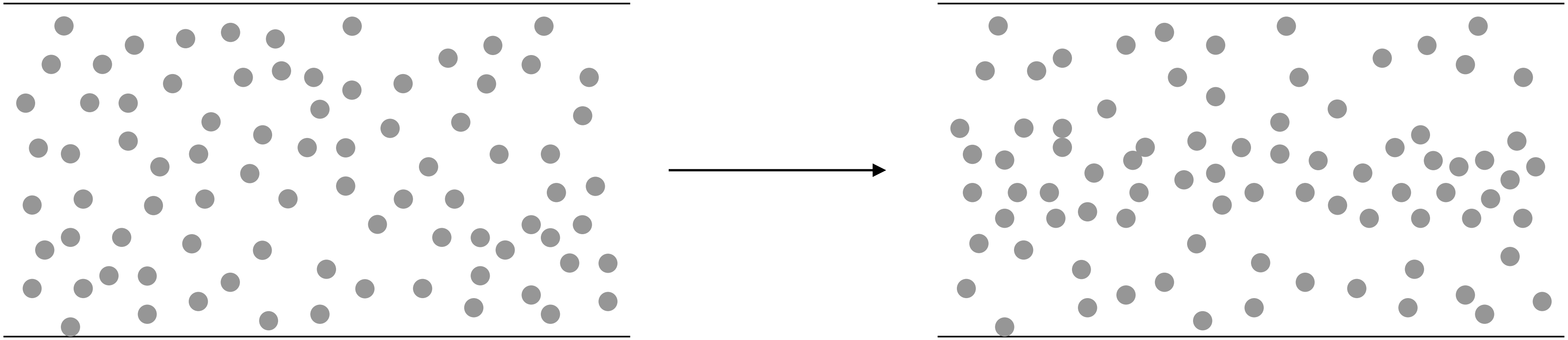
Migration in viscous noncolloidal suspensions

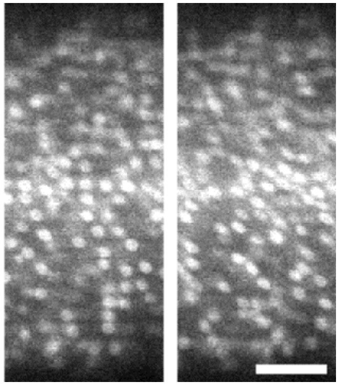
Leighton & Acrivos (1987), Phillips et al. (1992), …

Ex: pipe flow



Migration towards zones of low shear rate





Frank et al. (2003)

⇒ Need for a **diphasic description**

Shear-induced migration and normal stresses

diphasic description: $\overset{\text{suspension}}{\Sigma_{ij}} = \overset{\text{fluid}}{\sigma_{ij}^{f}} + \overset{\text{particles}}{\sigma_{ij}^{p}}$

Lhuillier (2009):
the relevant stress
driving migration is the
contact stress

stress equilibrium on the **particle phase**:

if $\nabla\boldsymbol{\sigma}^{p} \neq \mathbf{0} \rightarrow$ fluid filtration ; $\nabla\boldsymbol{\sigma}^{p}$ balanced by drag force

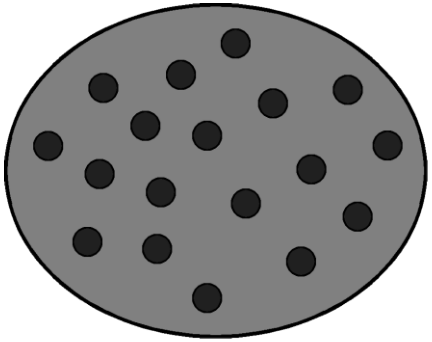
mass conservation → kinetic equation for the particle volume fraction

Nott and Brady, JFM (1994)
Mills and Snabre, J. Phys. II (1995)
Morris and Boulay, J. Rheol. (1999)
Lhuillier, Phys. Fluids (2009)
Nott et al., Phys. Fluids (2011)

Experimental setup

- PS beads : ø 140 µm, $\rho = 1.05$ g.cm^{-3}
- PEG: $\eta = 2.15$ Pa.s, $\rho = 1.05$ g.cm^{-3}

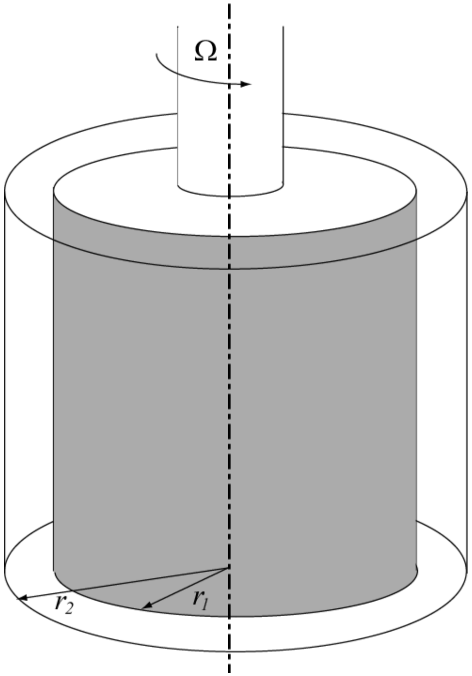
Volume fraction : from 5 to 56%



Same system as Boyer et al. 2011
and Garland et al. 2013
→ macroscopic behavior fully characterized
→ no free parameter for the model…

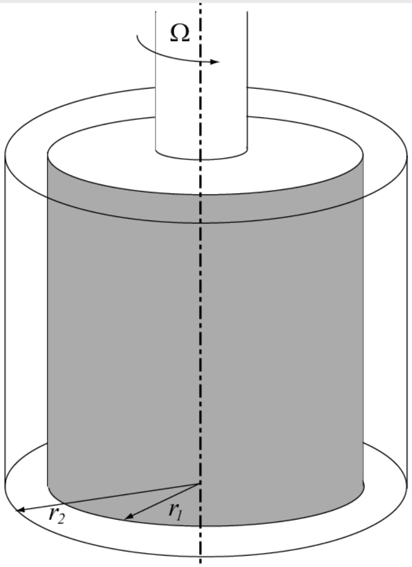
Geometry:

Wide gap Couette (R_{inner}=3cm, R_{outer}=5cm)

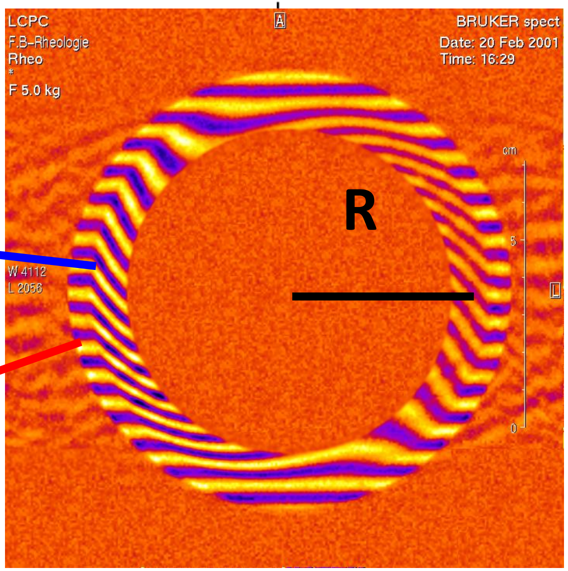


Experimental setup

MRI
Lab. Navier
Champs sur Marne



View from above



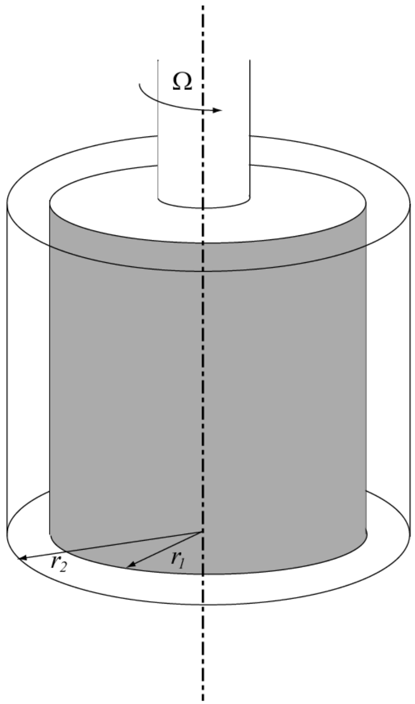
Torque measurement T

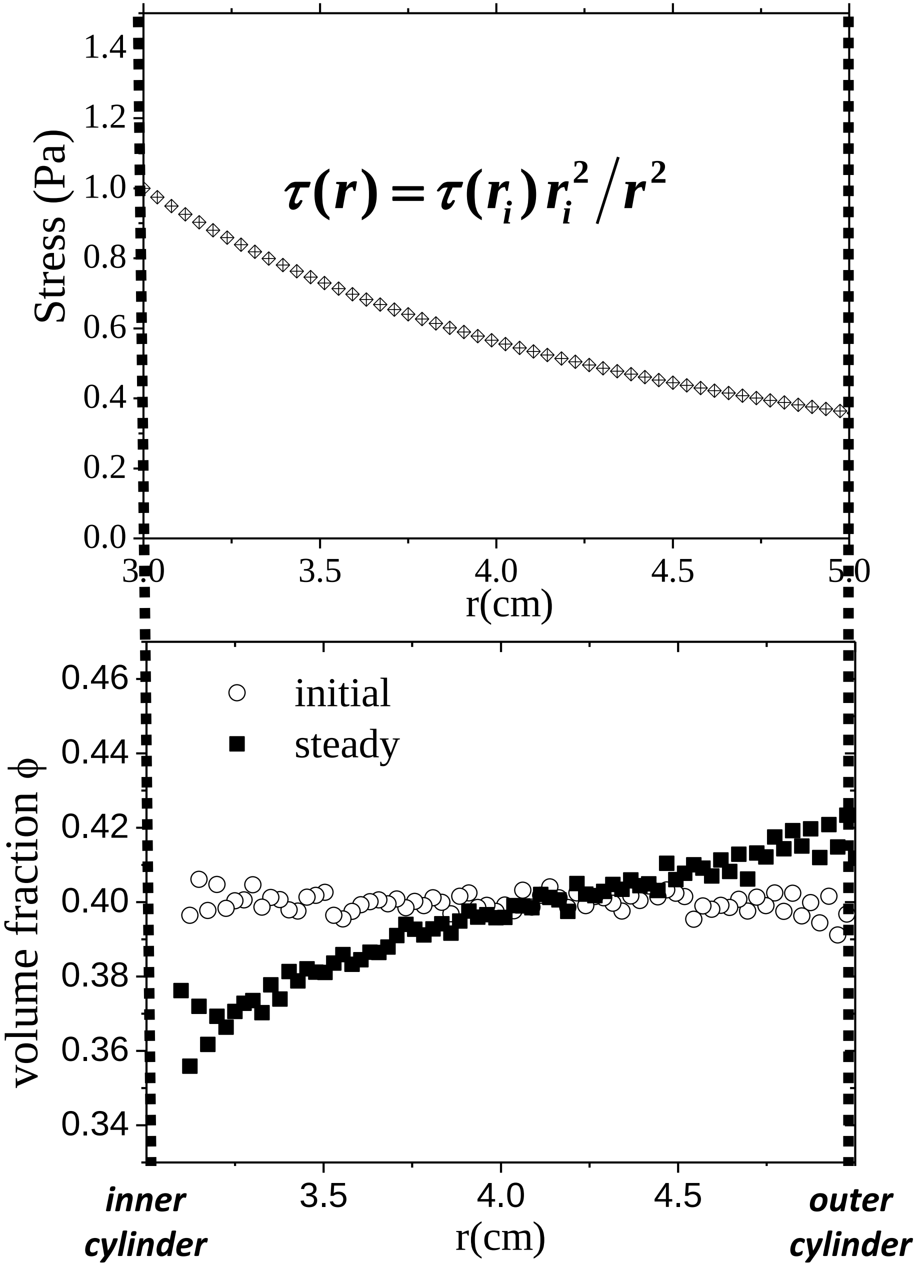
Velocity profile ***$V(R,t)$*** measurement
→ kinetics

Volume fraction profile $\phi(R)$ measurement
→ stationary profile

Steady-state: volume fraction

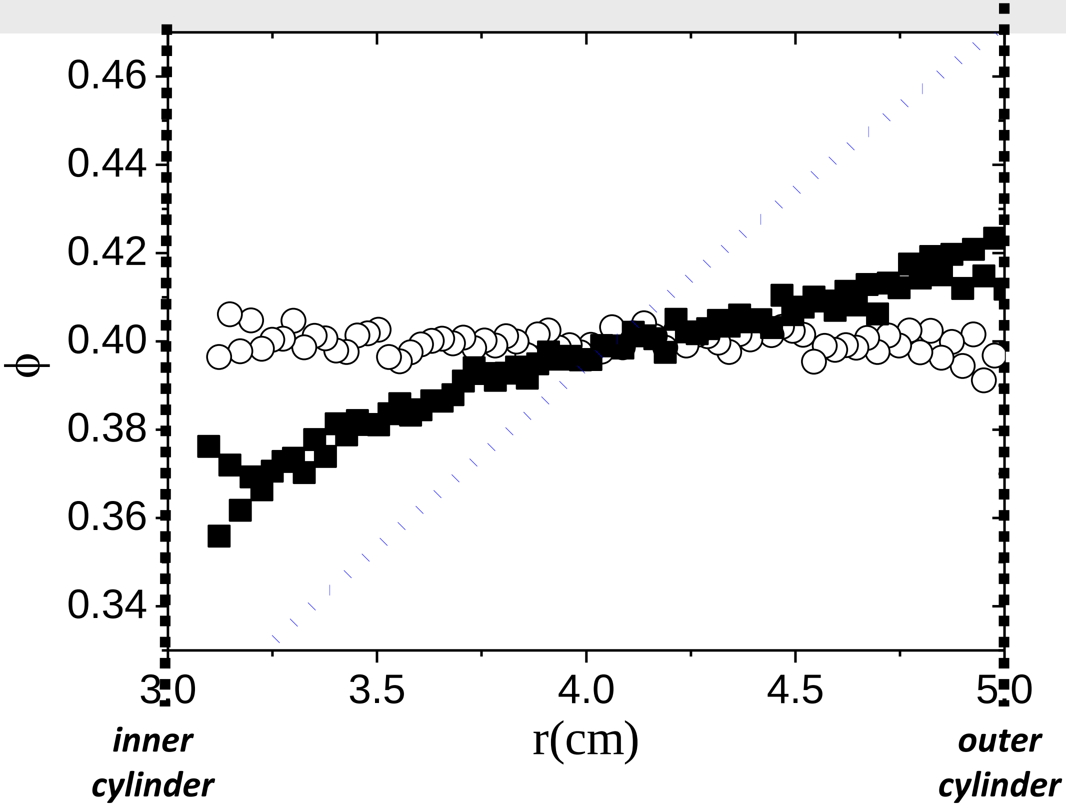
Ex : vol. fraction 40%





Migration towards zone of low shear rate

Migration in dense viscous suspensions: steady-state



normal stresses:

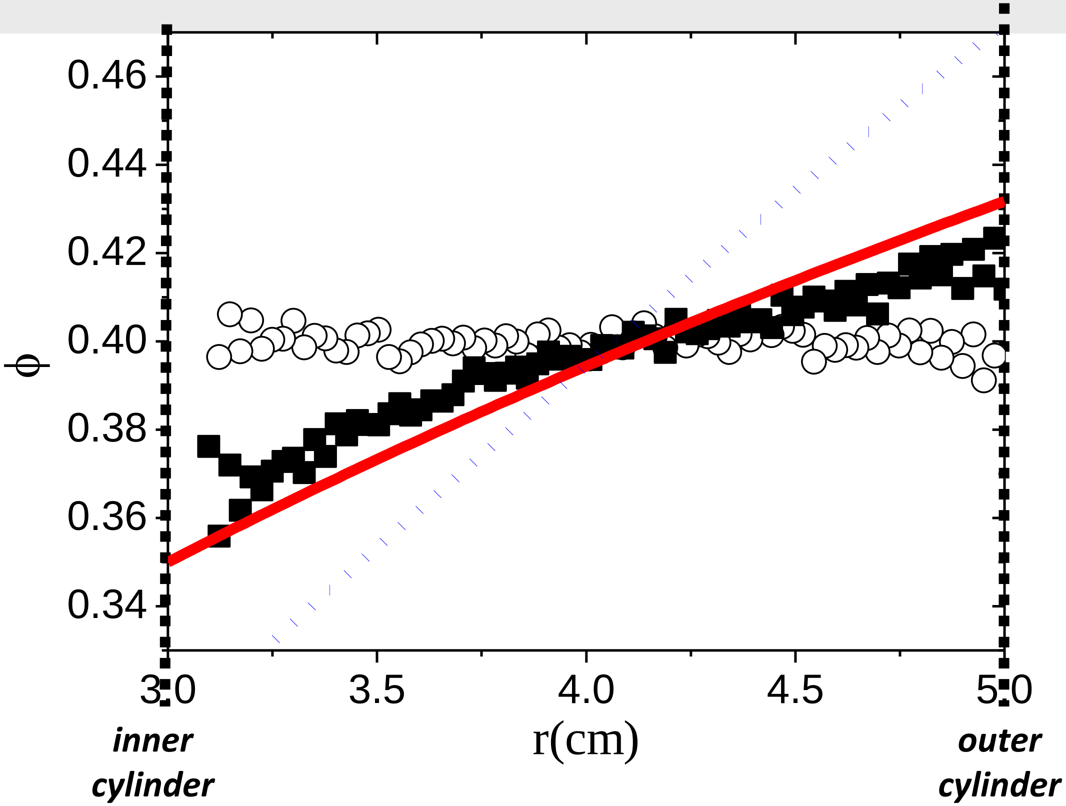
$$\underline{\underline{\sigma}}_n^p = \begin{bmatrix} \lambda_1^p & 0 & 0 \\ 0 & \lambda_2^p & 0 \\ 0 & 0 & \lambda_3^p \end{bmatrix} \eta_n(\phi)\eta_0\dot{\gamma}$$

Recent measurements suggest $\boxed{\lambda_1{}^p \approx \lambda_2{}^p \approx 1}$

Boyer et al. (2011), Dbouk et al. (2013)

Migration in dense viscous suspensions: steady-state

$$\boxed{\lambda_2^p >> \lambda_1^p}$$



normal stresses:

$$\underline{\underline{\sigma}}_n^p = \begin{bmatrix} \lambda_1^p & 0 & 0 \\ 0 & \lambda_2^p & 0 \\ 0 & 0 & \lambda_3^p \end{bmatrix} \eta_n(\phi)\eta_0\dot{\gamma}$$

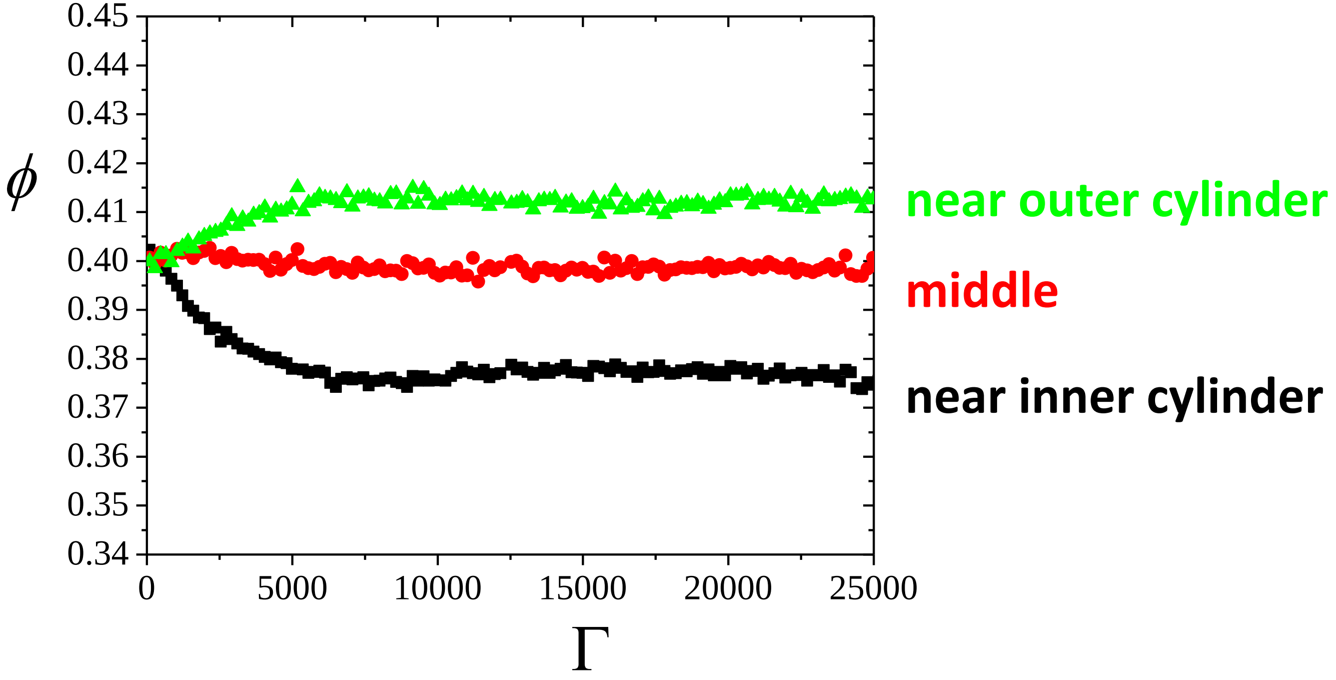
Doubt on what the measured particle stresses are…

Recent measurements suggest $\boxed{\lambda_1^p \approx \lambda_2^p \approx 1}$

Boyer et al. (2011), Dbouk et al. (2013)

Migration in dense viscous suspensions: kinetics

ϕ evolution in 3 different radial positions

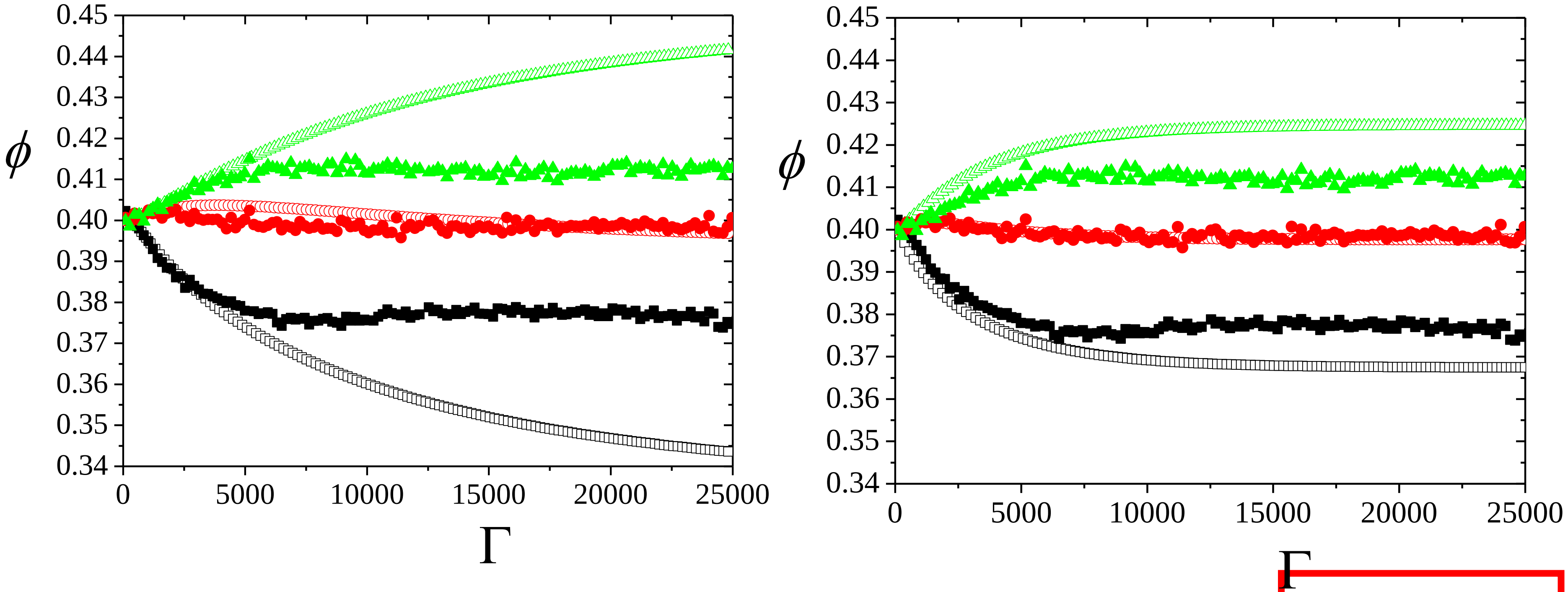


Migration in dense viscous suspensions: kinetics

$$\frac{\partial \phi}{\partial t} = -\frac{2}{9}\frac{a^2}{\eta}\frac{1}{r}\frac{\partial}{\partial r}\left[f(\phi)\left(r\frac{\partial \sigma^p{}_{rr}}{\partial r} - (\sigma^p{}_{\theta\theta} - \sigma^p{}_{rr}) \right) \right]$$

$$\underline{\underline{\sigma}}^p_n = \begin{bmatrix} \lambda_1^p & 0 & 0 \\ 0 & \lambda_2^p & 0 \\ 0 & 0 & \lambda_3^p \end{bmatrix} \eta_n(\phi)\eta_0\dot{\gamma}$$

ϕ evolution in 3 different radial positions



suggested values (Boyer et al. 2011)

$$\boxed{\lambda_1^p \approx \lambda_2^p \approx 1}$$

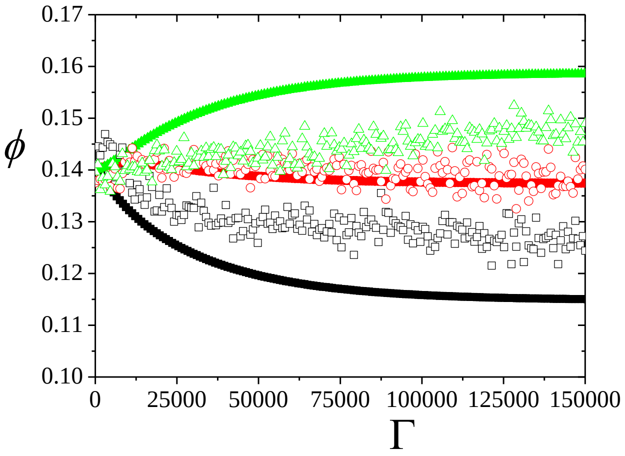
Possible values ?

$$\lambda_2^p >> \lambda_1^p$$

$$\lambda_2^p \sim 3-5$$

Other volume fractions: kinetics

model with $\lambda_1^p \approx \lambda_2^p \approx 1$



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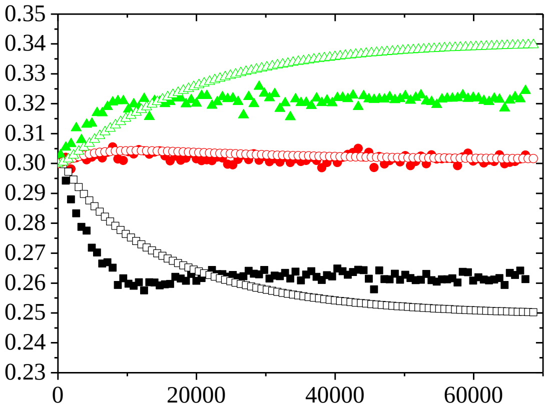

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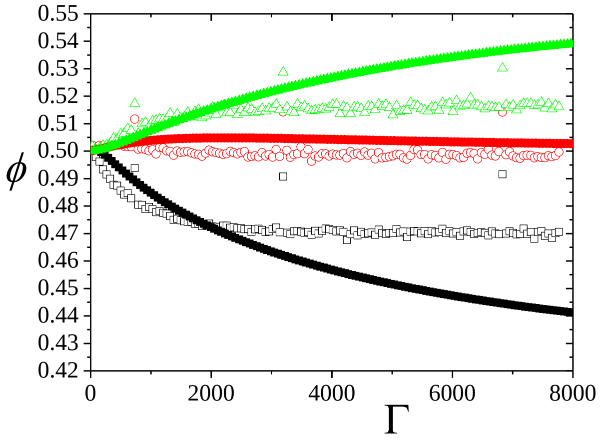

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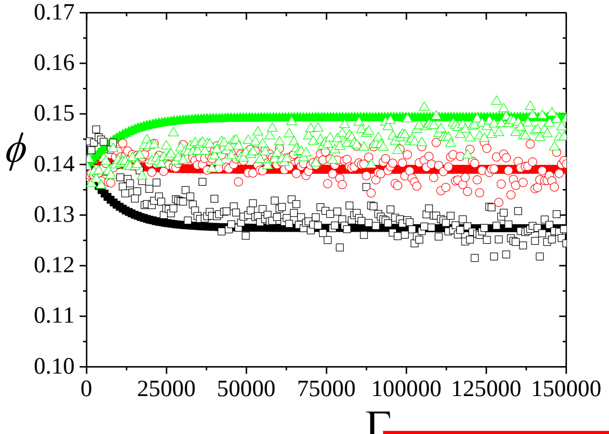
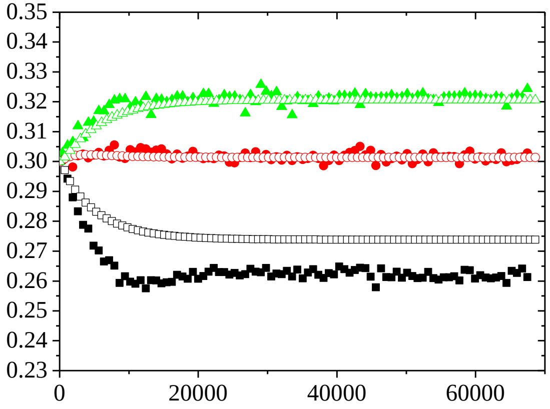
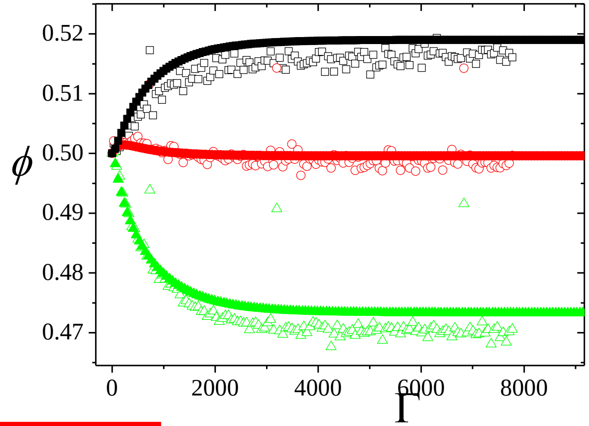






model with $\lambda_2^p >> \lambda_1^p, \quad \lambda_2^p \sim 3-5$

Other possibility: interphase force with gradients

diphasic description: $$\overset{\text{suspension}}{\Sigma_{ij}} = \overset{\text{fluid}}{\boldsymbol{\sigma}_{ij}^{f}} + \overset{\text{particles}}{\boldsymbol{\sigma}_{ij}^{p}}$$

Additional terms of interphase force in inhomogeneous field :

$$-\eta_f \phi \dot{\gamma} D_{ij} \frac{\partial \phi}{\partial x_j} + \eta_f \phi B_{ijkl} \frac{\partial^2 U_l}{\partial x_j \partial x_k}$$

Lhuillier, Phys. Fluids (2009)

Balance equations for the particle phase

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \langle \mathbf{u} \rangle^p) = 0,$$

$$\nabla \cdot \langle \boldsymbol{\sigma} \rangle^p + \mathbf{F}^{\mathrm{drag}} = 0,$$

Inter-phase drag force

$$\mathbf{F}^{\mathrm{drag}} \approx -\frac{9\eta_f}{2a^2} \frac{\phi}{f(\phi)} (\langle \mathbf{u} \rangle^p - \mathbf{u})$$

Migration equation

$$\begin{aligned} \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi &= -\nabla \cdot [\phi(\langle \mathbf{u} \rangle^p - \mathbf{u})] \\ &= -\frac{2a^2}{9\eta_f} \nabla \cdot [f(\phi) \nabla \cdot \langle \boldsymbol{\sigma} \rangle^p] \end{aligned}$$

Nott and Brady, JFM (1994)
Mills and Snabre, J. Phys. II (1995)
Morris and Boulay, J. Rheol. (1999)
Lhuillier, Phys. Fluids (2009)
Nott et al., Phys. Fluids (2011)

Other possibility: interphase force with gradients

Accounting for vol. frac. inhomogeneity in interphase force $- \eta_f \phi \dot{\gamma} D_{ij} \frac{\partial \phi}{\partial x_j}$

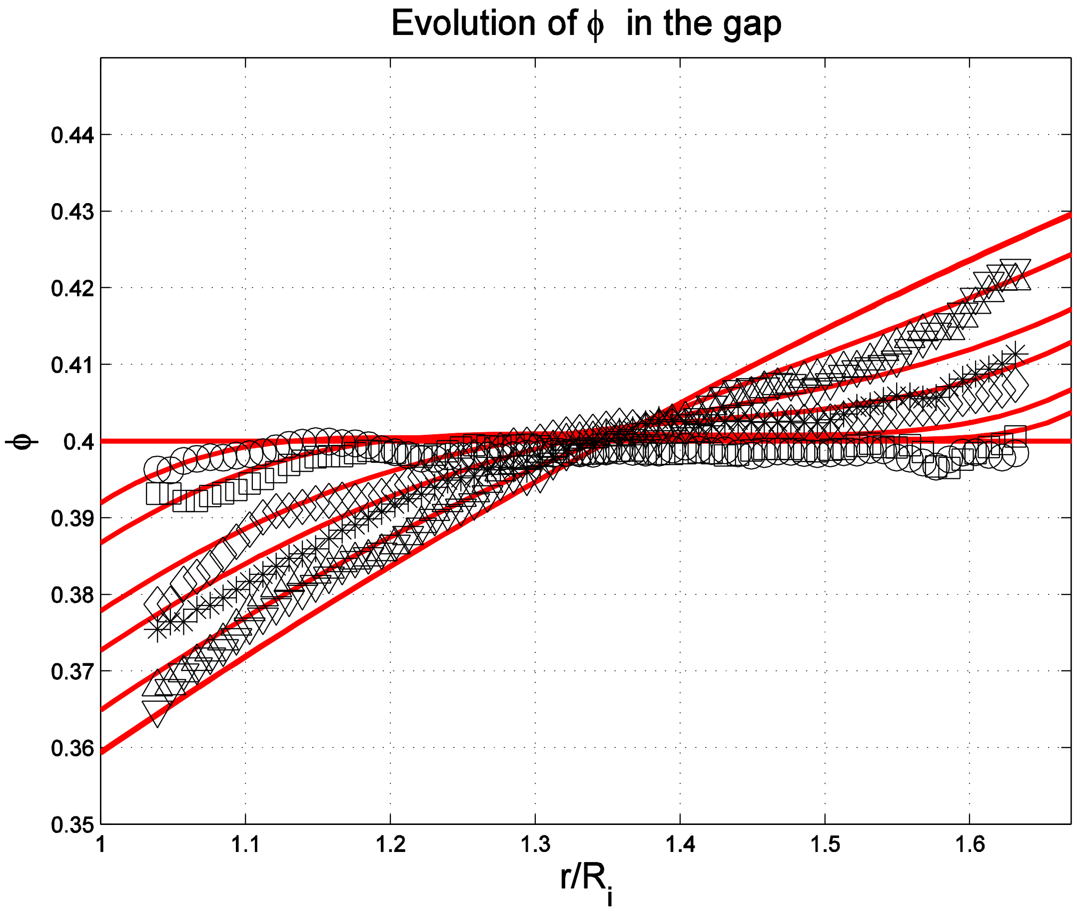
Lhuillier, Phys. Fluids (2009)

A **single (free…) parameter**

→ **Decrease of migration magnitude**

→ **Increase of kinetics**

Evolution of ϕ in the gap



Similar to adding a stress to the contact stress

Migration in a Couette cell: a complex 2D problem?

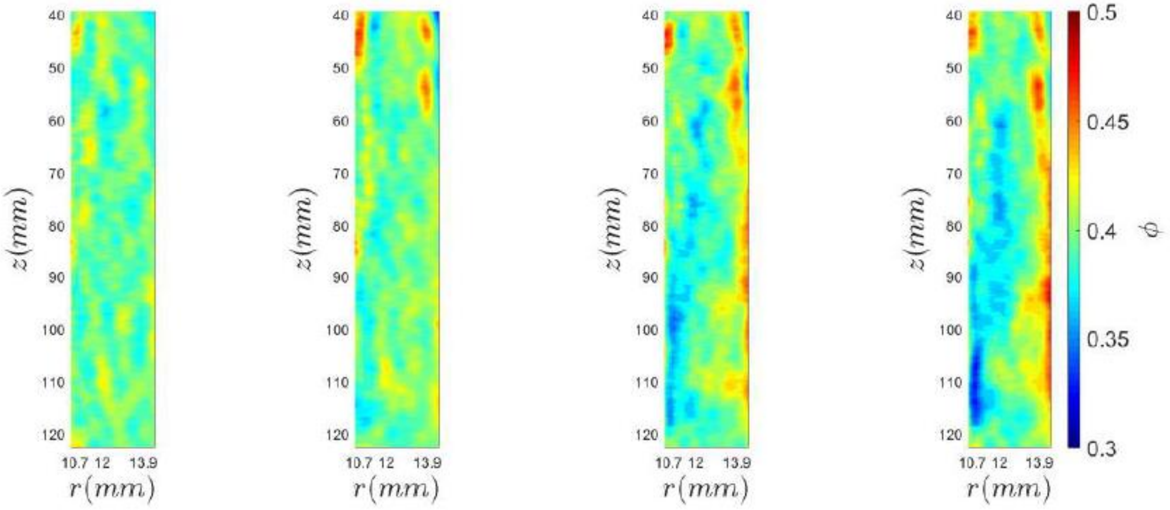
with A. Rashedi & S. Hormozi

Time evolution of the
2D volume fraction maps
in a wide gap Couette cell

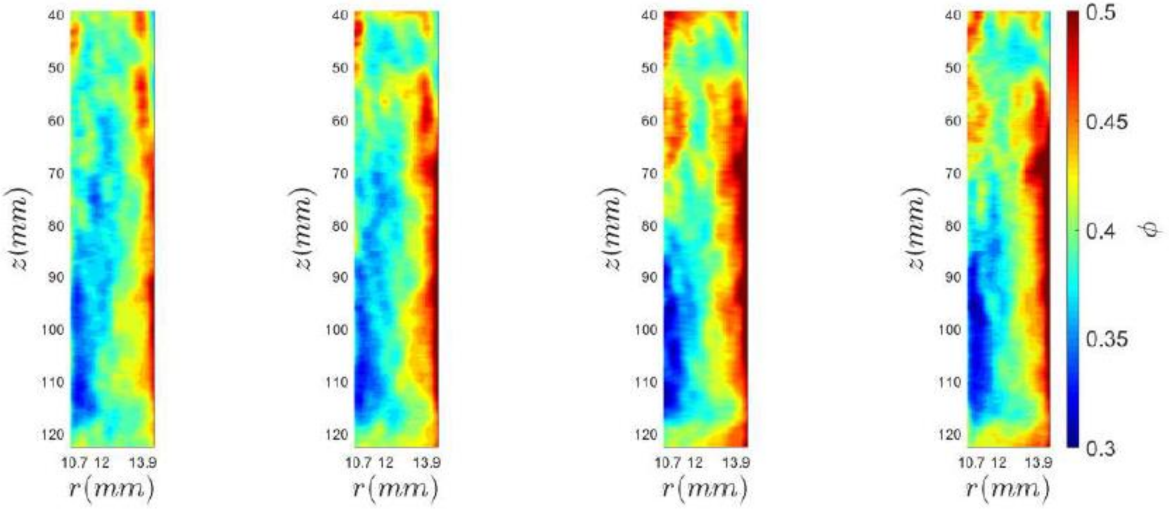
Height=25*gap

Unavoidable
z-inhomogeneities
due to imperfect
boundary conditions ?

Impact on the evolution
at long time only ?



(a) t=157s (b) t=232s (c) t=402s (d) t=602s



(e) t=877s (f) t=2877s (g) t=8377s (h) t=10627s

Shear-induced migration: strainscale

For a viscous suspension, **strainscale** $= \dfrac{(\text{gap})^2}{(\textit{particle size})^2} f(\phi)$

ϕ / ϕ_m	strainscale
0.15	100 000
0.5	10 000
0.85	2 000
0.96	50

case *investigated:*

100 particles in the gap → **strainscale**$=10000\ f(\phi)$

Strain scale seems to decrease down to 0 near jamming → unavoidable…

Nonlinear suspension: strainscale$= f(\phi, \dot{\gamma})$

Migration in viscous suspensions : Conclusion

- steady-state : migration **less important** than expected from model
- **kinetics: migration much faster** than expected from model
- Theory: **crucial role of contact stress**
- Experiments: is $\boldsymbol{\sigma^{p}_{22} \gg \sigma^{p}_{11}}$ **???**
- Possible **role of density gradients in the hydrodynamic force?**

Strain scale seems to decrease down to 0 near jamming
→ what about small particles? In the shear-thickening regime?

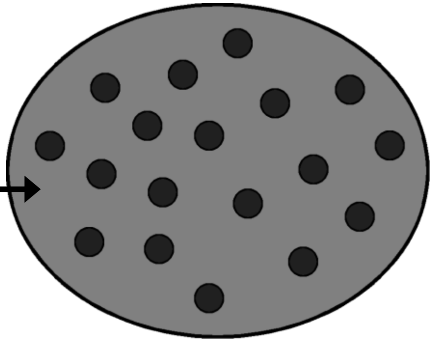
Shear-induced migration in viscous suspensions: test of model

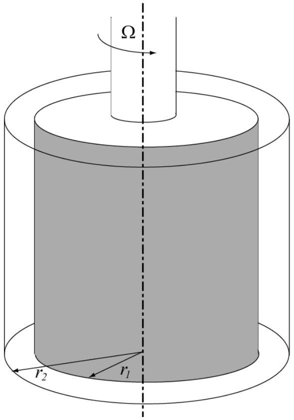
Shear-induced migration in shear-thickening suspensions:
(1) Role of interparticle contacts

Shear-induced migration in shear-thickening suspensions
(2) Jamming

A shear-thickening dense suspension

- *Polystyrene beads :* ***ø 40 µm****,* ρ = 1.05 g.cm^{-3}
- *Water + Cesium chloride*: ρ = 1.05 g.cm^{-3}, **η = 1 mPa.s**
- $\boldsymbol{\phi} = \mathbf{56}$ à $\mathbf{60\%}$





Geometry:

Wide gap Couette (R_{inner}=4cm, R_{outer}=6cm)

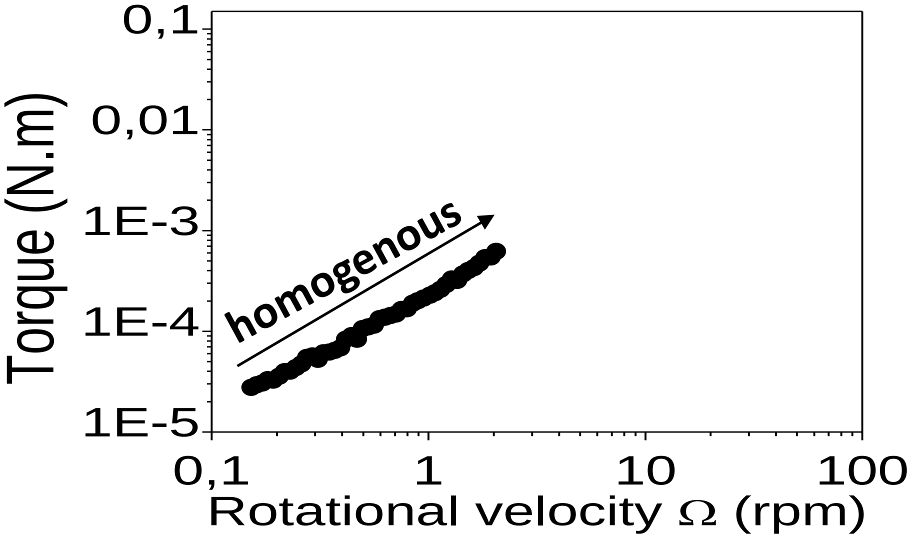
Migration ?

$$\text{strainscale} \propto \frac{(\text{gap})^2}{(\text{particle size})^2} \times f(\phi)$$

→ strainscale expected ~ 1000

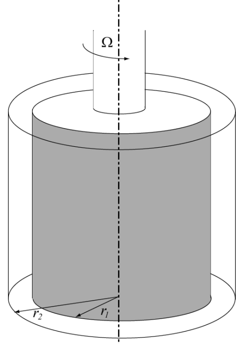
Macroscopic response

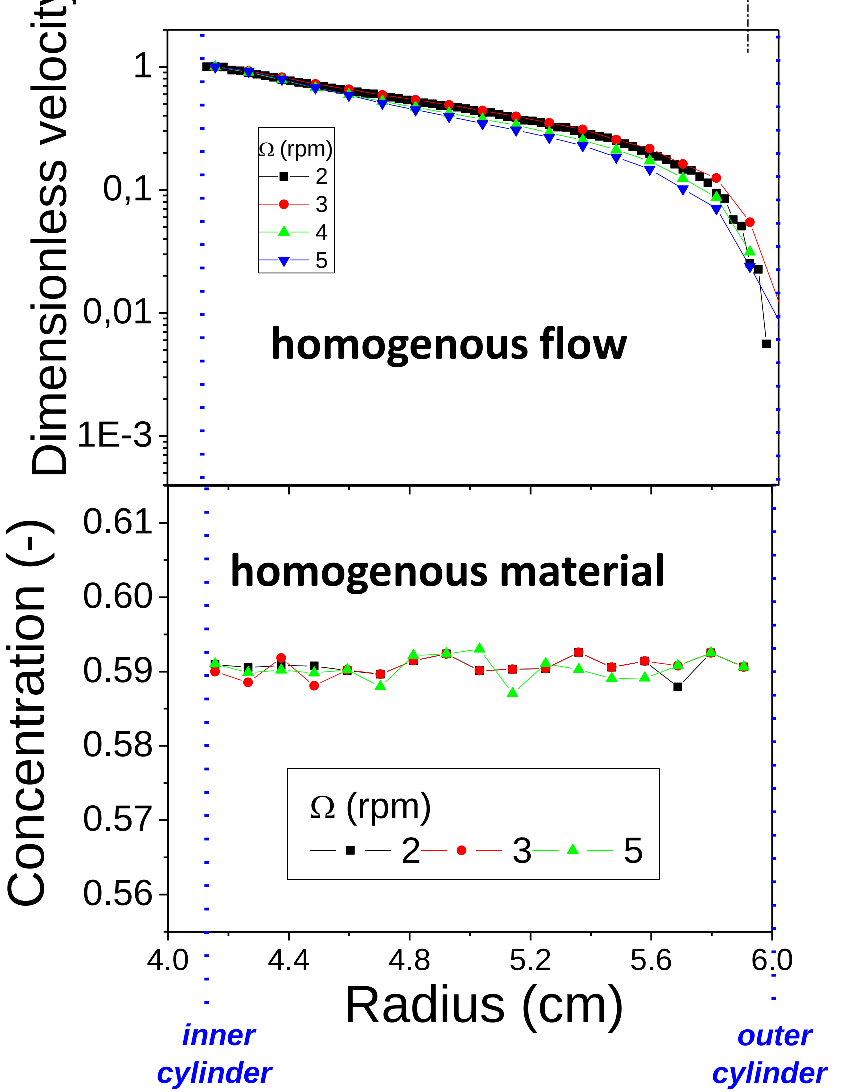
when increasing rotational velocity



Viscous response of the homogeneous material

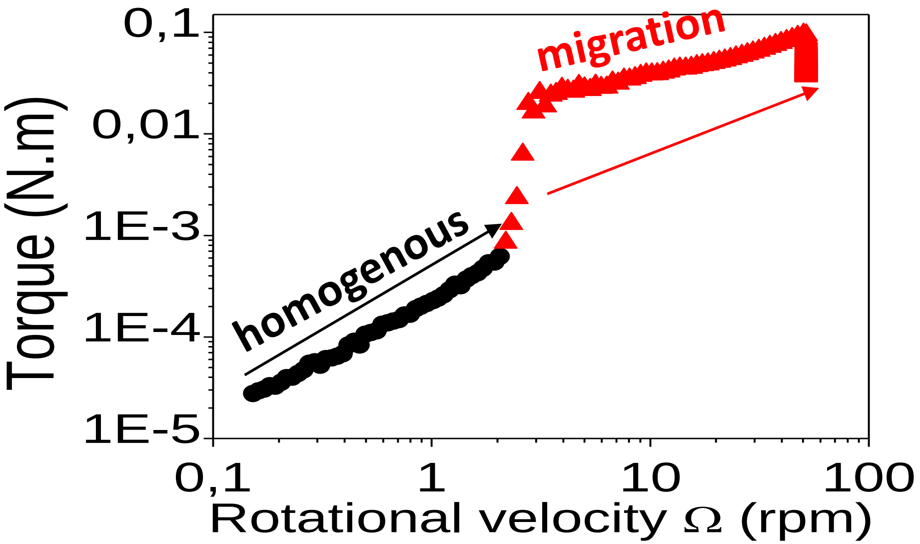
Local behavior





Macroscopic response

when increasing rotational velocity

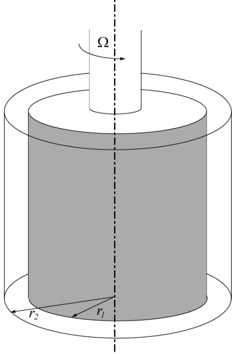


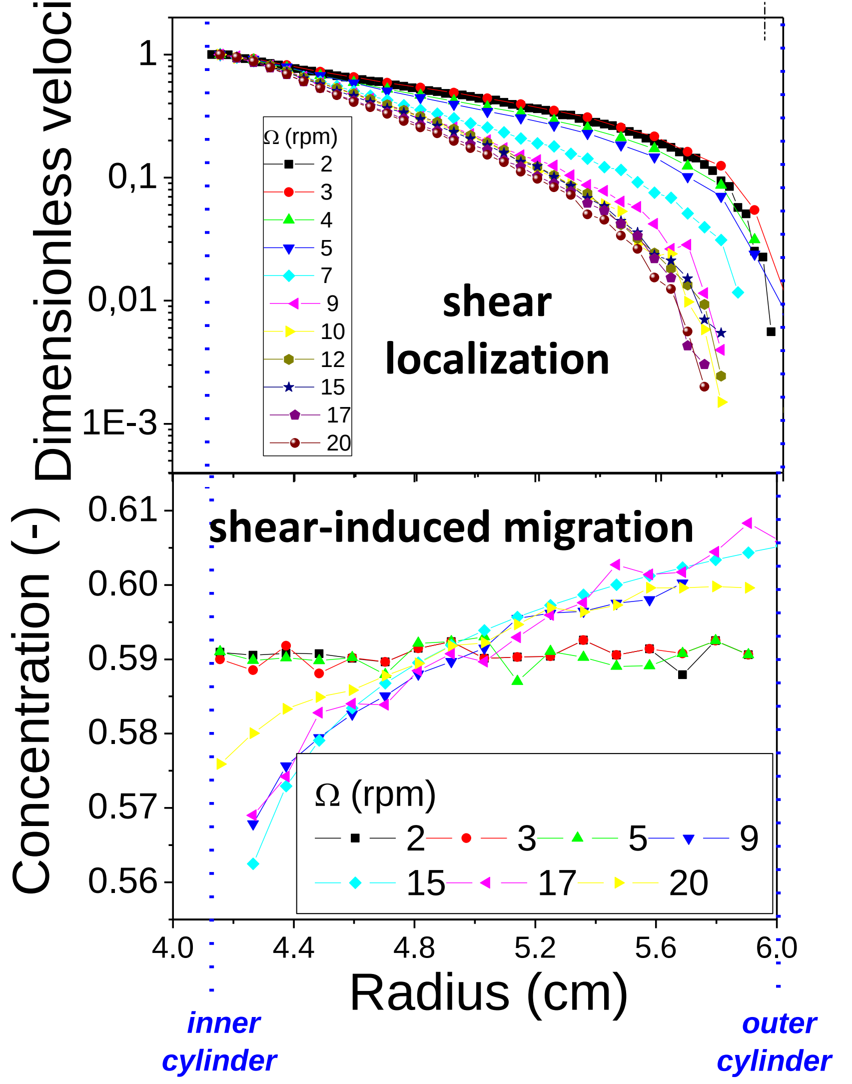
Discontinuous shear-thickening: transient response linked to migration

Strainscale<<1000

Consistent with the microscopic picture proposed by Seto, Mari, Wyart… (sudden emergence of contacts) but **what is the true local behavior?**

Local behavior

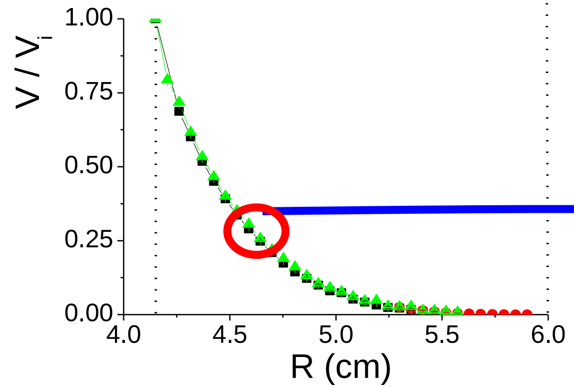




Local measurements of the intrinsic behavior

Ovarlez et al., J. Rheol. ***50****, 259-292 (2006)*
Fall et al. (2010)

Velocity profiles $\Rightarrow \dot{\gamma}(R,\Omega)$



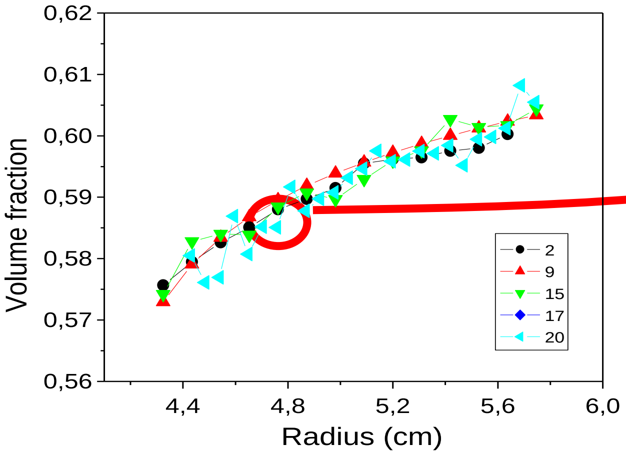
Local behavior at constant volume fraction 59%

Torque $\Rightarrow \tau(R,\Omega) = \dfrac{T(\Omega)}{2\pi H R^2}$

$$\boldsymbol{\tau}(\boldsymbol{\varphi}(\boldsymbol{R}),\dot{\boldsymbol{\gamma}}(\boldsymbol{R}))$$

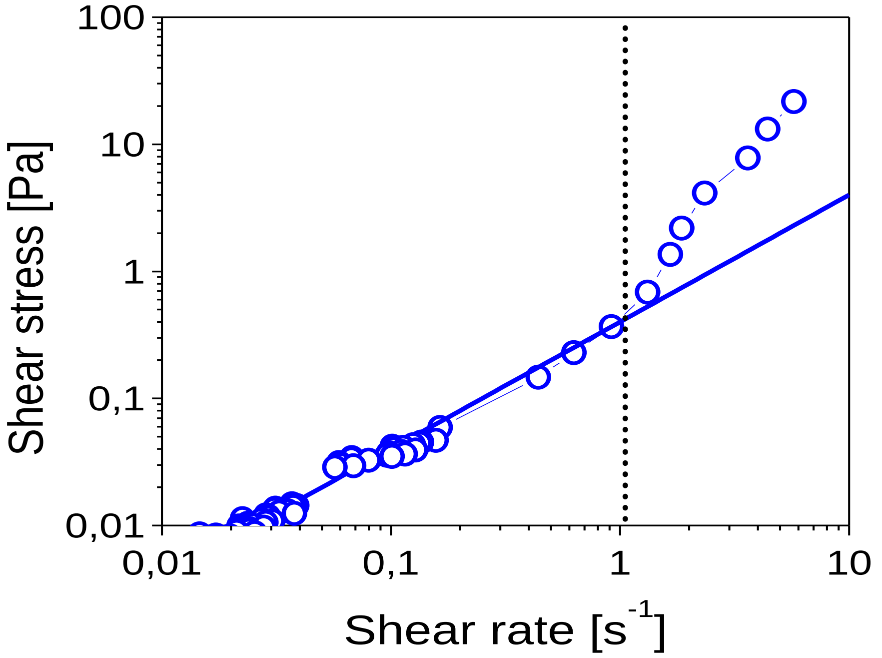
Concentration profiles

$\varphi = f(R,\cancel{\Omega}) \quad \Rightarrow \quad R \leftrightarrow \varphi$



Continuous shear-thickening: viscous → granular transition

Local measurement at constant volume fraction (59%)



$$\tau \prec \dot{\gamma} \longrightarrow \tau \prec \dot{\gamma}^2$$

Viscous

Viscous forces + Contact forces ;
negligible inertia

$$\mathbf{0} = \sum_j \boldsymbol{F}_{ij} + \boldsymbol{F}_i^{visc} \quad \Rightarrow \quad \boldsymbol{F}_{ij} \propto \dot{\gamma}$$

« Granular »

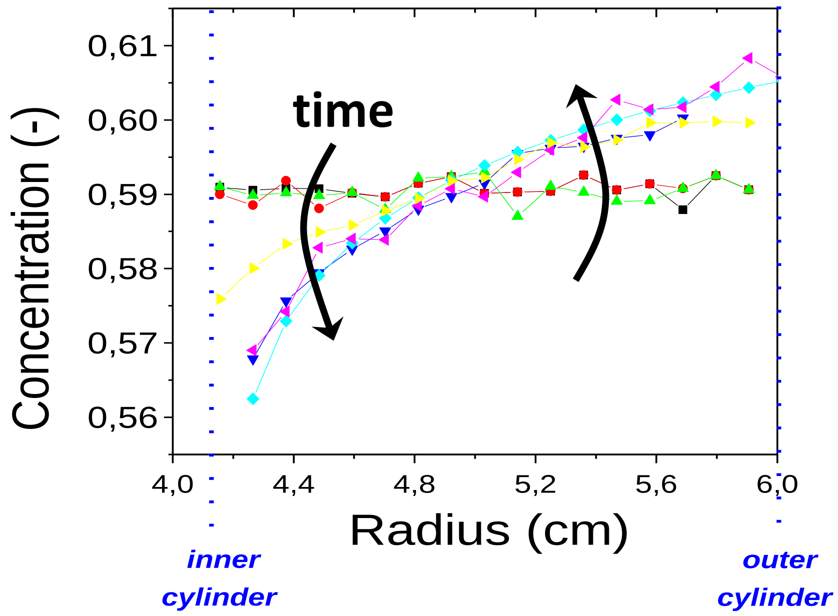
Contact forces + Grains inertia;
negligible viscous forces

$$\boldsymbol{m}\ \mathrm{d}^2\boldsymbol{r}_i / \mathrm{d}\boldsymbol{t}^2 = \sum_j \boldsymbol{F}_{ij} \quad \Rightarrow \quad \boldsymbol{F}_{ij} \propto \dot{\gamma}^2$$

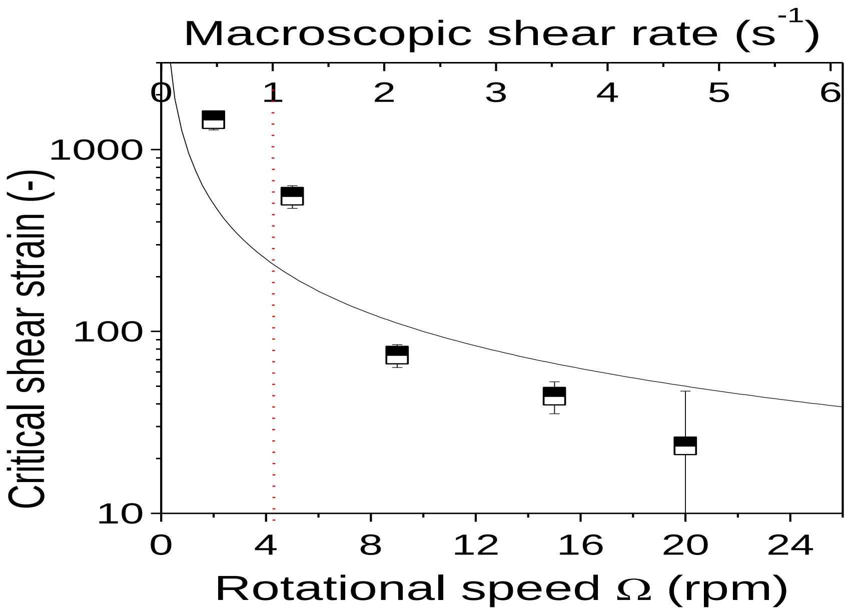
Lemaître et al, PRL 2002, Rheol. Acta 2009

Kinetics of migration

59% suspension of 40 microns beads



Strain at the end of migration



- ✓ **Slow** at low shear rate (strain ~1000)
- ✓ **“Instantaneous”** at high shear rate (strain <50)

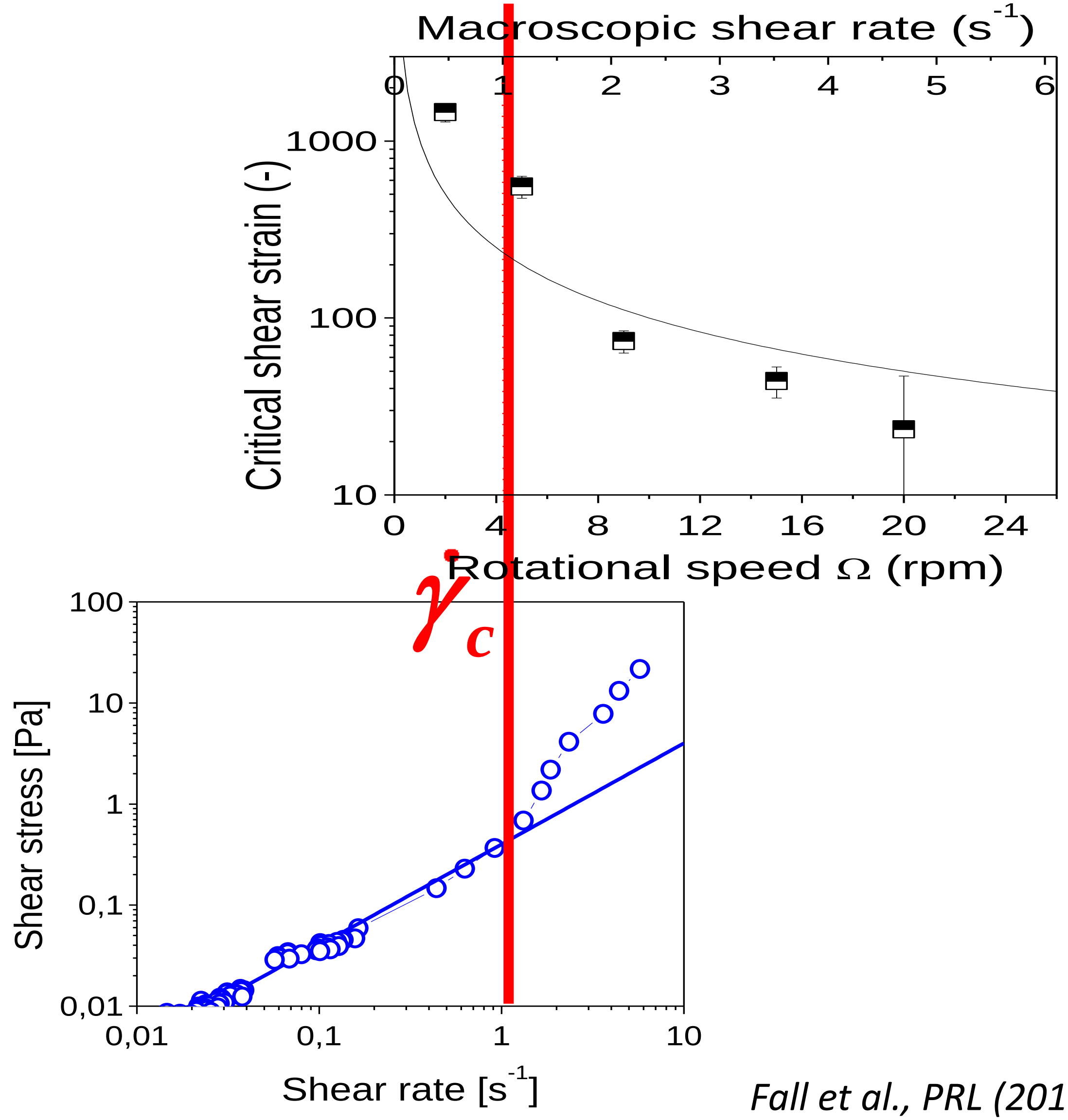
~~Theory for viscous suspensions~~

~~•*timescale =* $1/\dot{\gamma}$ → ***Unique strainscale***~~

~~•*strainscale expected ~ 1000*~~

Interplay between continuous shear-thickening / migration

Strain at the end of migration



Fall et al., PRL (2010)

Interplay between continuous shear-thickening / migration

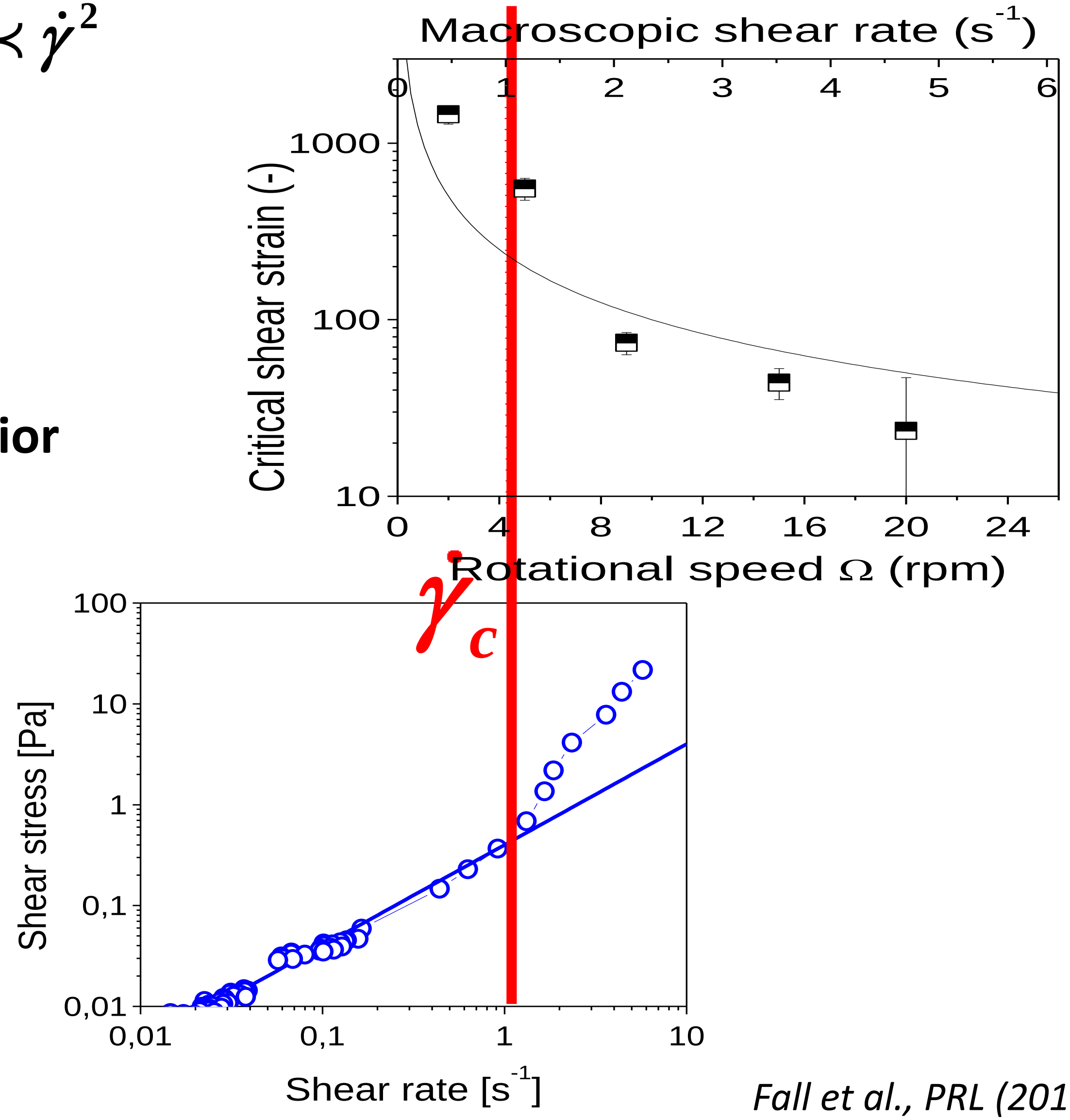
Normal stresses: $\sigma_{11}^{p}, \sigma_{22}^{p} \prec \dot{\gamma}^{2}$

⇒**Acceleration** of migration

Strainscale $\prec 1/\dot{\gamma}$

Signature of “granular” behavior

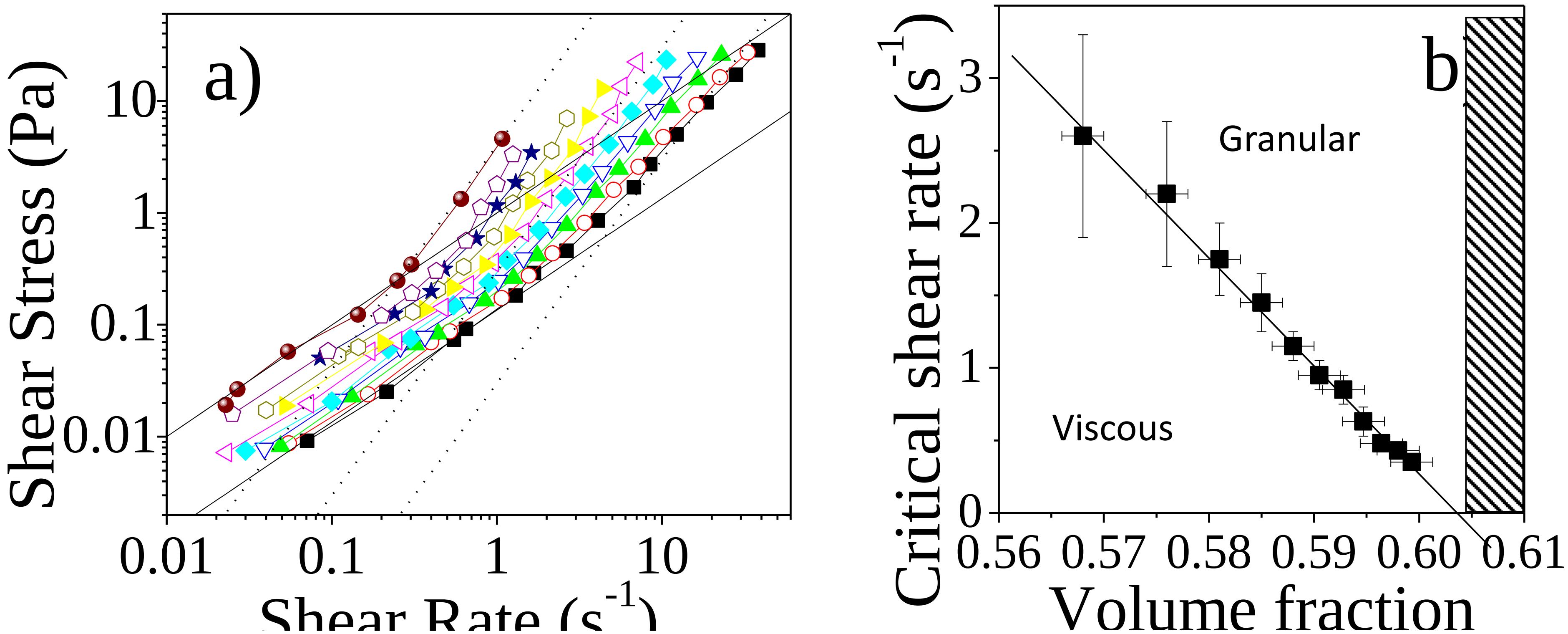
Strain at the end of migration



Fall et al., PRL (2010)

Shear-thickening: viscous → granular transition

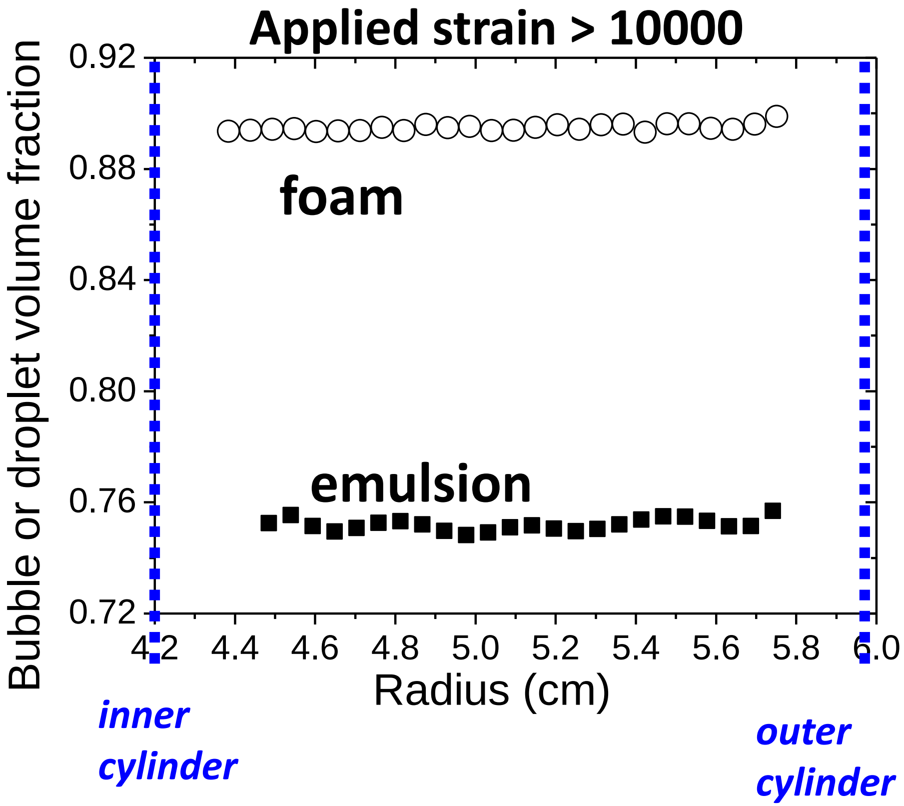
Vanishing of critical shear rate at jamming; similar to other thickening systems

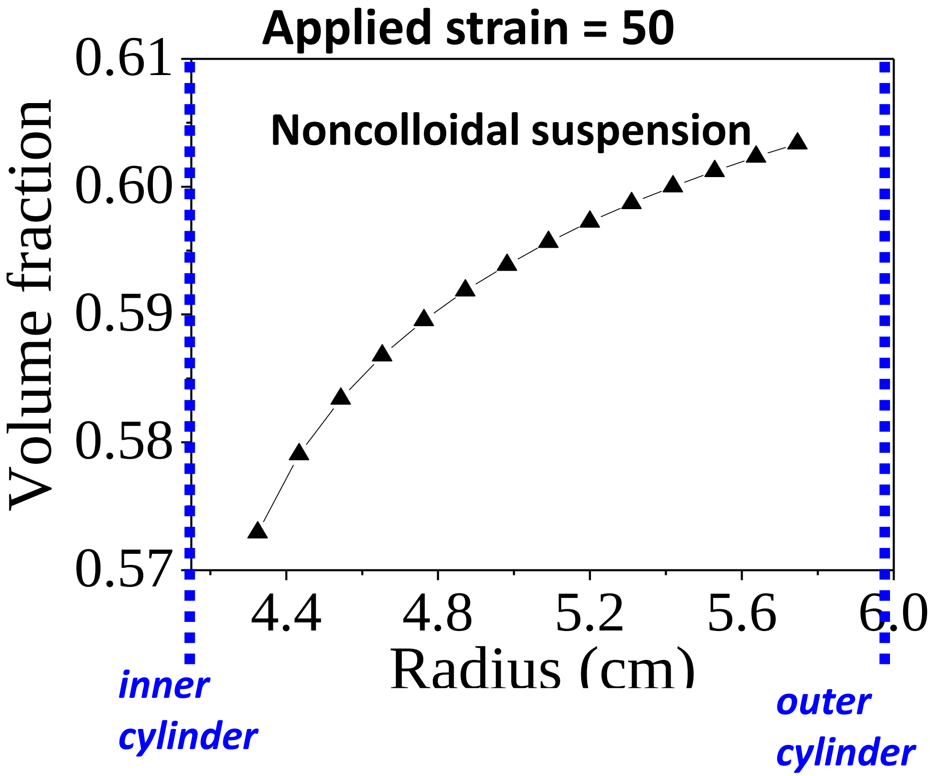


$$\left.\begin{array}{ll}\text{viscous regime:} & \Sigma_V(\phi)\propto(\phi_m-\phi)^{-1}\\ \text{granular regime:} & \Sigma_I(\phi)\propto(\phi_m-\phi)^{-2}\end{array}\right\}\text{transition}:\dot{\gamma}_c(\phi)\propto(\phi_m-\phi)$$

Fall et al., PRL (2010)

Dense suspensions of soft particles: ~~inhomogeneities?~~





Ovarlez et al., PRE 2008

Suspensions of soft particles remain homogeneous

→ no flow/concentration coupling mechanism

Consistent with diphasic model of Lhuillier *(Phys. Fluids 2009):*
migration driven by interparticle forces.

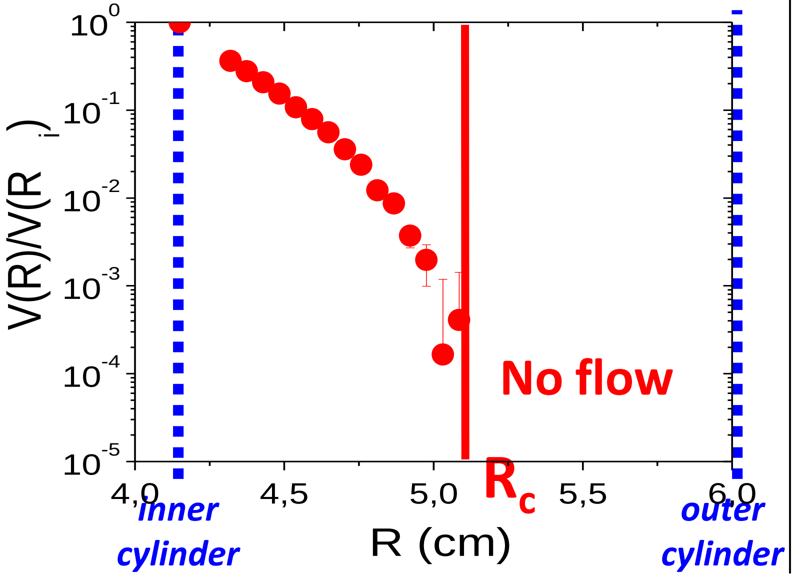
Shear-induced migration in viscous suspensions: test of model

Shear-induced migration in shear-thickening suspensions: (1) Role of interparticle contacts

Shear-induced migration in shear-thickening suspensions (2) Jamming

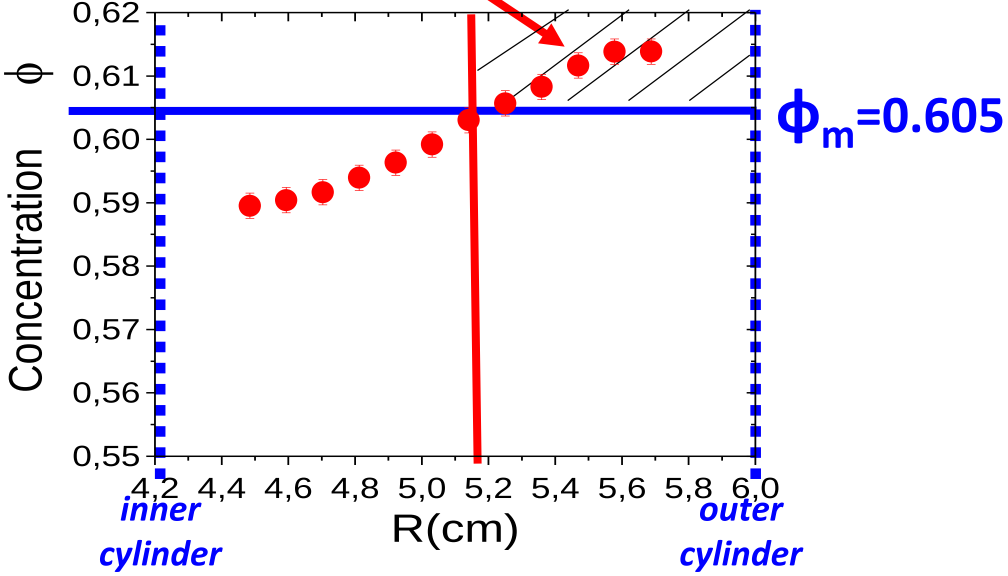
Shear induced migration and Shear induced jamming

Velocity profile in dense suspensions of ***<u>mean concentration</u>*** *60%*



Shear localization at high shear rates:
no flow allowed beyond a critical radius **R_c**

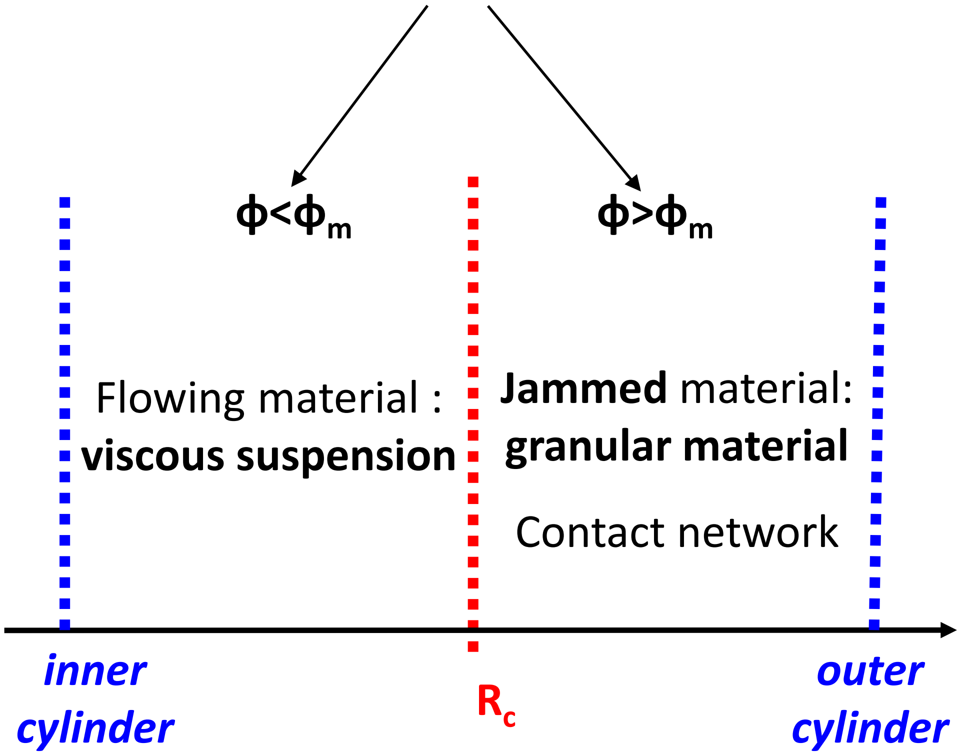
mean concentration 60%,
local concentration> ϕ_m
no flow allowed



Ovarlez et al., J. Rheol. (2006)
Fall et al., PRL (2010)

Shear induced migration and Shear induced jamming

Shear-induced migration generates 2 different zones



$\phi<\phi_m$

$\phi>\phi_m$

Flowing material :
viscous suspension

Jammed material:
granular material

Contact network

inner cylinder

R_c

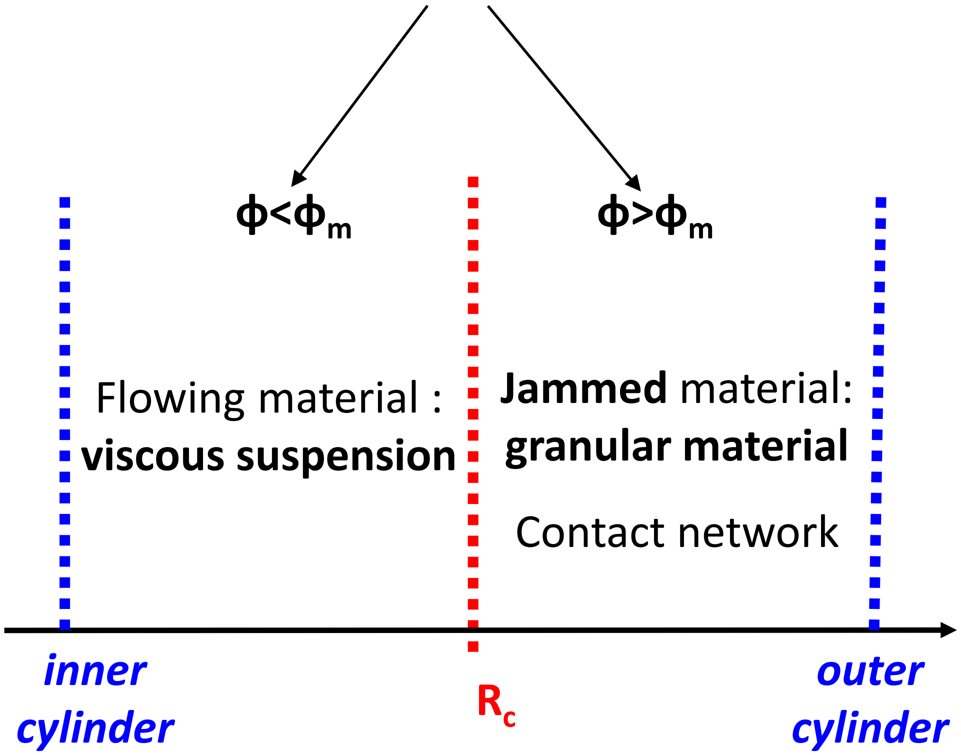
outer cylinder

Explains why one can found viscosity measurements above 60% in the literature
(the concentration in the flowing region may be much lower than the mean concentration)

Ovarlez et al., J. Rheol. (2006)
Fall et al., PRL (2010)

Shear induced migration and Shear induced jamming

Shear-induced migration generates 2 different zones



$\phi<\phi_m$

$\phi>\phi_m$

Flowing material : **viscous suspension**

Jammed material: **granular material**

Contact network

inner cylinder

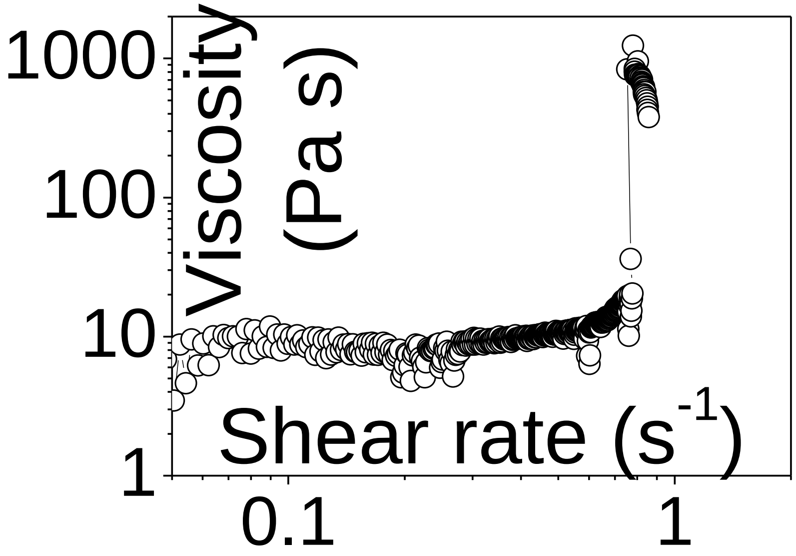
R_c

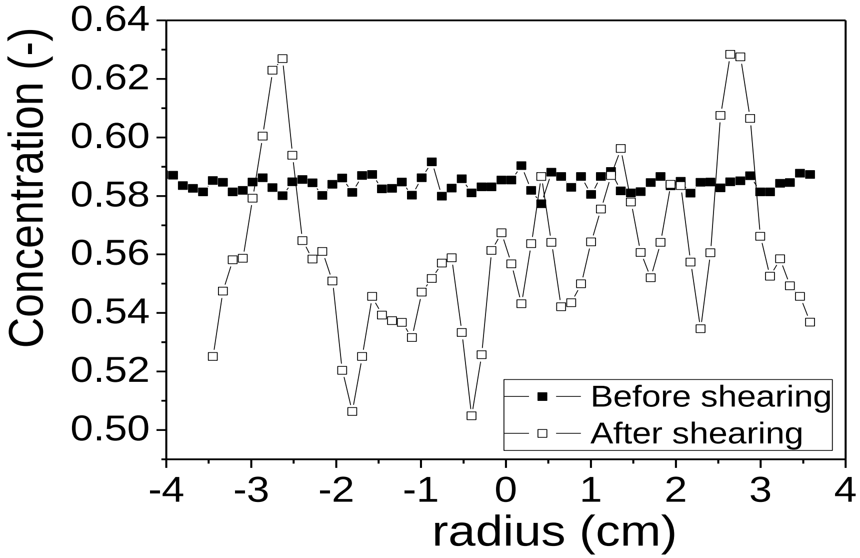
outer cylinder

A diphasic model accounting for the granular regime is necessary

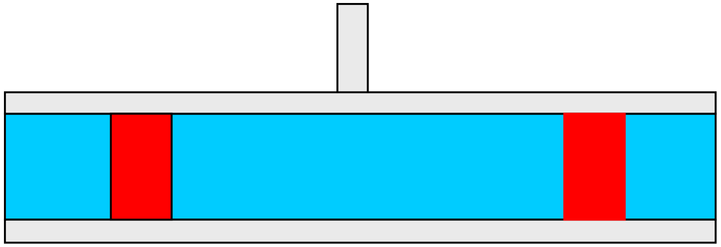
→ see Lecampion, Garagash, JFM (2014) !

Shear-induced jamming due to density changes





Friction of packs of jammed dense granular materials between the two plate surfaces leads to apparent viscosity jump

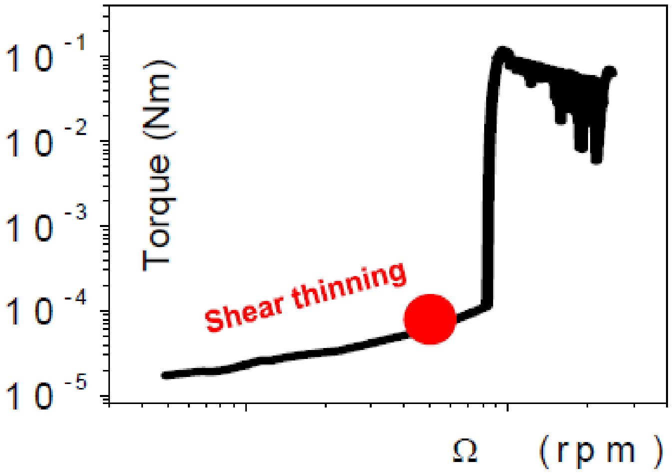


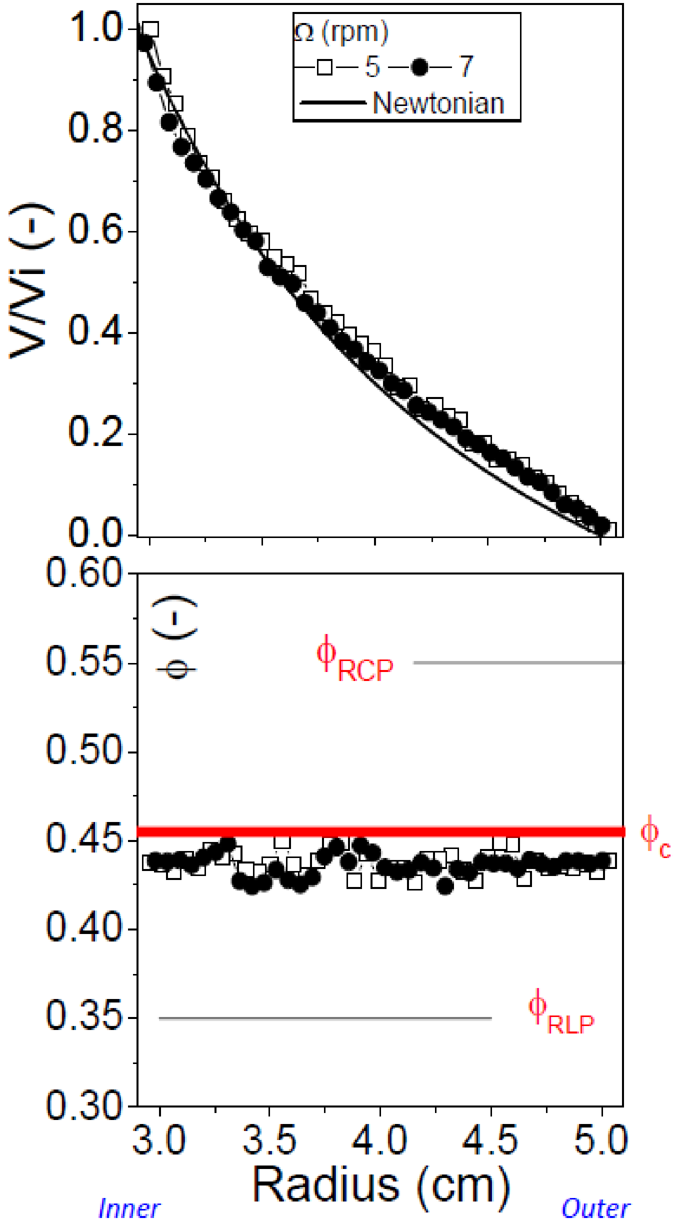
→ Not to be interpreted as an intrinsic behavior !

→ Same thing happens during a transient in the Couette cell?

Cornstarch suspension in a wide gap Couette

For a 43.9% mean volume fraction



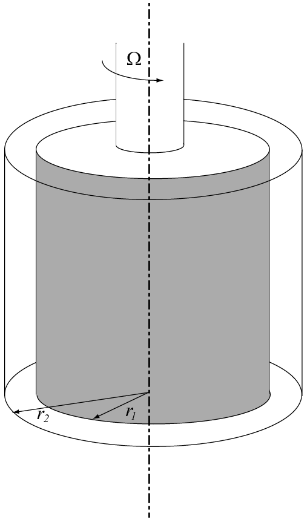


Inner cylinder

Outer cylinder

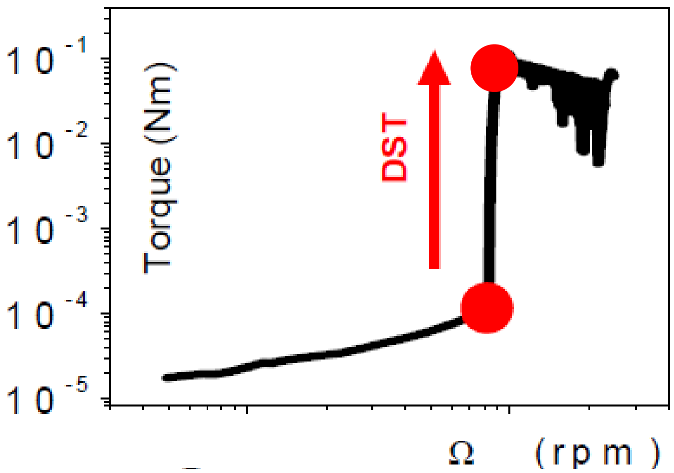
- $\Omega < \Omega_c$ ➔ Shear thinning

- Homogeneous flow



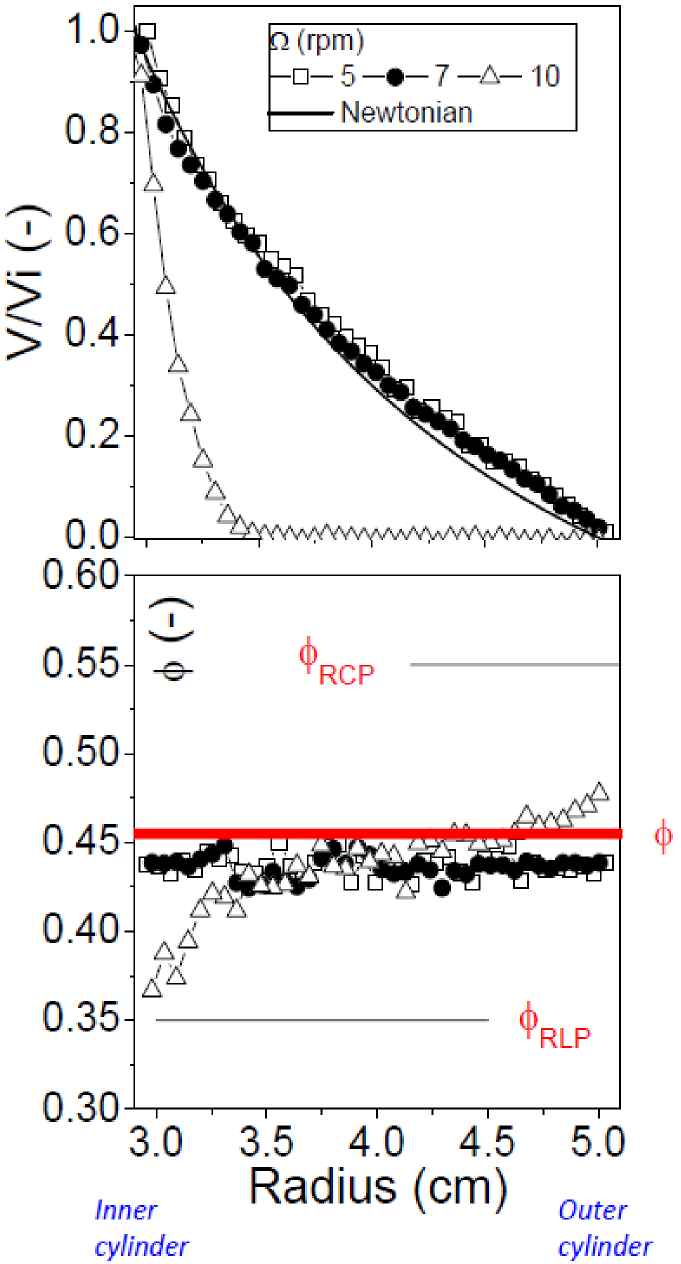
Cornstarch suspension in a wide gap Couette

For a 43.9% mean volume fraction

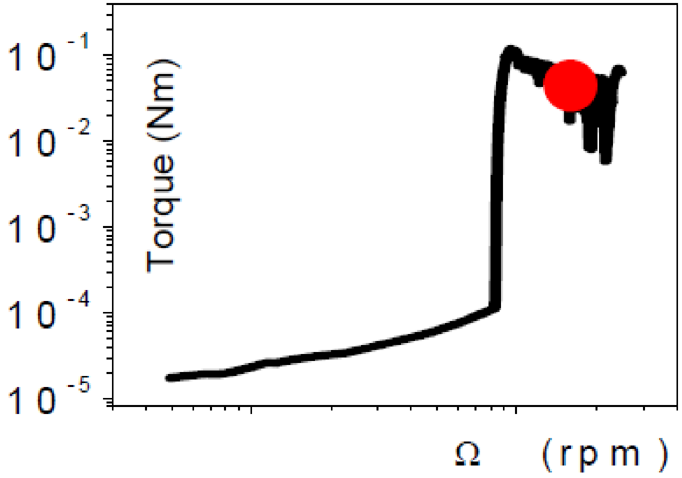


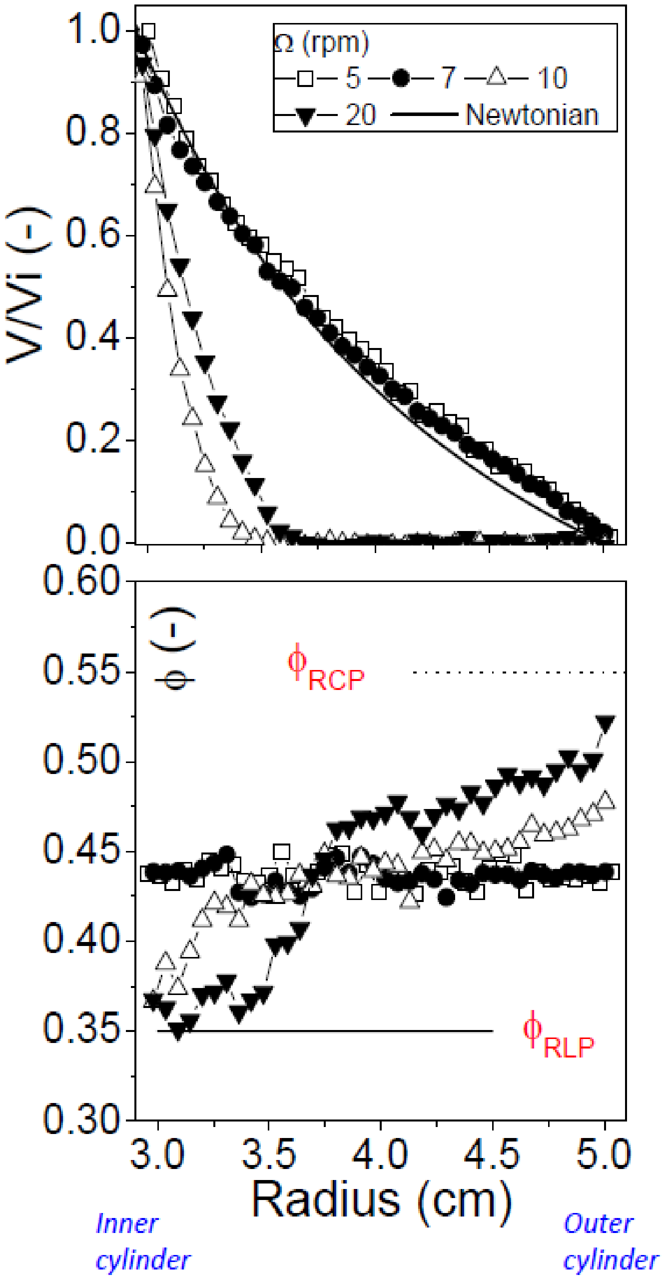
Sudden **shear localization** (creation of a **dead zone**)

Link with emergence of volume fraction **inhomogeneities**



Shear-thickening in cornstarch: local measurements

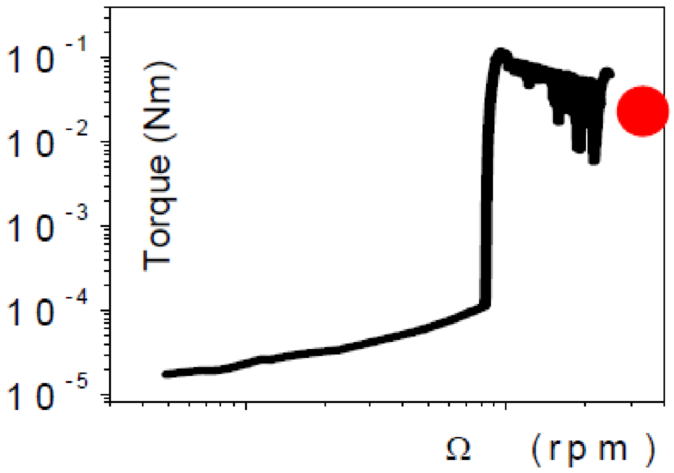

For a 43.9% mean volume fraction



- The sheared region grows up with Ω
- Broadening of the low-density region
- Compaction of the Jammed region towards ϕ_{RCP}

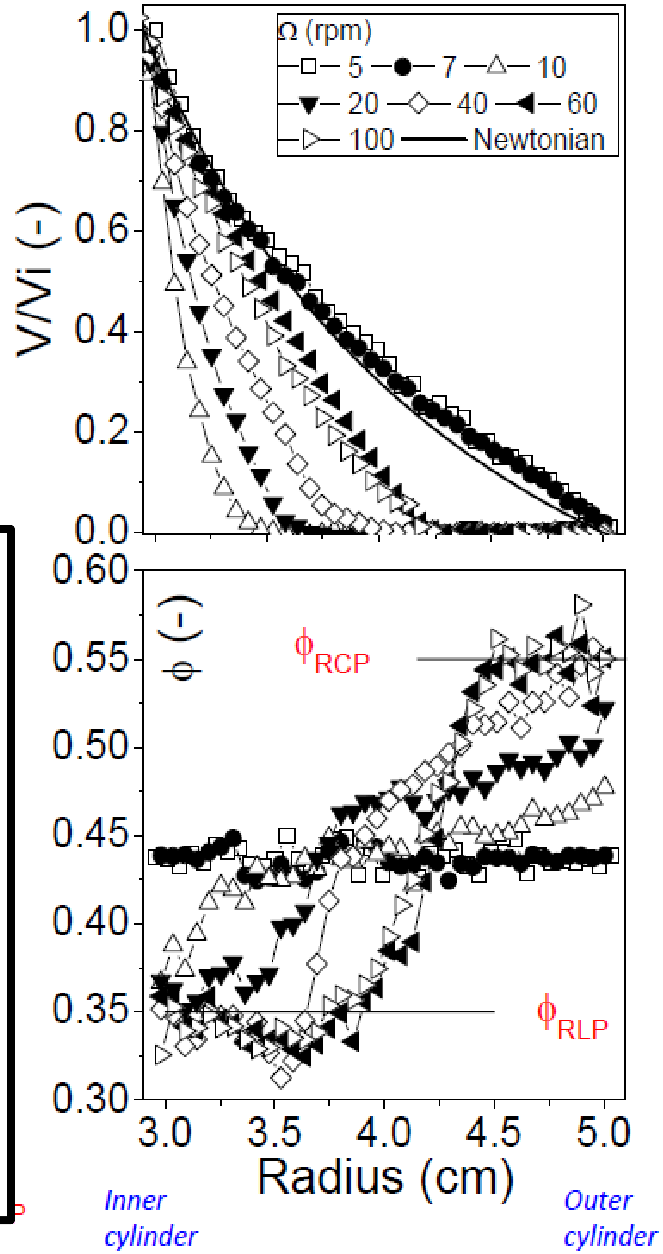
Shear-thickening in cornstarch: local measurements

For a 43.9% mean volume fraction



What can we learn on the local behavior from all this mess???

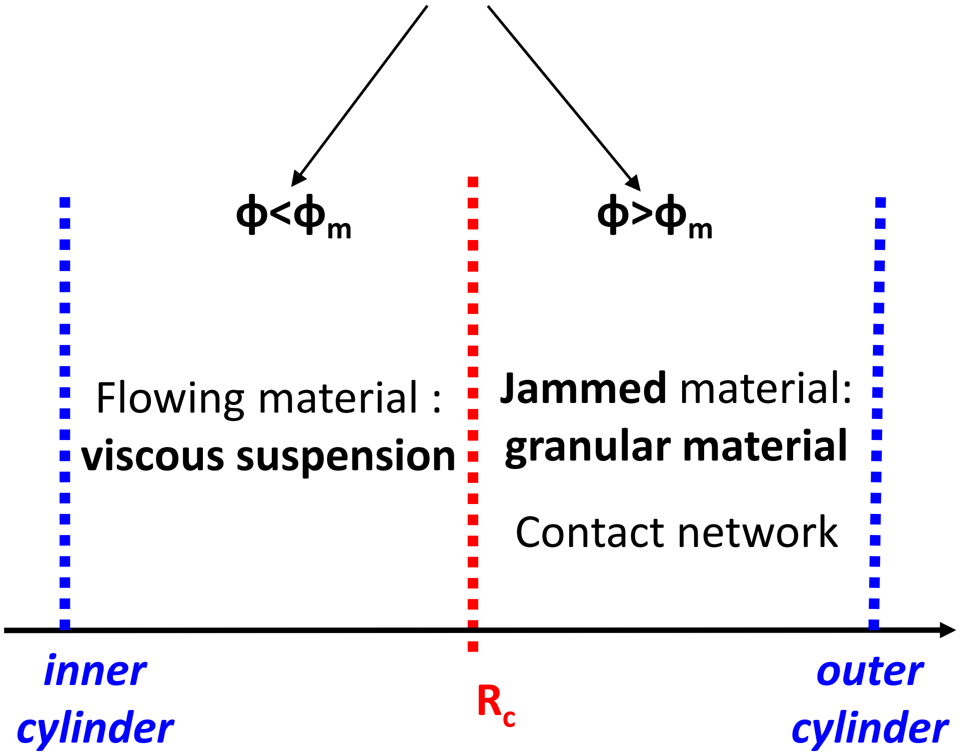
→ **Extract all $(\phi, \dot{\gamma}(\phi))$ to build a « state diagram »**



Fall et al., PRL (2015)

Shear induced migration and Shear induced jamming

Shear-induced migration can generate 2 different zones



$\phi<\phi_m$

$\phi>\phi_m$

Flowing material : **viscous suspension**

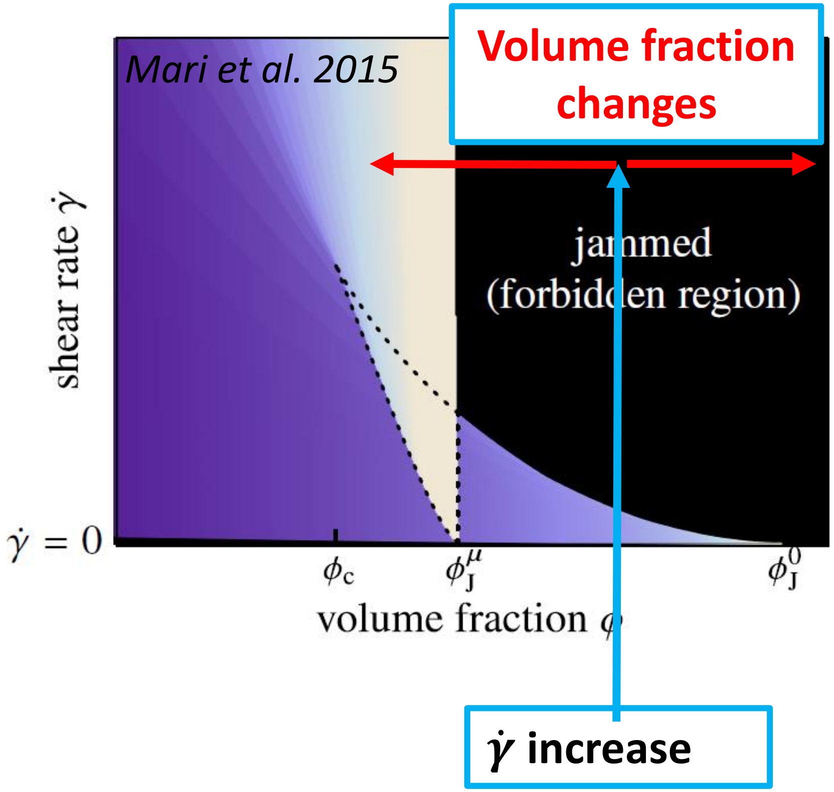
Jammed material: **granular material**

Contact network

inner cylinder

R_c

outer cylinder



Mari et al. 2015

Volume fraction changes

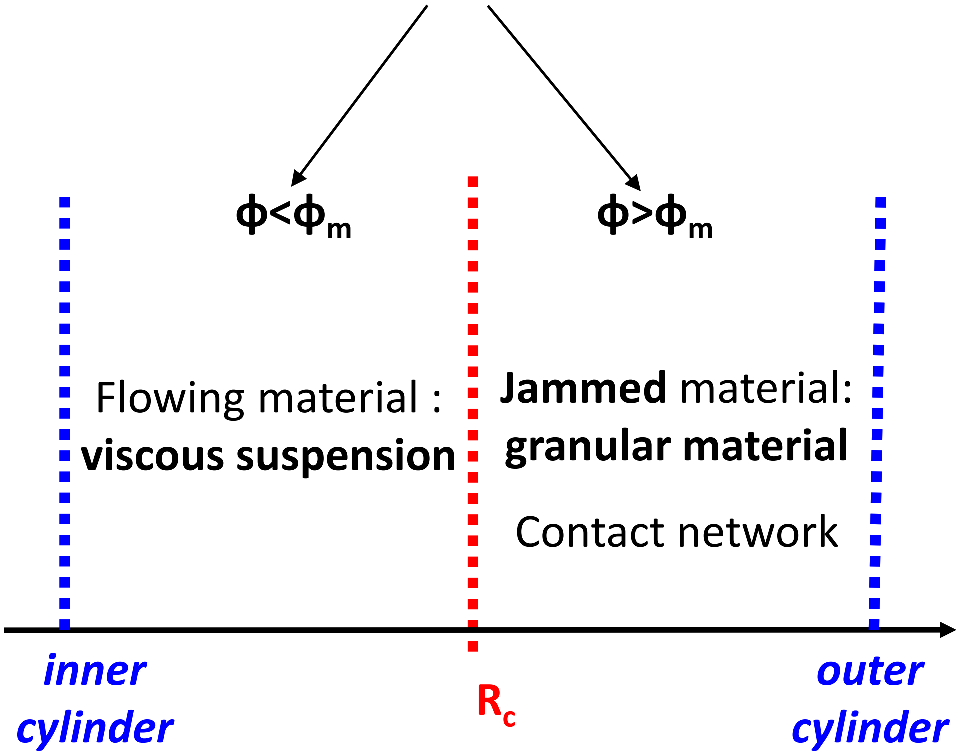
$\dot{\gamma}$ increase

Discontinuous shear thickening = (shear-?) jamming.

ϕ decreases locally to allow for flow near the moving boundary

Shear induced migration and Shear induced jamming

Shear-induced migration can generate 2 different zones



$\Phi<\Phi_m$

$\Phi>\Phi_m$

Flowing material : **viscous suspension**

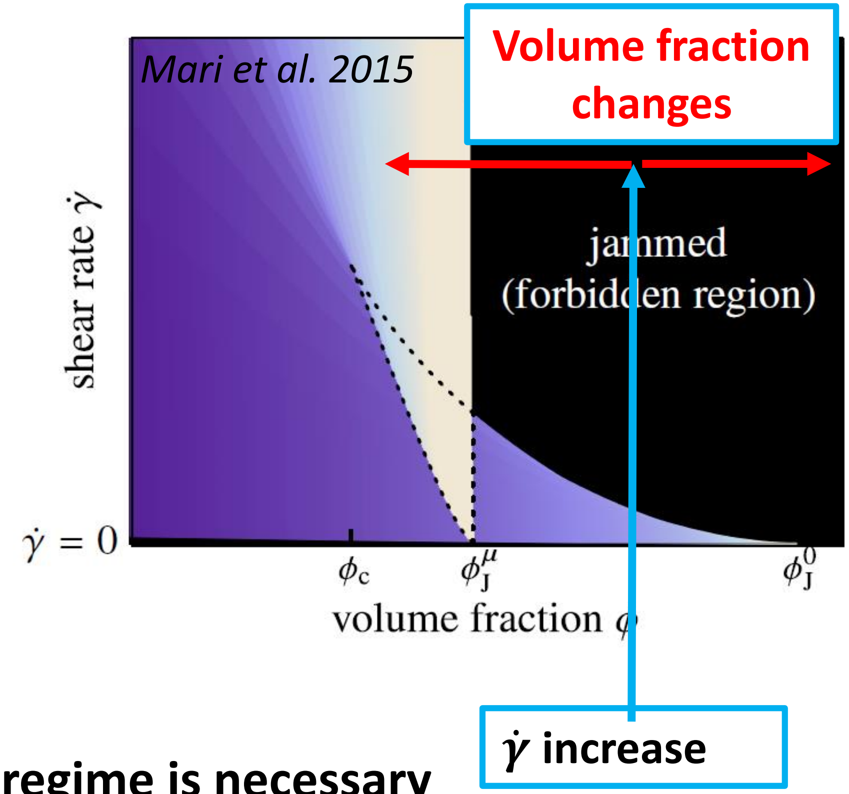
Jammed material: **granular material**

Contact network

inner cylinder

R_c

outer cylinder



Mari et al. 2015

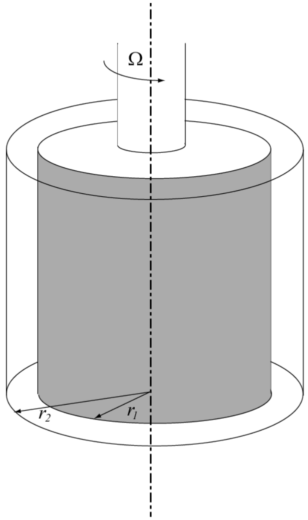
Volume fraction changes

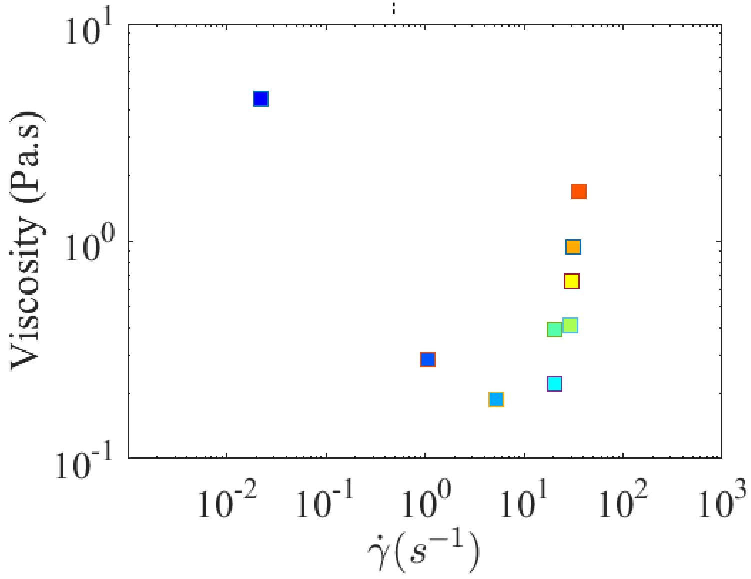
$\dot{\gamma}$ increase

A diphasic model accounting for the granular regime is necessary

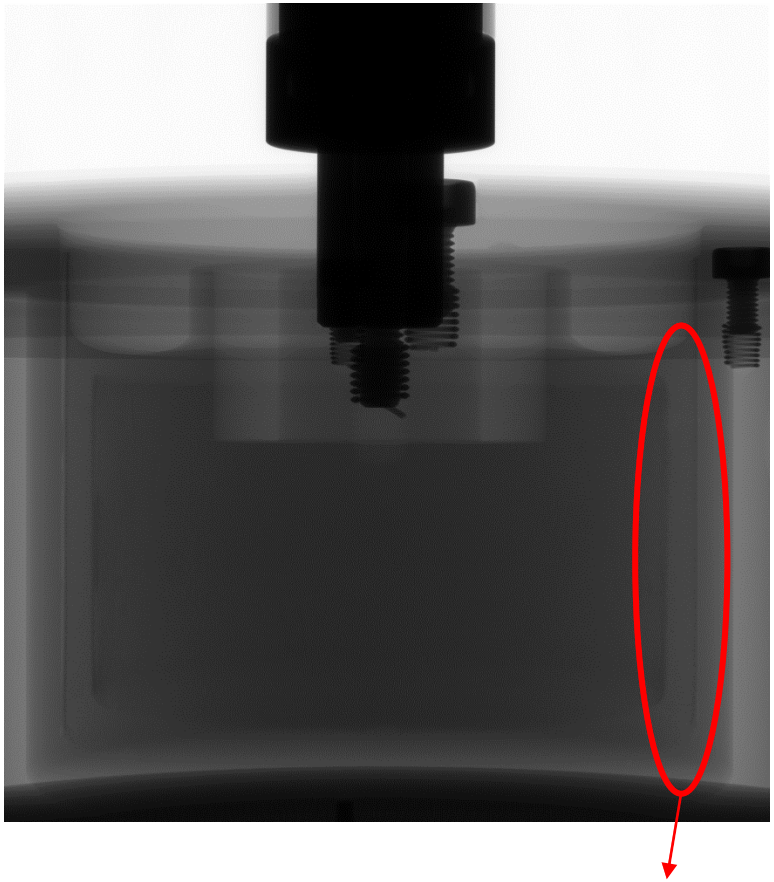
→ see Lecampion, Garagash, JFM (2014)

Rheology with X-ray imaging





@Placamat, Bordeaux



Volume fraction field $\boldsymbol{\phi(r,z,t)}$
extracted

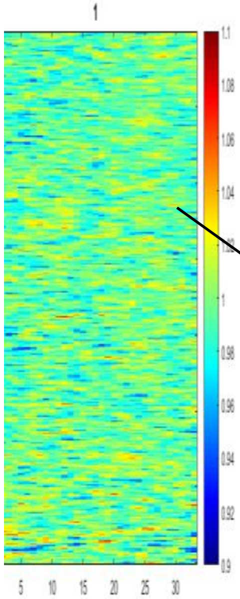
Real-time measurement during rheology experiment

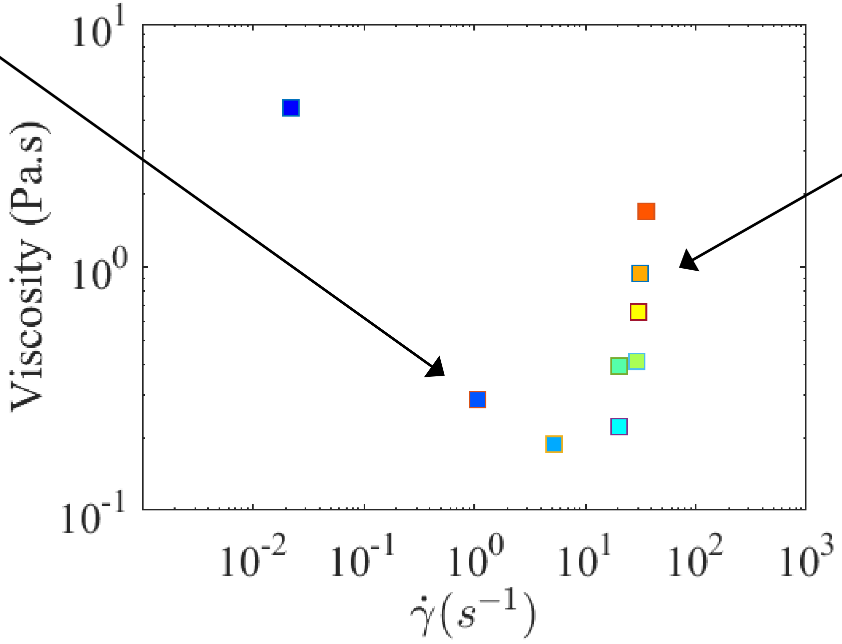
X-ray imaging: application to shear thickening

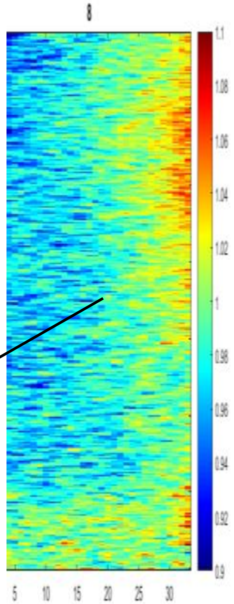
41% of **cornstarch in water/CsCl** in 1mm gap Couette *(stress inhomogeneity : 8%)*

Volume fraction field abruptly changes in the thickening regime

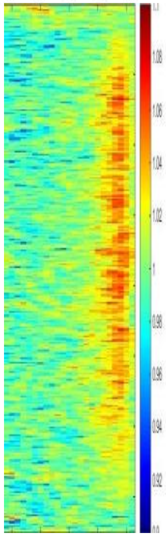
Homo geneous







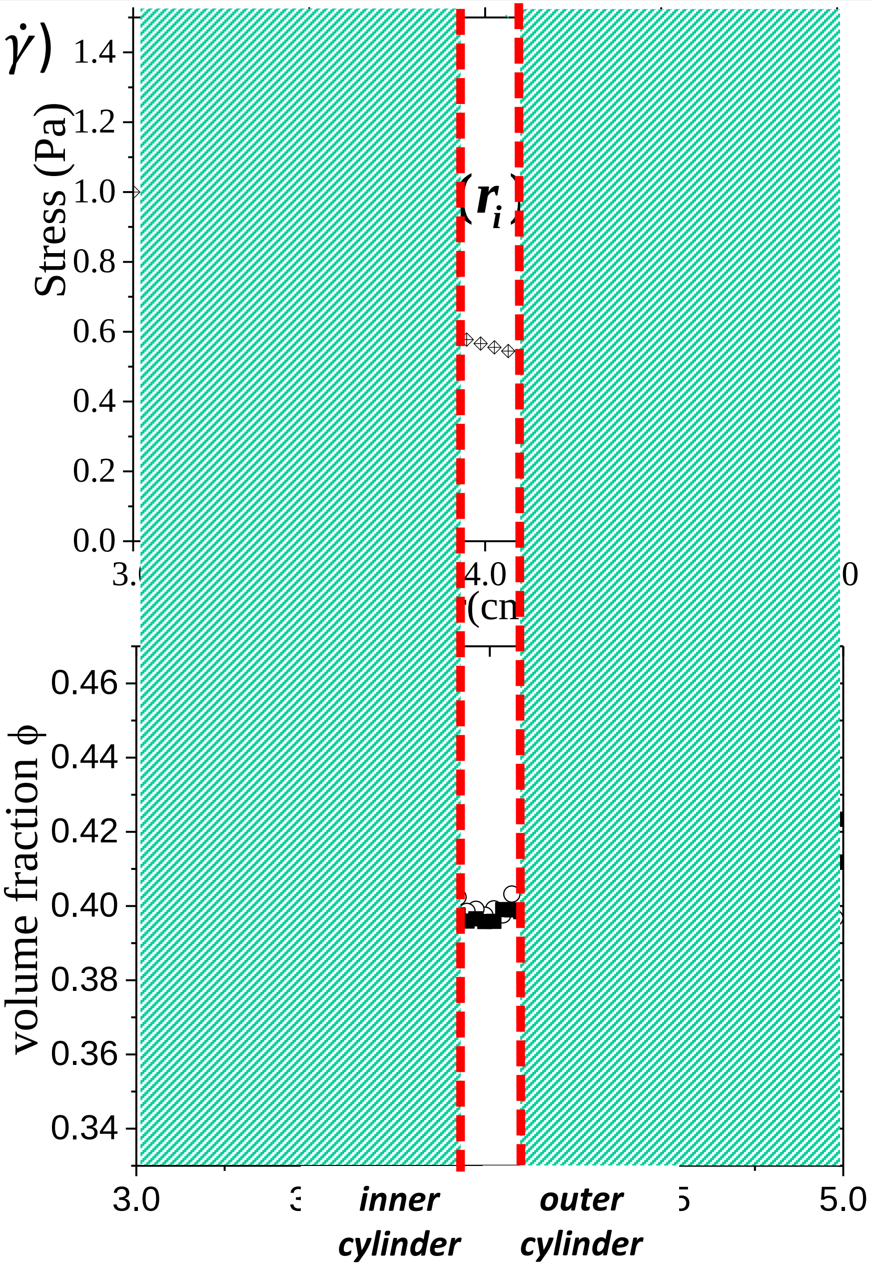
±5% variation in the gap



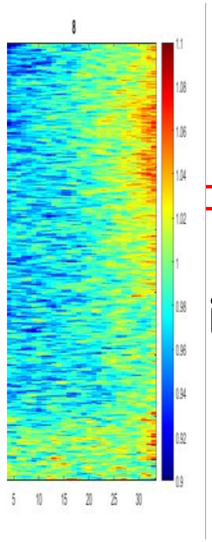
Same feature and magnitude in a $500\mu m$ gap, with smooth/rough boundaries

Steady-state: volume fraction in a thin gap Couette

Ex : viscous suspension ($\Sigma \propto \dot{\gamma}$)
40% of PS beads



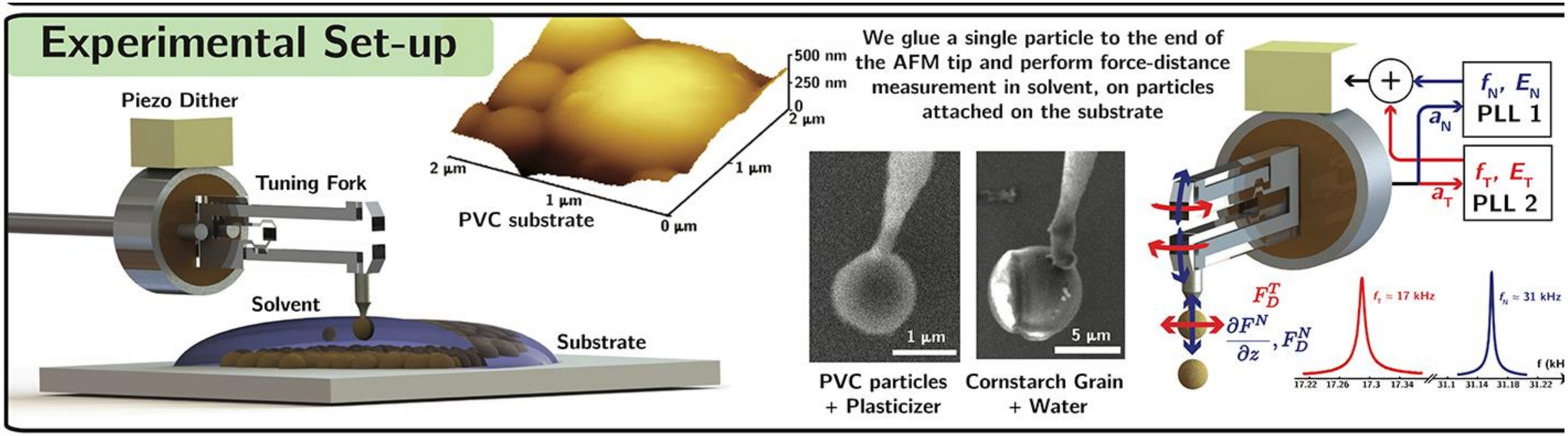
For a 8% stress inhomogeneity
(case for the thickening suspensions investigated)
$\pm 0.25\%$ relative vol. fraction expected



$\pm 5\%$ variation observed in cornstarch !

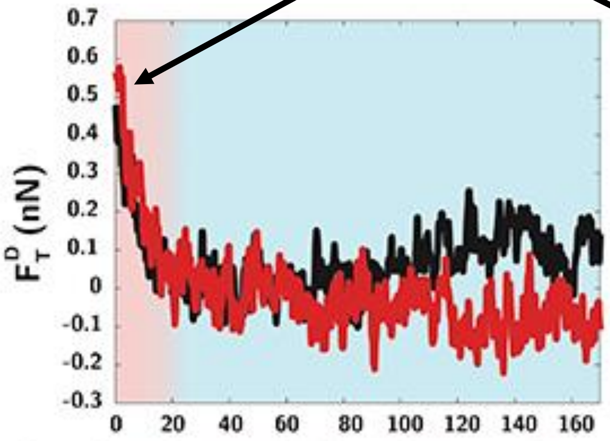
Now, physics can be discussed…

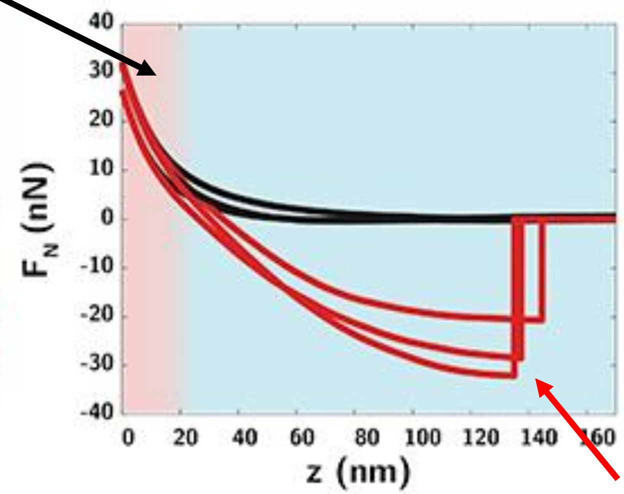
Force measurements between cornstarch particles



We glue a single particle to the end of the AFM tip and perform force-distance measurement in solvent, on particles attached on the substrate

Repulsion with friction





Hysteresis: adhesion force

Proposed mechanism

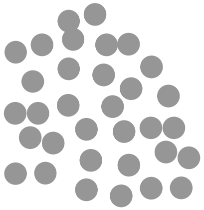
Not a revolution!…

…same idea as
Seto et al., PRL 2013
Wyart and Cates, PRL 2014…
… with adhesion

Transition from

Low stress / shear rate **:**
repulsion, frictionless particles

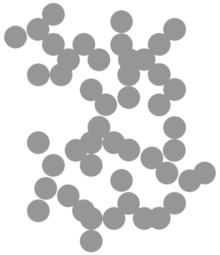
→ suspension of frictionless spheres with **viscous behaviour**



to

High stress/shear rate:
shear-induced adhesion

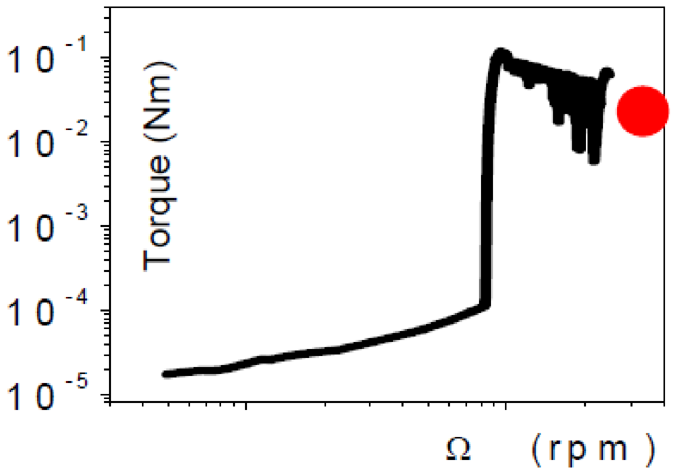
→ **cohesive granular material** (or colloidal gel) **with yield stress**



Order of magnitude, from adhesion force : $\tau_y \simeq 500\ Pa$ **consistent with post thickening behavior**

Linking yield stress behavior/local behavior

For a 43.9% mean volume fraction



Velocity profiles

of the material

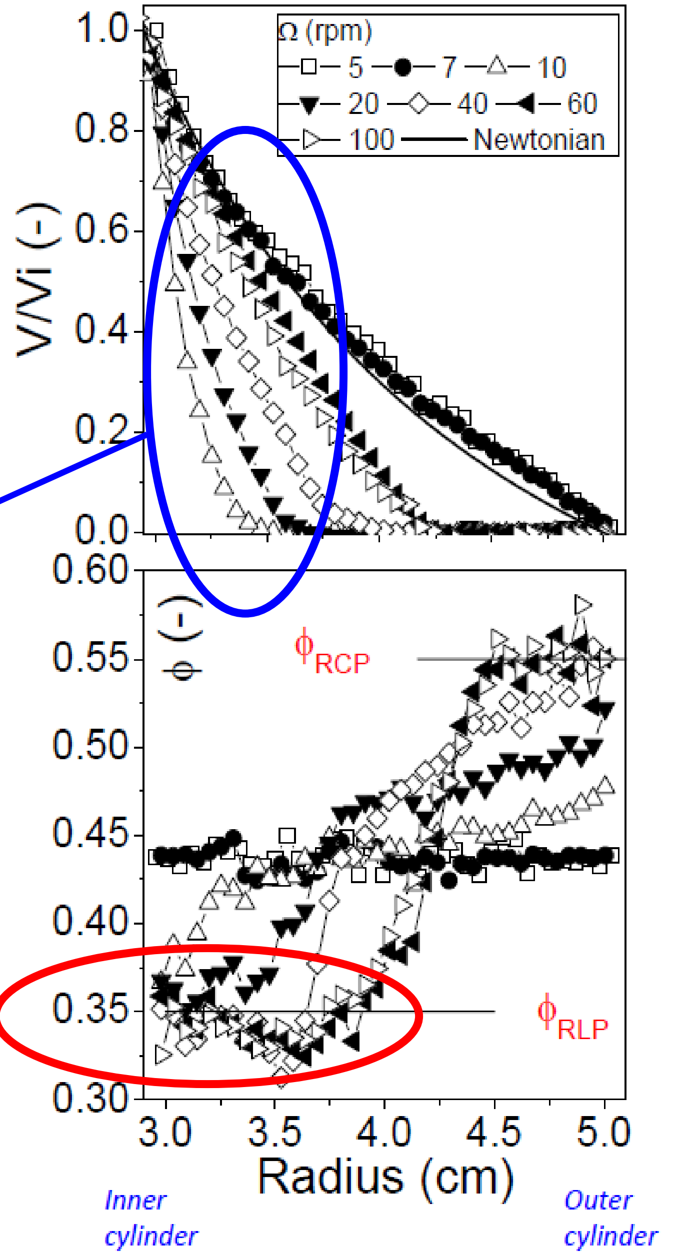
@ local vol. fraction ≃ 35%

consistent with

yield stress fluid behavior

(Herschel-Bulkley)

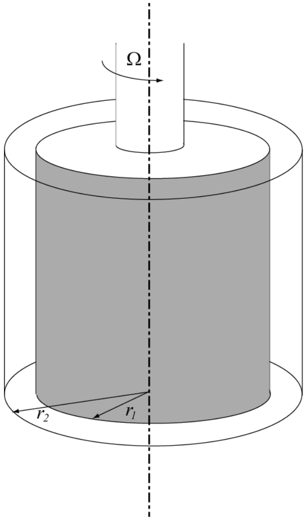
Yield stress ≃ hundreds of Pa

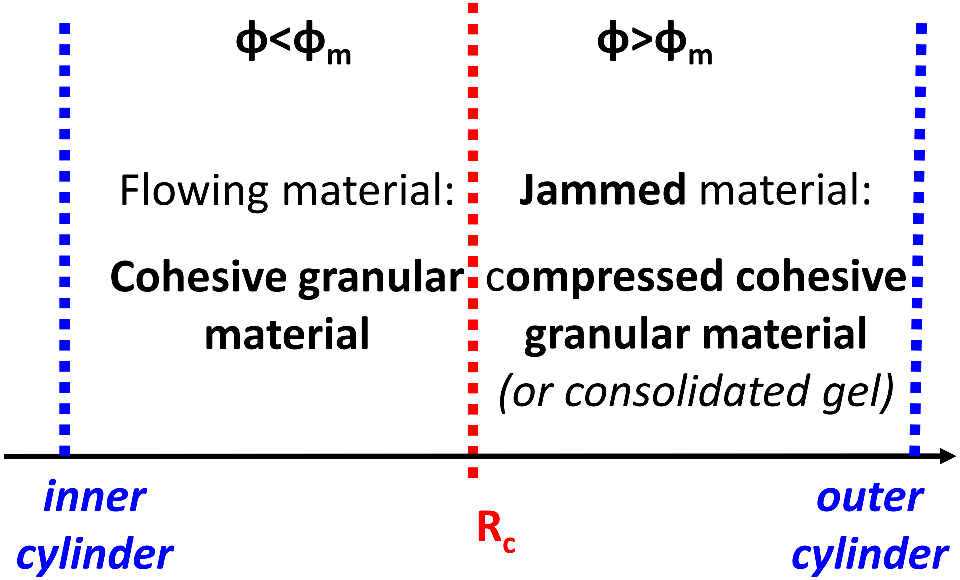


Fall et al., PRL (2015)

Linking shear-induced inhomogeneities/adhesion

- shear-thickening = shear-induced adhesion
- the cohesive granular material is irreversibly compressible up to ϕ_{rcp}





$\phi<\phi_m$

Flowing material:

Cohesive granular material

$\phi>\phi_m$

Jammed material:

compressed cohesive granular material

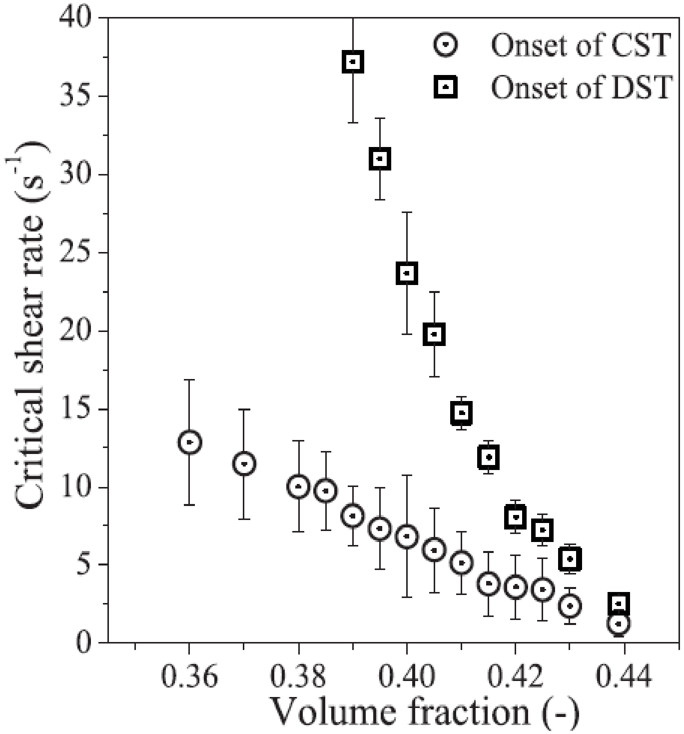
(or consolidated gel)

inner cylinder

R$_c$

outer cylinder

Accessible volume fractions



Jammed states

Compressed cohesive materials

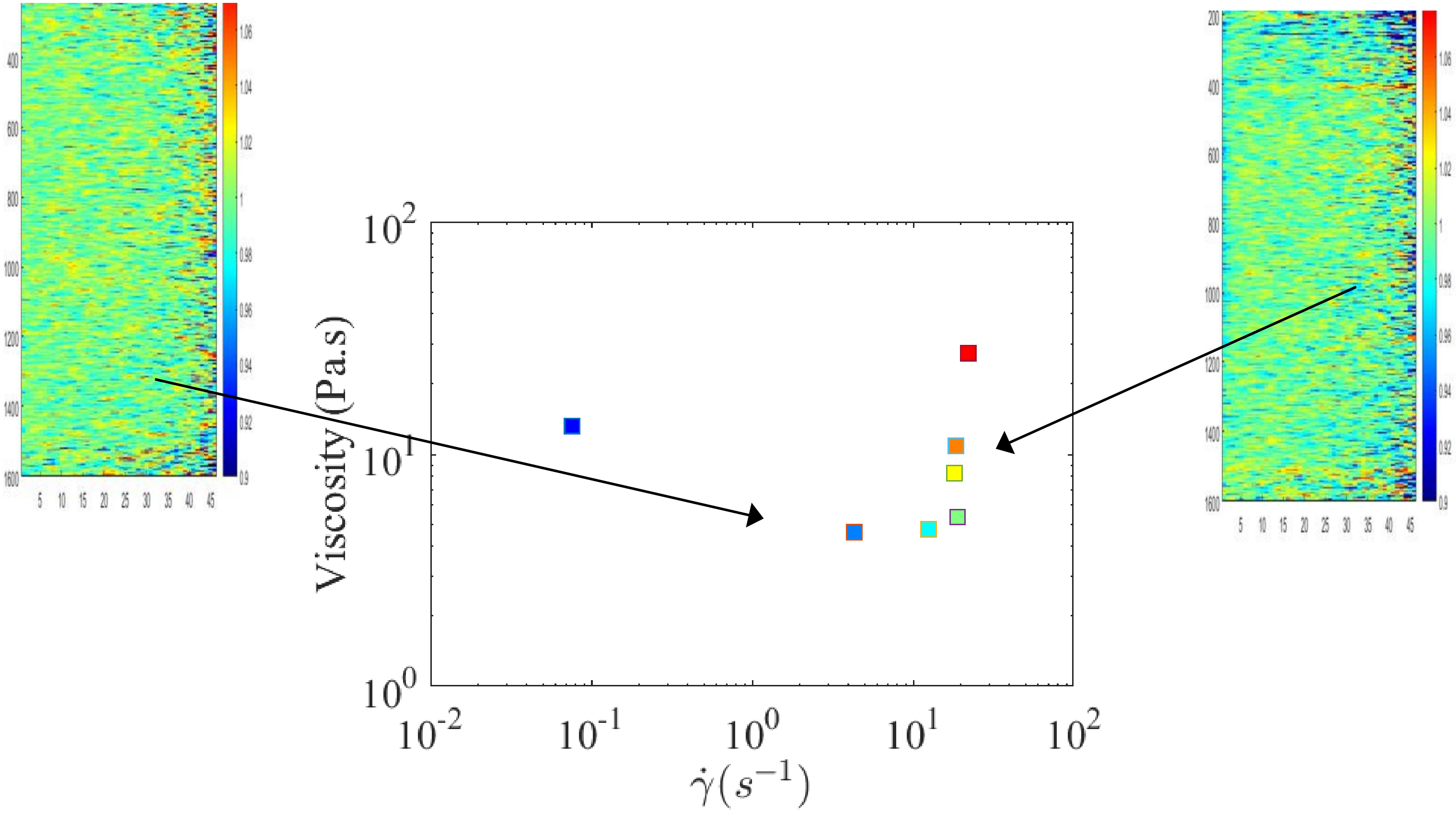
$\boldsymbol{\phi}$

$$\boldsymbol{\phi_0 = 45\%} \quad \neq \quad \boldsymbol{\phi_{rcp} = 55\%}$$

X-ray imaging: application to shear thickening

61% of **PVC particles in plasticizer** 1mm gap Couette *(stress inhomogeneity : 8%)*

Volume fraction fields in the gap of the Couette cell remain **constant**

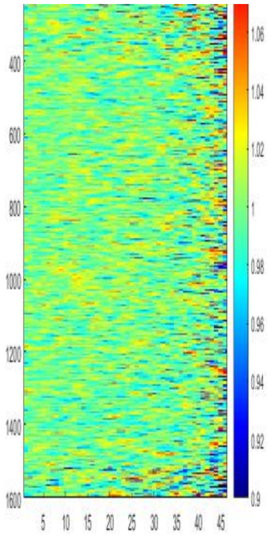


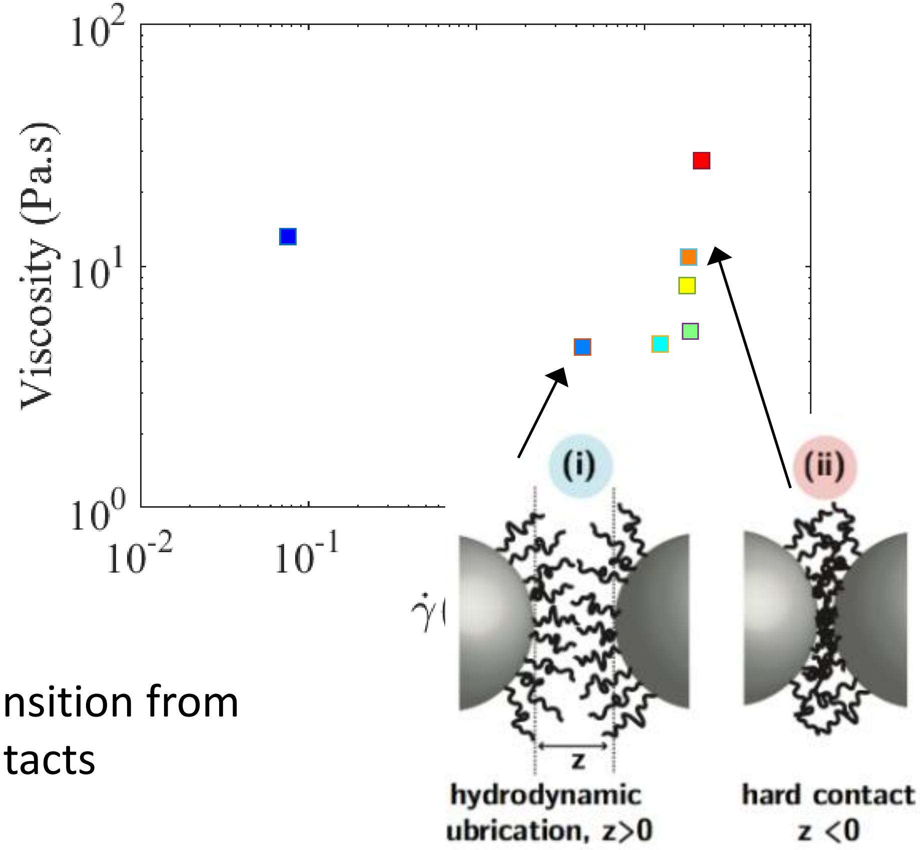
Discontinuous Shear Thickening is a viscosity jump
in the constitutive behavior of the homogeneous material

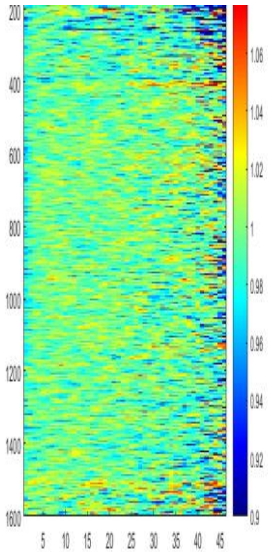
X-ray imaging: application to shear thickening

61% of **PVC particles in plasticizer** 1mm gap Couette *(stress inhomogeneity : 8%)*

Volume fraction fields in the gap of the Couette cell remain **constant**







Interpreted as transition from lubrication to contacts

Consistent with *Seto et al, PRL 2013* *Comtet et al., Nature communication 2017*

Conclusion

Strong interplay between migration / shear thickening / (Shear-) jamming

« Discontinuous shear-thickening »

jump in viscosity OR shear-induced yield stress OR shear jamming…

depends on interparticle forces

hard to know from only macroscopic measurements

Need for volume fraction measurements

Conclusion

Cornstarch:

- Revisit the origin of the **s-shape?**
  In which range of local vol. fraction does s-shape exist?
- Impacts on the surface of a Couette cell (*Peters et al., Nature 2016*)
  → **which volume fractions are truly investigated?**

Other systems investigated:

- **PMMA suspensions** → **inhomogeneous** at DST…
- **Silica suspensions** → **inhomogeneous** at DST…

} To be studied in more depth

- **PVC suspensions** (Comtet et al., Nature. Comm 2016) :
  « THE » canonical system!