Measurement Protocols for Many-body Properties in AMO Systems

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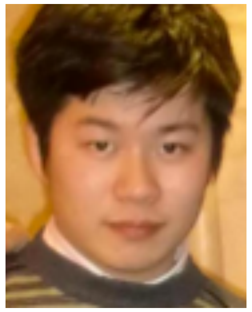
Designer Quantum Systems Out of Equilibrium
KITP Nov 2016

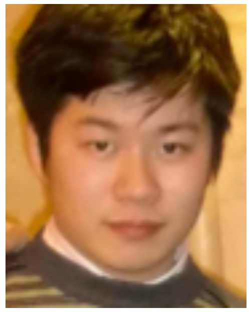
Given a many-body system (designer quantum system), what are the efficient ways to characterize and measure quantum states?

1. Measurement Protocol for the Entanglement Spectrum: cold atoms and circuit-QED

Hannes Pichler*, Guanyu Zhu*, Alireza Seif*, Peter Zoller and M.H. arXiv:1605.08624 (2016) to appear in PRX

2. Measurement of scrambling, out-of-time-order correlators

Guanyu Zhu, MH, and Tarun Grover arXiv: 1607.00079

Separable state:

$$|\Psi\rangle = |\uparrow\downarrow\rangle \qquad |\Psi\rangle\langle\Psi| = |\uparrow\downarrow\rangle\langle\downarrow\uparrow|$$

$$\mathrm{Tr}_2(|\Psi\rangle\langle\Psi|) = |\uparrow\rangle\langle\uparrow|$$

pure state

Entangled state:

$$|\Psi\rangle = \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

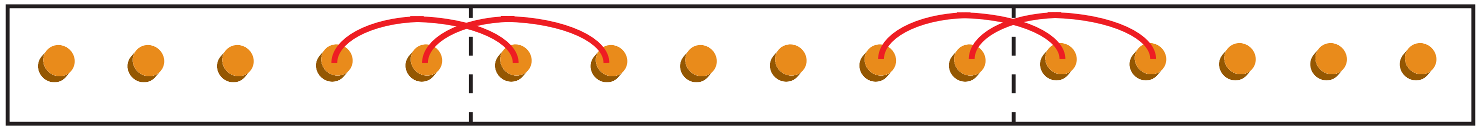
$$\mathrm{Tr}_2(|\Psi\rangle\langle\Psi|) = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$$

mixed state

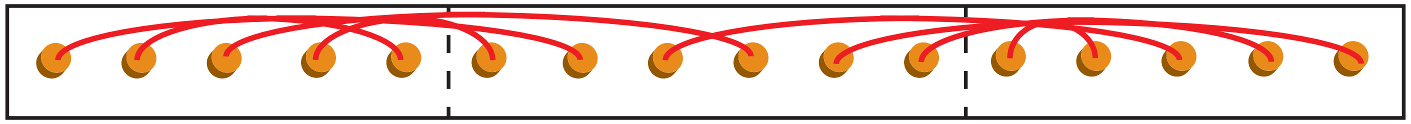
purity $\mathrm{Tr}(\rho^2)$ of the partially traced system (or in general entropy of it), tells us about the entanglement in the original system

Extension to many-body systems

$$S_A = -\mathrm{Tr}\{\rho_A \ln \rho_A\}$$



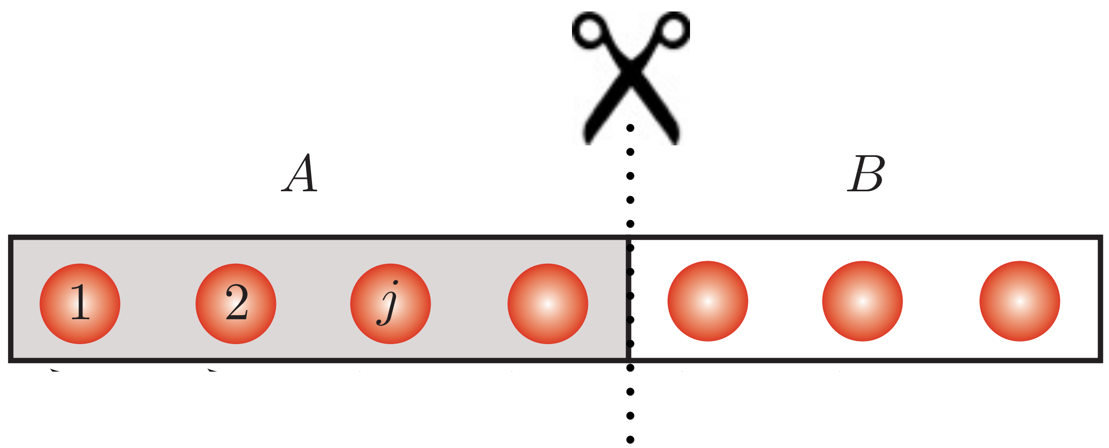
$S_A \sim L^{d-1}$ (Area Law)



$S_A \sim L^{d}$ (Volume Law)

Renyi entropy: $S_A^{(n)} = \dfrac{1}{1-n} \ln \mathrm{Tr}\{\rho_A^n\} \xrightarrow{\text{purity}} S^{(2)} = -\ln \mathrm{Tr} \rho^2$

Entanglement Spectrum
Entanglement Hamiltonian



Wavefunction: $|\,\phi\,\rangle$

$$\rho_A = Tr_B|\,\phi\,\rangle\langle\,\phi\,| = \sum_{\alpha=1}^{D} e^{-\xi_\alpha}|\,\chi_\alpha\,\rangle\langle\,\chi_\alpha\,|_A$$

$$\equiv e^{-H_E}$$

Entanglement Hamiltonian

Schmidt Eigenvalues: $\lambda_\alpha = e^{-\xi_\alpha} \in [0,1]$

Entanglement Spectrum: $\{\lambda_\alpha\}$ or $\{\xi_\alpha\}$

Entanglement "energy"

Li and Haldane PRL (2008) FQHE

Pollmann et al. PRB (2010) Haldane phase

A. Chandran, V. Khemani, and S. L. Sondhi, PRL (2014) How powerful?

N. Laflorencie Physics Report 643, 1-59 (2016) Recent review

Also Nielsen PRL (1999): entanglement transformation $|\psi\rangle \xrightarrow{LOCC} |\phi\rangle \Leftrightarrow \lambda[\phi] \succ \lambda[\phi]$

Recent experimental developments

- cold atoms on optical lattices

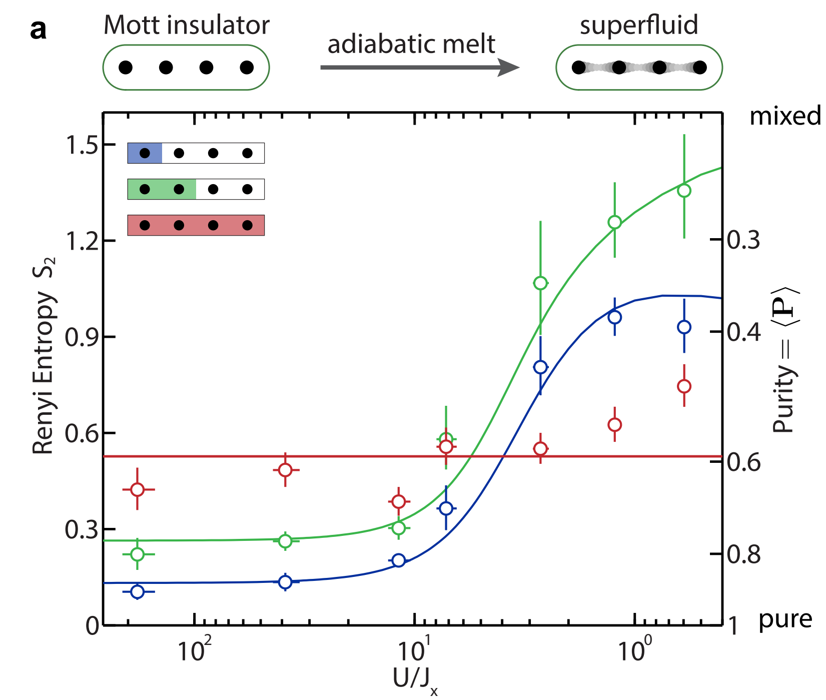
ARTICLE

Nature 528, 77 (2015)

doi:10.1038/nature15

Measuring entanglement entropy in a quantum many-body system

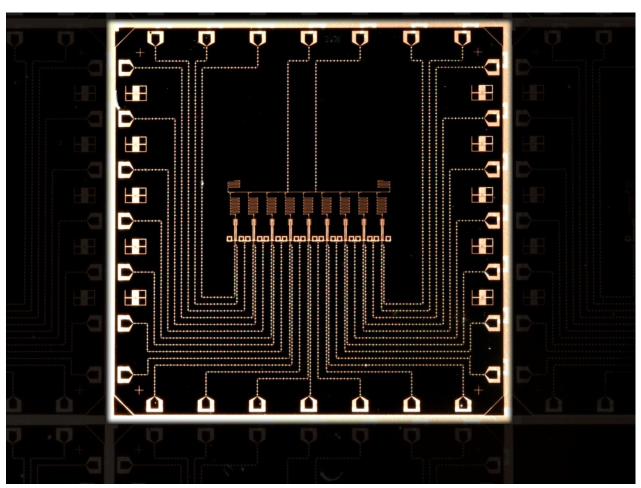
Rajibul Islam[1], Ruichao Ma[1], Philipp M. Preiss[1], M. Eric Tai[1], Alexander Lukin[1], Matthew Rispoli[1] & Markus Greiner[1]



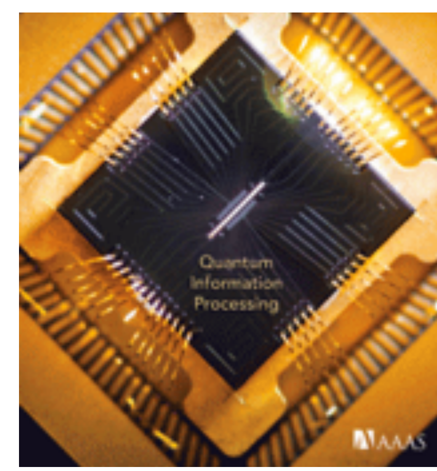
Daley, Pichler, Schachenmayer, Zoller, PRL (2012)
Abanin Demler PRL (2012)

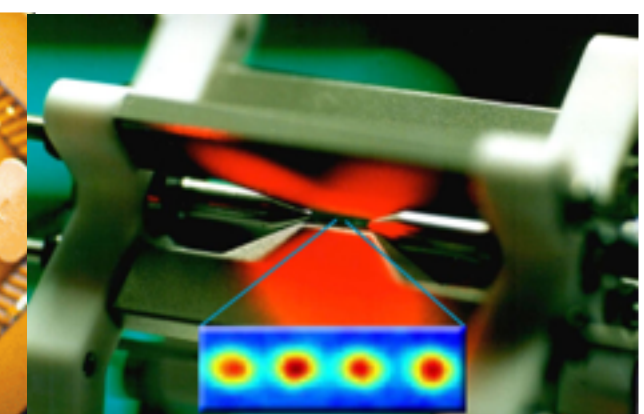
- circuit-QED

Google



- Ion traps





JILA, JQI, Innsbruck

➡ Challenges:

✓ Estimating a function from its moments is not usually efficient
H. Song, et al. PRB (2012), I. Klich and Levitov PRL (2009)

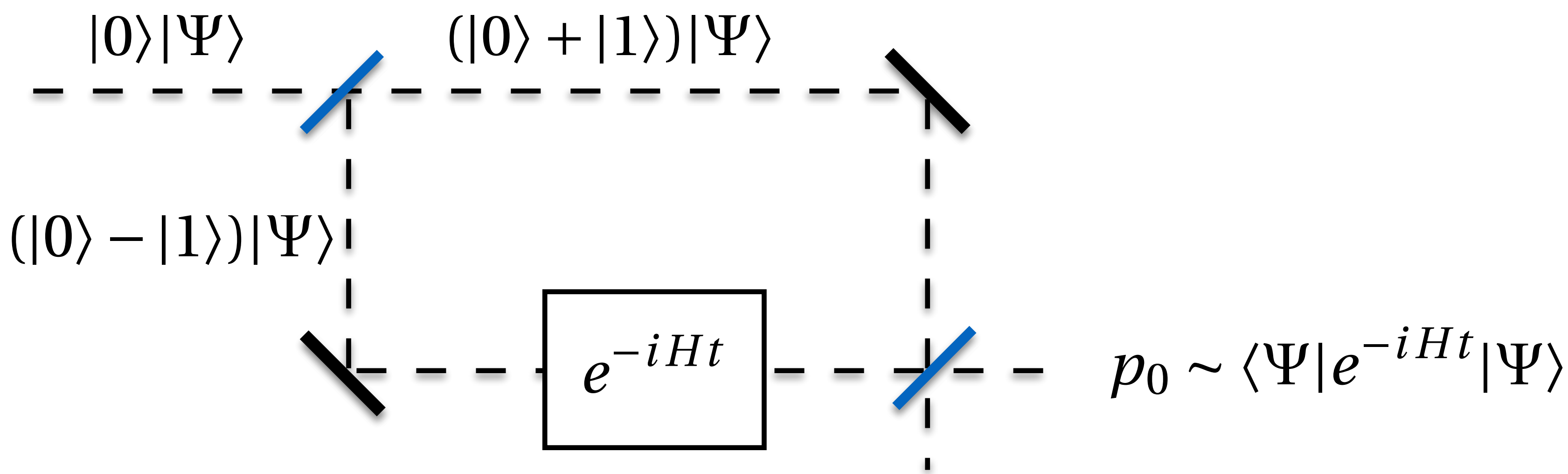
✓ Full quantum state tomography is very difficult, if not impossible

Spectroscopy of density matrix

Goal: Find the spectrum of the density matrix

Reminder: To find spectrum of the Hamiltonian, we access e^{-iHt}

Ramsey Spectroscopy



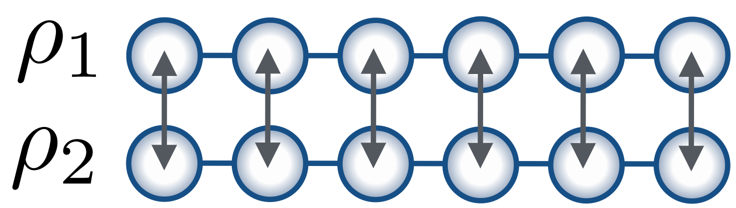
M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, and P. Zoller, PRL (2009).
M. Knap, A. Kantian, T. Giamarchi, I. Bloch,4,5 M. Lukin, and E. Demler PRL (2013)
M. Beverland, J. Haah, G. Alagic, G.Campbell, A. M. Rey, and A. Gorshkov (2016)

Spectroscopy of density matrix

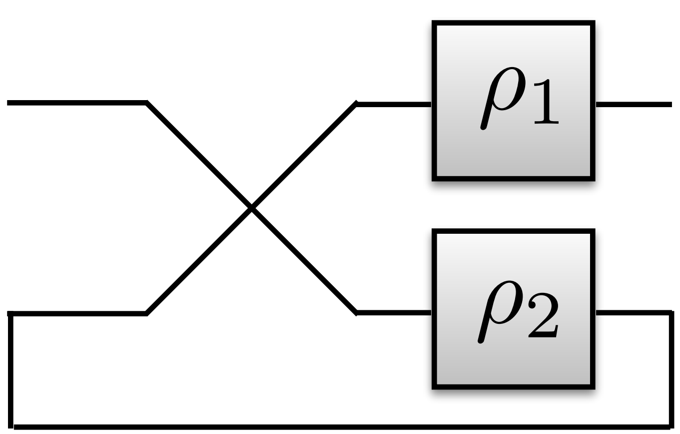
Solution: If we have access to $e^{-i\rho t}$, then we can find the spectrum of ρ

Trick one: Global SWAP between two copies simulates the evolution by the density matrix:

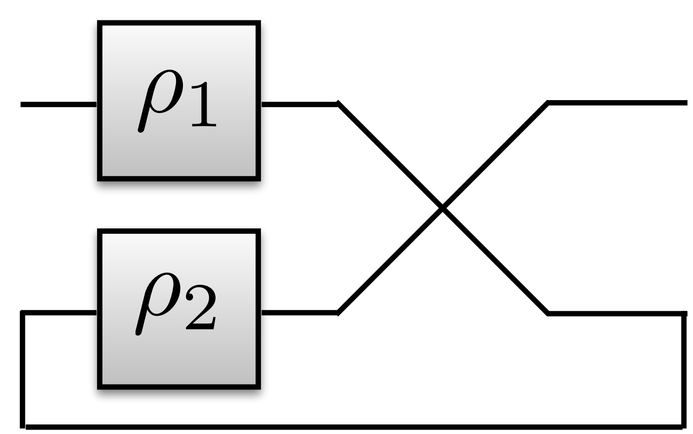
S. Lloyd et al. Nat. Phys. 10, 631 (2014)



$$\mathrm{tr}_2\{e^{-i\epsilon S}\rho_1 \otimes \rho_2\, e^{i\epsilon S}\} = \rho_1 - i\epsilon[\rho_2, \rho_1] + \mathcal{O}(\epsilon^2)$$



$= \rho_2\rho_1$



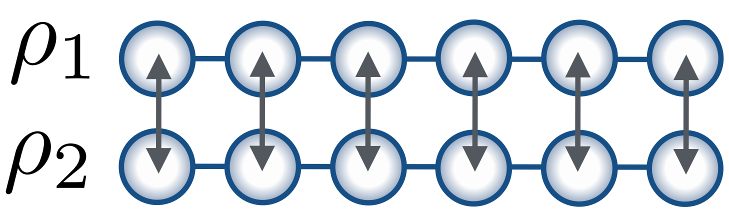
$= \rho_1\rho_2$

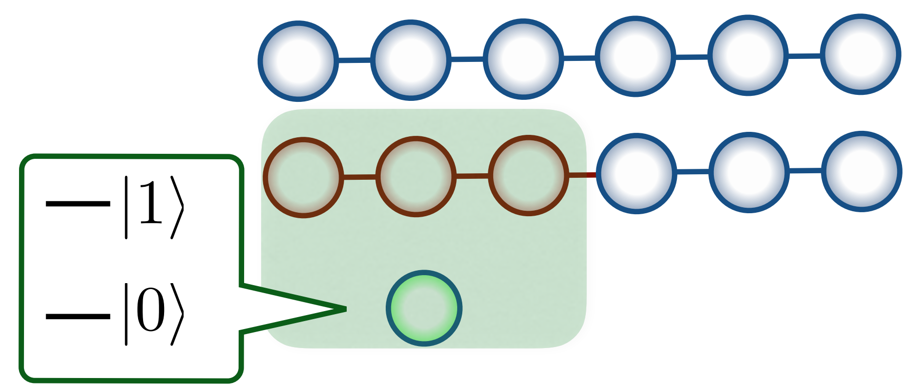
➡ Requirements:

✓ We need a global SWAP operator $S = \prod_{j \in A} S_j$, specifically $e^{i\epsilon S}$

✓ We do NOT want to use a Q-Fourier transform to get the spectrum

Trick two: controlled global SWAP can be achieved by dispersive coupling and the spectrum can be measured by a Ramsey scheme





SWAP: $\hat{a}_1 \leftrightarrow \hat{a}_2 \quad e^{i\frac{\pi}{2}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)}$

can be decomposed into tunneling and parity measurement

$$\hat{a}_{1,2} \to \frac{1}{\sqrt{2}}(\hat{a}_1 \pm \hat{a}_2) \qquad e^{i\pi \hat{a}_2^\dagger \hat{a}_2}$$

A dispersive Hamiltonian leads to parity operation:

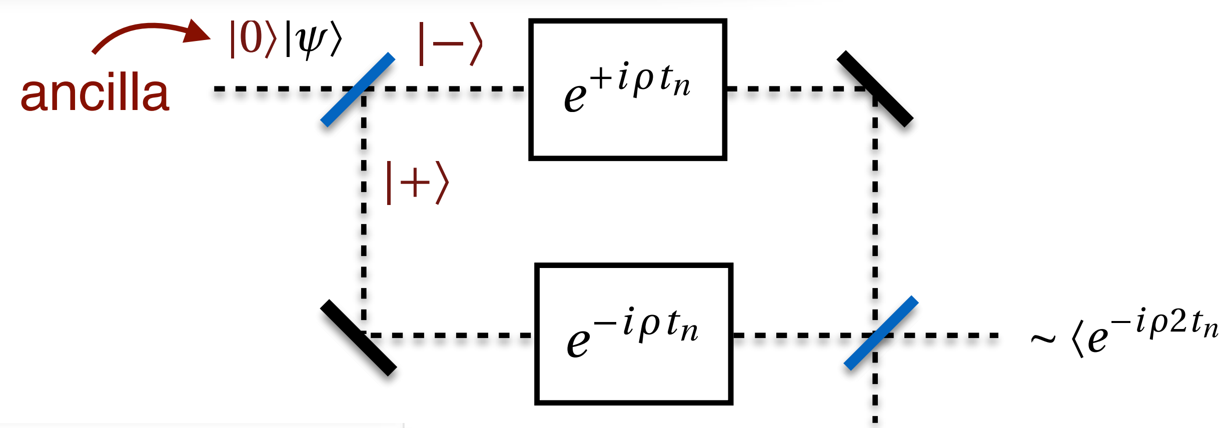
$$H_{\rm dis} = \chi |0\rangle\langle 0| \sum_{j \in \mathcal{A}} \hat{n}_{j,2}$$

$$U_{\rm c-phase} = |1\rangle\langle 1| \otimes \mathrm{I} + |0\rangle\langle 0| \otimes \boxed{(-1)^{\sum_{\rm j \in \mathcal{A}} \hat{\rm n}_{\rm j,2}}}$$

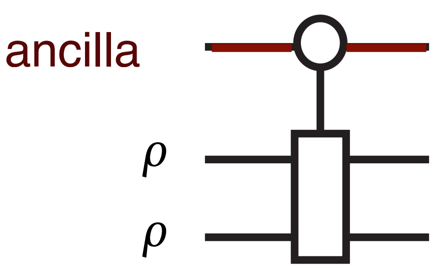
Trick three: infinitesimal rotation of the ancilla and SWAP, gives $e^{i\epsilon S}$

$$U_\epsilon = \exp(-i\epsilon(|0\rangle\langle 1| + |1\rangle\langle 0|))$$

Density matrix spectroscopy

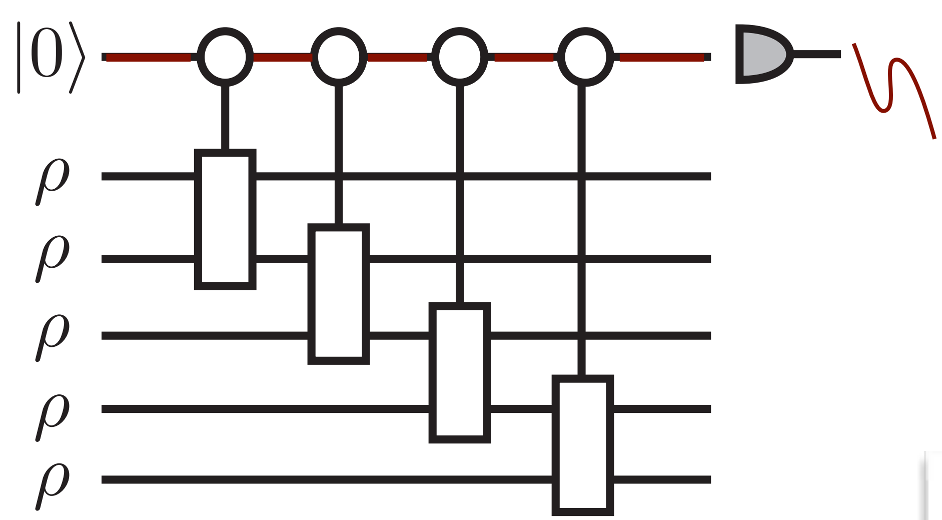


- **elementary step:** conditional $\exp(\pm i\epsilon S)$



$$U_{\text{step}} = |-\rangle\langle -| \otimes \exp(-i\epsilon S) + |+\rangle\langle +| \otimes \exp(+i\epsilon S)$$

$|\pm\rangle \sim |0\rangle \pm |1\rangle$



n copies

read out

conventional Ramsey:

$$\langle Z \rangle = \mathrm{tr}\{\rho \mathrm{e}^{-\mathrm{iHt}}\}$$

$n\epsilon$ 'time'

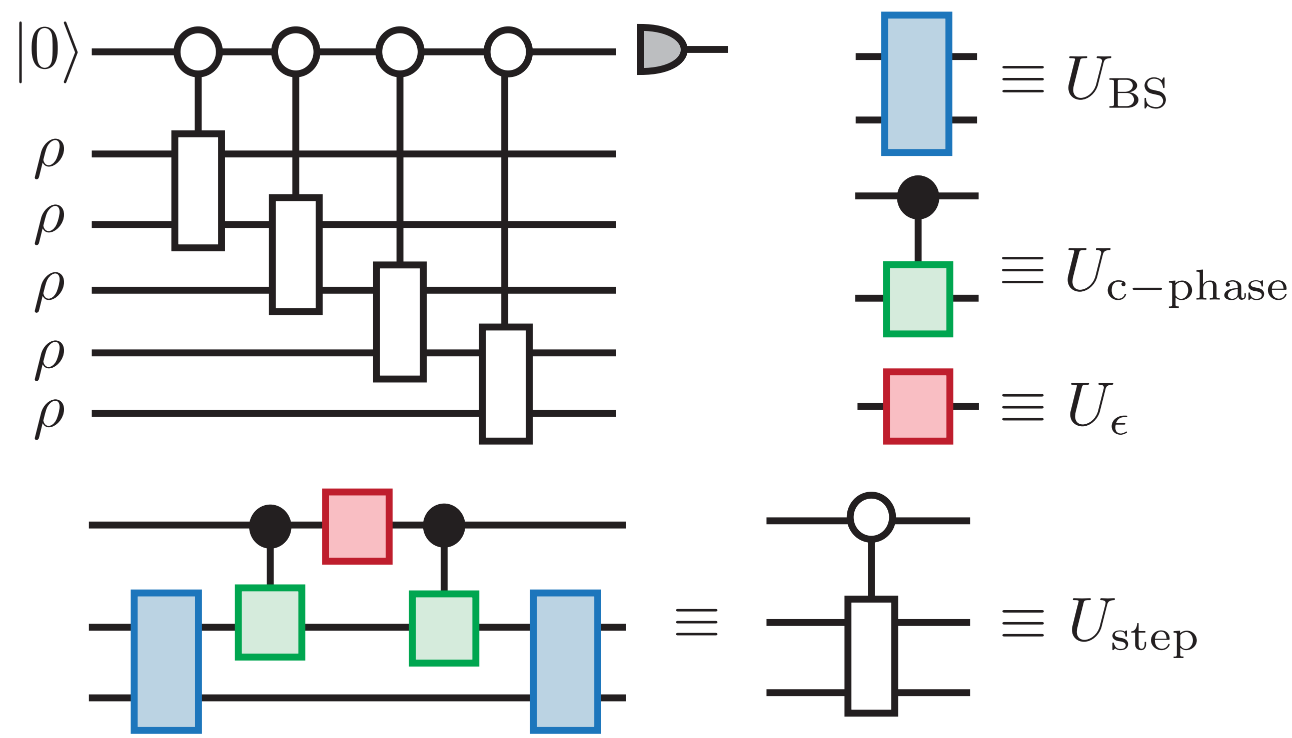
$$\langle Z \rangle_n = \frac{1}{2}\mathrm{tr}\{e^{-i2t_n\rho}\rho + \rho e^{i2t_n\rho}\}$$

$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

population difference

first system acts as ‘Hamiltonian’ for second system

Summary of protocol



(I) Initialize identical copies and the ancilla

(II) Stroboscopic application of

(a) tunneling between two copies for beamsplitter operation

(b) controlled-phase operation

(c) single qubit rotation on ancilla

(III) Measurement of ancilla in Z basis

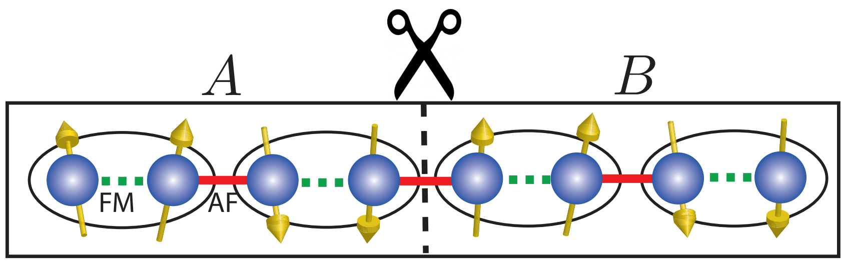
fourier transform give the eigenstates

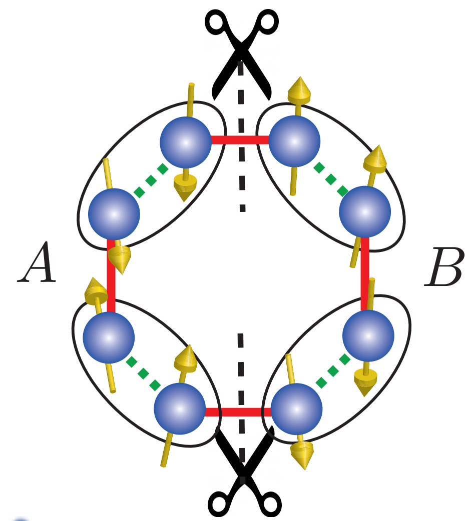
$$\langle Z\rangle_n = \mathrm{tr}\{\rho\cos(2t_n\rho)\} = \sum_\alpha \lambda_\alpha \cos(2t_n\lambda_\alpha)$$

- ☑ Spectrum via Fourier Transform
- ☑ Degeneracy from weights

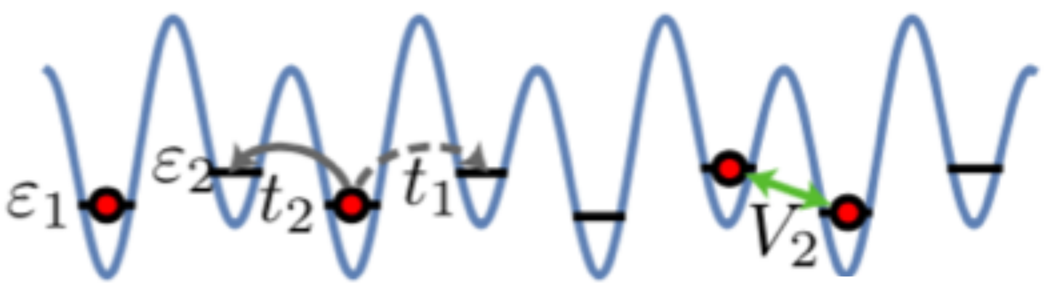
$$\rho = \sum_\alpha \lambda_\alpha |\Psi_\alpha\rangle\langle\Psi_\alpha|$$

Detecting topological degeneracy and entanglement gap of the Haldane phase



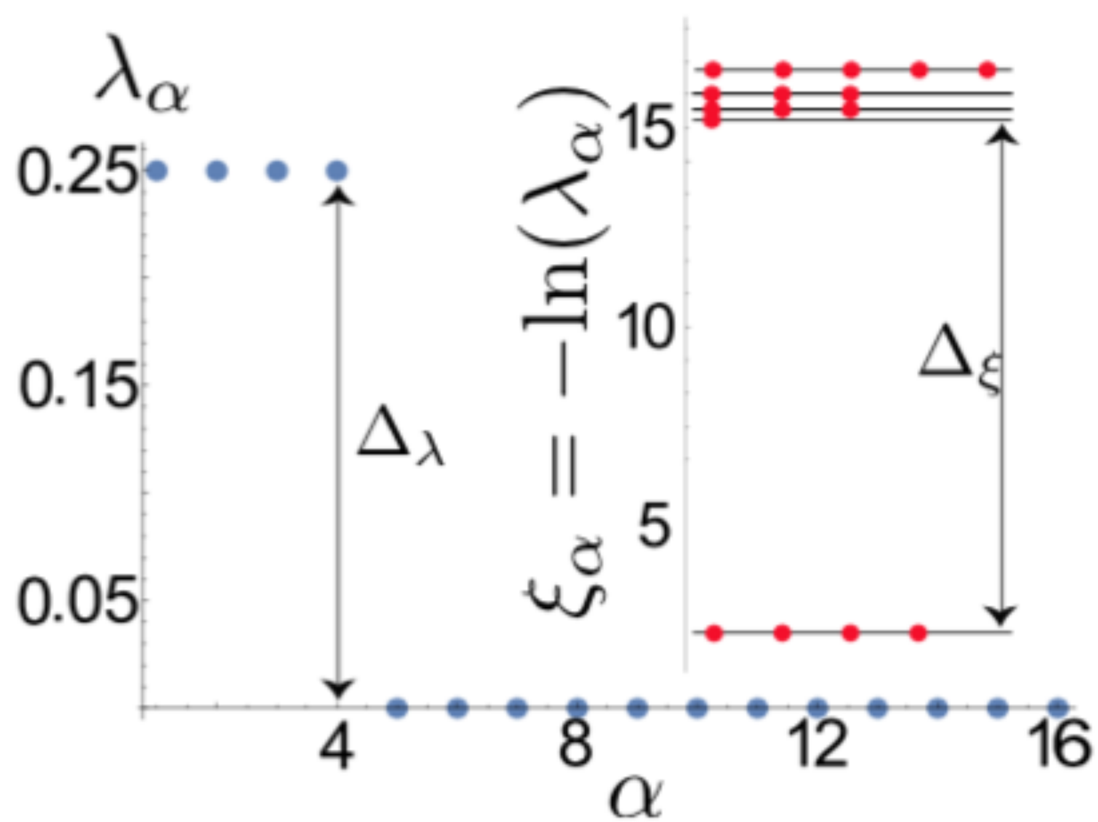


$$H = \sum_{j=1}^{N/2} [J_1(\vec{S}_{2j} \cdot \vec{S}_{2j+1}) - J_2(\vec{S}_{2j-1} \cdot \vec{S}_{2j})]$$



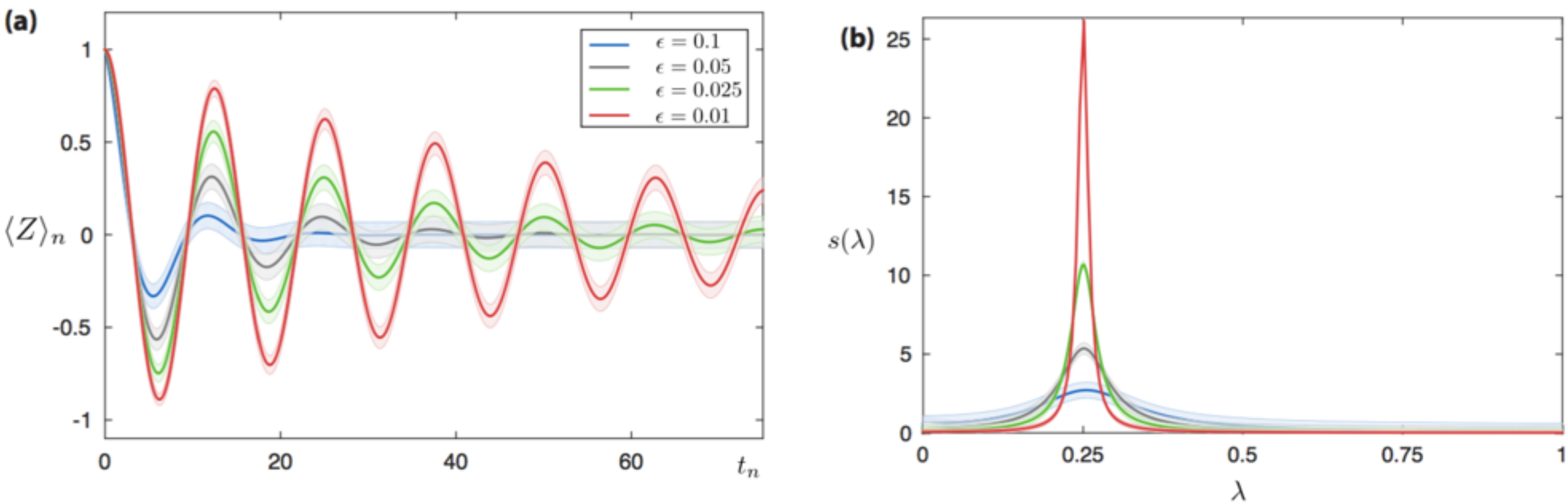
cold atom realization: extended-Bose-Hubbard model

e.g. Ferlaino's group Science (2016)



$J_1 = 1, J_2 = 0.1, N_{\text{site}} = 8$

Ramsey signal and Fourier transform



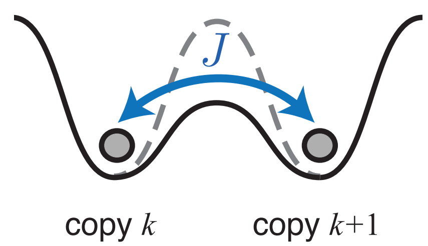
Signal decay due to 1st-order Trotter approximation

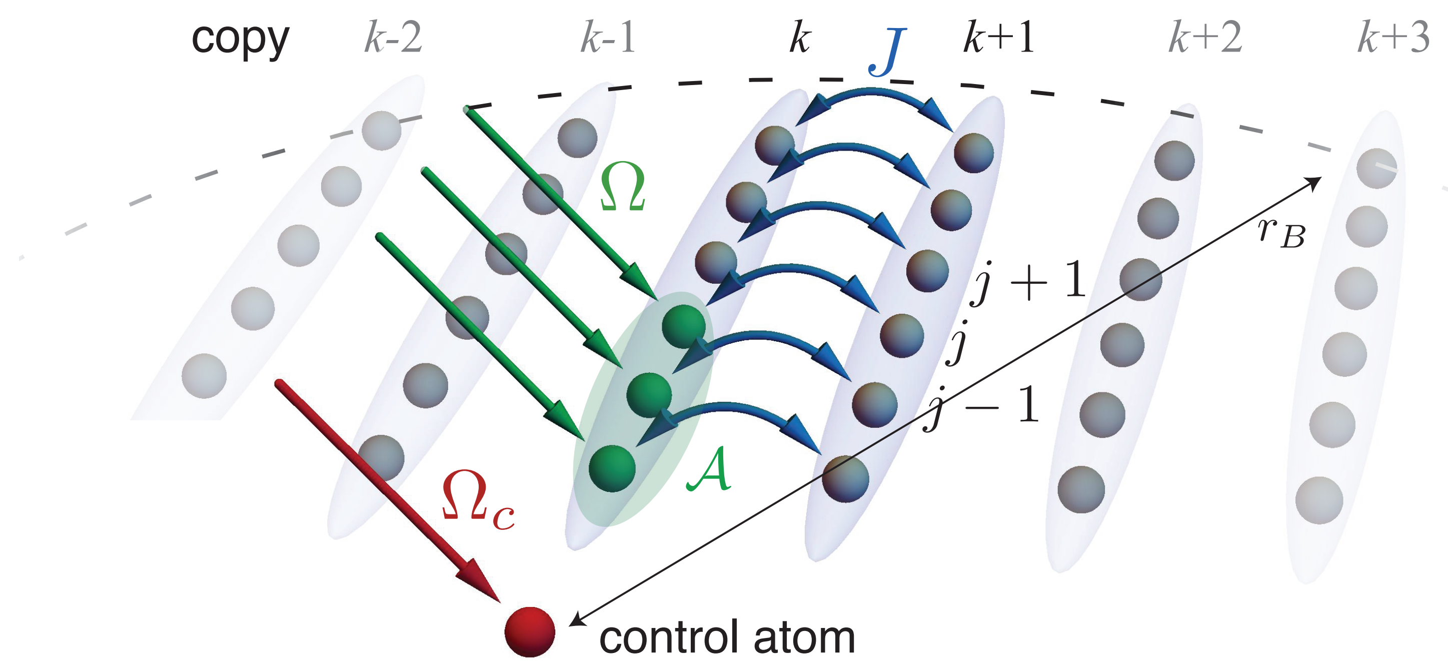
Minimum number of resets (stroboscopic steps)
to detect 4-fold degeneracy: ~50-100

Experimental realizations: atoms optical lattice

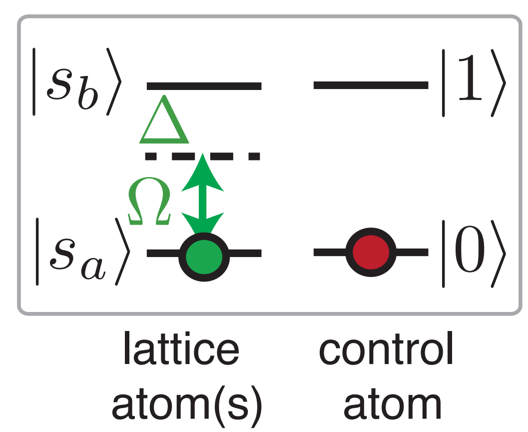
copies are different chains:

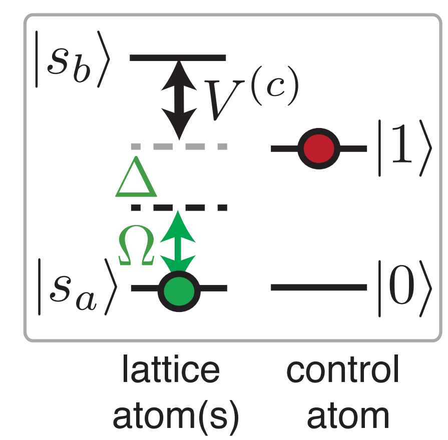
Tunneling :





control-phase gate by Rydberg dipole blockade
Saffman, Walker, Molmer RMP (2010)



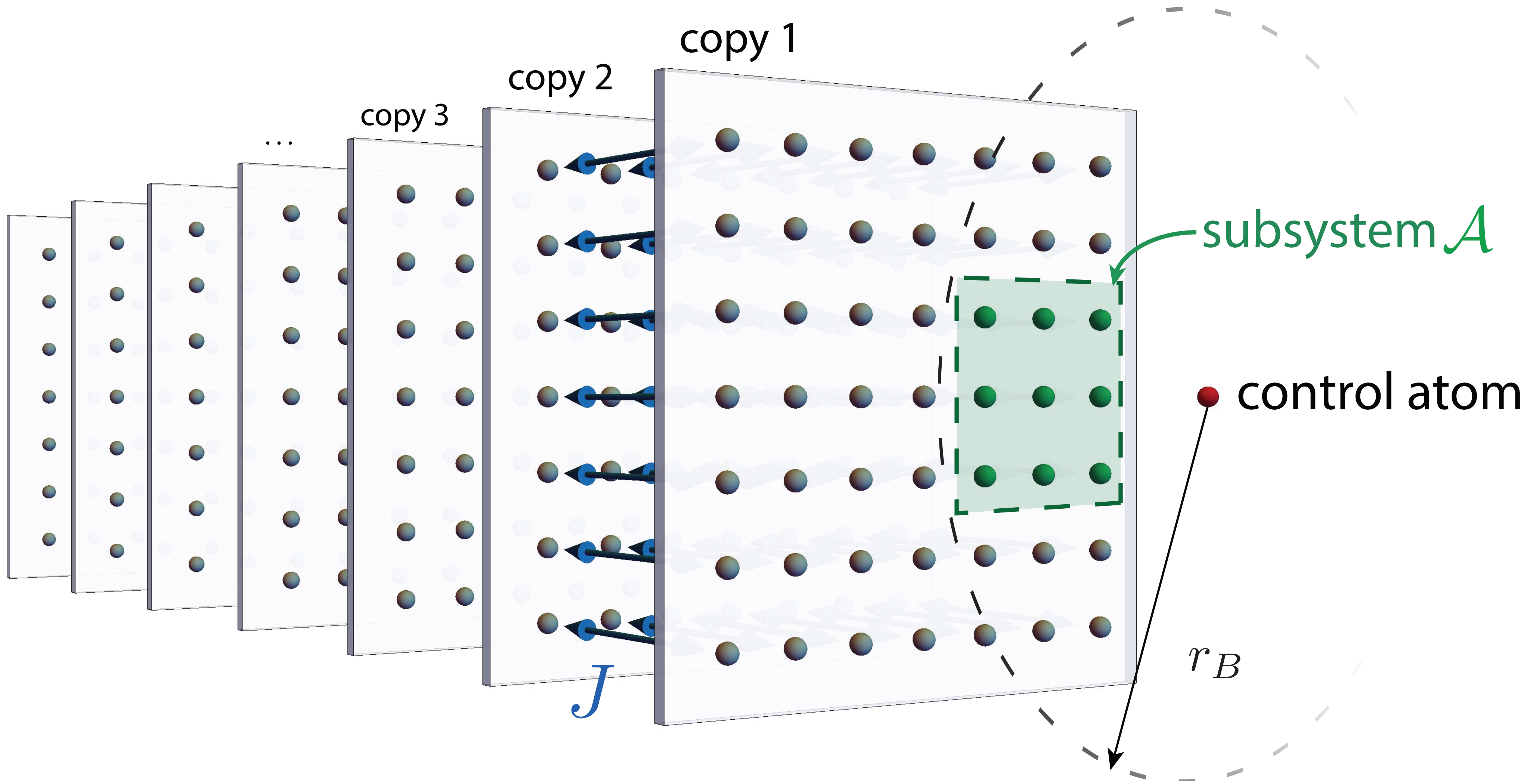


$$H = \sum_j -\Delta b^\dagger_{j,k} b_{j,k} + \Omega(b^\dagger_{j,k} a_{j,k} + a^\dagger_{j,k} b_{j,k}) + \sum_j V_j^{(c)} |\,1\,\rangle_c \langle\,1\,| \otimes b^\dagger_{j,k} b_{j,k} + \sum_{j,l} V_{j,l}^{(b)} b^\dagger_{j,k} b^\dagger_{l,k} b_{l,k} b_{j,k}$$

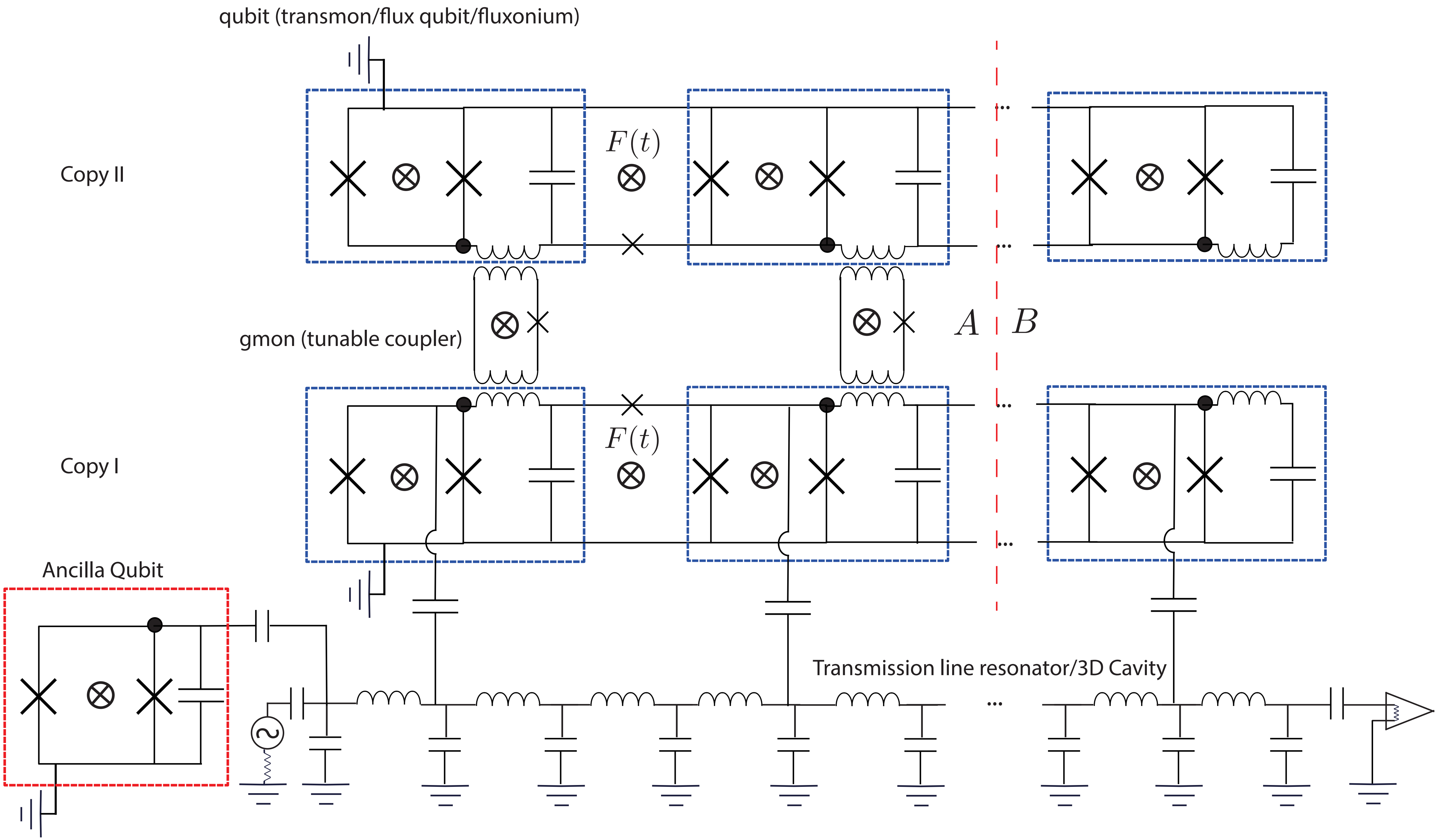
$$|V_j^{(c)}| \gg |\Delta| \gg |\Omega|\sqrt{N_{\mathcal{A}}} \qquad H_{\text{eff}} = \sum_j \frac{\Omega^2}{\Delta} |\,0\,\rangle\langle\,0\,|_c \otimes a^\dagger_{j,k} a_{j,k}$$

AC-Stark shift $\quad t_{\text{phase}} = \pi\Delta/\Omega^2$

Generalization to 2D



Experimental Realizations: circuit-QED



$$H_{\text{eff}} = \sum_j \frac{\Omega^2}{\Delta} a_c^\dagger a_c \otimes \sigma^z_{\text{I},j}$$

poster!

1. Measurement Protocol for the Entanglement Spectrum

Hannes Pichler*, Guanyu Zhu*, Alireza Seif*, Peter Zoller and M.H. arXiv:1605.08624 (2016)

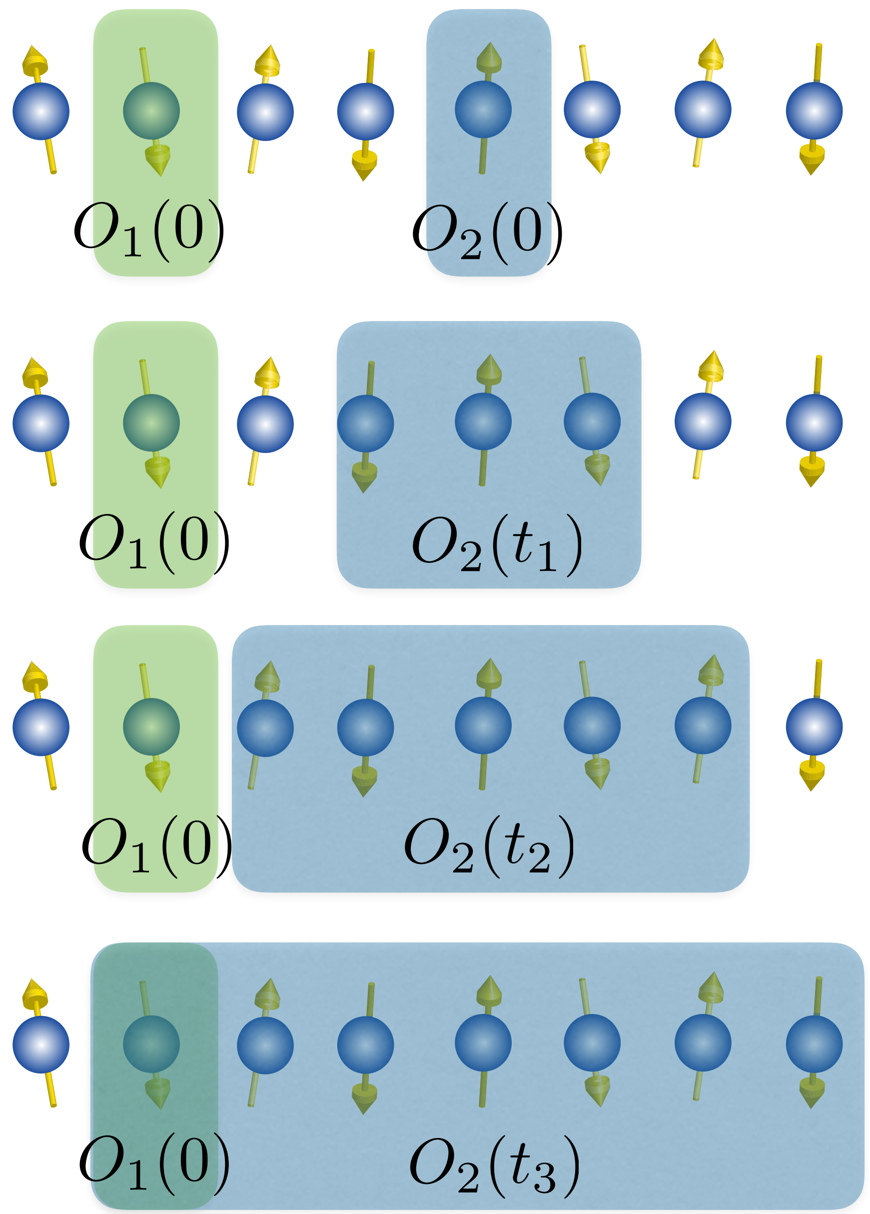
2. Measurement of scrambling, out-of-time-order correlators

Guanyu Zhu, MH, and Tarun Grover arXiv: 1607.00079

How fast a single spin get entangled to the rest of the system and the quantum information is scrambled?

Spreading of logical qubit in the Heisenberg picture:

$$O_2(t) = e^{iHt} O_2(0) e^{-iHt}$$



$$\langle |[O_2(t), O_1(0)]|^2 \rangle = 2[1 - \mathcal{R}e(C)]$$

$$C = \langle O_2(t) O_1(0) O_2(t) O_1(0) \rangle$$

One needs to measure the overlap:

$$\langle \psi | e^{iHt} O_2 e^{-iHt} O_1 e^{iHt} O_2 e^{-iHt} O_1 | \psi \rangle$$

J. Maldacena, S. H. Shenker, D. Stanford (2015)

Goal: Measure the many-body state overlap

Solution: Couple an ancilla and perform Ramsey interferometry, under forward and backward propagation in time.

- Simulate the forward and backward propagation by changing the parameters of the quantum simulator

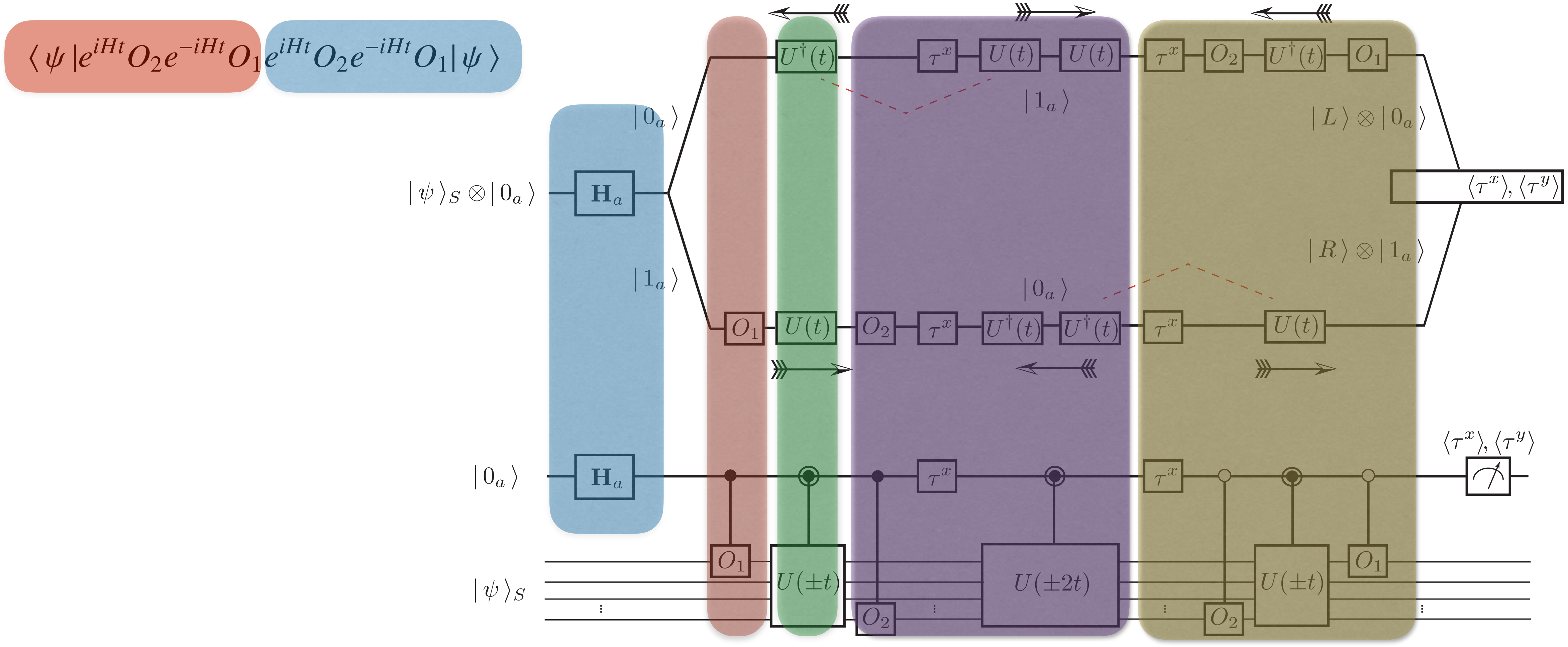
  Swingle et al. 1602.06271 Monika Schleier-Smith's talk
  N. Yao et al. arXiv:1607.01801 (two copies, only forward)
  M. Garttner et al. 1608.08938 Ana Maria Rey's talk

- Using the ancilla as a quantum clock to change the arrow of time.

$$H_{\mathrm{tot}} = (1 - 2a^\dagger a) \otimes H$$

$$U_{\mathrm{tot}}(t) = e^{-iH_{\mathrm{tot}}t} = e^{-iHt} \otimes |0_a\rangle\langle 0_a| + e^{iHt} \otimes |1_a\rangle\langle 1_a|$$

Important benefit: Calibration and benchmarking



$$|\psi\rangle_S \otimes |0_a\rangle$$

$$\frac{1}{\sqrt{2}}|\psi\rangle_S \otimes [|0_a\rangle + |1_a\rangle]$$

$$\frac{1}{\sqrt{2}}[O_1|\psi\rangle_S \otimes |1_a\rangle + |\psi\rangle_S \otimes |0_a\rangle]$$

$$\frac{1}{\sqrt{2}}[e^{-iHt}O_1|\psi\rangle_S \otimes |1_a\rangle + e^{iHt}|\psi\rangle_S \otimes |0_a\rangle]$$

$$\frac{1}{\sqrt{2}}[e^{2iHt}O_2e^{-iHt}O_1|\psi\rangle_S \otimes |0_a\rangle + e^{-2iHt}e^{iHt}|\psi\rangle_S \otimes |1_a\rangle]$$

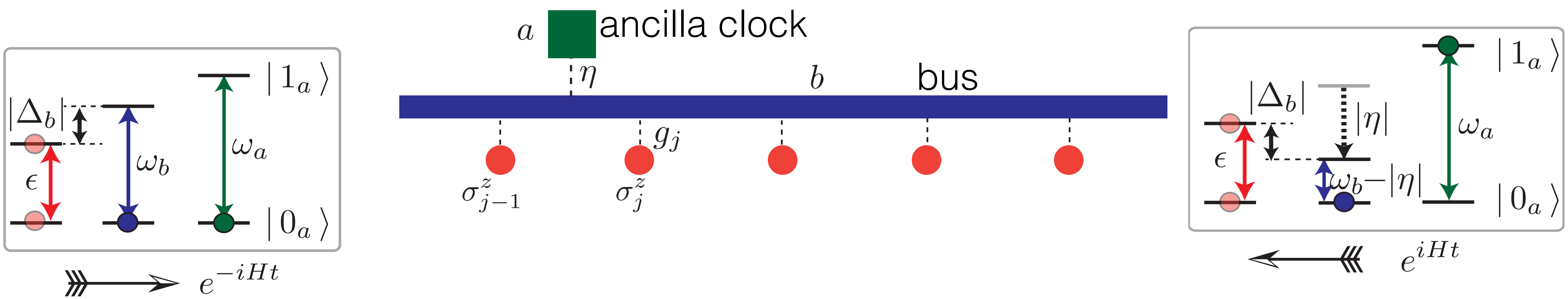
$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}[|R\rangle \otimes |1_a\rangle + |L\rangle \otimes |0_a\rangle]$$

$$|L\rangle \equiv O_1 e^{iHt} O_2 e^{-iHt} |\psi\rangle$$

$$|R\rangle \equiv e^{iHt} O_2 e^{-iHt} O_1 |\psi\rangle$$

$$\langle \tau^y \rangle_f = \mathrm{Im}[\langle L | R \rangle]$$

Implementation for an all-to-all spin model



Assuming that the clock can only take vacuum and one photon state:

$$H_0 = \sum_{n_a=0,1} [(\omega_b + \chi n_a) b^\dagger b + \omega_a n_a + \frac{1}{2}\epsilon \sum_j \sigma^z_{j,j+1}] | n_a \rangle\langle n_a |.$$

Energy of b depends on clock state

Treating the spin-boson coupling perturbatively:

$$V = \sum_j g_j(\sigma_j^+ b + \text{H.c.})$$

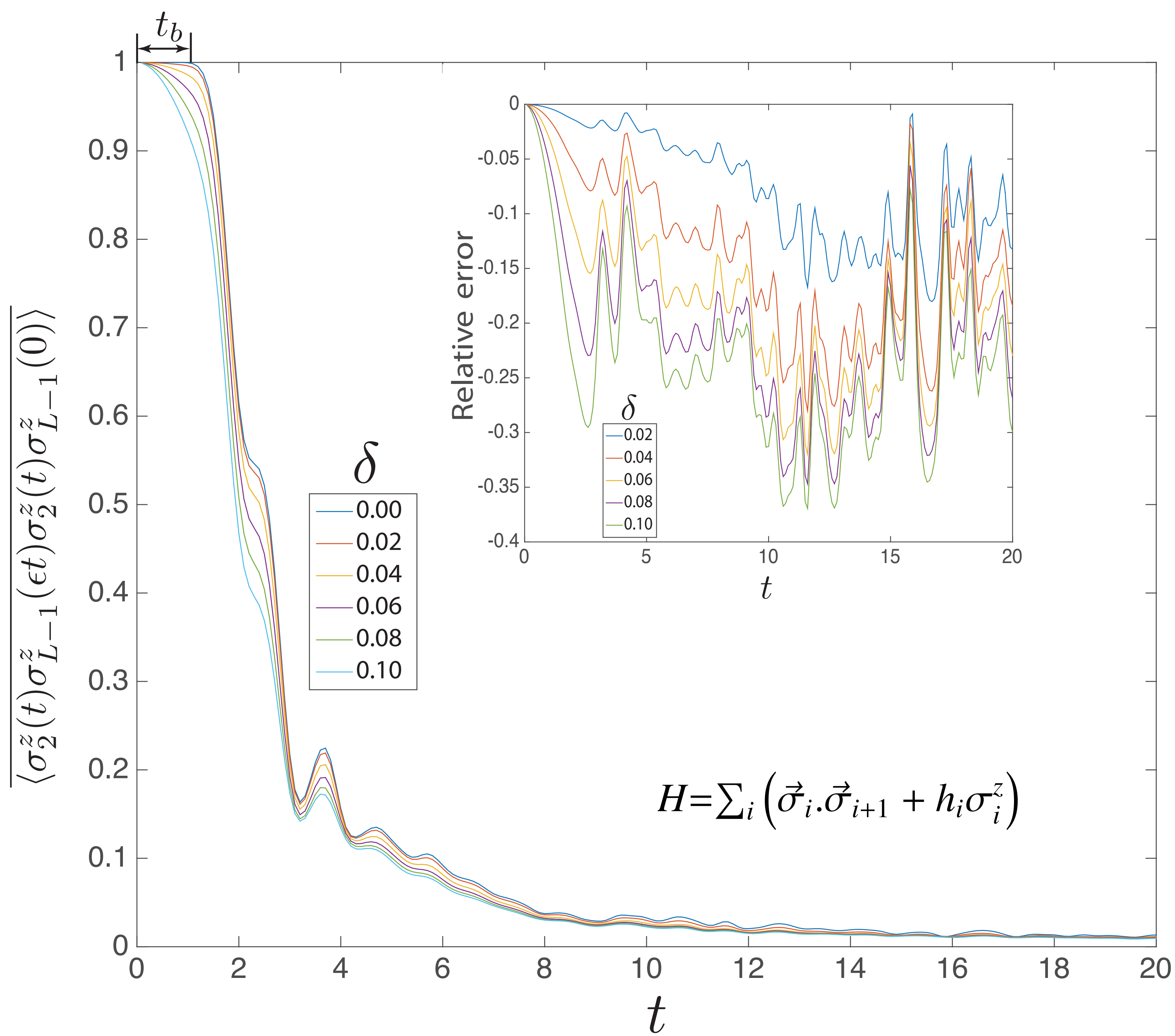
$$H_{\text{eff}} = H_0 + \Big[\sum_{j,j'} \frac{g_j g_{j'}}{\Delta_{b,n_a}} \sigma_j^+ \sigma_{j'}^- + \sum_j \frac{1}{2} \frac{g_j^2}{\Delta_{b,n_a}} \sigma_j^z \Big] | n_a \rangle\langle n_a |$$

$$\eta = 2(\epsilon - \omega_b) \equiv 2\Delta_b$$

$$\tilde{H}_{\text{eff}} = (1 - 2a^\dagger a) \Big[\sum_{j<j'} \frac{g_j g_{j'}}{\Delta_b} (\sigma_j^+ \sigma_{j'}^- + \text{H.c.}) + \sum_j \frac{1}{2} \frac{g_j^2}{\Delta_b} \sigma_j^z \Big] + \mathcal{O}\left(\frac{g_j^4}{\Delta_b^3} \right).$$

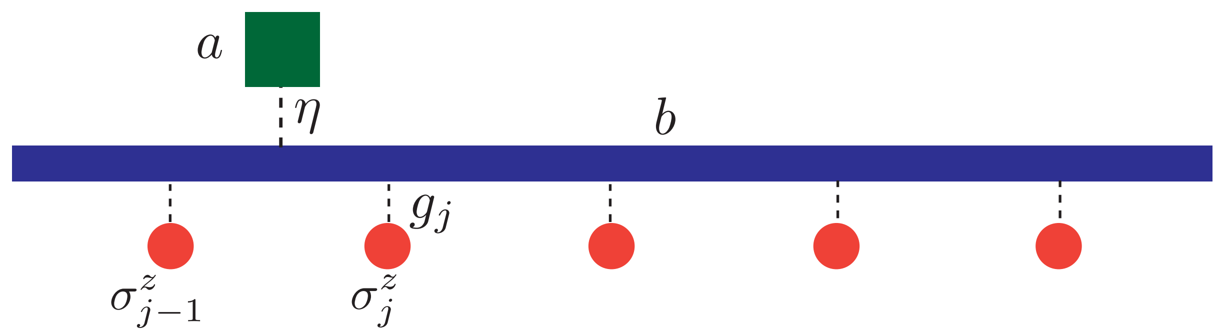
classical switch vs. quantum clock

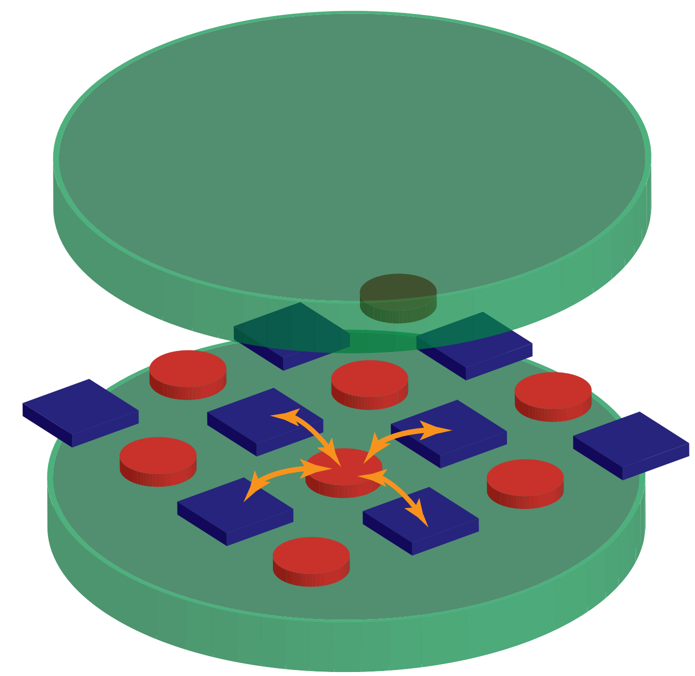
$$1 - e^{\lambda(t-x/v_b)}$$



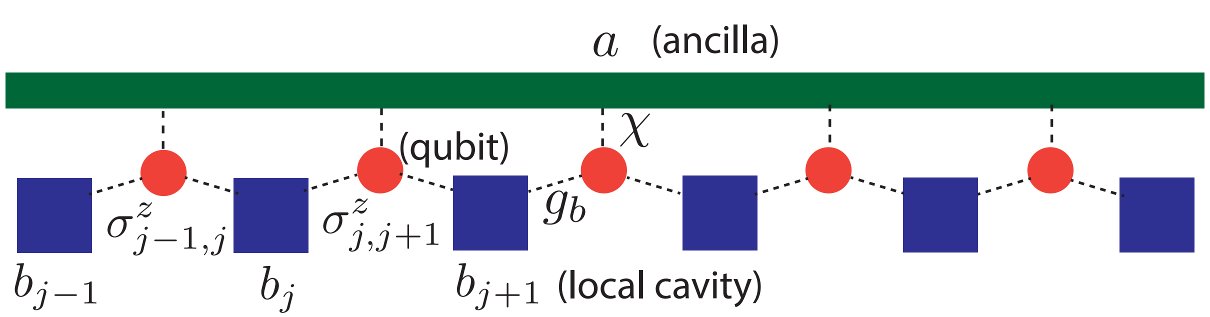
implementation in circuit-QED and generalization

all-to-all coupling



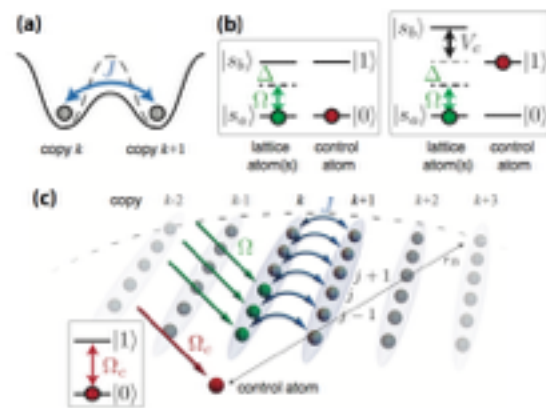


nearest neighbor coupling



Summary

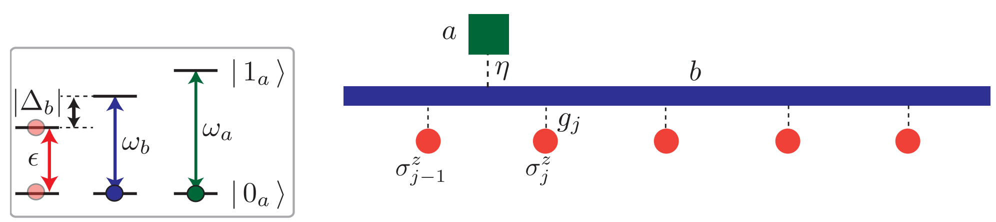
1. Measurement Protocol for the Entanglement Spectrum: cold atoms and circuit-QED



Hannes Pichler*, Guanyu Zhu*, Alireza Seif*, Peter Zoller and M.H. arXiv:1605.08624 (2016) to appear in PRX

poster!

2. Measurement of scrambling, out-of-time-order correlators



Guanyu Zhu, MH, and Tarun Grover arXiv: 1607.00079