Designing Novel Interacting Topological States, Time-Dependent Driving Protocols and Bi-Partite Measurements

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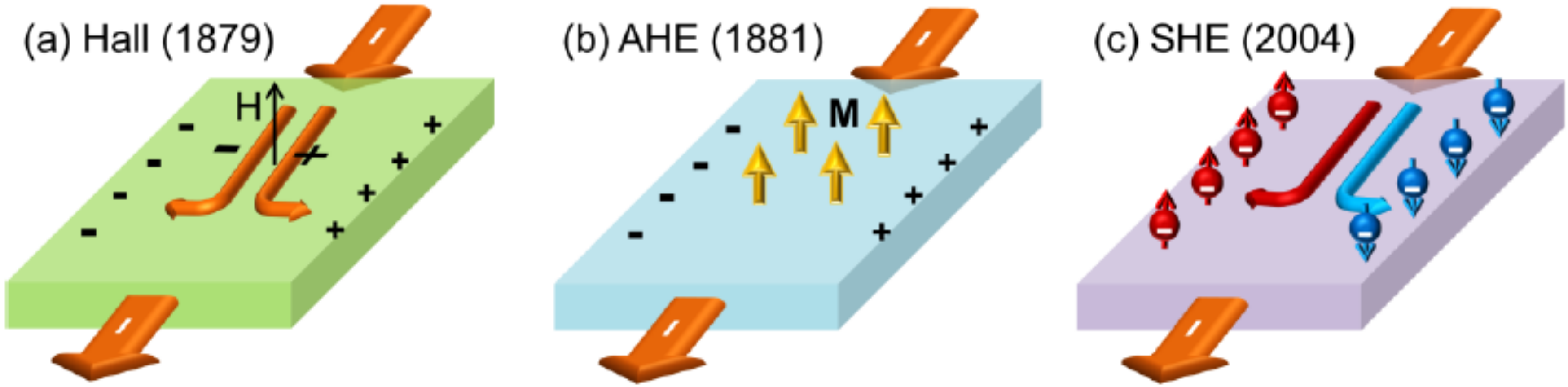
KITP Santa Barbara, November 14-18 2016

Designer Quantum Systems Out of Equilibrium
Coordinators: Dmitry Abanin, Alexey Gorshkov, Ehud Altman, Victor Galitski

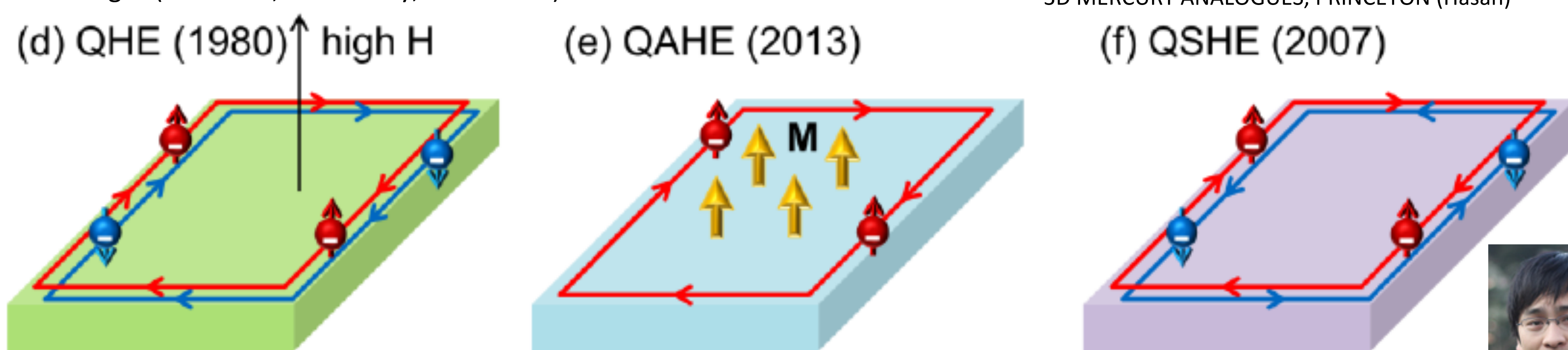
Funded by Labex PALM, Paris-Saclay
DFG Forschegruppe unit, Germany, FOR2014

Topological states of matter: magnetic fields and spin-orbit coupling



Von Klitzing, Dorda, Pepper;
fractional charges (Grenoble, CEA Saclay, Weizmann)

REALIZED AT WURZBURG IN HGTE (Molenkamp)
3D MERCURY ANALOGUES, PRINCETON (Hasan)



W. Wu, DMFT
China & Yale 2011

C. Z. Chang and M. Li, Topical Review, arXiv:1510.01754
From material science, to cold atoms and photons

Stable towards interactions: exemples S. Rachel & KLH Kane-Mele-Hubbard model 2010 QSH; D. Pesin & L. Balents, 3D (2010)
C. Varney, K. Sun, M. Rigol, V. Galitski (Maryland) 2010 QAH

Several works to appear:

- Exploring novel topological states of **bosonic systems**
  Mott, FQHE in ladder systems: measure fractions? (to appear)
  Haldane bosonic analogues, FFLO physics

- Importance of driving protocols; new states
  also new probes « sensing »

- Bi-partite measurements (fluctuations, dynamical Chern number)

Entanglement measure, measure of « topology »
2-pole measurement on the Bloch sphere:
Dissipation effects on topology ; in & out of equilibrium
Strongly entangled limit (paper just submitted on arXiv)

Cold Atoms:

Jaksch & Zoller 2003

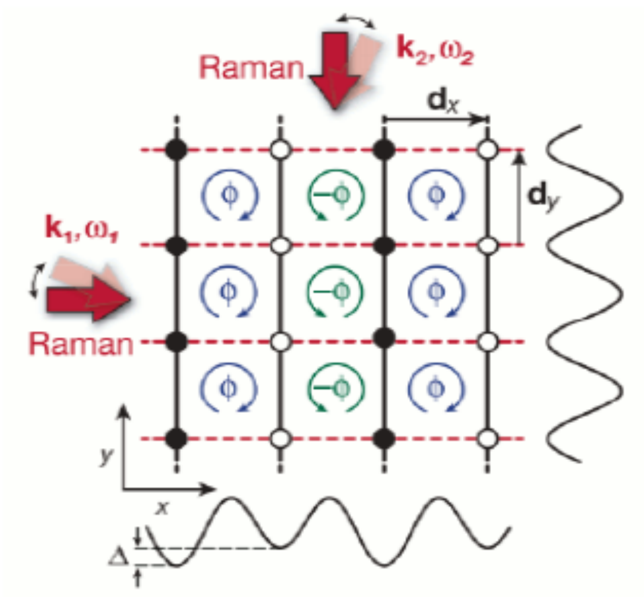
A. L. Fetter RMP 2009; J. Dalibard, F. Gerbier, G. Juzeliunas, P. Ohberg RMP 2011;
I. Bloch et al. Nature (2012); Juzeliunas & Spielman NJP (2012);...
D. Cocks, P. Orth, S. Rachel, M. Buchhold, KLH, W. Hofstetter PRL 2012

- Ways to implement magnetic fields & gauge fields

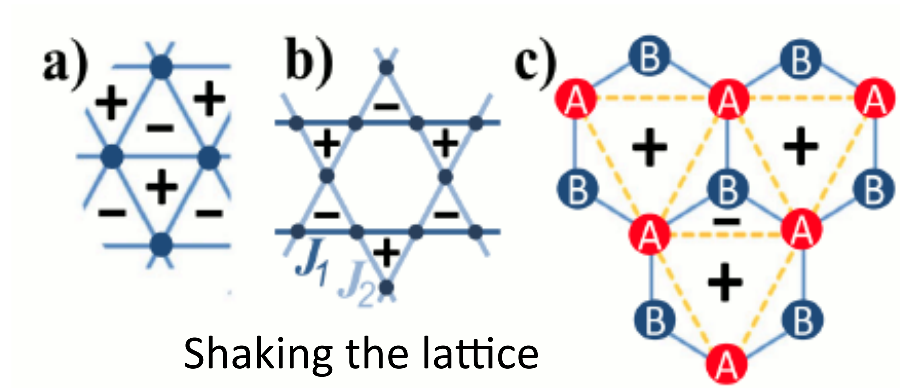
N. Goldman et al. Phys. Rev. Lett. 103, 035301 (2009)

M. Aidelsburger et al. arXiv:1110.5314 (Muenich's group, PRL)

J. Struck et al. arXiv:1203.0049 (Hamburg's group)



Laser-assisted tunneling in optical superlattice PRL 107, 255301 (2011)



Shaking the lattice

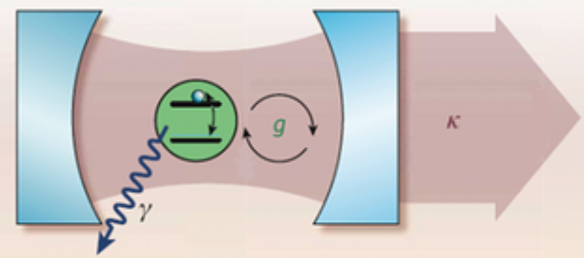
Floquet Topological Insulators:
Reviews: J. Cayssol, B. Dora, F. Simon, R. Moessner, arXiv:1211.5623
N. Goldman, J. Dalibard, PRX 2014

Cavity & Circuit QED: 1 mode of light …

Coupling atoms to the EM field

- atoms can couple to the EM field via dipole moment
- coupling strength can be enhanced by confining field to a cavity

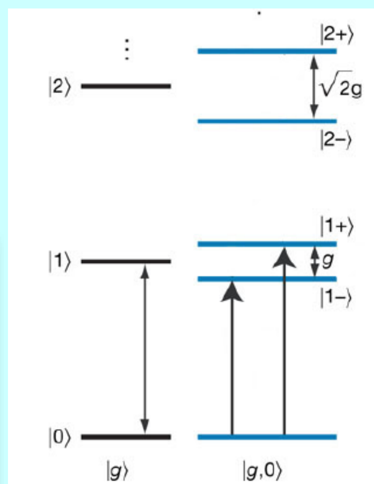
cavity QED, ENS Paris and Collège De France



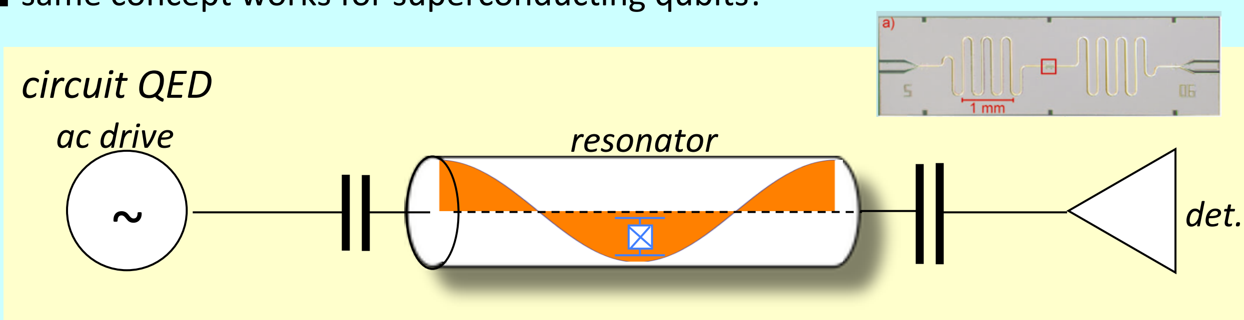
2g = vacuum Rabi frequency
γ = atomic relaxation rate
κ = photon escape rate

Jaynes-Cummings Hamiltonian

$$H = \frac{1}{2}\omega_a\sigma_z + \omega_r a^\dagger a + g\left(\sigma_- a^\dagger + \sigma_+ a\right) + \left(H_{\mathrm{drive}} + H_{\mathrm{baths}}\right)$$



- same concept works for superconducting qubits!

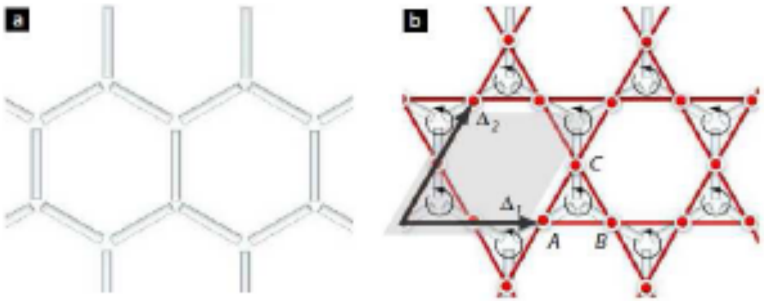


J. M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys. **73**, 565 (2001)
R. J. Schoelkopf, S. M. Girvin, Nature **451**, 664 (2008) and A. Blais et al.; D. Vion et al. (SPEC Saclay) 2002; J. Martinis

Developments with light

KLH, L. Henriet, A. Petrescu, K. Plekhanov, G. Roux and M. Schiro arXiv:1505.00167 & Compte Rendu Académie Sciences 2016; F. Appas, T. Goren, KLH in progress

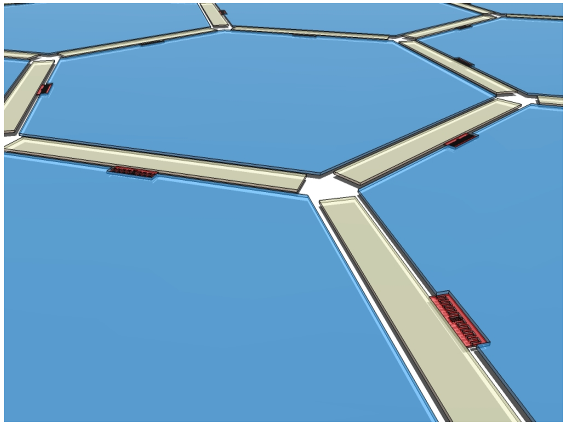
Also in cold atoms (L. Tarruell); polaritons



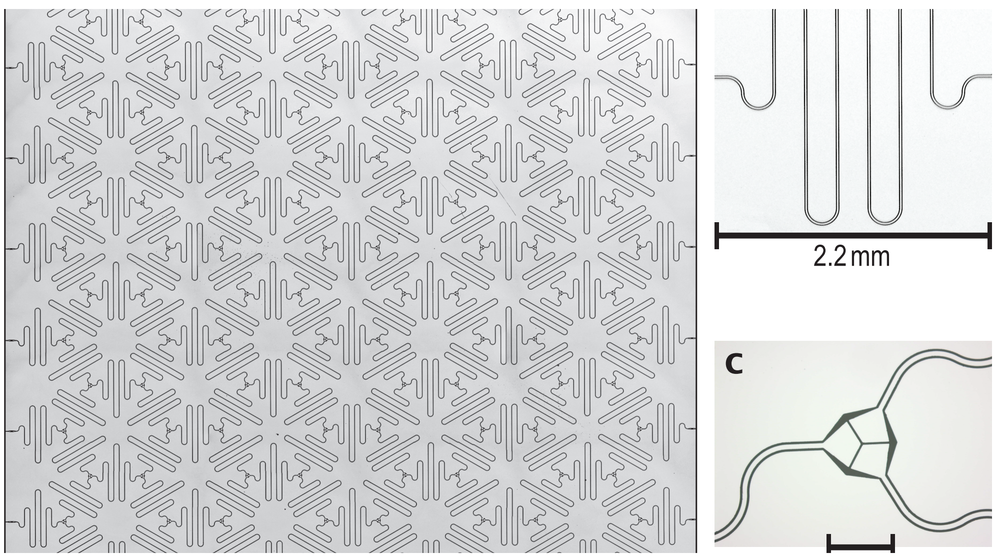
Experiments
Pannetier
Grenoble, 1980’

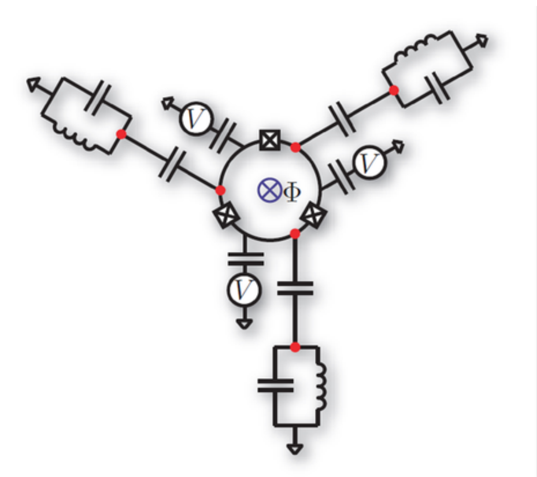
Y. Xiao et al. 1981
Superconducting wires
Aluminium

J. Koch et al. PRA 2010



A. Houck lab, Princeton



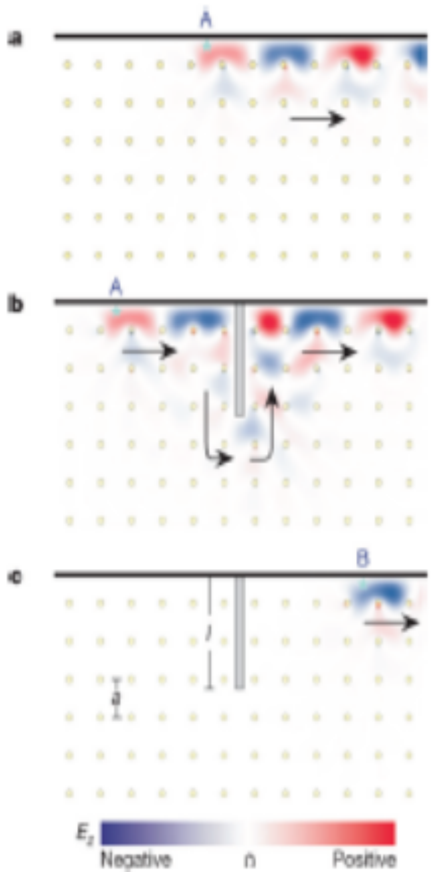


Superconducting cQED networks (also J. Gabelli; J. Esteve Orsay)
Photon arrays microwave (A. Houck, J. Koch, H. Tureci, Nature Physics)

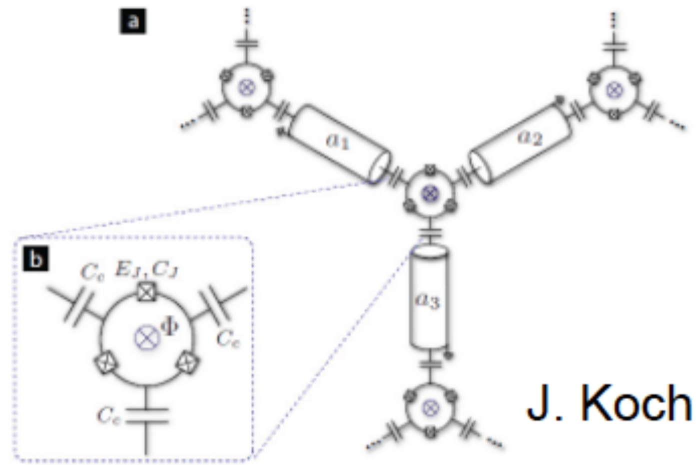
Progress at Santa Barbara, 2016
P. ROUSHAN ET AL. (J. MARTINIS)

Artificial Gauge Fields with Light

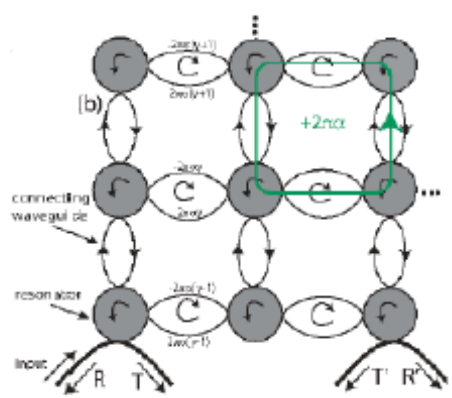
Review:
I. Carusotto
& C. Ciuti
RMP 2012



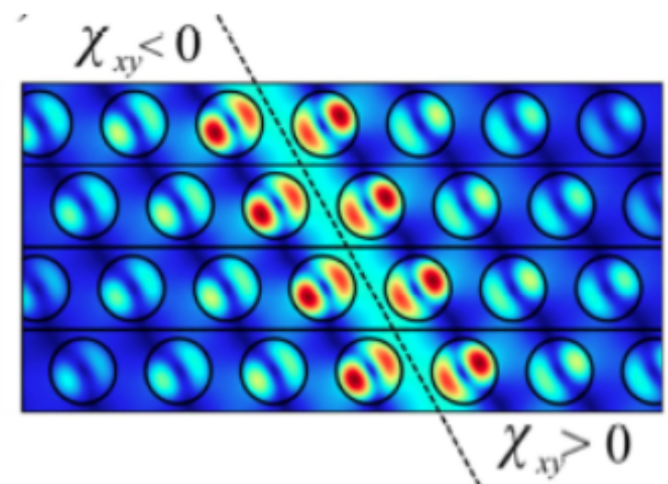
Haldane-Raghu, PRL 2008
Z. Wang et al. Nature 2009



J. Koch et al, PRA 2010



M. Hafezi, E. Demler, M. Lukin, J. Taylor 2011



A. MacDonald et al. 2012

Systems of interacting photons: Theory surveys

► M. Hartmann et al., Laser & Photonics Review 2, 527 (2008)
A. Tomadin & R. Fazio, J. Opt. Soc. Am B 27, A130 (2010)
J. Larson ; I. Carusotto and C. Ciuti, RMP 2012

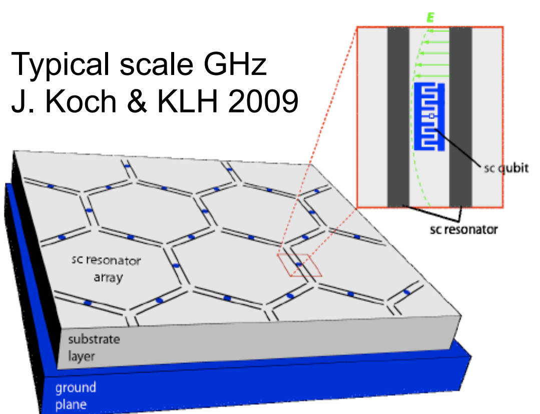
Interacting photons:

M. Lukin, E. Demler et al:
Fermionizing light

*realizations: superfluidity of polaritons **Stanford at Grenoble-EPFL, LKB ENS, LPN Marcoussis, PIttsburg***

* photonic band gap cavities
* arrays of silicon micro-cavities
* fibre based cavities
* cQED Array current realization

Typical scale GHz
J. Koch & KLH 2009

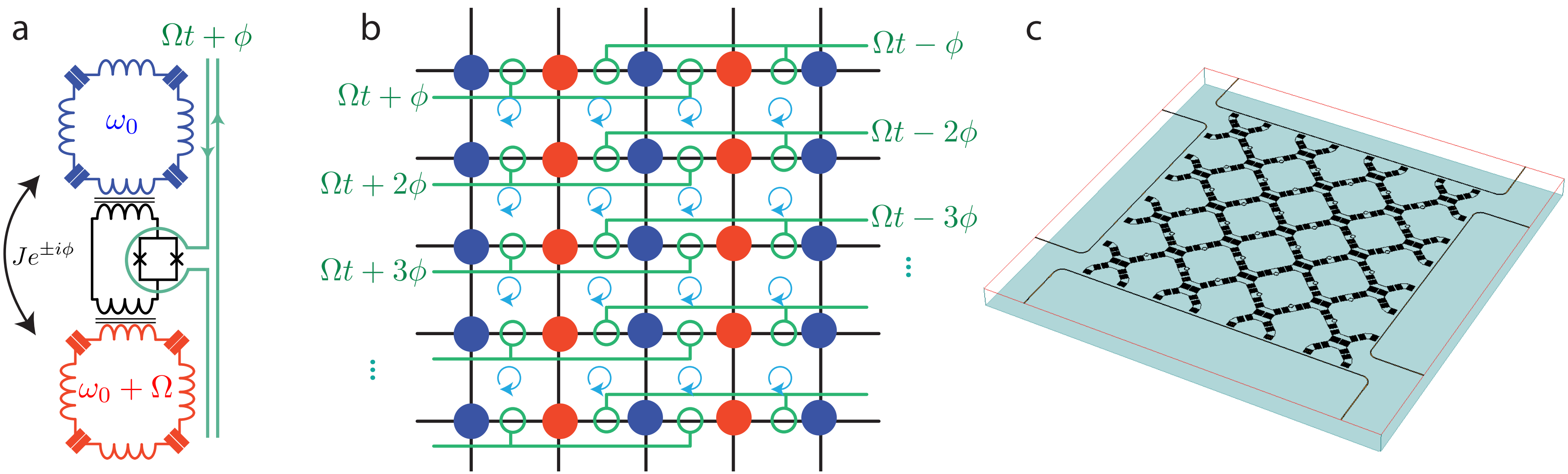


some pros and cons

+ tunability
+ access to single lattice site
- must be treated as open system
+ interesting: transitions between different steady states

Other Way to Produce Gauge Fields

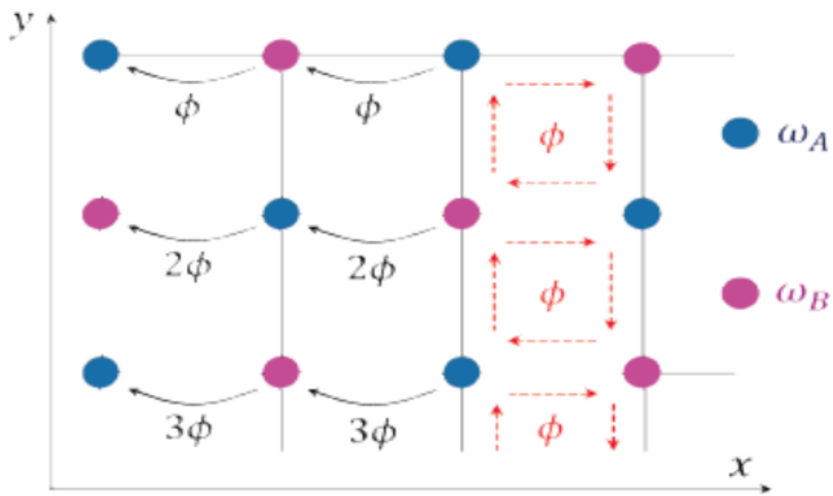
Uniform Magnetic Field



K. Fang, Z. Yu and S. Fan Nature Photonics 6, 782 (2012)
Seems feasible to realize in cQED arrays (J. Gabelli, J. Esteve)

Magnetic field for neutral particles

Cours Jean Dalibard collège de France



$$H = \omega_A \sum_i a_i^\dagger a_i + \omega_B \sum_i b_i^\dagger b_i + \sum_{\langle i;j \rangle} V \cos(\Omega t + \phi_{ij})(a_i^\dagger b_j + h.c).$$

Close to resonance

Analogy Two-level systems
coupled to light: « Floquet theory »

$$c_{i(j)} = e^{[i\omega_{A(B)} t c_{i(j)}^\dagger c_{i(j)}]} a_i(b_j)$$

$$H_{eff} = \sum_{\langle i;j \rangle} \frac{V}{2}(e^{-i\phi_{ij}} c_i^\dagger c_j + e^{i\phi_{ij}} c_j^\dagger c_i).$$

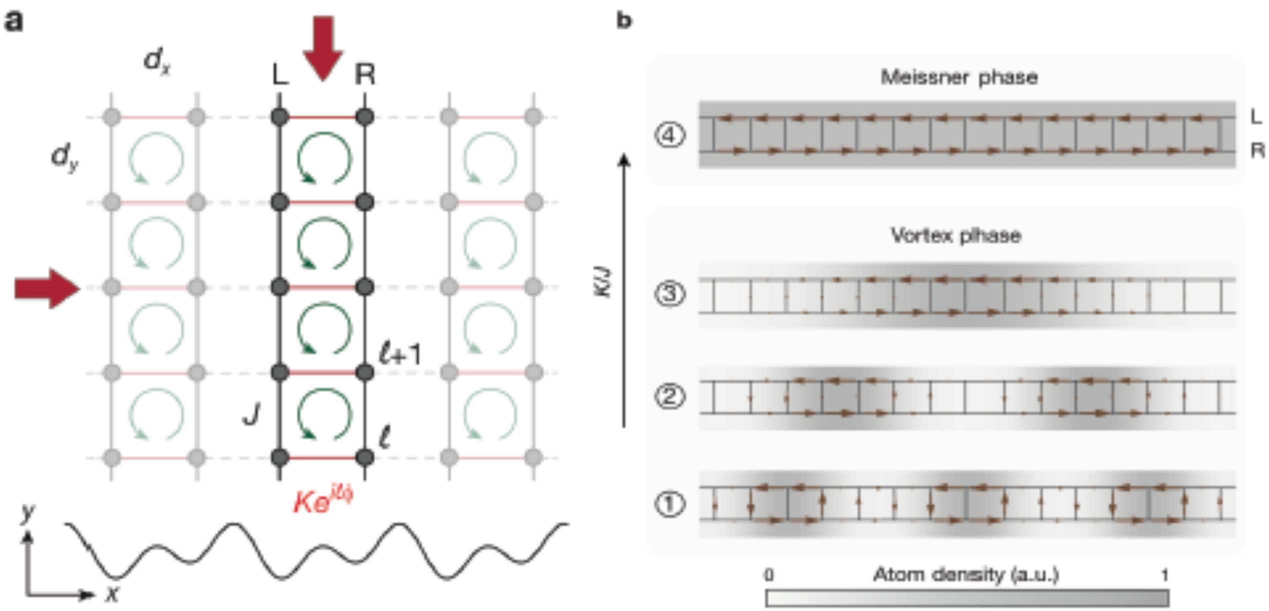
$$\int_i^j \mathbf{A}_{eff} \cdot d\mathbf{l} = \phi_{ij}$$

Lattice gauges theories: uniform magnetic field (QHE)

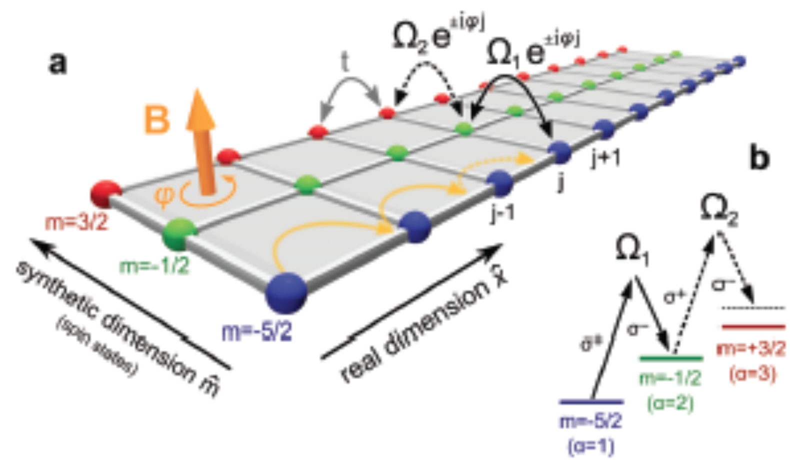
Meissner effect and IQHE in 1D ladder

BOSONS: currents screen the flux superfluids

Theory
E. Orignac & T. Giamarchi 2001



FERMIONS: IQHE: chiral edge modes



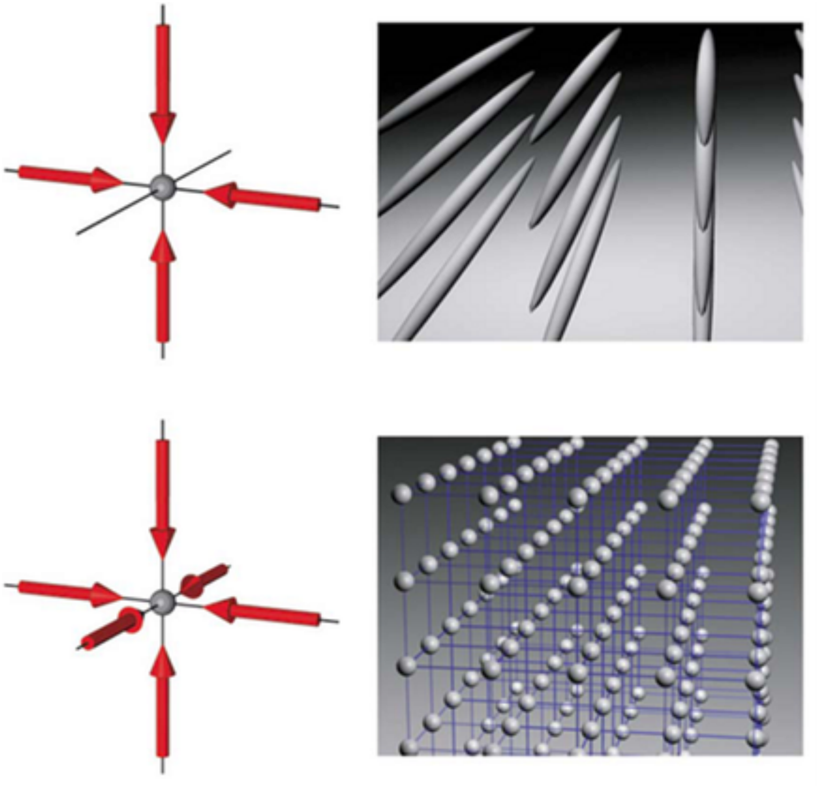
Theory by
P. Zoller et al.
and R. Fazio et al.

^{87}Rb Meissner/vortex transition [Atala et al. 2014],

^{173}Yb fermion IQHE[Mancini et al. 2015]. See also [Stuhl et al. 2015]

Bose-Hubbard model

I. Bloch, J. Dalibard, W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)

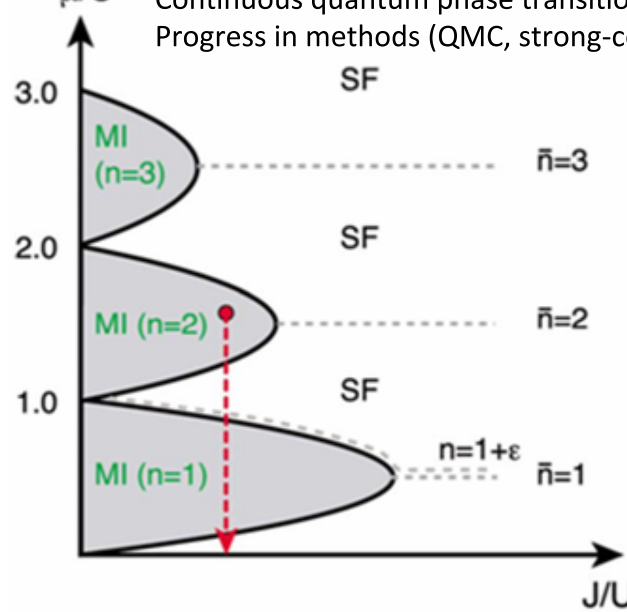


D. Jaksch et al., Phys. Rev. Lett. **81**, 3108 (1998)
M. Greiner et al., Nature **415**, 39 (2002)
Relation with cuprates: KLH & T. M. Rice, review 2009

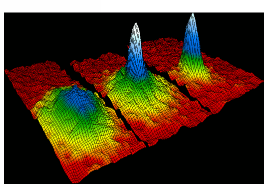
e.g., realization of the Bose-Hubbard model:

$$H = \sum_j [-\mu a_j^\dagger a_j + \frac{1}{2} U n_j (n_j - 1)] - J \sum_{\langle i,j \rangle} (a_j^\dagger a_i + \mathrm{h.c.})$$

M.P.A. Fisher et al., PRB 40, 546 (1989)
Continuous quantum phase transition (second order) in 2D
Progress in methods (QMC, strong-coupling, …)

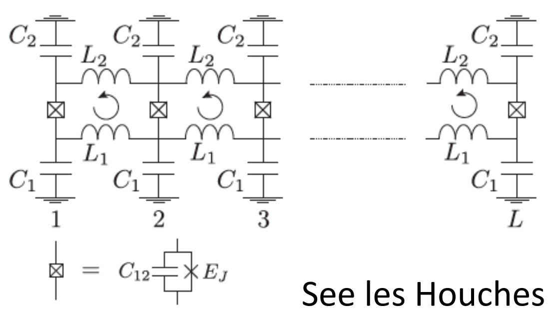


Nobel prize 2001



Bose-Einstein condensate

Also Josephson junction arrays

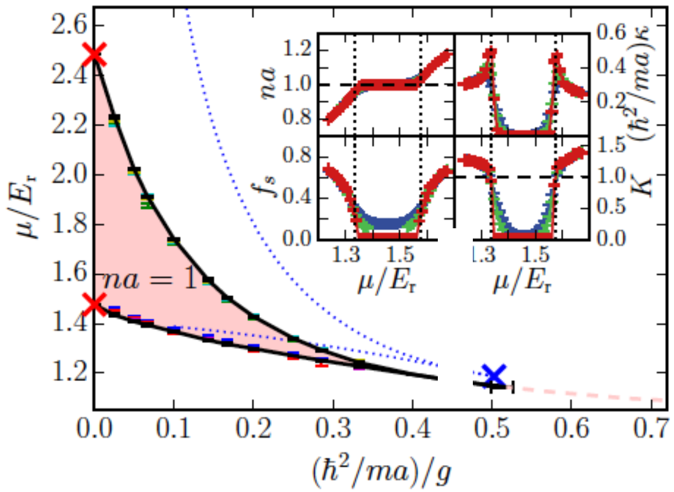


See les Houches lectures: D. Esteve & D. Vion; M. Devoret; J. Martinis & Kevin Osborne 2004

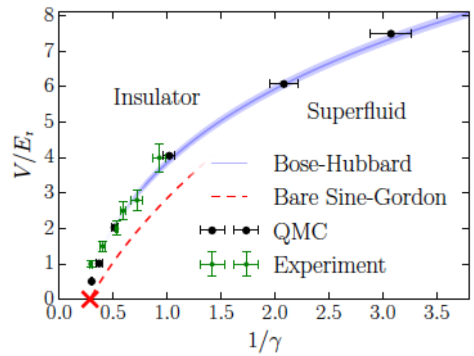
Mott 1D....

disorder ; Alain Aspect,
Vincent Josse, Philippe Bouyer
Juliette Billy...
Anderson localization

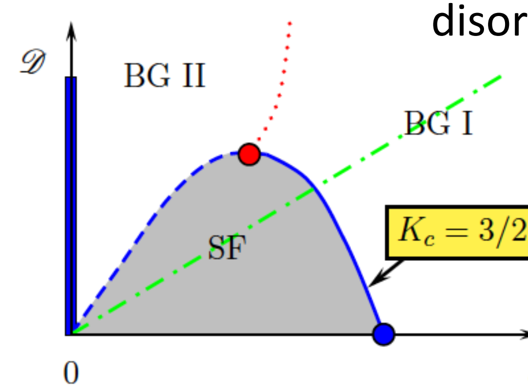
Kosterlitz-Thouless transition



1D: interactions are included
in fluid Luttinger description
K is the Luttinger parameter
Haldane 1981 (K=1 Tonks limit)



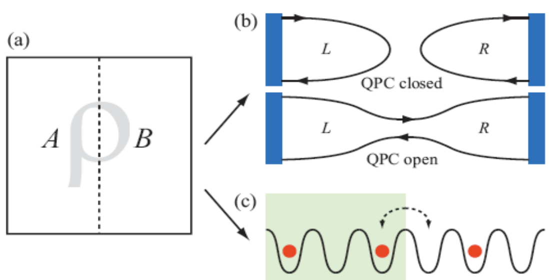
disorder



H. J. Schulz & T. Giamarchi 1988
Z. Ristivojevic, A. Petkovic, (Toulouse)
T. Giamarchi, P. Le Doussal (ENS) 2012
E. Altman, Y. Kafri, A. Polkovnikov, G. Refael

2016
Experiment Modugno, Florence. Theory & numerics T. Giamarchi (Geneva), L. Sanchez Palencia (Institut Optique, LCFIO)

New probes: bi-partite entanglement
Entropies, entanglement spectrum
Linked with conformal field theory
(John Cardy, P. Calabrese)



Critical coupling strength

Kc=2

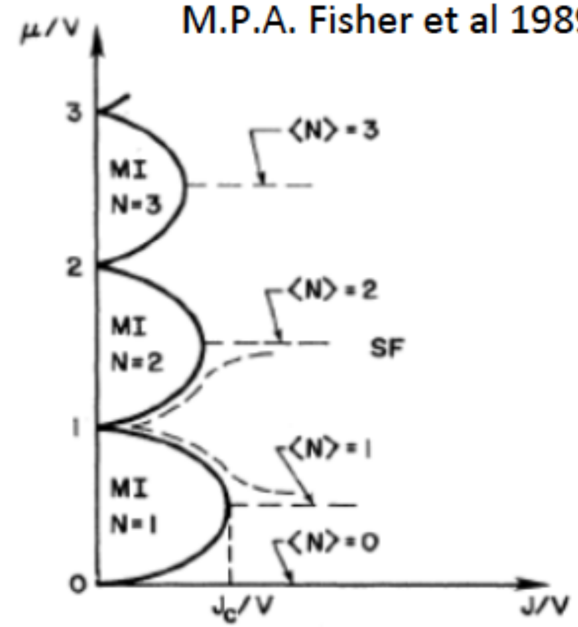
Year	Reference	Technique	Observable	Estimate
1991	Krauth [5]	(approximate) Bethe Ansatz		$1/(2\sqrt{3}) \simeq 0.2887$
1992	Batrouni *et al.* [6]	QMC	Superfluid stiffness	0.2100(100)
1994	Elesin *et al.* [7]	Exact Diagonalization	Gap	0.2750(50)
1996	Kashurnikov *et al.* [8]	QMC	Gap	0.3000(50)
1999	Elstner *et al.* [9]	Strong coupling	Gap	0.2600(100)
2000	Kühner *et al.* [10]	DMRG	Correlation function	0.2970(100)
2008	Zakrzewski *et al.* [11]	Time Evolving Block Decimation	Correlation function	0.2975(5)
2008	Laüchli *et al.* [12]	DMRG	von Neuman entropy	0.2980(50)
2008	Roux *et al.* [13]	DMRG	Gap	0.3030(90)
2011	Ejima *et al.* [14]	DMRG	Correlation function	0.3050(10)
2011	Danshita *et al.* [15]	Time Evolving Block Decimation	Excitation spectrum	0.3190(10)
2011	This work	DMRG	Bipartite Fluctuations	0.2989(2)

See later
PhD H. Francis Song, Yale 2011
PhD Loic Herviou CPHT 2016

S. Rachel, N. Laflorencie (Toulouse), H. F. Song, and K. Le Hur 108, 116401 (2012)

Mott Physics in Boson Systems: Lattice Effects

M.P.A. Fisher et al 1989



Bose-Hubbard model of a single lattice boson:

$$H = -t \sum_{<ij>} b_i^\dagger b_j + \sum_i \frac{U}{2} n_i (n_i - 1) - \mu n_i$$

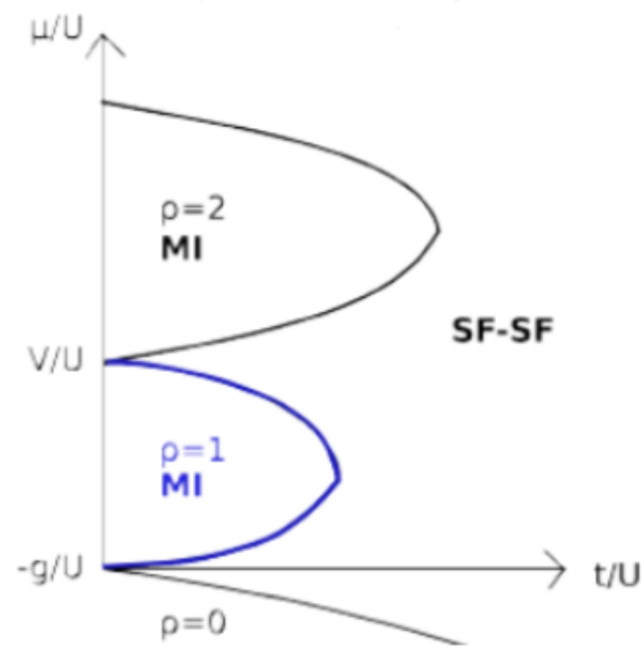
Two-species Bose-Hubbard model:

$$H = -t \sum_{\alpha=1,2} \sum_{<ij>} b_{\alpha i}^\dagger b_{\alpha j} + \sum_{\alpha i} \frac{U}{2} n_{\alpha i} (n_{\alpha i} - 1) - \mu n_{\alpha i}$$

$$+ \sum_i V_\perp n_{1i} n_{2i} - g \sum_i b_{1i}^\dagger b_{2i} + H.c.$$

Mott at ρ=1

Interchain coherence:
Meissner effect



Multicomponent systems: active field in cold atoms

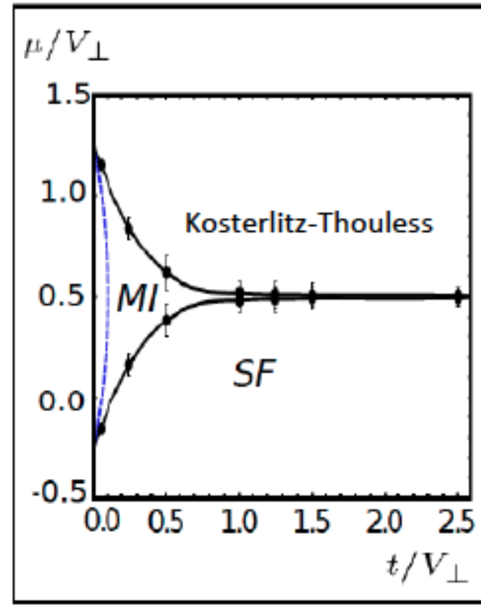
e.g. E. Altman, W. Hofstetter, E. Demler, M. Lukin 2003

Route for Chiral Mott Insulator: Spin Meissner Effect

Mott insulating phase of total density:

$$\rho = b_1^\dagger b_1 + b_2^\dagger b_2$$

A. Petrescu and KLH, PRL 2013



Relative density exhibits fluctuations.

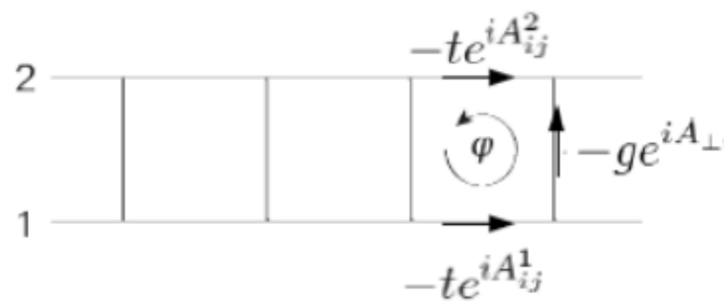
$$\sigma^z = b_1^\dagger b_1 - b_2^\dagger b_2$$

(At ρ=1, spin ½ exchange Hamiltonian)

$$\langle j_\parallel \rangle = -2J_{xx}\,\mathrm{phase}_{ij}$$

XXZ chain mapping
In transverse field

$$J_{xx} = \frac{t^2}{V_\perp}$$



Example: Ladder System

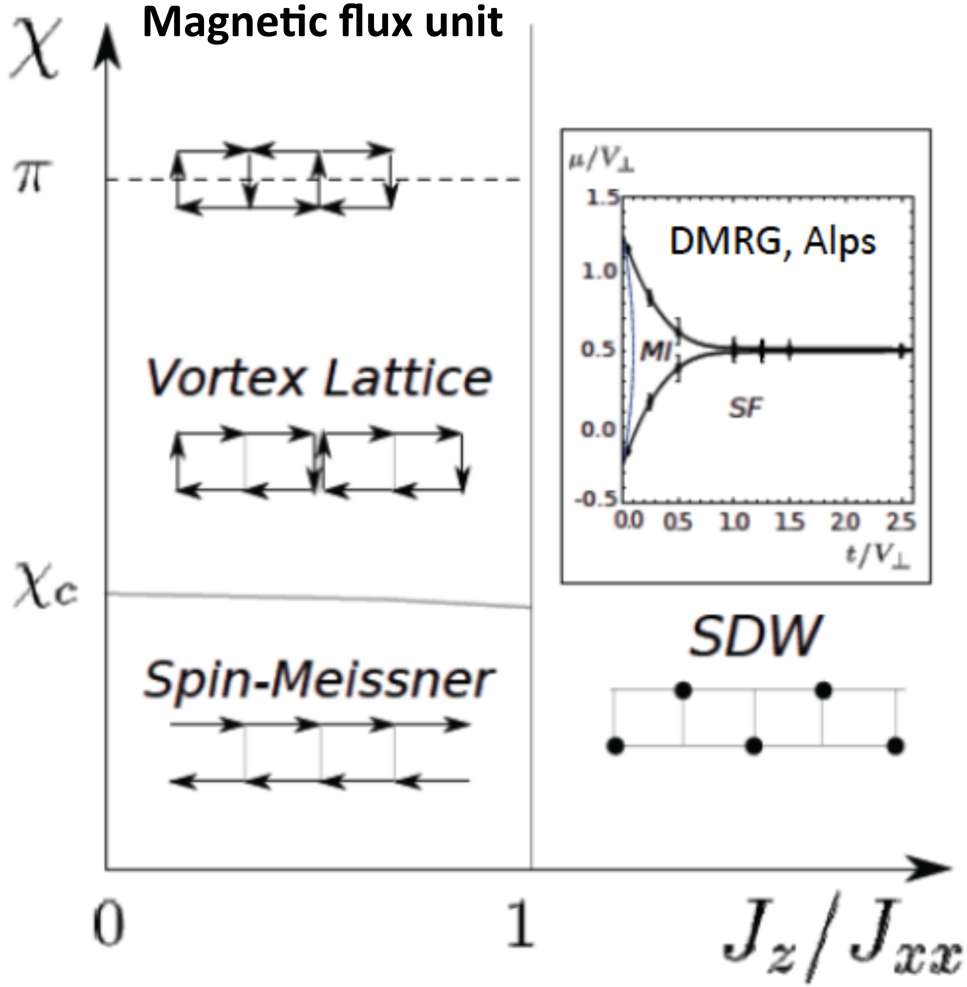
at other filling
A. Tokuno & A. Georges, 2014

Spin Meissner Effect

Similar Hamiltonian different contexts
I. Garate and I. Affleck

Chiral Mott insulator Arya Dhar et al. PRA A 85, 041602 (2012)

Analogy to DDW phase in high-Tc systems



See also
M. Piraud et al
arXiv:1409.7016

Collaboration on DMRG with Guillaume Roux

Possible transitions to Laughlin states

Teo – Kane, PRB 2014

$$J_z = t^2\left(-\frac{2}{U} + \frac{1}{V_\perp}\right)$$

PRL 2013

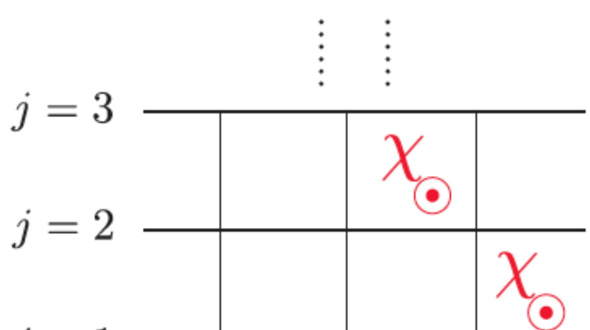
Bosonic Mott insulator with Meissner Currents Alex Petrescu and KLH, arXiv:1306.5986

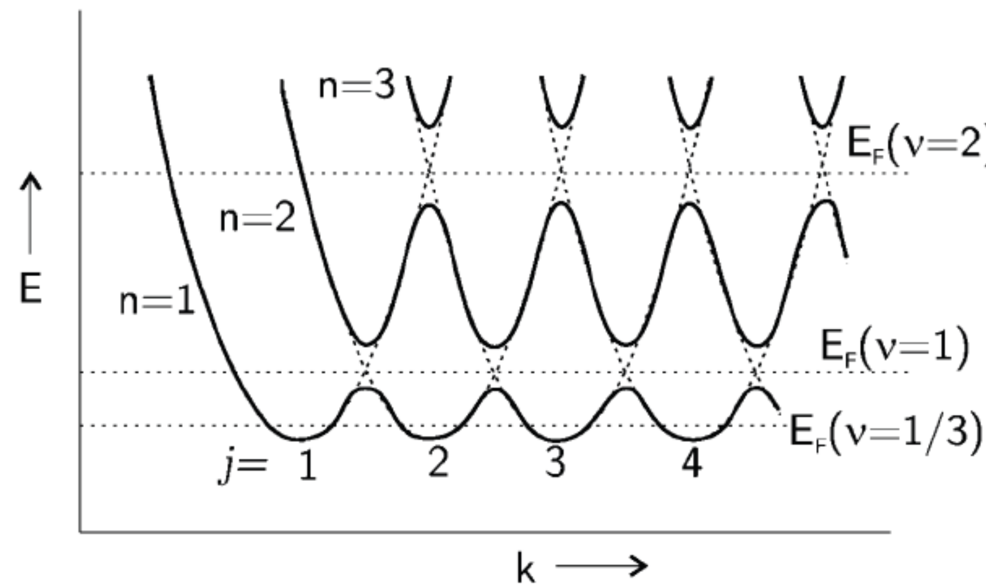
Quantum Hall physics

Wire construction of quantum Hall state

Kane's construction for quantum wires

$$\nu = N_{boson}/N_{\phi}$$





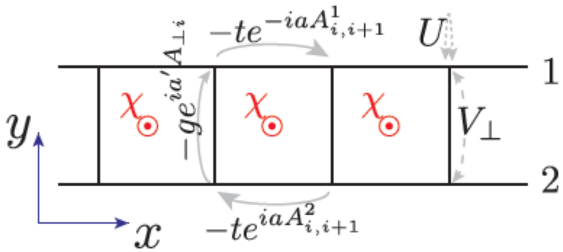
Bosons: hard-core limit

Duality spins-fermions: Jordan-Wigner transformation

(node in Jastrow wavefunction & compete with Meissner & vortex)

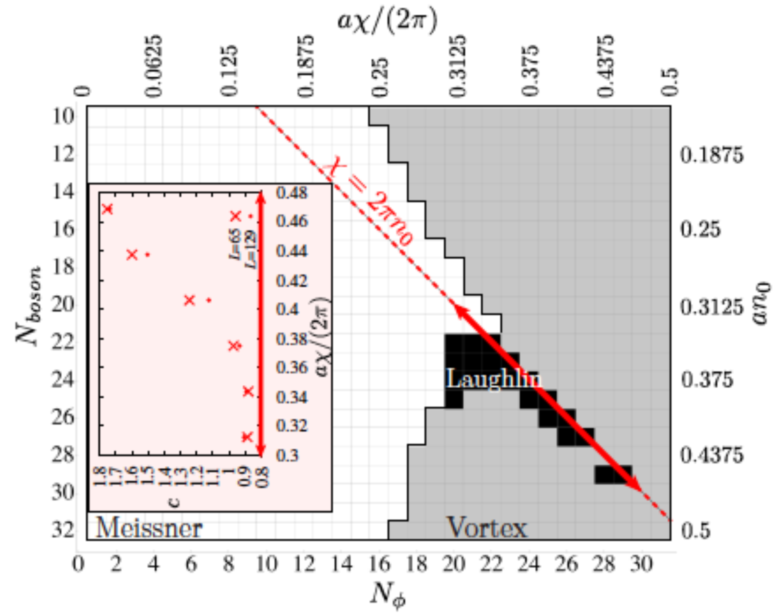
FQHE bosons: 2-leg ladder?

C. L. Kane, Lubensky, Mukhopadyay; Teo & Kane, classification of quantum Hall phases in ladders
Numerical results support bosonic LAUGHLIN PHASE for hard-core bosons with V=0 **finite** systems



A. Petrescu & KLH, PRB 2015 (analytics : V needed for infinite systems)
A. Petrescu, M. Piraud, I. McCulloch, G. Roux, KLH, **to appear**

DMRG Small densities



Laughlin phase: chiral edge modes with fractional charges
Bipartite fluctuations confirm Laughlin phase theoretically and numerically

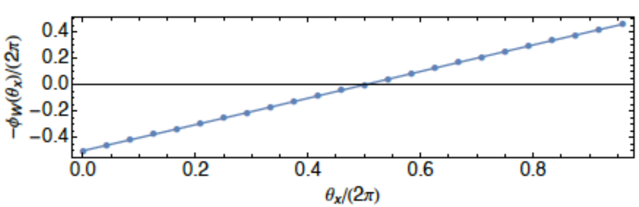
Measurement in quantum wires of fractional charges

H. Steinberg, G. Barak, A. Yacoby, L. N. Pfeiffer, K. W. West
B. Halperin and K. Le Hur, 2008; see also E. Berg, Y. Oreg, E.-A. Kim, F. von Oppen

K.V. Pham, M. Gabay, P. Lederer, 2000; Safi & Schulz, 1995
Application topological insulators edge modes: Ion Garate & KLH, 2012

$$\sigma_{xy} = \frac{1}{d}\frac{1}{2\pi}\int_0^{2\pi} d\theta_x \frac{\partial}{\partial\theta_x}\phi_W(\theta_x) = \frac{1}{d}\frac{1}{2\pi}\left[\phi_W(\theta_{x,N_x}) - \phi_W(\theta_{x,0})\right].$$

Torus geometry: gap the edges
Thouless Laughlin pump (Joel Moore,
Experiment in Muenich, Bloch's group
Zak phase (work D. Abanin, E. Demler)
measures the polarization « 1/2 »



See also F. Grusdt – M. Honing 2014

Ground state	Meissner	Vortex	Laughlin
c	1	2	1
N_V	1		>1

H. Francis Song, Stephan Rachel, Christian Flindt, Israel Klich
Nicolas Laflorencie, Karyn Le Hur (review)
Phys. Rev. B **85**, 035409 (2012)

Application to Renyi entropies and quantum Hall systems
A. Petrescu, H. F. Song, S. Rachel, Z. Ristivojevic, C. Flindt,
N. Laflorencie, I. Klich, N. Regnault, K. Le Hur (review)
J. Stat. Mech. (2014) P10005

$$\hat{N}_A \rightarrow \hat{S}_z = \sum_{i \in A} \hat{S}_z^i$$

Number (spin) fluctuations:

$$\mathcal{F}_A = \langle (\hat{N}_A - \langle \hat{N}_A \rangle)^2 \rangle.$$

- Symmetric (cf. entanglement entropy):

$$\mathcal{F}_A = \mathcal{F}_B.$$

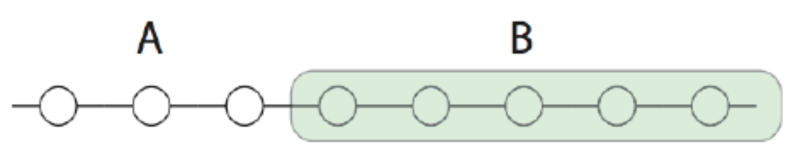
- Zero for a product state

$$\mathcal{F}_A = 0 \text{ if } |\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle.$$

- Luttinger liquids:

$$\pi^2 \mathcal{F}(x) = \frac{1}{\pi^2} \langle [\phi(x) - \phi(0)]^2 \rangle \sim K \ln \frac{x}{a}.$$

Relation to
- Correlation functions
- bi-partite quantum Fisher



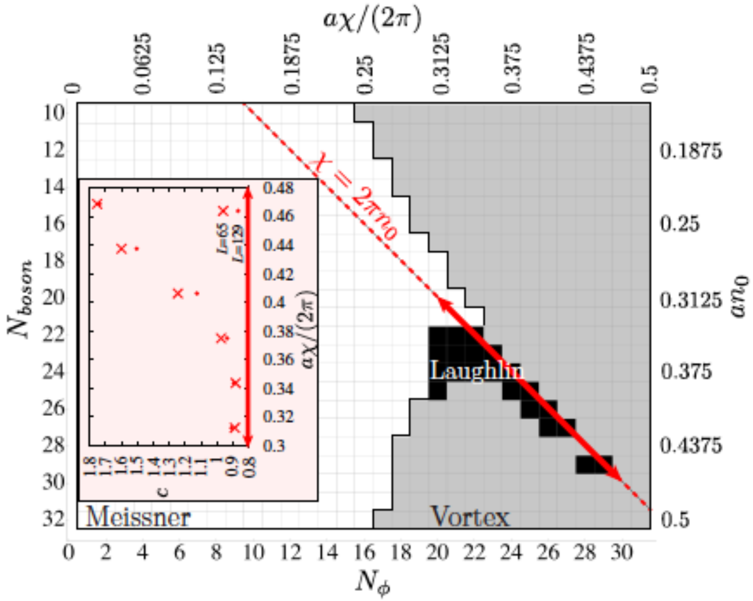
K=1 free fermions (Klich-Levitov)
K=ν for fractional QH edges
(Fradkin et al)

H. Francis Song, Stephan Rachel & Karyn Le Hur, PRB 2010

Bi-partite fluctuations: probe of Laughlin phase

A. Petrescu, M. Piraud, I. McCulloch, G. Roux, KLH, to appear

$$\mathcal{F}^{\pm}(\ell) \equiv \langle [N^{\pm}(\ell)]^2 \rangle_{\text{conn}}.$$

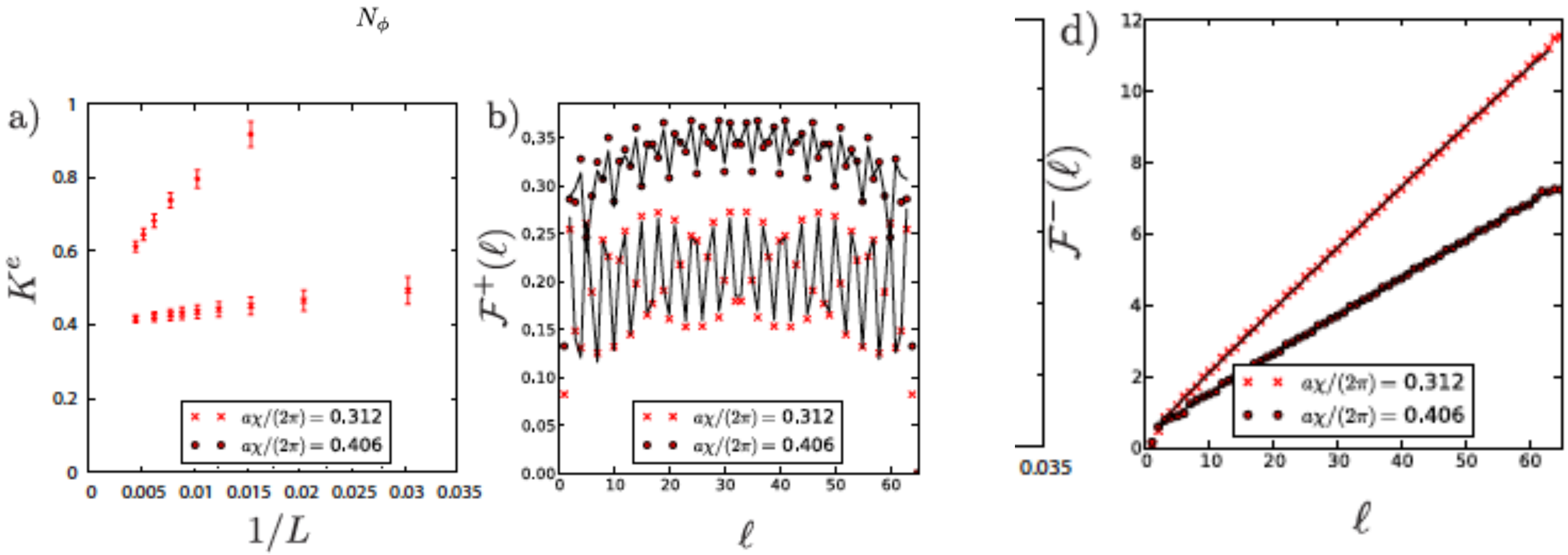


$$\mathcal{F}^{+}_{Laughlin}(\ell) = \frac{K^e}{2\pi^2} \log\left[d(\ell|L)\right] + c + db_E(\ell).$$

Filling factor ½ with « definitions »
Small backscattering between edges Ke=2/5
(quasi-SPT; class A analogue T. Neupert et al. 2014)

$$\mathcal{F}^{-}_{Laughlin}(\ell) = \frac{1}{2\pi^2 K^e} \log\left[d(\ell|L)\right] + b\ell + c + db_E(\ell),$$

b probes the « gap » in bulk



Loic Herviou, C. Mora & KLH, PRB 2016 (Majorana Kitaev ladders)

$$H = -\sum_i \left(J \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^x \right)$$

Quantum phase transition at $J = \Delta$

Model exactly solvable through Jordan-Wigner

Model with no total charge conservation

Relation to Kitaev model CRITICAL model

CFT allows to compute bi-partite fluctuations

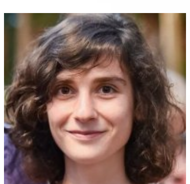
$$\iint \langle \sigma^x(a)\, \sigma^x(b) \rangle_c \, da\, db$$

DMRG Check

General formula $\alpha x + \beta \ln \frac{x}{a} + \dots$

$\downarrow$

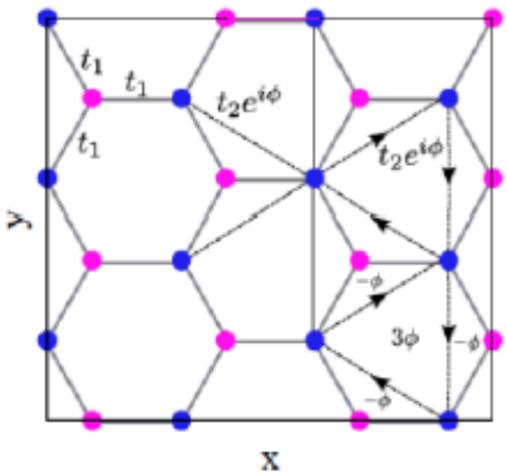
$\beta < 0$ here ($c = \frac{1}{2}$ theory)

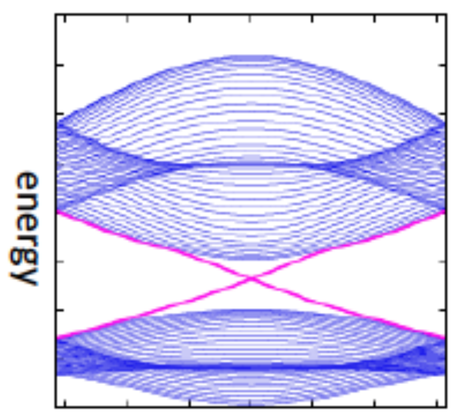


Kitaev magnetic ladders: Ariane Soret, Fan Yang, KLH in progress

Quantum Anomalous Hall Effect

F. D. M. Haldane 1988



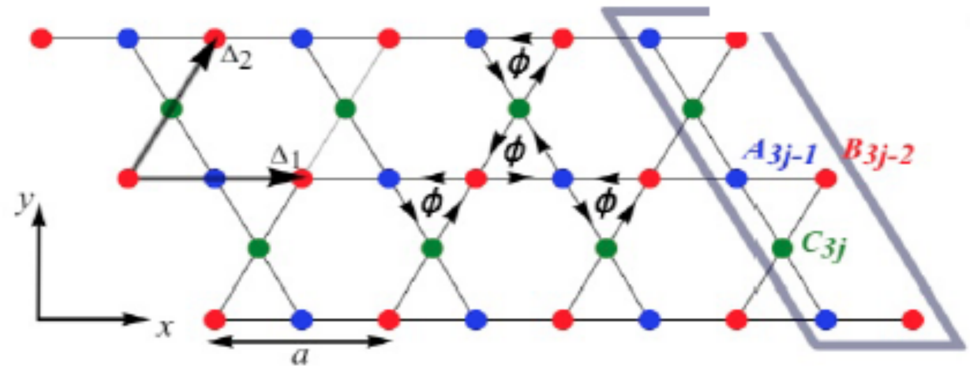


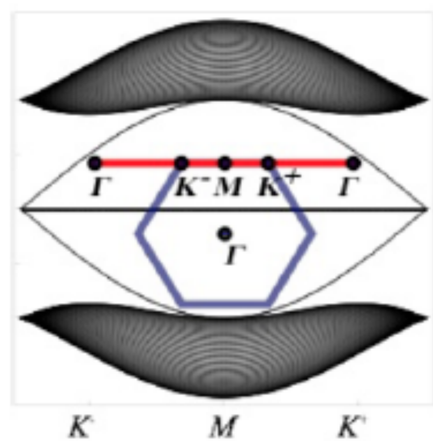
Graphene
+gap

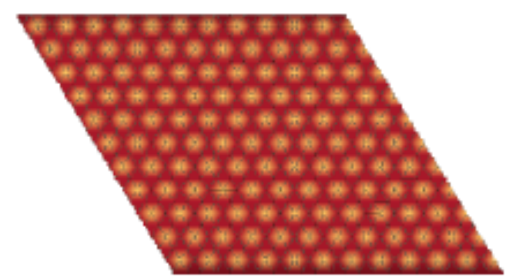
Kagome version:

A. Petrescu, A. A. Houck and KLH, 2012

See also J. Koch, A. Houck, KLH, S. Girvin 2010







+ disorder effects

Other Experimental observations

Jotzu et al. arXiv:1406.7874

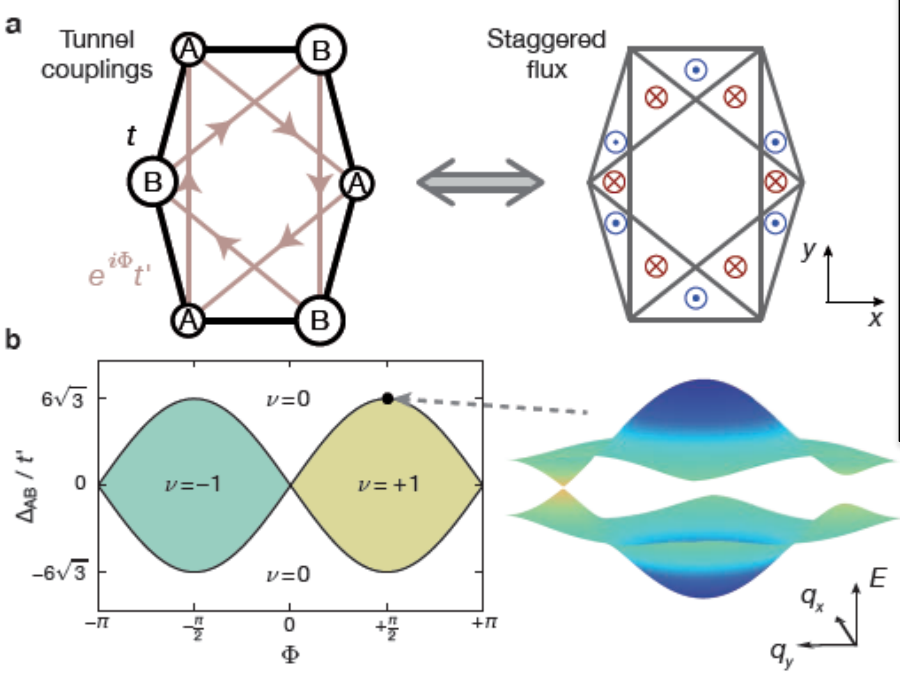
- Ultra-cold atoms – see for example Esslinger's experiment (ETH)
- Ultra-cold atoms: importance of Floquet-type point of view

<u>Modulation of optical lattice: lattice shaking</u>

$$\mathbf{r}_{\text{lat}} = -A\Big(\cos(\omega t)\mathbf{e}_x + \cos(\omega t - \varphi)\mathbf{e}_y\Big),$$

$$\boldsymbol{F}(t) = -m\ddot{\boldsymbol{r}}_{\text{lat}}(t)$$

$$\hat{H}_{\text{lat}}(t) = \sum_{\langle ij \rangle} t_{ij}\hat{c}_i^\dagger \hat{c}_j + \sum_i \left(\boldsymbol{F}(t)\cdot \boldsymbol{r}_i\right)\hat{c}_i^\dagger \hat{c}_i$$



Talks on Monday

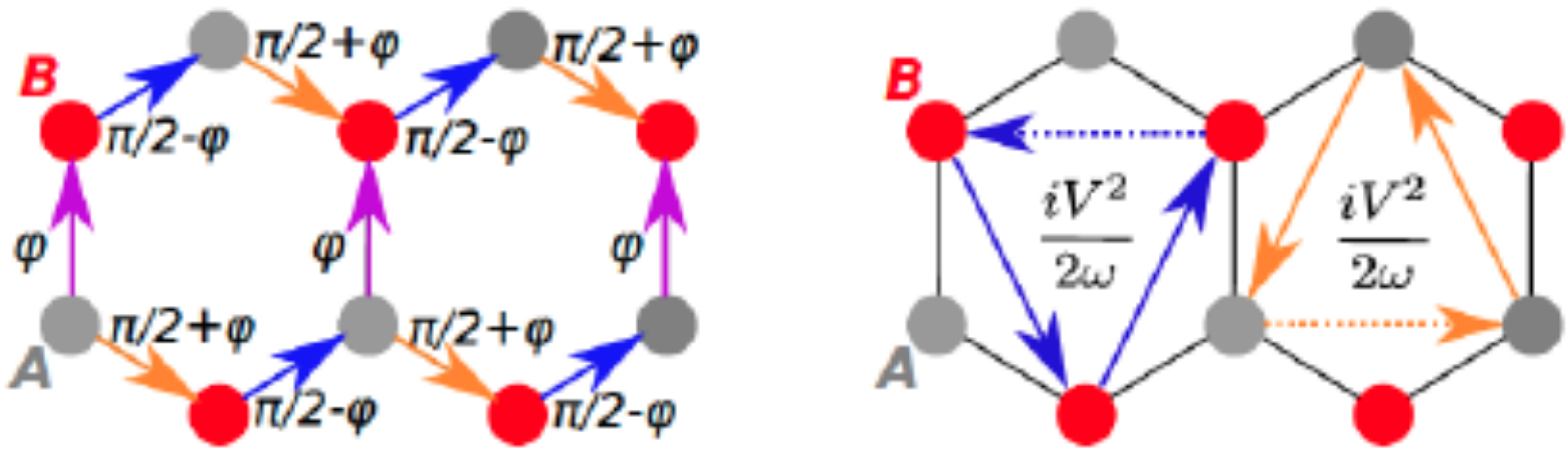
$$\hat{U}(T,t_0) = \mathcal{T} e^{-i\int_{t_0}^{t_0+T} \hat{H}(t)dt} = e^{-iT\hat{H}_{\text{eff}}(t_0)}$$

See also work by Kitagawa, Demler et al.

T : Hamiltonian periodic in time

Anisotropic version allowed

Laser-Raman assisted tunneling, high-frequency Floquet expansion



Computation (ED) of edge currents and Chern number for free particles

Important point: conservation zero net flux in a unit cell (one can build closed loop with Pi-flux with 4 sites in a unit cell)

Validity of Floquet Hamiltonian + interactions on the honeycomb lattice

K. Plekhanov (PhD student LPTMS & CPHT), G. Roux, KLH, 2016
arXiv:1608.00025

Chiral Bosonic Phases on the Haldane Honeycomb Lattice

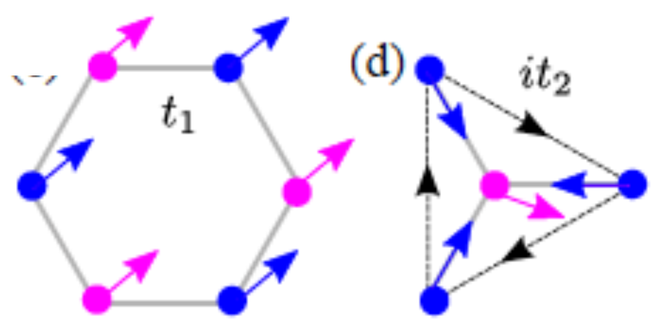
I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB 2015)

$$\mathcal{H} = \mathcal{H}_{\mathrm{H}} + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) - \mu \sum_i \hat{n}_i,$$

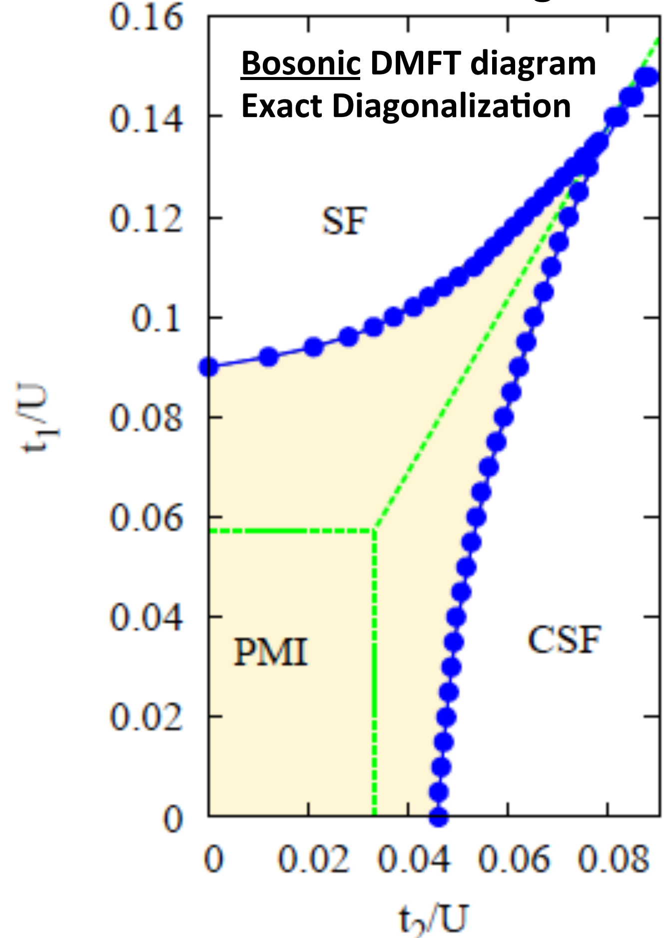
Phase-angle variables $b_i^\dagger = \sqrt{n}e^{i\theta_i}$

chiral SF:
nonuniform phase,
plaquette currents

SF:
uniform phase,
“Meissner current”



Unit filling

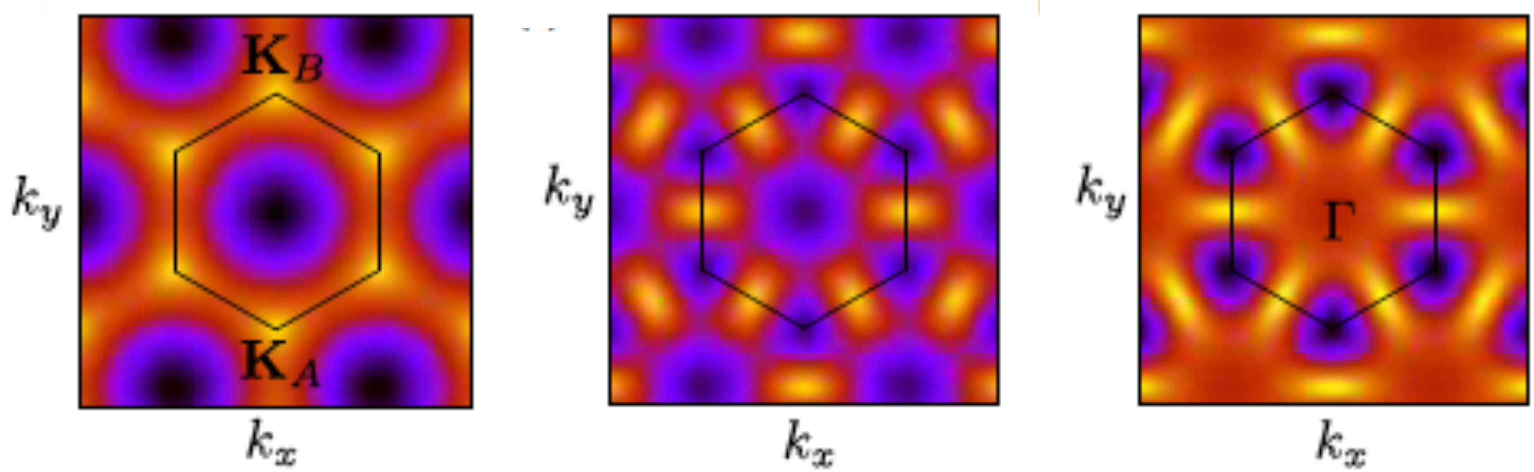


Similar models on square lattice:
L. K. Lim, C. M. Smith and A. Hemmerich,
Phys. Rev. Lett. 100, 130402 (2008) and PRA 2010

Condensation of Bosons

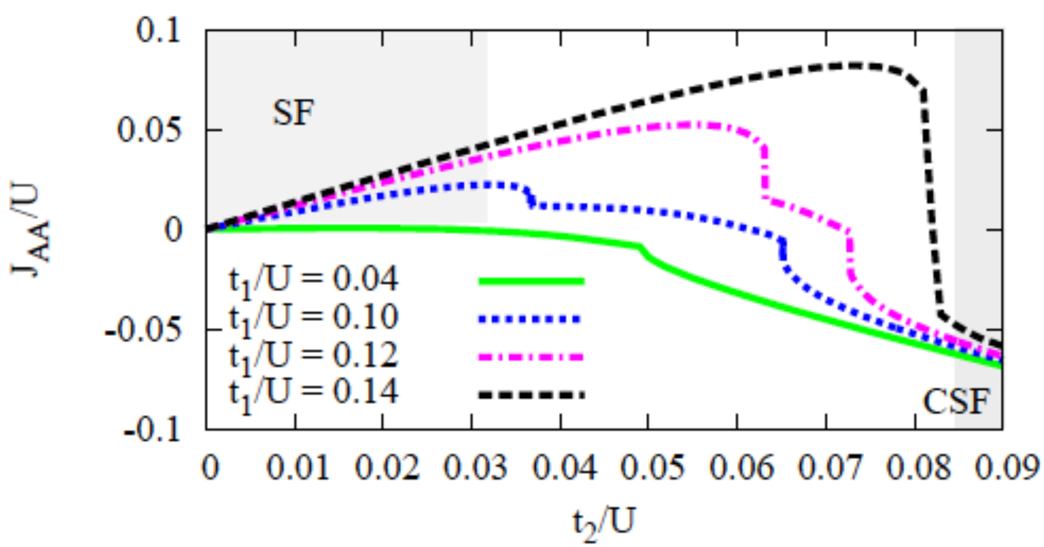
I. Vidanovic Vasic, A. Petrescu, K. Le Hur, W. Hofstetter, arXiv:1408.1411 (PRB 2015)

SF CSF



$$J_{AA}^{SF} = -2\,n\,t_2\,\mathrm{Im}\exp(-i\pi/2) = 2nt_2$$

Nature of the transition changes with Anisotropy : Plekhanov, Roux, Le Hur 2016



$$J_{AA}^{CSF} = -2\;\mathrm{Im}\left(t_2 e^{i\phi}\left\langle \hat{b}^\dagger_{Ai}\hat{b}_{Aj}\right\rangle\right)$$
$$= -2t_2 n \sin\left[\phi - \mathbf{K}_A\cdot(\mathbf{r}_i - \mathbf{r}_j)\right] = -nt_2$$

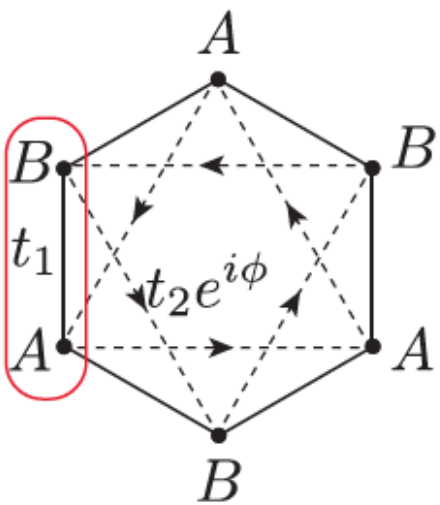
FFLO analogue in Heisenberg-Kitaev doped models

Tianhan Liu, Cécile Repellin,
Benoît Douçot, Nicolas Regnault
Karyn Le Hur, PRB Rapid Comm (just appeared)

No topological Mott « yet » but rich Mott

Strong coupling perturbation theory

A. Petrescu & KLH

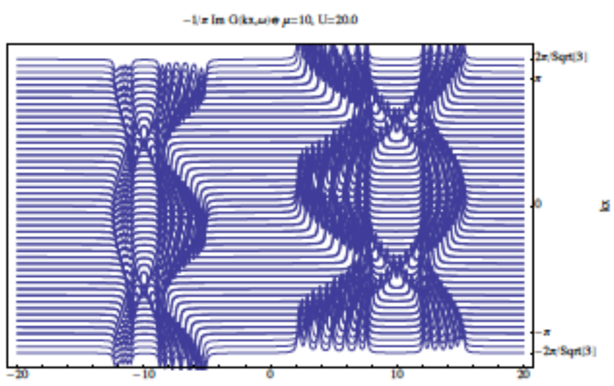


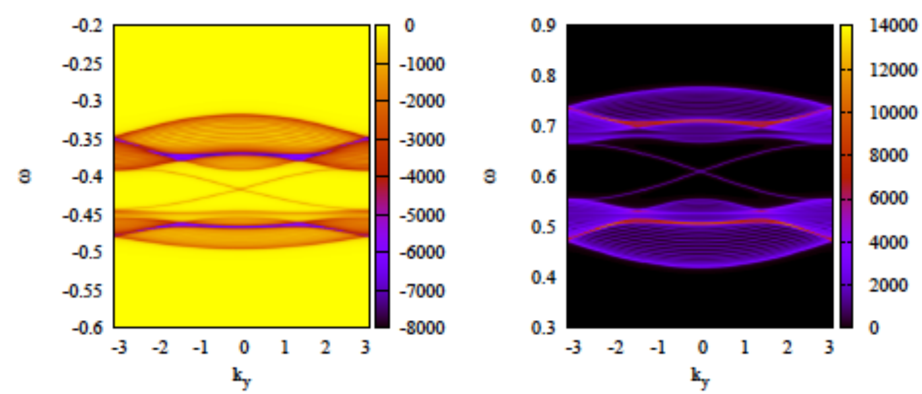
$$G^{-1}(i\omega,k) = g^{-1}(i\omega) - h_k.$$

$g(i\omega)$ = local cluster Green's function

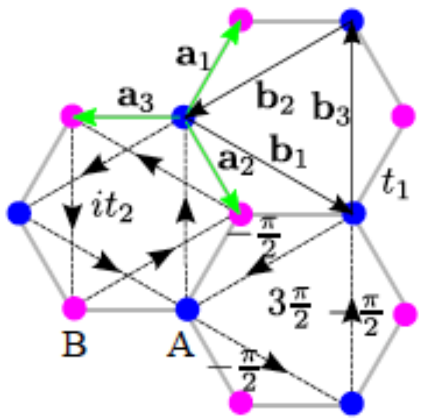
$G(i\omega,k)$ = approximate Green's function

DMFT (I. Vasic & W. Hofstetter)





2-component version promising: Rajbir Nirwan, K. Plekhanov, A. Petrescu, I. Vasic, G. Roux, KLH, W. Hofstetter, in progress



$$\mathcal{H}_{\mathrm{H}}(\mathbf{k}) = -\mathbf{d}\,(\mathbf{k}) \cdot \hat{\sigma},$$

We have introduced the field $\psi(\mathbf{k}) = (b_A(\mathbf{k}), b_B(\mathbf{k}))^T$ of Fourier transforms of the annihilation operators for bosons on sublattices A and B. We wrote $\mathcal{H}_{\mathrm{H}}$ in the basis of Pauli matrices $\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of

$$\mathbf{d}(\mathbf{k}) = \left(t_1 \sum_i \cos \mathbf{k}\, \mathbf{a_i}, t_1 \sum_i \sin \mathbf{k}\, \mathbf{a_i}, -2t_2 \sum_i \sin \mathbf{k}\, \mathbf{b_i} \right).$$

Realized in cold atoms:

Group of T. Esslinger, 2014

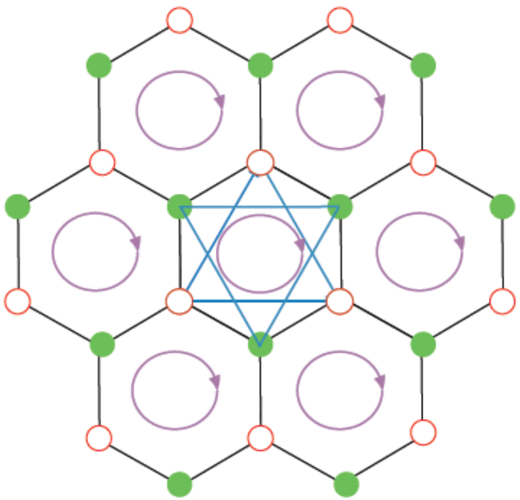
arXiv:1406.7874

The non-trivial topology of the Bloch bands translates to a nonzero winding number of the map $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ from the torus (the first Brillouin zone) to the unit sphere.

$$\mathcal{C}_- = \frac{1}{4\pi} \int_{\mathrm{BZ}} d\mathbf{k}\, \hat{\mathbf{d}} \cdot \left(\partial_1 \hat{\mathbf{d}} \times \partial_2 \hat{\mathbf{d}} \right)$$

This is the Chern number of the lower Bloch band, and takes the value $\mathcal{C}_- = 1$. The formula for the upper band is obtained by replacing $\hat{\mathbf{d}}$ by $-\hat{\mathbf{d}}$, and leads to $\mathcal{C}_+ = -1$.

Berry curvature & 2-level systems



$$\Phi^{+}(\boldsymbol{k}) = \begin{pmatrix} u_1^{+}(\boldsymbol{k}) \\ u_2^{+}(\boldsymbol{k}) \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_{\boldsymbol{k}}}{2} e^{i\phi_{\boldsymbol{k}}} \\ \sin\frac{\theta_{\boldsymbol{k}}}{2} \end{pmatrix},$$

$$\Phi^{-}(\boldsymbol{k}) = \begin{pmatrix} u_1^{-}(\boldsymbol{k}) \\ u_2^{-}(\boldsymbol{k}) \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta_{\boldsymbol{k}}}{2} e^{-i\phi_{\boldsymbol{k}}} \\ -\cos\frac{\theta_{\boldsymbol{k}}}{2} \end{pmatrix},$$

$$\mathcal{A}^{\alpha}(\boldsymbol{k}) = i \sum_{a=1}^{2} (u_a^{\alpha})^{*} \nabla_{\boldsymbol{k}} u_a^{\alpha},$$

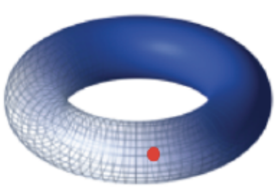
$$F_{xy}^{\alpha} = [\nabla_{\boldsymbol{k}} \wedge \mathcal{A}^{\alpha}(\boldsymbol{k})]_z = \partial_{k_x} A_y^{\alpha} - \partial_{k_y} A_x^{\alpha}.$$

$$C^{\alpha} = \frac{1}{2\pi} \int_{\mathrm{BZ}} dk\, F_{xy}^{\alpha}(\boldsymbol{k}),$$

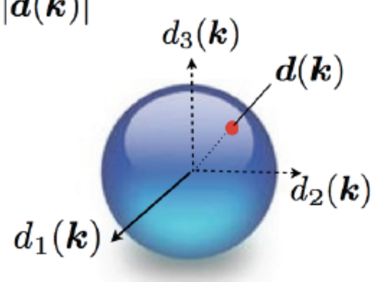
$$C^{-} = \frac{1}{4\pi} \int_{\mathrm{BZ}} dk\, \sin\theta_{\boldsymbol{k}} \left(\frac{\partial \theta_{\boldsymbol{k}}}{\partial k_x} \frac{\partial \phi_{\boldsymbol{k}}}{\partial k_y} - \frac{\partial \phi_{\boldsymbol{k}}}{\partial k_x} \frac{\partial \theta_{\boldsymbol{k}}}{\partial k_y} \right)$$

Brillouin zone

$\boldsymbol{k}$ → $\hat{\boldsymbol{d}}(\boldsymbol{k}) = \boldsymbol{d}(\boldsymbol{k})/|\boldsymbol{d}(\boldsymbol{k})|$



Bloch sphere

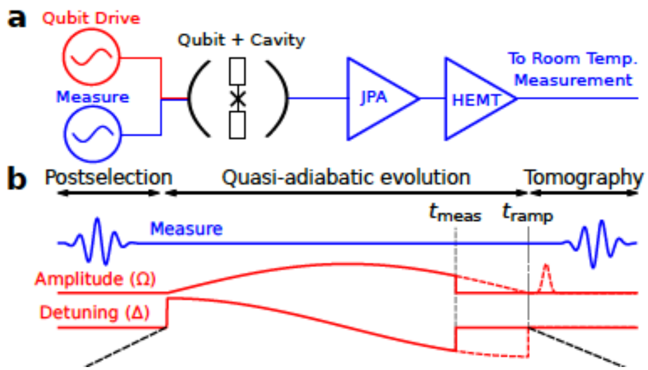


$$\hat{\boldsymbol{d}}(\boldsymbol{k}) = \frac{\boldsymbol{d}(\boldsymbol{k})}{|\boldsymbol{d}(\boldsymbol{k})|} = \begin{pmatrix} \cos\phi_{\boldsymbol{k}} \sin\theta_{\boldsymbol{k}} \\ \sin\phi_{\boldsymbol{k}} \sin\theta_{\boldsymbol{k}} \\ \cos\theta_{\boldsymbol{k}} \end{pmatrix},$$

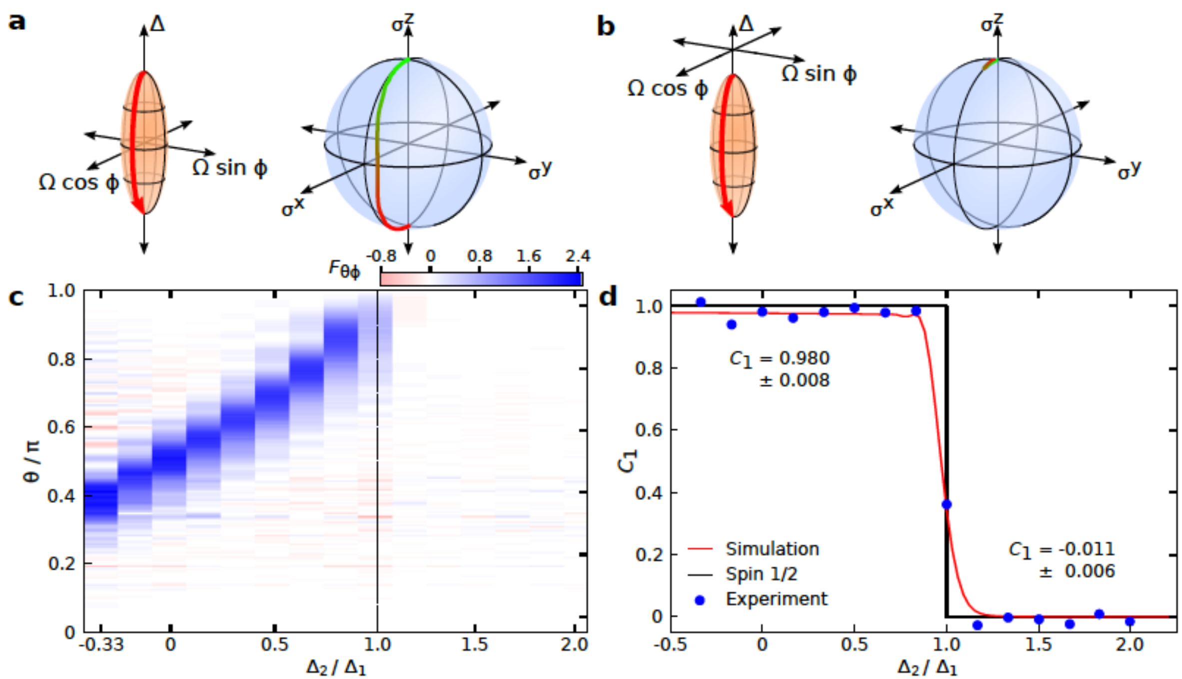
Review by J. Cayssol académie des sciences 2013, arXiv:1310.0792

spin-1/2 analogue:
Landau-Zener in curved space

Konrad Lehnert group (Colorado)
D. Schroer et al. PRL 2014
P. Roushan et al. Nature (John Martinis, Santa Barbara) 2014



$$H/\hbar = \frac{1}{2}\left[\Delta\, \sigma_z + \Omega\, \sigma_x \cos\phi + \Omega\, \sigma_y \sin\phi\right] \ ,$$



$$\Delta = \Delta_1 \cos\theta + \Delta_2 \ , \ \Omega = \Omega_1 \sin\theta$$

ARP protocole

$$\theta(t) = \pi t / t_{\text{ramp}}$$

$$F_{\theta\phi} = \frac{\langle \partial_\phi H \rangle}{v_\theta} = \frac{\Omega_1 \sin\theta}{2 v_\theta} \langle \sigma^y \rangle,$$

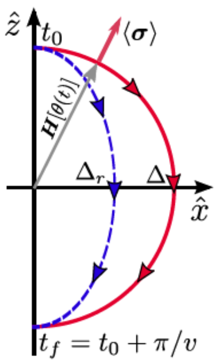
$$C_1 = \int_0^\pi F_{\theta\phi} d\theta \, .$$

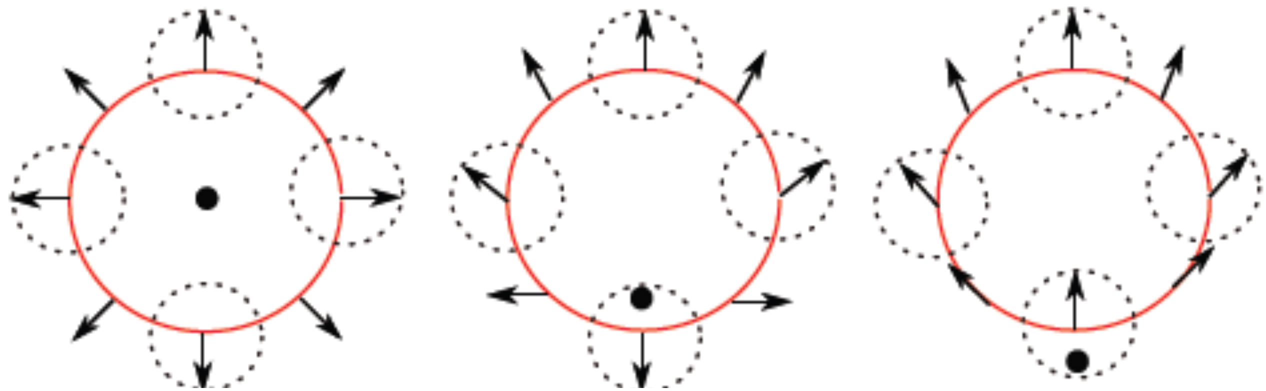
Tramp 1micro.s
Theory by Polkovnikov
Gritsev, de Grandi

Influence of a Caldeira-Leggett bath

$$\mathcal{H}_{TLS} = -\frac{1}{2}\vec{d}.\vec{\sigma},$$

$$\text{where } \vec{d} = (H\sin\theta\cos\phi, H\sin\theta\sin\phi, H_0 + H\cos\theta)^T.$$





Yale 2011

Bi-partite measurement

$$C = \frac{\langle\sigma^z(\theta=0)\rangle - \langle\sigma^z(\theta=\pi)\rangle}{2}.$$

$$\mathcal{H}_{diss} = \sigma^z \sum_k \frac{\lambda_k}{2}(b_k + b_k^\dagger) + \sum_k \omega_k \left(b_k^\dagger b_k + \frac{1}{2}\right).$$

$J(\omega) = \sum_i |\lambda_i|^2 m_i \delta(\omega-\omega_i) = J(\omega) \propto \alpha\omega$

Loic Henriet, A. Sclocchi, Peter P. Orth, KLH to appear

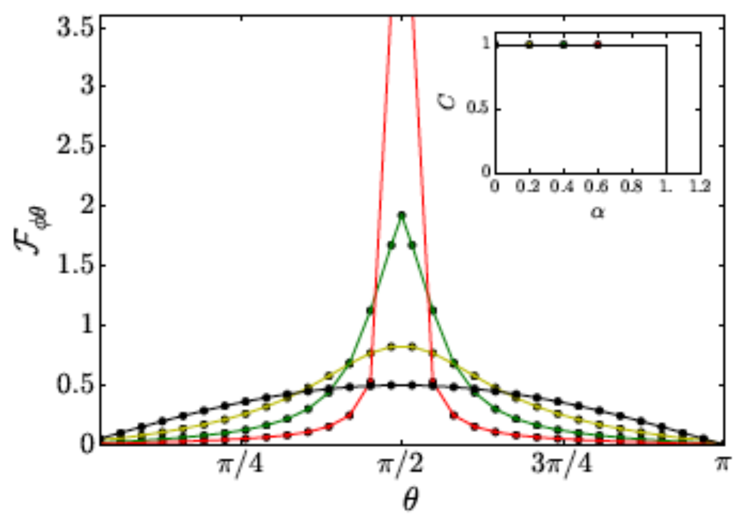
CPHT 2016

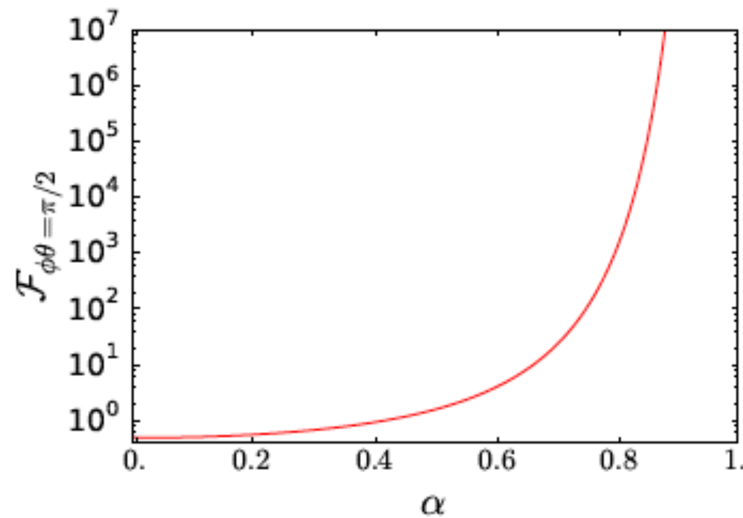
Equilibrium results:

Equilibrium Chern number « **quantized to one** »

$$|g\rangle = \frac{1}{\sqrt{p^2+q^2}}\left[pe^{-i\phi}|\uparrow_z\rangle \otimes |\chi_\uparrow\rangle + q|\downarrow_z\rangle \otimes |\chi_\downarrow\rangle\right],$$

Variational method
1 polaron expansion (Silbey-Harris)





Bethe Ansatz calculation (KLH, 2008 based on Cedraschi & Buttiker Annals of Physics paper)

$$\mathcal{F}_{\phi\theta} = -\partial_\theta \langle \sigma^z \rangle /2.$$

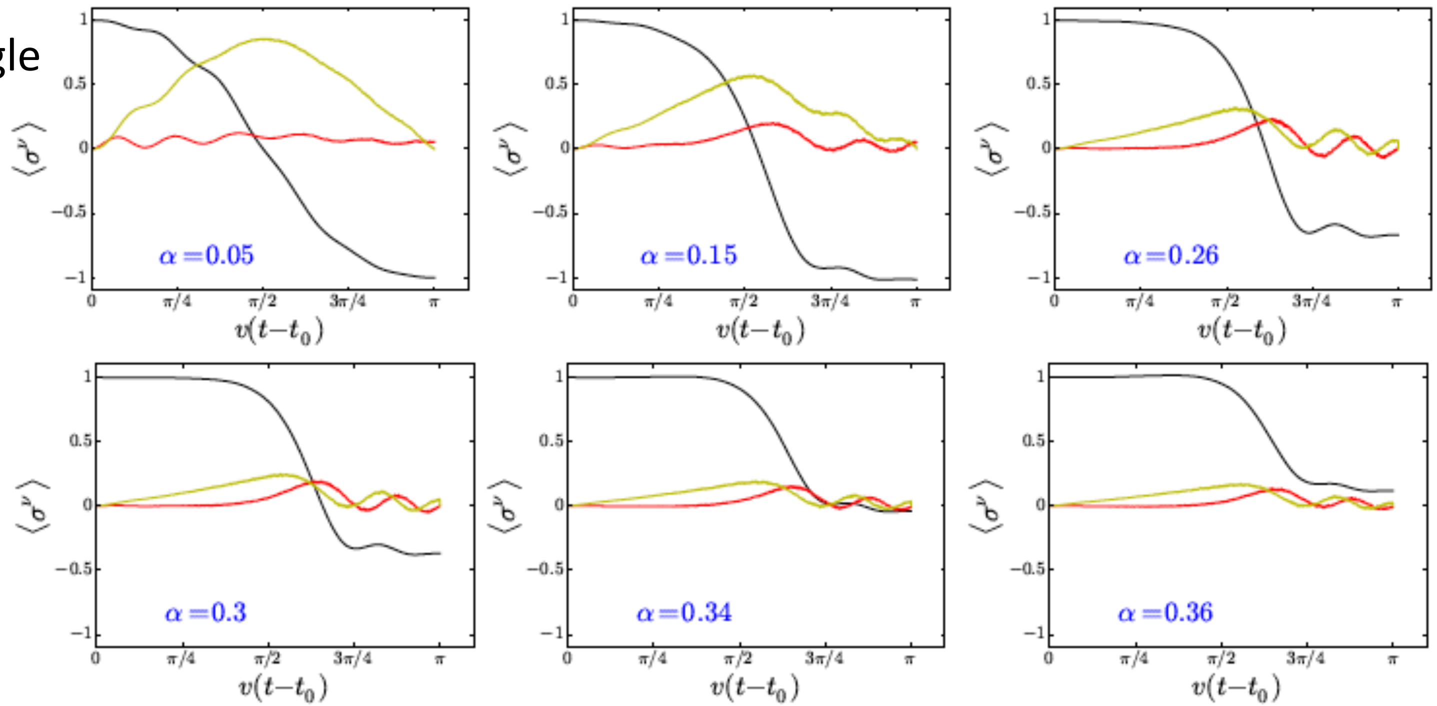
$$\mathcal{F}_{\phi\theta=\pi/2} = F(\alpha)\left(\frac{\omega_c}{H}\right)^{\frac{\alpha}{1-\alpha}},$$

In Agreement with Kosterlitz-Thouless transition at α=1
Antiferromagnetic-Ferromagnetic transition of Kondo model

Driven effects: stochastic approach

Polar angle = vt

(1 pulse)



[12] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys **59**, 1 (1987).
[13] P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B **1**, 4464 (1970).

Trace out the bath

Hubbard-Stratonovitch transformation

[35] P. P. Orth, A. O. Imambekov, and K. Le Hur, Phys. Rev. A **82**, 032118 (2010).
[36] P. P. Orth, A. O. Imambekov, and K. Le Hur, Phys. Rev. B **87**, 014305 (2013).
[37] L. Henriet, Z. Ristivojevic, P. P. Orth, and K. Le Hur, Phys. Rev. A **90**, 023820 (2014).
[38] L. Henriet and K. Le Hur, Phys. Rev. B **93**, 064411 (2016).
[39] J. Cao, L. W. Ungar, and G. A. Voth, The Journal of Chemical Physics **104**, 4189 (1996).
[40] J. T. Stockburger and C. H. Mac, J. Chem. Phys. **110**, 4983 (1999).
[41] J. T. Stockburger and H. Grabert, Phys. Rev. Lett. **88**, 170407 (2002).
[42] G. B. Lesovik, A. O. Lebedev, and A. O. Imambekov, JETP Lett. **75**, 474 (2002).

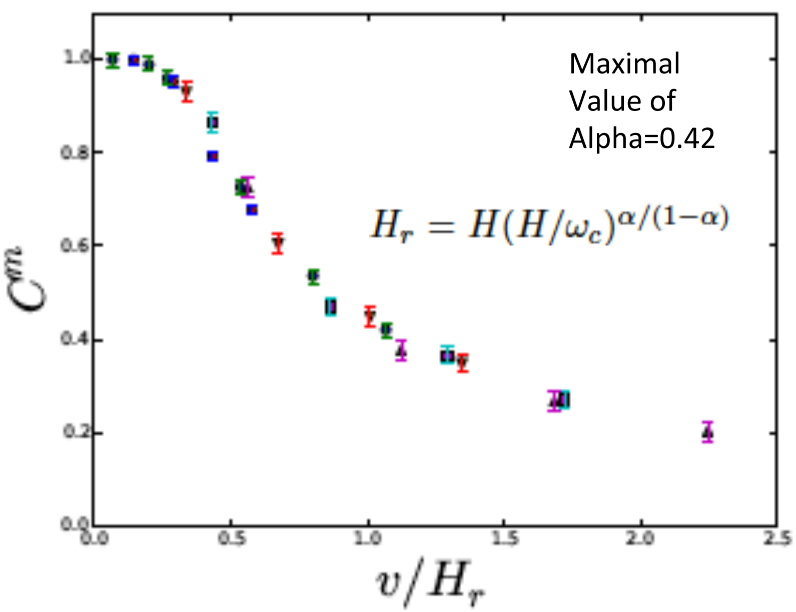
« purely dynamical Chern number »

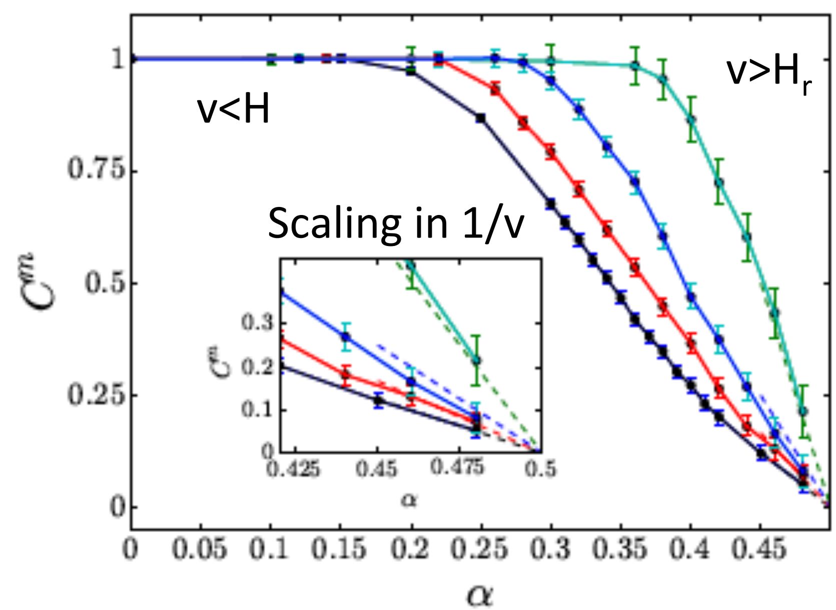
$$C = \underbrace{\frac{\langle \sigma^z(t_0)\rangle - \langle \sigma^z(t_f = t_0 + \pi/v)\rangle}{2}}_{C^m} + o(v).$$

$v \ll H_r,$

Slow (adiabatic) versus fast (non-adiabatic)

Effective Boltzmann-Gibbs description for the spin:
Negative temperature when population inversion
Spin stays « up » on the Bloch sphere





α =1/2 exactly solvable point (Toulouse)
resonant level model (Keldysh)

Effective 1 mode model

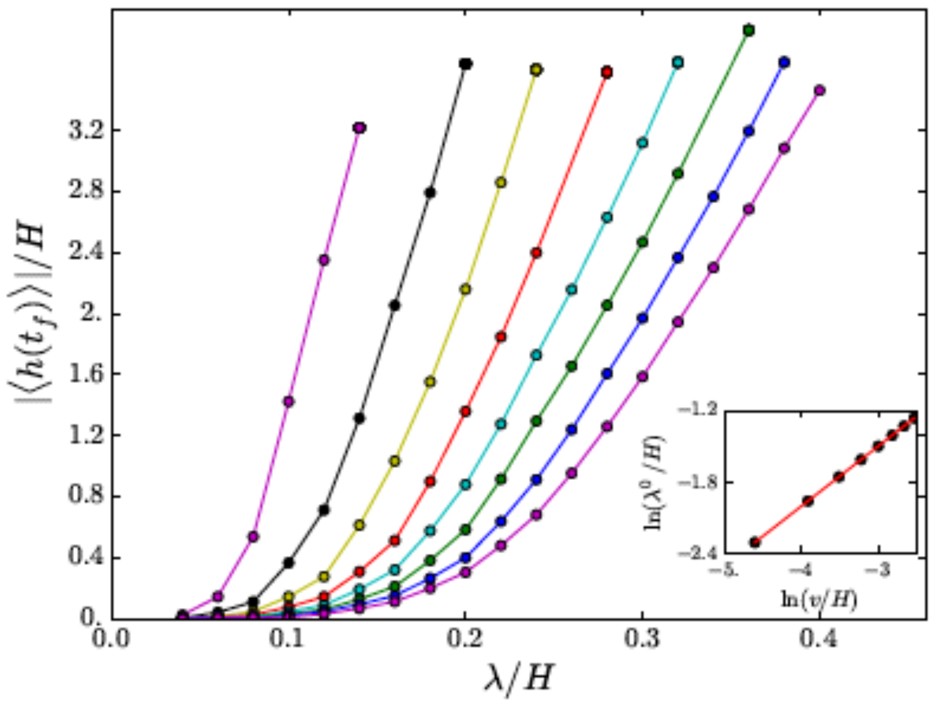
Induced field compensates the applied magnetic field: « photon-emission »
Loic Henriet, A. Sclocchi, Peter P. Orth, KLH to appear

$$H_{toy} = \frac{H}{2}\cos vt\sigma^z + \frac{H}{2}\sin vt\sigma^x + \frac{\lambda}{2}\sigma^z(b+b^\dagger) + vb^\dagger b.$$

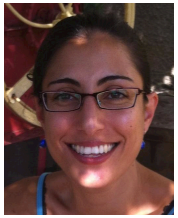
h induced

$$\lambda = \sqrt{2\alpha v H}$$

Driven Super-radiance



$<\sigma_z>$ is obtained from the stochastic approach
Dynamics of « b » solved with Master equation



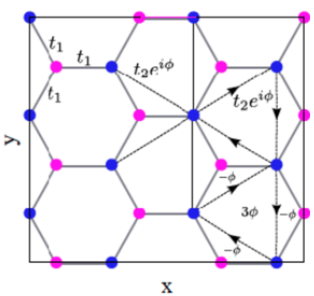
2 pulses

Ramsey Interferometry with environment, QPC: T. Goren, KLH, E. Akkermans **to appear**

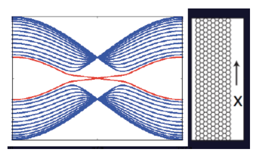
Quantum Simulators :

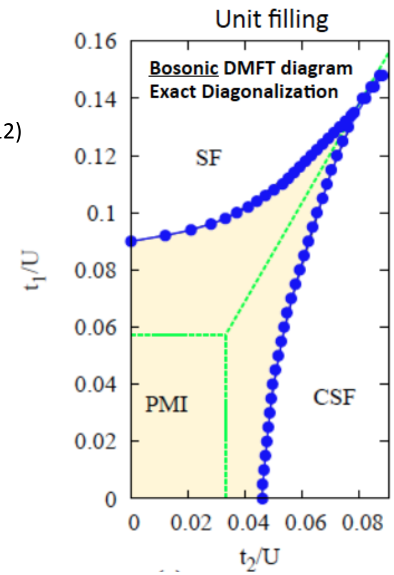
Quantum Hall phases, topological insulators, spin liquids (Kagome, Kitaev model, spin-1 chain)
symmetry protected phases, bosons and superconductors, Majoranas, …

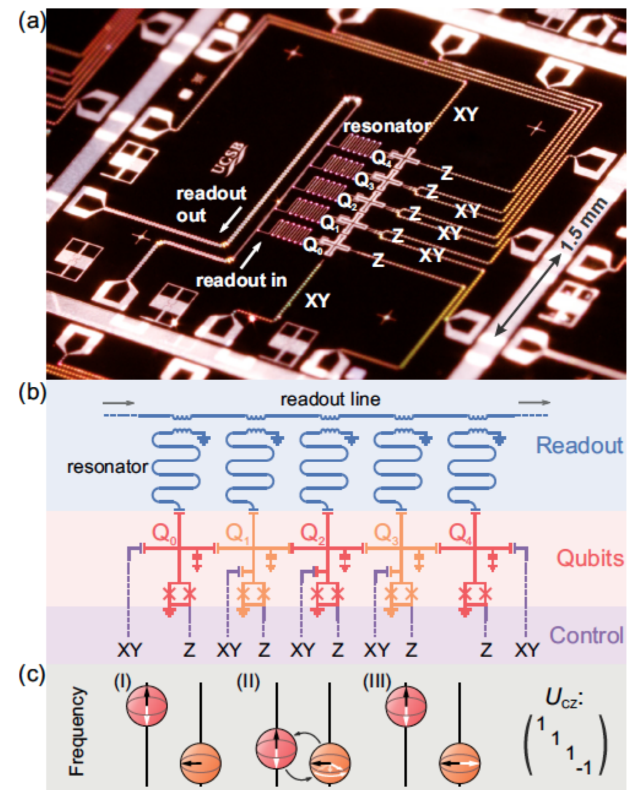
J. Martinis group



Collaboration CPHT & Frankfurt
With Walter Hofstetter
Guillaume Roux, LPTMS
CDMFT fermions (W. Wu et al 2012)





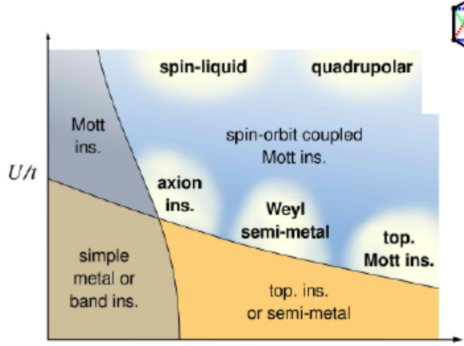


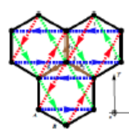
Chern insulators on graphene
realized experimentally in ultra-cold atoms
Photons and quantum materials
Many theorists involved (ENS Lyon, Bordeaux, ENS Paris, CPHT, LPS Orsay, UPMC, Cergy Pontoise, Toulouse,…)

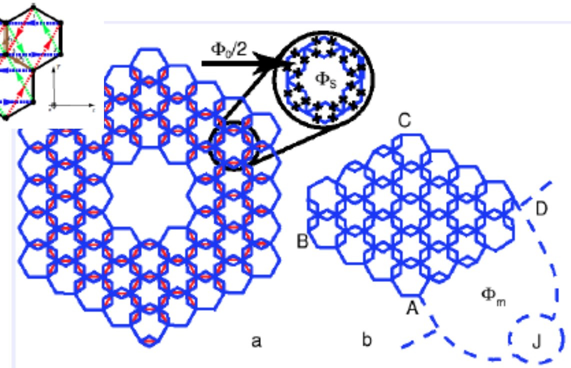
Simulating Quantum Materials (iridates) with Spin-orbit coupling (link with high energy and gauge theories: Chern-Simons models, gravitation),
L. Balents; Jackeli - Khaliulin

Protected qubits & Majoranas
Guichard, Buisson superconducting networks
Theory Benoît Douçot, Julien Vidal, Lev Ioffe
Implementing the Kitaev toric code;
Majorana analogues (Barbara Terhal)

Developments in engineering gates
Efforts in quantum graphs, walks in curve space
P. Arrighi, F. Debbasch, M.-E. Brachet







Some Developments of numerical efforts,
DMRG, ED, DMFT, QMC, stochastic approaches,…
D. Poilblanc (Toulouse) PEPS methods
Entanglement spectrum of Li and Haldane,
Numerically N. Regnault (ENS)

Students and Post-docs involved in talk : thanks

Sherbrooke & Yale
2002 - 2011

Picture Jean-Francois Dars, Anne Papillault, CNRS

More Informations at:

https://www.cpht.polytechnique.fr/cpht/lehur/Karyn.LeHur.html

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Francis H. Song (now New-York University post-doc, Yale 2012)
Tianhan Liu (UPMC and X, 2015 co-direction B. Douçot, now Cambridge post-doc)
Alexandru Petrescu (Yale and Ecole Polytechnique 2015, now Princeton post-doc)
Peter P. Orth (Yale 2010, now Minneapolis soon at Ames lab Faculty position)
Prasenjit Dutt (Yale PhD 2013, now Mathematical Finance at Stamford UBS)
Wei Wu (Yale 2010 and China, now collège De France Paris post-doc)
Emilie Dupont (Sherbrooke 2006); Michel Pioro-Ladriere (Sherbrooke 2004)

Post-doctoral (senior) associates :

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Ion Garate (Yale, now professor Sherbrooke)
Zoran Ristivojevic (CPHT, now Toulouse CNRS), Jens Koch (Yale, now Northwestern professor
Marie Piraud (Muenich), Marco Schiro (Princeton, Columbia now CNRS IPHT),
Ivana Vasic Frankfurt with W. Hofstetter, now Serbie Faculty)
special thanks to Adilet Imambekov

Ecole Polytechnique 2015