Entanglement dynamics following quantum quenches: Applications to 2d Floquet chern Insulator and 3d critical system

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Supported by DOE-BES and NSF- DMR

Quantum quench

$$\left|\Psi(t)\right\rangle = e^{-iH_f t}\left|\Phi_{H_i}\right\rangle$$

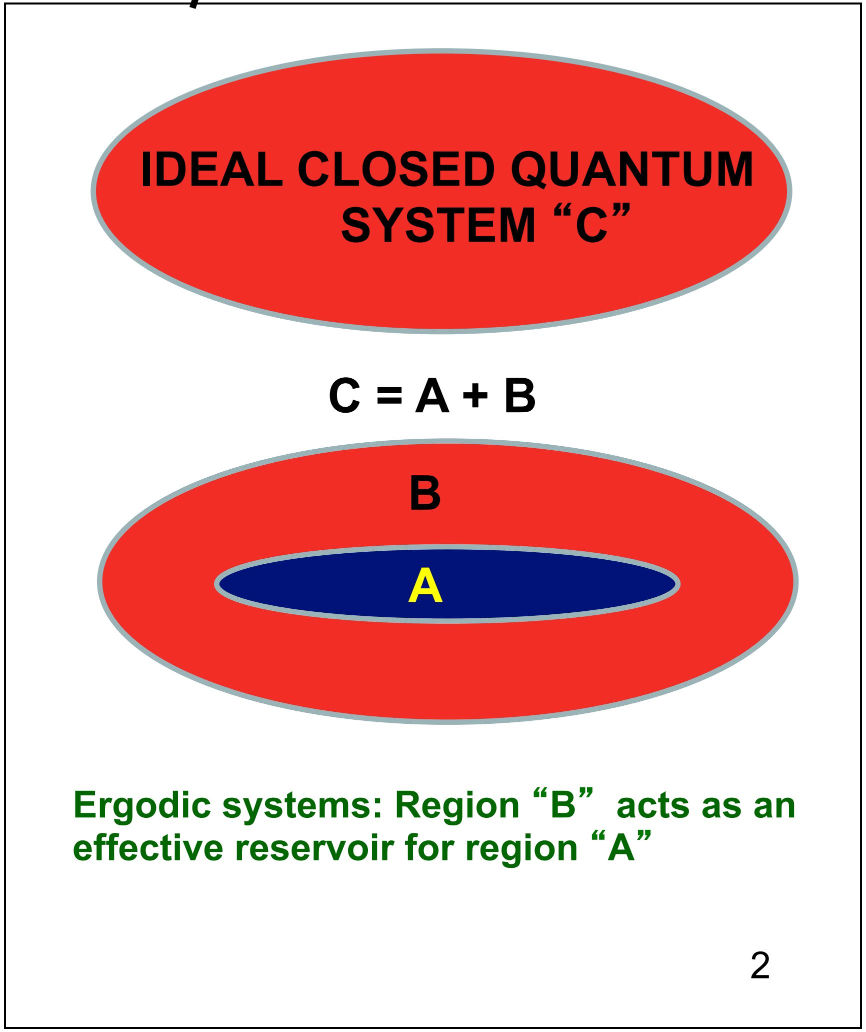
What happens when Hi is topological but Hf is not, what if Hi supports one phase (magnetically disordered….) and Hf supports another phase (critical or magnetically ordered).

Density matrix:

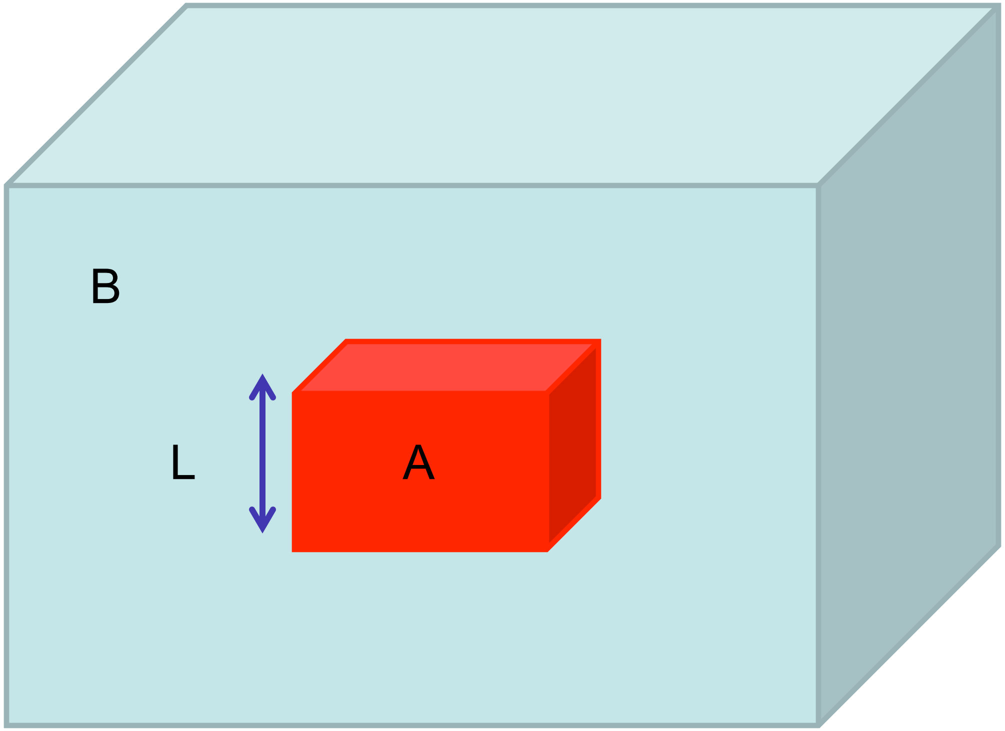
$$\rho(t) = \left|\Psi(t)\right\rangle\left\langle\Psi(t)\right|$$

A useful concept: Reduced density matrix

$$\rho_A(t) = Tr_B\left[\rho\right]$$



What does the entanglement entropy tell us?



$$\rho_A = Tr_B\big[|\psi\rangle\langle\psi|\big]$$

$$S_A = -Tr_B\big[\rho_A \ln \rho_A\big]$$

J. Eisert, M. Cramer, and M. B. Plenio, Rev. Mod. Phys. **82**, 277 (2010).
B. Zeng, X. Chen, D.-L. Zhou, and X.-G. Wen, arXiv:1508.02595 (2015).
P. Calabrese and J. Cardy, Journal of Statistical Mechanics: Theory and Experiment **2005**, P04010 (2005).
P. Calabrese and J. Cardy, Journal of Statistical Mechanics: Theory and Experiment **2007**, P10004 (2007).
U. Schollwöck, Annals of Physics **326**, 96 (2011), january 2011 Special Issue.

Ground state of Hamiltonians:
Area law for short-range interaction, area law violated for long-range interactions.

$$S_A = O(L^{d-1})$$

d: spatial dimensions

Excited states: Volume law

$$S_A = O(L^d)$$

Area-law => efficient numerical simulations using matrix product states (eg: DMRG)

Non-ergodic states (many body localized states) show area law entanglement entropy even for excited eigen-states.

Entanglement Spectrum has even more information

Entanglement spectrum: Eigenvalues of the reduced density matrix

$$\rho_A(t) = Tr_B[\rho]$$

A spectroscopic tool when there is no conventional order-parameter, examples being topological order.

Reflects the bulk-boundary correspondence in topological systems where an entanglement cut in a spatially extended system now hosts "edge-states".

MORE ON THIS LATER!

M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006).

A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006).

H. Li and F. D. M. Haldane, Phys. Rev. Lett. **101**, 010504 (2008).

L. Fidkowski, Phys. Rev. Lett. **104**, 130502 (2010).

What about dynamics such as pure-states evolving due to a quantum quench at time t=0?

Concrete results only in 1-dimension and that too for short-ranged interactions.

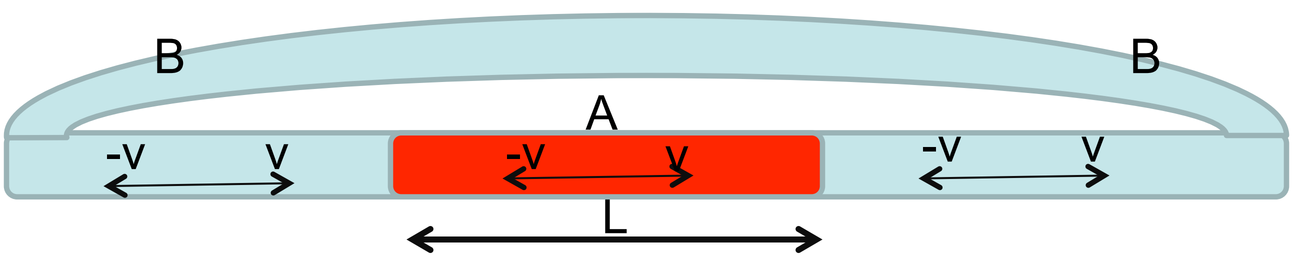
Entanglement growth in time reflects Lieb-Robinson bounds (the maximum speed "v" at which information propagates).

$$S_A = a + vt, t < \frac{L}{2v}$$

$$S_A = O(L), t > \frac{L}{2v}$$

P. Calabrese and J. Cardy, Journal of Statistical Mechanics: Theory and Experiment **2007**, P10004 (2007).

Quasi-particle picture: After a quench, entangled quasi-particles pairs are emitted everywhere.



At time such that L=2vt, the region A has got completely entangled with the region B.

Outline of talk:

Quenches in higher spatial dimensions (d>1).

A). Quenches in d=2 when final Hamiltonian has topological order due to periodic drive (Floquet Chern Insulator). Signatures of it in transport, ARPES as well as in the entanglement statistics.

Daniel Yates, Yonah Lemonik, AM, PRB (2016), in print

Daniel J. Yates, Yonah Lemonik, and Aditi Mitra, *Entanglement properties of Floquet Chern insulators*, arXiv:1602.02723, in review at PRL.

Hossein Dehghani and Aditi Mitra, *Occupation probabilities and current densities of bulk and edge states of a Floquet topological insulator*, arXiv:1601.05732, in review at PRB.

Hossein Dehghani and Aditi Mitra, *Floquet topological systems in the vicinity of band crossings: Reservoir induced coherence and steady-state entropy production*, arXiv:1512.00532, in review at PRB.

Hossein Dehghani and Aditi Mitra, *Optical Hall conductivity of a Floquet Topological Insulator*, Phys. Rev. B **92**, 165111 (2015).

Hossein Dehghani, Takashi Oka, and Aditi Mitra, *Out of equilibrium electrons and the Hall conductance of a Floquet topological insulator*, Phys. Rev. B **91**, 155422 (2015).

Hossein Dehghani, Takashi Oka, and Aditi Mitra, *Dissipative Floquet Topological Systems*, Phys. Rev. B **90**, 195429 (2014).

B). Quenches near a critical point (d>2). Looking for universal physics both in conventional quantities such as correlation functions, as well as in entanglement entropy and spectrum

Yonah Lemonik, AM, PRB (2016)

Alessio Chiocchetta*, Marco Tavora*, Andrea Gambassi, and Aditi Mitra, *Short-time universal scaling in an isolated quantum system after a quench*, Phys. Rev. B **91**, 220302(R) (2015). ***Authors contributed equally**.

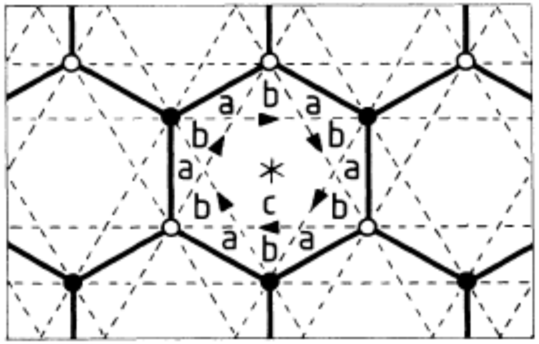
Anna Maraga, Alessio Chiocchetta, Aditi Mitra, Andrea Gambassi, *Aging and coarsening in isolated quantum systems after a quench: Exact results for the quantum $O(N)$ model with $N \to \infty$*, Phys. Rev. E **92**, 042151 (2015).

Yonah Lemonik and Aditi Mitra, *Entanglement properties of the critical quench of $O(N)$ bosons*, arXiv:1512.02749, in review at PRL.

Alessio Chiocchetta, Marco Tavora, Andrea Gambassi, and Aditi Mitra, *Short-time universal scaling and light-cone dynamics after a quench in an isolated quantum system in d spatial dimensions*, arXiv:1604.04614, in review at PRB.

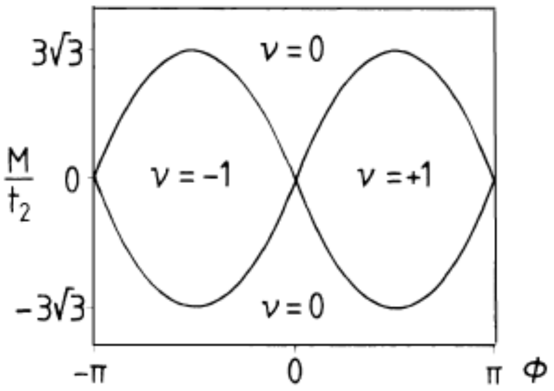
Chern insulator: Quantum Hall effect in zero magnetic field

Haldane, 1988



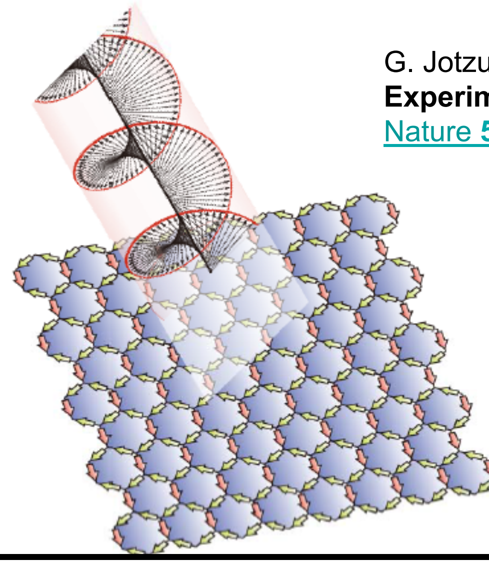
Time reversal symmetry broken by a staggered (but net zero) magnetic flux.

Bands with nonzero Chern number and protected chiral edge modes.



Haldane model can be realized experimentally using time-periodic drives!

Graphene irradiated with circularly polarized laser



G. Jotzu, T. Esslinger et al.:
Experimental realization of the topological Haldane model
Nature **515**, 237-240 (2014).

Oka and Aoki PRB 2009 in graphene
Other models:
Kitagawa et al PRB 2011,
Lindner et al Nature 2011
Yao, MacDonald, Niu et al PRL 2007

$$U(T+t,t) = e^{-iH_{eff}T}$$

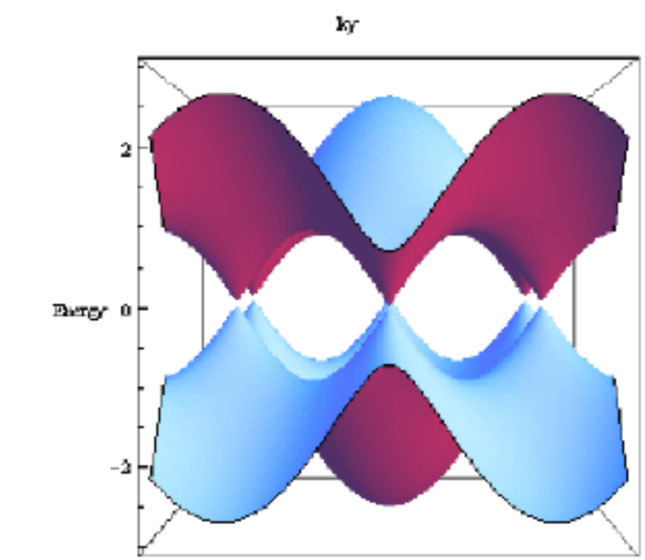
$$\boxed{H_{eff} \approx k_x \sigma_x \tau_z + k_y \sigma_y + \frac{A_0^2}{\Omega} \sigma_z \tau_z}$$

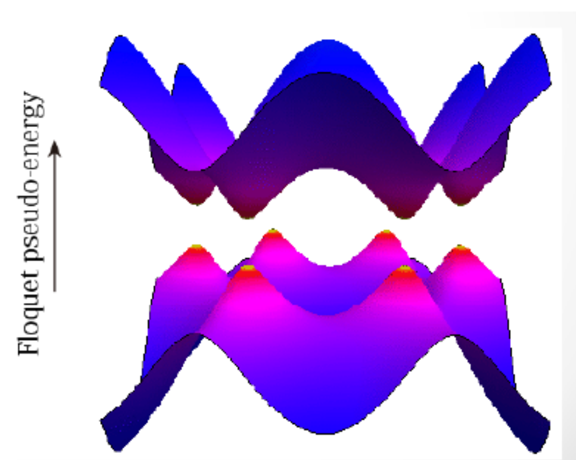
σ:Sublattice index

τ:K,K' points

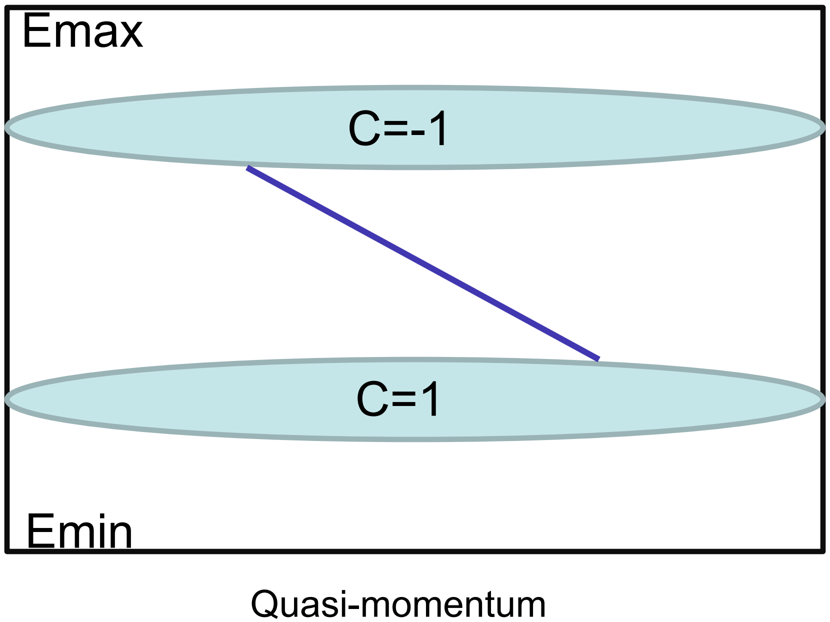
Breaks time-reversal

H_{eff} **Maps onto the Haldane model**

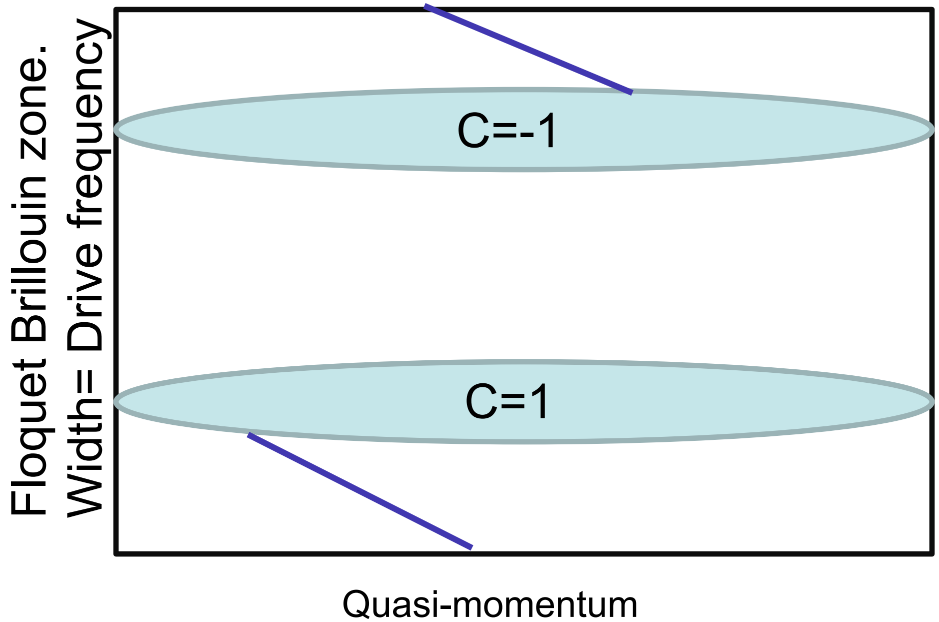




STATIC CHERN INSULATOR:
Energy bands



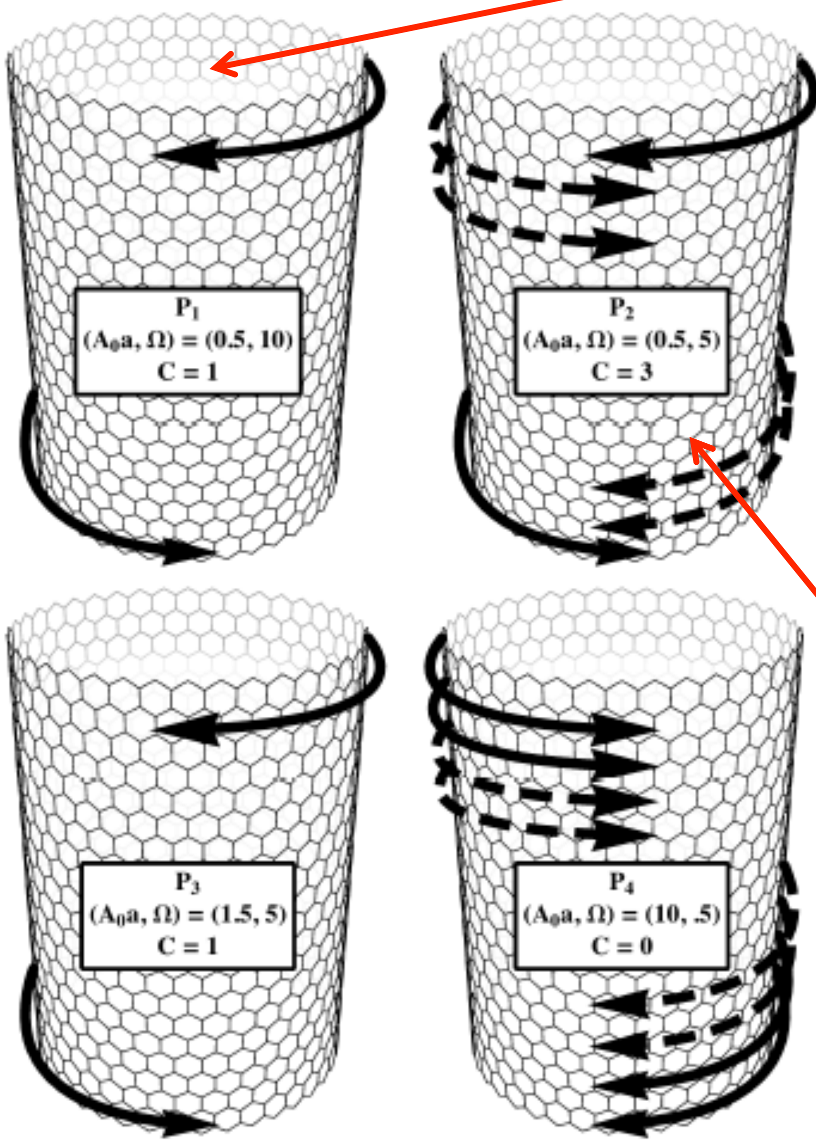
FLOQUET CHERN INSULATOR:
Quasi-energy band



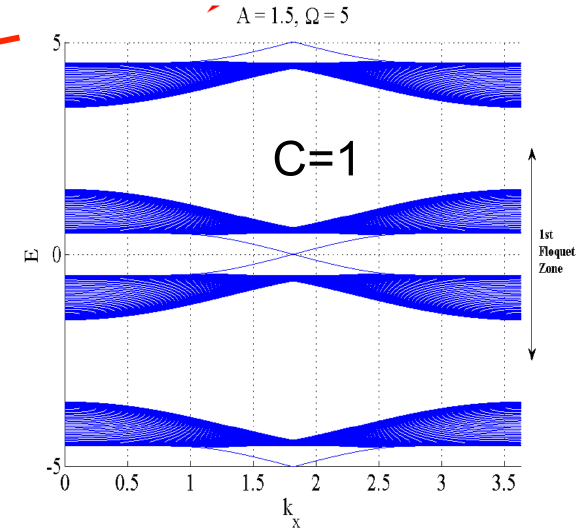
For C=1, there are two possible ways in which topological edge-states connect the two bands.

For Floquet Chern Insulators, C= Difference between the number of chiral edge-modes above and below the quasi-energy band
(see: Rudner, Lindner, Berg, Levin PRX, 2013)

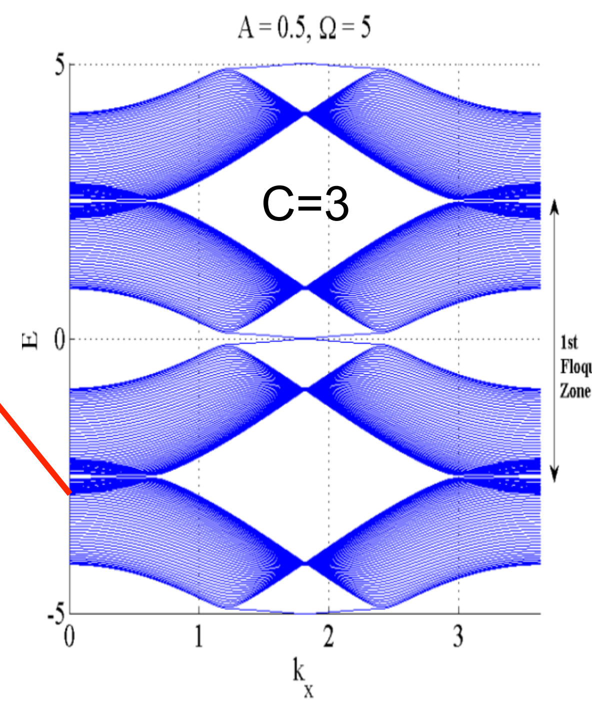
Examples of some topological phases of the Floquet Chern Insulator



Solid line: Edges at the center of the Floquet zone
Dashed line: Edge-states at at Floquet zone-boundaries

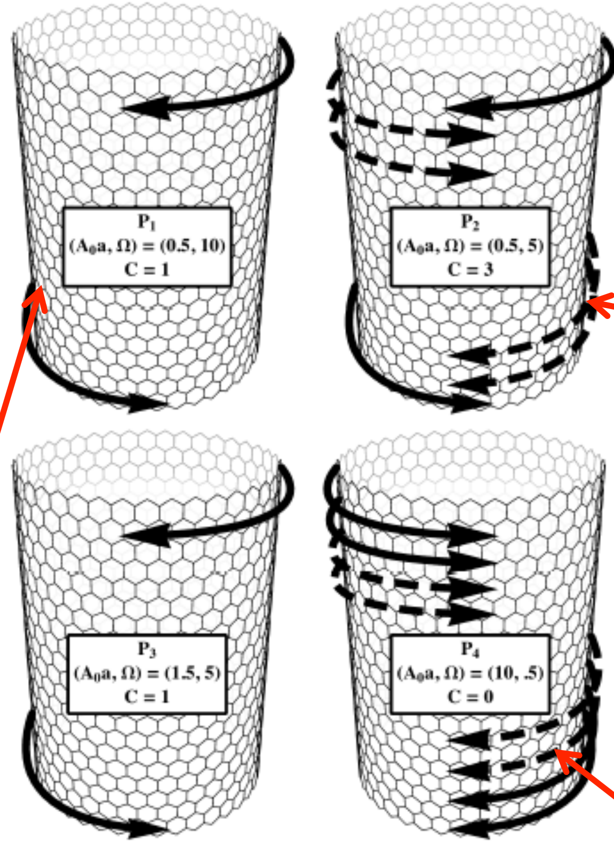


Graphene band width = 6



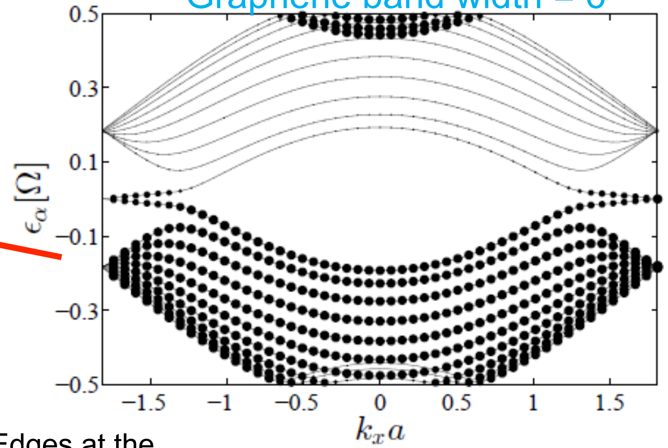
Dehghani and Mitra, 2016

Band occupation for closed system for a laser quench



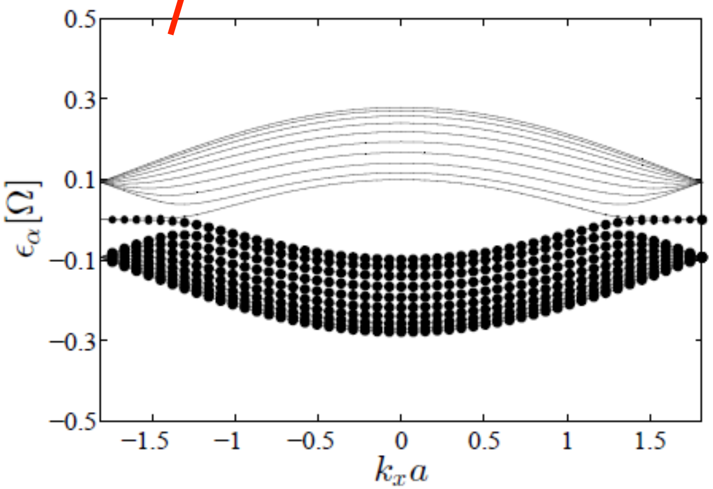
High frequency limit a lot like Quenched Haldane Model: Ciao, Cooper, Bhaseen PRL 2015

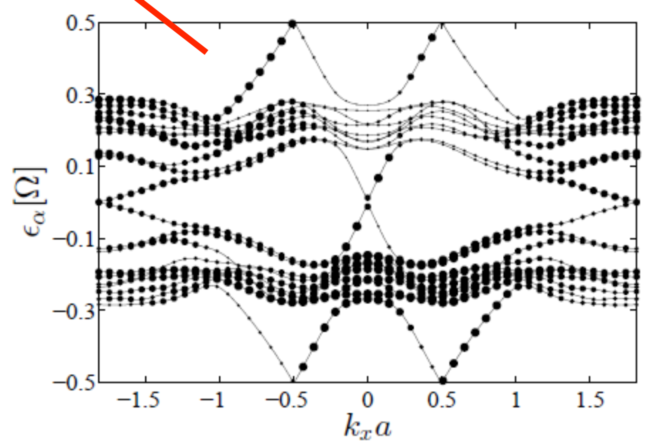
Graphene band width = 6



Solid line: Edges at the center of the Floquet zone
Dashed line: Edge-states at at Floquet zone-boundaries

Size of filled circles: population





Physical Observables : dc Hall conductance/conductivity

$$j_i(t) = \int dt' \sigma_{ij}(t,t') E_j(t')$$

Conductance = conductivity* L^{d-2}

Kubo formula for dc conductance (after time-averaging over laser cycle): Similar to Anomalous Hall conductance except that Berry curvature is time-averaged over a laser cycle.

$$\sigma_{xy}(\omega = 0) = \frac{e^2}{h} \int d^2k \overline{\Omega}_k (\rho_{kd} - \rho_{ku})$$

$$\overline{\Omega}_k = \frac{1}{T} \int_0^T dt \Omega_k(t)$$

$$\Omega_k(t) = \frac{1}{\pi} \mathrm{Im}\left[\left\langle \partial_x \varphi(t) \middle| \partial_y \varphi(t) \right\rangle \right]$$

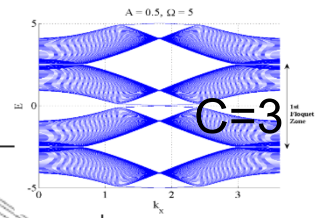
$$|\rho_{ku} - \rho_{kd}| = 1, \sigma_{xy}(\omega = 0) \rightarrow \frac{Ce^2}{h}$$

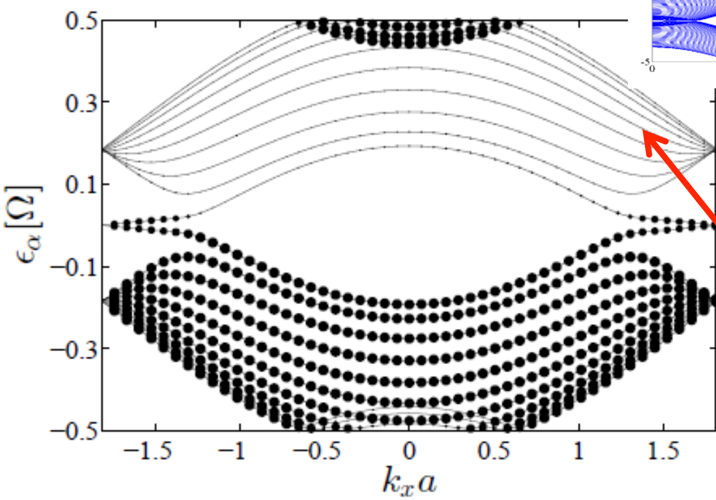
Dehghani, Oka, and Mitra, 2015

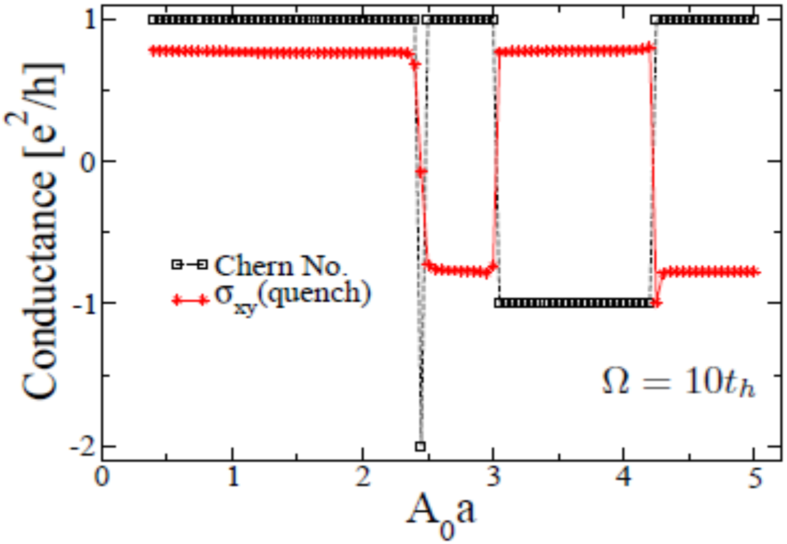
Dehghani, Oka, and Mitra, 2015

Hall conductance: chern number vs quench case (closed system)

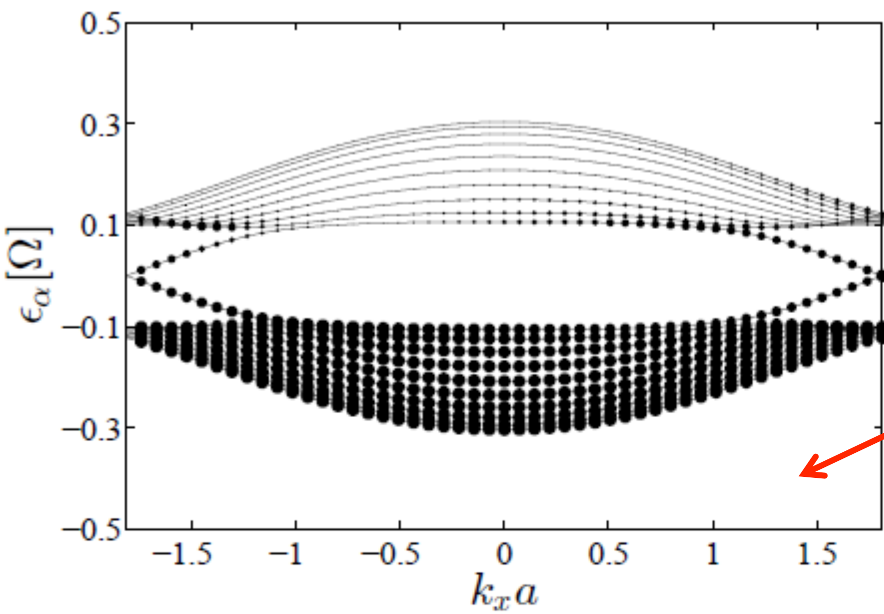
Graphene band width = 6 th

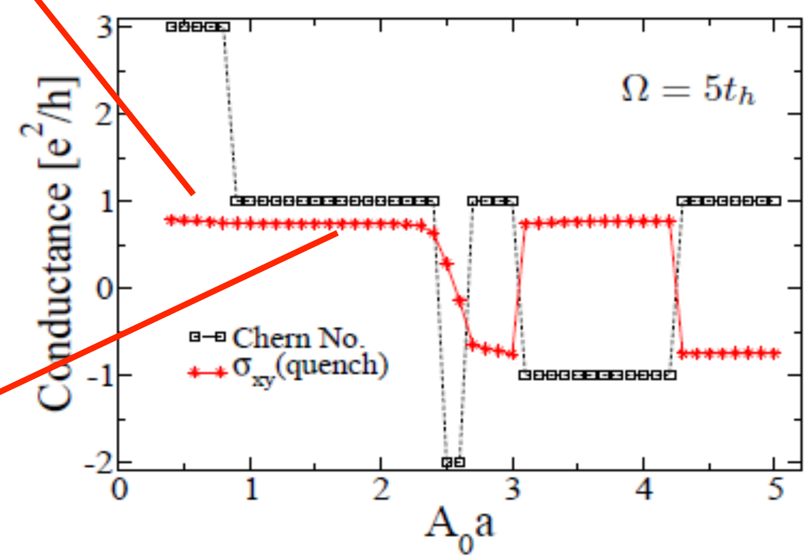






(a)





What about topological protection of Floquet edge states?

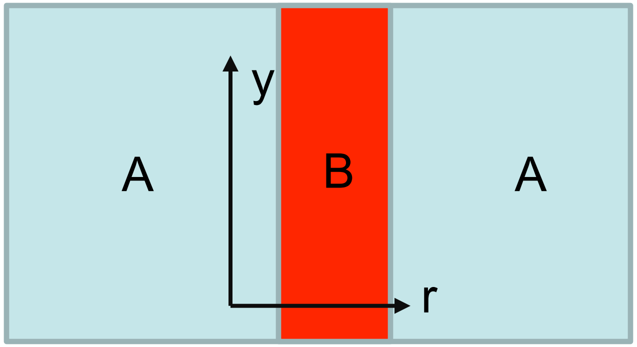
Next: A picture in terms of entanglement properties.

Entanglement spectrum: Diagonalization of reduced density matrix or equivalently the correlation matrix (Wick's theorem)

I. Peschel and V. Eisler, Journal of Physics A: Mathematical and Theoretical **42**, 504003 (2009).

$$C_{rr'} = \mathrm{Tr}\left[\rho c_r^\dagger c_{r'}\right]$$

$$C_{rr'}\left(t \to \infty\right) = \tfrac{1}{2}\delta_{rr'} + D_{rr'}$$



For a spatially invariant system, eigenvalues of C are simply the occupation probabilities.

Entanglement cut: Semi-infinite strip, Translationally invariant in y.

$$D_{rr'} \equiv \int_{BZ} d^2k\ \delta\rho_k M(k) e^{ik\cdot(r-r')},$$

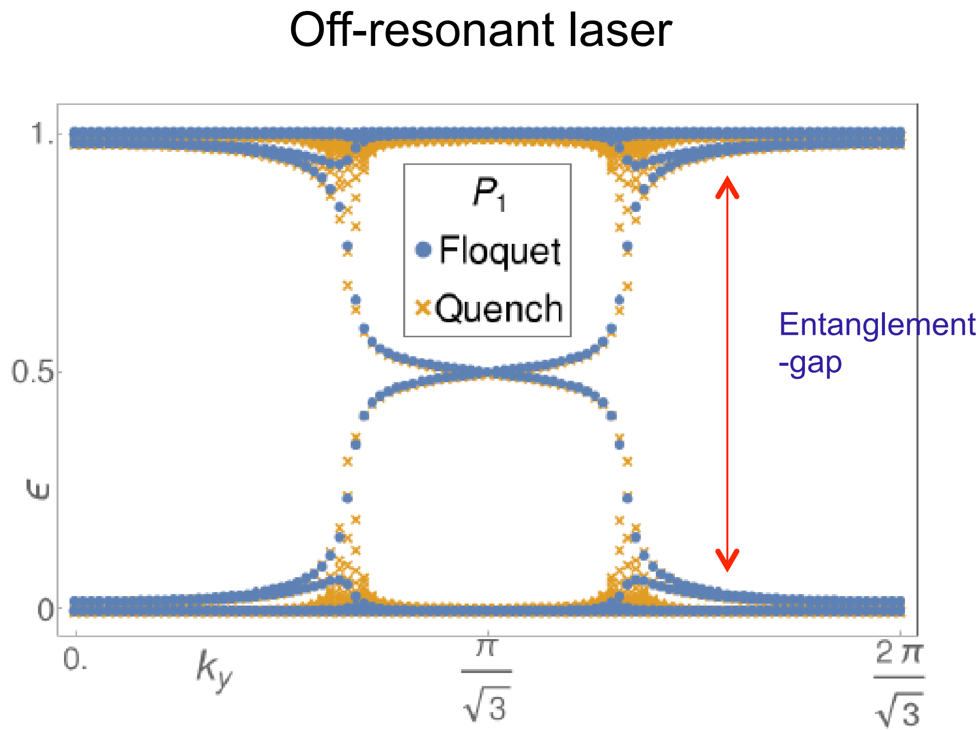
$$M(k) \equiv |a_k(t)\rangle\langle a_k(t)| - |b_k(t)\rangle\langle b_k(t)|,$$

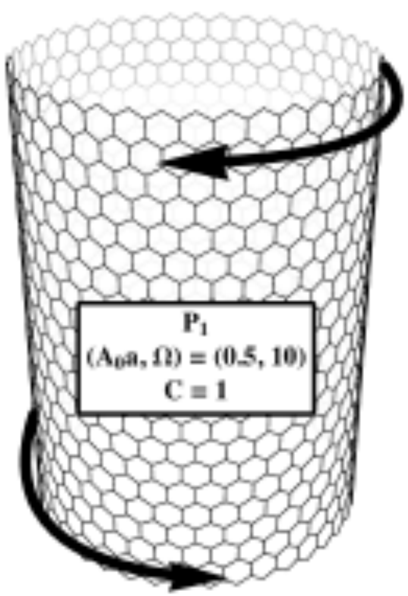
$$\delta\rho_k = \frac{\rho_{k,\mathrm{down}} - \rho_{k,\mathrm{up}}}{2}.$$

arXiv:1602.02723 **Entanglement properties of Floquet Chern insulators** Daniel J. Yates, Yonah Lemonik, Aditi Mitra

A part of the entanglement spectrum is simply the occupation of the bulk bands but now projected on the ky axis.

Entanglement spectrum: off-resonant laser.





Solid line: Edges at the
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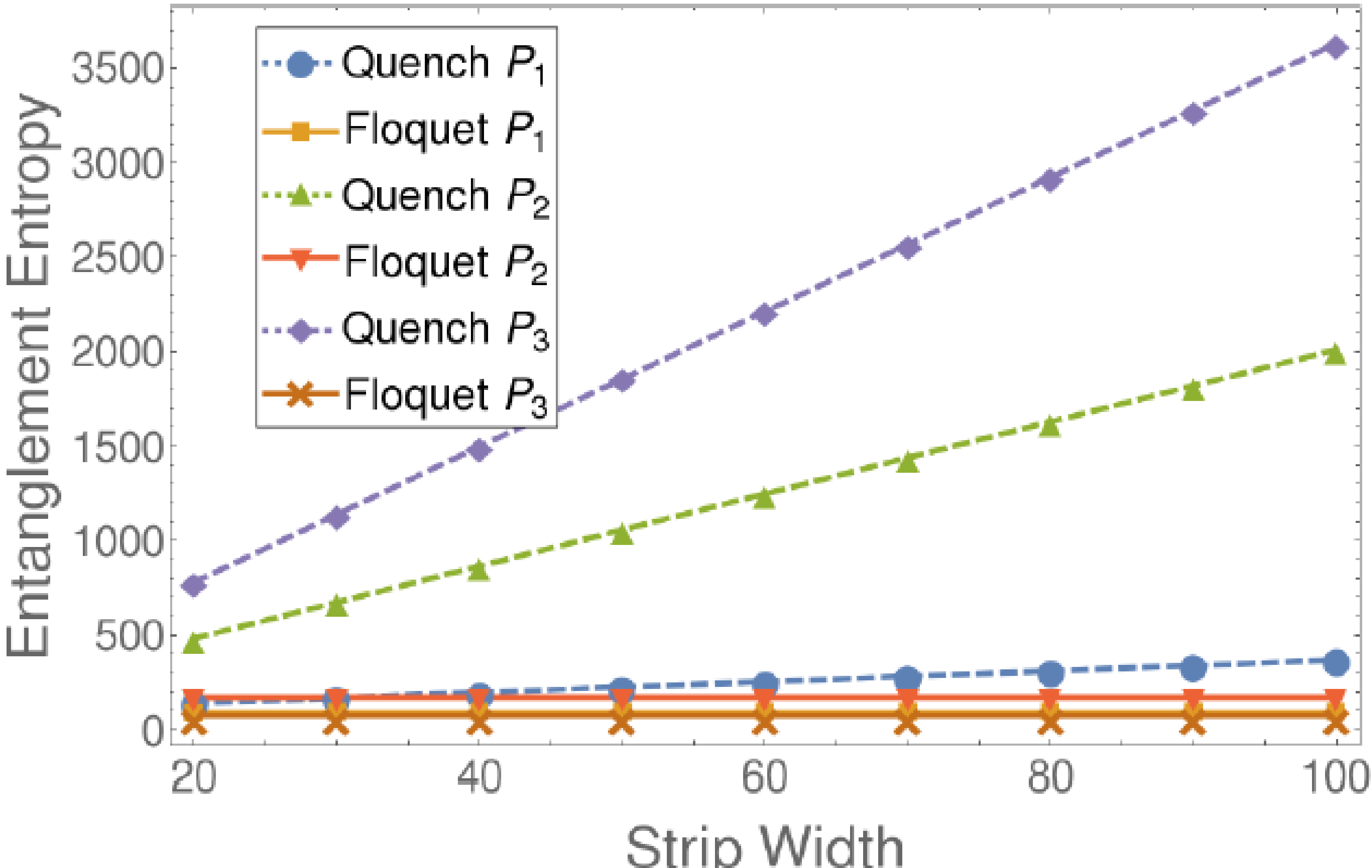
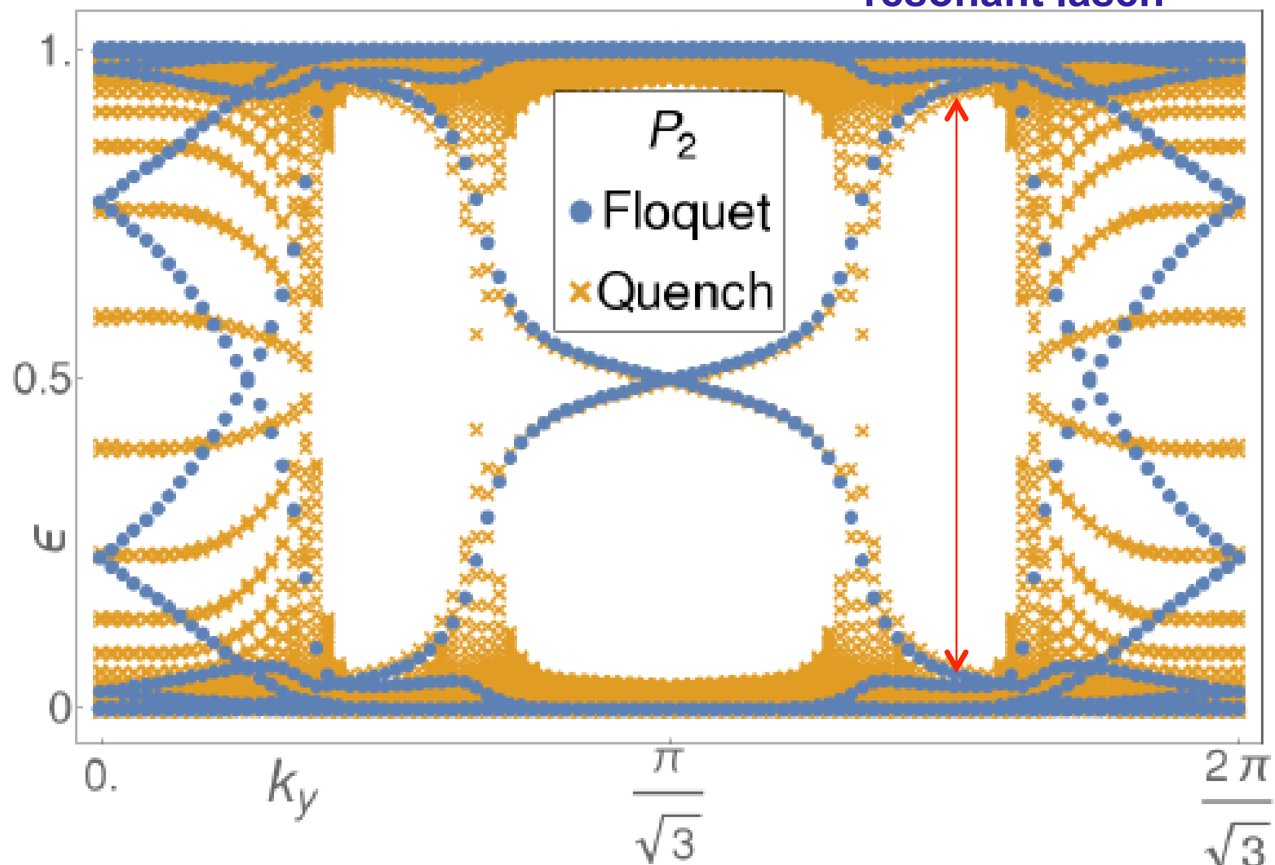
arXiv:1602.02723 Entanglement properties of Floquet Chern insulators
Daniel J. Yates, Yonah Lemonik, Aditi Mitra



$$S_A = \sum \left[\epsilon \ln \epsilon + (1-\epsilon) \ln (1-\epsilon) \right]$$

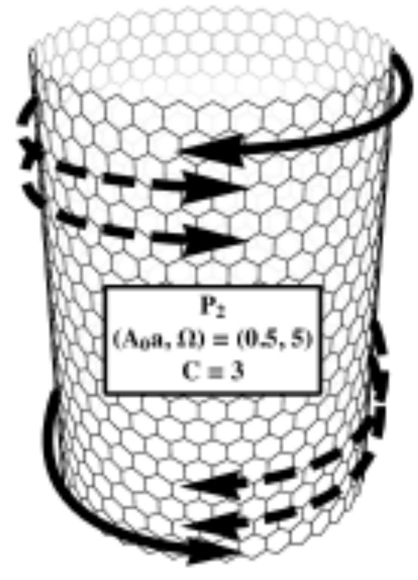
Resonant laser

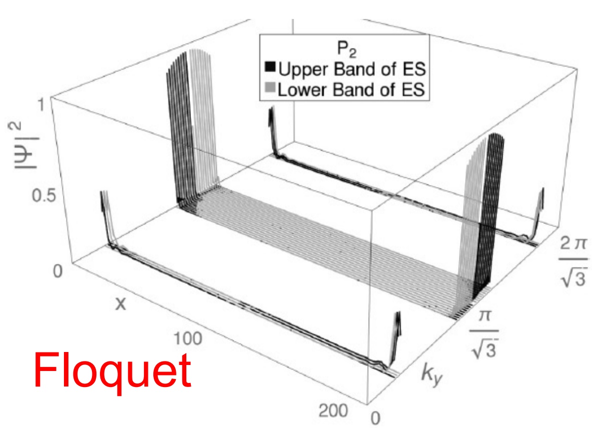
Entanglement spectrum: not all edge-states visible for the resonant laser.



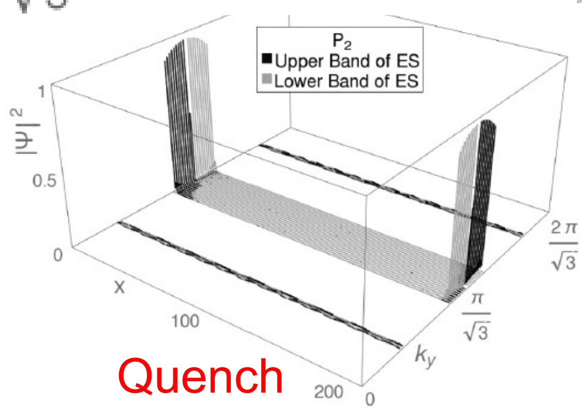
Entanglement -gap

Solid line: Edges at the center of the Floquet zone
Dashed line: Edge-states at at Floquet zone-boundaries



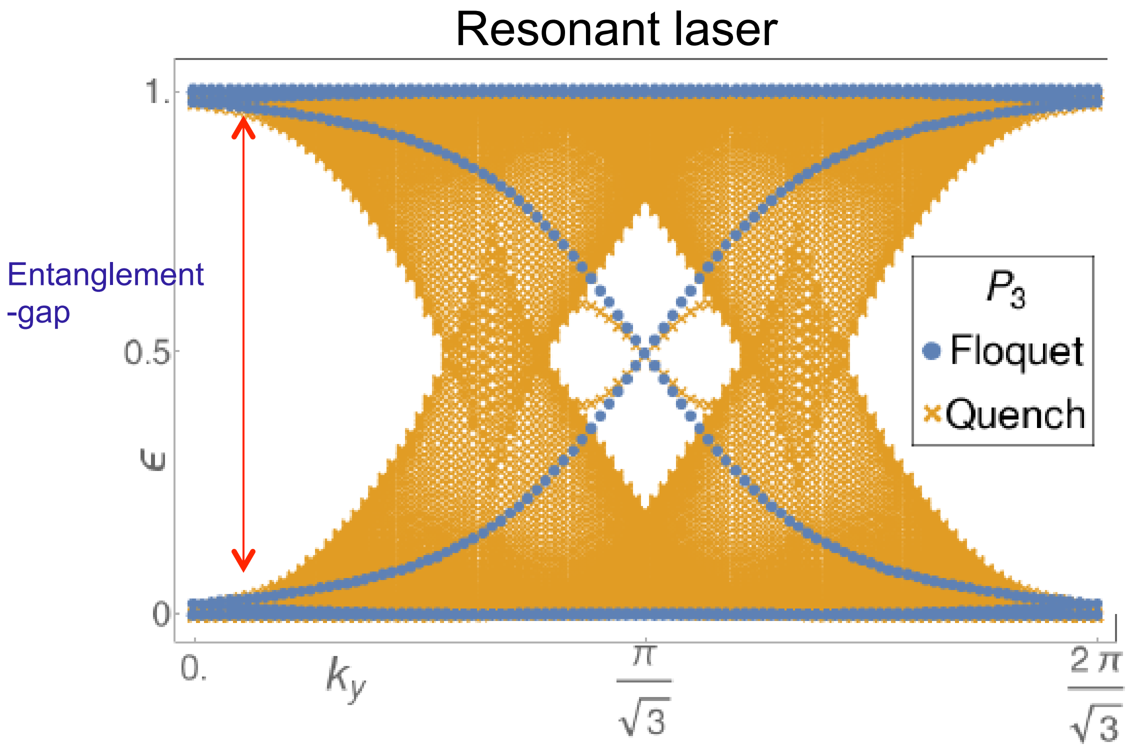


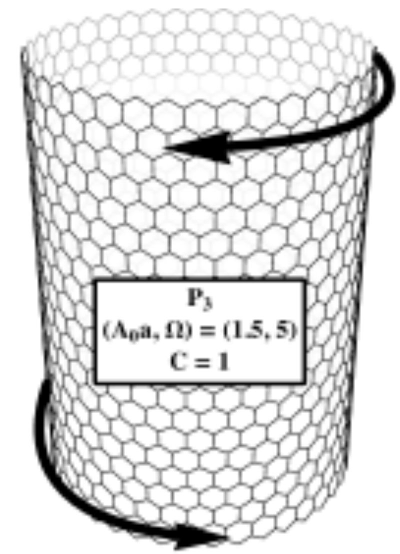
Floquet



Quench

Closing of the entanglement gap due to laser induced excitations





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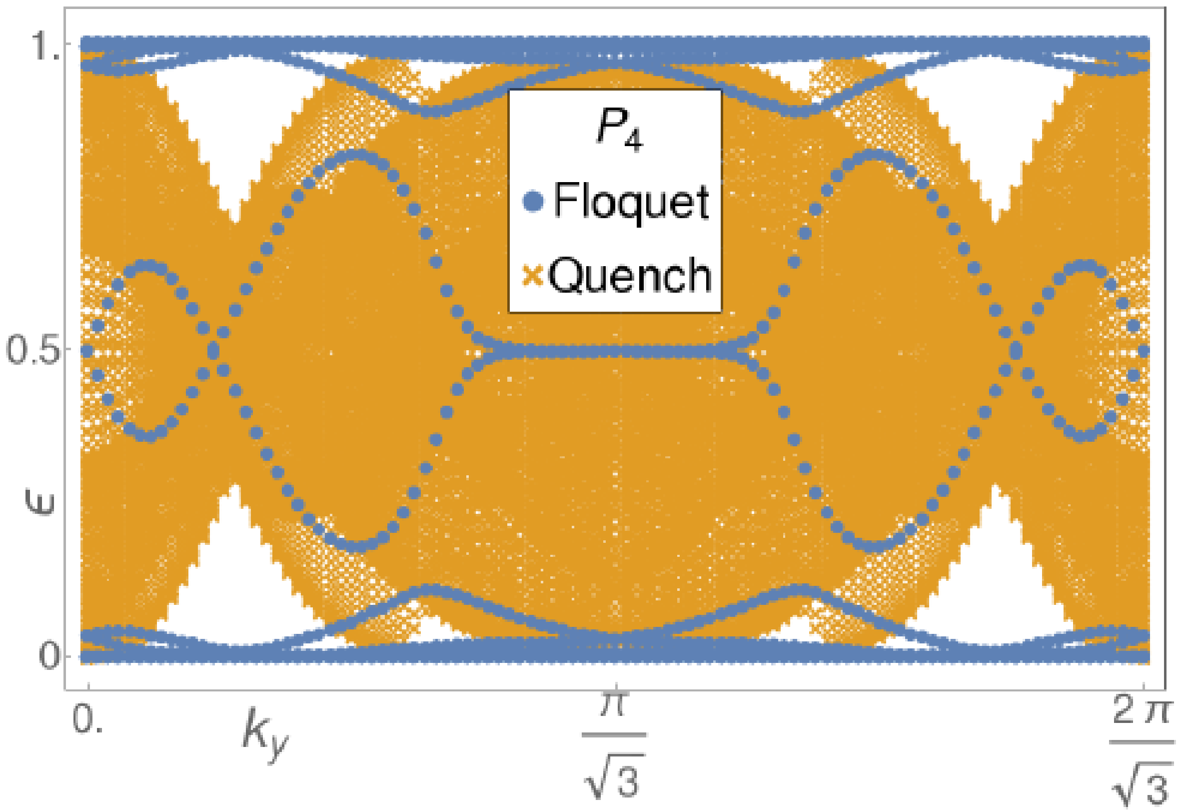
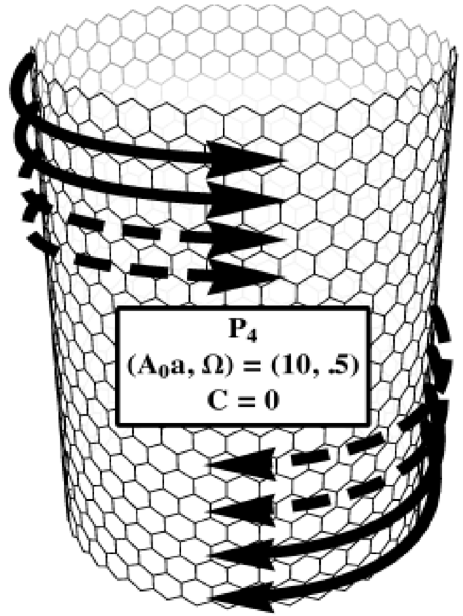
For a resonant laser, edge states co-exist with bulk excitations. Thus these states are no longer protected as they can hybridize with the bulk states.

Resonant laser





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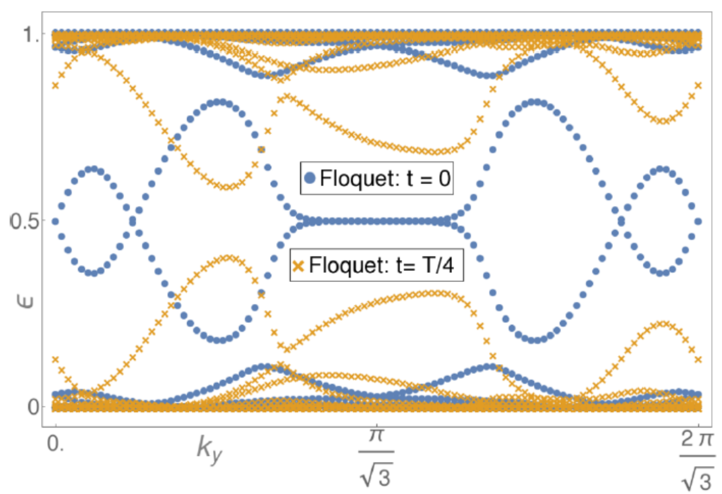
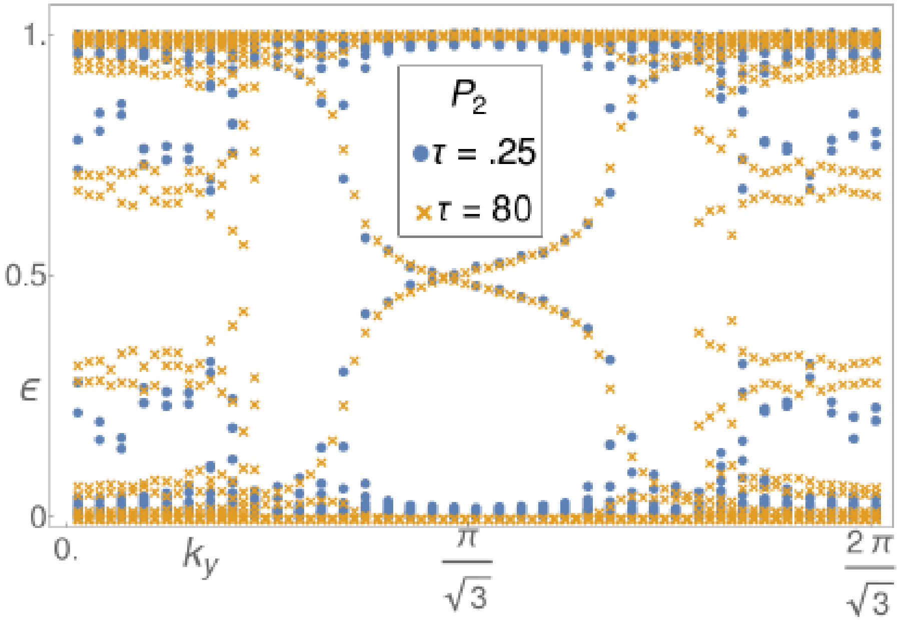


Entanglement spectrum for slow quench: No adiabatic theorem for the resonant laser.



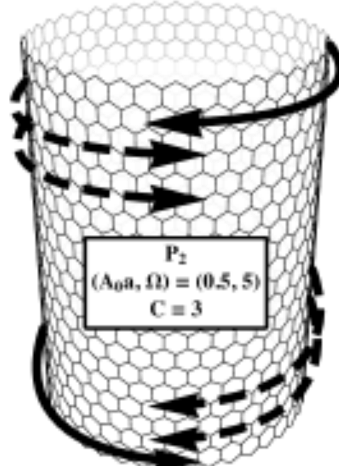


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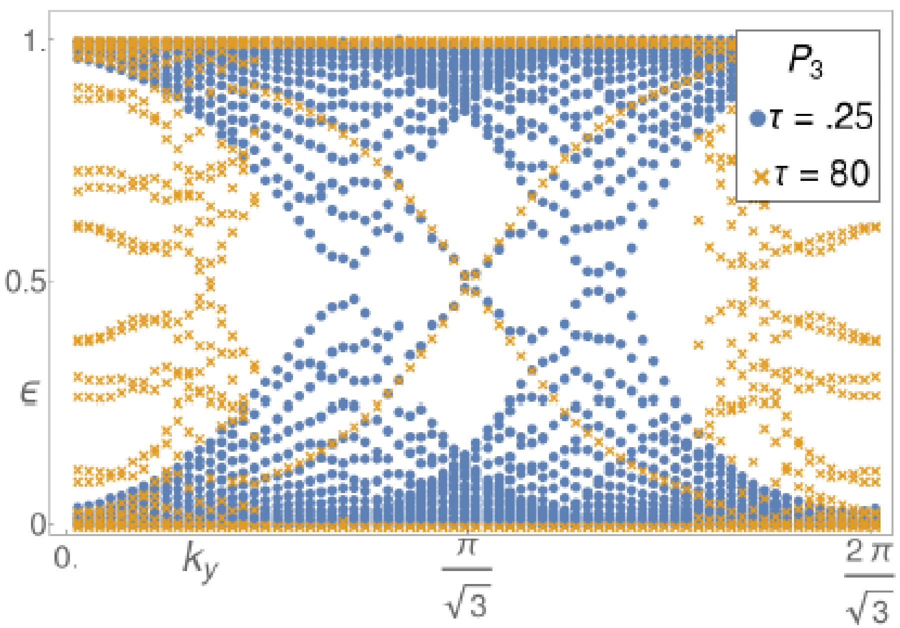
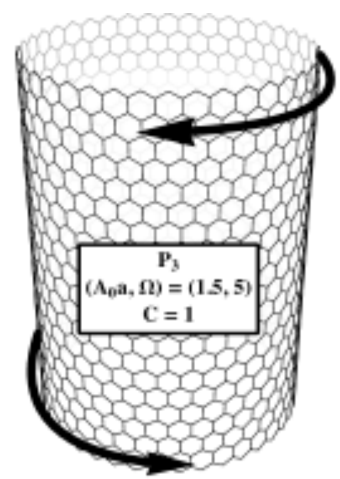
FIG. 3: Entanglement spectrum (ES) resulting from a numerically propagated state for a strip width of 20 cells. The time constants τ correspond to the ramp speed of the laser light, of the form $\frac{A_0}{2}\left[\tanh(\frac{t}{\tau})+1\right]$. ES shown at time $t = 700.5T$ for fast ramp, and at time $t = 800.5T$ for slow ramp, where T is the laser period.



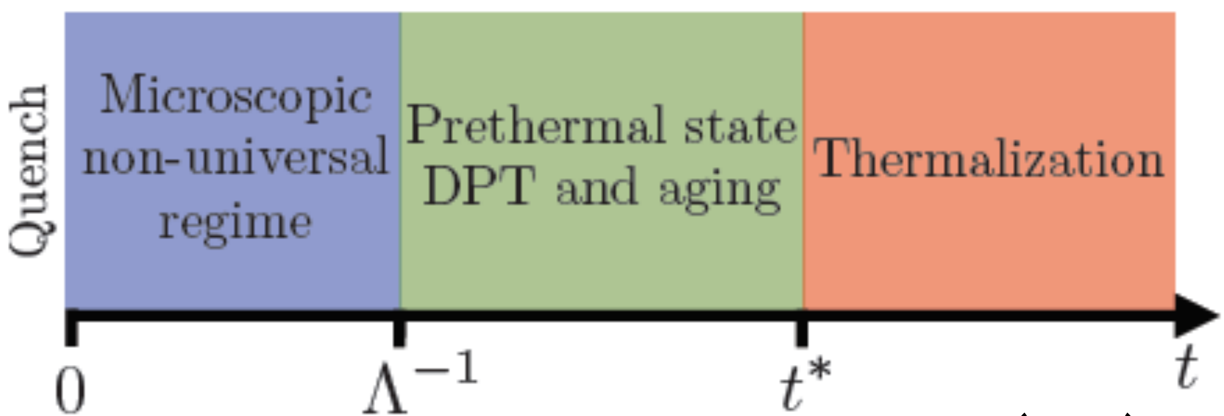


Next topic: Non-topological but interacting system. Dynamics in a critical prethermal phase

Alessio Chiocchetta, Marco Tavora, Andrea Gambassi, and Aditi Mitra, *Short-time universal scaling and light-cone dynamics after a quench in an isolated quantum system in d spatial dimensions*, arXiv:1604.04614, in review at PRB.

Anna Maraga, Alessio Chiocchetta, Aditi Mitra, Andrea Gambassi, *Aging and coarsening in isolated quantum systems after a quench: Exact results for the quantum O(N) model with* $N \to \infty$, Phys. Rev. E **92**, 042151 (2015).

Alessio Chiocchetta*, Marco Tavora*, Andrea Gambassi, and Aditi Mitra, *Short-time universal scaling in an isolated quantum system after a quench*, Phys. Rev. B **91**, 220302(R) (2015). *Authors contributed equally.



$$t^* \approx O(N)$$

Model

t< 0 $$H_i = \int d^d x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}\Omega_0^2\varphi^2 \right]$$

A quantum quench at t=0 where the mass has been suddenly changed, and interactions (anharmonicities) become important.

t> 0 $$H_f = \int d^d x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}r\varphi^2 + \frac{u}{4!N}\varphi^4 \right]$$

N=Number of components of $\vec{\varphi}$

We will be interested in the deep quench limit: $\Omega_0^2 >> r \approx 0$

The model is capable of thermalizing, yet the system can get trapped in a metastable state described by an effective H*. Universal features in the time-evolution controlled by H* and memory of the initial state. Aging in a quantum system: power-law rather than exponential decay of correlations controlled by universal exponents.

Results at long-wavelengths (qt, qt' <<1). Quench at time=0

Origin of exponent: Scaling dimension of a small magnetic field applied before quench. Can also be thought of as a boundary (in time) critical exponent.

$$\theta_N = \frac{\varepsilon}{4}\left(\frac{N+2}{N+8}\right) + O(\varepsilon^2); \varepsilon = 4 - d$$

Response function: $G_R(q=0, t >> t') \approx t\left(t'/t\right)^{\theta_N}$

Correlation function: $iG_K(q=0, t >> t') \approx (t')^{2-2\theta_N}\left(t'/t\right)^{\theta_N - 1}$

Aging: No single time-scale determines the relaxation. Relaxation depends on the waiting time t' at which the system has been perturbed.
"Quantum" aging due to ballistic rather than diffusive excitations, leading to differences in the temporal behavior of aging.

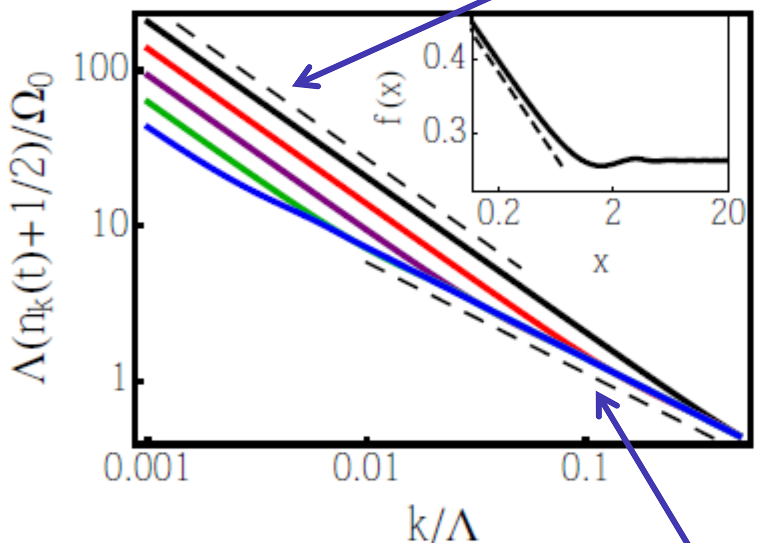
Chiocchetta, Tavora, Gambassi, AM, PRB 2015 and 2016

Boson distribution function at the non-thermal fixed point shows universal power-laws

Chiocchetta, Tavora, Gambassi, AM, PRB 2015 and 2016

$$\boxed{n(k) \approx \frac{1}{k}}$$

Short times kt <1



Increasing time

$$\boxed{n(k) \approx \frac{1}{k^{1-2\theta_N}}}$$

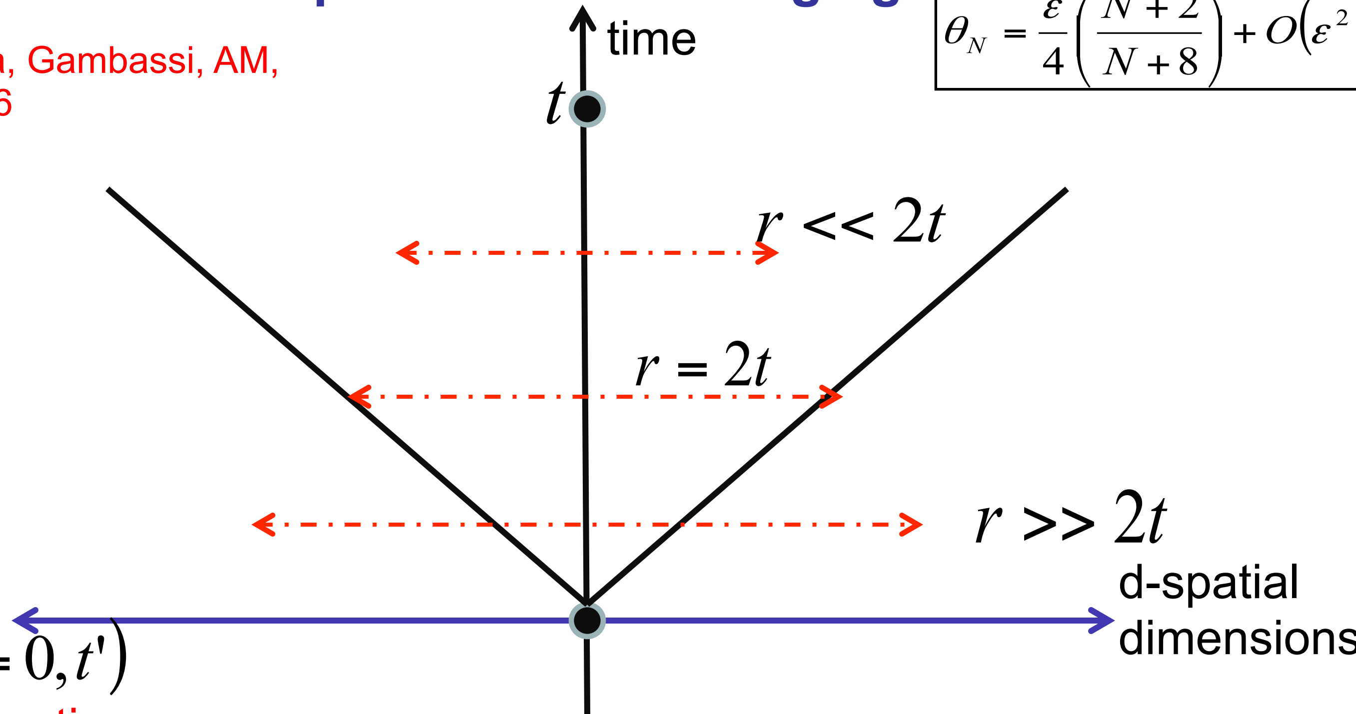
Long times kt >1

$$\boxed{\theta_N = \frac{\varepsilon}{4}\left(\frac{N+2}{N+8}\right) + O\left(\varepsilon^2\right), \varepsilon = 4-d}$$

Light-cone dynamics in the interacting problem in d>2-spatial Dimensions: Universal exponent related to aging.

$$\theta_N = \frac{\varepsilon}{4}\left(\frac{N+2}{N+8}\right) + O\left(\varepsilon^2\right); \varepsilon = 4-d$$

Chiocchetta, Tavora, Gambassi, AM, PRB 2015 and 2016



$$G_R\left(r,t;r'=0,t'\right)$$

Response function
(for quench at time=0)

$$G_R(r \neq t-t') = 0$$

$$G_R[r = t-t', (\Lambda t' >> 1)] = \frac{1}{r^{(d-1)/2}}$$

$$G_R[r = t-t', (\Lambda t' << 1)] = \frac{(t')^{\theta_N}}{r^{(d-1)/2}}$$

Equal-time correlation function
(for quench at time 0)

$$G_K\left(2t << r\right) = 0$$

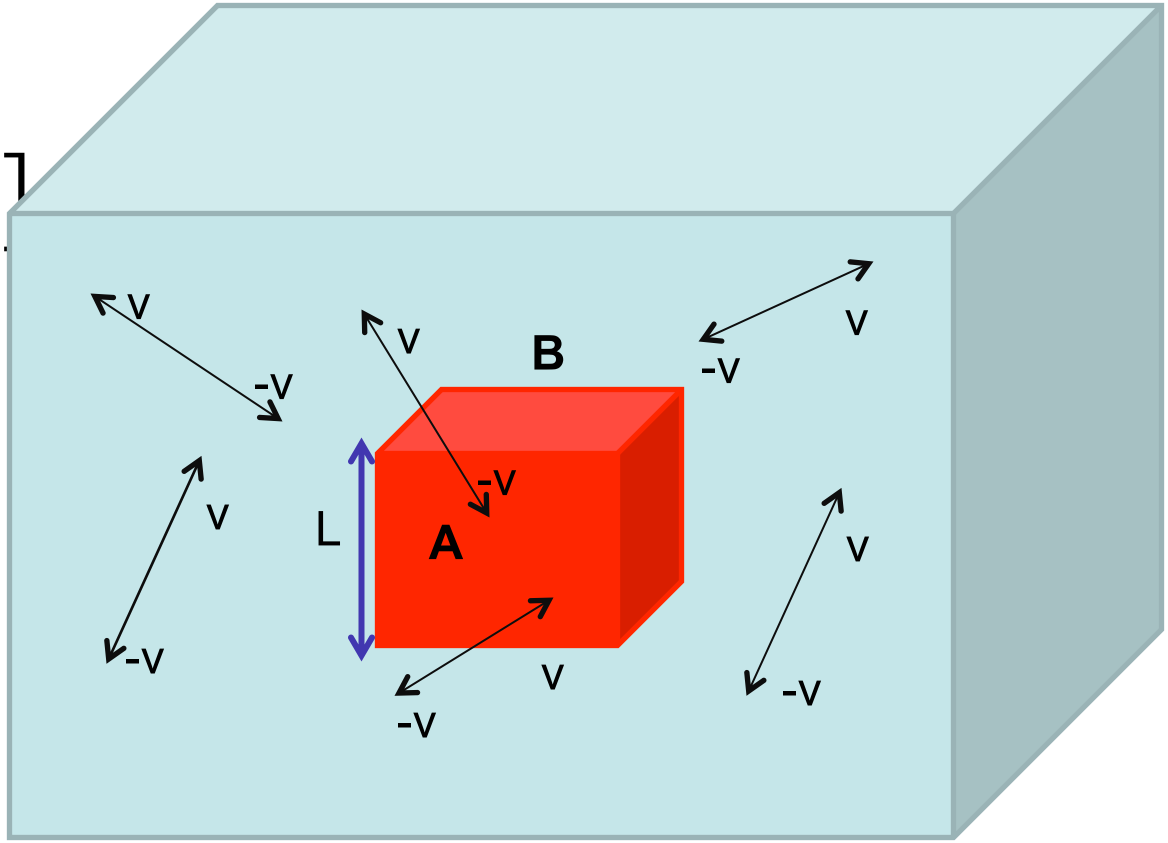
$$G_K(2t = r) \propto \frac{1}{r^{(d-2)/2}}$$

$$G_K(2t >> r) \propto \frac{1}{r^{d-2+2\theta_N}}$$

Entanglement dynamics for O(N) bosons in d=3:

$$\rho_A(t) = Tr_B\left[|\psi\rangle\langle\psi|\right]$$

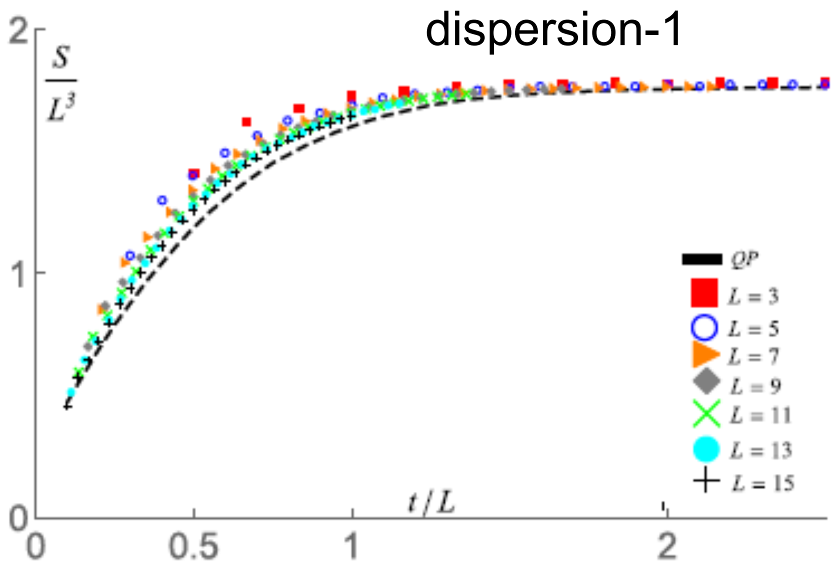
$$S_A(t) = -Tr_B\left[\rho_A \ln \rho_A\right]$$

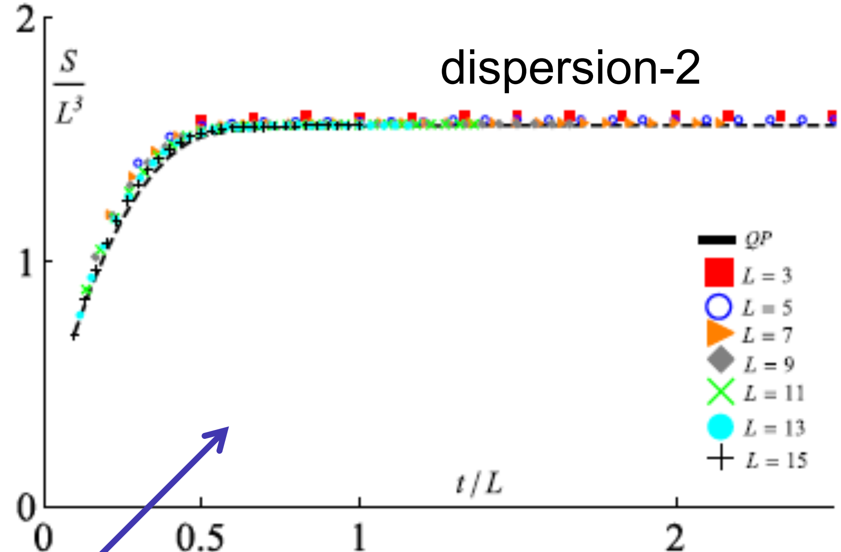


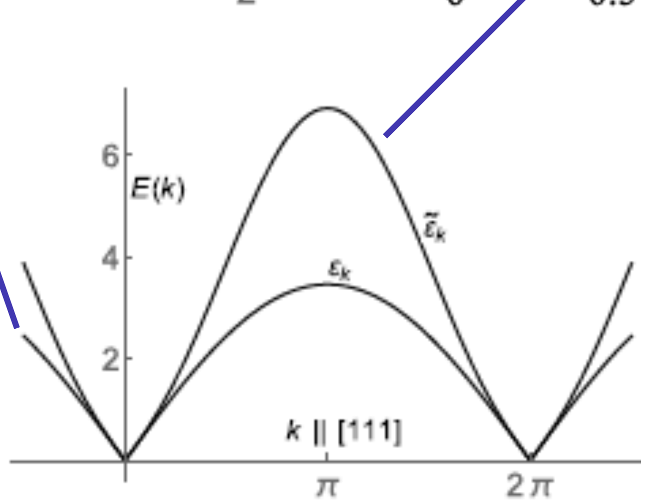
Main idea: For N= infty, Wicks theorem holds so that the reduced density matrix can be constructed from the two point correlation functions in region-A.

$$\tilde{C}_{ij}(t) \equiv \begin{pmatrix} C_{\Pi\phi}(r_i, r_j; t) & C_{\phi\phi}(r_i, r_j; t) \\ -C_{\Pi\Pi}(r_i, r_j; t) & -C_{\Pi\phi}(r_i, r_j; t) \end{pmatrix}$$

Entanglement entropy after a quench in d=3: Good agreement with quasi-particle picture and qualitatively different from d=1 due to difference in geometry (cube vs line.)





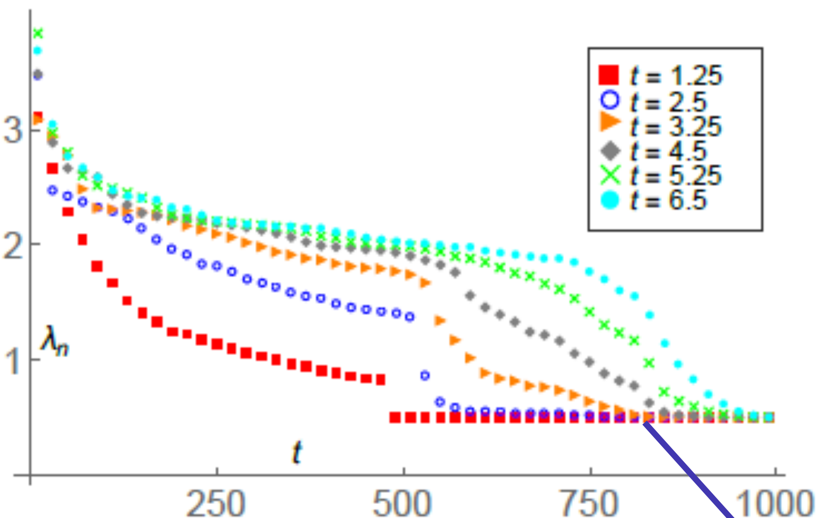


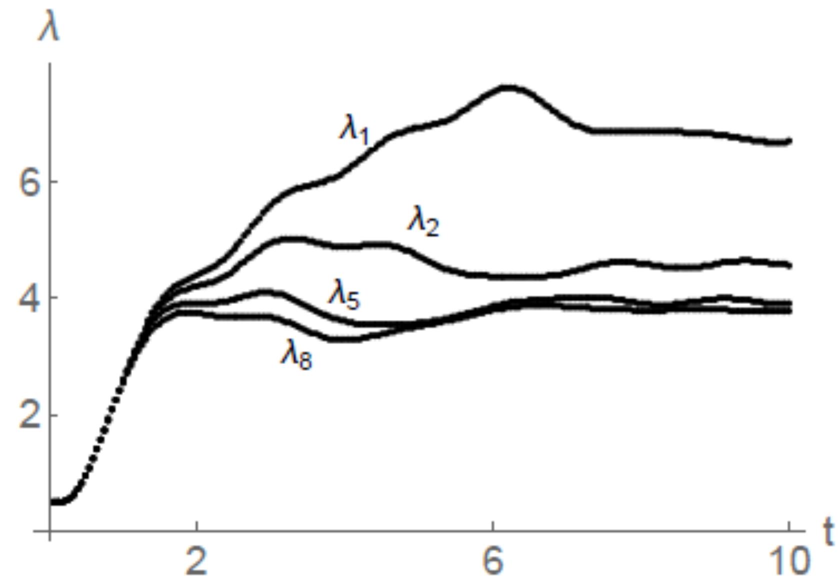
$$\rho_A(t) = Tr_B\left[|\psi(t)\rangle\langle\psi(t)|\right]$$

$$S(t) = -Tr\left[\rho_A \ln \rho_A\right]$$

Time-evolution of the entanglement spectrum (eigenvalues of the reduced density matrix)

$$S = \sum_i \left[\left(\lambda_i + \frac{1}{2}\right) \log\left(\lambda_i + \frac{1}{2}\right) - \left(\lambda_i - \frac{1}{2}\right) \log\left(\lambda_i - \frac{1}{2}\right) \right]$$





Remnants of the initial state

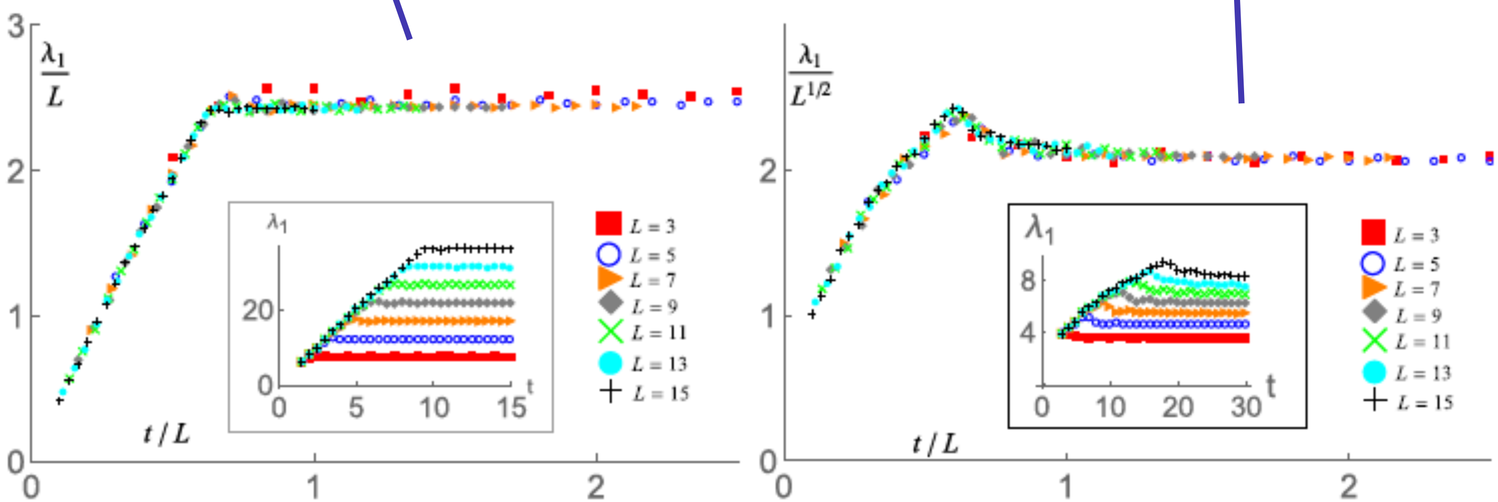
Entanglement spectrum (d=3) : sensitive to aging critical-exponent

Free gapless system

$\theta_\infty = 0$

Interacting critical quench

$\theta_\infty = \frac{1}{4}$



$$\lambda(L,t) = L^{1-2\theta_N} W(t/L)$$

Summary:

Floquet systems are now being realized in many different ways.
Closed quantum systems using cold-atoms, surface states of 3D topological insulators, photonic waveguides….

Ideal quantum limit for the Hall conductance achievable provided the laser is highly off-resonant.

Entanglement spectrum indicates whether edge-states might be protected or not from scattering off bulk states.

For a quench to a critical state, universal exponents are revealed by studying "low-lying" entanglement eigenvalues. The entanglement entropy itself is dominated by microscopic non-universal features and can be understood in terms of ballistically propagating quasi-particles.

Hall conductance: chern number vs open case