Many-body localized phase: dynamics and efficient numerical simulation

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[PRL 117, 160601(2016)]

SynQuant Conference

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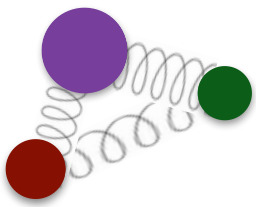
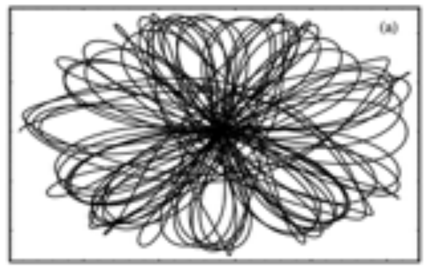
Ergodicity and integrability

Classical

Ergodic systems

chaos → ergodicity

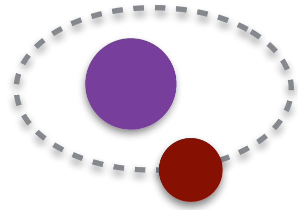


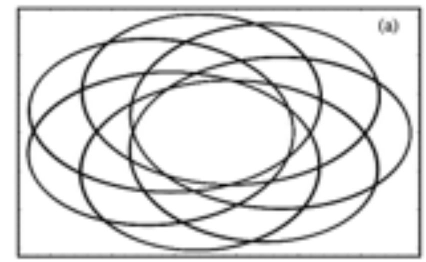


Integrable systems

stable to weak perturbations

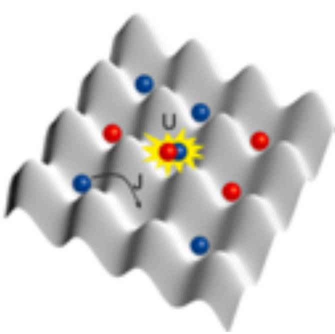
[Kolmogorov-Arnold-Moser theorem]

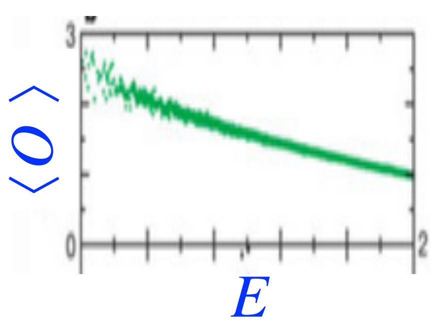




Quantum

Thermalizing phases

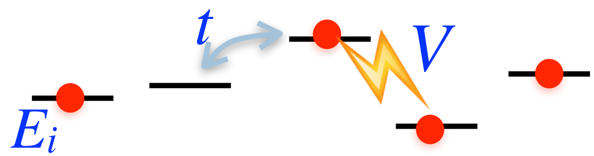




MBL phases

“toric cow” of non-ergodic systems

MBL: generic non-ergodic phase



- MBL = localized phase with interactions

[Anderson, Fleishman'80]

Perturbative arguments: [Basko, Aleiner,Altshuler'05] [Gorniy, Polyakov,Mirlin'05]

Numerical evidence: [Oganesyan,Huse'08] [Znidaric,Prosen'08] [Pal,Huse'10]

- Revived interest in MBL:

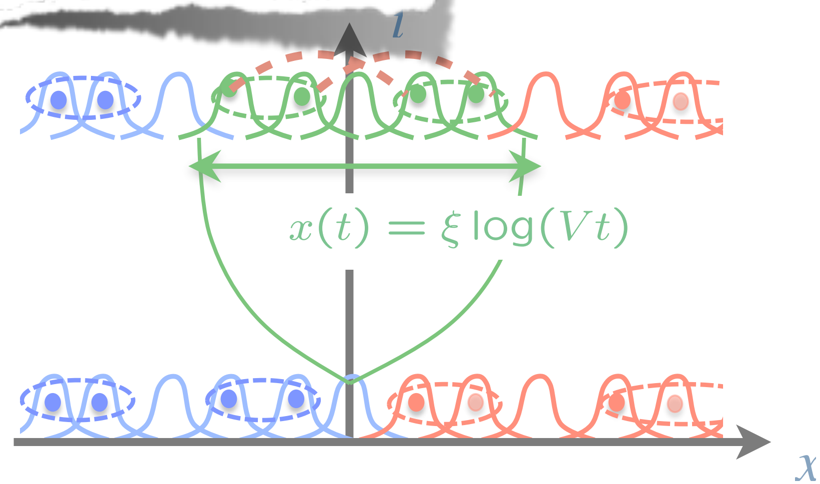
  - ✳ Experiments in cold atoms, ion chains…
  - ✳ Emergent integrability →universal non-ergodic dynamics
  - ✳ Breakdown of statistical mechanics →symmetry breaking at $T=\infty$,…

Towards understanding of MBL phase

- I. Dynamics in MBL phase
  - ✳ Local integrals of motion
  - ✳ Entanglement growth and dephasing
  - ✳ New probes of dephasing dynamics
- II. Highly excited MBL eigenstates
  - ✳ Structure of entanglement spectrum
  - ✳ Efficient numerical simulation with MPS

    [MS, Michailidis,Abanin,Papic, PRL 117, 160601(2016)]
- III. Summary and Outlook

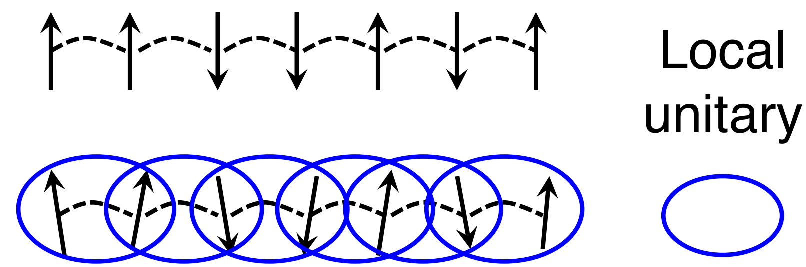
I. Dynamics in MBL phase



Constructing local integrals of motion

$$H_0 = \sum_i h_i S_i^z + J_z S_i^z S_{i+1}^z$$

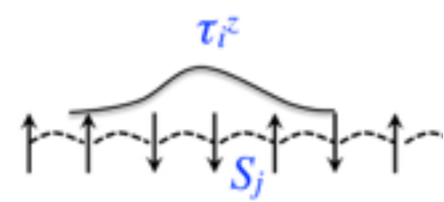
$$H = H_0 + \sum_i J_\perp (S_i^+ S_{i+1}^- + h.c.)$$



Sequence of local unitaries: $U^\dagger H U = H_{\text{diag}}$

Local integrals of motion $\hat{\tau}_i^z = U^\dagger \hat{S}_i^z U$

$$[\hat{\tau}_i^z, H] = 0$$



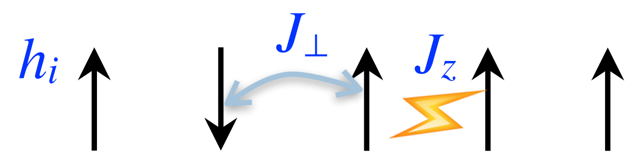
Effective spins form complete set
Emergent integrability

[MS, Papic, Abanin PRL'13]
[Huse, Nandkishore, Oganesyan PRB'14]
[Imbrie'14, PRL'16]
strong disorder RG:[Vosk&Altman,PRL'13]

Universal Hamiltonian of MBL phase

- If model is in MBL phase, apply sequence of local unitaries

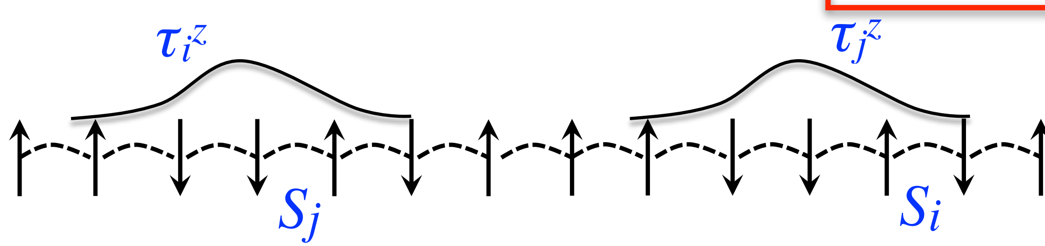
$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + h_i S_i^z$$



- Hamiltonian expressed via $\tau_i = U^\dagger \, S_i \, U$

$$\hat{H} = \sum_i H_i \tau_i^z + \sum_{ij} H_{ij} \tau_i^z \tau_j^z + \sum_{ijk} H_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

$$H_{ij} \propto \exp(-|i-j|/\xi)$$



- Effective spins cannot relax→ no transport

  Interactions → dephasing& relaxation

Entanglement growth from dephasing

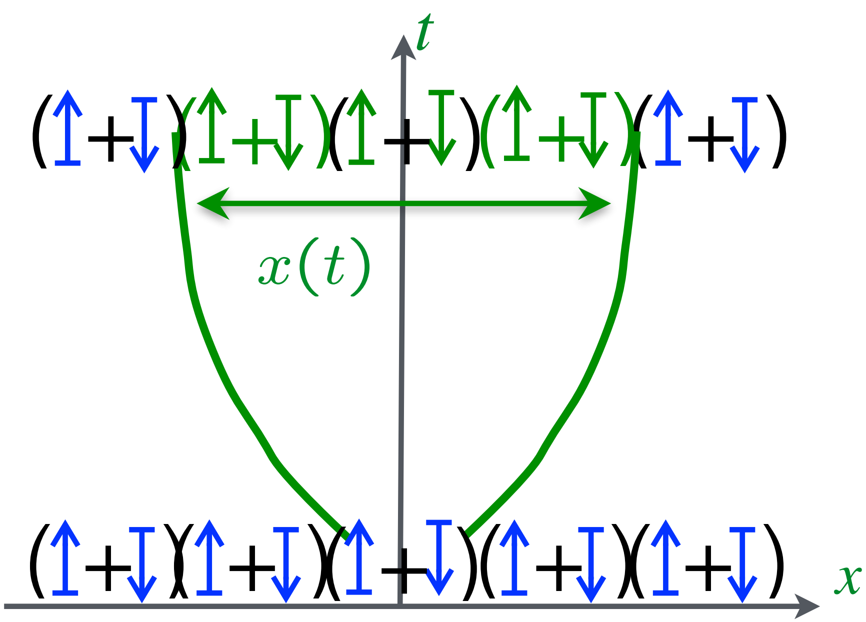
$$\hat{H} = \sum_i H_i \tau_i^z + \sum_{ij} H_{ij} \tau_i^z \tau_j^z + \dots \qquad H_{ij} \propto J e^{-|i-j|/\xi}$$

- **Phases** randomize on distance $x(t)$:

$$t H_{ij} = t J \exp(-x/\xi) \sim 1$$

$$\downarrow$$

$$x(t) = \xi \log(Jt)$$

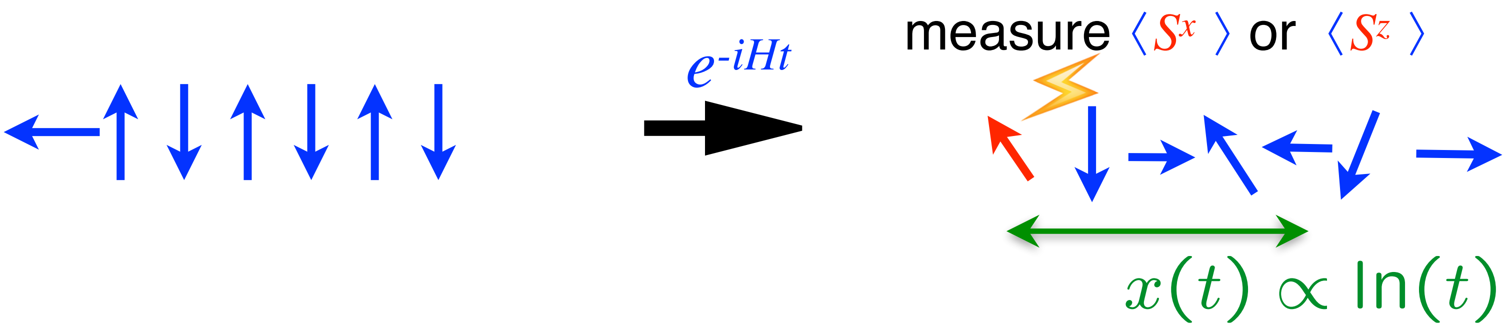


- Logarithmic growth of entanglement [MS, Papic, Abanin, PRL'13]

[Znidaric,Prosen,Prelovsek,PRB'08] [Bardarson,Pollmann,Moore,PRL'12]

Q: How to probe in experiment?

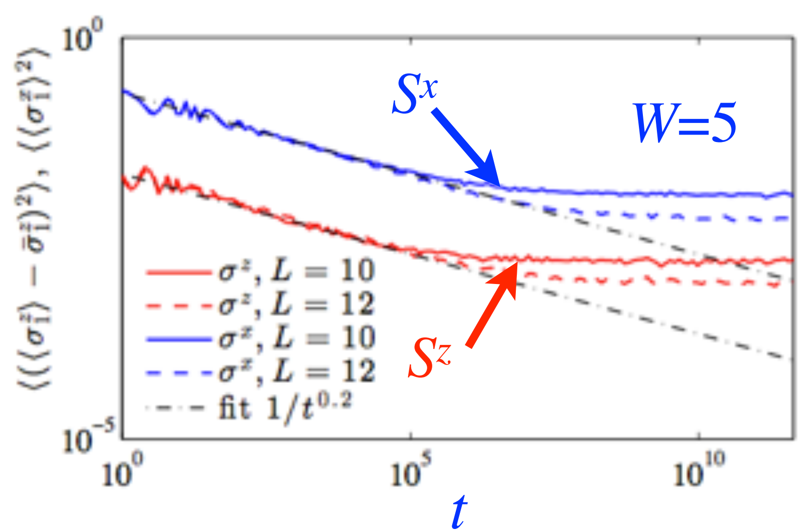
Local observables in a quench



- $\langle \tau^x(t) \rangle = \rho_{\uparrow\downarrow}(t) = \sum[N(t) = 2^{x(t)}$ oscillating terms]
- Decay of oscillations of $\langle \tau^x(t) \rangle$: $|\langle \tau^x_k(t) \rangle| \propto \frac{1}{\sqrt{N(t)}} = \frac{1}{(tJ)^a}$

$$\left|\langle \hat{\mathcal{O}}(t) \rangle - \langle \mathcal{O}(\infty) \rangle\right| \sim \frac{1}{t^a}$$

memory of initial state



[MS, Papic, Abanin, PRB'14]

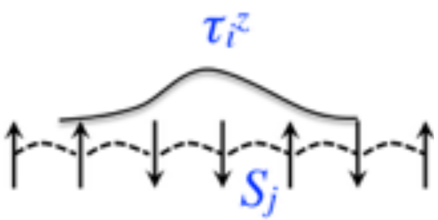
Other probes of dephasing

- Modified spin echo protocol
  Quantum revivals, …

[MS,Knap,et al.,PRL'14]

[Vasseur, Parameswaran,Moore, PRB'15]

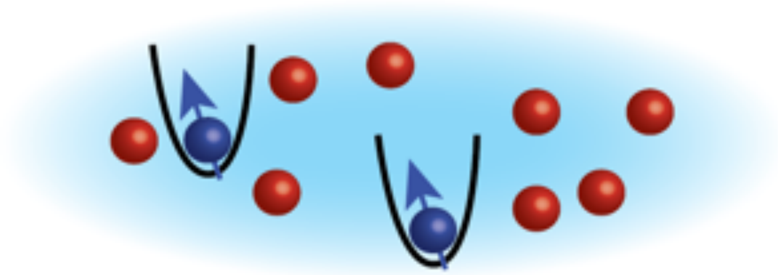
All protocols assume: $\sigma_1^z \approx \tau_1^z$



- How to probe "operator expansion"?

$$\sigma_1^z = \sum \alpha_i \tau_i^z + \sum_{ij} \beta_{ij} \tau_i^+ \tau_j^- + \sum_{ijk} \alpha_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

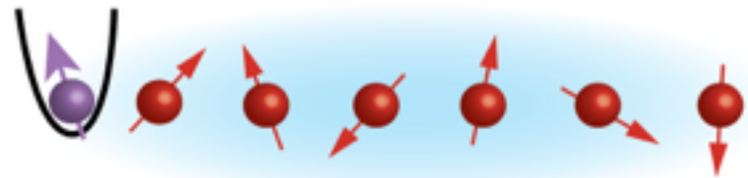
- **Next:** orthogonality catastrophe



[MS&Abanin, in preparation]

Ramsey interferometry & operator expansion

- Impurity spin coupling $H_{int} = g\sigma^z_{\text{imp}}\sigma^z_1$



- Initialize along x, measure

$$\langle\sigma^x_{\text{imp}}(t)\rangle = \text{Re}\langle\psi_0|e^{i(H+g\sigma^z_1)t}e^{-i(H-g\sigma^z_1)t}|\psi_0\rangle$$

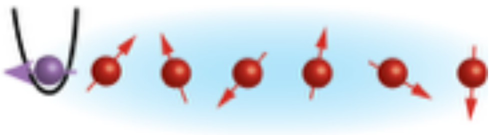
- Non-trivial dynamics comes from expansion:

$$\sigma^z_1 = \sum\alpha_i\tau^z_i + \cancel{\sum_{ij}\beta_{ij}\tau^+_i\tau^-_j} + \sum_{ijk}\alpha_{ijk}\tau^z_i\tau^z_j\tau^z_k + \dots$$

$$\langle\sigma^x_{\text{imp}}(t)\rangle \approx \text{Re}\langle\psi_0|e^{2itg(\sum\alpha_i\tau^z_i+\sum_{ijk}\alpha_{ijk}\tau^z_i\tau^z_j\tau^z_k+\dots)}|\psi_0\rangle$$

- Dephasing → power law decay of fluctuations $|\langle\sigma^x_{\text{imp}}(t)\rangle| \propto \frac{1}{t^b}$

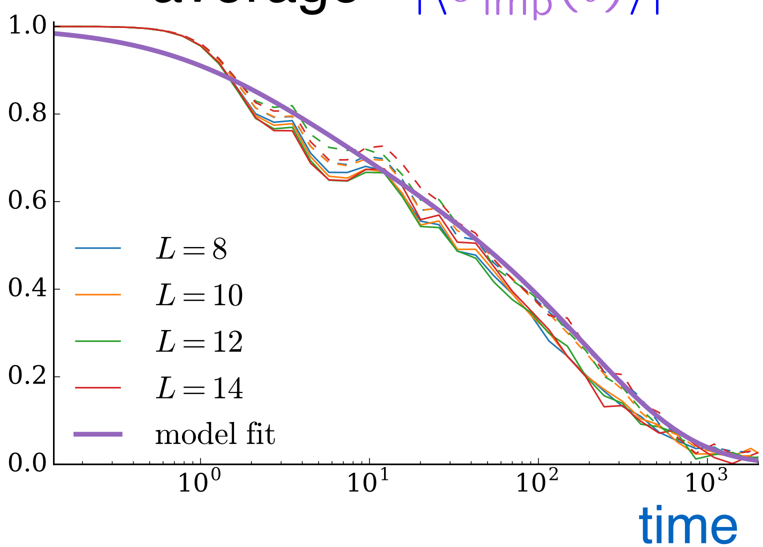
Average vs fluctuations



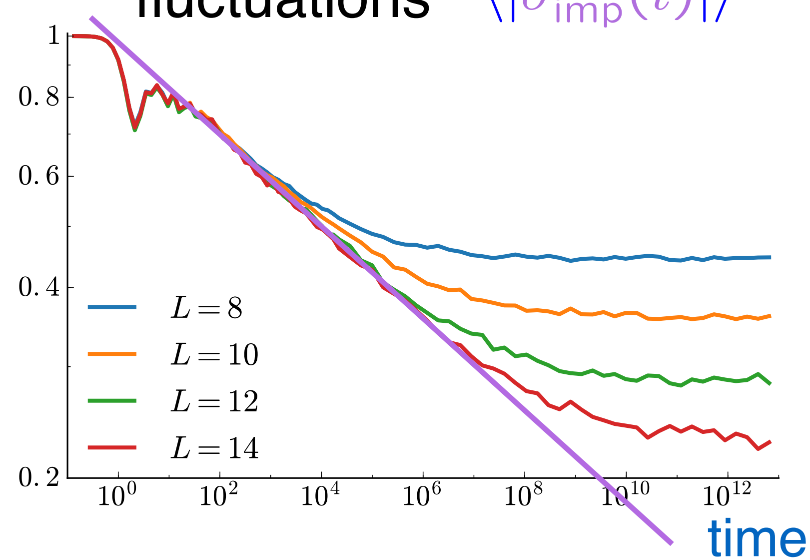
$$\langle \sigma^x_{\text{imp}}(t) \rangle \approx \text{Re}\langle \psi_0 | e^{2itg(\sum \alpha_i \tau_i^z + \sum_{ijk} \alpha_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots)} | \psi_0 \rangle$$

- Average: $\boldsymbol{\alpha}_i$ = matrix elements → universal function
- Fluctuations: dephasing → power law decay

average $|\langle \sigma^x_{\text{imp}}(t) \rangle|$

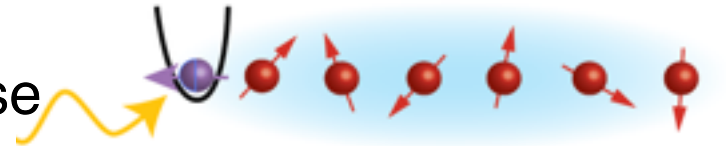


fluctuations $\langle |\sigma^x_{\text{imp}}(t)| \rangle$



Off-diagonal terms in operator expansion

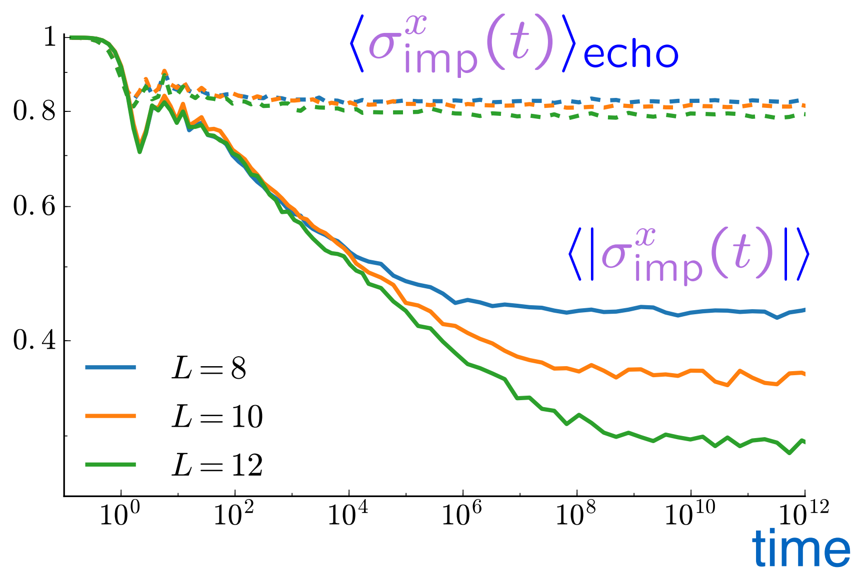
- Spin echo protocol:



$$\langle \sigma^x_{\text{imp}}(t)\rangle_{\text{echo}} = \text{Re}\langle\psi_0|e^{i(H-g\sigma_1^z)t}e^{i(H+g\sigma_1^z)t}e^{-i(H-g\sigma_1^z)t}e^{-i(H+g\sigma_1^z)t}|\psi_0\rangle$$

$$\sigma_1^z = \sum \alpha_i \tau_i^z + \sum_{ij} \beta_{ij}\tau_i^+\tau_j^- + \sum_{ijk} \alpha_{ijk}\tau_i^z\tau_j^z\tau_k^z + \dots$$

- Spin-flip terms are less important:



[MS&Abanin, in preparation]

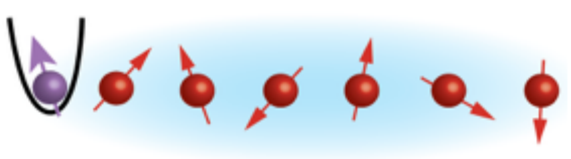
Probes of dephasing dynamics

- **Global probes**: quench, modified spin echo,…:

$$|\langle \hat{\mathcal{O}}(t)\rangle - \langle \mathcal{O}(\infty)\rangle| \sim \frac{1}{t^a}$$

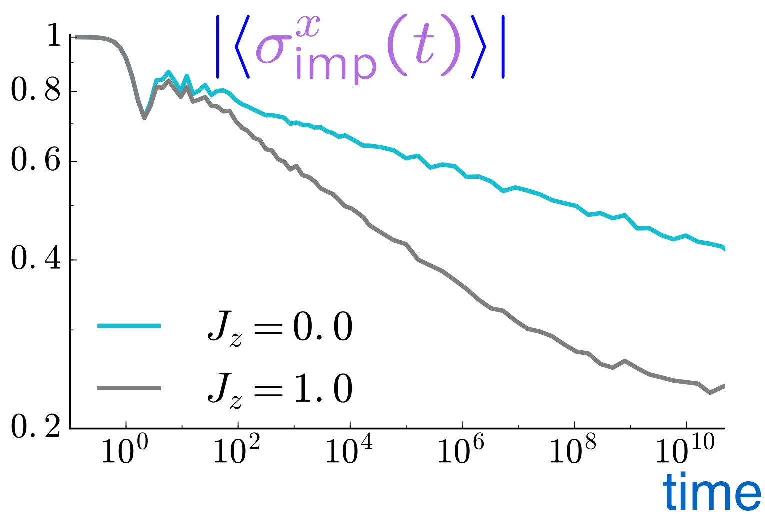
$a \neq 0$ ↔ presence of interactions

- **Local probes**: orthogonality catastrophe,…:

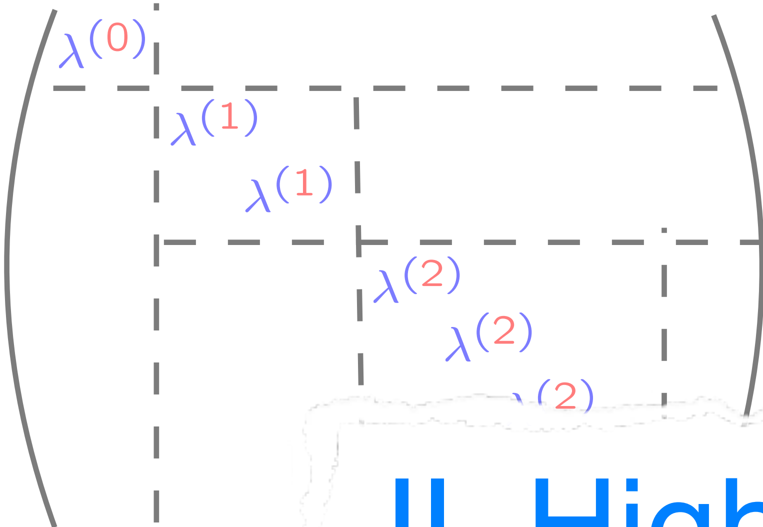


$$|\langle \sigma^x_{\rm imp}(t)\rangle| \propto \frac{1}{t^b}$$

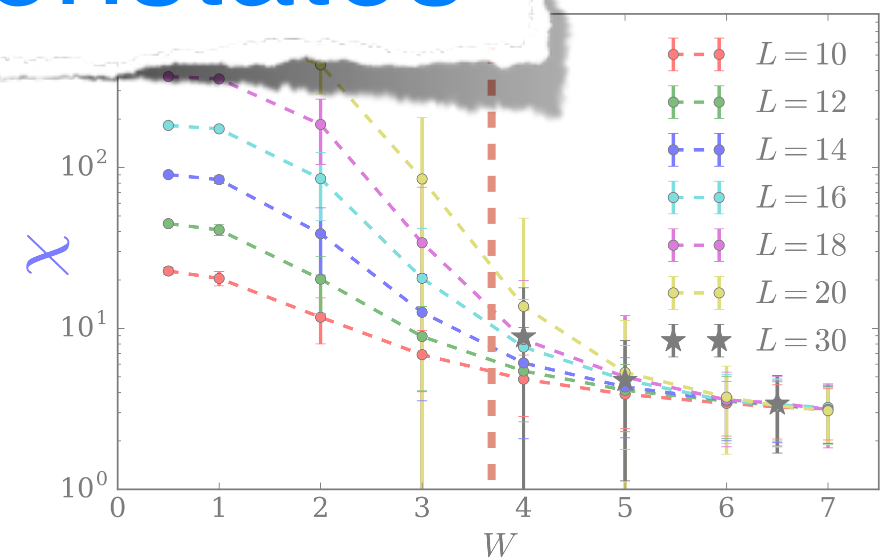
decay depends on interactions



Experimental challenge: access fluctuations

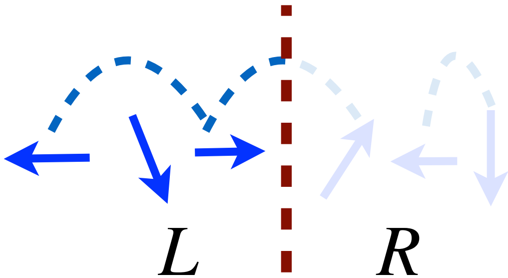


II. Highly excited MBL eigenstates



From entanglement entropy to spectrum

- "Quantumness" of the pure state:



trace out $R \to$

$$\rho_L = \mathrm{Tr}_R \, |\psi\rangle\langle\psi|$$

- Entanglement entropy: $S_{\mathrm{ent}} = -\sum_i \lambda_i \log \lambda_i$

  ✳ ground states: probes topological order

  [Levin&Wen], [Kitaev&Preskill]

  ✳ excited states: probes ergodicity

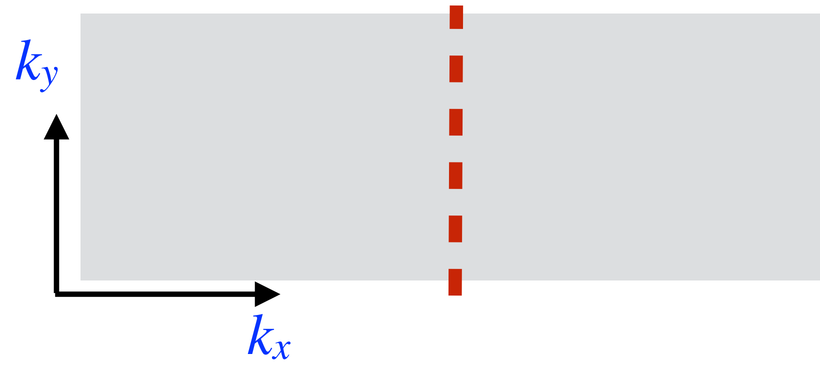
- Beyond entanglement? More information in $\{\lambda_{\mathrm{i}}\}$

  [Li & Haldane]

Organization of entanglement spectrum

- Quantum Hall wave function:

  k_y to organize ES
  [Li & Haldane]



- MBL phase: conserved quantities label ES

$$|\uparrow\uparrow\uparrow\uparrow\rangle = c_0\,|\uparrow\uparrow\rangle|\uparrow\uparrow\rangle + e^{-\kappa}\underbrace{|\uparrow\downarrow\rangle|\uparrow\uparrow\rangle}_{r=1} + e^{-2\kappa}\underbrace{|\uparrow\downarrow\rangle|\downarrow\uparrow\rangle}_{r=2} + \dots$$

$$+\; e^{-4\kappa}\underbrace{|\downarrow\downarrow\rangle|\downarrow\downarrow\rangle}_{r=4} + \;.....$$

- Coefficients decay as $\left|C_{\uparrow\dots\uparrow\underbrace{\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow}_{r}\uparrow\dots\uparrow}\right| \propto e^{-\kappa r}$

Power-law entanglement spectrum

- Hierarchical structure of $\rho_L = \sum_{r=0}^{L/2} |\psi^{(r)}\rangle\langle\psi^{(r)}|$

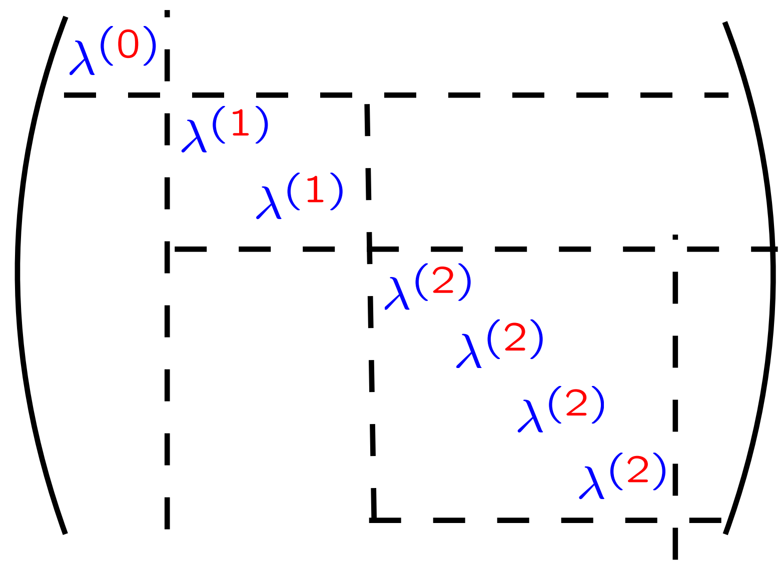
$$\langle\psi^{(r)}|\psi^{(r)}\rangle \propto e^{-2\kappa r}$$ but non-orthogonal

- Orthogonalize perturbatively

$$\lambda^{(r)} \propto e^{-4\kappa r}$$

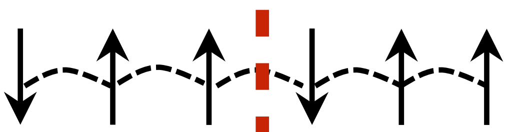
multiplicity is 2^r

$$\lambda_k \propto \frac{1}{k^\gamma}, \qquad \gamma \approx \frac{4\kappa}{\ln 2}$$

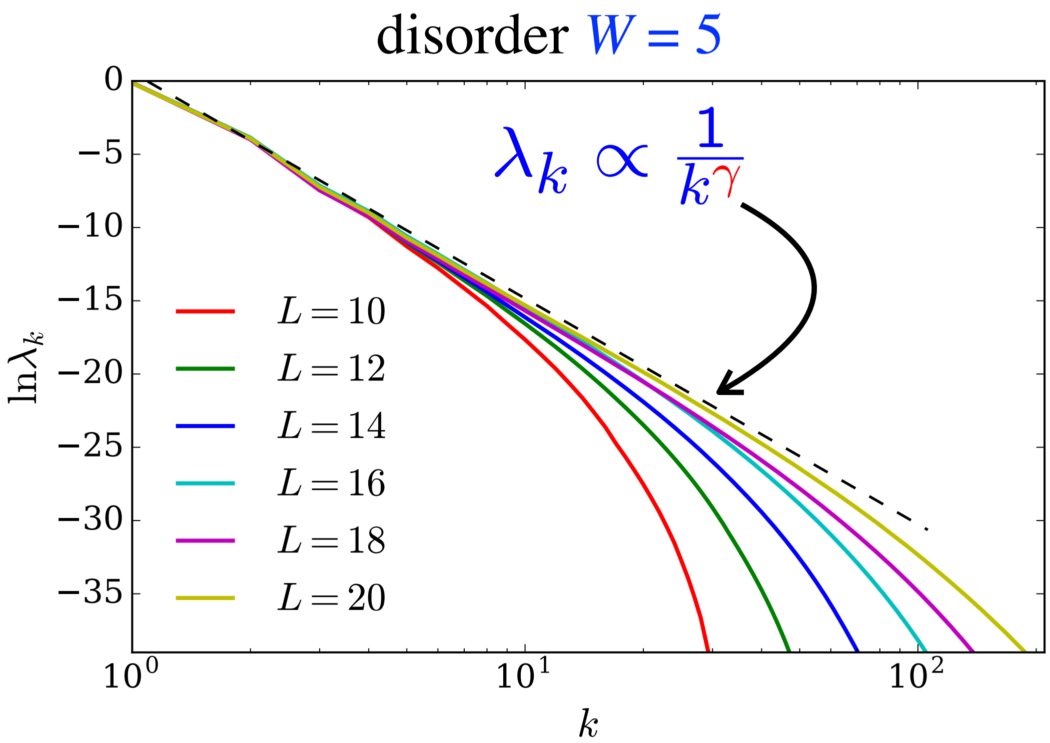


Numerics for XXZ spin chain

- Spin chain in random field: $J_\perp = J_z = 1$



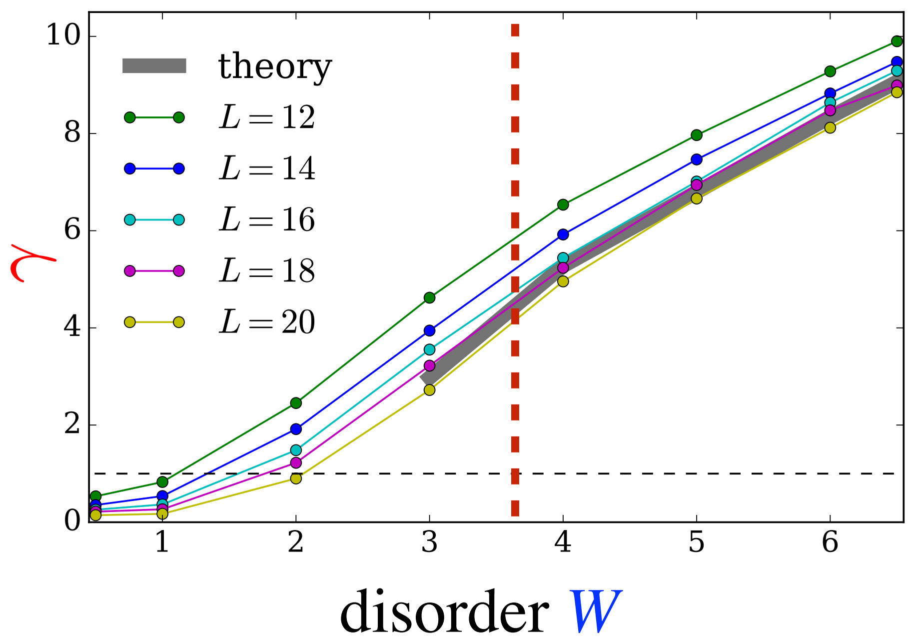
$$H = \sum_i (h_i S_i^z + J_\perp S_i^+ S_{i+1}^- + h.c.) + \sum_i J_z S_i^z S_{i+1}^z$$



[MS, Michailidis, Abanin, Papic, PRL 117, 160601(2016)]

Decay of entanglement spectrum

- γ controls decay of entanglement spectrum $\lambda_k \propto \frac{1}{k^\gamma}$



$$\gamma \approx \frac{4\kappa}{\ln 2}$$

perturbation theory

$$\kappa = 2\kappa' + \ln 2$$

$$\mathcal{G}(L) \propto e^{-\kappa' L}$$

Thouless conductance for MBL

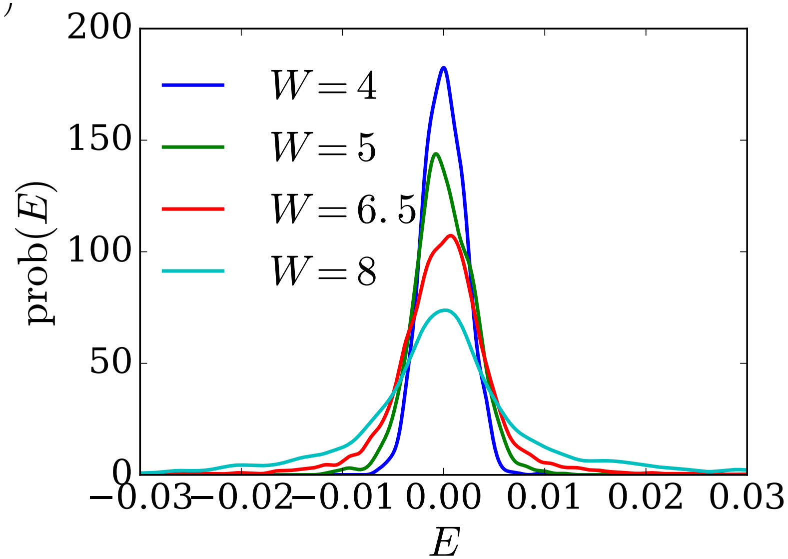
[MS,Papic,Abanin,PRX'15]

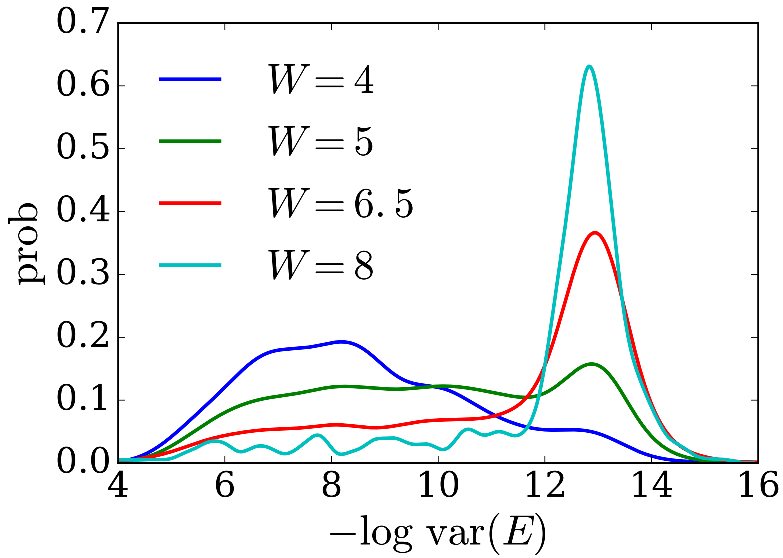
- Large value of γ → MPS description! $\frac{1}{\chi^{\gamma-1}} \approx \frac{1}{400^3} \approx 10^{-7}$

Implementation of MPS algorithm

- Goal: access highly excited states
- "Shift-invert": $H \rightarrow \frac{1}{(H-E)^2}$
- 50 DMRG-type sweeps; solve $|\psi_i\rangle = (H-E)^2|\psi_{i+1}\rangle$
- Conjugate gradient → large bond dimensions χ=400

more details:
[PRL 117, 160601(2016)]



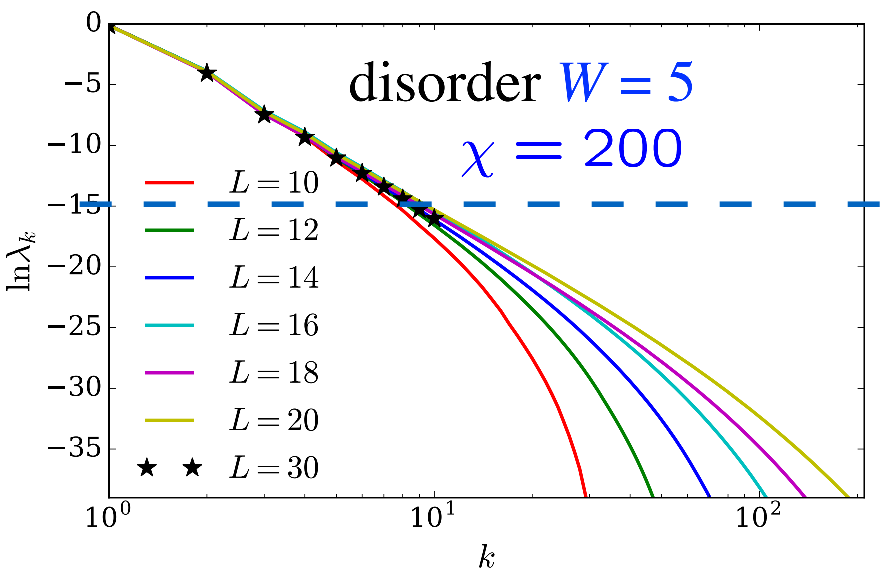


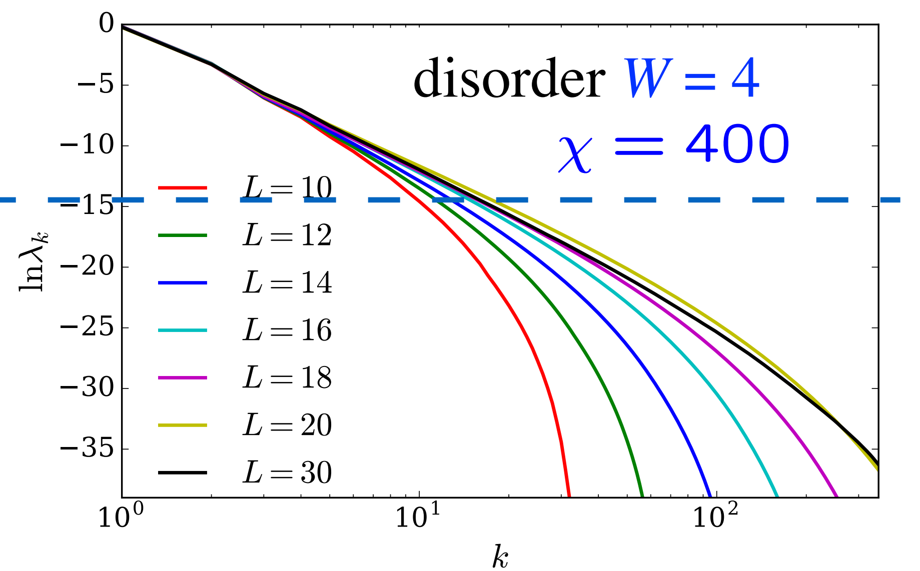
also: [Yu et al arXiv:1509.01244] [Lim&Sheng arXiv:1510.08145] [Pollmann et al arXiv:1509.00483] [Kennes&Karrasch arXiv:1511.02205]

Entanglement spectrum as a test

- Large bond dimensions are necessary close to transition
- DMRG underestimates entanglement spectrum for

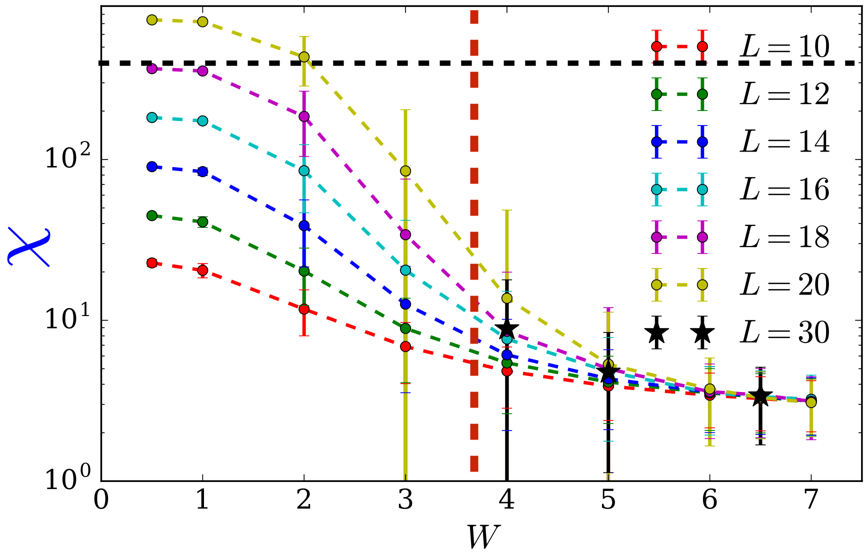
$$\lambda_k \geq e^{-15} \approx 10^{-6}$$





Estimates for the bond dimension

- To converge S_{ent} up to 1%:



- χ=400 → eigenstates close to MBL transition

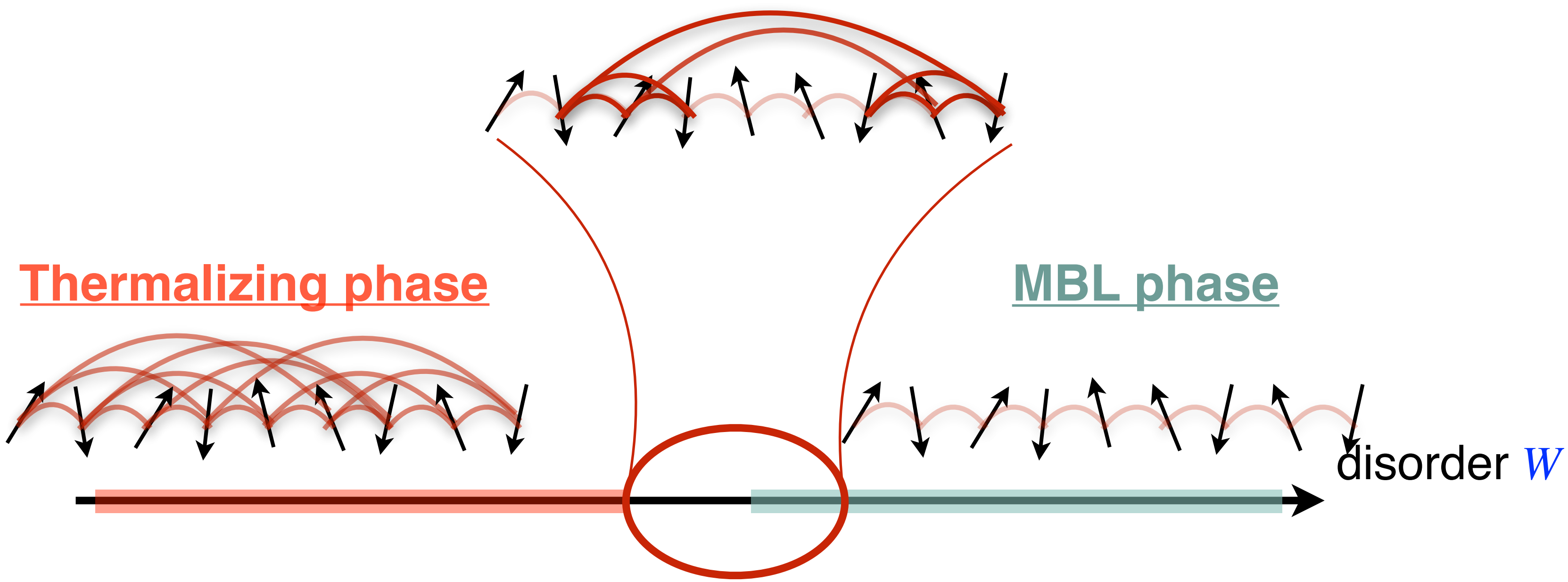
Q: What can we learn from this?

Future directions

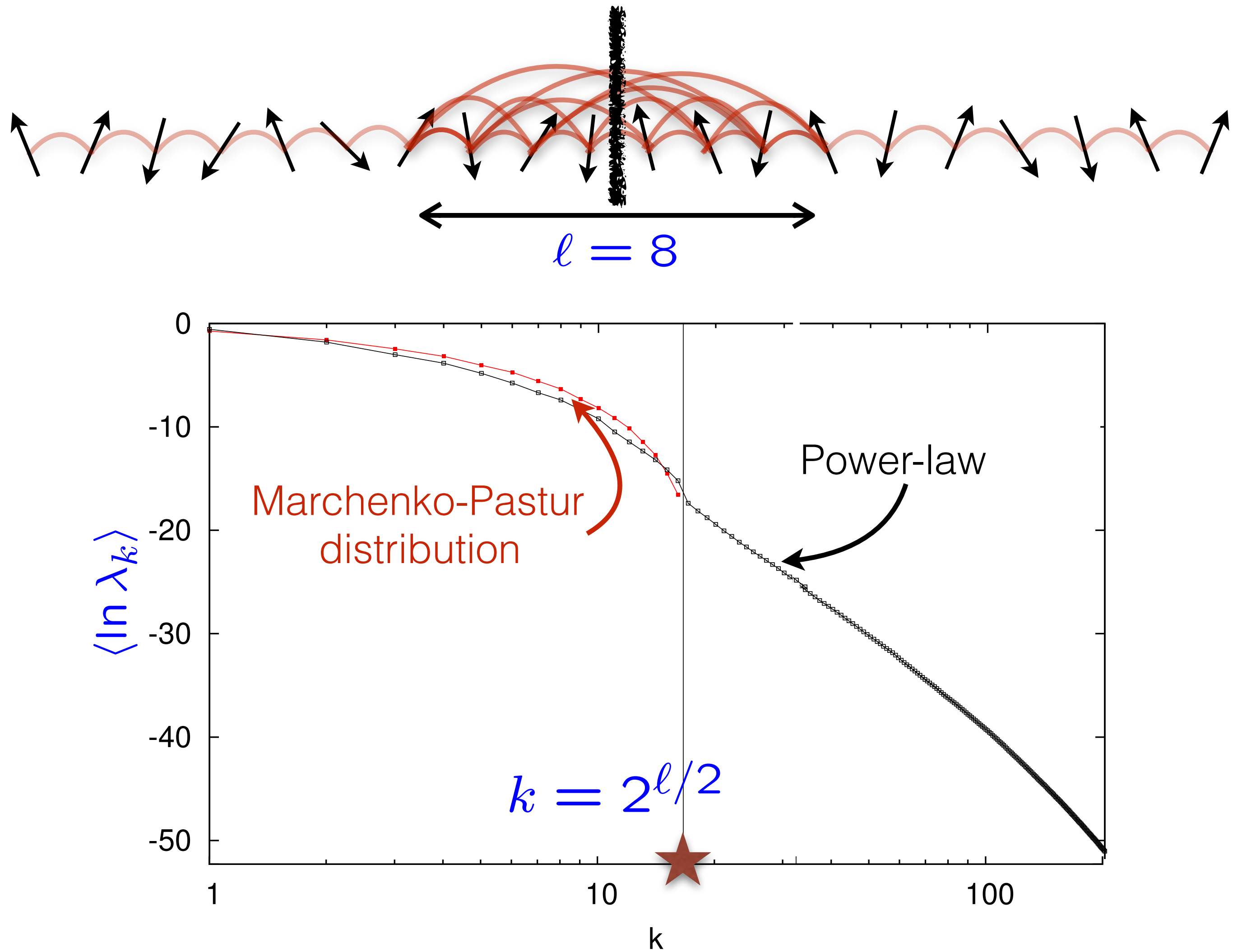
- Phase transitions within MBL phase
- MBL with fermions, S>1/2, bosons, etc.
- Structure of many-body resonances that drive transition?

phenomenological RG: [Vosk,Huse,Altman,PRX'15] [Potter,Vasseur,Parameswaran,PRX'15]

exact diagonalization: [Khemani et al, arXiv:1607.05756]



“Hot region” inside MBL phase



Identify structure of generic resonance from ES?

Acknowledgments

Alexios Michailidis
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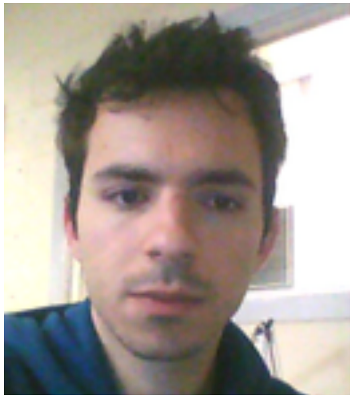

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GORDON AND BETTY MOORE FOUNDATION

Outline and perspectives

- Orthogonality catastrophe in MBL phase :

  →power-law decay

  →probe relation between $\widehat{\tau}_i$ and $\widehat{S}_i$

  [MS,Abanin in preparation]

- Power-law entanglement spectrum in MBL $\lambda_k \propto \frac{1}{k^{\gamma}}$

  →power $\gamma \leftrightarrow$ scaling of matrix elements

  →MPS algorithm close to transition

  [MS,Alexios,Abanin,Papic,PRL'16]