

Driven-dissipative polariton quantum fluids in and out of equilibrium

Marzena Szymańska

Designer Quantum Systems Out of Equilibrium
KITP, November 2016

Acknowledgements

Group:



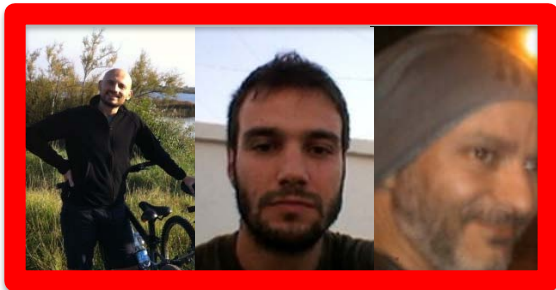
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P. Comaron



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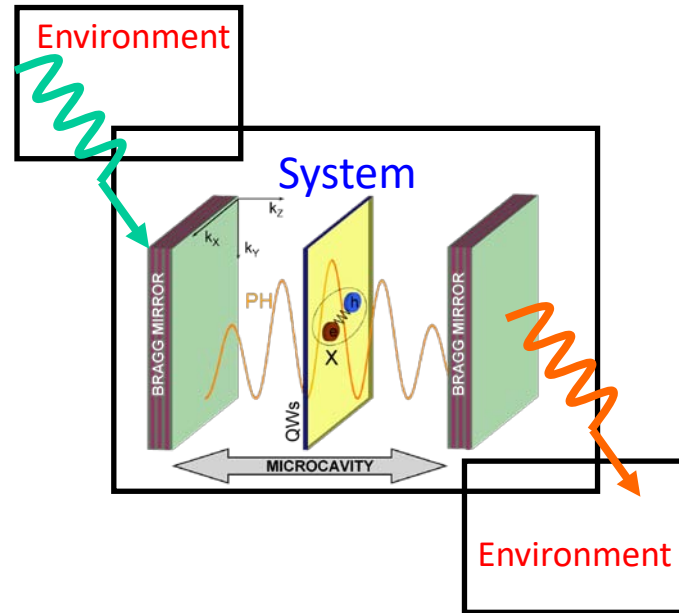
I. Carusotto

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Research Council

Driven-dissipative Condensates



2D Light-matter condensates with drive and decay

Polaritons

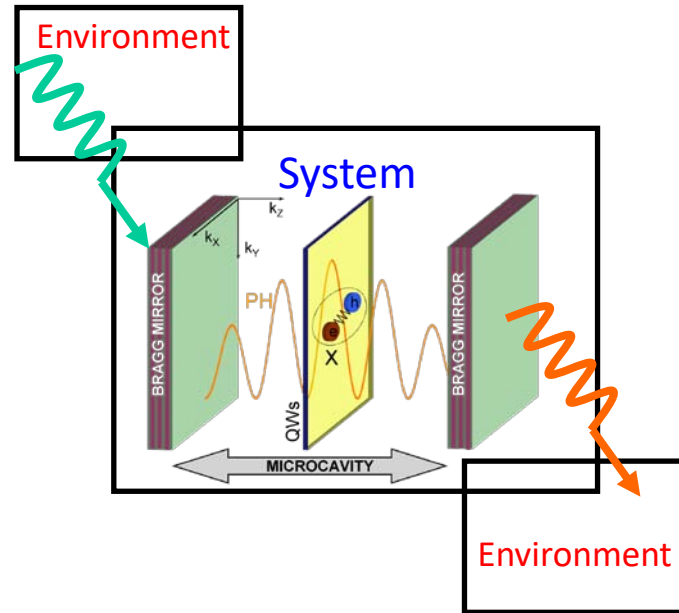
Photon BEC

Circuit QED systems

Atoms in cavities

....

Driven-dissipative Condensates



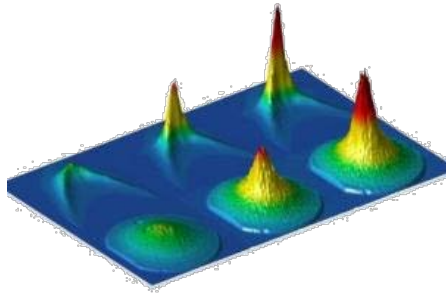
2D Light-matter condensates with drive and decay

Can thermal equilibrium be achieved?

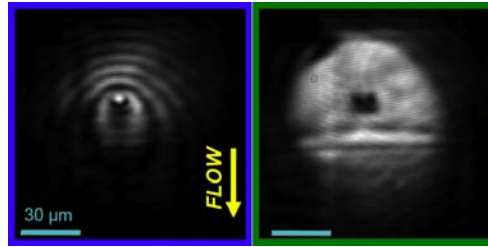
Can non-equilibrium but non-trivial phases be engineered?

State of the art: experiments

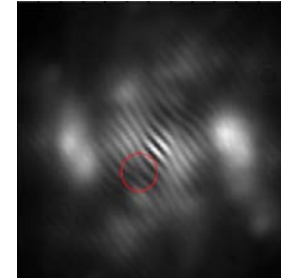
✧ Condensation, superfluidity and vortices



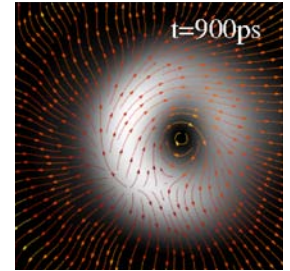
[Kasprzak et al., *Nature* 2006]



[Amo et al. *Nature Phys.* 2009]



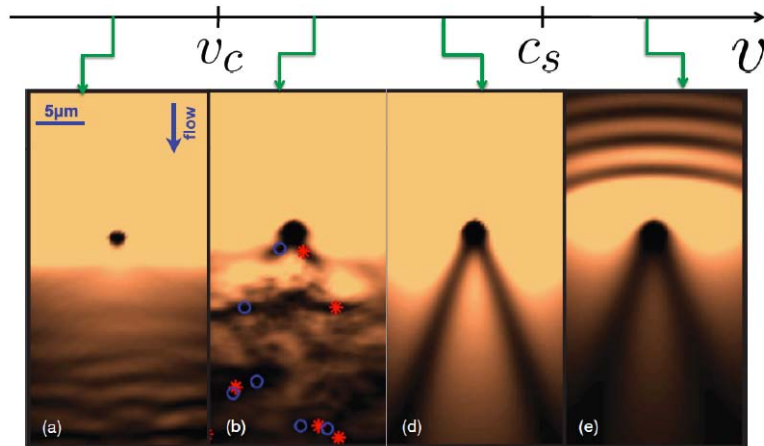
[K. G. Lagoudakis et al, *Nature Physics* 2008]



[Sanvitto et al., *Nature Phys.* 2010]

✧ Hydrodynamics (nucleation of V-AV pairs, solitons in the wake of an obstacle), quantum turbulence, pattern formation

Superfluidity vortices and solitons Cherenkov waves

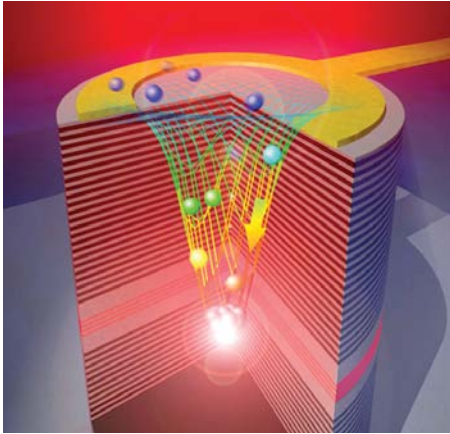


[Pigeon et al., *PRB* (2011)]

- [Nardin et al., *Nature Phys.* 2011]
- [Sanvitto et al., *Nature Photonics* 2011]
- [Grosso et al., *PRL* 2011]
- [Amo et al., *Science* 2011, *Nature* 2009]
- [Wertz et al., *Nature Phys.* 2010]

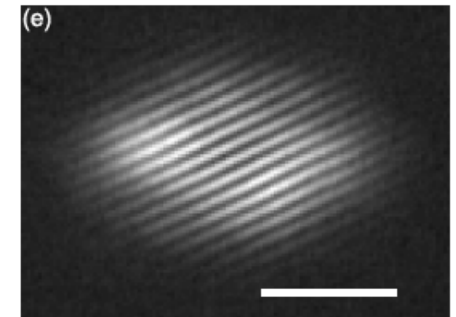
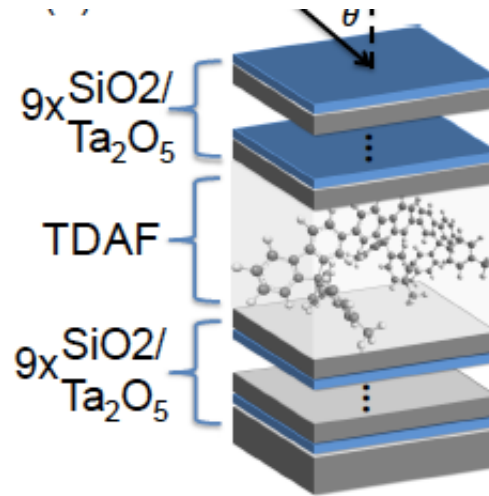
State of the art: towards applications

✧ Electrically pumped polariton laser



[Schneider et al., *Nature* 2013]

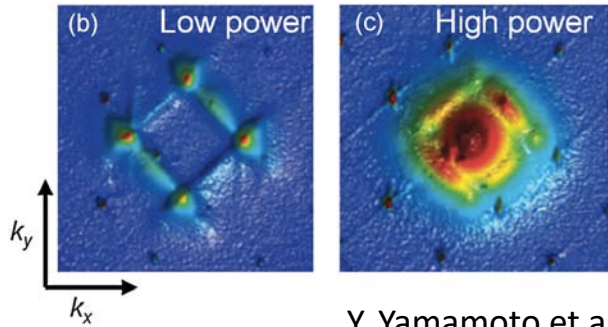
✧ Room temperature condensation and superfluidity in organics



Kena-Cohen's group 2015

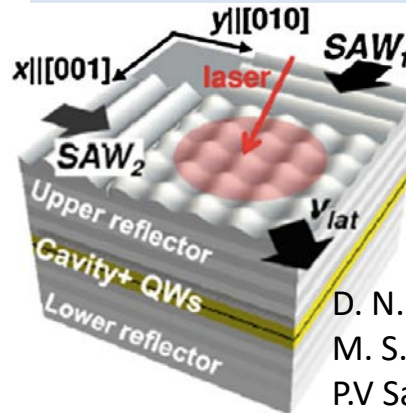
State of the art: lattices

Gold deposition



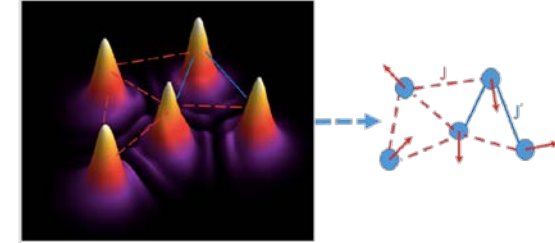
Y. Yamamoto et al.

Surface acoustic waves



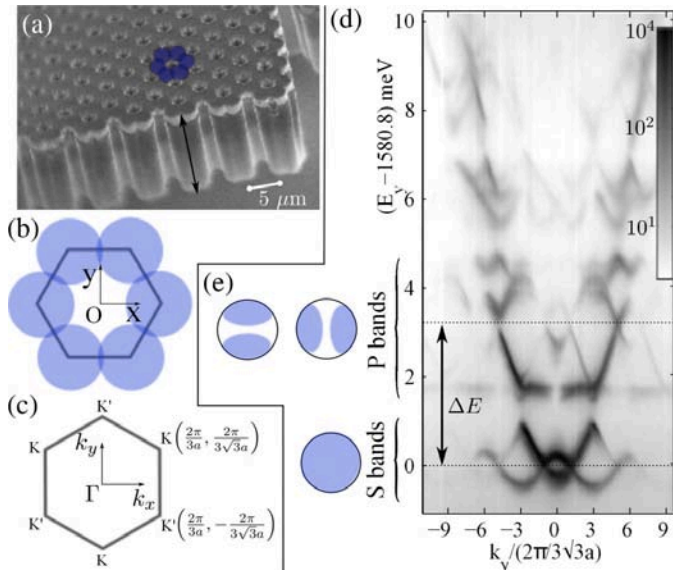
D. N. Krizhanovskii,
M. S. Skolnick,
P.V Santos et al.

Pump modulation



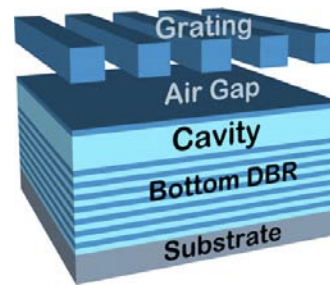
P. G. Lagoudakis
N. Berloff

Micropillars



A. Amo, J. Bloch et al.

Sub-wavelength Gratings



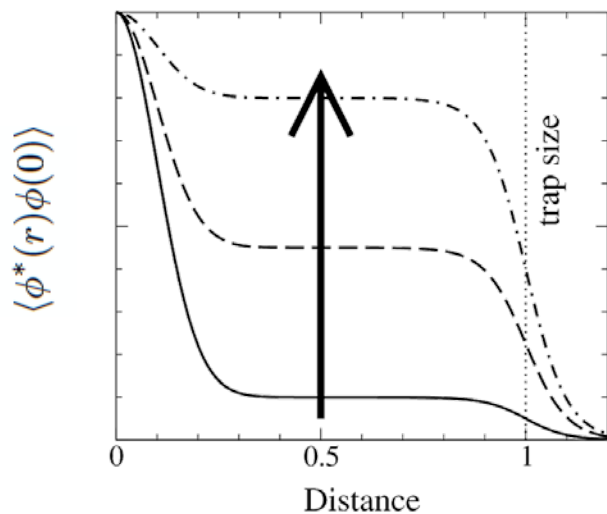
H. Deng et al.

Observed

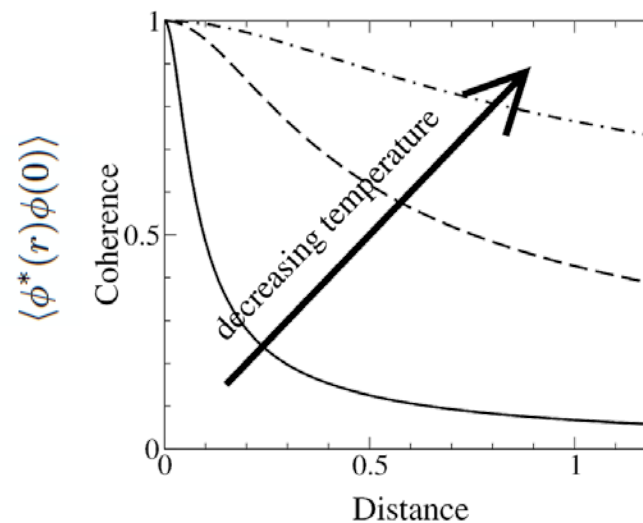
- Edge states
- Spin-orbit coupling
- Dirac cones
- Magnetic monopoles
- Condensation in flat band
- BH model
- XY Hamiltonian
- ...

What Type of “Condensate”?

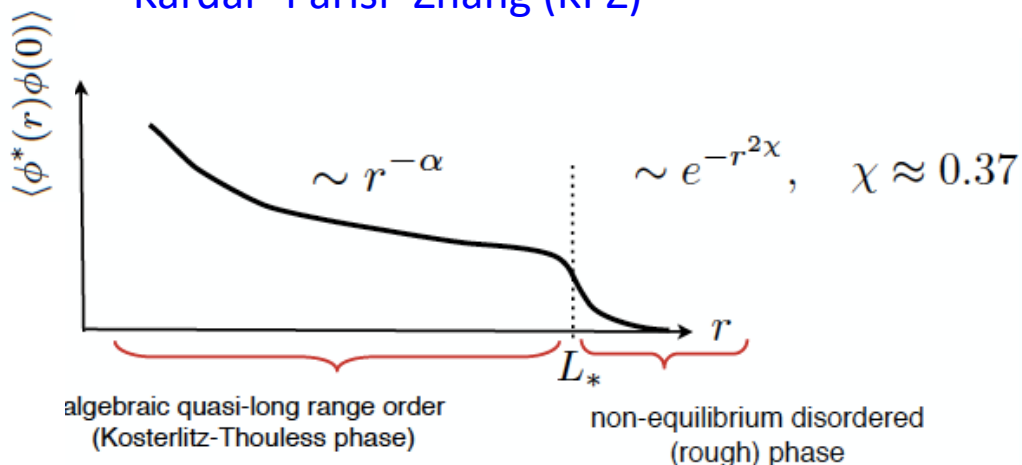
Finite-size BEC



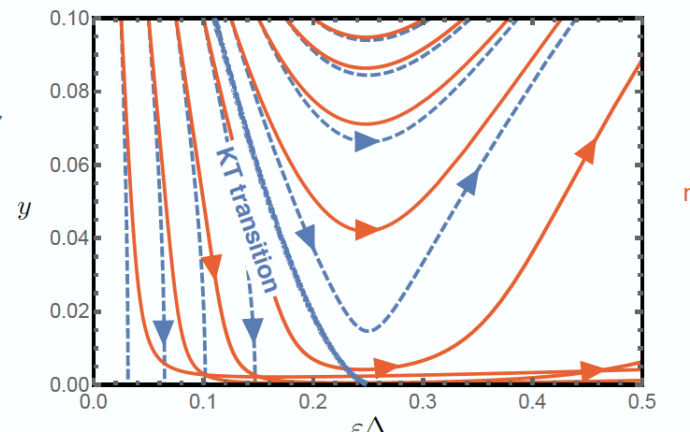
BKT



Kardar–Parisi–Zhang (KPZ)

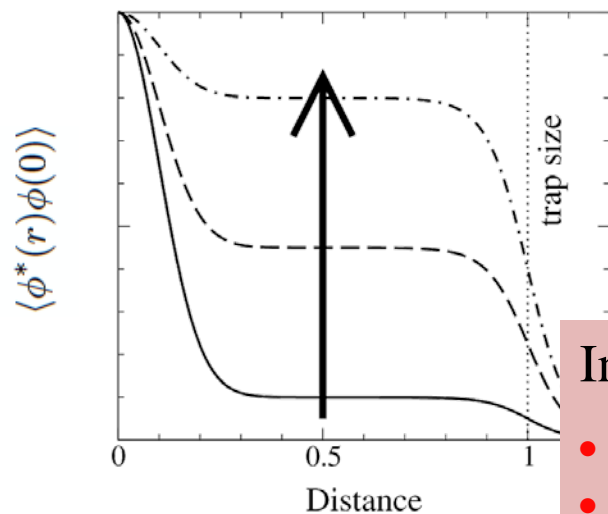


Disordered Phase

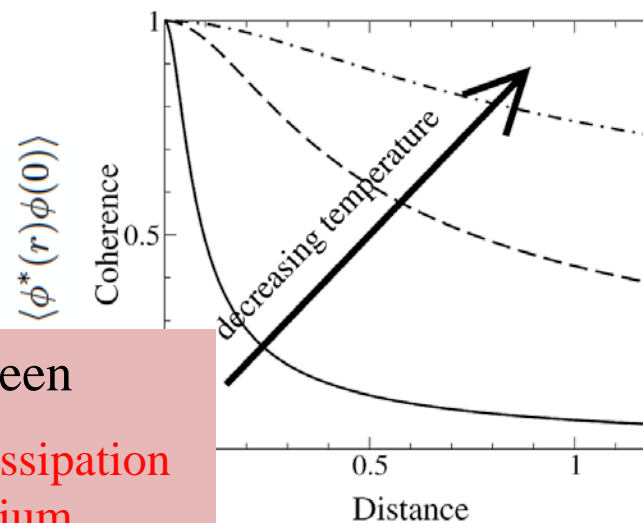


What Type of “Condensate”?

Finite-size BEC



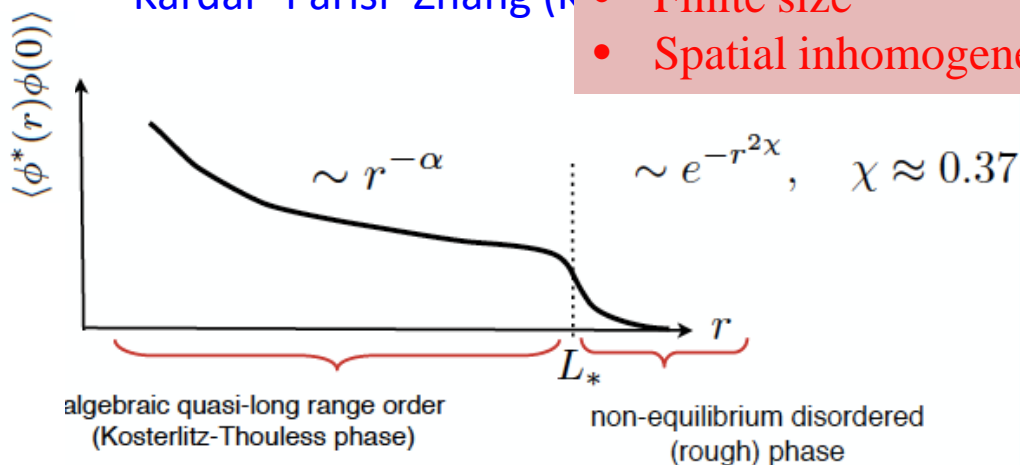
BKT



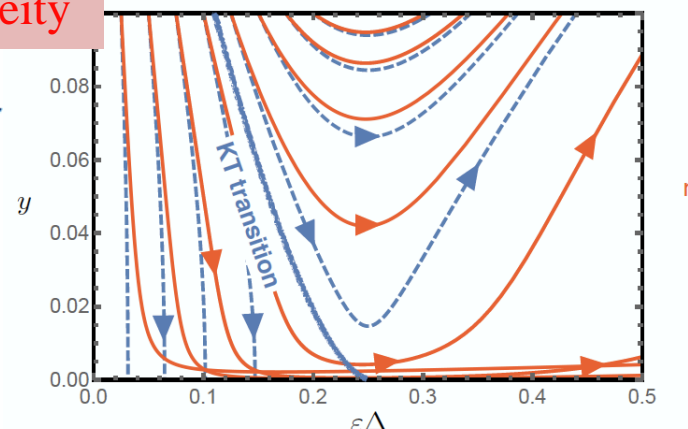
Interplay between

- Drive and Dissipation
- Non-equilibrium
- Finite size
- Spatial inhomogeneity

Kardar–Parisi–Zhang (KPZ)



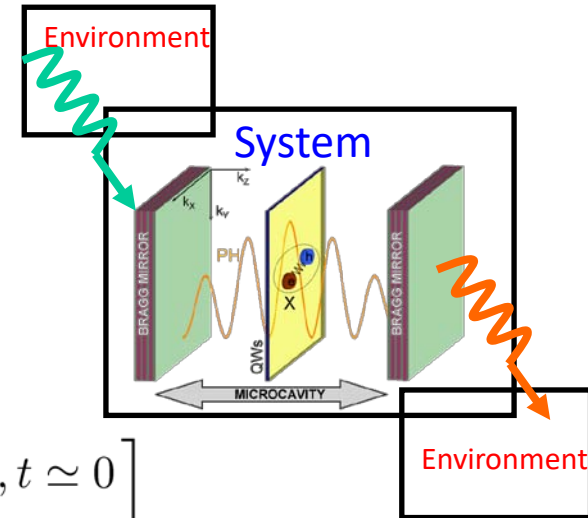
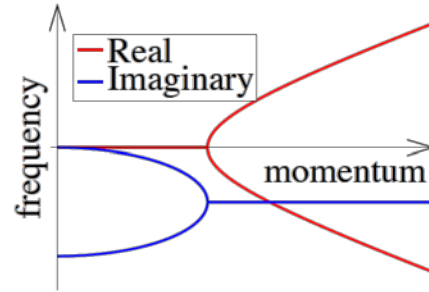
Disordered Phase



Spatial and Temporal Coherence

Driven-dissipative System

- ✧ Dimensionality: 2D
- ✧ Modes: diffusive
- ✧ Occupation: non-thermal



From Keldysh field theory

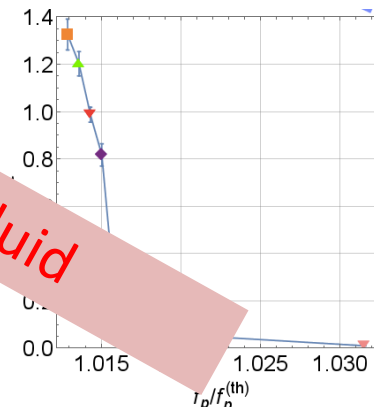
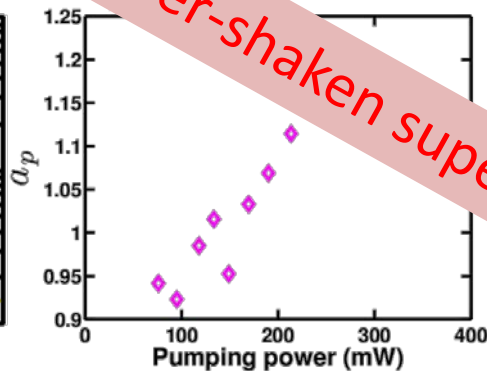
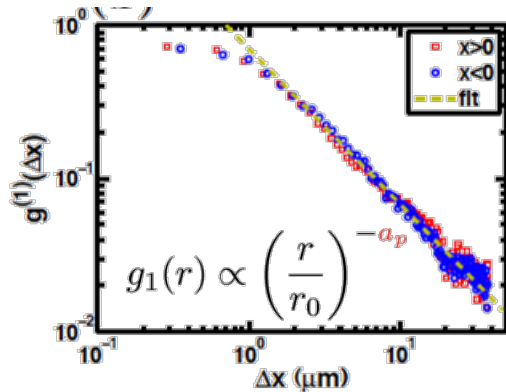
$$\langle \psi^\dagger(\mathbf{r}, t) \psi(0, 0) \rangle \simeq |\psi_0|^2 \exp \left[-\alpha \begin{cases} \ln(r/r_0) & r \rightarrow \infty, t \simeq 0 \\ \frac{1}{2} \ln(c^2 t / \gamma_{\text{tot}} r_0^2) & r \simeq 0, t \rightarrow \infty \end{cases} \right]$$

$\alpha(\text{pump, decay, density})$

[Szymańska et al., *PRL* 2006; *PRB* 2007]

$$\alpha_t = 1/2\alpha_s$$

From stochastic dynamics, KMT and early experiments



Faster decay possible
than equilibrium
upper limit

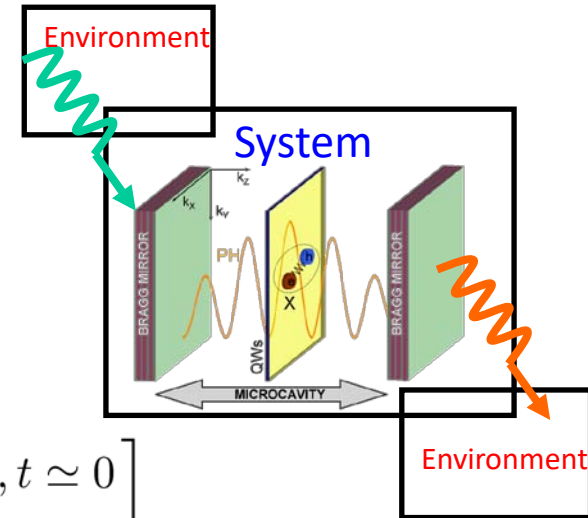
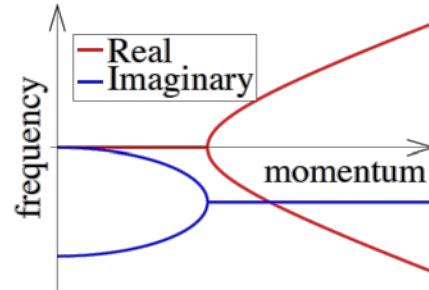
PNAS (2012),
PRX (2015)

Over-shaken superfluid

Spatial and Temporal Coherence

Driven-dissipative System

- ✧ Dimensionality: 2D
- ✧ Modes: diffusive
- ✧ Occupation: non-thermal



From Keldysh field theory

$$\langle \psi^\dagger(\mathbf{r}, t) \psi(0, 0) \rangle \simeq |\psi_0|^2 \exp \left[-\alpha \begin{cases} \ln(r/r_0) & r \rightarrow \infty, t \simeq 0 \\ \frac{1}{2} \ln(c^2 t / \gamma_{\text{tot}} r_0^2) & r \simeq 0, t \rightarrow \infty \end{cases} \right]$$

α (pump, decay, density)

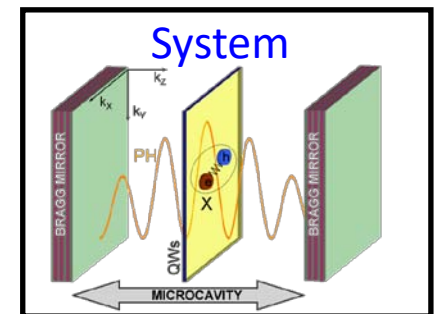
$$\alpha_t = 1/2\alpha_s$$

[Szymańska et al., *PRL* 2006; *PRB* 2007]

Closed System

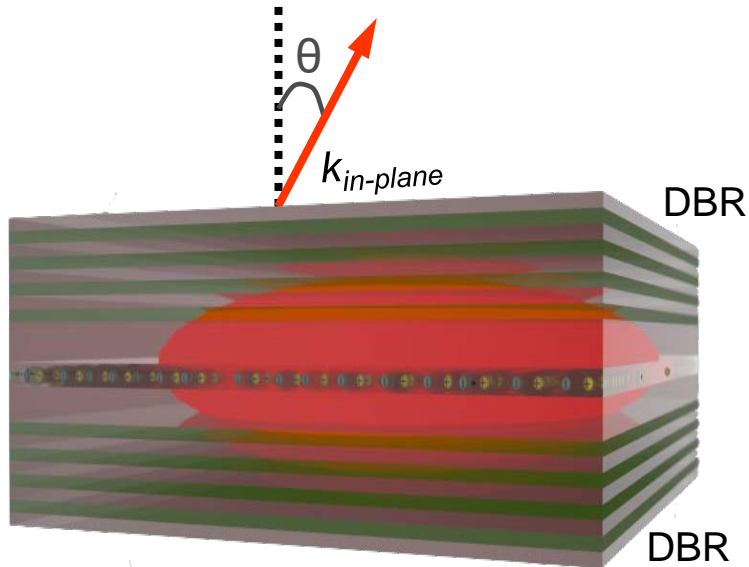
- ✧ Dimensionality: 2D
- ✧ Modes: linear
- ✧ Occupation: thermal

$$\alpha_{s,t} = k_B T / n_s < 1/4$$



Long Lifetime Microcavity

K. West, L. N. Pfeifer
Studied in Snoke's and Sanvitto's groups



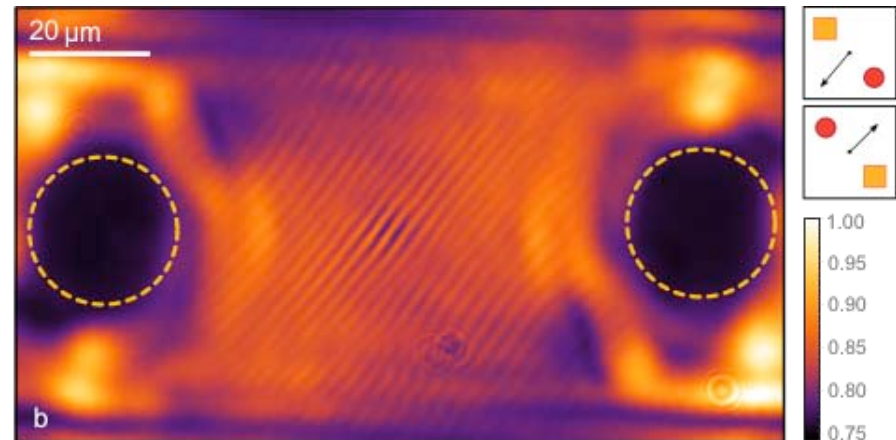
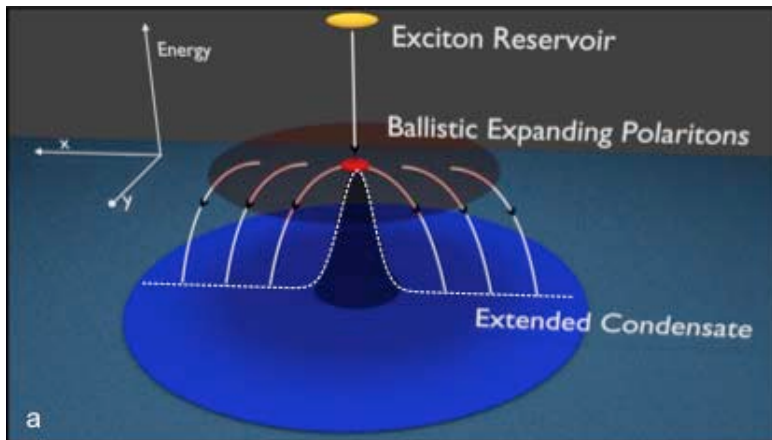
12 QWs

Rabi: 16 meV

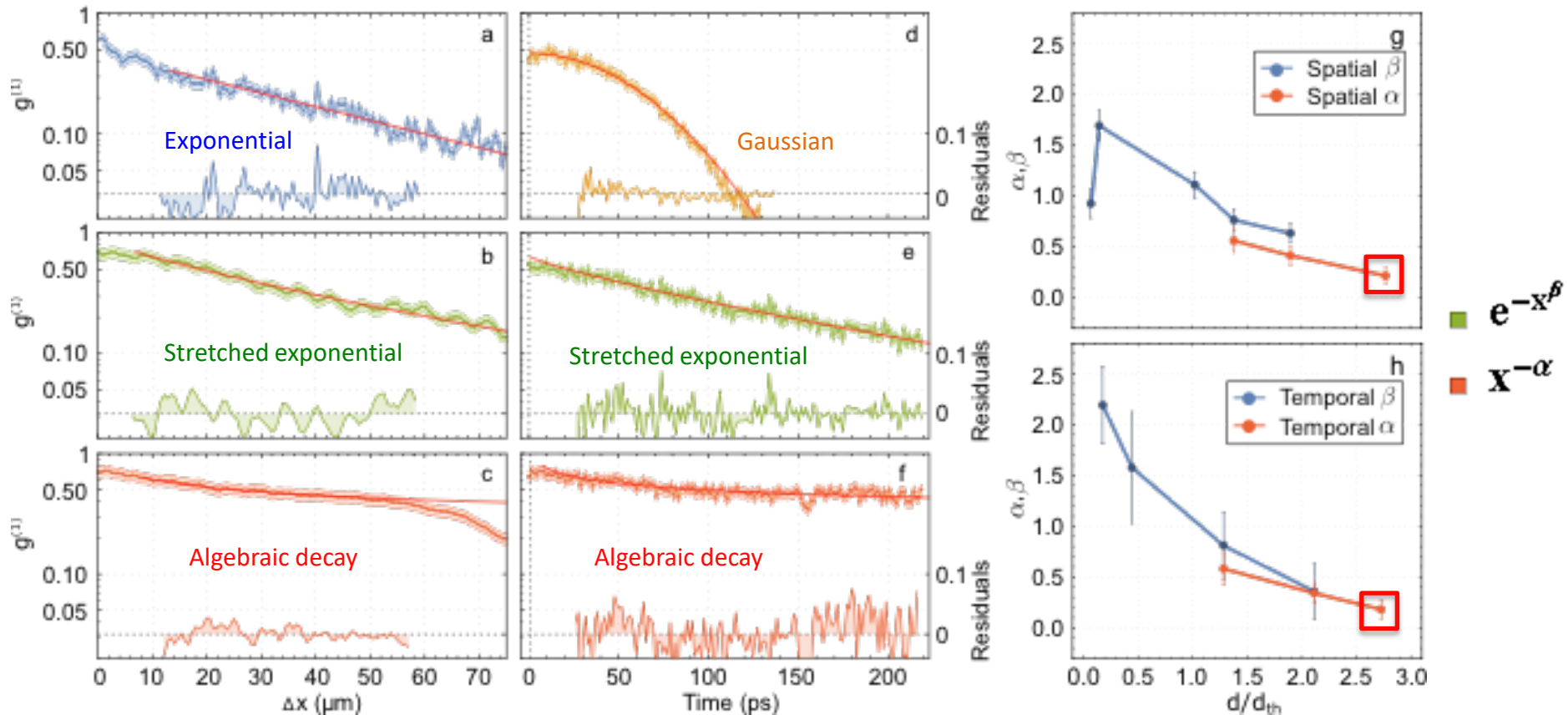
t_p : 100 ps

$Q > 100.000$

Spatially homogeneous



Spatial and Temporal Coherence - Experiment



$$\alpha_s = \alpha_t < 1/4$$

Crossover from exponential to algebraic decay
of coherence with equilibrium exponents

Spatial and Temporal Coherence - Theory

$$i d\psi(\mathbf{r}, t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r}, t)|^2 + i(\gamma - \kappa - \Gamma|\psi(\mathbf{r}, t)|^2) \right] \psi(\mathbf{r}, t) dt + dW$$

See Sebastian Diehl's talk

$$\frac{\gamma}{1 + \frac{|\psi(\mathbf{r}, t)|^2}{n_s}}$$

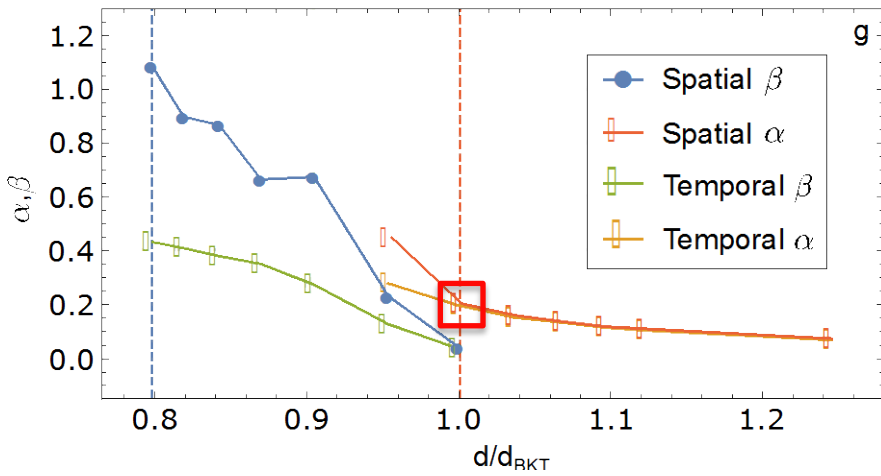
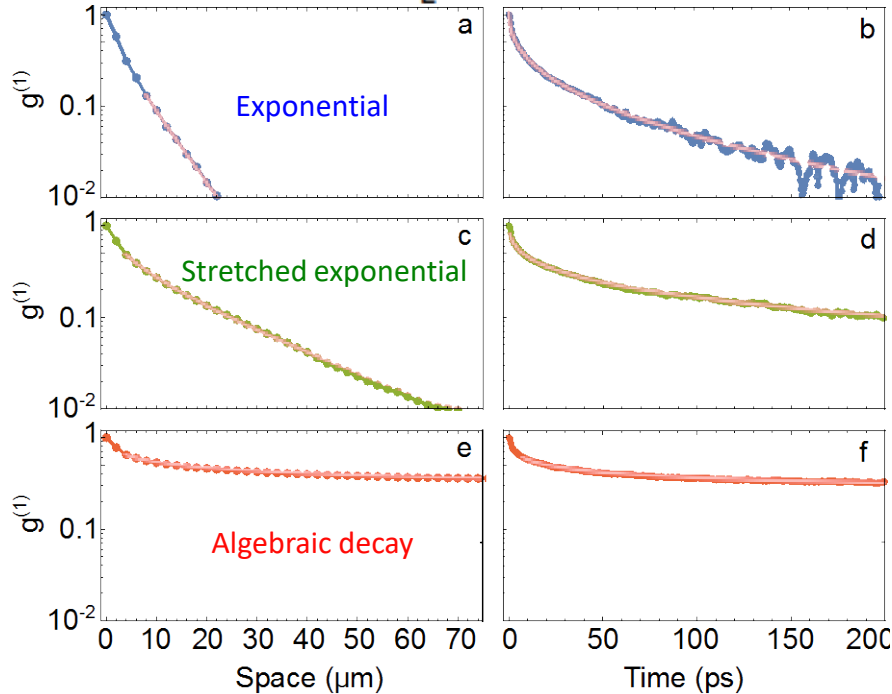
more non-linear drive

$$\langle dW^*(\mathbf{r}', t) dW(\mathbf{r}, t) \rangle = \frac{\gamma + \kappa + \Gamma|\psi(\mathbf{r}, t)|^2}{dV} \delta_{\mathbf{r}, \mathbf{r}'} dt$$

$$\frac{\gamma}{1 + \frac{|\psi(\mathbf{r}, t)|^2}{n_s}} + \kappa$$

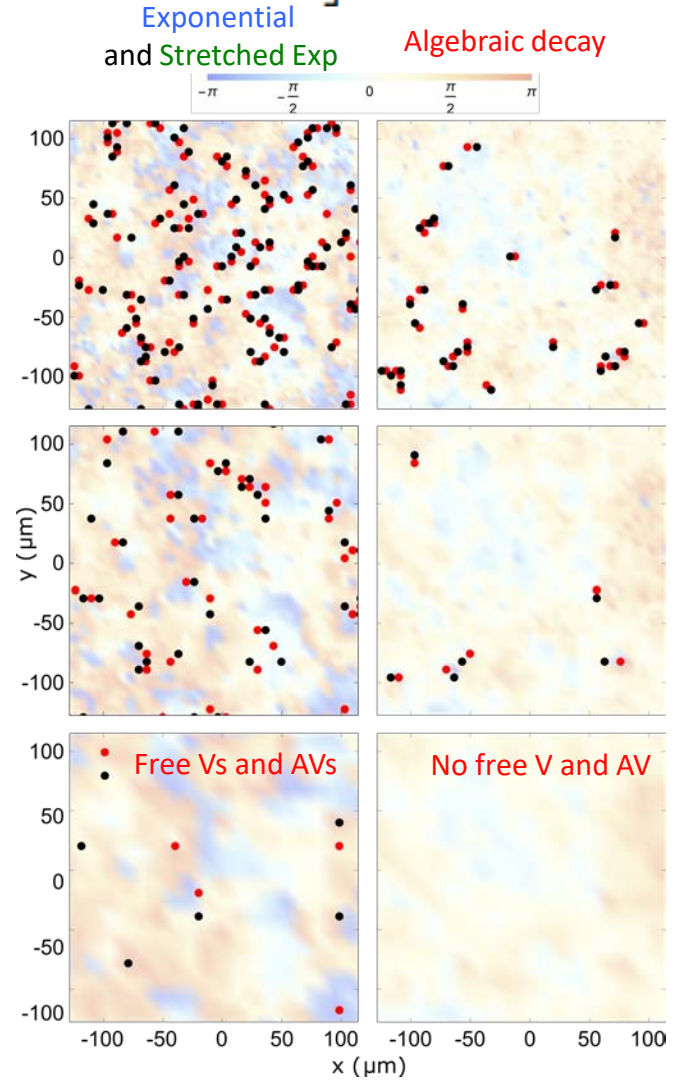
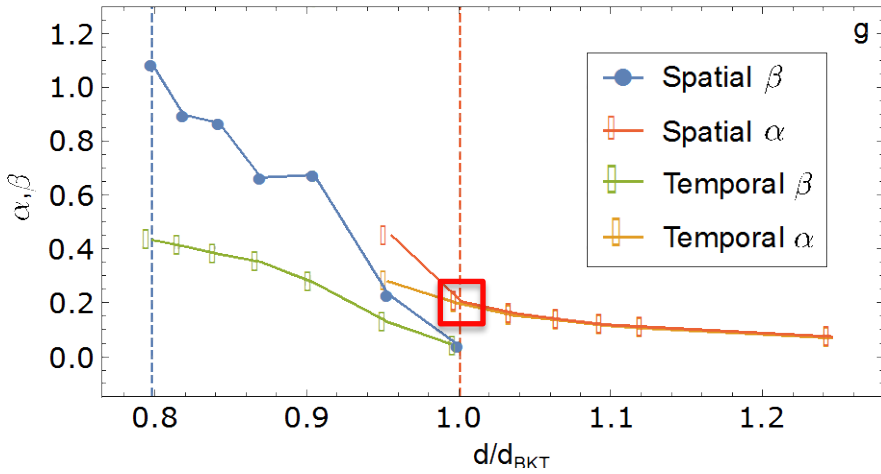
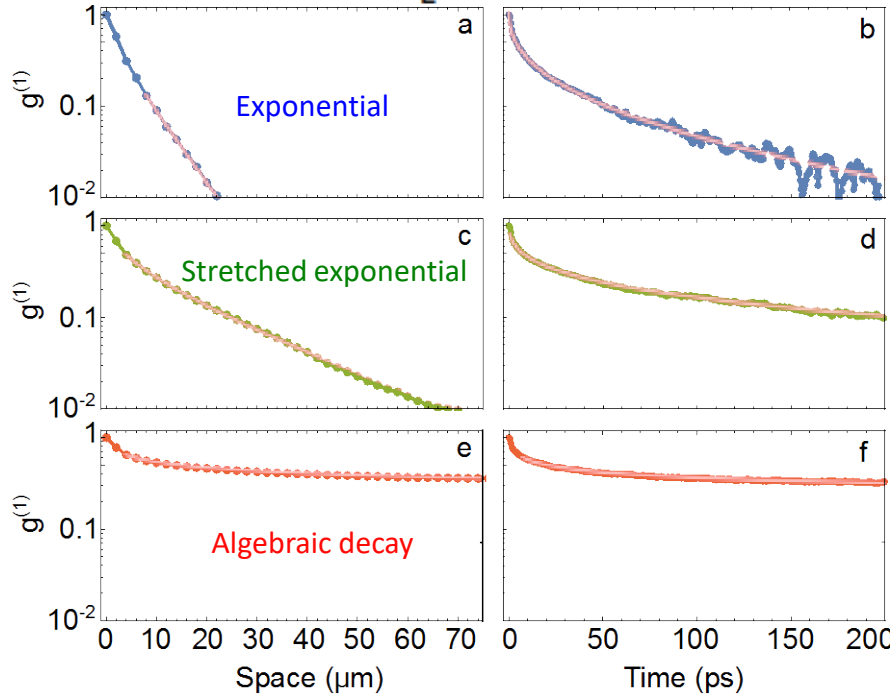
For experimental parameters

$$\alpha_s = \alpha_t < 1/4$$



Spatial and Temporal Coherence - Theory

$$i d\psi(\mathbf{r}, t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r}, t)|^2 + i(\gamma - \kappa - \Gamma|\psi(\mathbf{r}, t)|^2) \right] \psi(\mathbf{r}, t) dt + dW$$



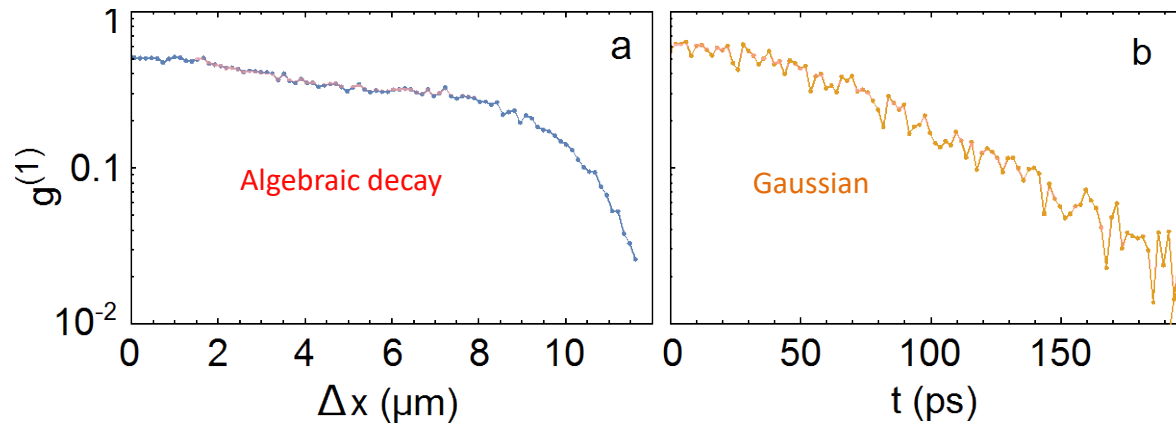
BKT crossover

Coarse-graining

Spatial and Temporal Coherence – Photon Laser

Measurements in a bad cavity (short lifetime) sample
in a weak coupling regime

In a lasing VCSEL coherence shows power law decay in space but Gaussian in time



Importance of consistent picture in space and
time correlations

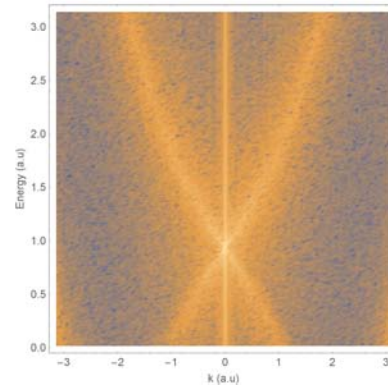
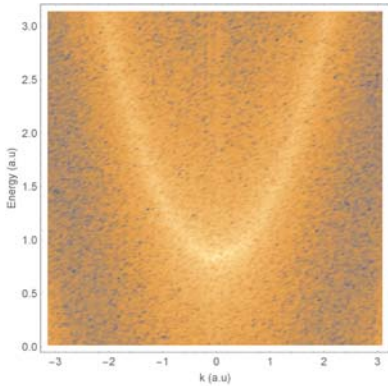
Spectrum of Excitations

$$E(\omega) = -i\gamma \pm \sqrt{\omega_k(\omega_k + 2u) - \gamma^2}$$

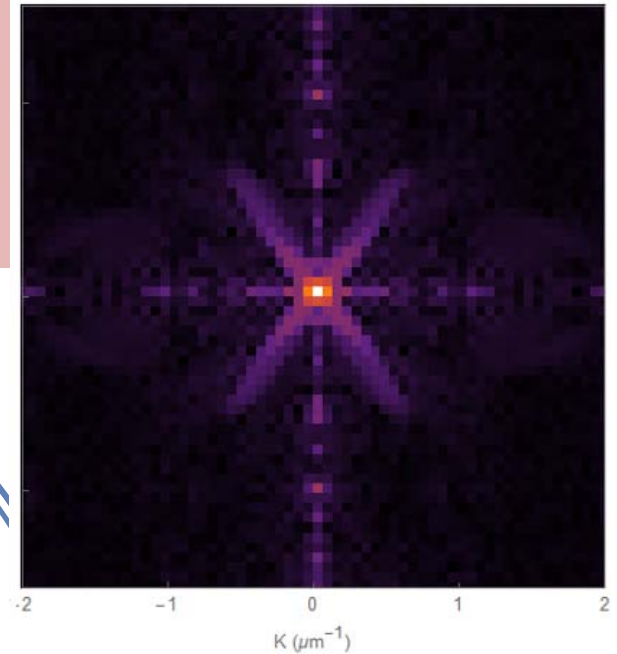
$$\gamma = 0.01$$

$$u = 5$$

Theoretical Fourier transform of $g^1(r,t)$



Experimental Fourier transform of $g^1(r,t)$



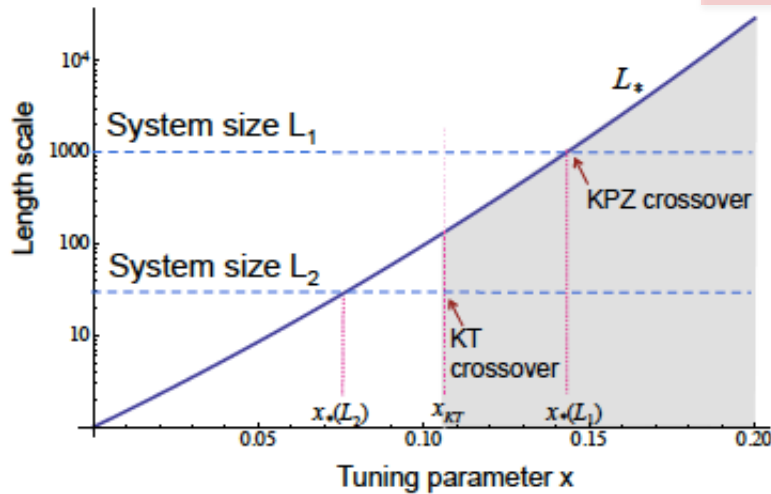
Equilibrium-like BKT
despite dissipative
driven system

See also David Snoko's work on BE
distribution

Kardar–Parisi–Zhang (KPZ) Theory

[Altman et al, *PRX* 2015]

$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + \bar{\zeta}(\mathbf{x}, t)$$



KPZ non-linearity

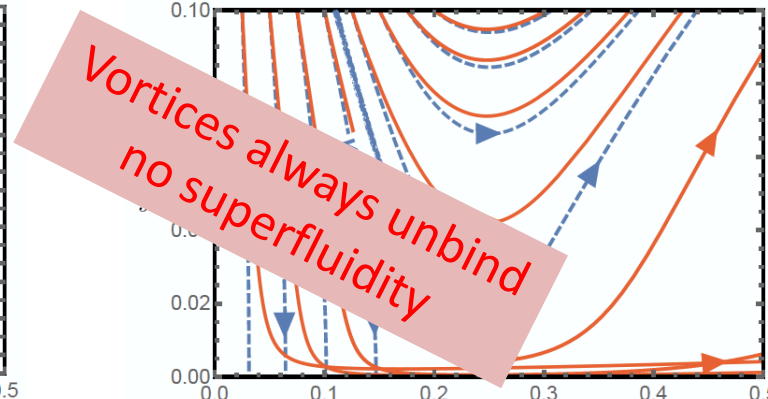
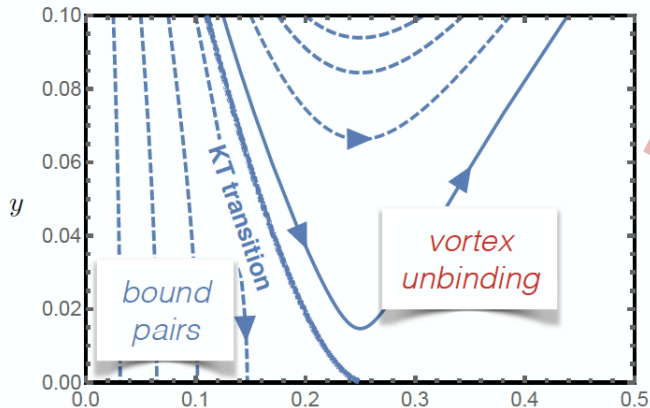
See Sebastian Diehl's talk

At large distances

$$\langle \phi^*(r) \phi(0) \rangle \sim e^{-r^{2\chi}}, \quad \chi \approx 0.37$$

Stretched exponential (faster than algebraic) decay of coherence but **superfluidity survives**

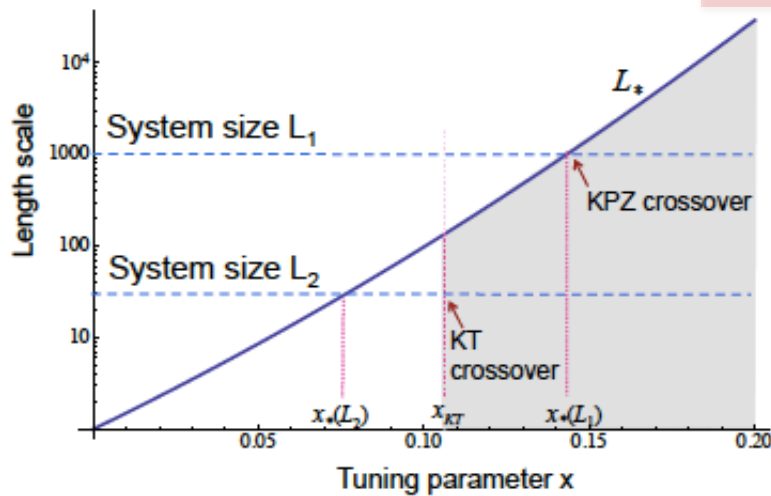
Adding vortices



Kardar-Parisi-Zhang (KPZ) Theory

[Altman et al, *PRX* 2015]

$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + \bar{\zeta}(\mathbf{x}, t)$$



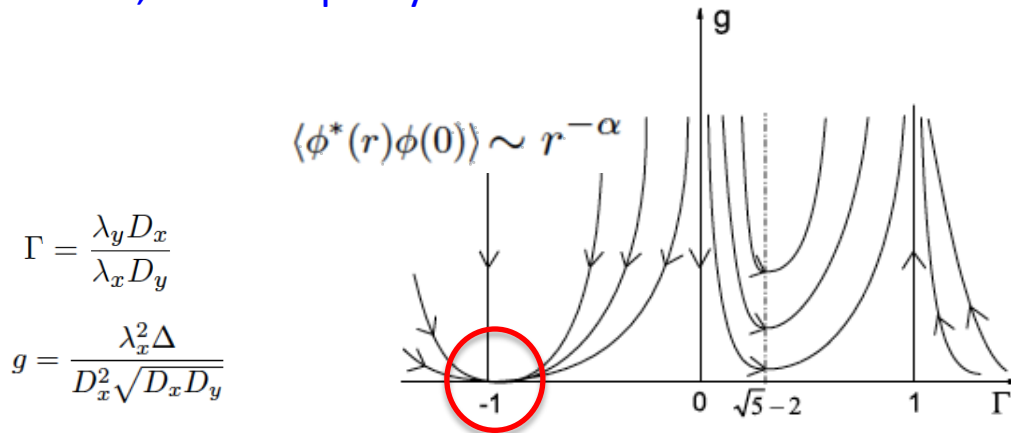
KPZ non-linearity

At large distances

$$\langle \phi^*(r) \phi(0) \rangle \sim e^{-r^{2\chi}}, \quad \chi \approx 0.37$$

Stretched exponential (faster than algebraic) decay of coherence but superfluidity survives

However, anisotropic system flows to a Gaussian fixed point power-law at any scale

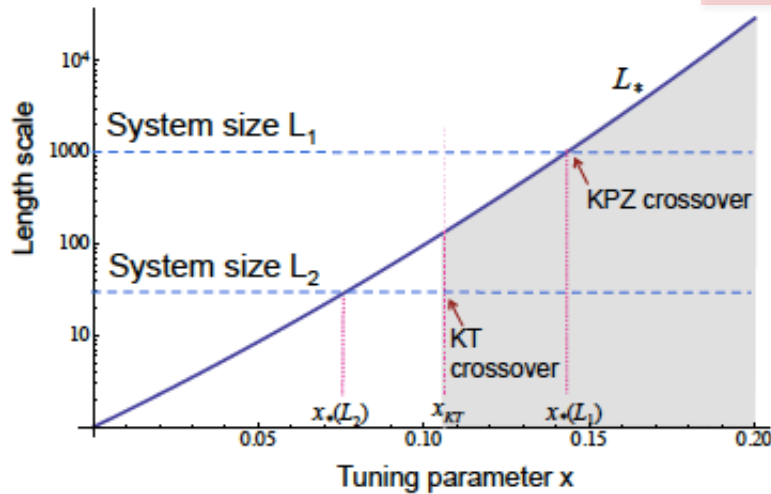


How to realise in experiment?

Kardar-Parisi-Zhang (KPZ) Theory

[Altman et al, PRX 2015]

$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + \bar{\zeta}(\mathbf{x}, t)$$



KPZ non-linearity

At large distances

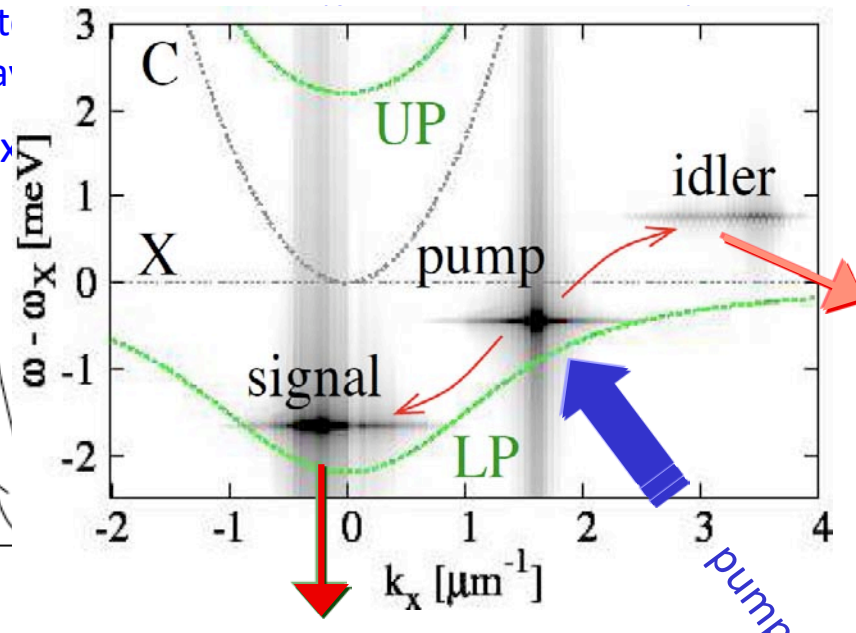
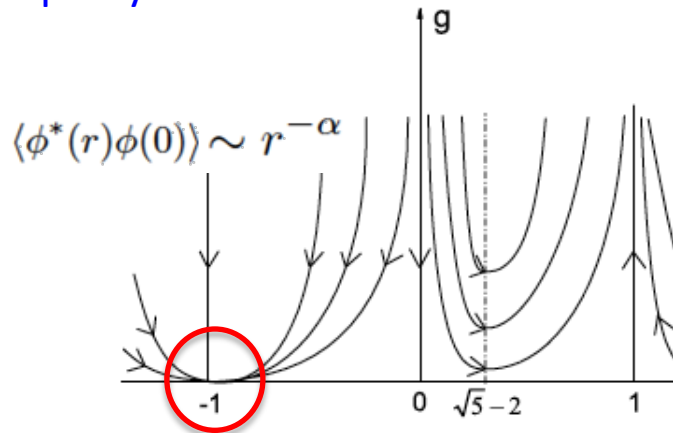
$$\langle \phi^*(r) \phi(0) \rangle \sim e^{-r^{2\chi}}, \quad \chi \approx 0.37$$

Stret
deca

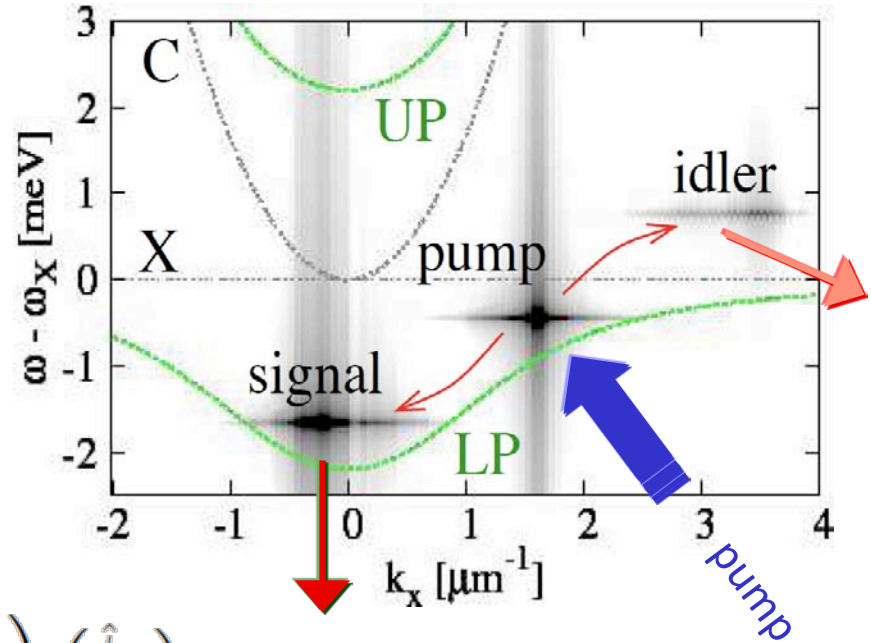
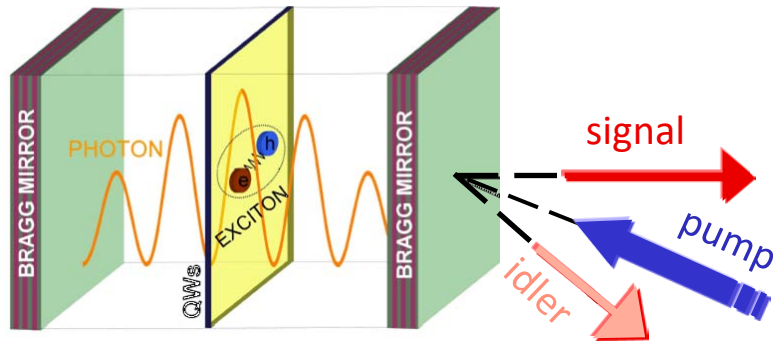
However, anisotropic system flows to a Gaussian fixed point

$$\Gamma = \frac{\lambda_y D_x}{\lambda_x D_y}$$

$$g = \frac{\lambda_x^2 \Delta}{D_x^2 \sqrt{D_x D_y}}$$



Playing with Spatial Anisotropy: OPO



$$\hat{H}_S = \int d\mathbf{r} \begin{pmatrix} \hat{\psi}_X^\dagger & \hat{\psi}_C^\dagger \end{pmatrix} \begin{pmatrix} \frac{-\nabla^2}{2m_X} + \frac{g_X}{2} |\hat{\psi}_X|^2 & \frac{\Omega_R}{2} \\ \frac{\Omega_R}{2} & \frac{-\nabla^2}{2m_C} \end{pmatrix} \begin{pmatrix} \hat{\psi}_X \\ \hat{\psi}_C \end{pmatrix}$$

$$\hat{H}_{SB} = \int d\mathbf{r} \left[F(\mathbf{r}, t) \hat{\psi}_C^\dagger(\mathbf{r}, t) + \text{H.c.} \right] + \sum_{\mathbf{k}} \sum_{l=X,C} \left\{ \zeta_{\mathbf{k}}^l \left[\hat{\psi}_{l,\mathbf{k}}^\dagger(t) \hat{B}_{l,\mathbf{k}} + \text{H.c.} \right] + \omega_{l,\mathbf{k}} \hat{B}_{l,\mathbf{k}}^\dagger \hat{B}_{l,\mathbf{k}} \right\}$$

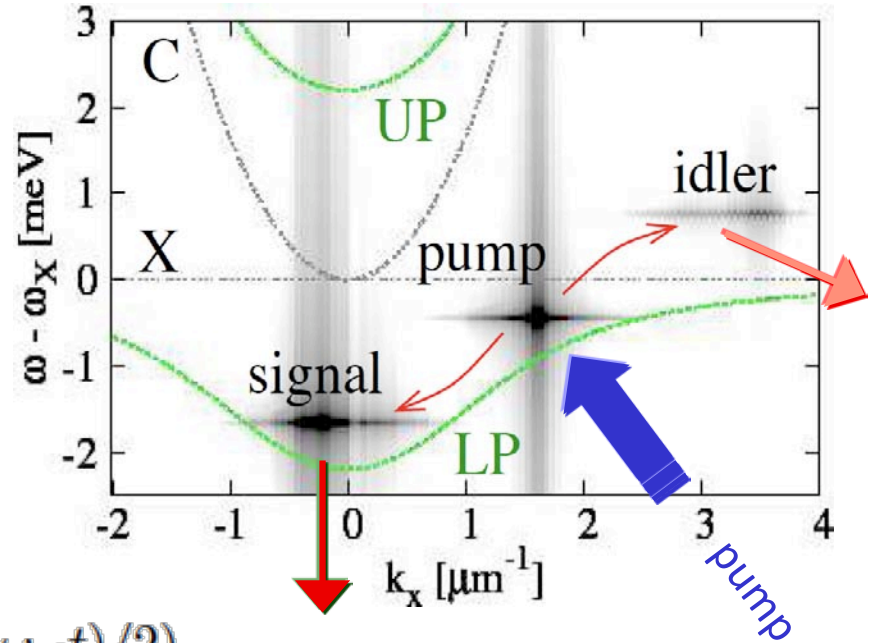
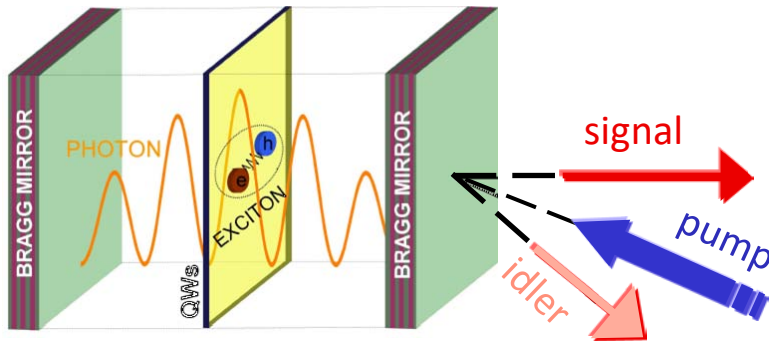
✧ Non-thermal occupation

✧ Signal phase is completely free and idler phase locked to signal via pump

$$2\varphi_p = \varphi_s + \varphi_i$$

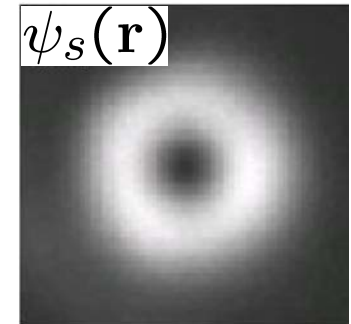
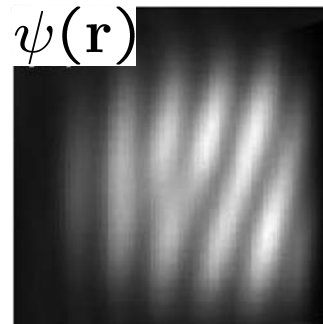
✧ Spontaneous U(1) symmetry breaking gapless and diffusive Goldstone mode

Playing with Spatial Anisotropy: OPO



$$|\psi_{LP}(\mathbf{r}, t)|^2 = \sum_j \rho_j^2 + 2 \left(\sqrt{\rho_s \rho_p} \cos(\phi_s - \phi_p + (\mathbf{k}_{si} \cdot \mathbf{r} - \omega_{si}t)/2) + \sqrt{\rho_p \rho_i} \cos(\phi_i - \phi_p - (\mathbf{k}_{si} \cdot \mathbf{r} - \omega_{si}t)/2) + \sqrt{\rho_s \rho_i} \cos(\phi_s - \phi_i + \mathbf{k}_{si} \cdot \mathbf{r} - \omega_{si}t) \right)$$

- ✧ Time crystal
- ✧ **Vortices:** dislocations in density wave and time crystal
- ✧ After filtering in momentum: usual vortices



Stochastic Description

- From Keldysh action by ignoring the RG irrelevant terms (quantum fluctuations of order higher than second) and using MSR formalism

[Sieberer et al PRL (2013)]

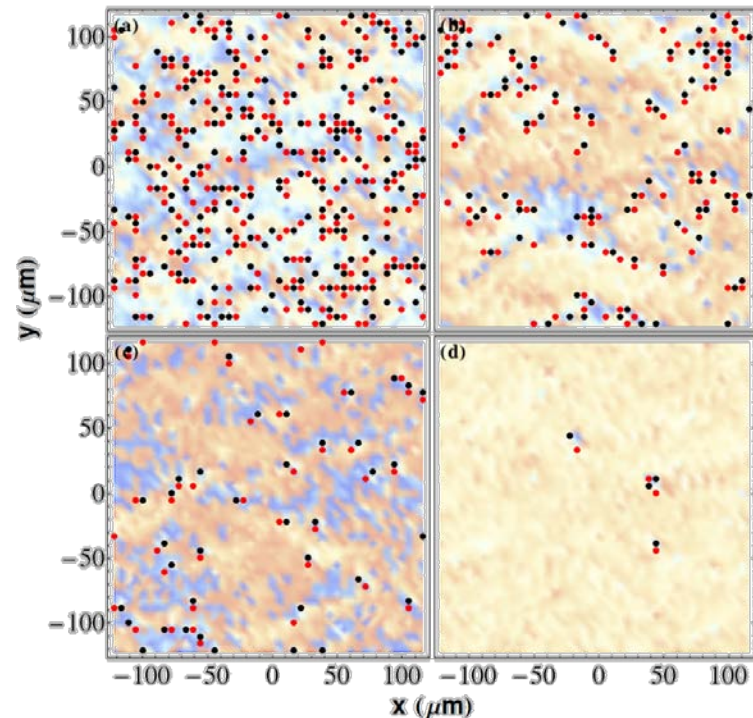
$$i \begin{pmatrix} d\psi_X \\ d\psi_C \end{pmatrix} = \left\{ \begin{pmatrix} \omega_X - i\kappa_X + g_X(|\psi_X|^2 - \frac{1}{\Delta V}) & \Omega_R/2 \\ \Omega_R/2 & \frac{\nabla^2}{2m_c} - i\kappa_C \end{pmatrix} \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} + \begin{pmatrix} 0 \\ F_p \end{pmatrix} \right\} dt + \begin{pmatrix} \sqrt{\kappa_X} dW_X \\ \sqrt{\kappa_C} dW_C \end{pmatrix}$$

$$F_p(\mathbf{r}, t) = \mathcal{F}_p(\mathbf{r}) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

dW - Wiener noise delta correlated in space and time

Observables: MC averages over noise

Exact numerical solution



Low density:

Vortex/antivortex
proliferation

Medium density:

V/AV pairing

High density:

V/AV annihilation, no
vortices

[G. Dagvadorj et al, PRX (2015)]

Stochastic Description

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[Sieberer et al PRL (2013)]

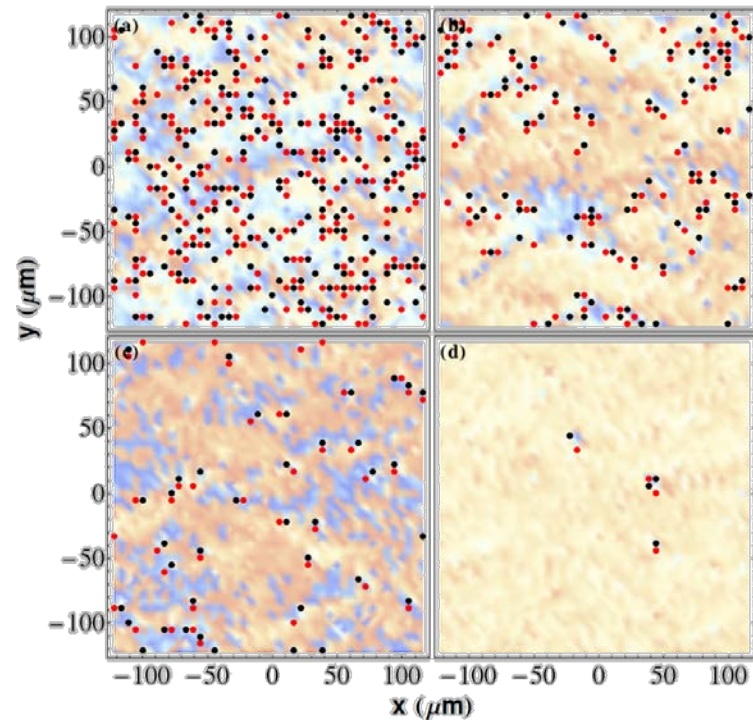
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$$F_p(\mathbf{r}, t) = \mathcal{F}_p(\mathbf{r}) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

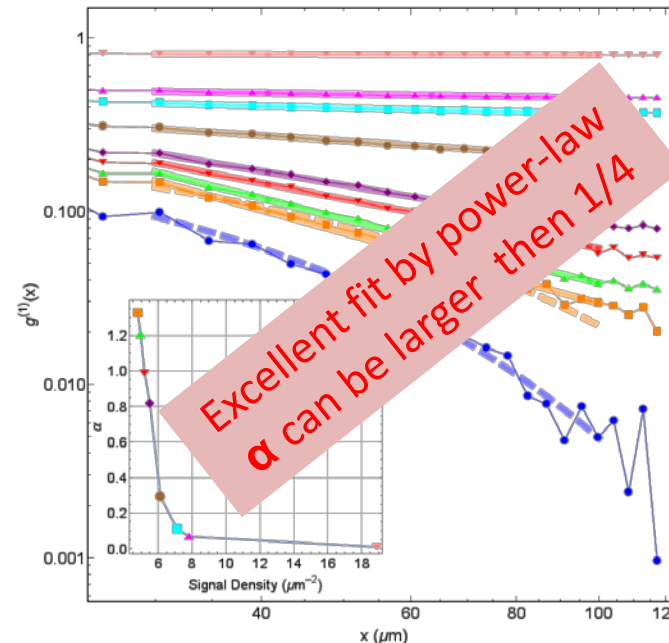
dW - Wiener noise delta correlated in space and time

Observables: MC averages over noise

Exact numerical solution



$$g_s^{(1)}(\mathbf{r}) = \frac{\langle \psi_s^*(\mathbf{r}+\mathbf{R}) \psi_s(\mathbf{R}) \rangle}{\sqrt{\langle \psi_s^*(\mathbf{r}) \psi_s(\mathbf{r}) \rangle \langle \psi_s^*(\mathbf{r}+\mathbf{R}) \psi_s(\mathbf{r}+\mathbf{R}) \rangle}}$$



[G. Dagvadorj et al, PRX (2015)]

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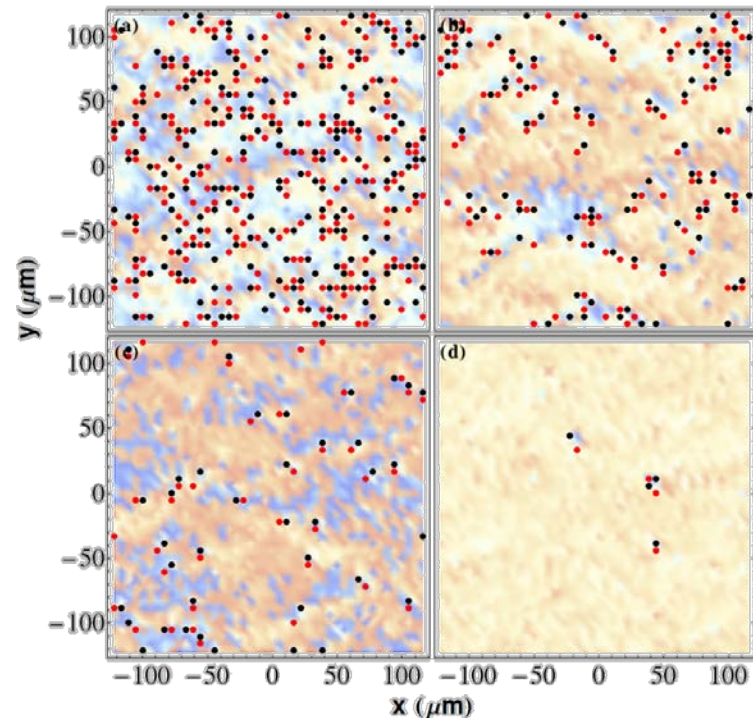
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$$F_p(\mathbf{r}, t) = \mathcal{F}_p(\mathbf{r}) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

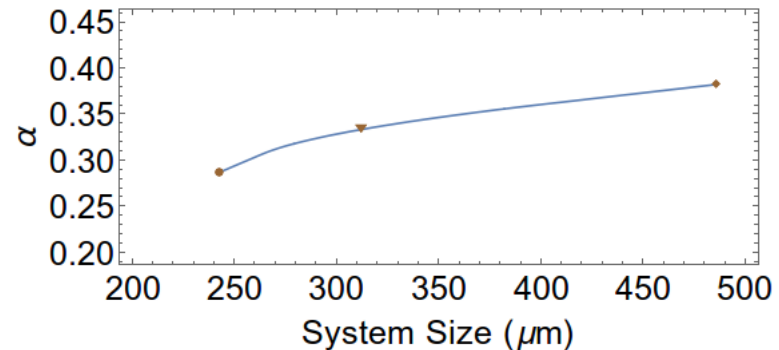
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$$g_s^{(1)}(\mathbf{r}) = \frac{\langle \psi_s^*(\mathbf{r}+\mathbf{R}) \psi_s(\mathbf{R}) \rangle}{\sqrt{\langle \psi_s^*(\mathbf{r}) \psi_s(\mathbf{r}) \rangle \langle \psi_s^*(\mathbf{r}+\mathbf{R}) \psi_s(\mathbf{r}+\mathbf{R}) \rangle}}$$



Excellent fit by power-law
 α can be larger than 1/4

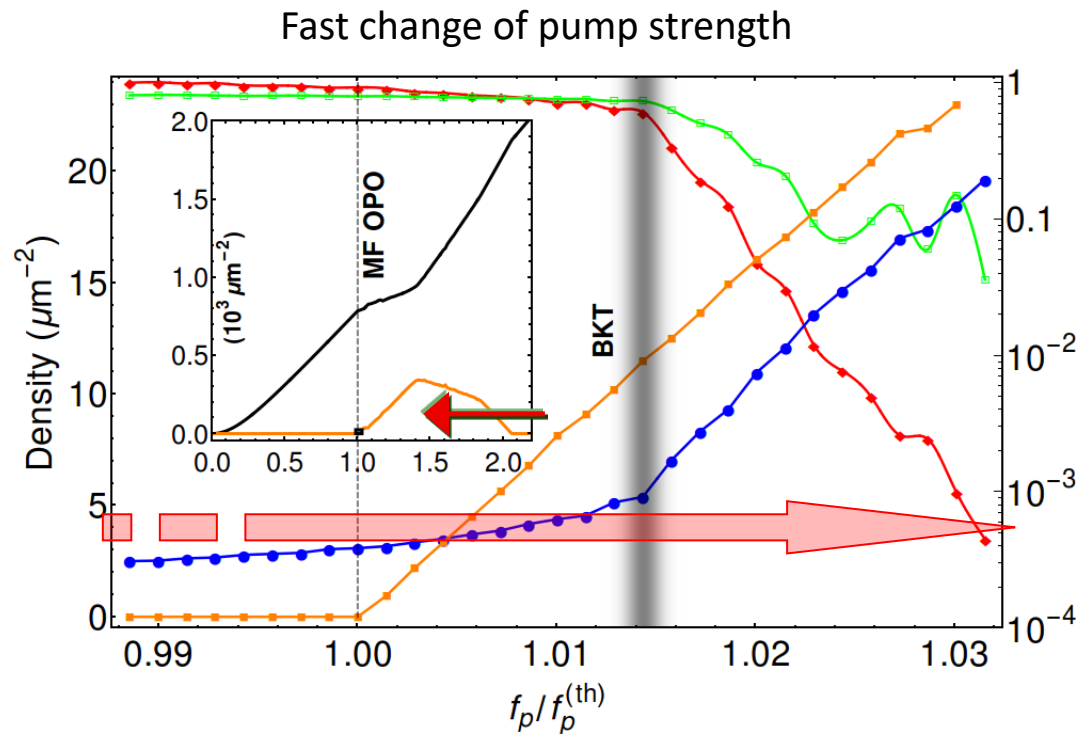
[G. Dagvadorj et al, PRX (2015)]

Quench Dynamics

[P. Comaron, G. Dagvadorj, A. Zamora, et al, in preparation]

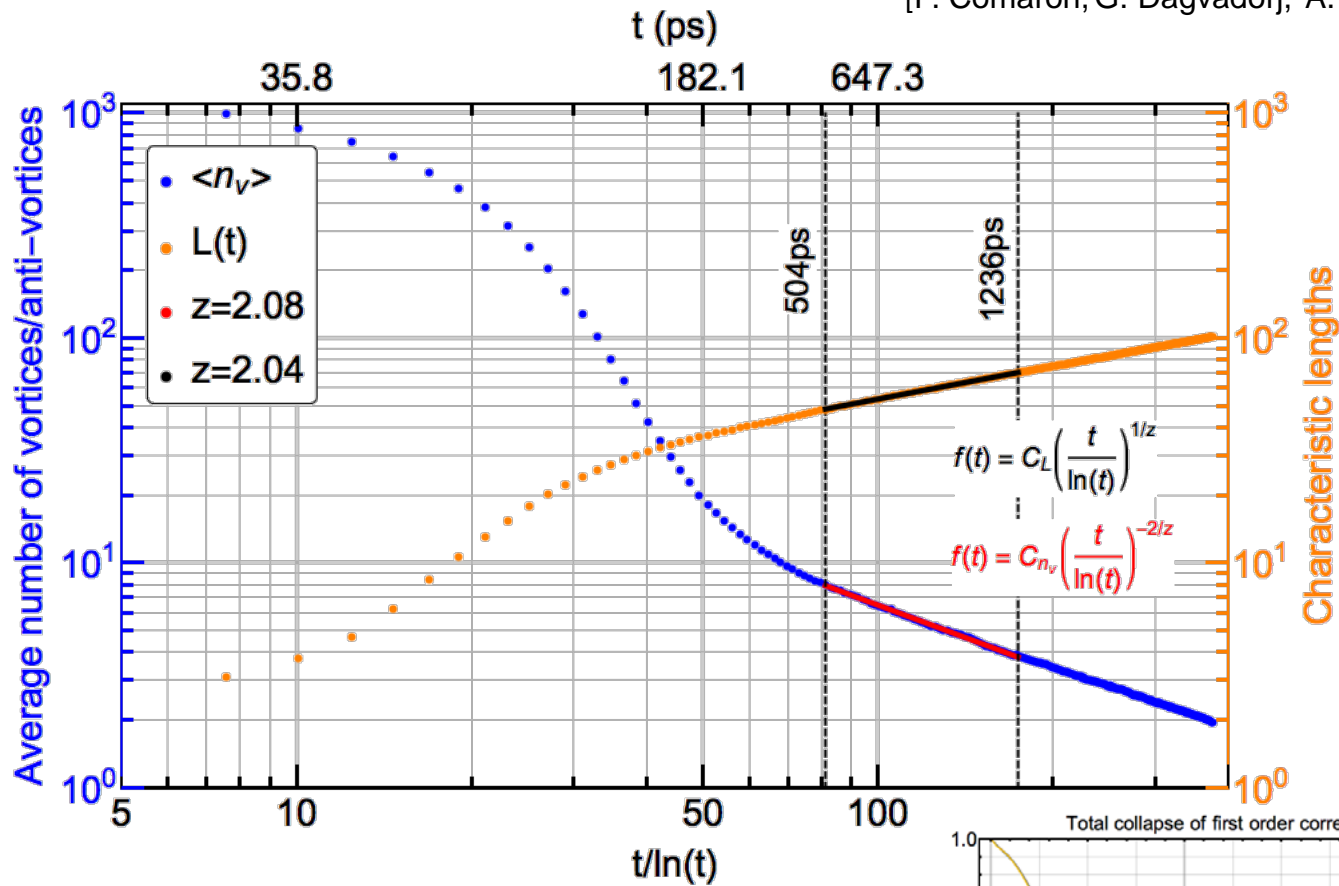
Dynamics after infinitely fast quench

if universal can reveal the critical exponents of the transition

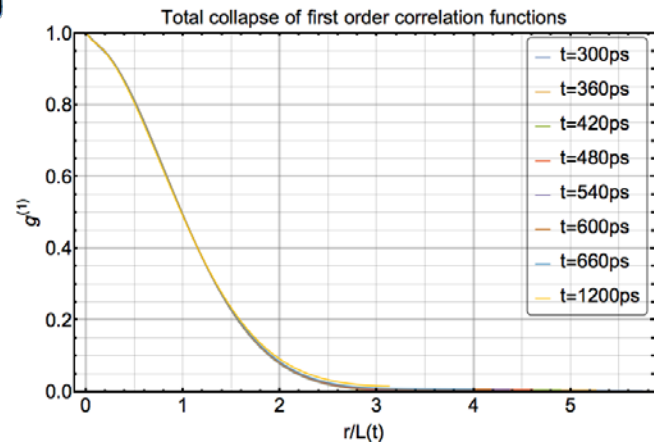


Dynamical Critical Exponents

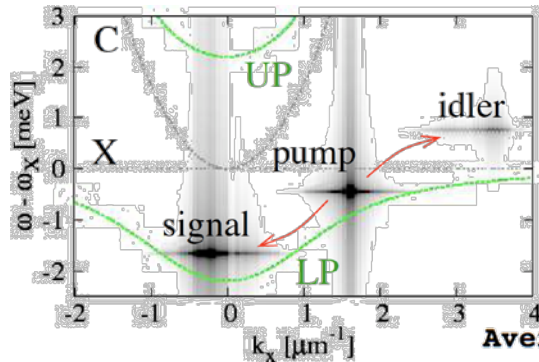
[P. Comaron, G. Dagvadorj, A. Zamora, et al, in preparation]



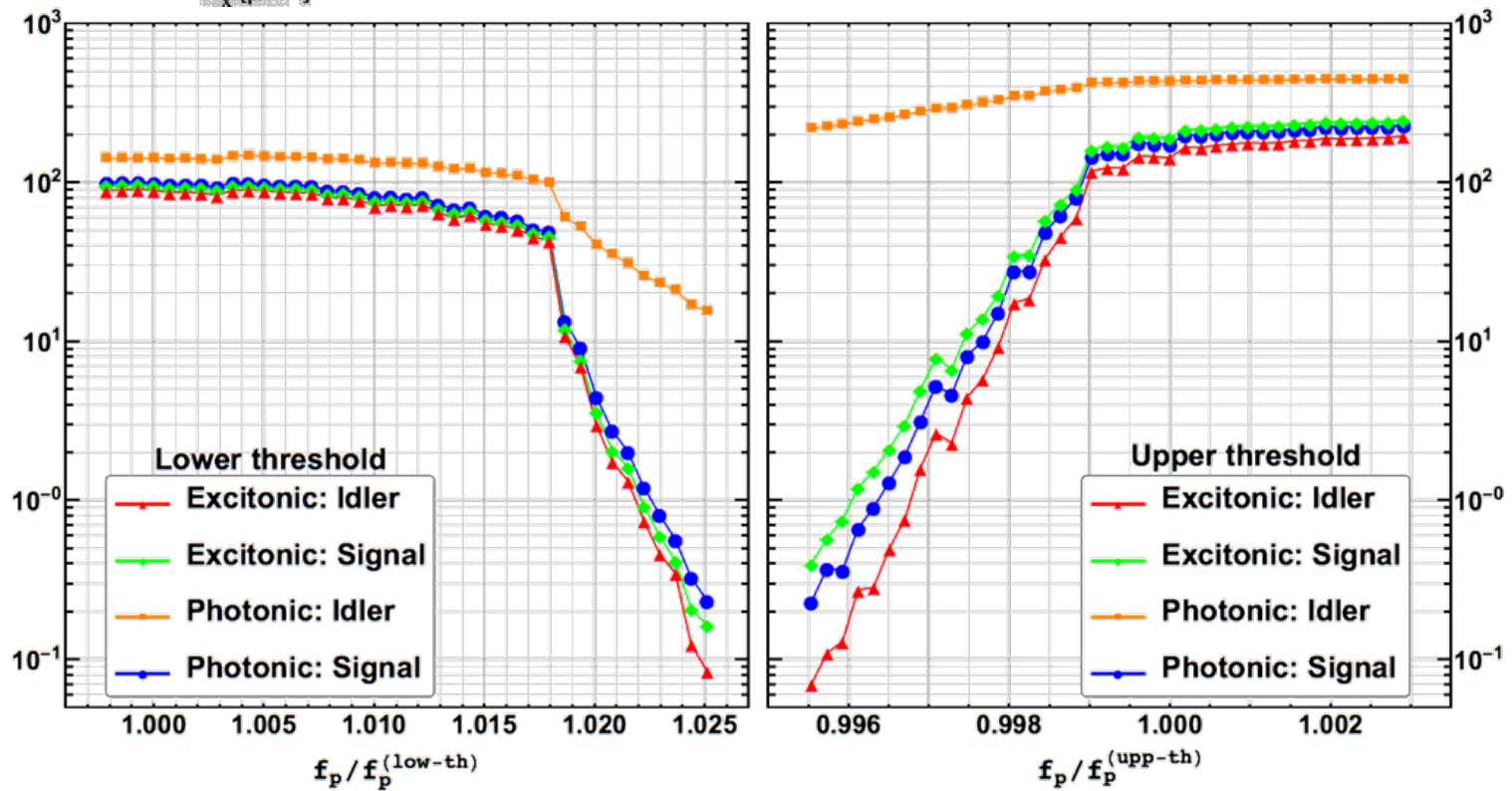
Critical exponent $z=2$
 as in equilibrium XY model
 i.e. equilibrium BKT



Multicomponent Condensate



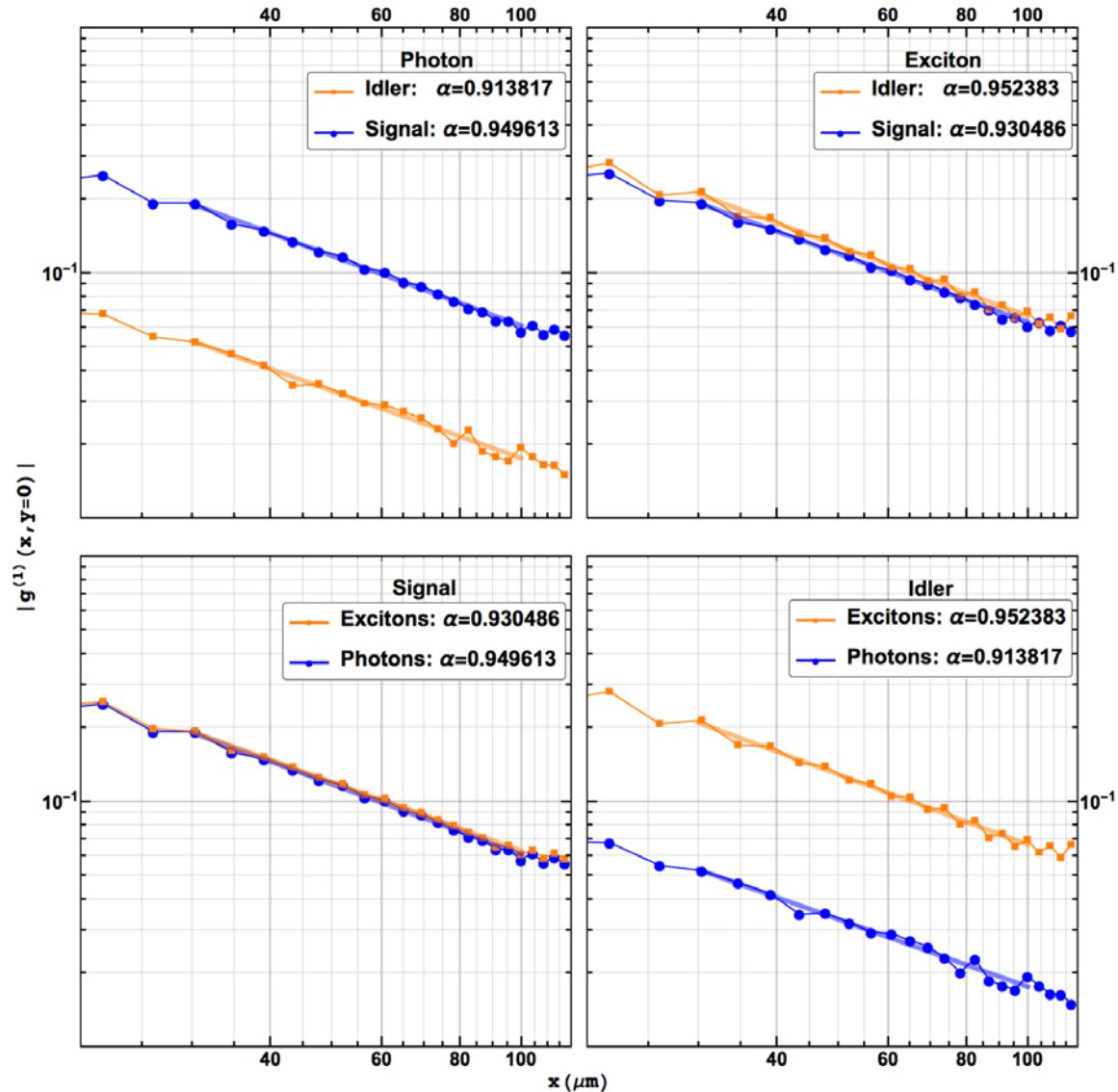
Four coupled fields:
excitons, photons, signal, idler



Phase transition at the same point for all fields

despite much lower density for photonic idler

Spatial Coherence for All Components



The same exponent of the power-law decay of $g_1(r)$

despite much lower density for photonic idler

Spinor Case and Fractional Vortices

- ✧ Polariton system is spinor: excitons +1 and -1, two polarisations of light

TE-TM splitting

$$i\frac{\partial\psi_{\pm}}{\partial t} = \left(-\frac{\nabla^2}{2m_{\phi}} - i\kappa_C\right)\psi_{\pm} + \frac{\Omega_R}{2}\phi_{\pm} +$$

$$\chi\left(\frac{\partial}{\partial x} \mp i\frac{\partial}{\partial y}\right)^2\psi_{\mp} + \frac{1}{2}\chi_0\psi_{\mp} + F_{\pm} + \sqrt{\kappa_C}dW_{C,\pm}$$

$$i\frac{\partial\phi_{\pm}}{\partial t} = \left(-\frac{\nabla^2}{2m_{\psi}} - i\kappa_X\right)\phi_{\pm} + \frac{\Omega_R}{2}\psi_{\pm}$$

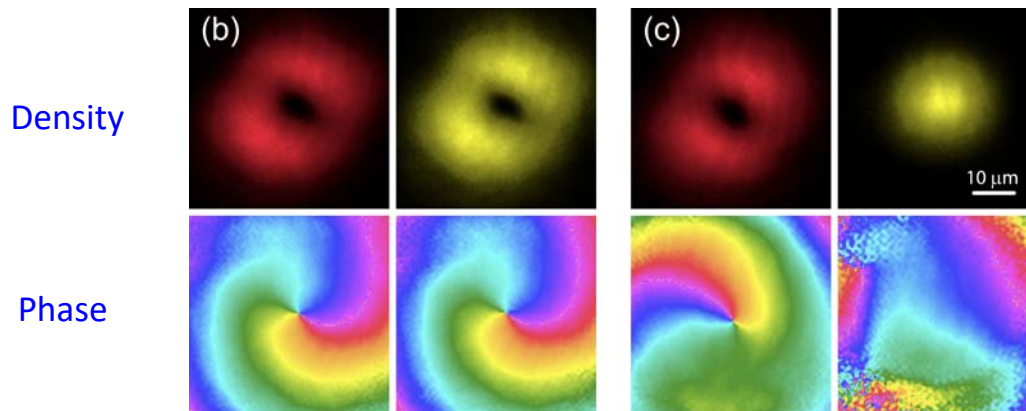
$$+\alpha_1|\phi_{\pm}|^2\phi_{\pm} - \alpha_2|\phi_{\mp}|^2\phi_{\pm} + \sqrt{\kappa_X}dW_{X,\pm}$$

Anisotropy splitting

Interactions between different spins

- ✧ Full vortices and fractional vortices present

Vortex in both polarisations Vortex in one polarisation



“Full” vortex

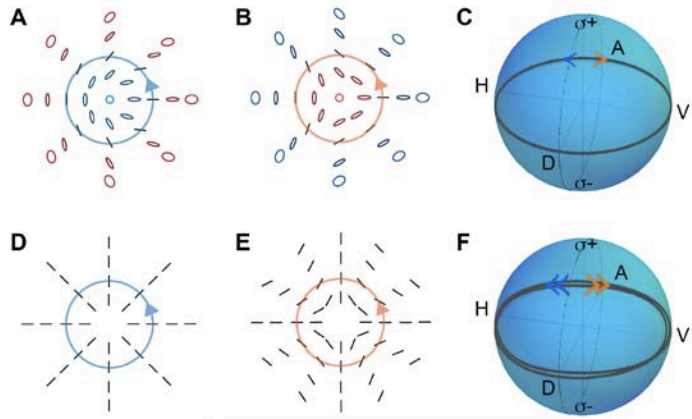
“Half” vortex

Skyrmions and Spin Vortices

[Donati et al. to appear in PNAS]

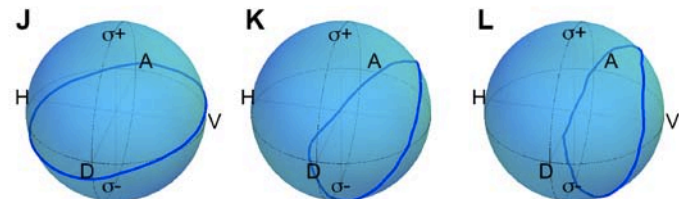
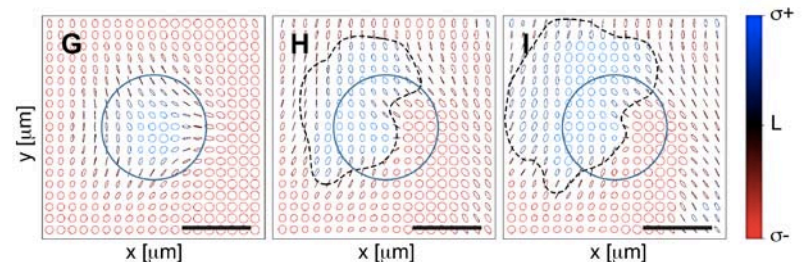
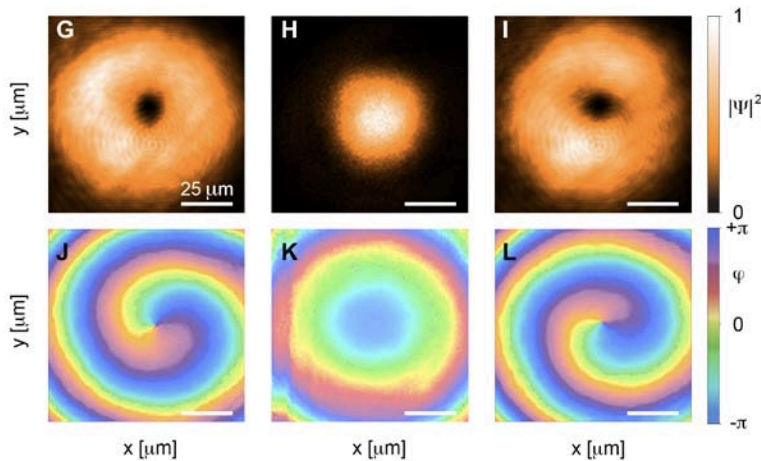
Spin and orbital angular momentum
are both quantized and mixed

σ^-	LG ₀₊₁	LG ₀₀	LG ₀₋₁
σ^+	Phase	Lemon	Hed
LG ₀₊₁	Star	Gaussian	Lemon
LG ₀₀	Hyp	Star	Phase



Polarization anisotropy leads to complex dynamics

$$i\frac{\partial\psi_{\pm}}{\partial t} = \left(-\frac{\nabla^2}{2m_{\phi}} - i\kappa_C\right)\psi_{\pm} + \frac{\Omega_R}{2}\phi_{\pm} + \chi\left(\frac{\partial}{\partial x} \mp i\frac{\partial}{\partial y}\right)^2\psi_{\mp} + \frac{1}{2}\chi_0\psi_{\mp} + F_{\pm}$$



Keldysh Action for OPO

Projected into LP sub-space and limited to three modes

$$\psi_{lp}(\mathbf{r}, t) = \psi_s e^{i(\mathbf{k}_s \mathbf{r} - \omega_s t)} + \psi_i e^{i(\mathbf{k}_i \mathbf{r} - \omega_i t)} + \psi_p e^{i(\mathbf{k}_p \mathbf{r} - \omega_p t)}$$

Keldysh Action

$$S_C = \int dt d^2 \mathbf{r} \left\{ -(F_p^* \psi_p^Q + F_p \bar{\psi}_p^Q) + \sum_{j=s,p,i} \left[\frac{1}{X_j^2} (\bar{\psi}_j^C \bar{\psi}_j^Q) \begin{pmatrix} 0 & [D_0^A]_j^{-1} \\ [D_0^R]_j^{-1} & [D_0^{-1}]_j^K \end{pmatrix} \begin{pmatrix} \psi_j^C \\ \psi_j^Q \end{pmatrix} - g_x \left((2(|\psi_s^C|^2 + |\psi_i^C|^2 + |\psi_p^C|^2) - |\psi_j^C|^2) \psi_j^C \bar{\psi}_j^Q + \text{c.c.} \right) - g_x \left(2\psi_s^C \psi_i^C \bar{\psi}_p^C \bar{\psi}_p^Q + (\psi_p^C)^2 (\bar{\psi}_i^C \bar{\psi}_s^Q + \bar{\psi}_s^C \bar{\psi}_i^Q) + \text{c.c.} \right) \right] \right\},$$

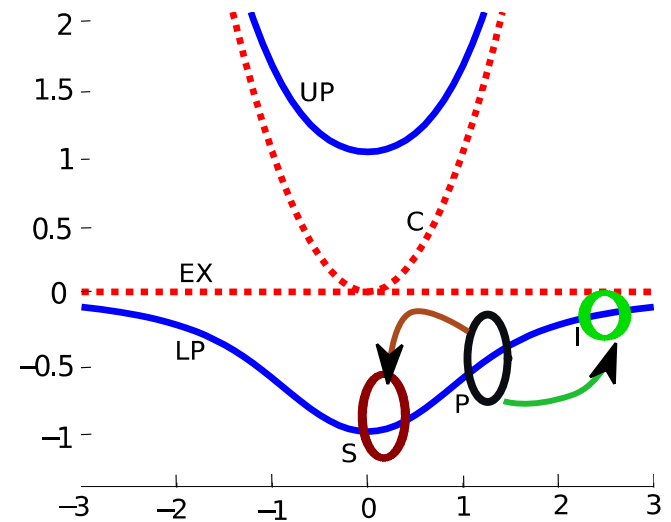
$$F_p(\mathbf{r}, t) = \mathcal{F}_p(\mathbf{r}) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

Polariton dispersion: non-quadratic and **anisotropic**

$$[D_0^A]_j^{-1} = i\partial_t - \omega_{lp}(\mathbf{k}_j - i\nabla) - i\gamma_j$$

$$\omega_{lp}(\mathbf{q}) = \frac{1}{2} \left(q^2 + \delta_{CX} - \sqrt{(q^2 + \delta_{CX})^2 + 4} \right)$$

$$\delta_{CX} \equiv \omega_C(0) - \omega_X$$



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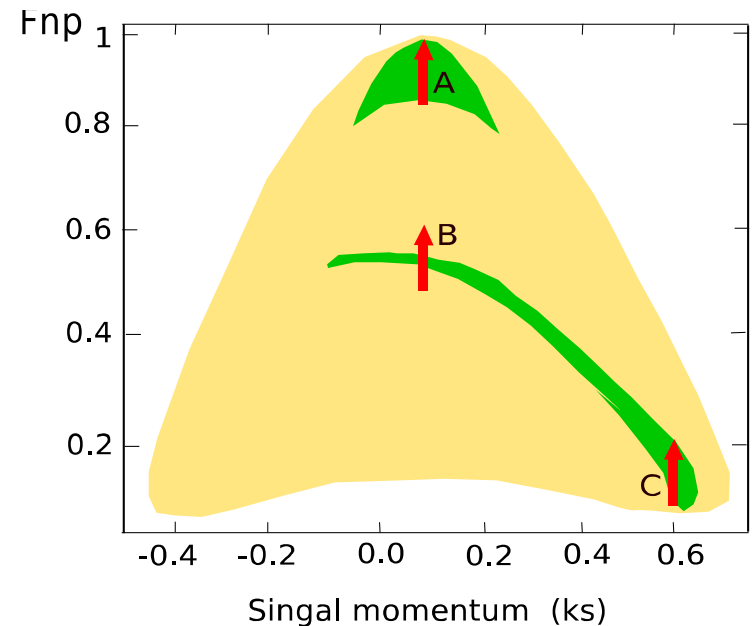
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$$\delta_{CX} \equiv \omega_C(0) - \omega_X$$



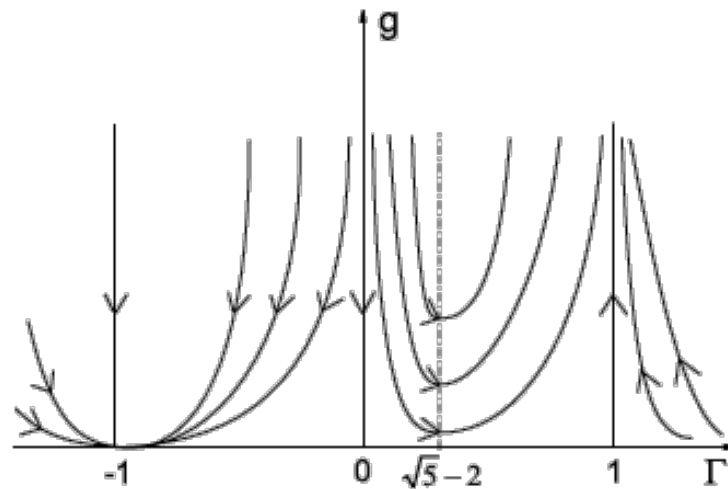
KPZ for OPO – Drift Term

$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + B_x \partial_x \theta + B_y \partial_y \theta + \bar{\xi}$$

[A. Zamora, L. Sieberer, S. Diehl, et al. in preparation]

The drift term can be eliminated with Galilean boost.

In the moving frame standard KPZ with RG flow:



$$g \equiv \lambda_x^2 \Delta / (D_x^2 \sqrt{D_x D_y})$$

$$\Gamma = \frac{\lambda_y D_x}{\lambda_x D_y}$$

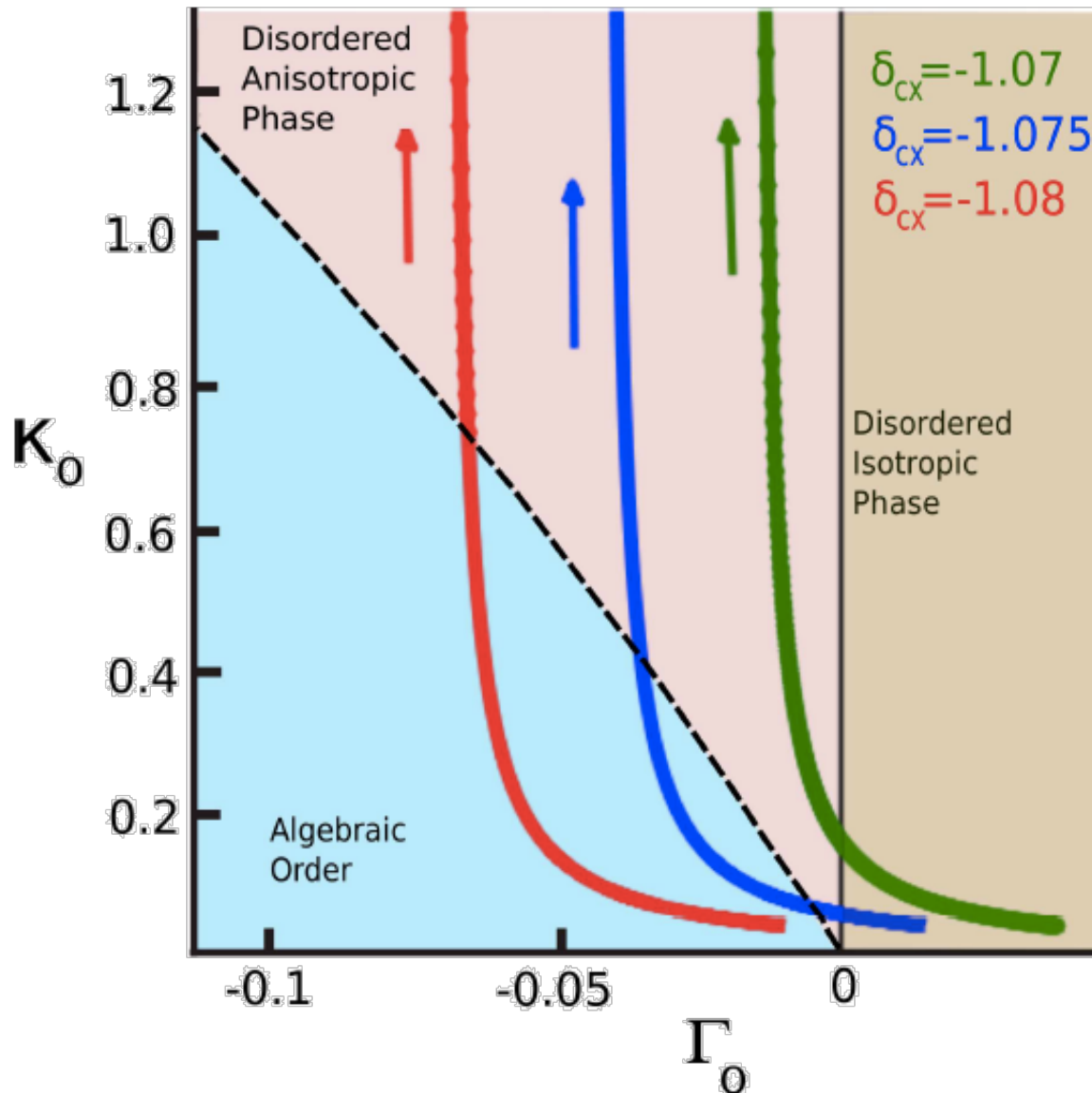
There is some influence on $g_1(t) = \exp[-C/2]$

$$C(0, t) \rightarrow (Bt)^{2\chi} F\left(\frac{t}{(Bt)^{z'}}\right) \rightarrow \begin{cases} t^{2\chi/z'} & \text{if } t \ll B^{z'}/(1-z') \\ t^{2\chi} & \text{if } t \gg B^{z'}/(1-z') \end{cases}$$

Faster temporal decay at long times

Infinite System – Driving Across Universalities

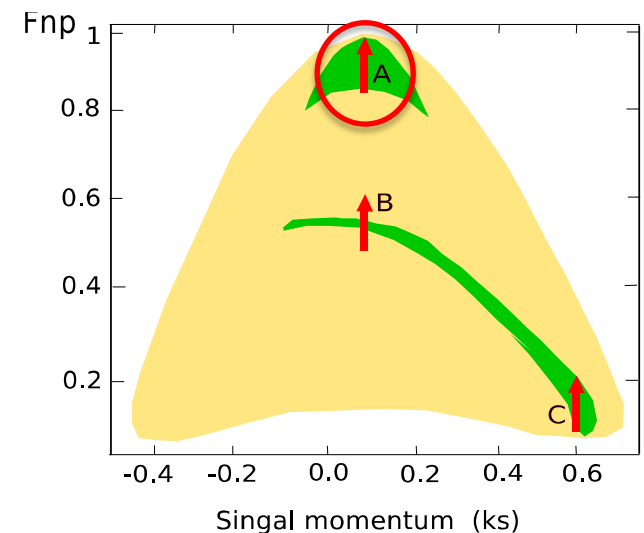
[A. Zamora, L. Sieberer, S. Diehl, et al. in preparation]



Negative detuning

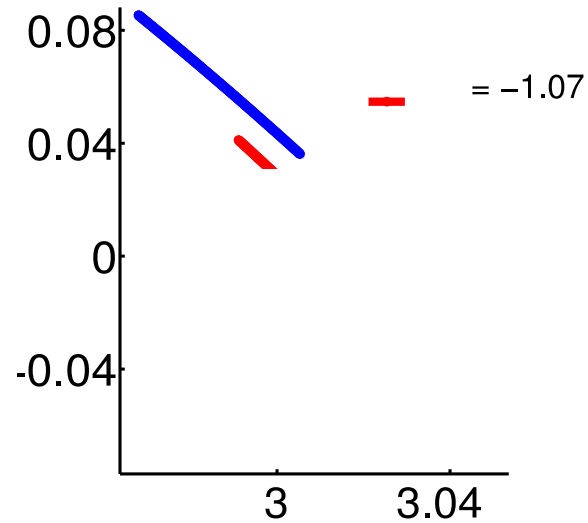
By increasing drive we move from **non-equilibrium** to **equilibrium** fixed point

Two different universality classes as the drive is increased

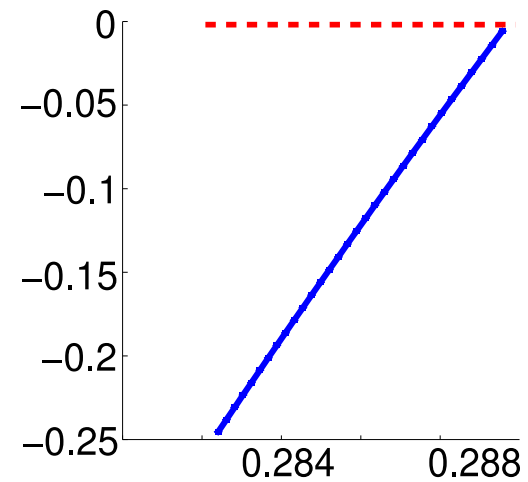


How to Achieve Large Anisotropy?

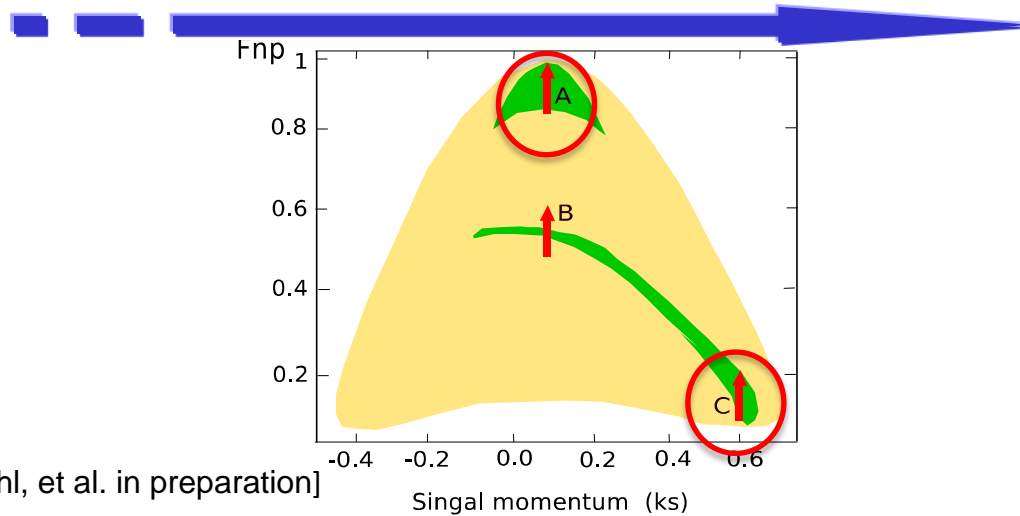
Small k_s but Increasing detuning



At zero detuning by increasing k_s

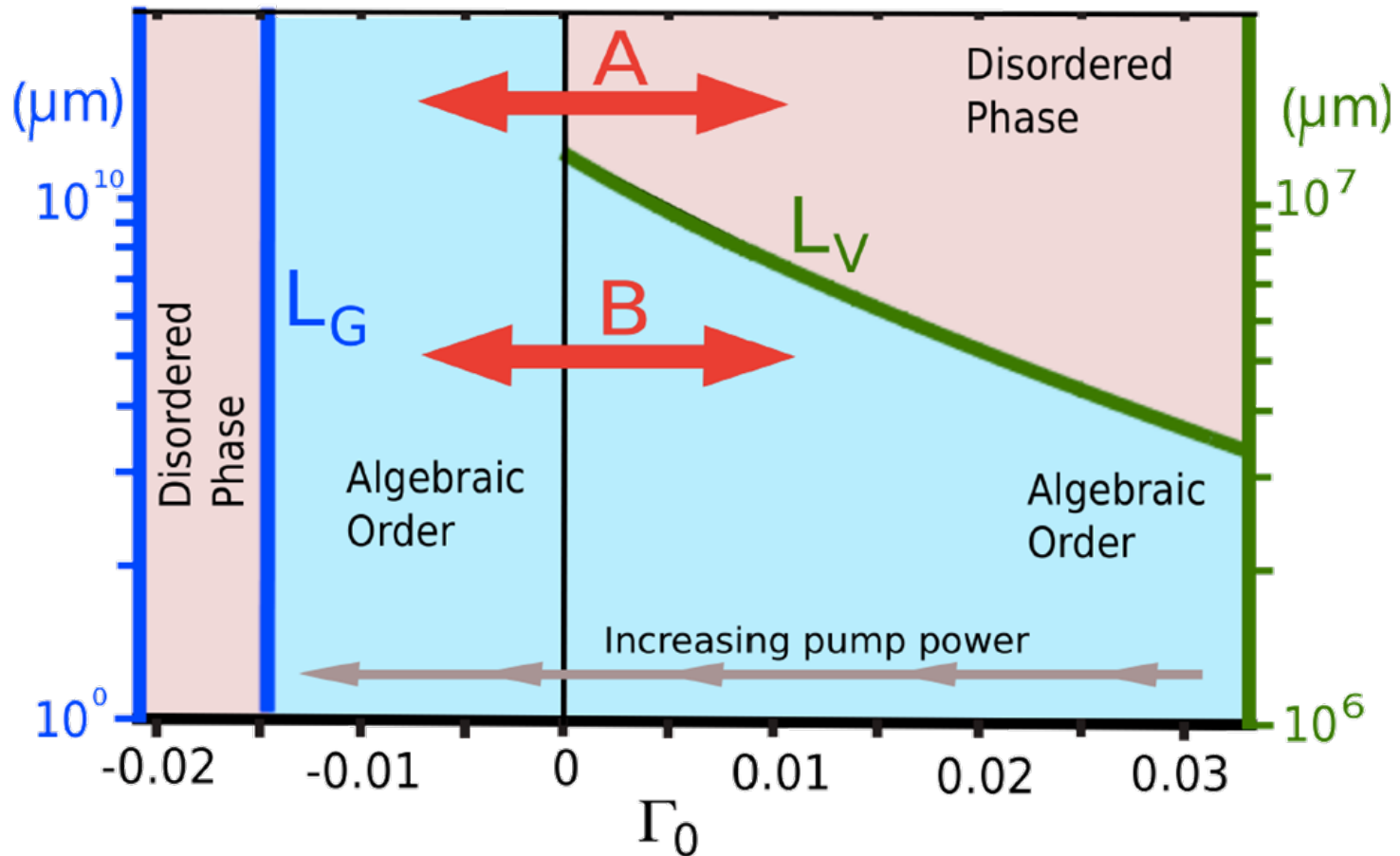


Increasing pump power



Driving Across Universalities in a Finite System

[A. Zamora, L. Sieberer, S. Diehl, et al. in preparation]

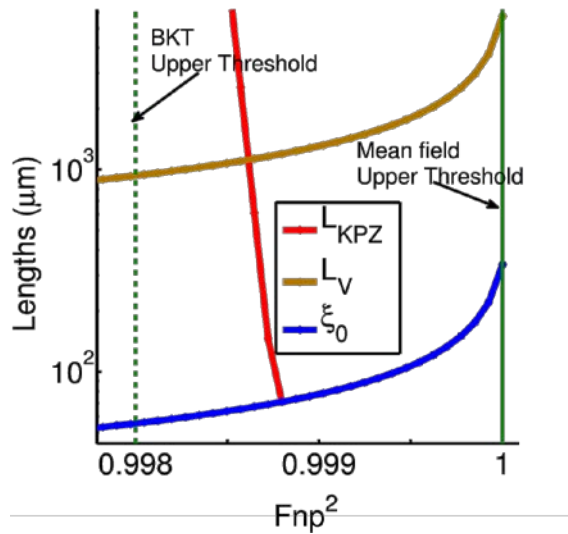


Equilibrium physics dominates: transition between algebraic order and exponential decay of coherence (disorder) BKT-like in systems up to meters

Worst quality samples needed for the transition A

Searching for the KPZ Phase

[A. Zamora, L. Sieberer, S. Diehl, et al. in preparation]



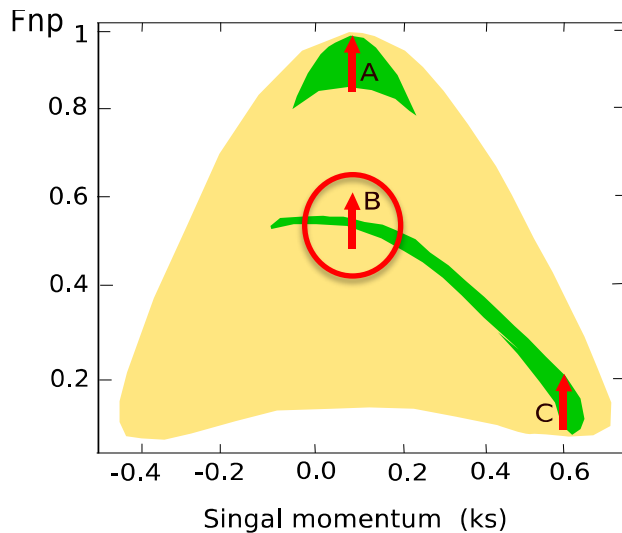
Away from threshold:

L_{KPZ} astronomical

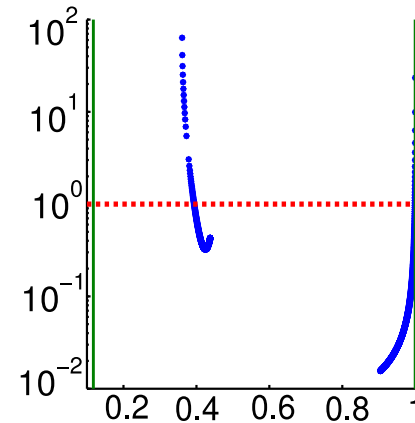
Very close to threshold

L_{KPZ} reasonable and $L_{\text{KPZ}} < L_V$ only extremely close to threshold i.e. below BKT transition

Note: analytics not valid in this regime



Middle of the OPO regime

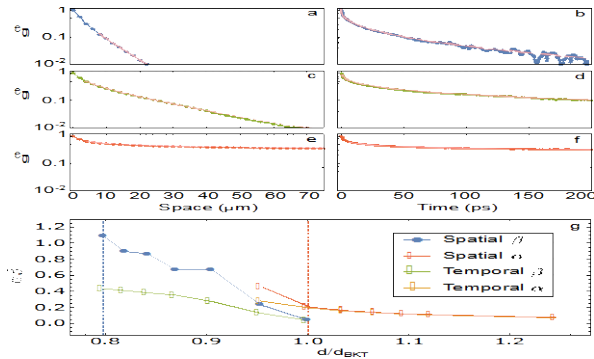


Large g, even > 1

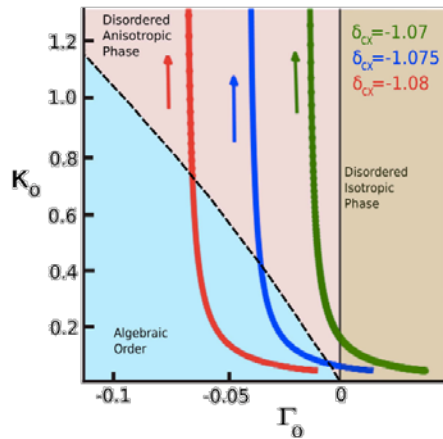
KPZ at all length-scales?

Conclusions

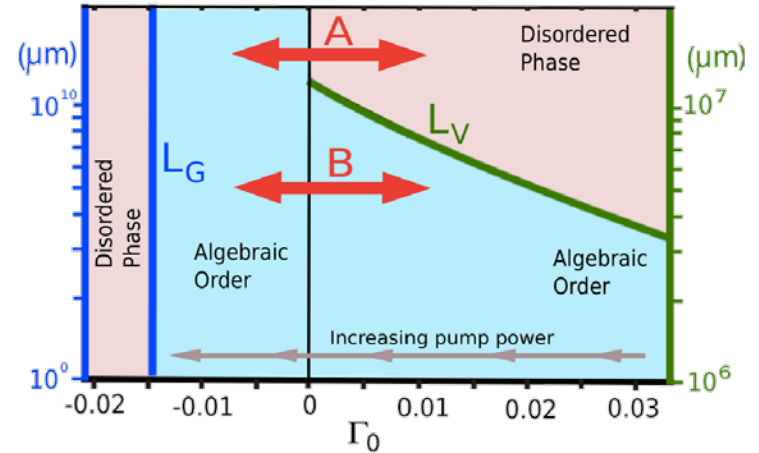
- Best microcavities indistinguishable from closed systems in equilibrium



- Anisotropy and dissipation – as in OPO – different phases possible



- Current microcavities “too good” for their size: equilibrium-like physics dominates



- But OPO shows a regime of strong KPZ non-linearity i.e. KPZ order at all length-scales?

