Out of Equilibrium Analogues of Symmetry Protected Topological Phases of Matter

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Quantum Phases of matter

- In equilibrium context, quantum phases are characterised by the long distance correlation behaviour of their low-lying states. Correlation behaviour changes sharply across phase boundaries. e.g., paramagnets have short range correlations, while ferromagnets exhibit long range order.

- With MBL a new idea appears. Notion of ‘eigenstate phase’. All of the eigenstates of a many body Hamiltonian have a common quantum order in an eigenstate phase (e.g., long range order, string order, edge modes).

- Eigenstate phases have dynamical signatures. (Huse et al ’13; see also Pekker et al, Vosk et al, Chandran et al ’14, Bahri et al ’15)

Phases of matter

Both the equilibrium and MBL problems involve characterising the eigenstates of local time independent Hamiltonians.

What new phenomena/phases can arise for driven systems i.e., systems with time varying Hamiltonians? Is there a notion of 'eigenstate phase' in the driven context? What are its dynamical signatures?

!?!

Floquet systems in brief

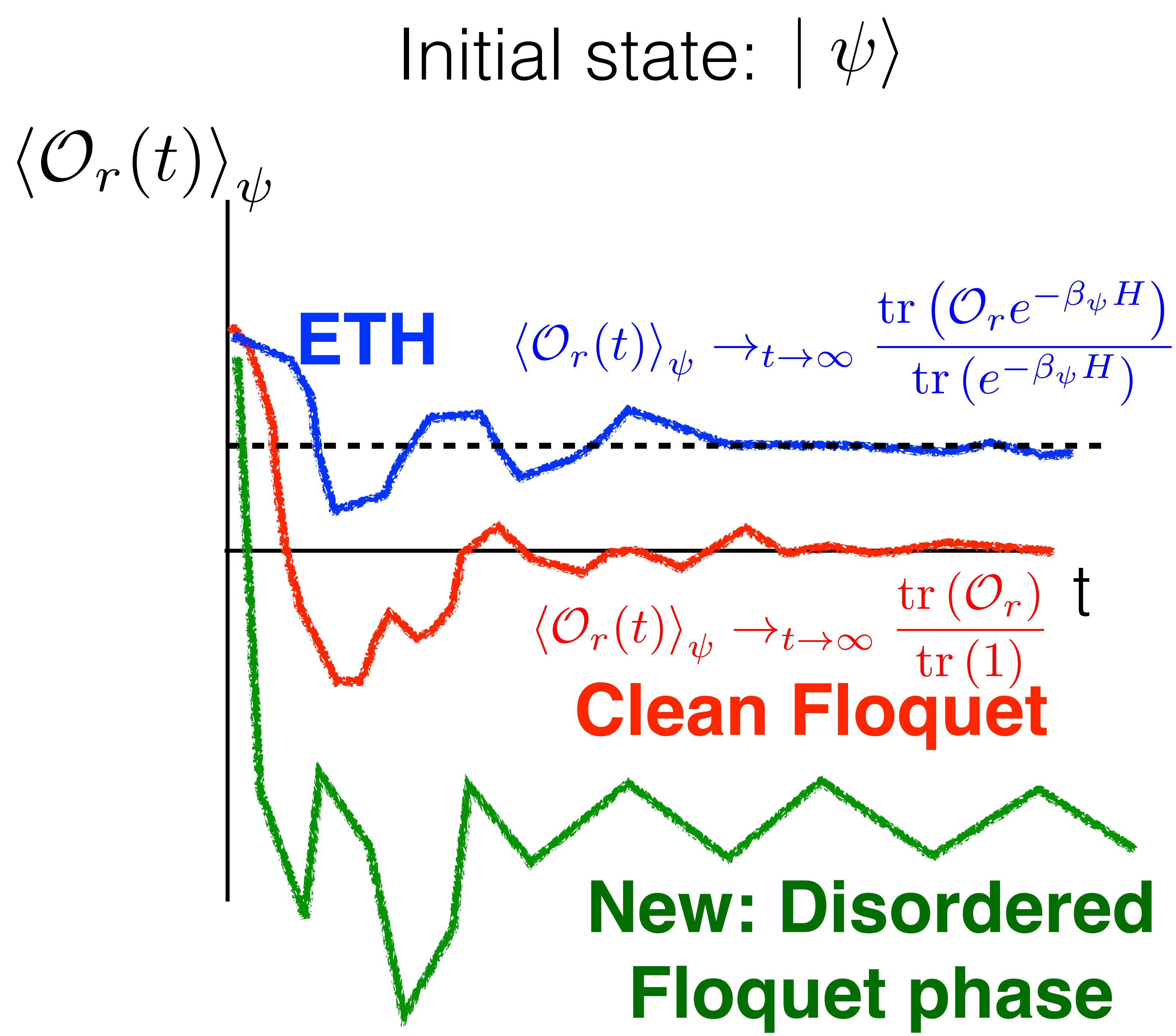
- Floquet systems: Time periodic Hamiltonians H(t)=H(t+T).
- With sufficient disorder in H(t), there is a sharp notion of phase of matter for periodically driven systems (Khemani et al 2015).
- 'Floquet phase' is characterised by the long time behaviour of local observables. This in turn is determined by the eigenstate properties of

$$U_f = \mathcal{T} e^{-i \int_0^T H(t') dt'}$$

- Disorder necessary…

Why introduce disorder?

- **Clean interacting** Floquet systems generically thermalise to `infinite temperature'. Local observables exhibit trivial dynamics at long times. (Lazarides & Moessner '15, Abanin et al. '14,'15, D'Alessio & Rigol '15)
- In **disordered interacting** Floquet systems, this `trivial' long time behaviour can be avoided e.g., observables can show persistent, large, **universal** oscillations at long times.



Today

- There is a sharp definition of 'phase of matter' for periodically driven disordered systems encoded in the long time behaviour of local observables.

- **We present a whole new class of disordered Floquet interacting SPT quantum phases in 1D and a classification. Many of the new phases can only arise in driven systems.**

- Similarly we have classification of disordered symmetry broken Floquet phases in all dimensions (these are time crystals — won't discuss these today). (von Keyserlingk + Sondhi PRB **93**, 245146 (2016))

- These SPT phases have clear experimental signatures — can be probed by modifying existing techniques for identifying MBL ordered states (e.g., Vosk & Altman '14, Bahri et al. '15).

1D SPT classification

- **INPUT:** Global symmetry group G (finite+Abelian). Symmetric 1D time periodic Hamiltonians H(t)=H(t+T).

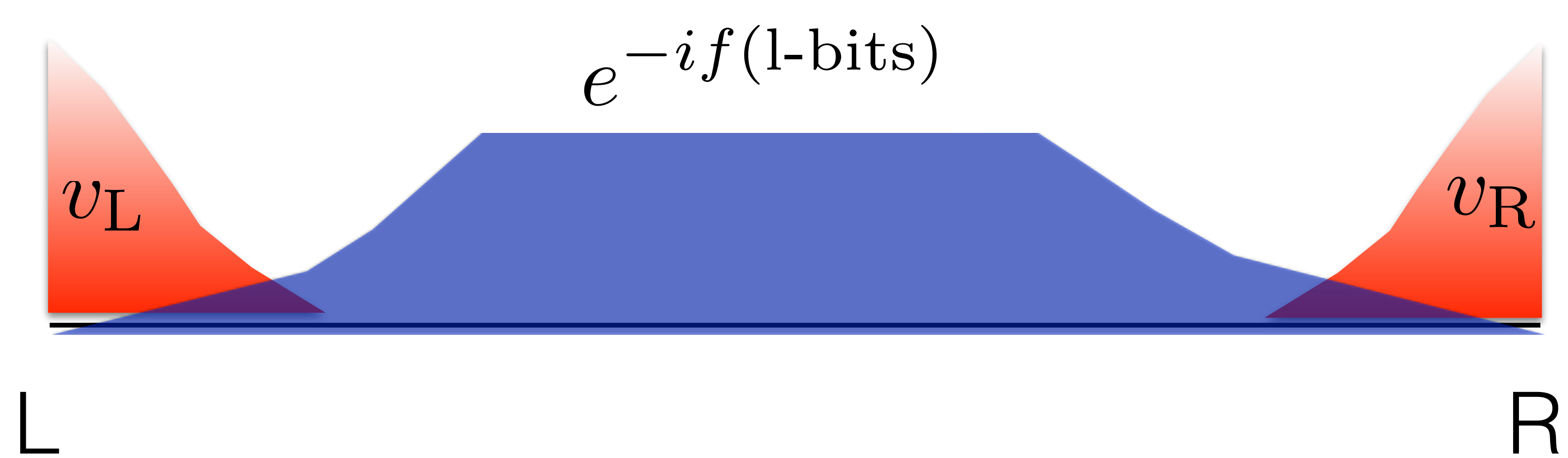
$$U_f = \mathcal{T} e^{-i \int_0^T H(t')dt'}$$

- We are interested in H(t) sufficiently disordered such that U_f has a (complete) set of bulk local conserved quantities (l-bits).

- Classification asks: What properties of U_f are robust to sufficiently small (symmetric) perturbations to U_f in the bulk and arbitrary (symmetric) modifications at the edge.

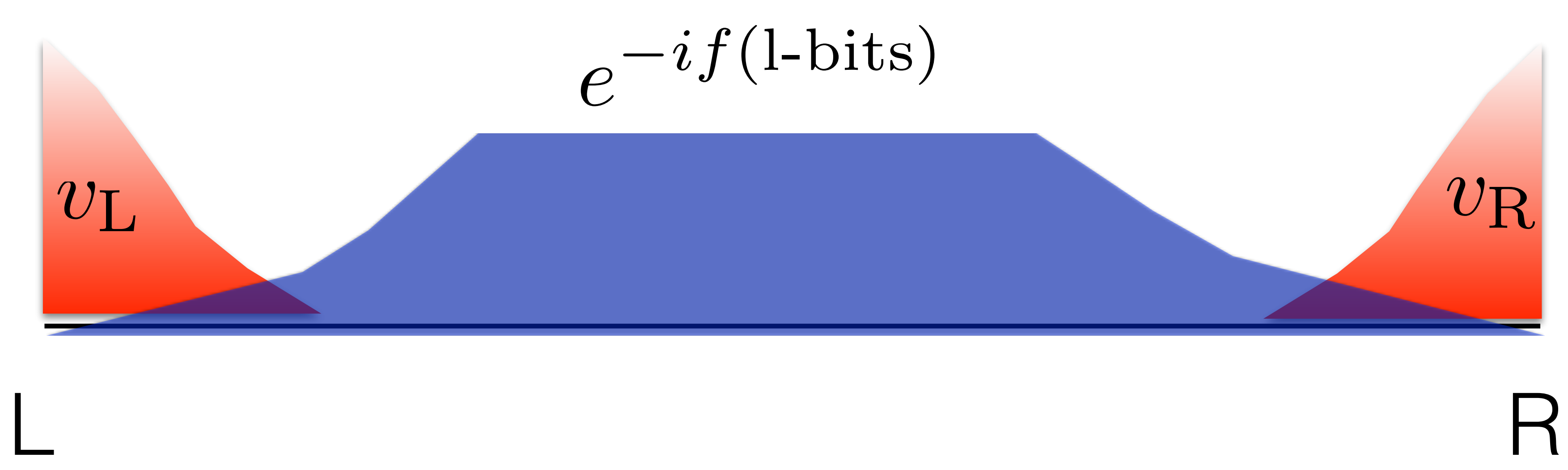
Structure of MBL drives on systems with edges

- We argue that U_f takes form $U_f = e^{-i\,f}\, v_L\, v_R$ on a system with boundary, where:
  - f is a local functional of bulk l-bits — i.e., a standard local MBL-SPT Hamiltonian.
  - v_L, v_R are unitaries localised near the Left,Right edges of the system respectively, commuting with f.



Output

- **OUTPUT:** Classification looks like Cl(G) x A(G) where
  - **Cl(G) = undriven interacting SPT classification encoded by f.**
  - **A(G) = (twisted) 1D representation of G characterising the symmetry properties $v_{L,R}$. We call this the pumped charge.**



Structure of MBL drives

- Pumped charge is labelled by A(G) — roughly the set of 1D representations of G. For unitary symmetry groups, this is just the commutation relation between $v_{L,R}$ and the global symmetry.

- Roughly speaking, all floquet SPT drives can be decomposed into a canonical form: 'evolve with SPT'+`pump charge from one edge to the other'

$$U_f = \underbrace{v_L v_R}_{\text{pump charge}} \times \underbrace{e^{-iH_{\text{mbl,SPT}}}}_{\text{evolve with MBL-SPT}}$$

Example: ‘Class D’

- **Undriven setting:** 1D superconductor. Fermion parity symmetry — no further symmetries. Two phases, exemplified by the Kitaev and trivial states. Classification is $\mathrm{Cl}=\mathbb{Z}_2$.

- **Floquet setting:** the number of phases doubles. There is a: $\mathrm{Cl}\times A(G)=\mathbb{Z}_2\times\mathbb{Z}_2$ classification. $A(G) = \mathbb{Z}_2$.

- Agrees with noninteracting results (Kitigawa et al `10, Jiang et al `11, Rudner et al `13, Rudner+Nathan `15).

Example: 'Class BDI'

- **Undriven setting:** 1D superconductor with fermion parity + time reversal. Eight phases — this is the celebrated $\mathrm{Cl} = \mathbb{Z}_8$ interacting classification.
- **Floquet setting:** The number of phases quadruples. We get a $\mathbb{Z}_8 \times \mathbb{Z}_4$ classification.
- New set of states.

Further SPT results

- If G finite abelian, and degrees of freedom bosonic then: Cl = $H^2(G)$, A(G) = $H^1(G)$. This gives a net classification $H^2(G) \times H^1(G)$.

- More generally our classification can be interpreted as showing that Floquet SPTs in 1D are classified by the projected representations of G extended by time translation group $\mathbb{Z}$. (Else+Nayak, Potter et al. `16).

One example, in more detail…

Ising SPT (G=$\mathbb{Z}_2$)

$$P \equiv \prod_r X_r$$

- **Undriven setting:** There is only one phase with unbroken P symmetry, i.e., Cl=$\mathbb{Z}_1$. An example hamiltonian in this phase

$$H = -\sum_r h_r X_r$$

- The corresponding paramagnetic phase has short range correlations and no interesting edge modes. In our Floquet classification, however, the number of phases doubles. There is a $\mathbb{Z}_2$= ClxA(G) classification. A(G) = $\mathbb{Z}_2$.

The non-trivial Floquet Ising SPT: **0π PM**

- Example: trivial Ising Floquet phase. An example on system with boundary

$$U_{\mathrm{f,triv}} = e^{-it_1 \sum_r h_r X_r}$$

- Example: Non-trivial phase (called **0π PM**) has a pumped charge. So it takes the form

$$U_{\mathrm{f},0\pi} = \underbrace{Z_1 Z_N}_{v_{\mathrm{L}} v_{\mathrm{R}}} e^{-it_1 \sum_{r=2}^{N-1} h_r X_r}$$

- where v_L and v_R are Ising odd and localised to the left and right edges of the system respectively. I will now engineer an Ising symmetric drive corresponding to this non-trivial unitary.

Ising Floquet SPT: **0π PM**

- Look at the piecewise constant hamiltonian (extended periodically with period T).

$$H_{0\pi}(t) = \begin{cases} \sum_r h_r X_r & 0 < t < t_1 \\ \sum_r Z_r Z_{r+1} & t_1 < t < T = t_1 + \frac{\pi}{2} \end{cases}$$

- On a system without boundary the Floquet unitary takes the form.

$$e^{-i\frac{\pi}{2}\sum_r Z_r Z_{r+1}} e^{-it_1 \sum_r h_r X_r} \propto e^{-it_1 \sum_r h_r X_r}$$

- On a system with boundary, the Floquet unitary takes the required form:

$$U_{\mathrm{f},0\pi} = Z_1 Z_N e^{-it_1 \sum_{r=2}^{N-1} h_r X_r}$$

Some algebra

$$
\begin{aligned}
e^{-\frac{i\pi}{2}\sum_{r=1}^{N-1} Z_r Z_{r+1}} &= \prod_{r=1}^{N-1} e^{-i\frac{\pi}{2} Z_r Z_{r+1}} \\
&\propto \prod_{r=1}^{N-1} Z_r Z_{r+1} \\
&= Z_1 Z_N
\end{aligned}
$$

Use $e^{\frac{i\pi}{2}\sigma} = i\sigma$ for $\sigma^2 = 1$.

0π PM is a symmetry protected phase

- Henceforth we will work with particular unitary $U_{f,0\pi}$ to illustrate some of the physics of the 0π phase. But the statements I will make apply to a whole phase, with only minor modifications.The essential physics of this model is robust to arbitrary local Ising symmetric perturbations, which obey the periodicity of the underlying Floquet drive:

$$H_{0\pi}(t) \rightarrow H_{0\pi,\lambda}(t) = H_{0\pi}(t) + \lambda V(t)$$

$$[P, V(t)] = 0 \;\; V(t+T) = V(t)$$

Experimental signatures

- Consider operator X_1. Note that it obeys an equation of motion:

$$X_1(nT) \equiv U_{\mathrm{f}}^{-n} X_1 U_{\mathrm{f}}^{n} = (-1)^n X_1$$

$$U_{\mathrm{f},0\pi} = Z_1 Z_N e^{-it_1 \sum_{r=2}^{N-1} h_r X_r}$$

- Hence, X_1 shows oscillations provided the initial state has some expectation value

$$\langle X_1(nT) \rangle_{\psi} = (-1)^n \langle X_1(0) \rangle_{\psi}$$

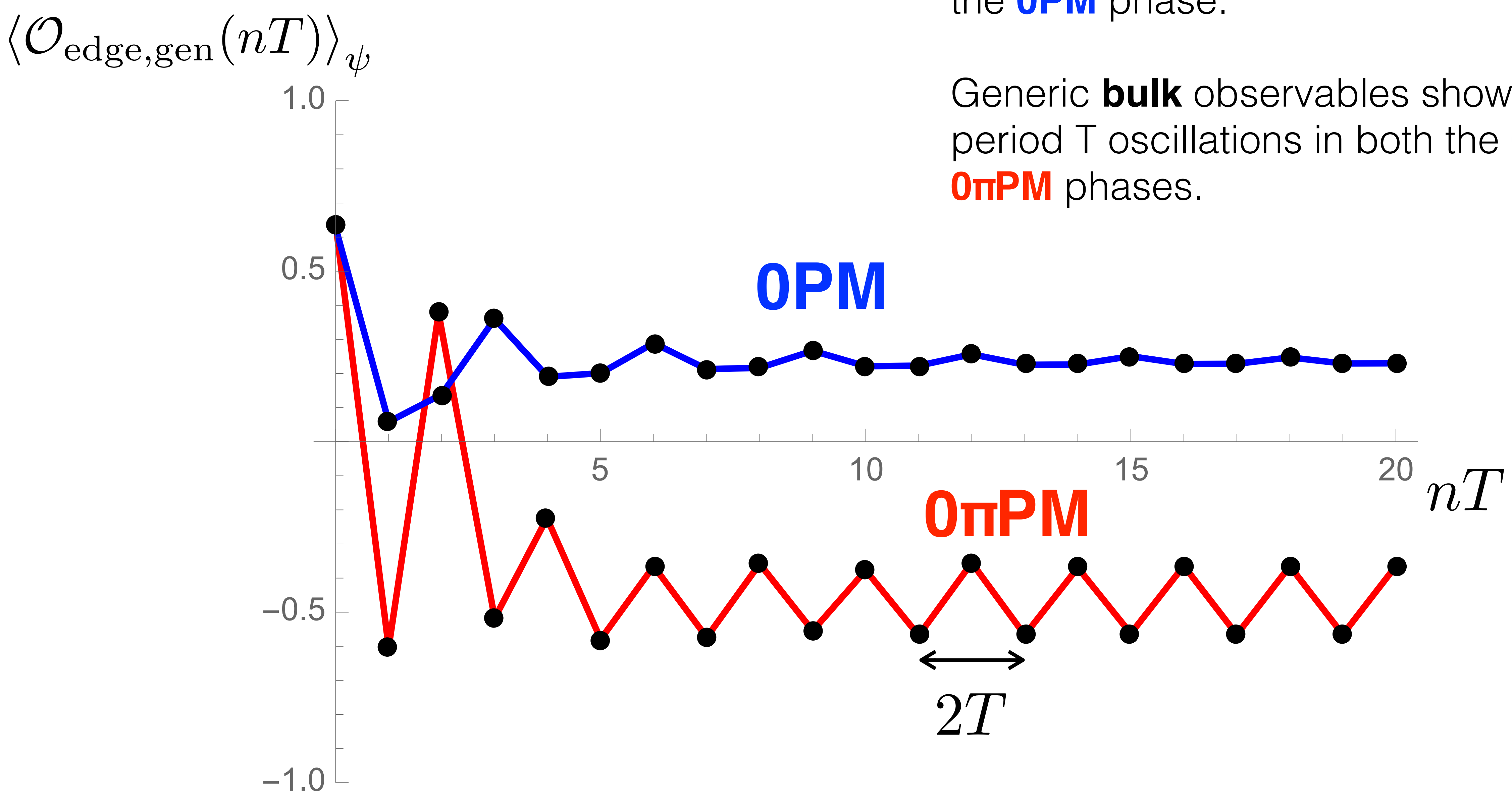
- This operator oscillations with period 2T.

See also 1610.07611 Potirniche et al.

Experimental signatures (cartoon)

Generic **edge** observables show persistent period 2T oscillations at late times in the **0πPM**. They show period T oscillations in the **0PM** phase.

Generic **bulk** observables show persistent period T oscillations in both the **0PM** and **0πPM** phases.



Conclusion

- Sharp definition of ‘phase of matter’ for periodically driven disordered systems.
- We present a whole new family of Floquet analogues of interacting SPT quantum phases in 1D, many of which can only be realised in the driven setting.
- Phases characterised by the long time behaviour of bulk and edge observables.

Thanks!

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Related work

- Time crystals: Khemani et al1508.03344, CvK & SLS 1602.06949, Else et al 1603.08001, CvK+VK+SLS:1605.00639, Yao et al 1608.02589. Monroe group and Lukin group experiments!
- SPTs: Else, Nayak: arXiv:1602.0480. Potter, Morimoto, Vishwanath, et al.: 1602.05194, 1610.07611. Roy, Harper: 1602.08089,1609.06303
- The new SPT phases are related to those seen in the clean non-interacting setting (see Nathan & Rudner '15, and references therein).
- Already some higher dimensional generalisations using very similar ideas. e.g., arXiv: 1609.00006 (Hoi Chun Po et al.)

Assume there is a local Ising symmetric unitary W_λ such that

$$U^{\lambda}_{\mathrm{f},0\pi} = O_{LR} e^{-if_{\mathrm{bulk}}(\{X_{r,\lambda}\})}$$
$$X_{r\lambda} = W_\lambda^{-1} X_\lambda W_\lambda$$

where O_{LR} is supported at the boundaries of the system. Existence of this W_λ is akin to supposing stability of the bulk paramagnetic order. Locality of $U^{\lambda}_{\mathrm{f},0\pi}$ and W_λ implies that O_{LR} factorises as $O_{LR} = v_L v_R$ in the thermodynamic limit, and that f is a local functional. The whole unitary is Ising even as is f, so $P v_L v_R P^{-1} = v_L v_R$. As P is itself a local circuit it follows that

$$\begin{aligned} P v_L P^{-1} &= e^{i\theta} v_L \\ e^{i\theta} &= \pm 1 \end{aligned}$$

Another expression for this phase in the thermodynamic limit

$$e^{i\theta} = \left[U^{\lambda}_{\mathrm{f},0\pi} : W_\lambda^{-1} \left(\prod_{1 \le r < s} X_r \right) W_\lambda \right] = \pm 1$$

When $\lambda = 0$ we get $e^{i\theta} = -1$. $U^{\lambda}_{\mathrm{f},0\pi}$ changes continuously with λ. If W_λ also changes continuously, then the phase stays fixed because -1 cannot change to 1 continuously. This argument breaks down when e.g., W_λ can no longer be chosen to be local.