

Model Many-body Systems Based on Quantum Dots

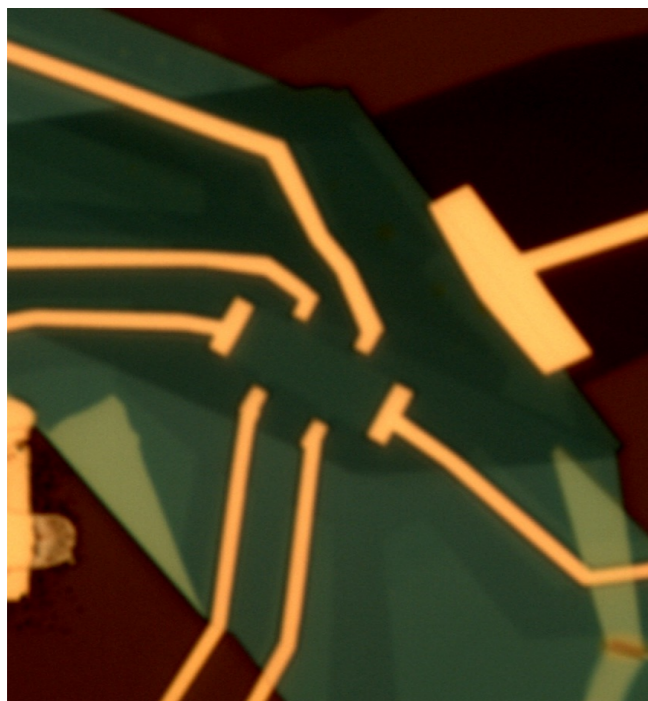
David Goldhaber-Gordon
Stanford University

KITP SYNQUANT Workshop
November 1, 2016

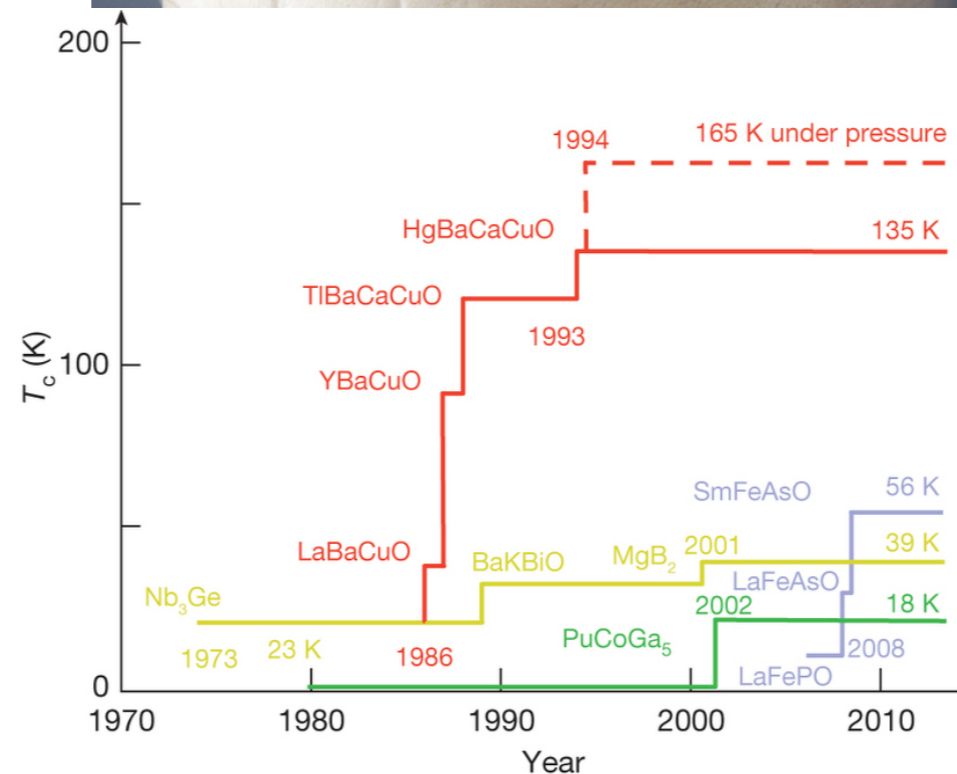
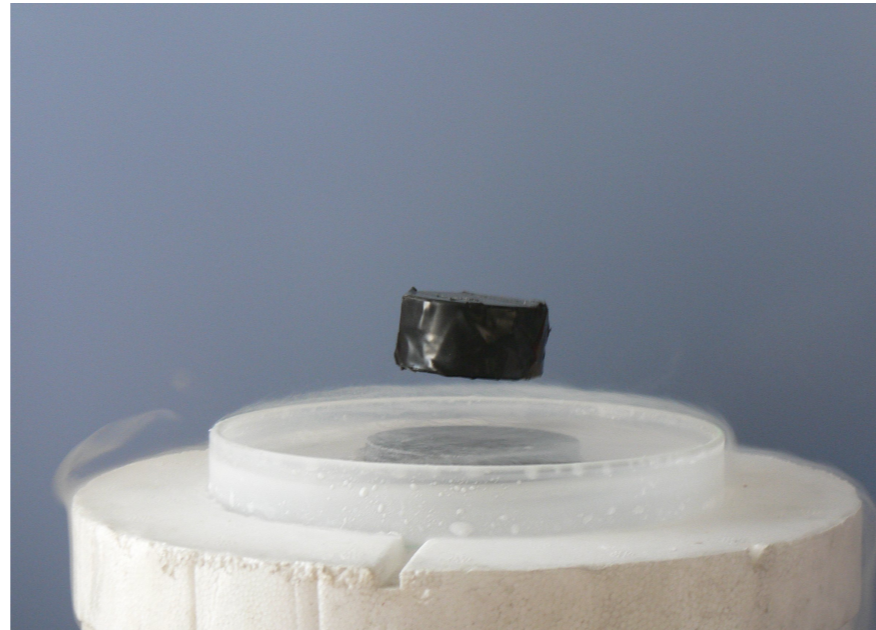
How to beat Ohm's Law

Three possible approaches.

Make the best conductor

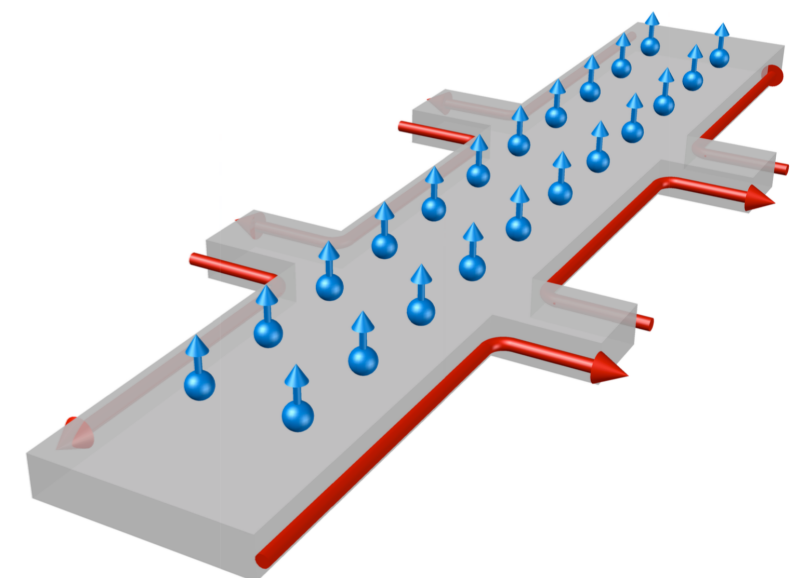


Make a superconductor



B. Keimer *et al.*, Nature, 2015

Set some rules



Outline

- **Symmetry and renormalization**
- Quantum dots and the Kondo effect
- Emergent $SU(4)$ -symmetric Kondo effect in a double quantum dot
- Universal crossover between quantum critical and Fermi liquid states in a quantum dot coupled to a “metallic grain”

Calculations in this talk by G. Zaránd, C. P. Moca, I. Weymann

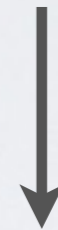
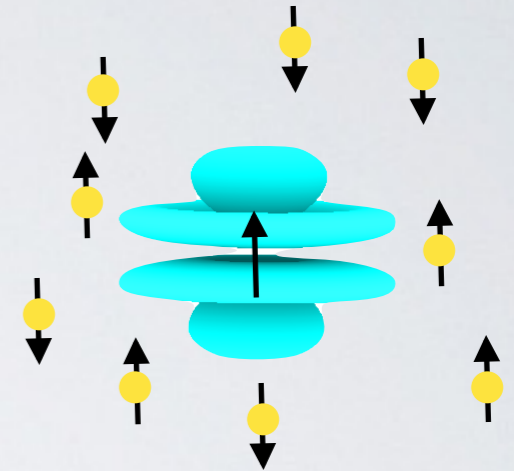
Outline

- Symmetries in physics
- **Quantum dots and the Kondo effect**
- Emergent $SU(4)$ -symmetric Kondo effect in a double quantum dot
- Universal crossover between quantum critical and Fermi liquid states in a quantum dot coupled to a “metallic grain”

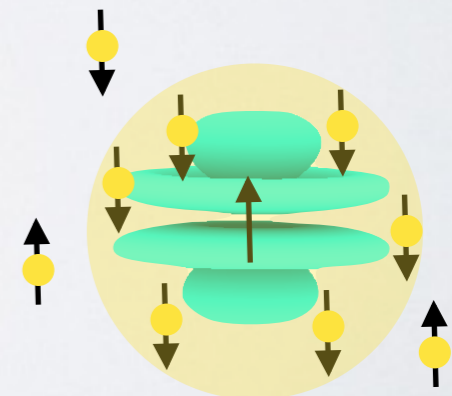
Kondo effect

- Itinerant electrons screen a localized moment
- Emergence of a dynamical energy scale, the Kondo temperature T_K
- Formation of a many-body singlet when $T \ll T_K$
- Describes low-energy physics of Anderson impurity model

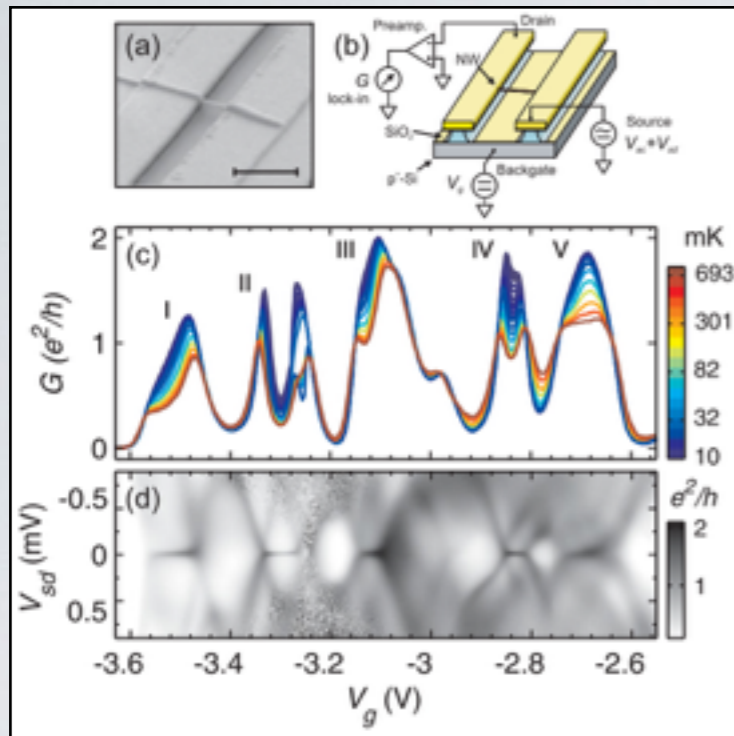
$T > T_K$



$T \ll T_K$

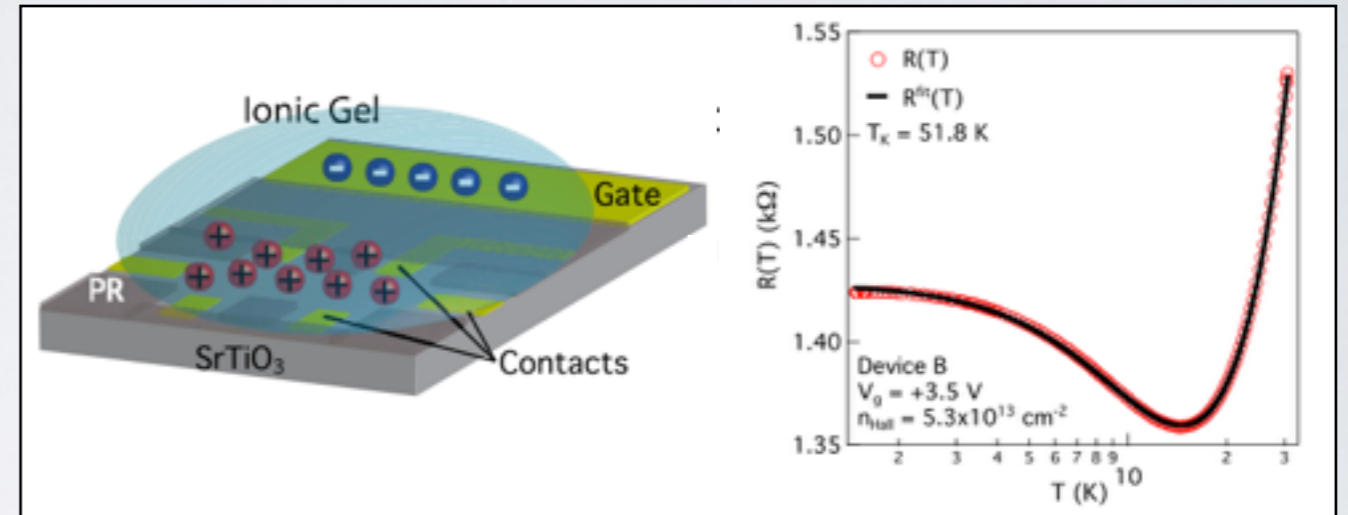


Kondo effect



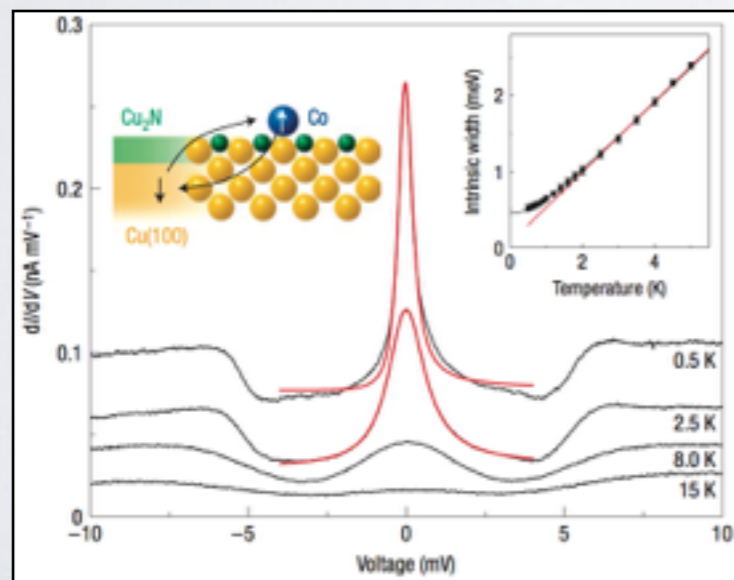
InAs nanowires

Kretinin, A.V., et al, *PRB* 84, 245316 (2011)



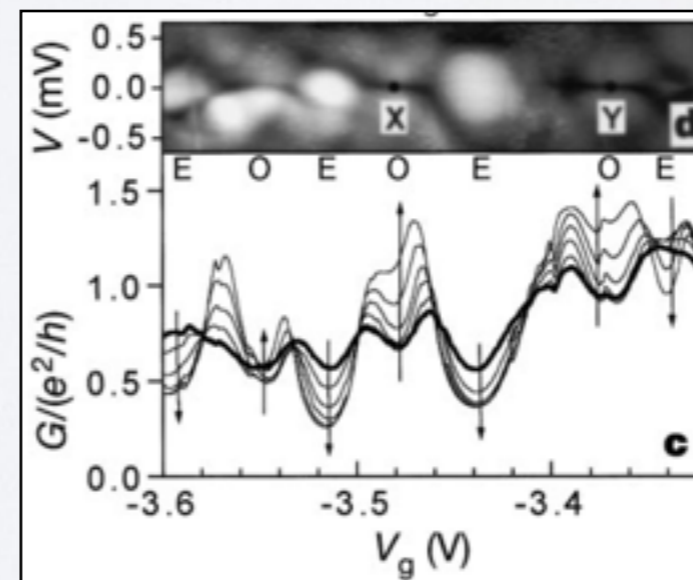
Electrolyte gate controlled SrTiO₃

Lee, M., et al, *PRL* 107, 256601 (2011)



Magnetic adatoms on surfaces

Otte, A. F., et al, *Nat. Phys.* 4 (2008)

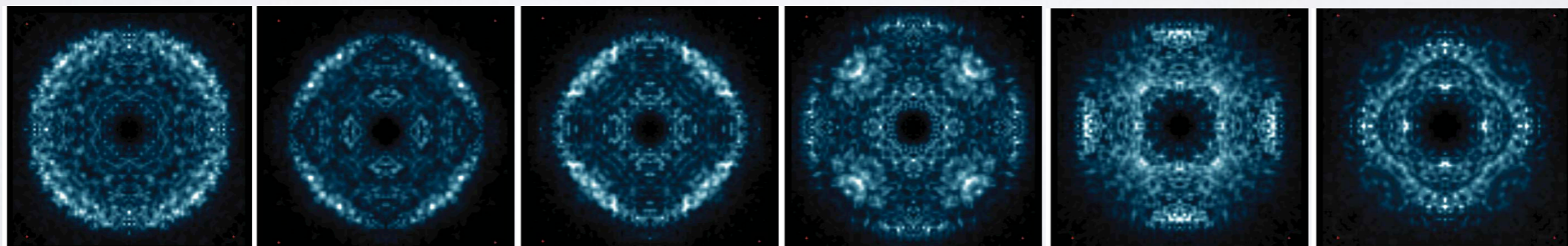
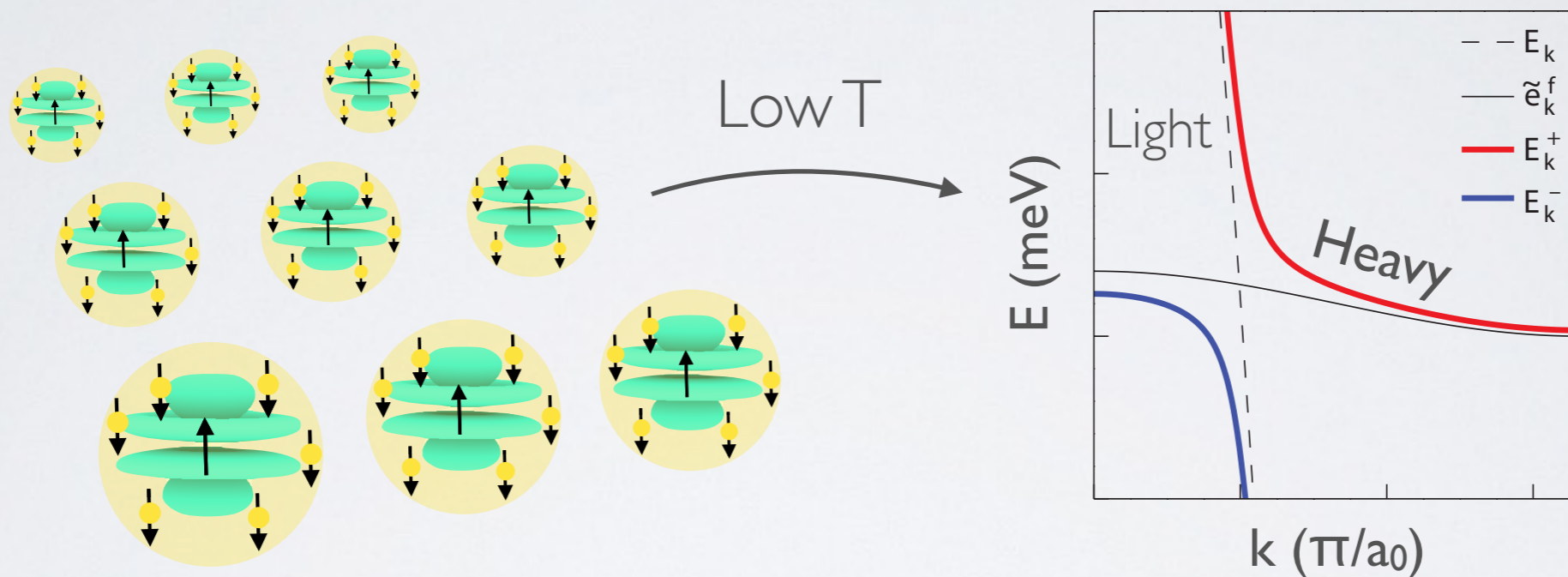


Carbon nanotubes

Nygård, J., et al, *Nature* 408 (2000)

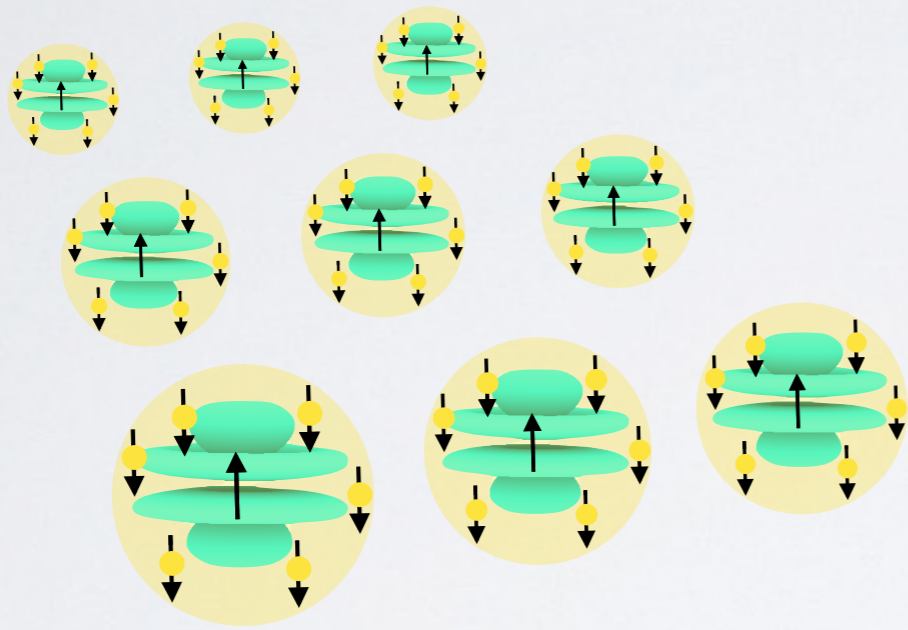
Adding more local spins

Heavy fermion systems



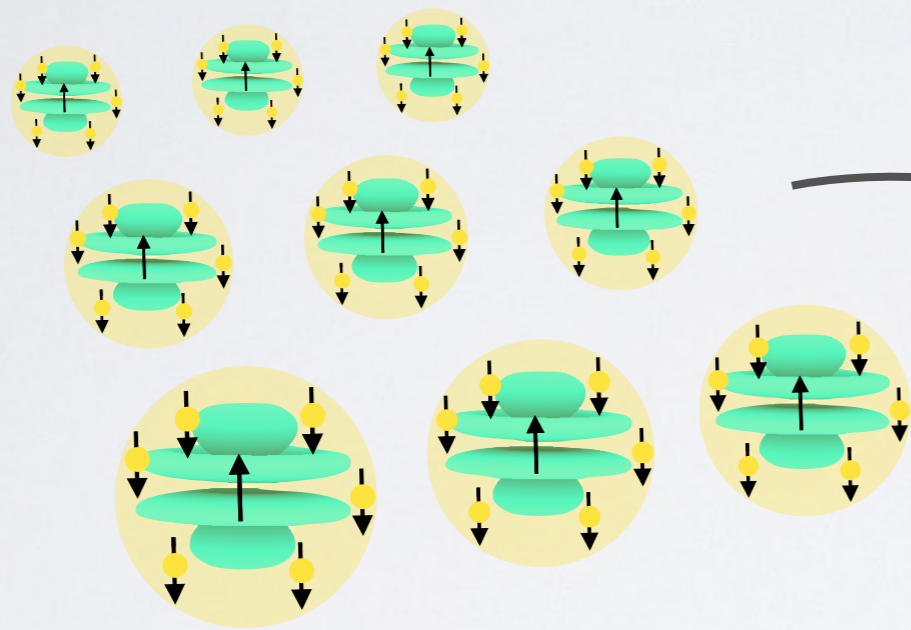
A. R. Schmidt, *et al.*, Nature 465, 570 (2010)

Adding more local spins



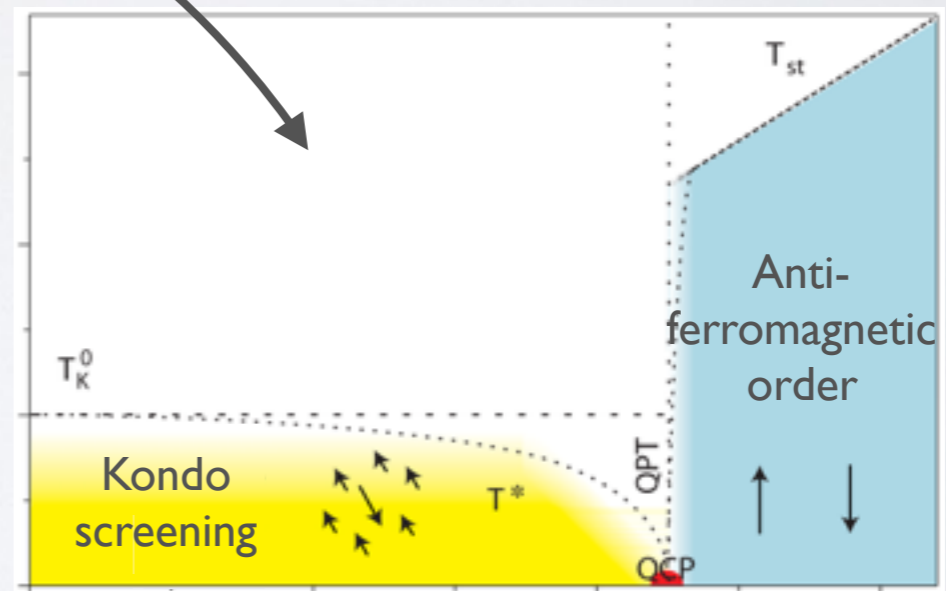
Adding more local spins

Quantum critical points?



+ RKKY interaction

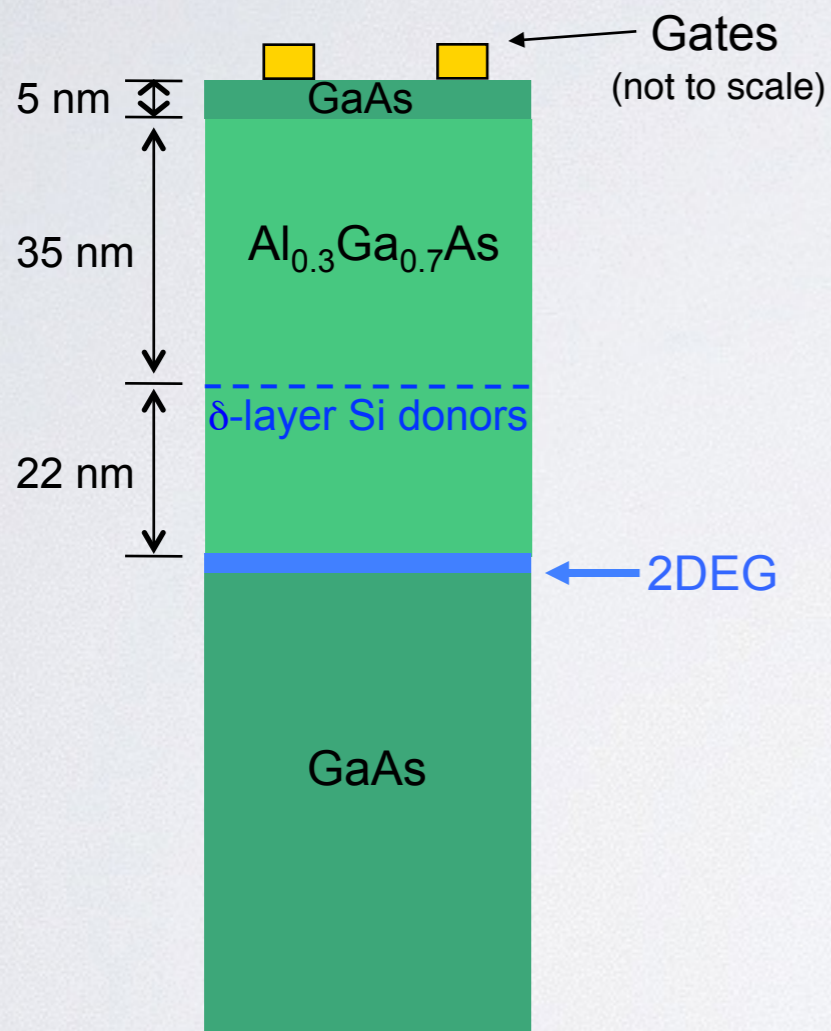
Energy



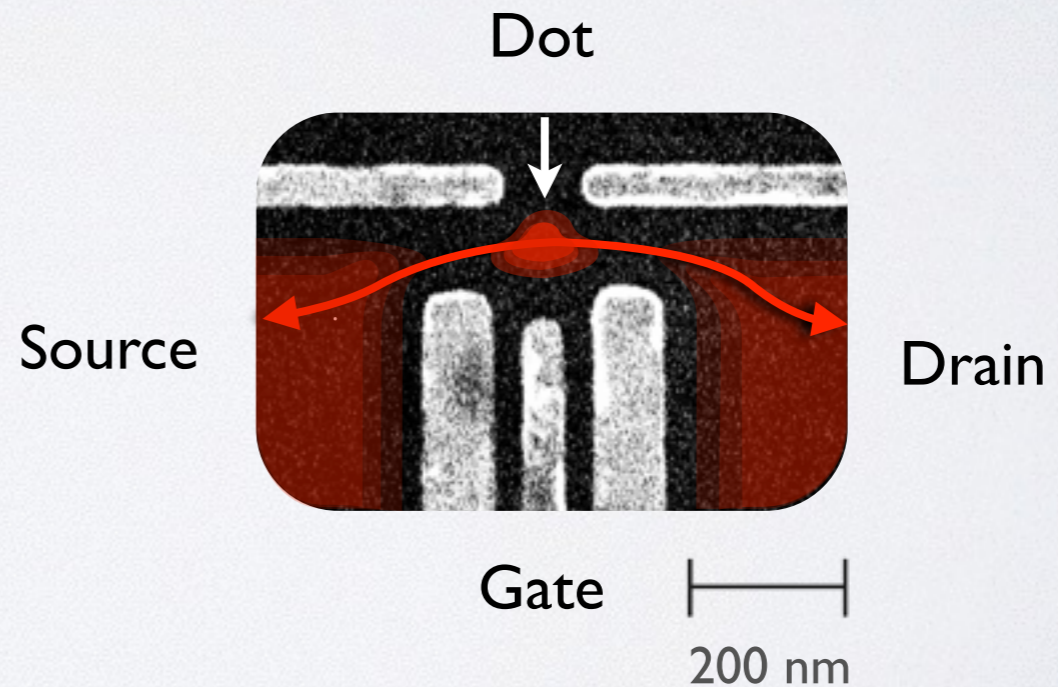
Interaction strength

Adapted from Bork, et al., Nature Physics 7, 11 (2011).

Quantum dots

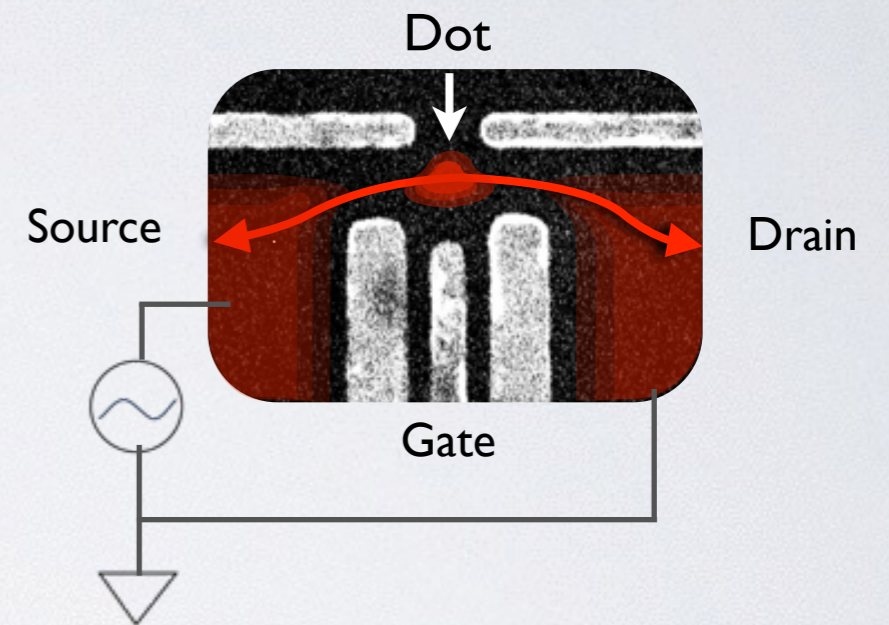
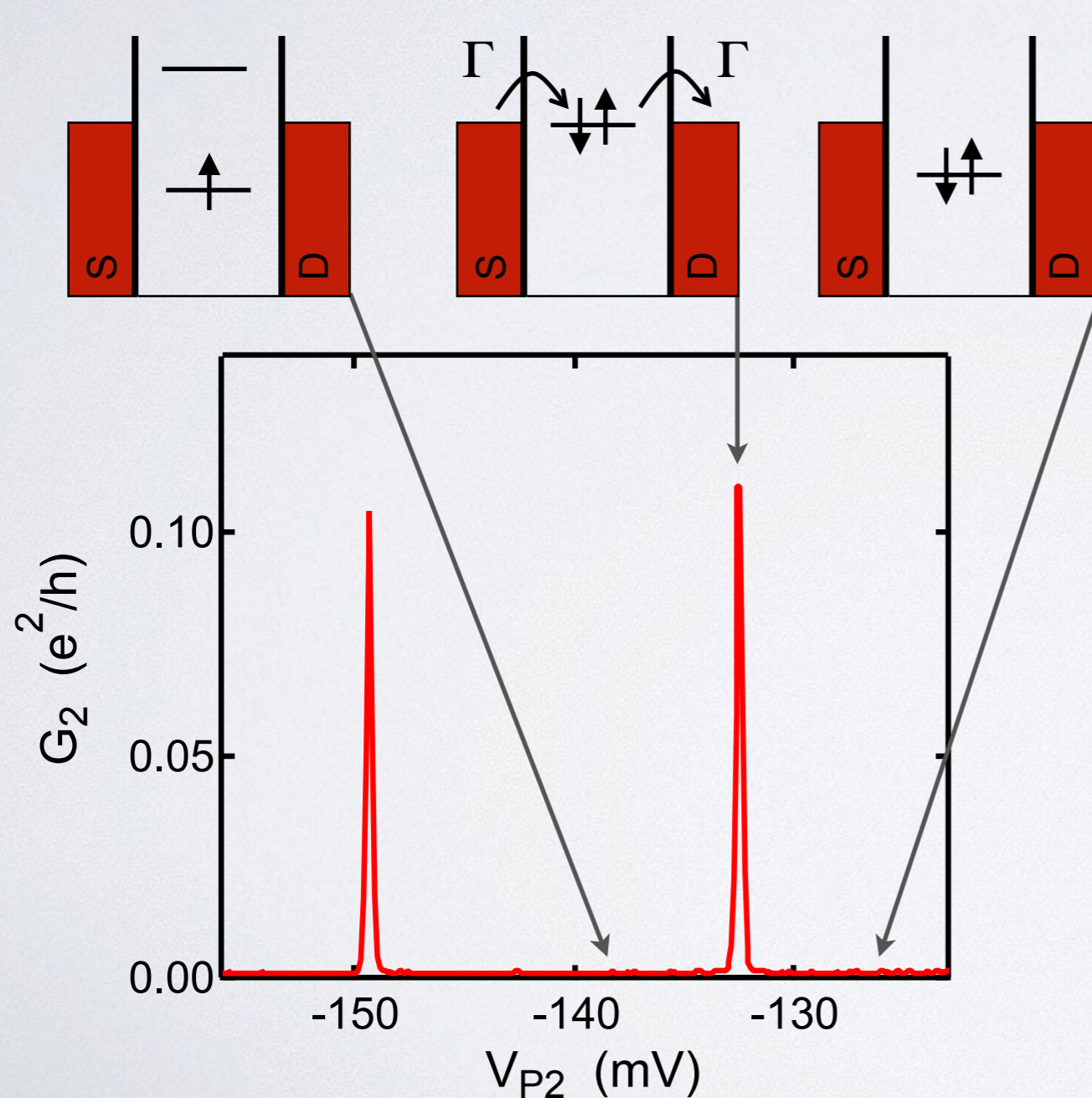


$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{\sigma} \epsilon_d d_{\sigma}^\dagger d_{\sigma} + U n_{d\uparrow} n_{d\downarrow} + \sum_{k\sigma} (V_k d_{\sigma}^\dagger c_{k\sigma} + \text{h.c.})$$



Transport through quantum dots

Coulomb blockade

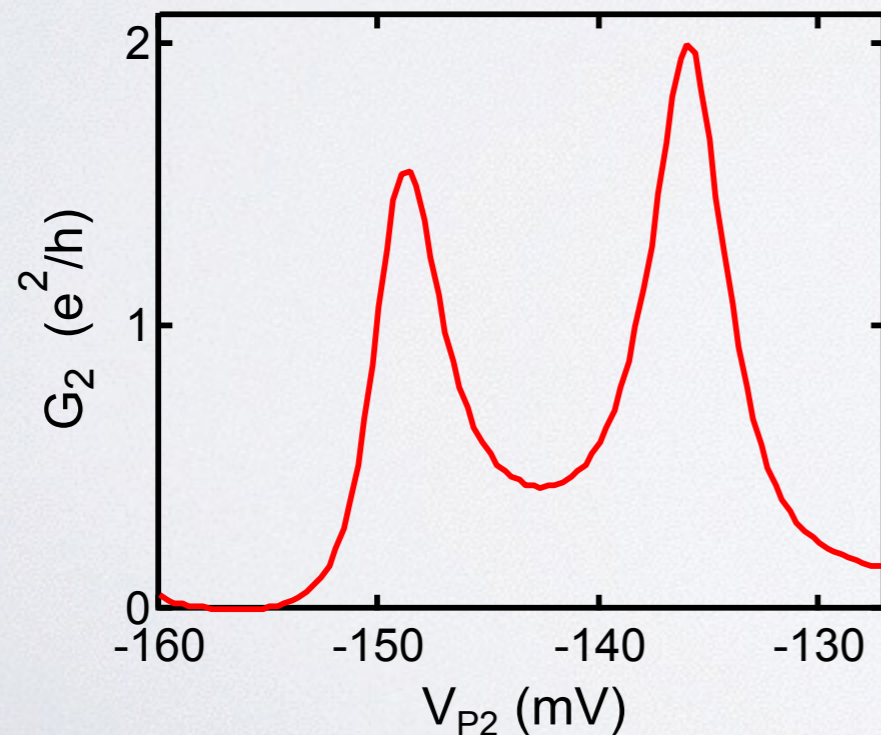
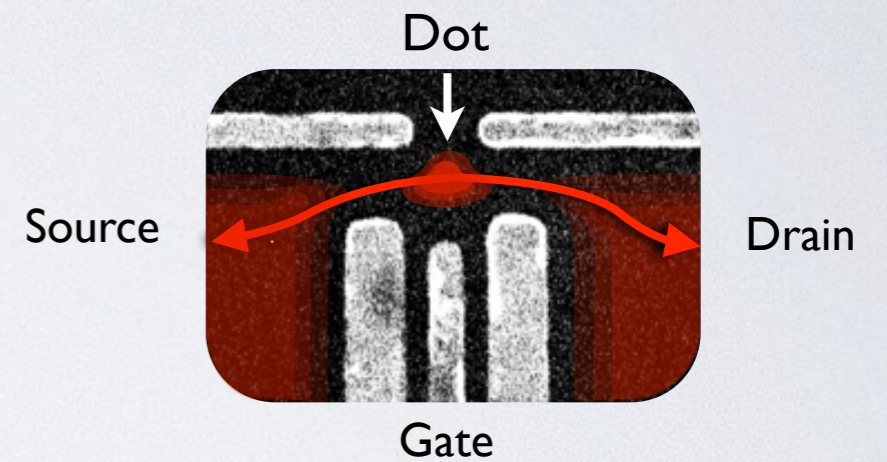
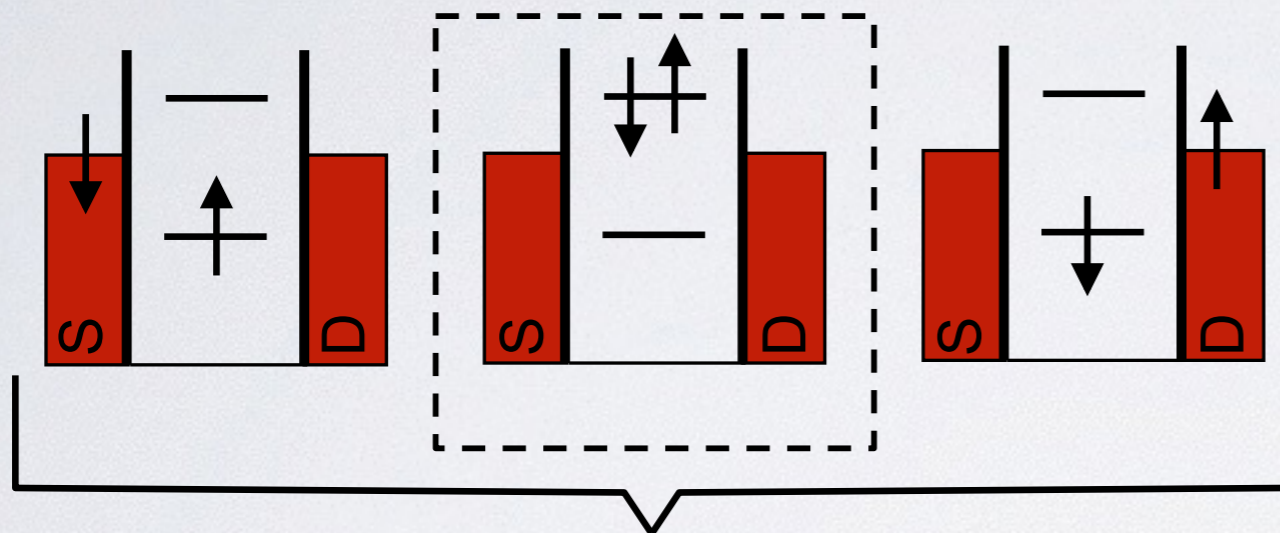


Charging energy: U
Dot-lead tunnel rate: Γ

$$\Gamma/U \ll 1$$

Transport through quantum dots

Kondo effect

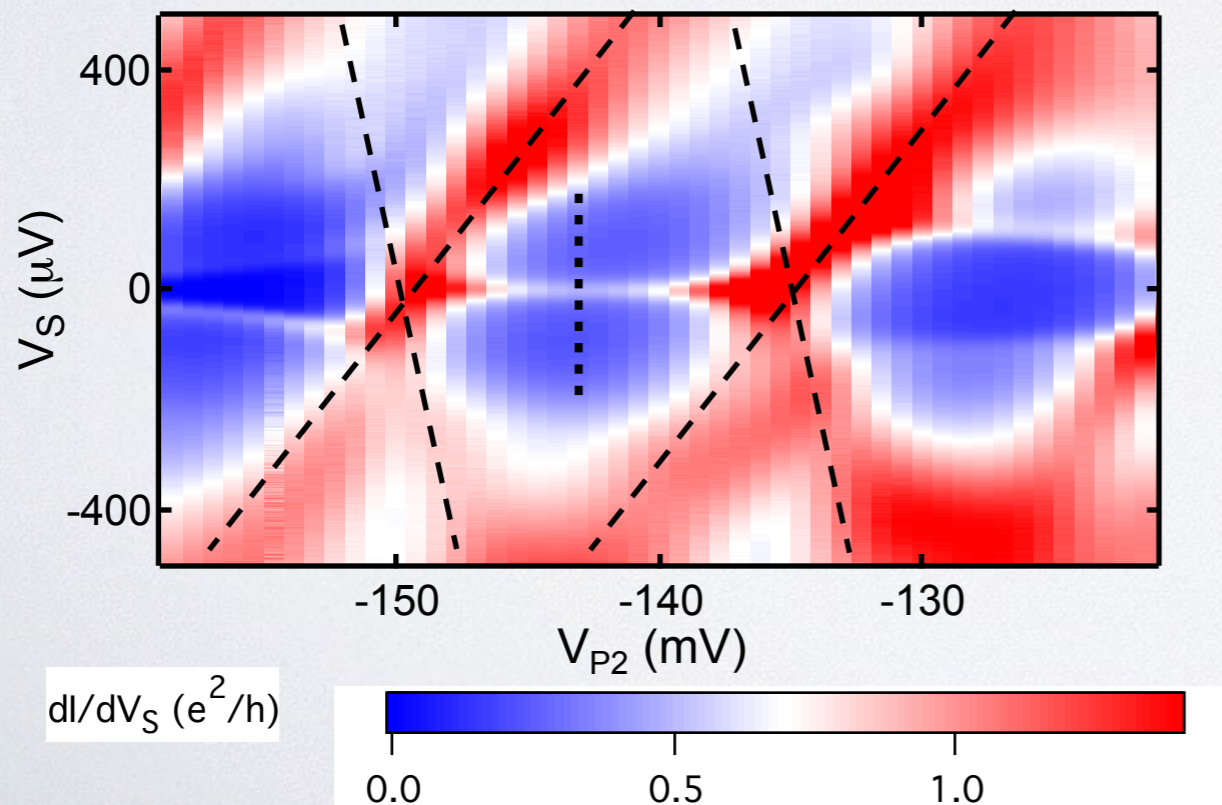
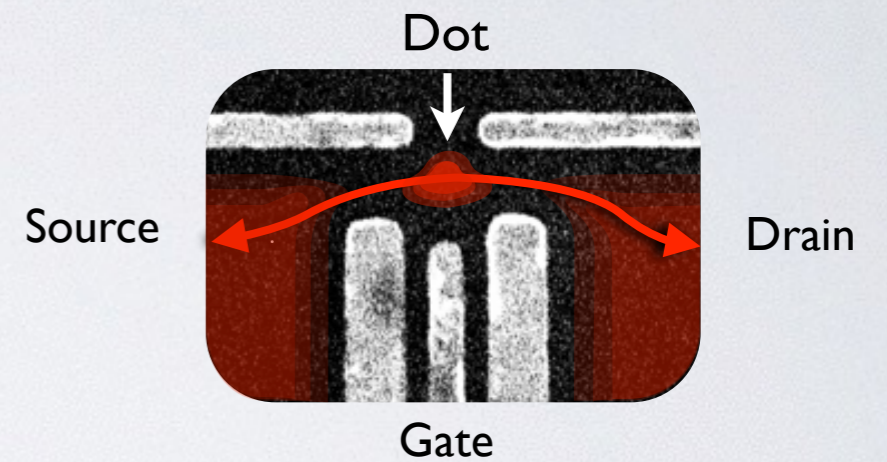
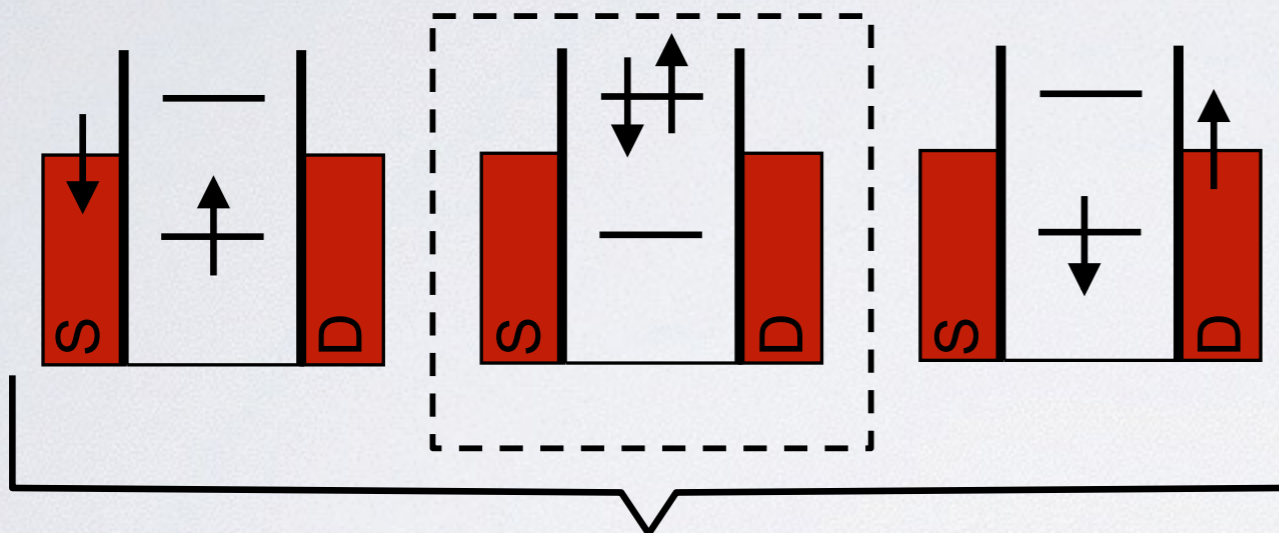


Charging energy: U
Dot-lead tunnel rate: Γ

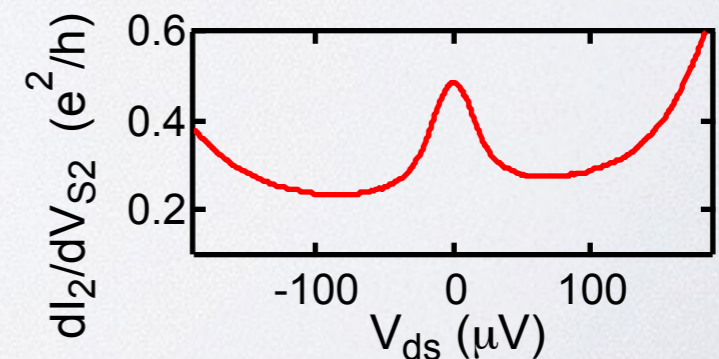
$$\Gamma/U \sim 0.2 \text{ to } 0.3$$

Transport through quantum dots

Kondo effect



Zero-bias conductance enhancement implies Kondo



Challenge

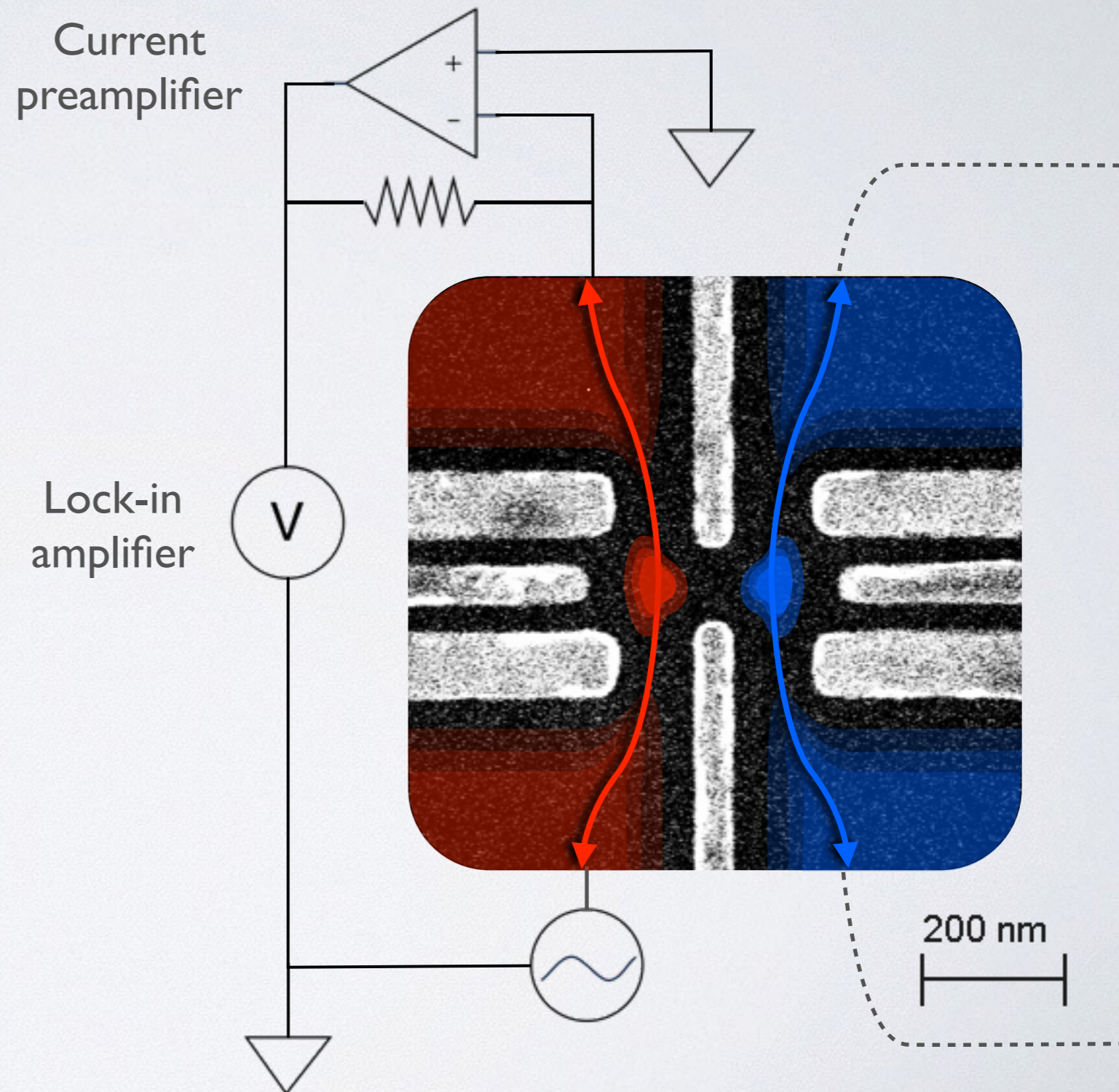
- Can we realize more complicated Kondo effects to address the physics of exotic material systems?

Outline

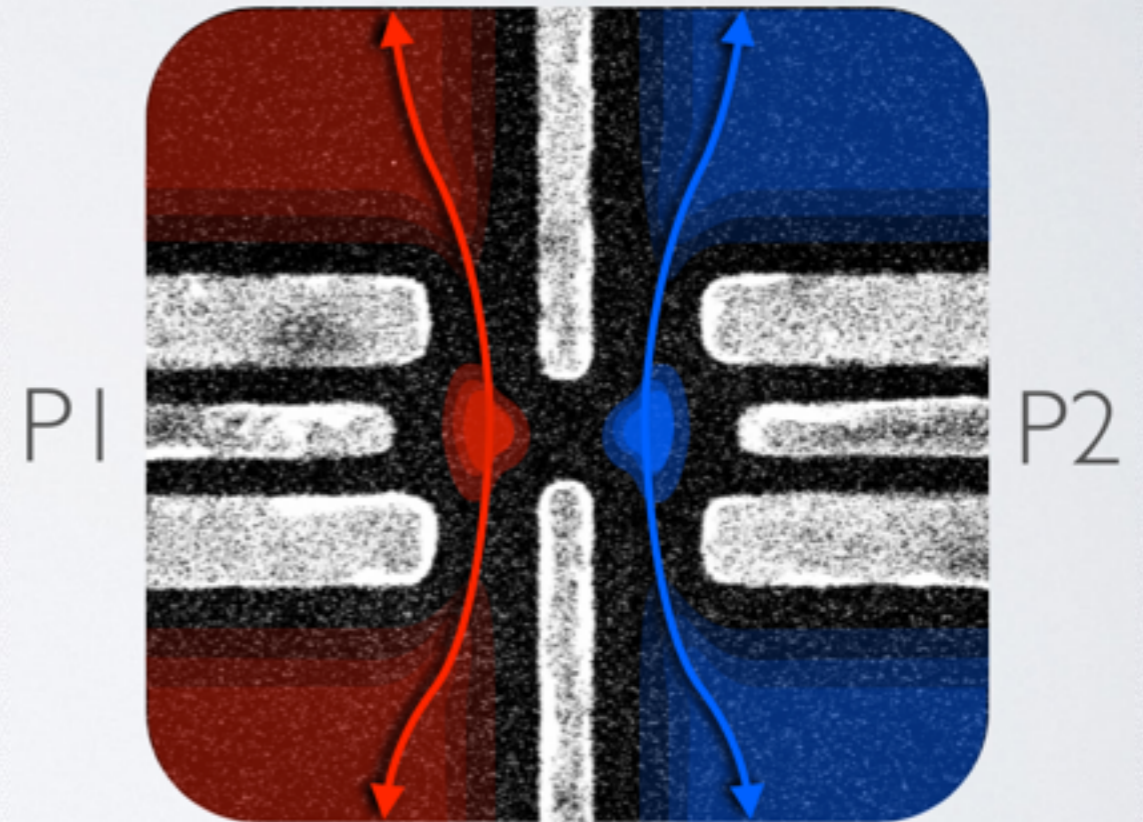
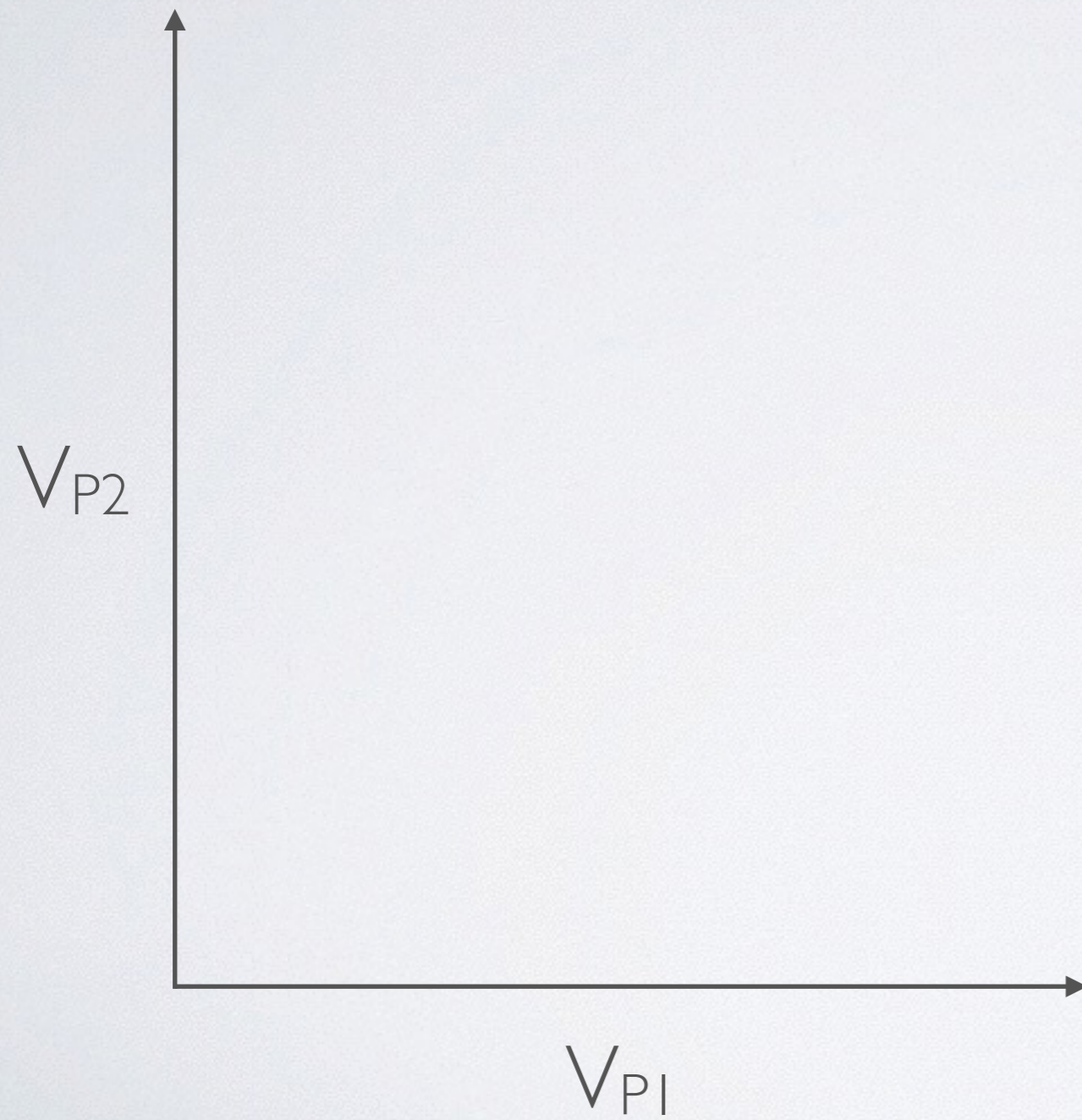
- Symmetries in physics
- Quantum dots and the Kondo effect
- **Emergent $SU(4)$ -symmetric Kondo effect in a double quantum dot**
- Universal crossover between quantum critical and Fermi liquid states in a quantum dot coupled to a “metallic grain”

Measurement scheme

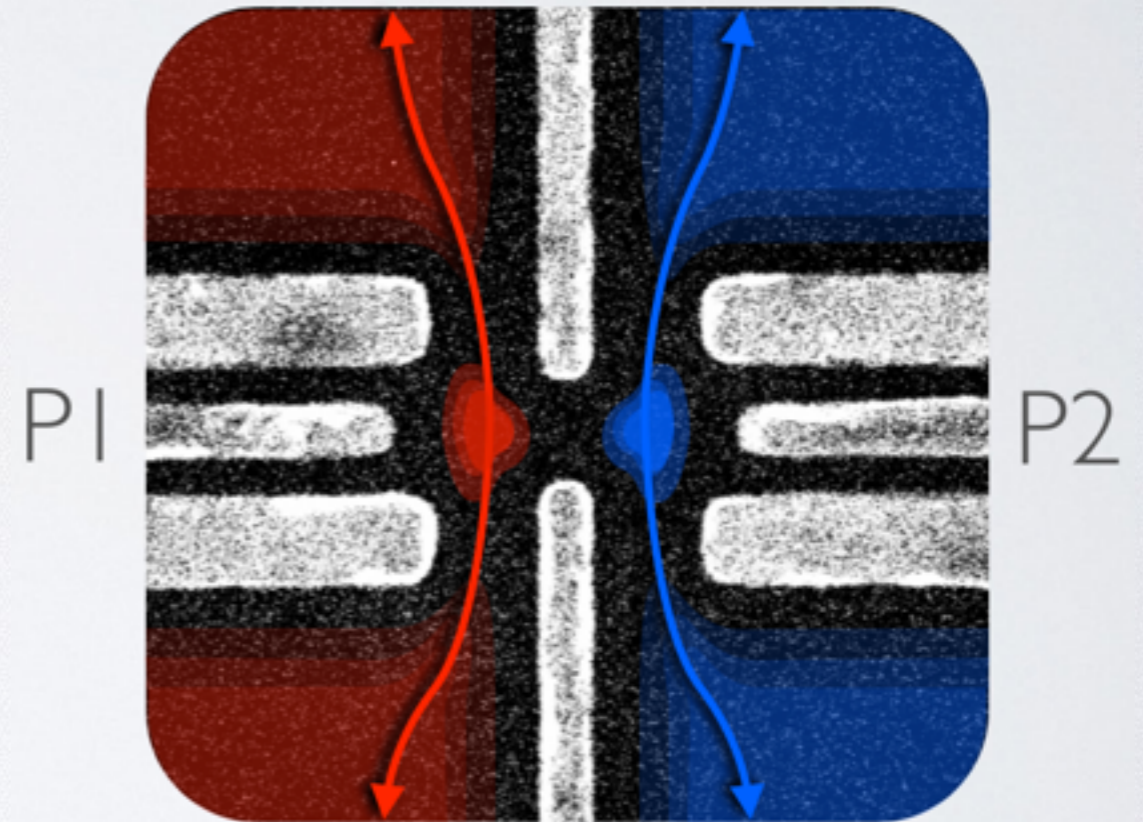
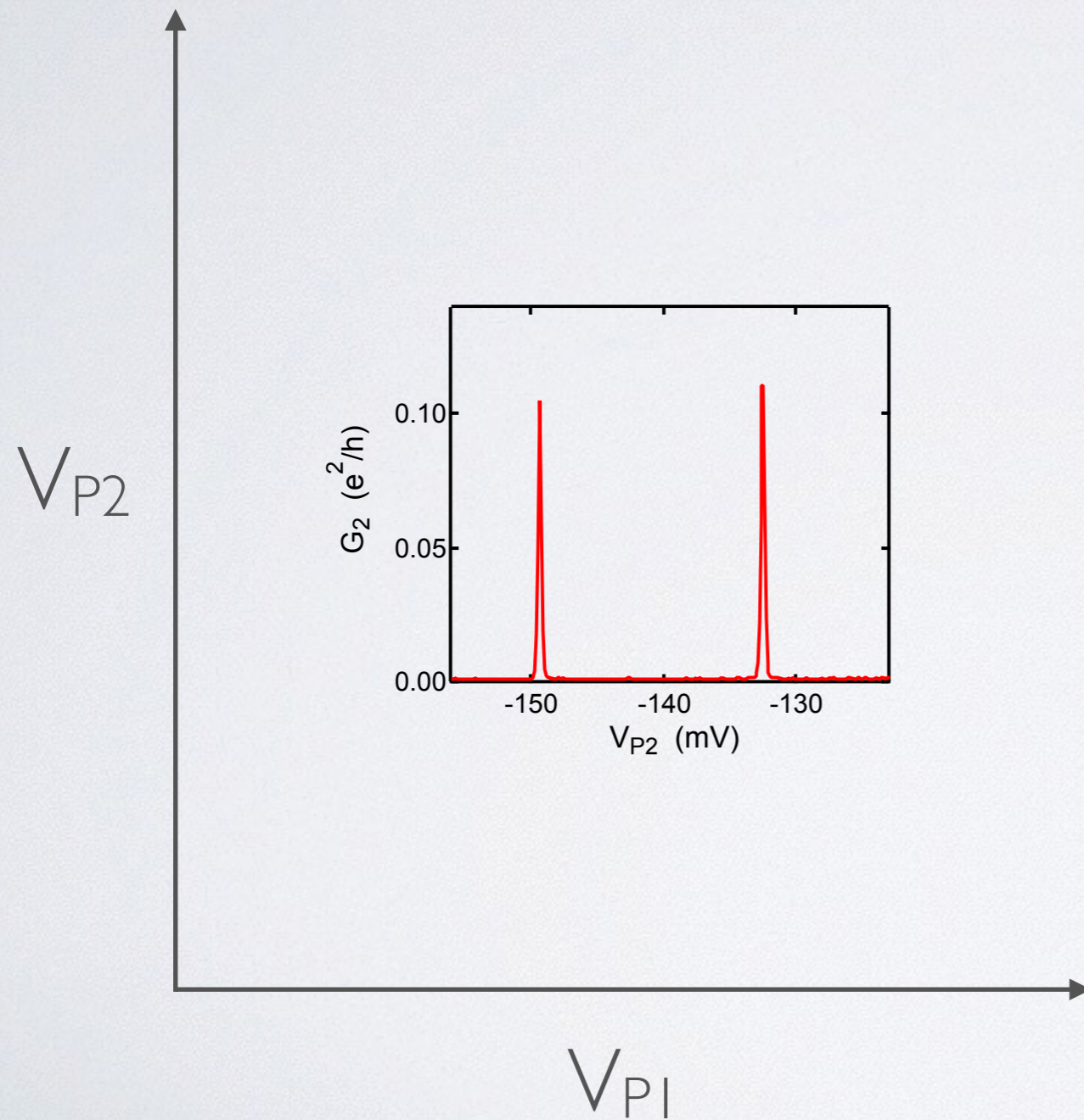
- Simultaneously measure conductance through each dot in parallel



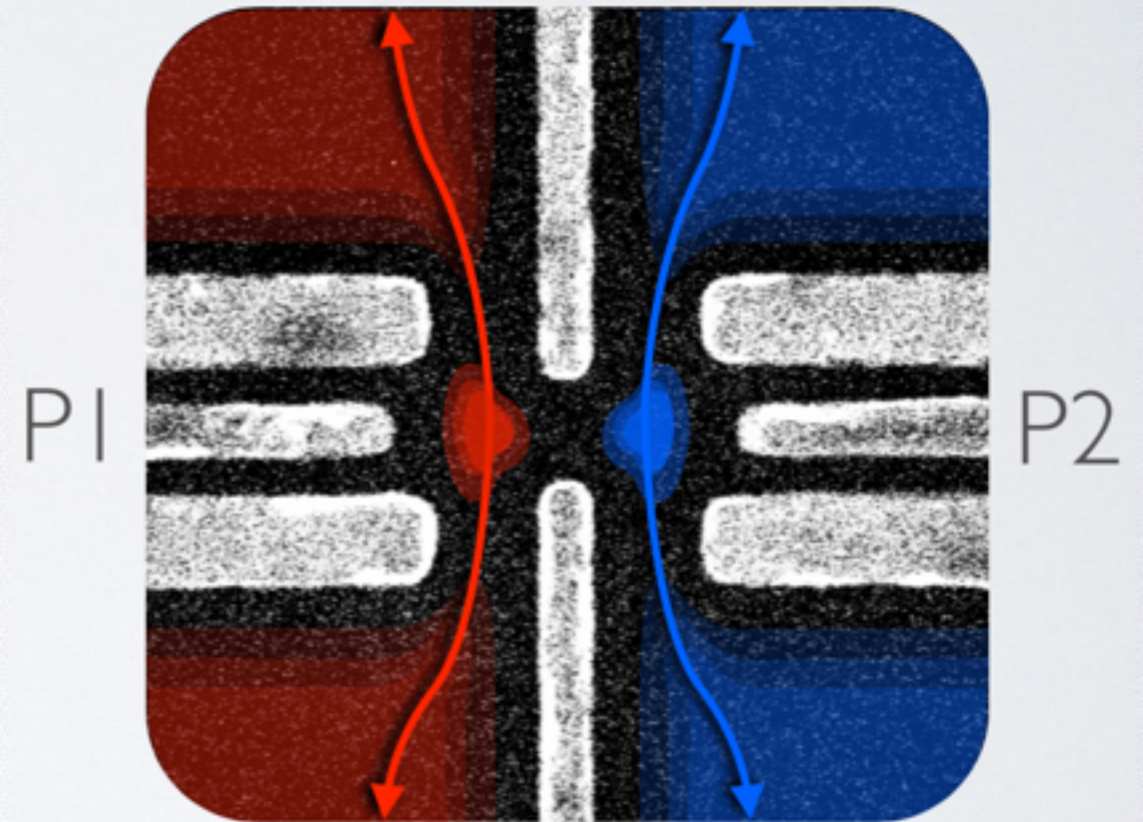
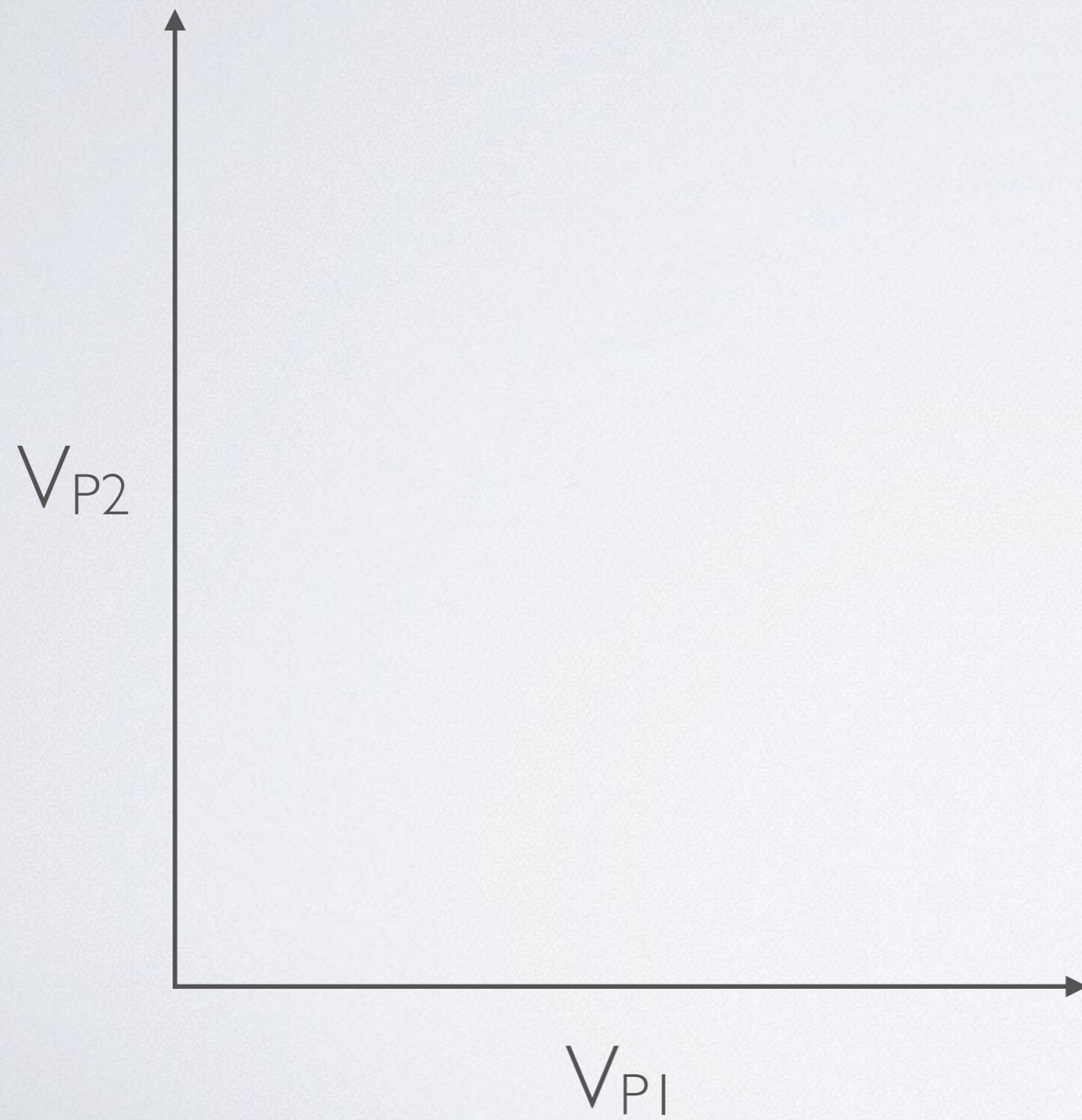
Charge stability diagram



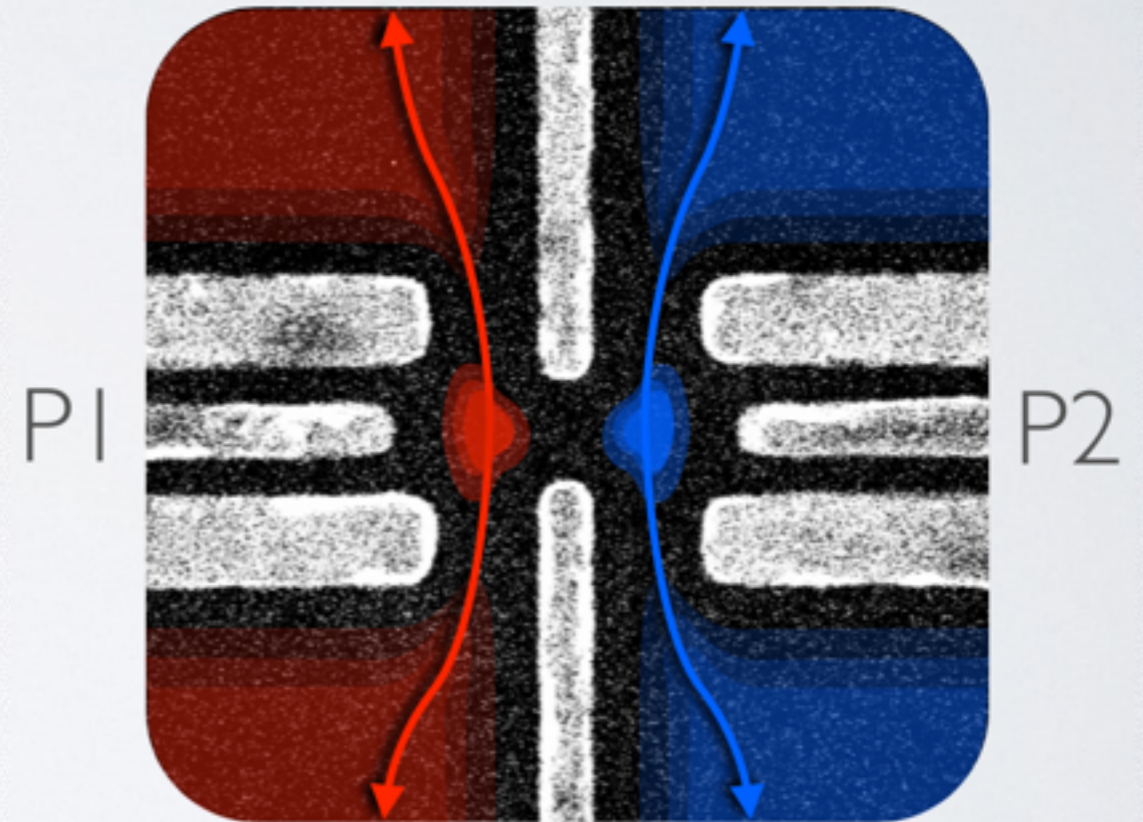
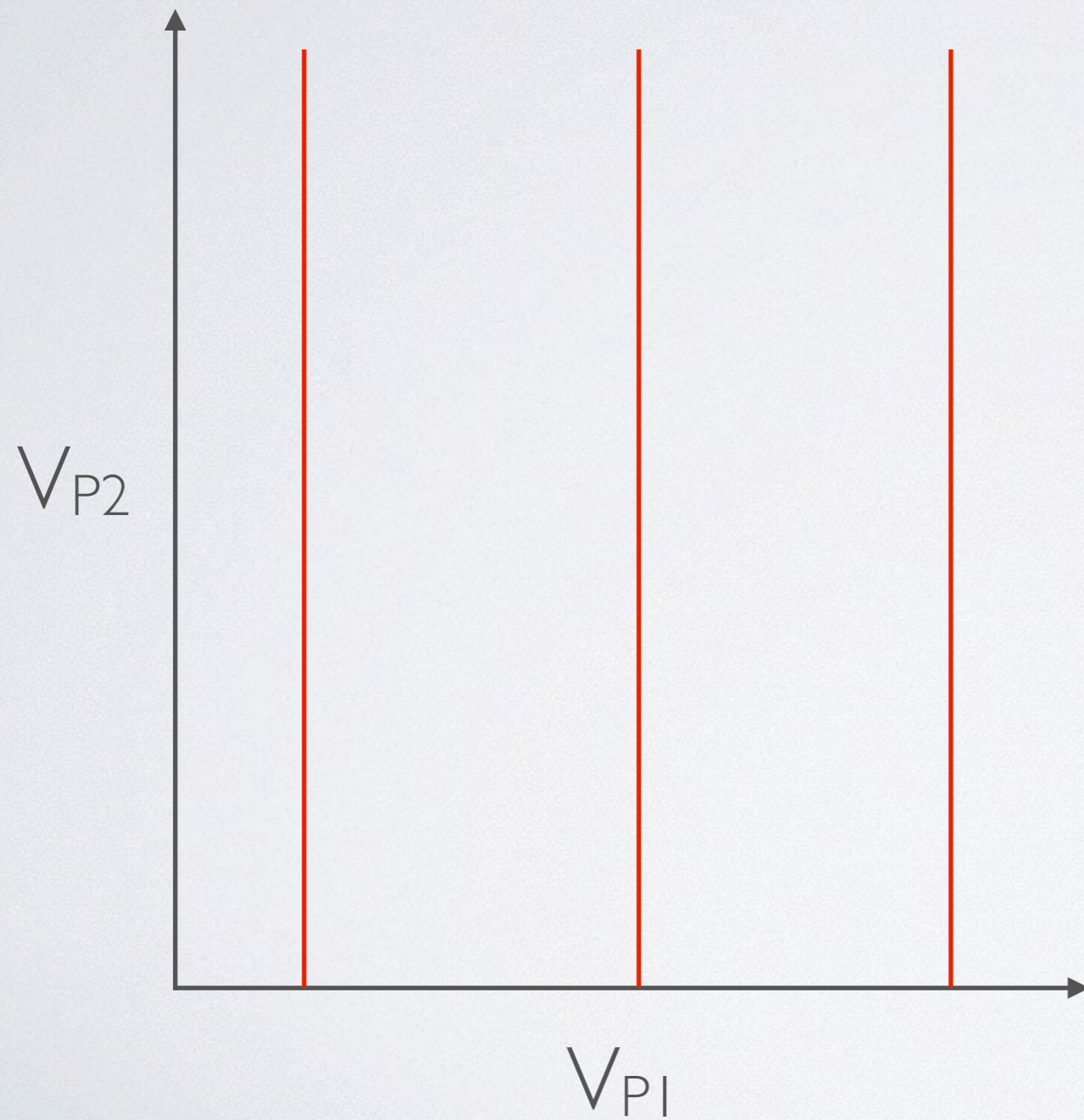
Charge stability diagram



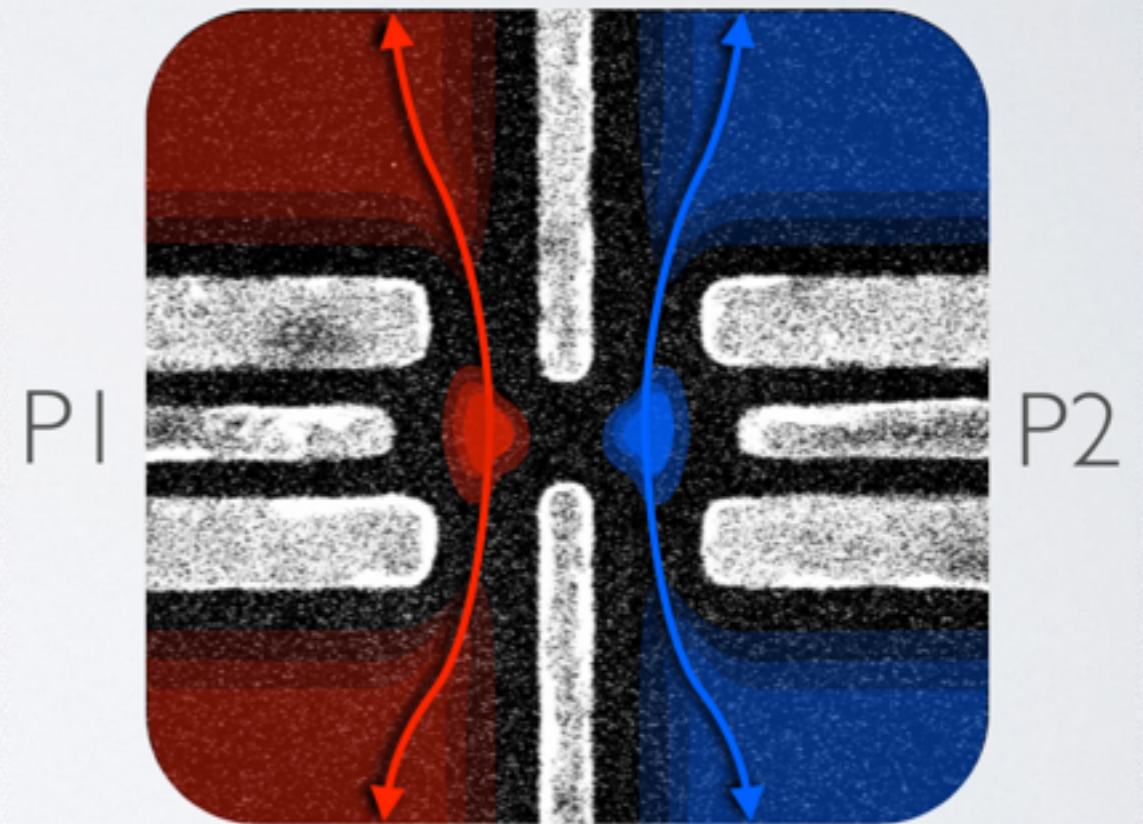
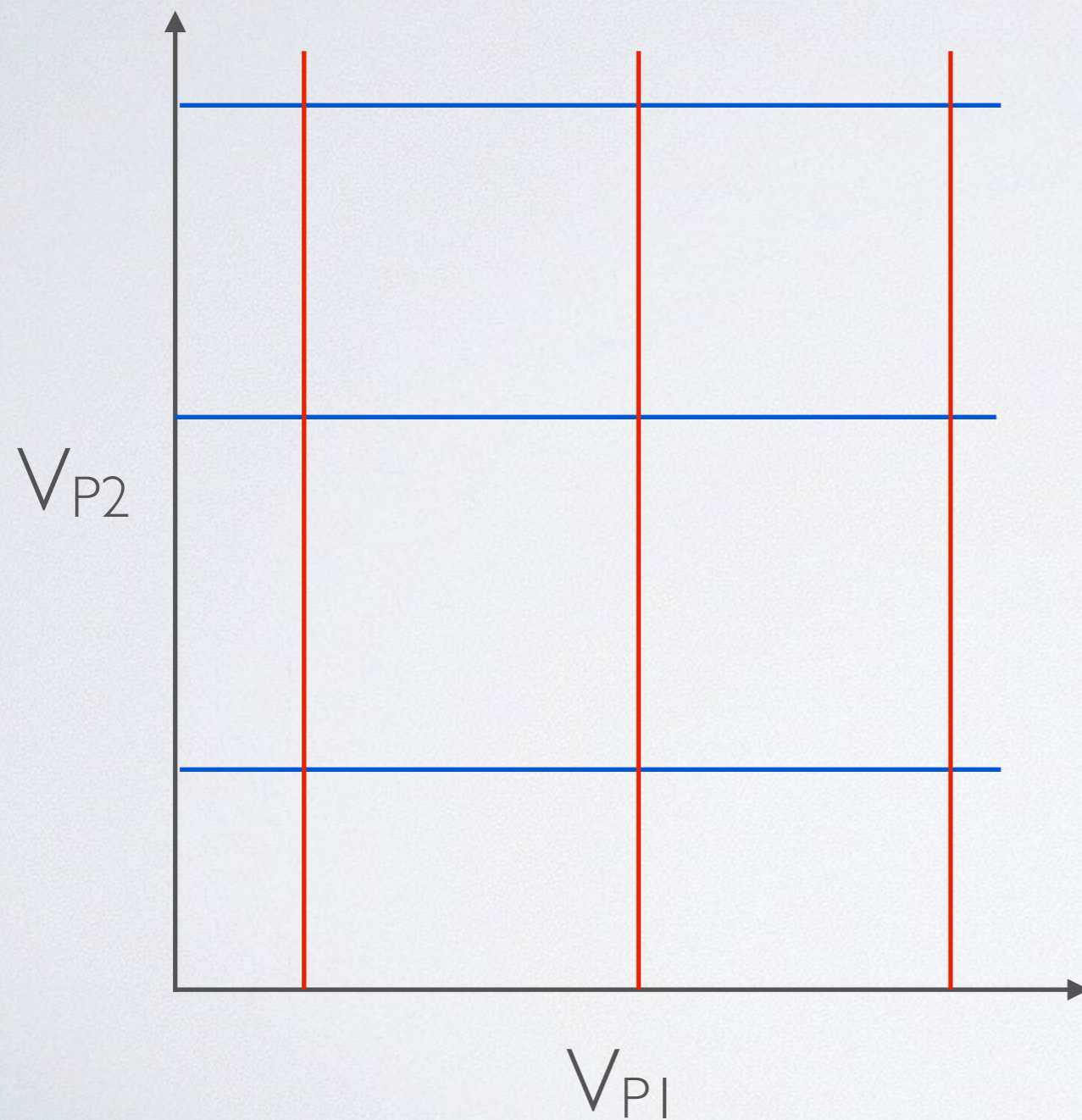
Charge stability diagram



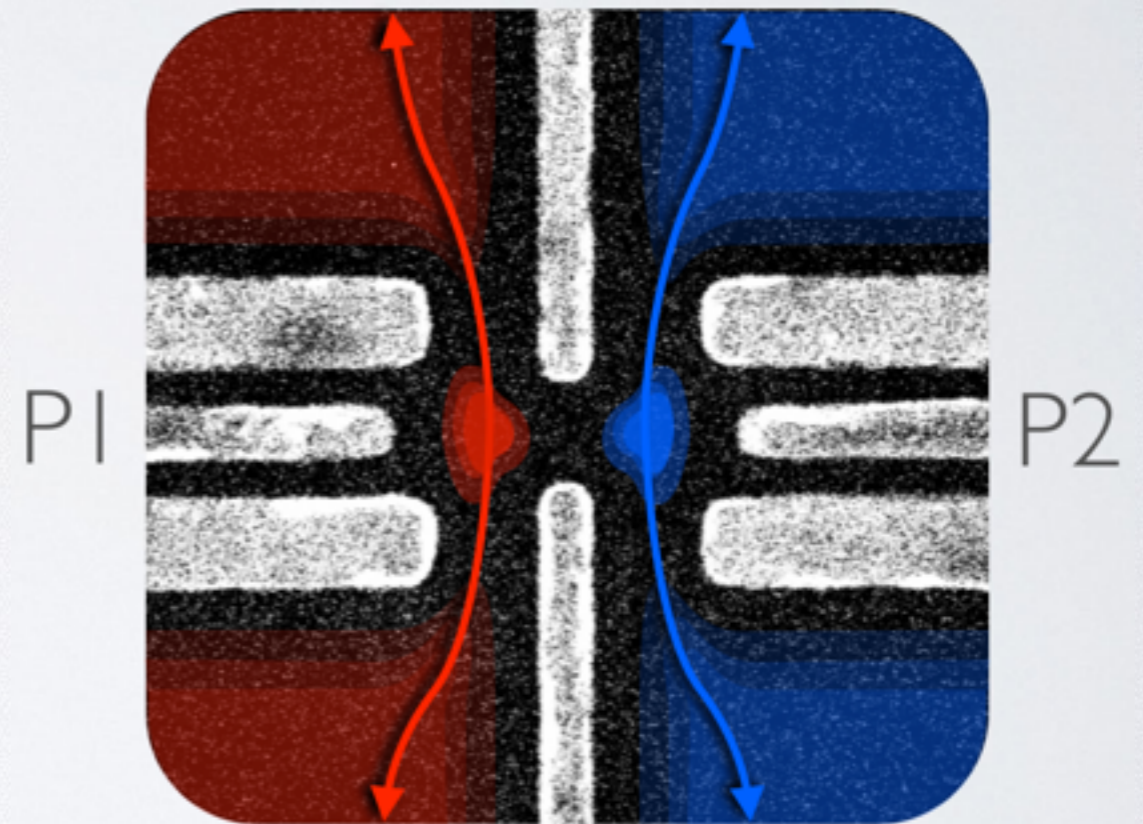
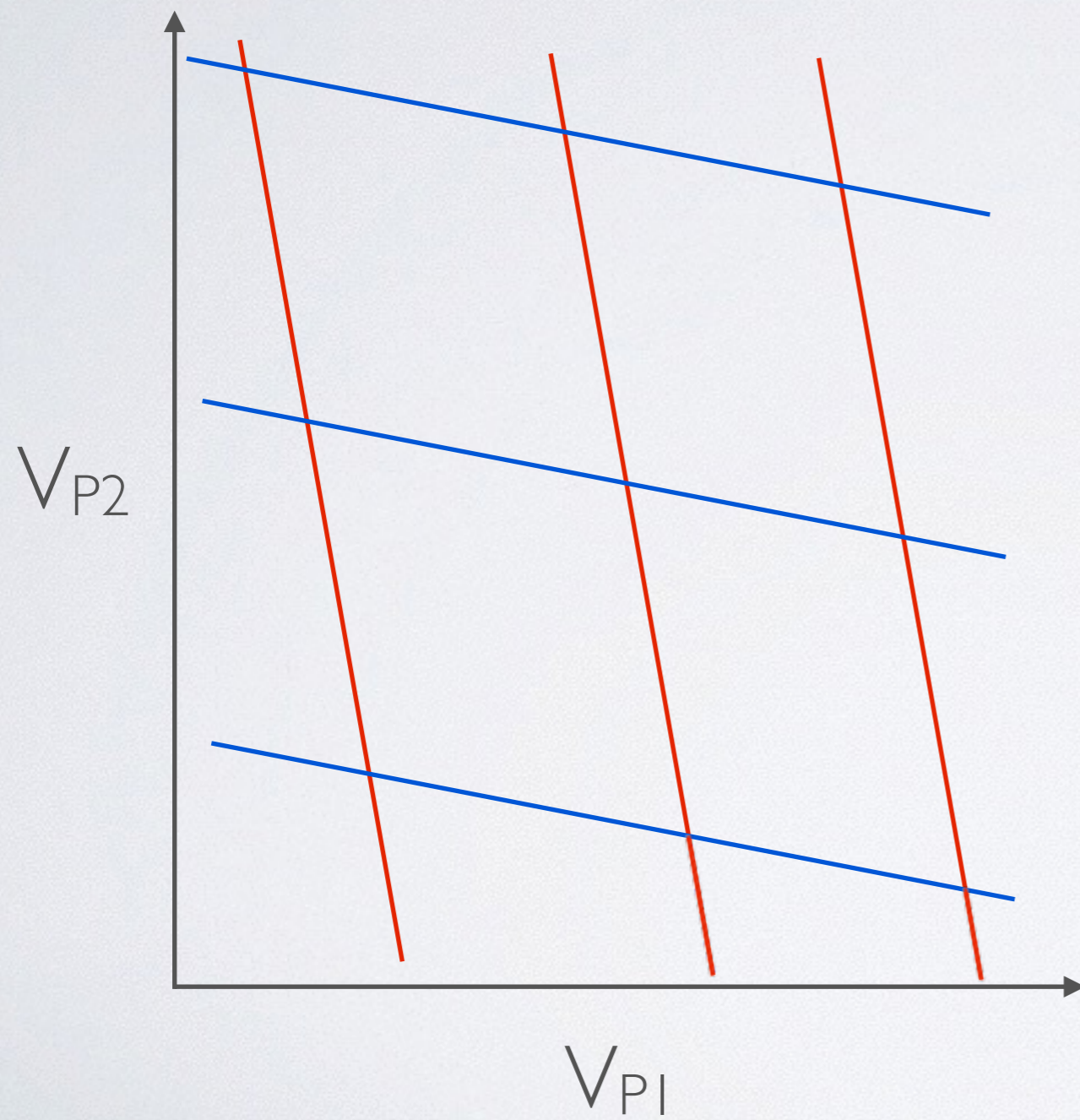
Charge stability diagram



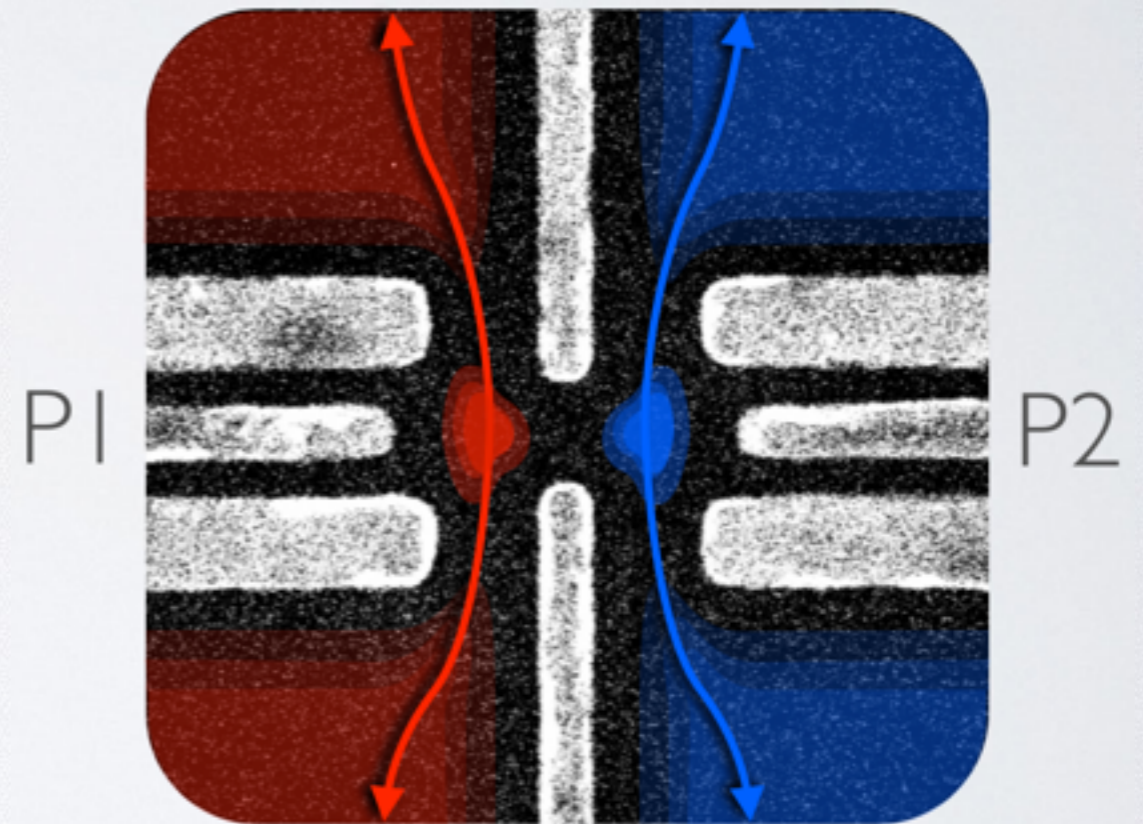
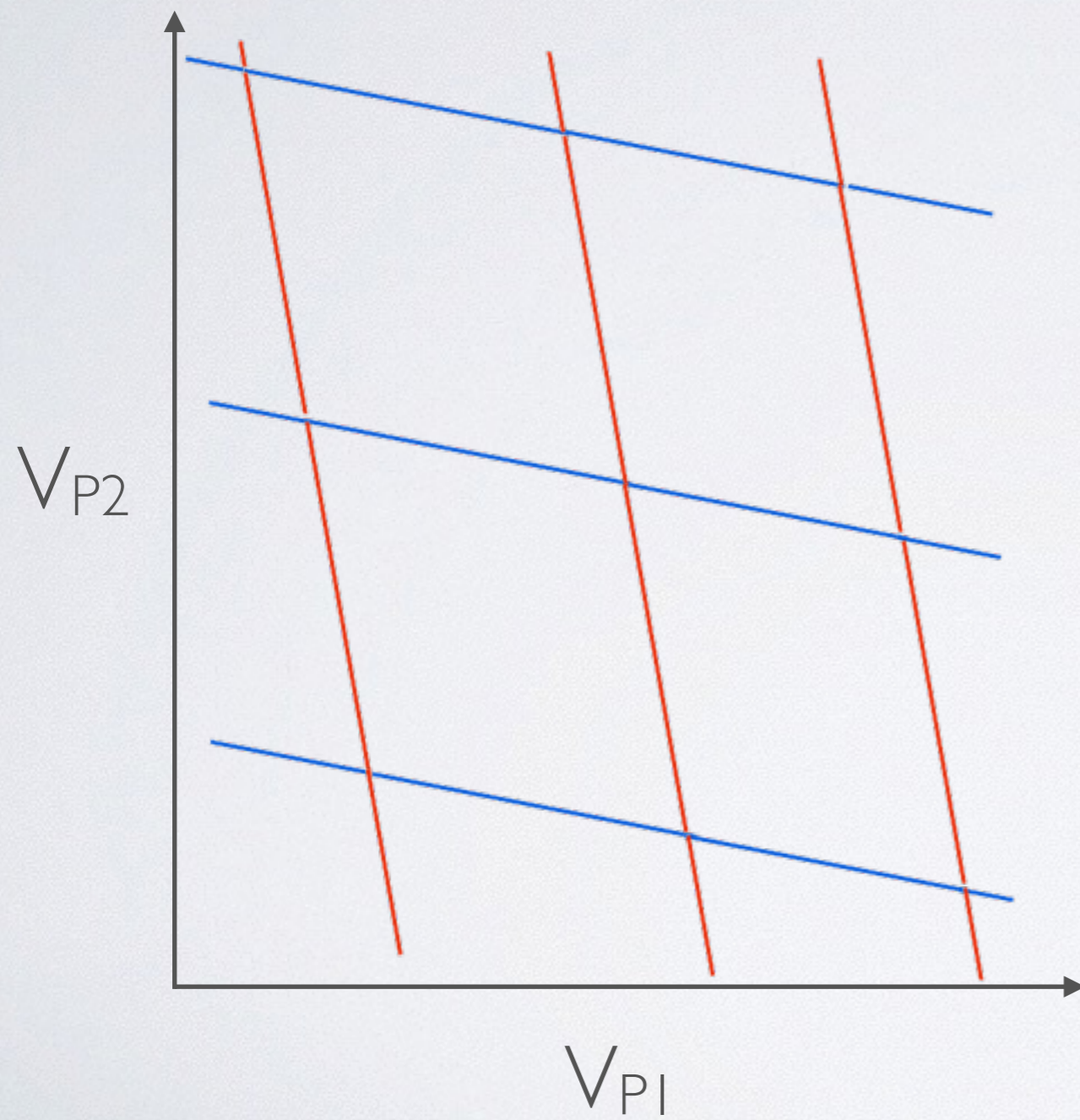
Charge stability diagram



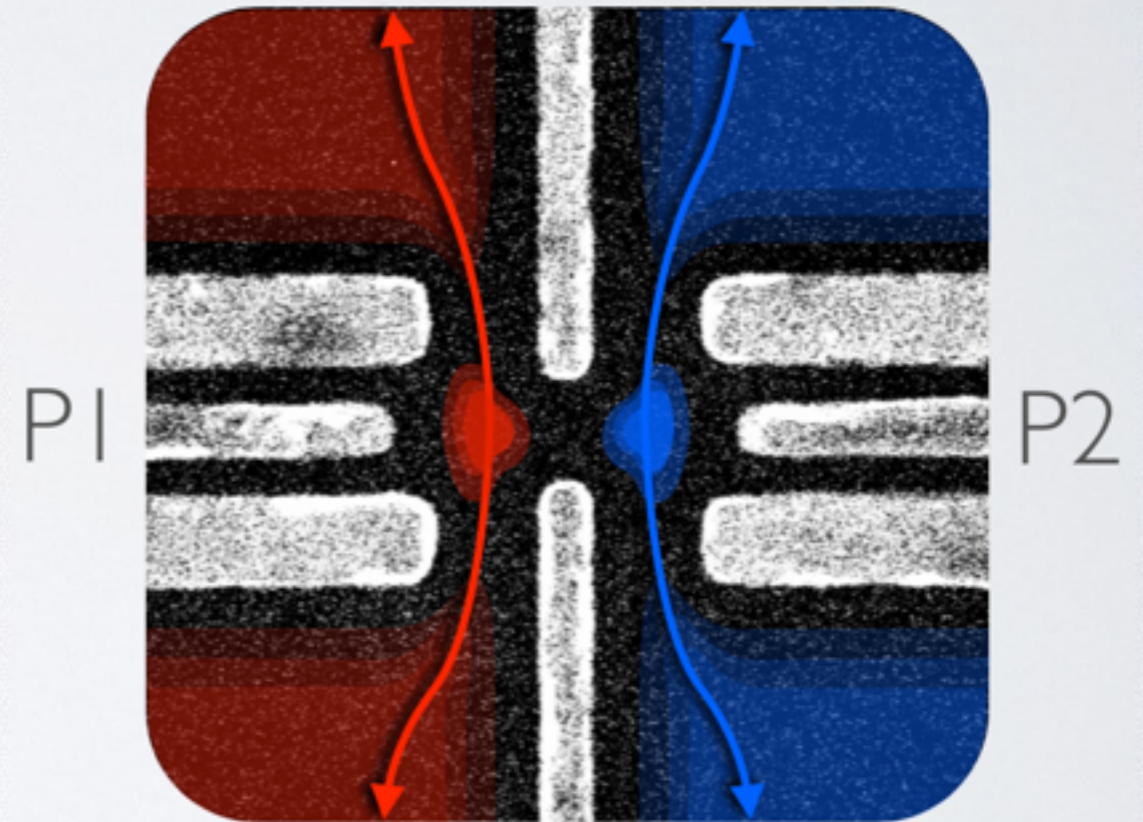
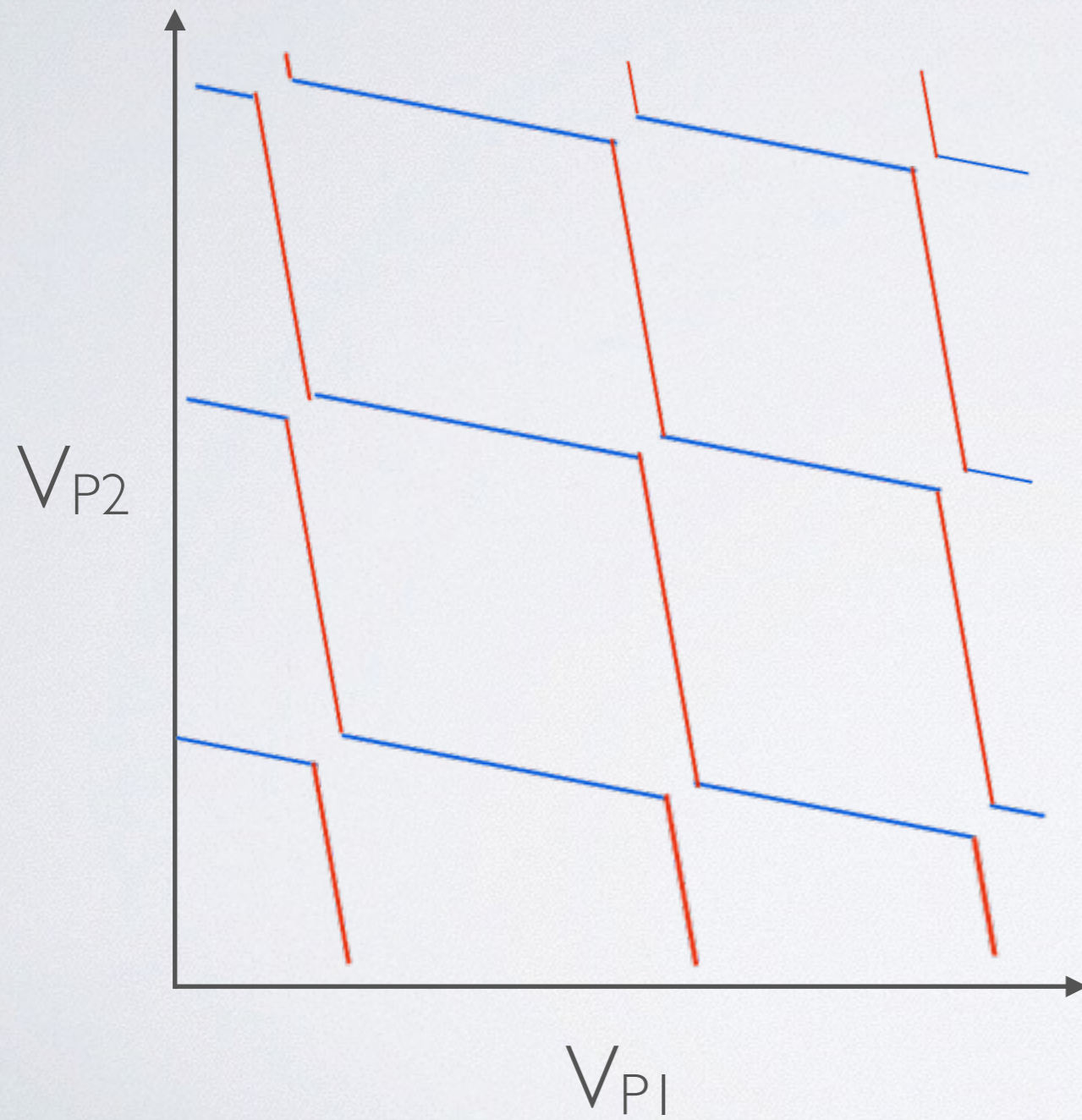
Charge stability diagram



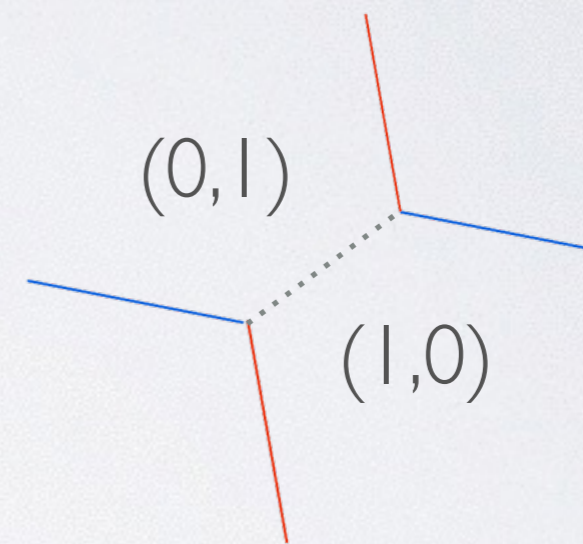
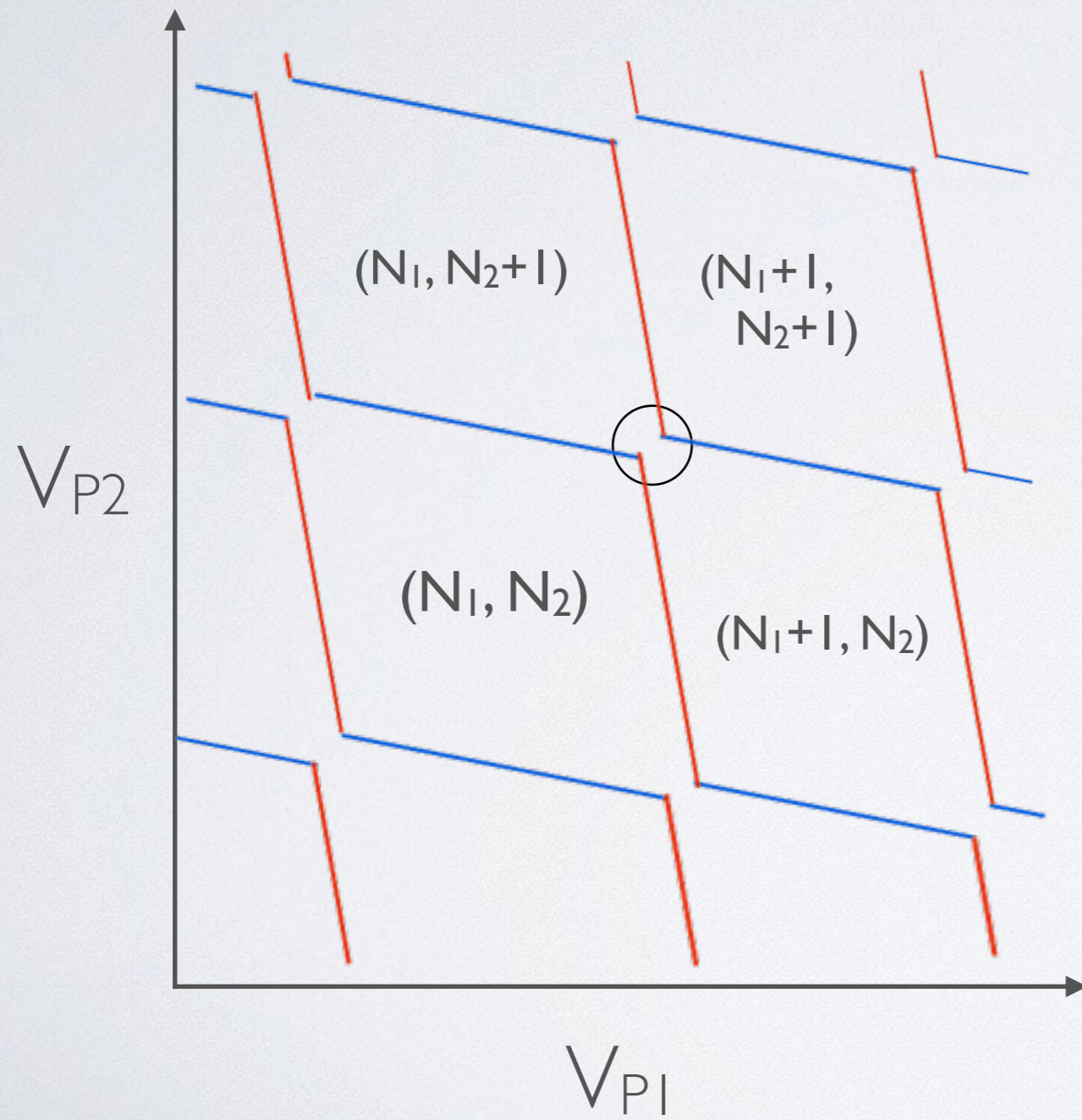
Charge stability diagram



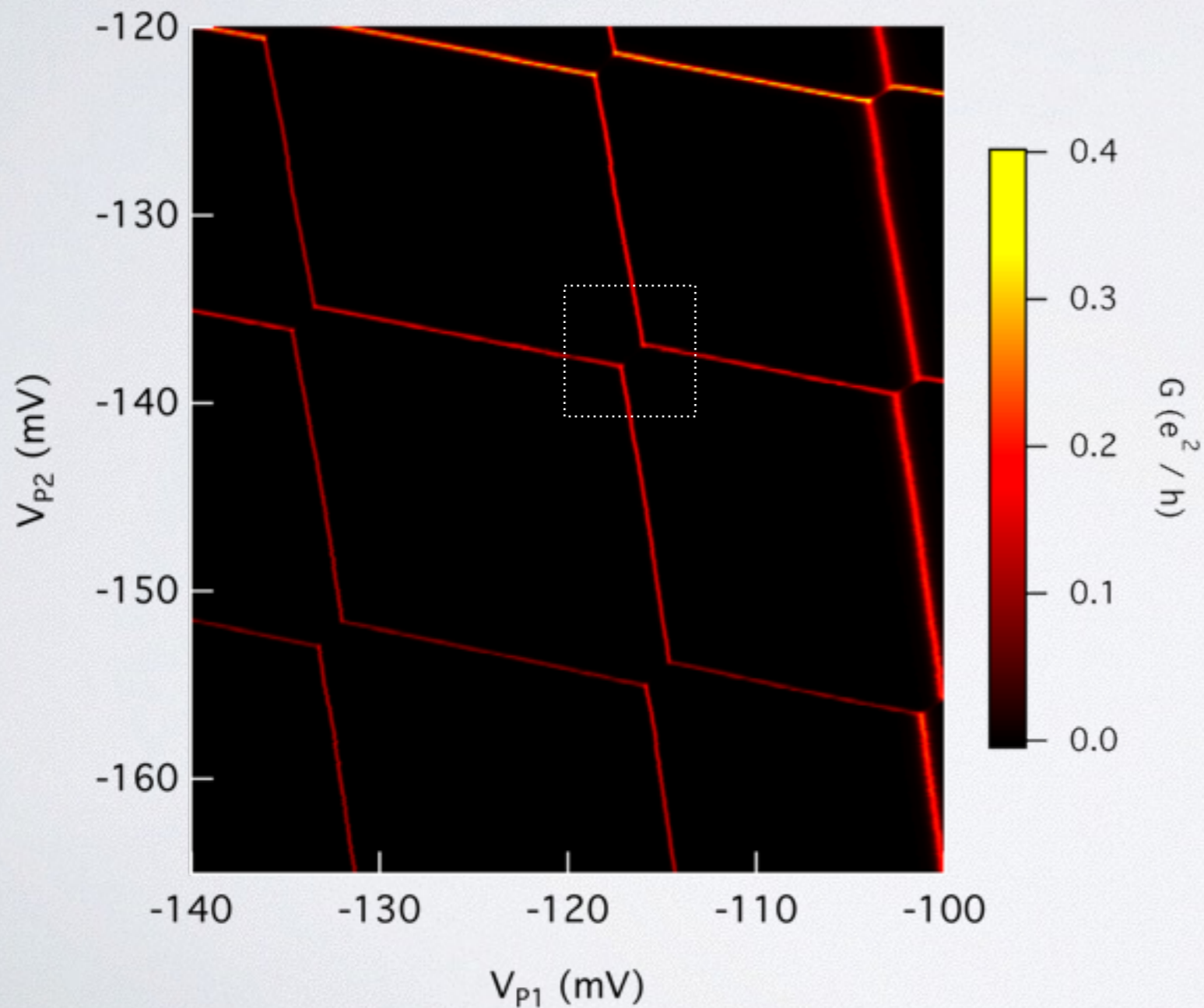
Charge stability diagram



Charge stability diagram



Charge stability diagram

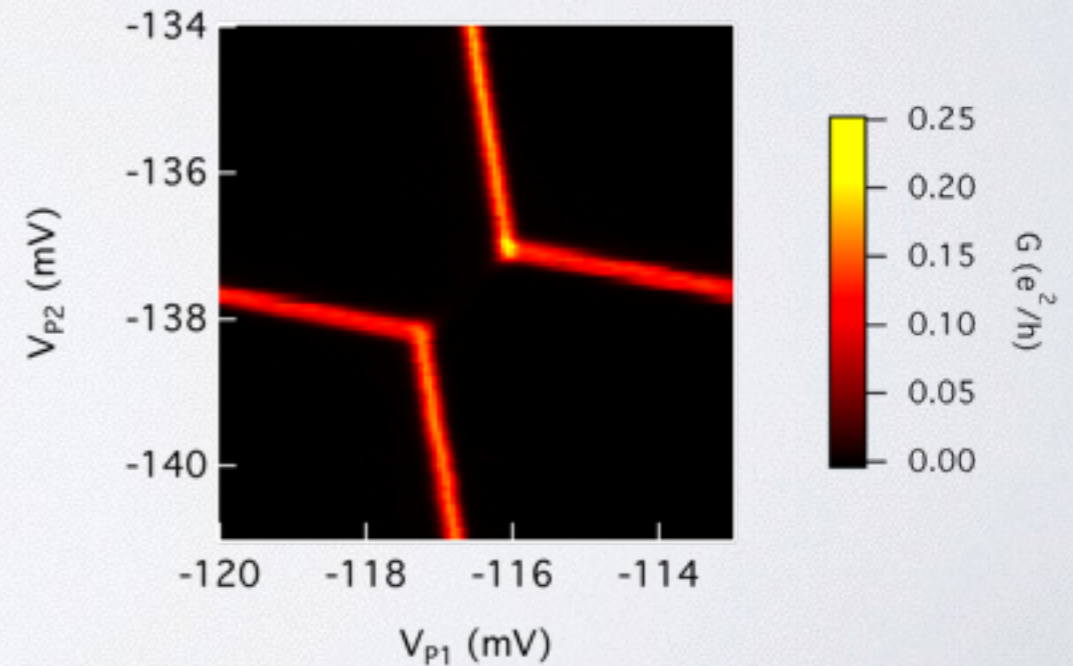


$$T_e = 1.7 \mu\text{eV} \text{ (20 mK)}$$

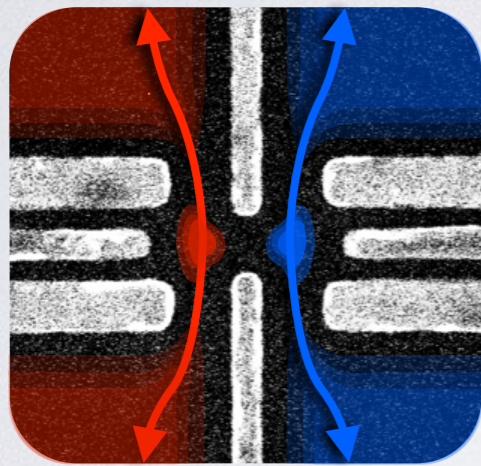
$$\Gamma_1 \sim \Gamma_2 \sim 20 \mu\text{eV}$$

$$U' \sim 100 \mu\text{eV}$$

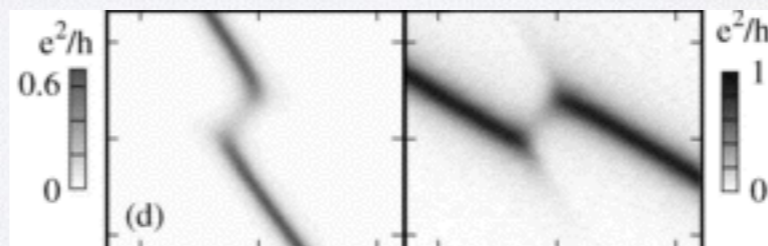
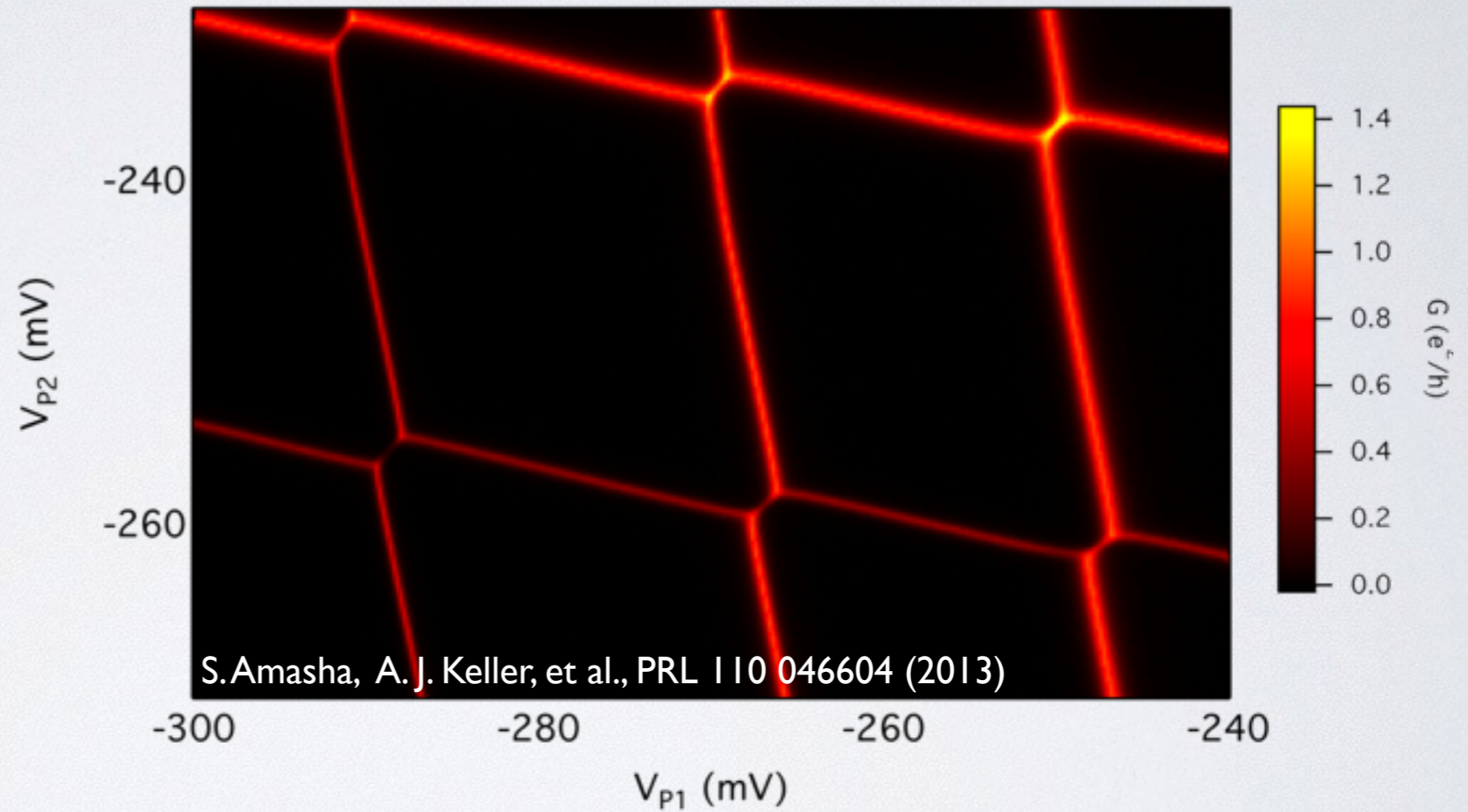
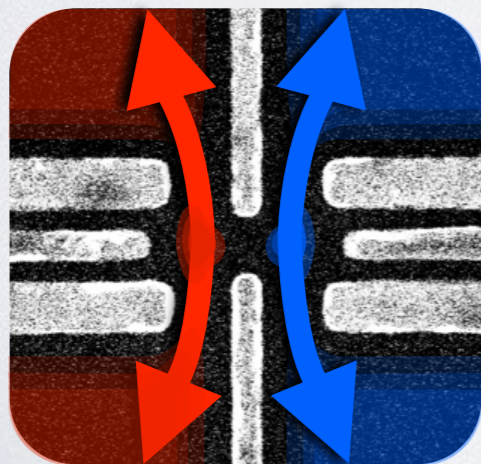
$$U \sim 1 \text{ meV}$$



A Kondo effect between triple points?



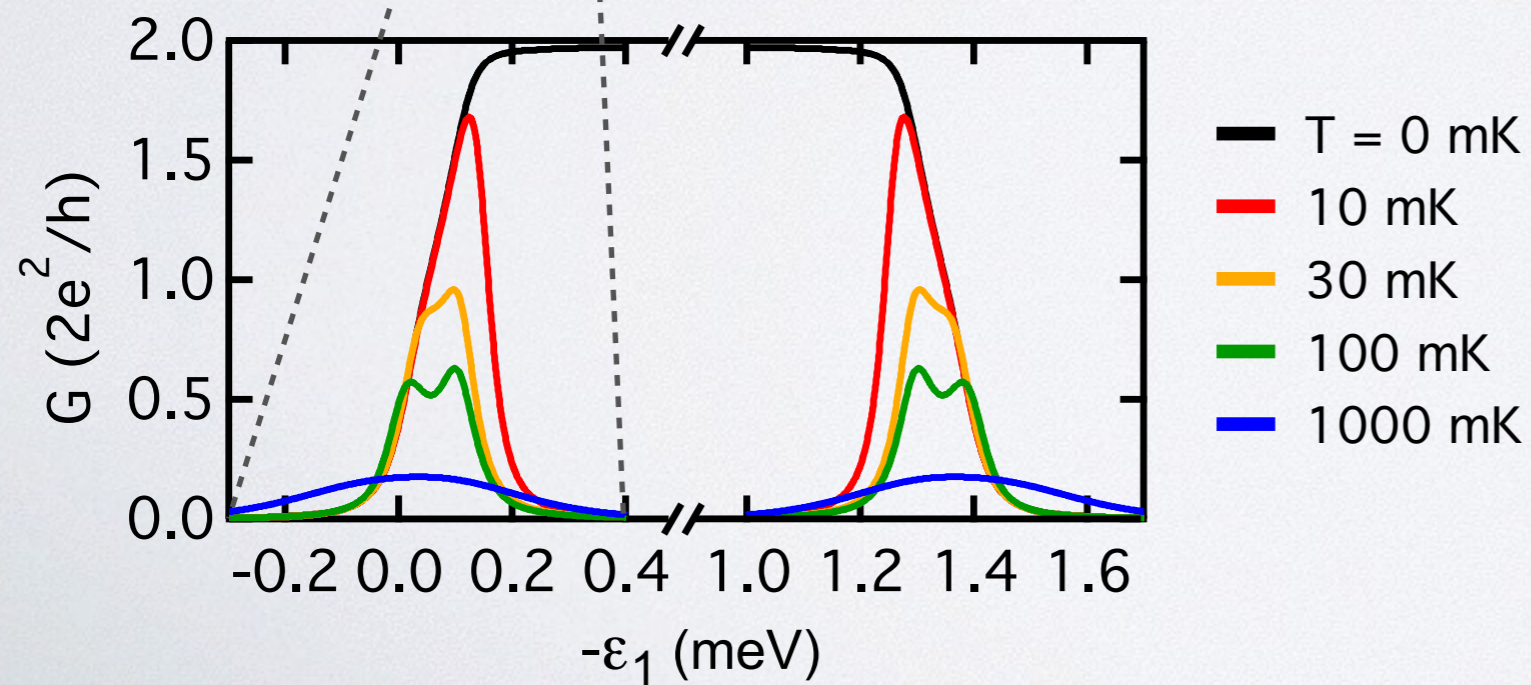
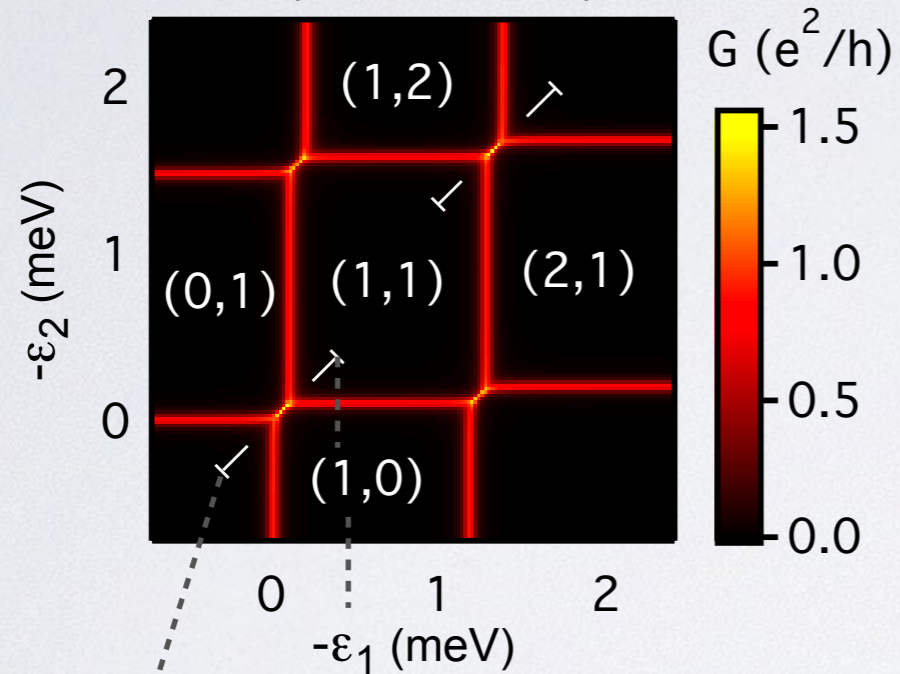
Increasing Γ ↓



First observed by Hübner, et al,
PRL 101 186804 (2004).

A four-fold degenerate state

(calculations)



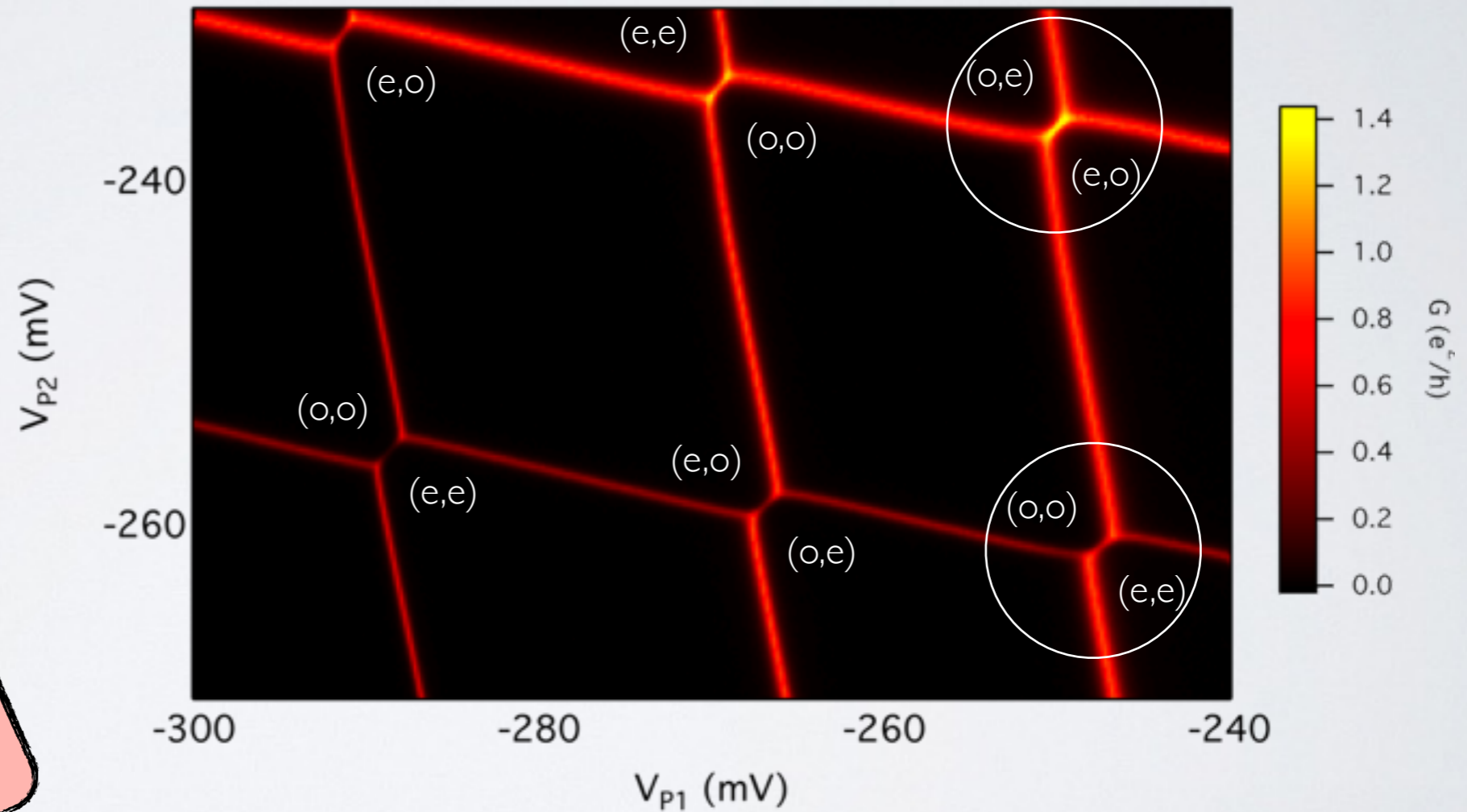
Degeneracies

$(o,e)-(e,o)$
degeneracies:

$(\uparrow,0)$ $(\downarrow,0)$
 $(0,\uparrow)$ $(0,\downarrow)$

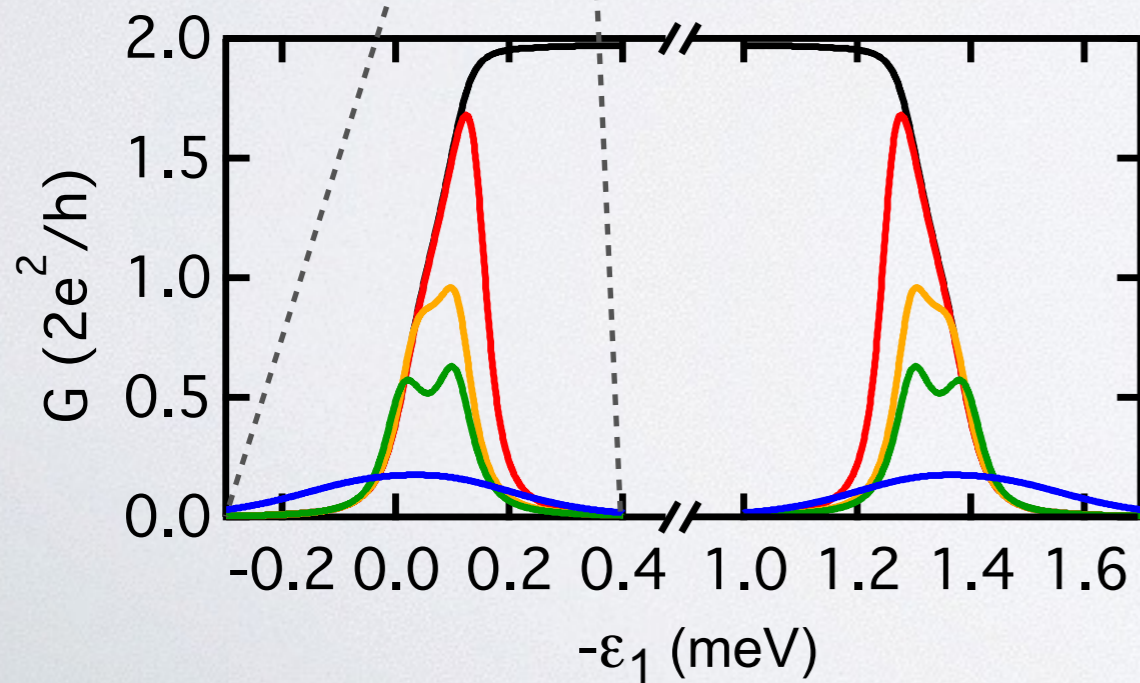
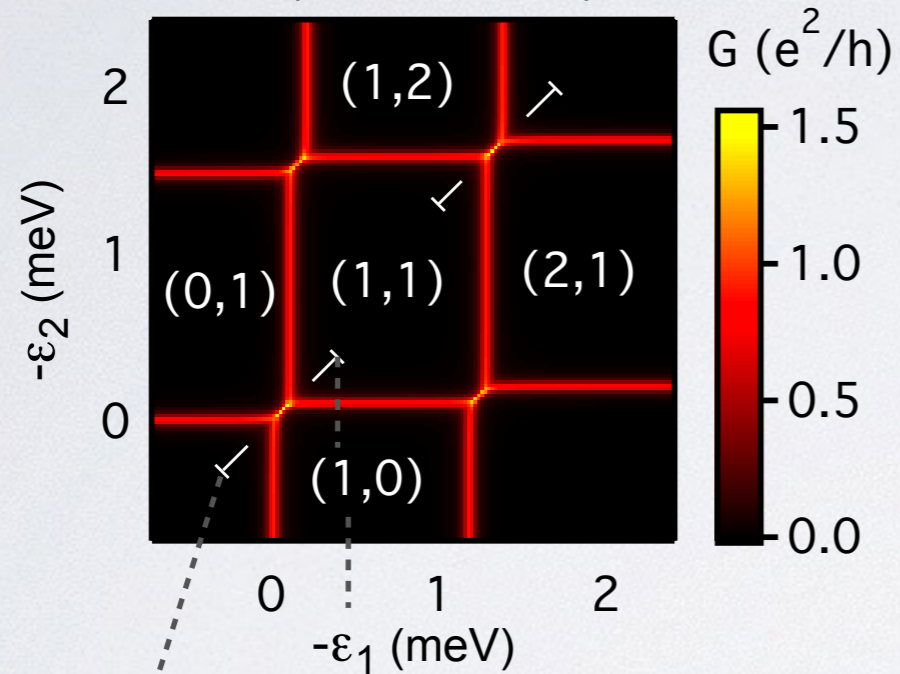
$(o,o)-(e,e)$
degeneracies:

(\uparrow,\uparrow) (\downarrow,\uparrow)
 (\uparrow,\downarrow) (\downarrow,\downarrow)
and
 $(0,2)$

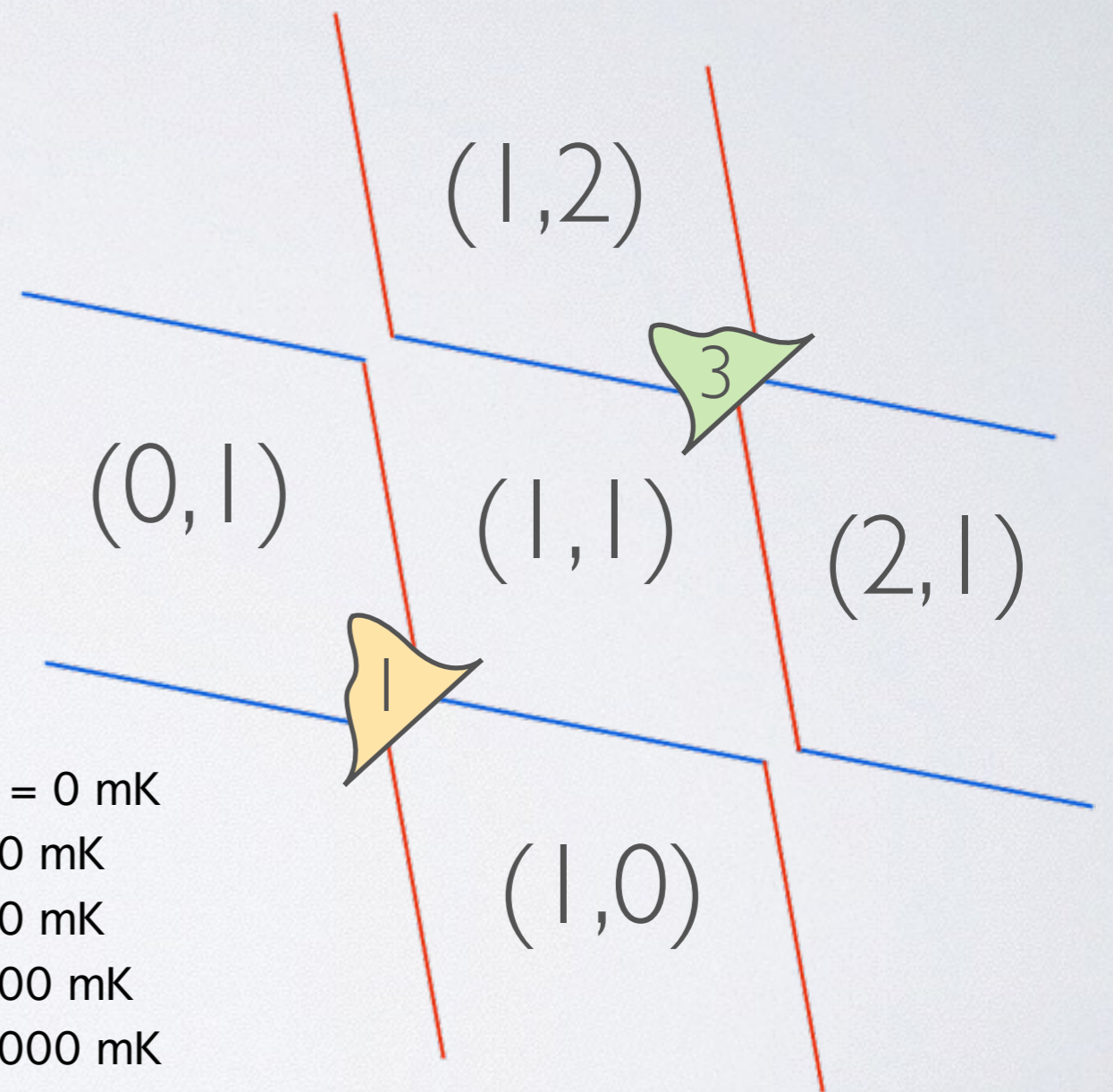


A four-fold degenerate state

(calculations)

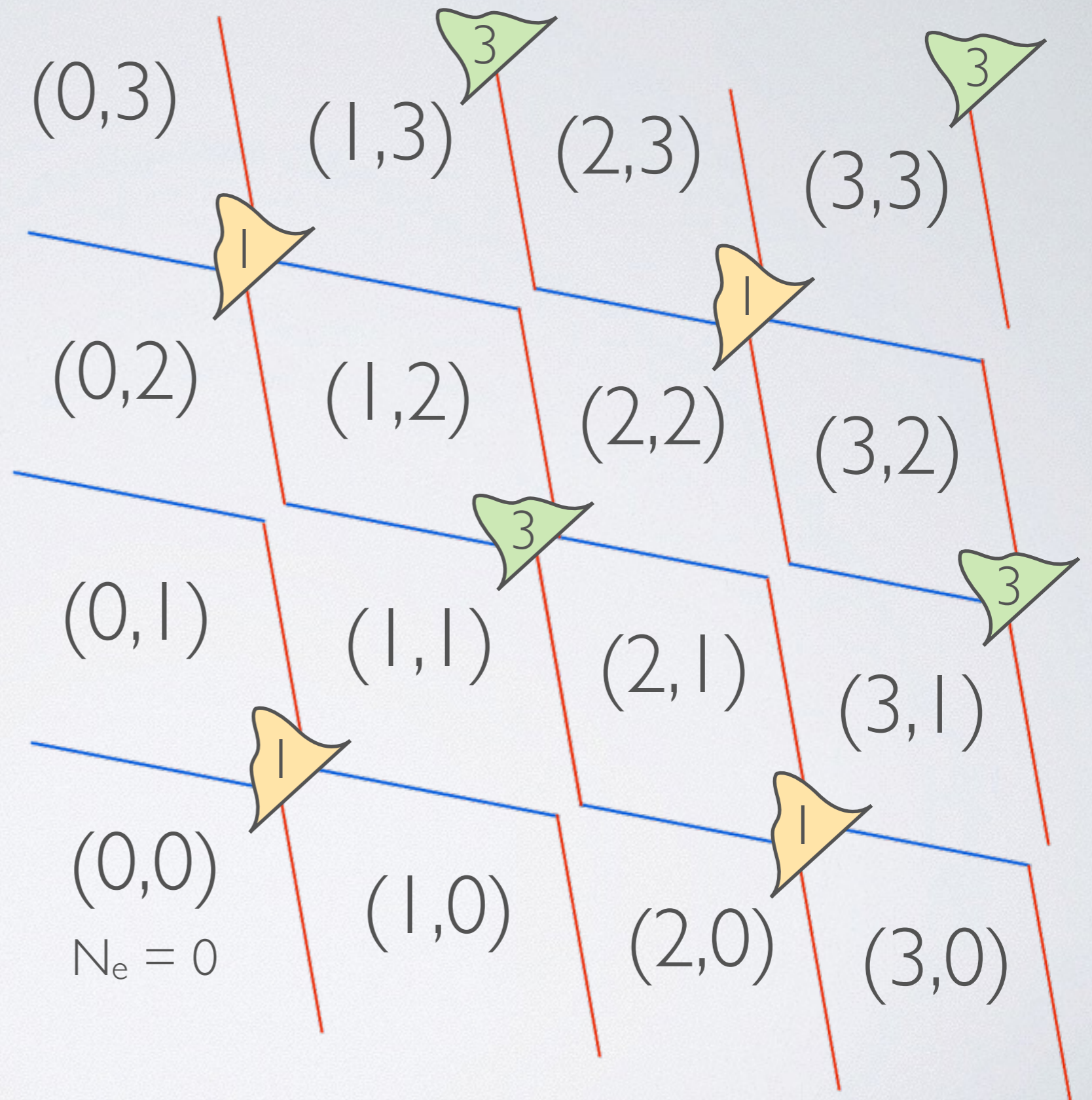


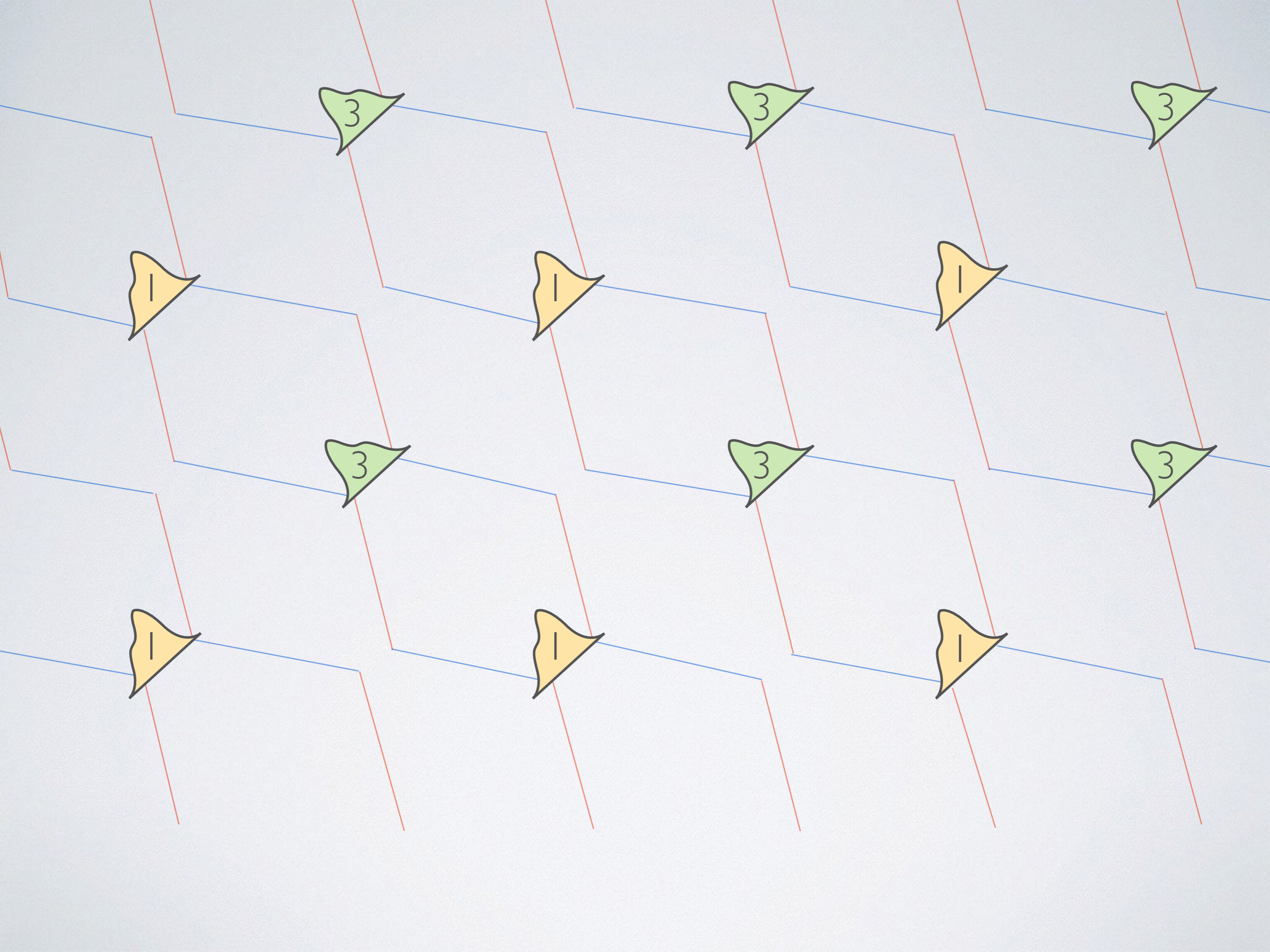
- $T = 0$ mK
- 10 mK
- 30 mK
- 100 mK
- 1000 mK

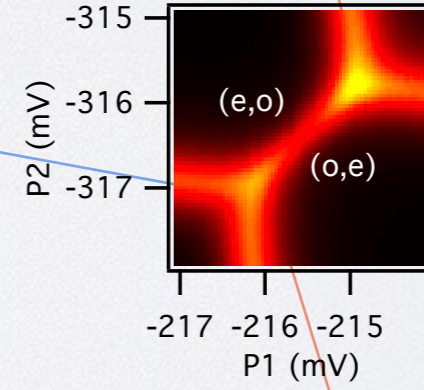
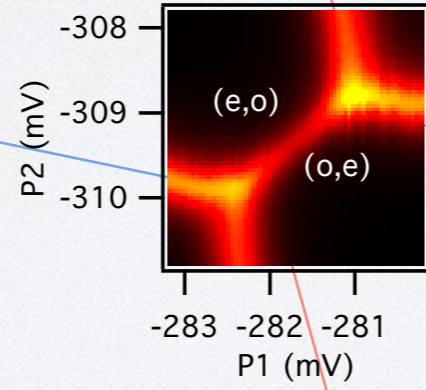
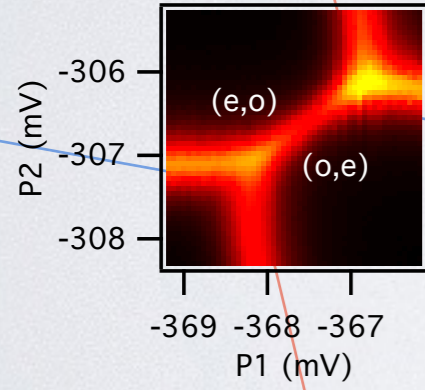
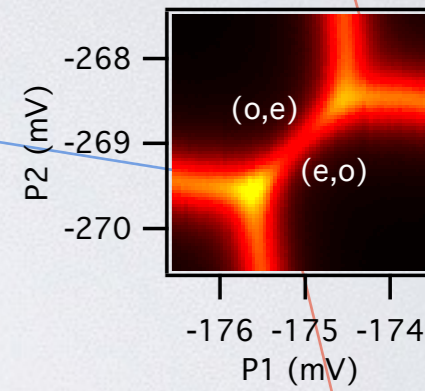
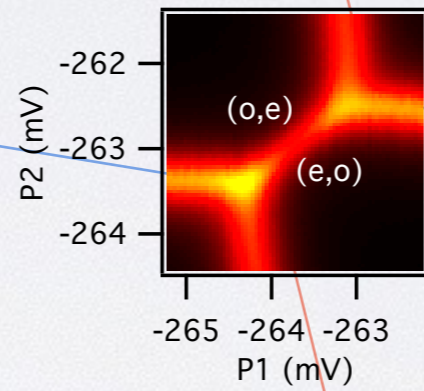
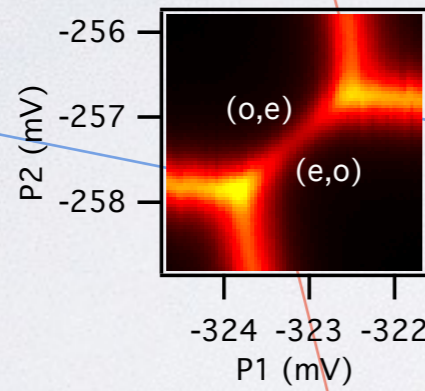
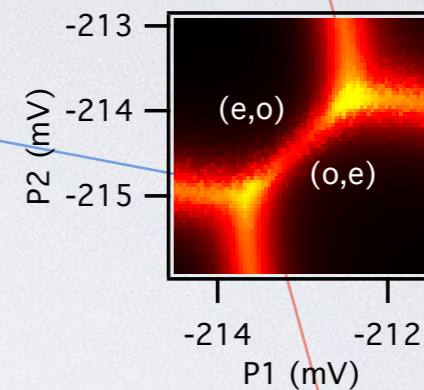
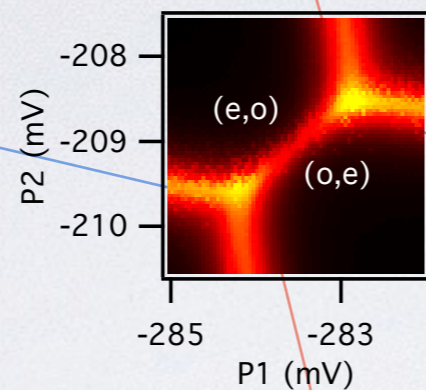
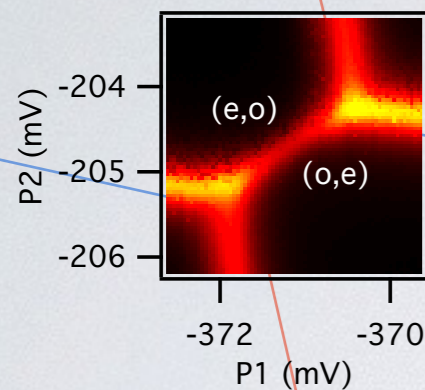
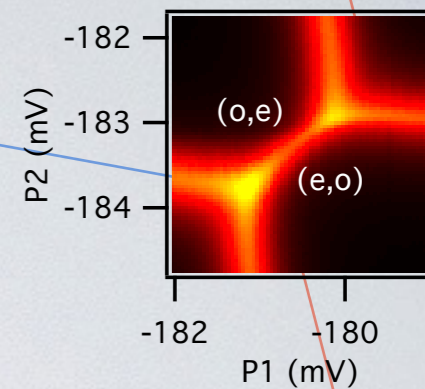
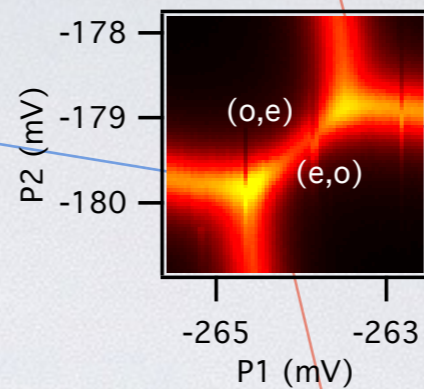
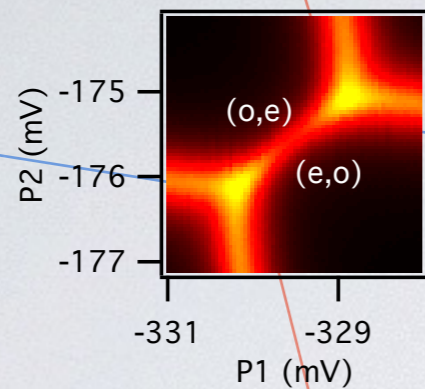


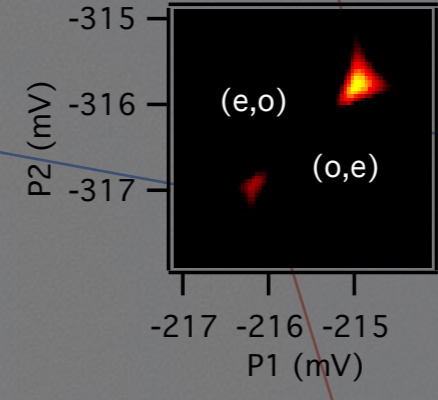
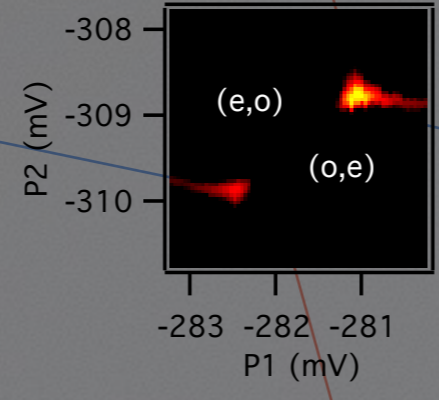
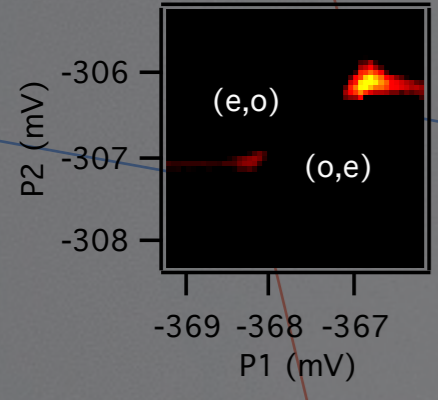
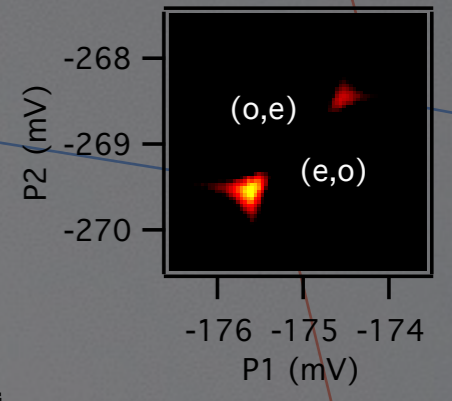
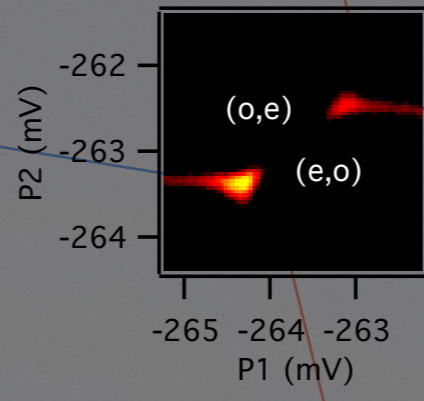
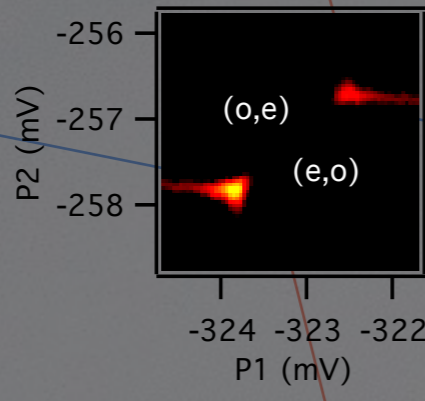
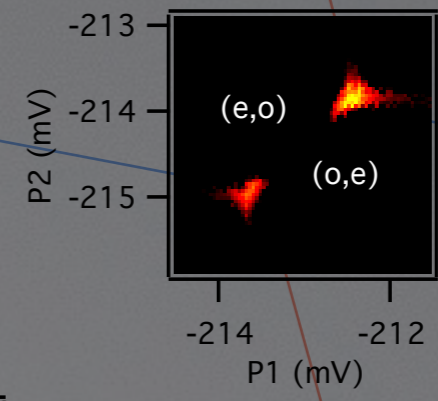
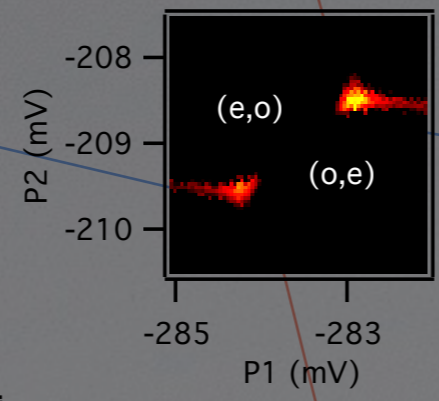
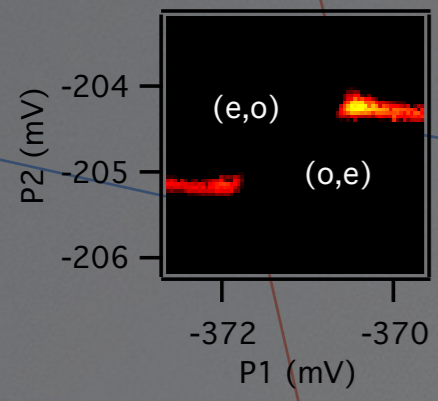
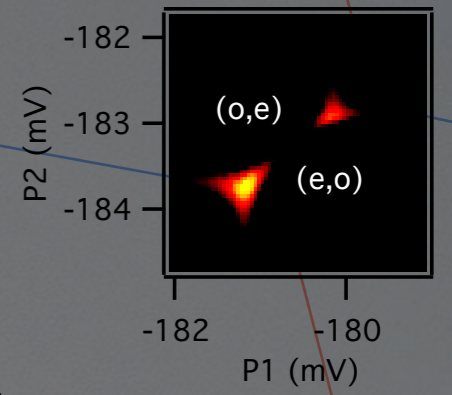
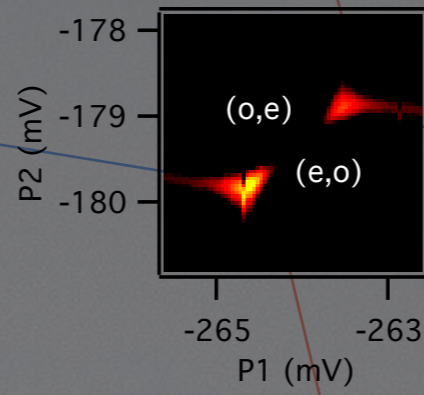
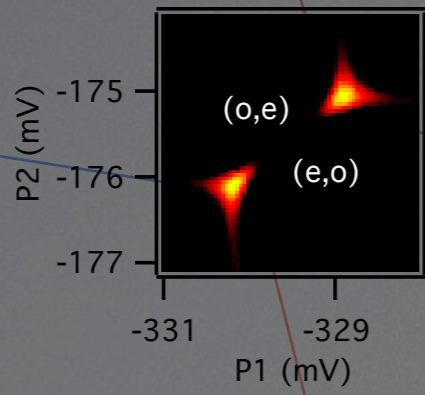
Adding two electrons to either dot should not change the physics

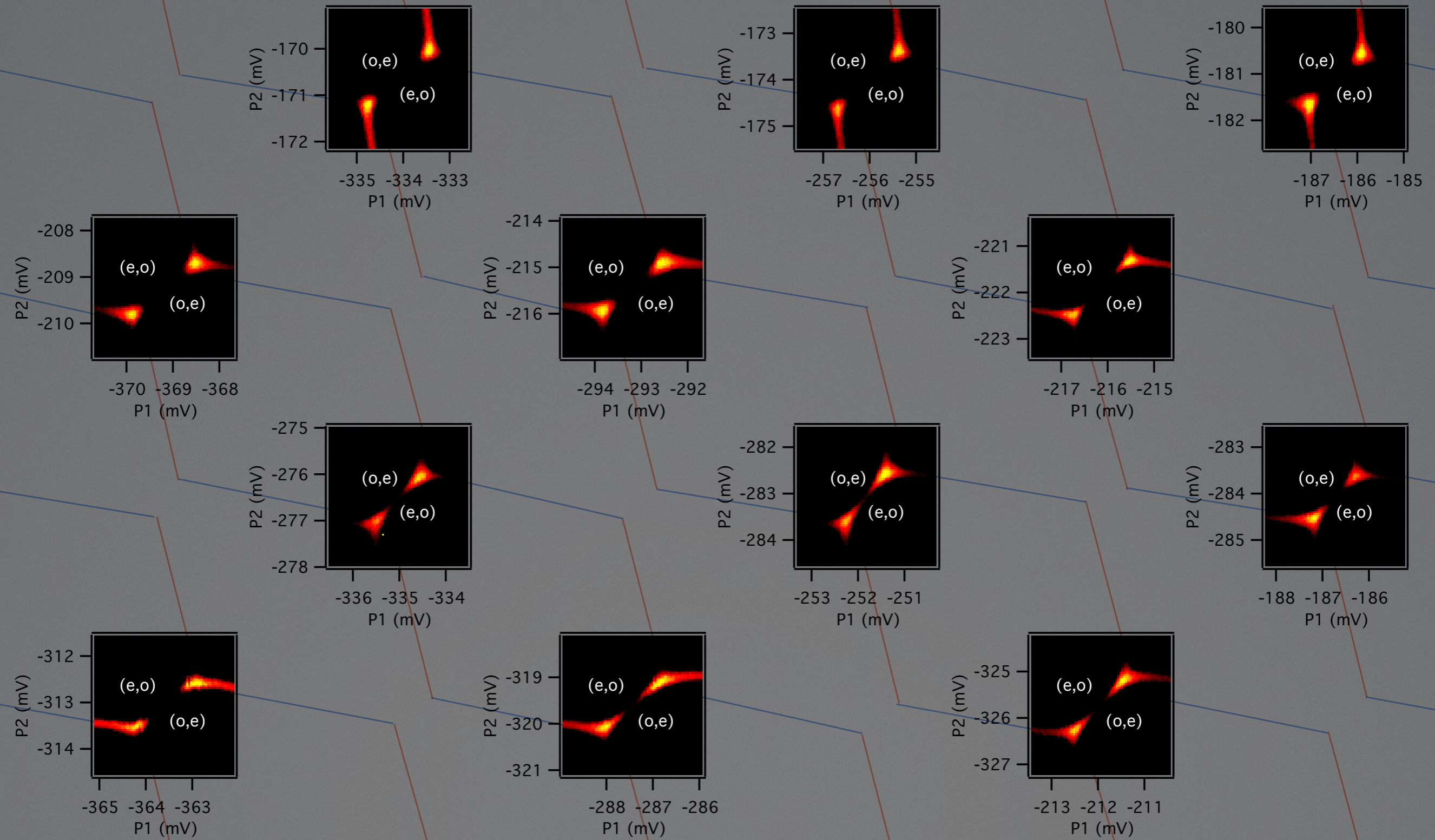
Conductance should always "point to" the (odd, odd) hexagon











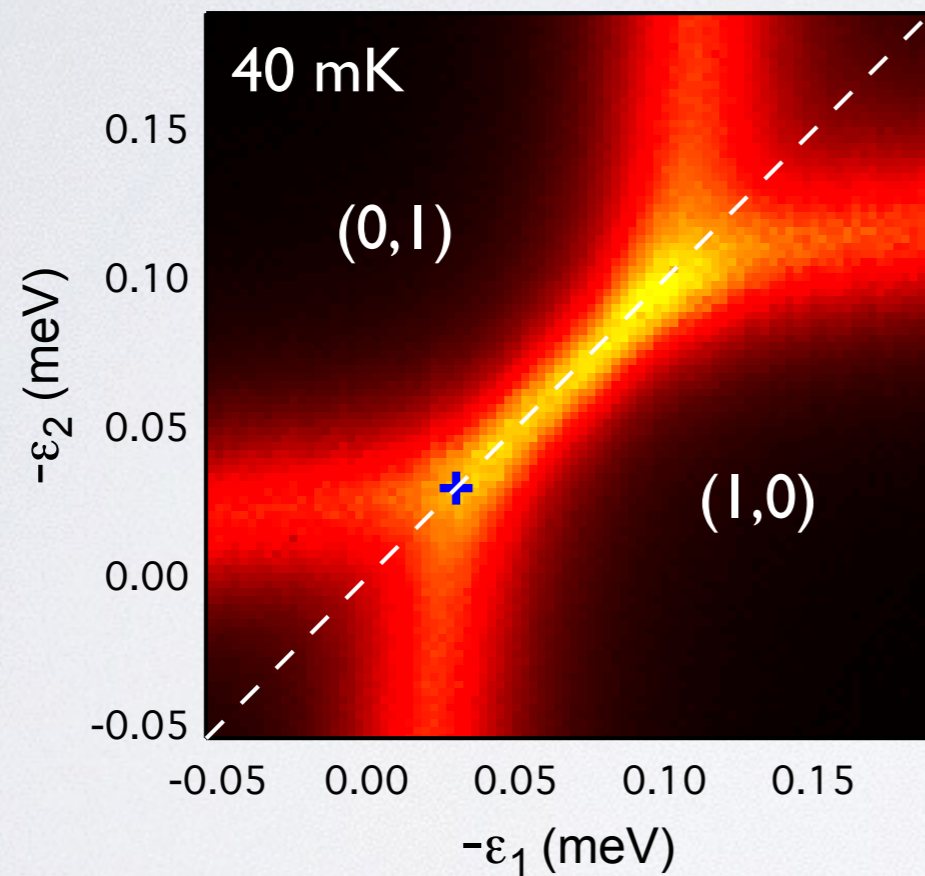
2 Tesla magnetic field

Establishing a correspondence between experiment and theory

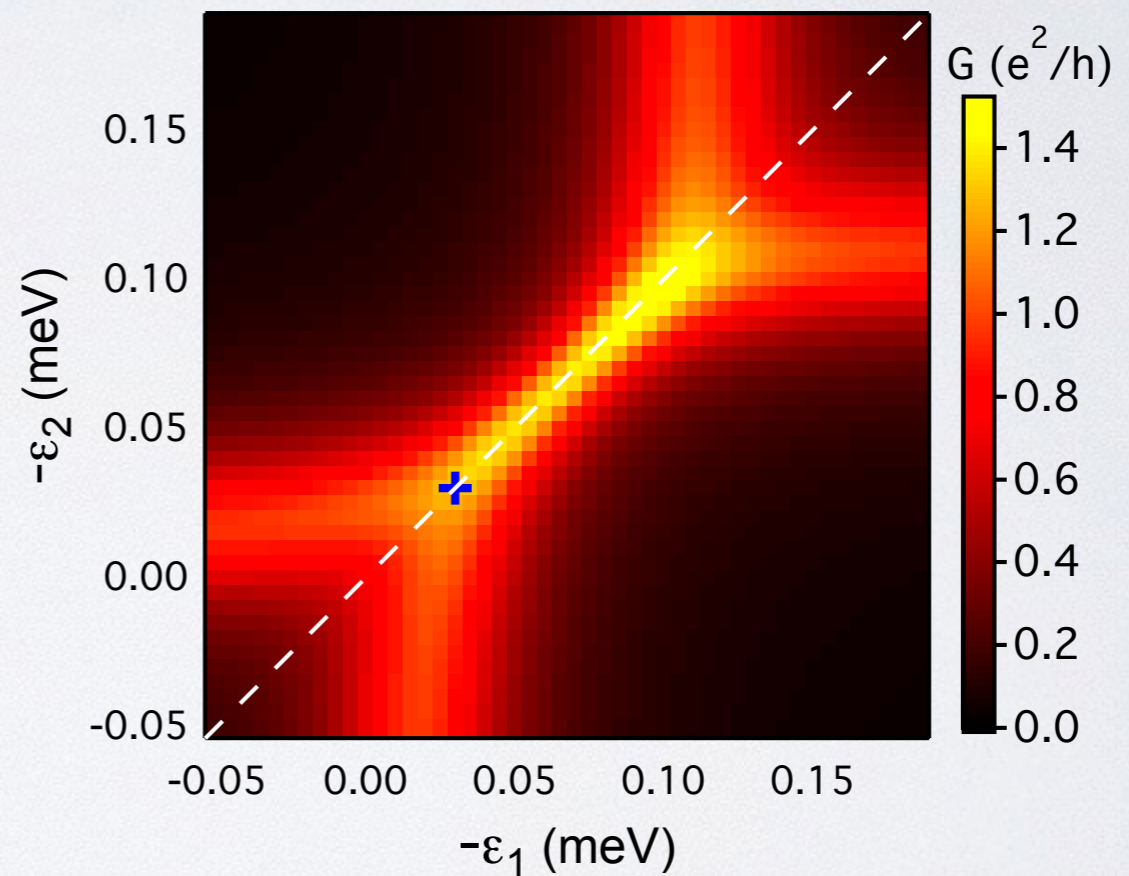
- Agreement between simple theoretical models and experiments gives confidence in the interpretation of experimental data
- This correspondence helps us compare experimental data to the prediction of universal models

Establishing a correspondence between experiment and theory

Experiment



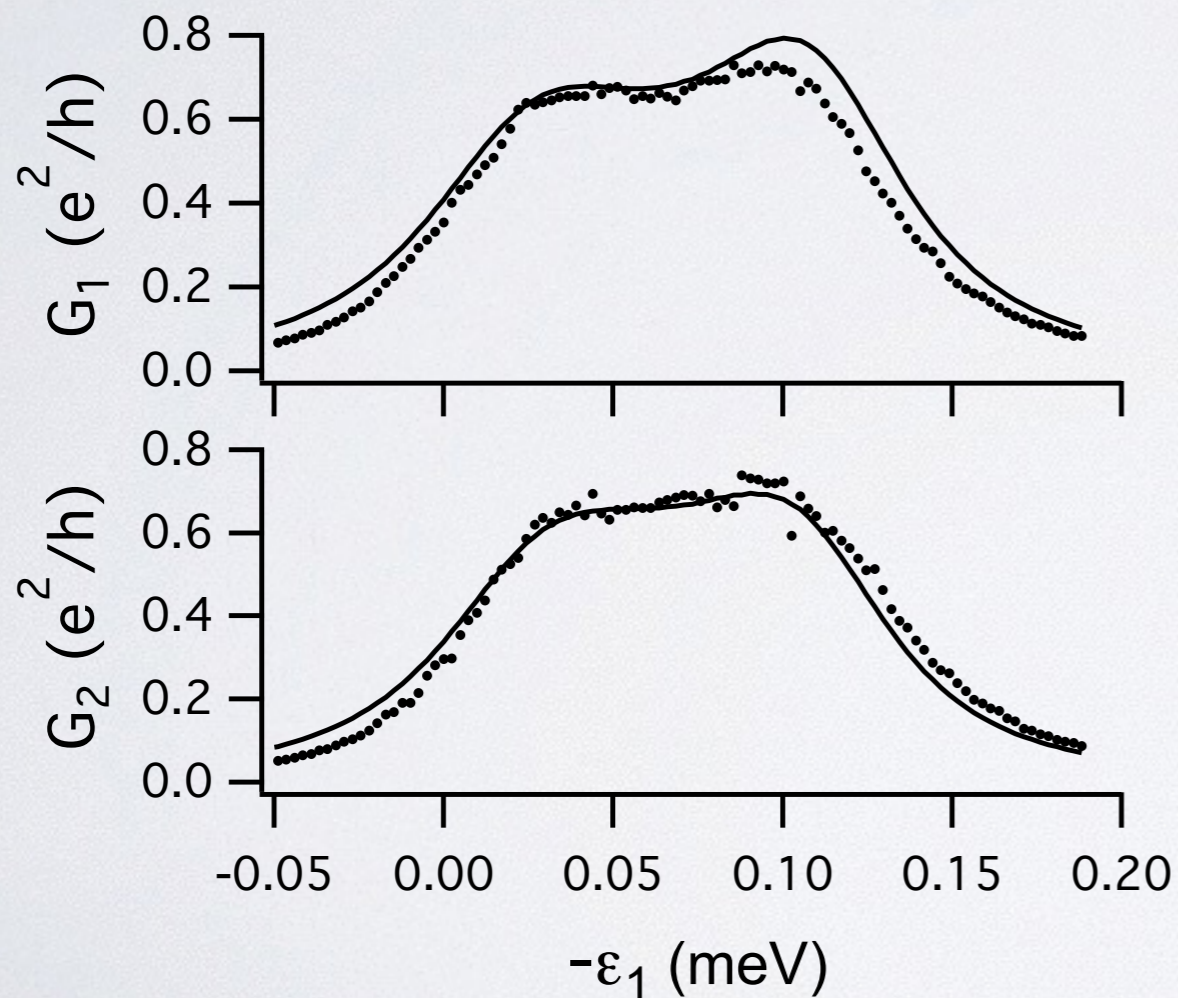
NRG



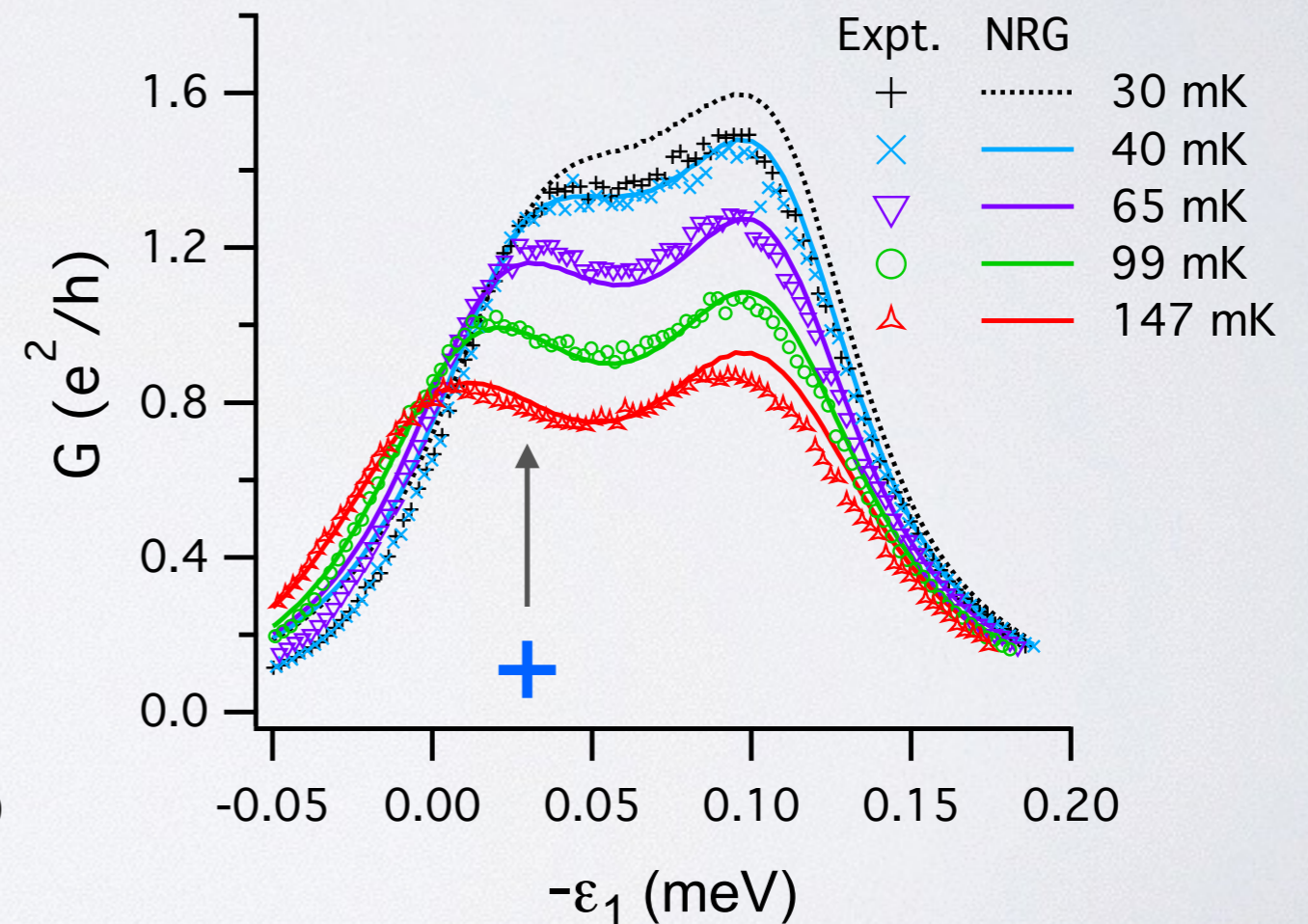
$N = 1$ LBTP

Establishing a correspondence between experiment and theory

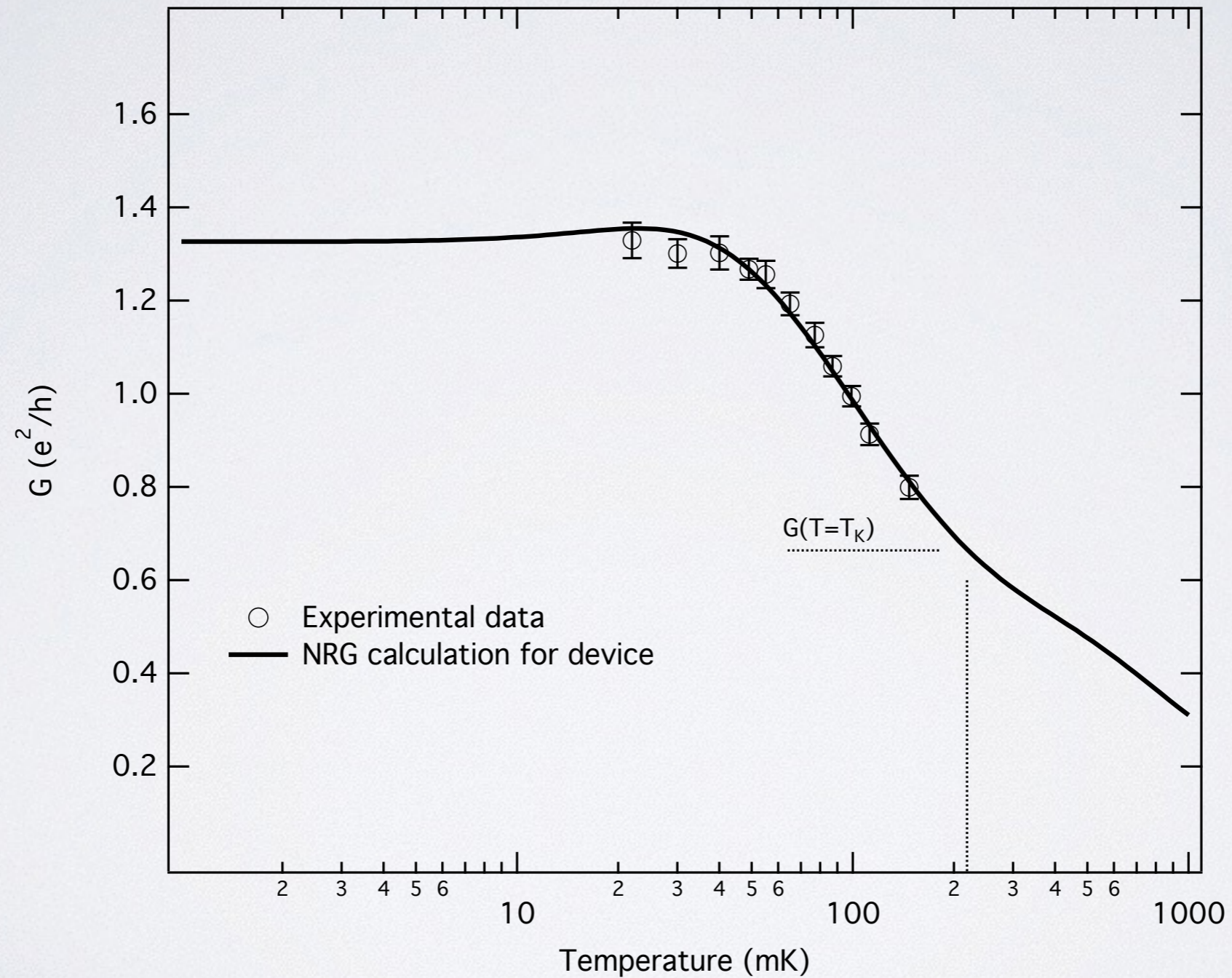
Orbital state-resolved conductances at 40 mK



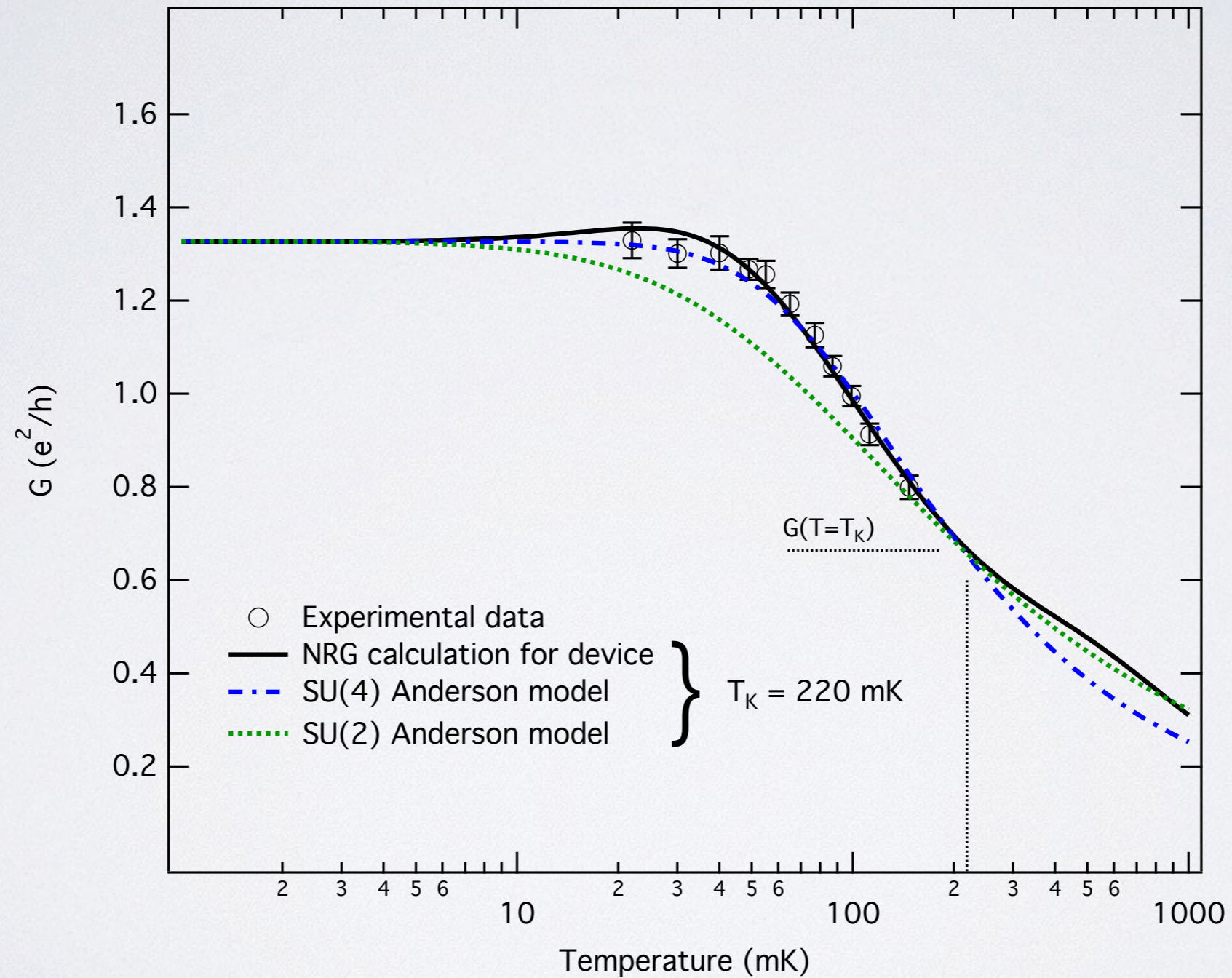
Temperature dependent conductances



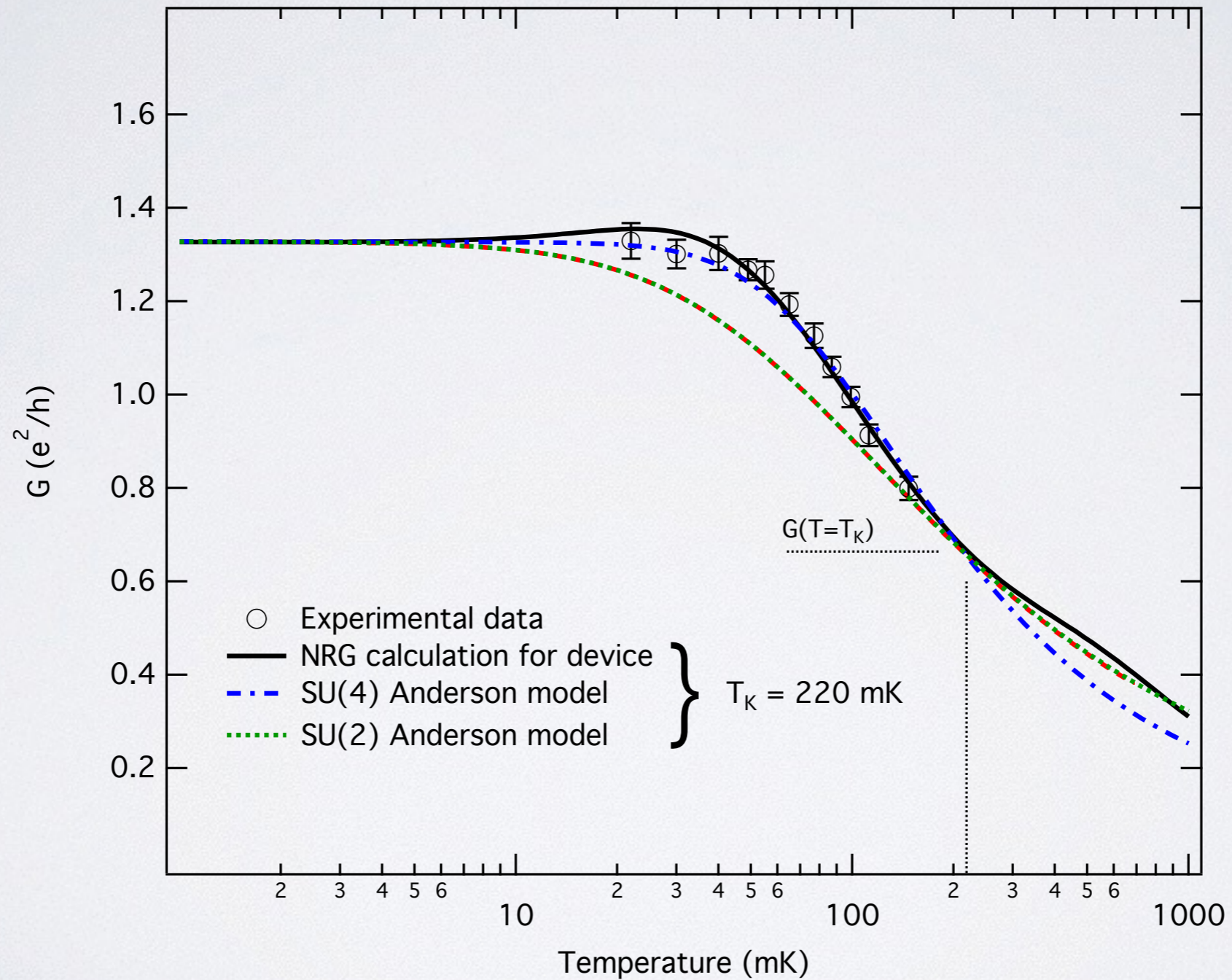
Comparing to universal models



Comparing to universal models



Comparing to universal models

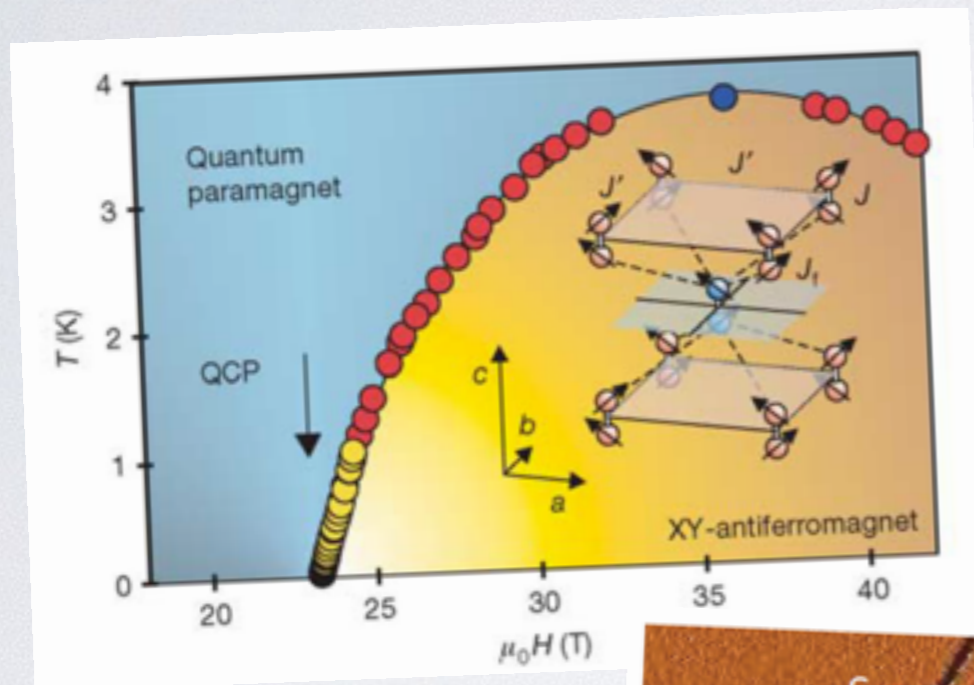


Outline

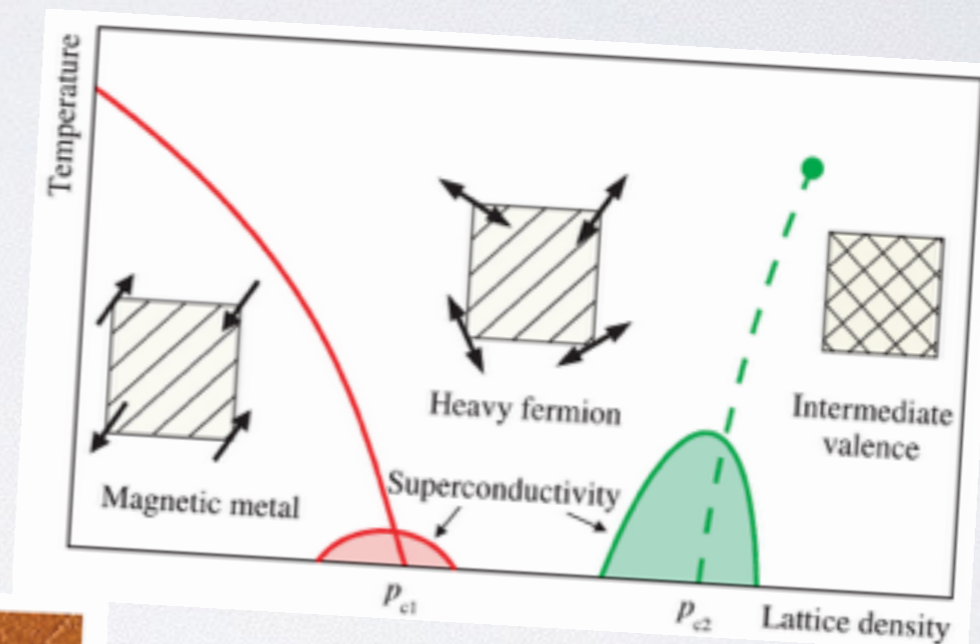
- Symmetries in physics
- Quantum dots and the Kondo effect
- Emergent $SU(4)$ -symmetric Kondo effect in a double quantum dot
- **Universal crossover between quantum critical and Fermi liquid states in a quantum dot coupled to a “metallic grain”**

Quantum criticality

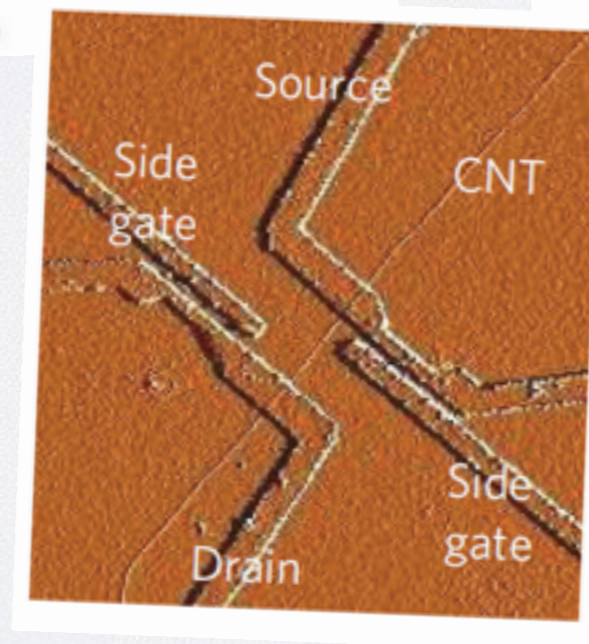
Materials design and device design as complementary approaches



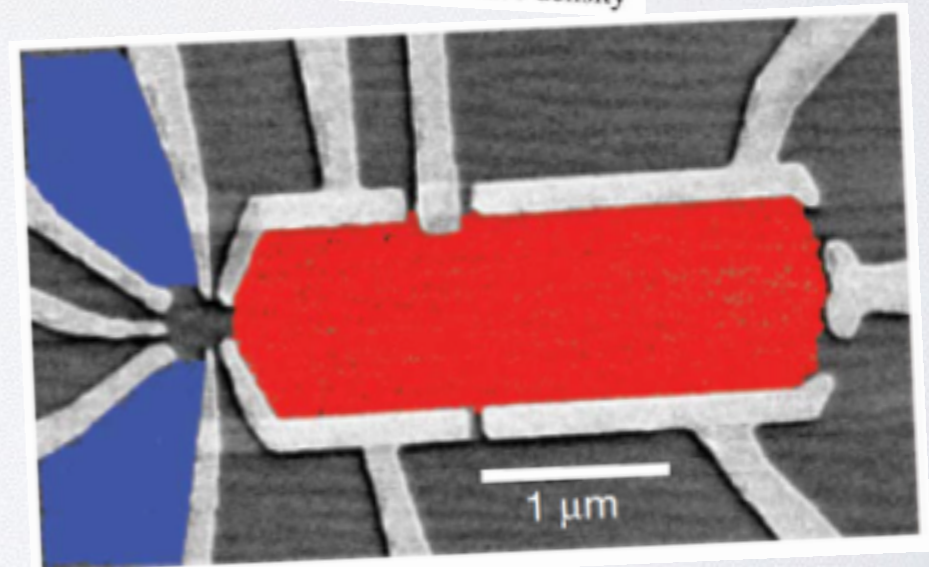
S. E. Sebastian *et al.*,
Nature **441**, 617 (2006).



H. Q. Yuan *et al.*,
Science **302**, 2104 (2003).

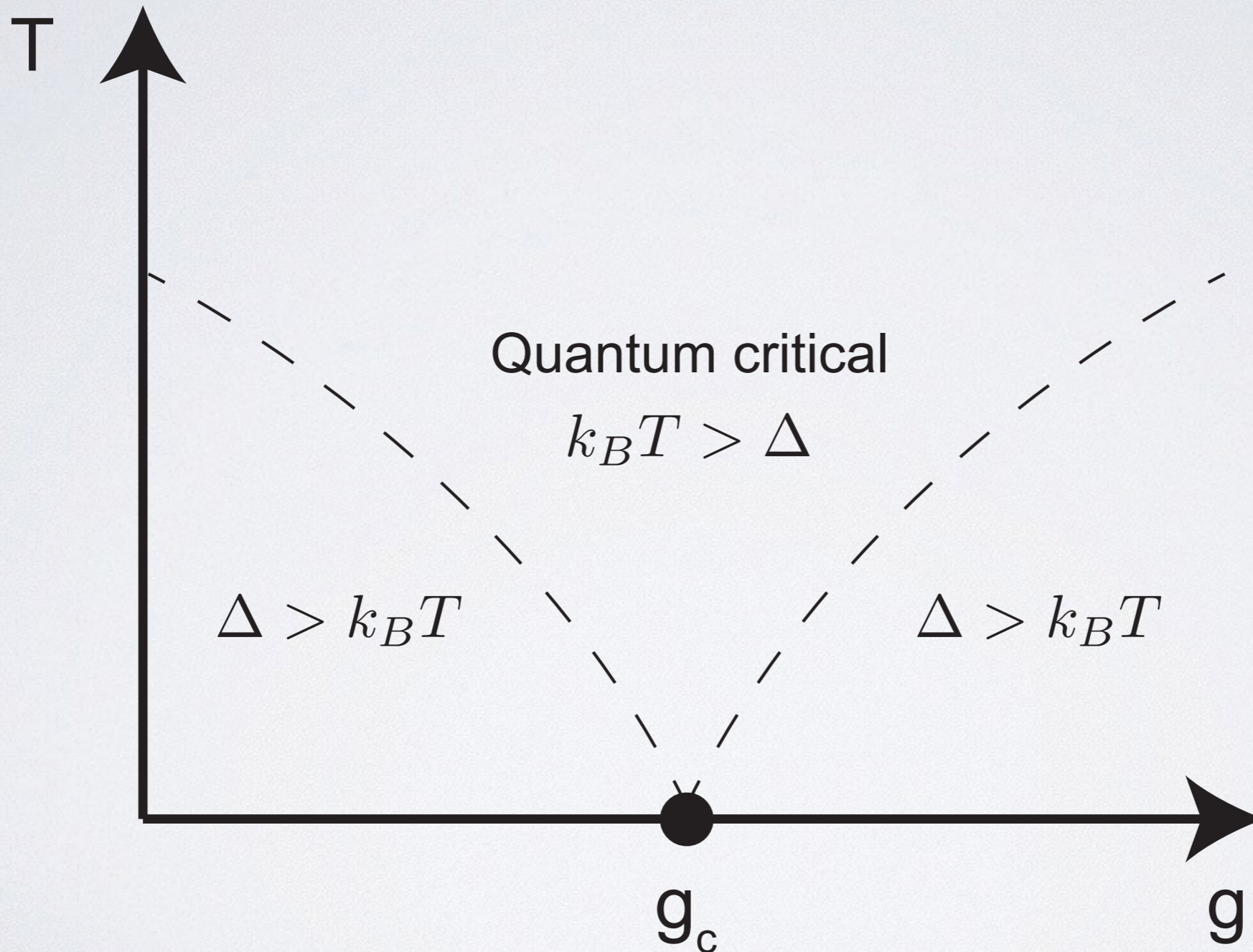


H. Mebrahtu *et al.*,
Nature Phys. **9**, 732 (2013).



R. Potok *et al.*, Nature **446**, 167 (2007).

Quantum criticality

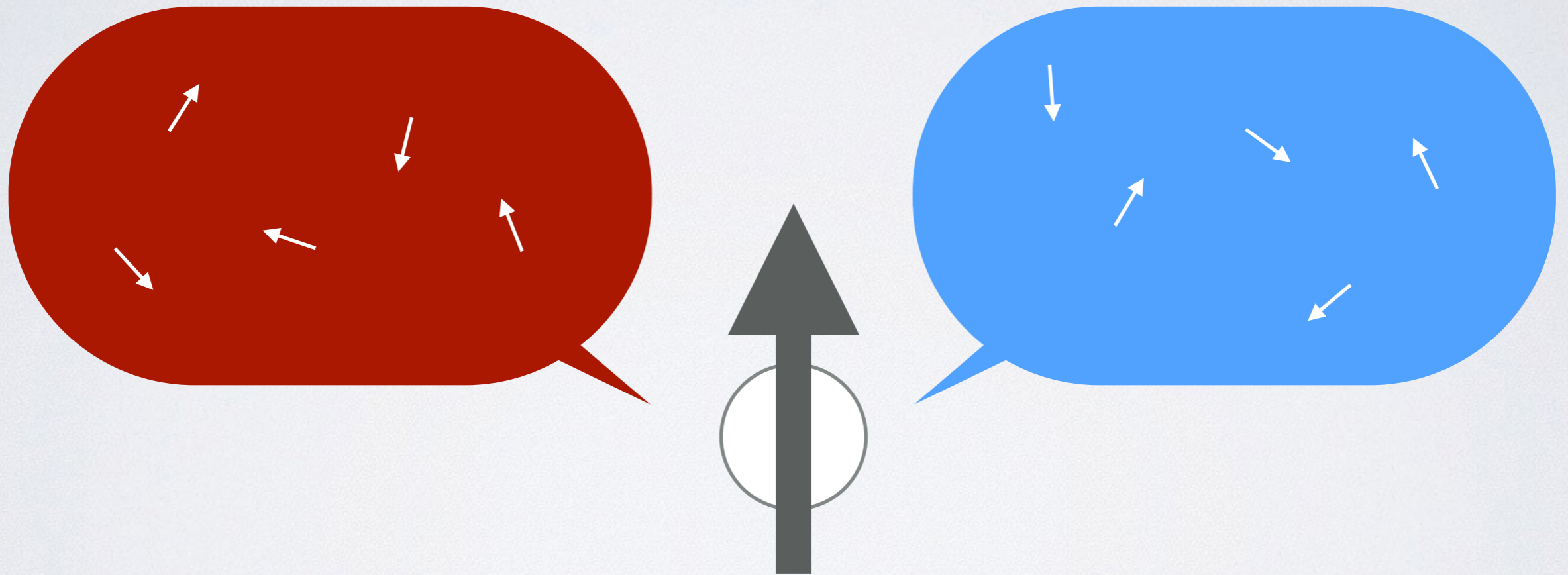


Chakravarty, Halperin, Nelson 1989

Figure: S. Sachdev, *Quantum Phase Transitions* 2nd ed. (2011)

Two-channel Kondo effect

A model quantum phase transition



Two-channel Kondo effect, by way of *Office Space* (1999)

Boss #1



Boss #2



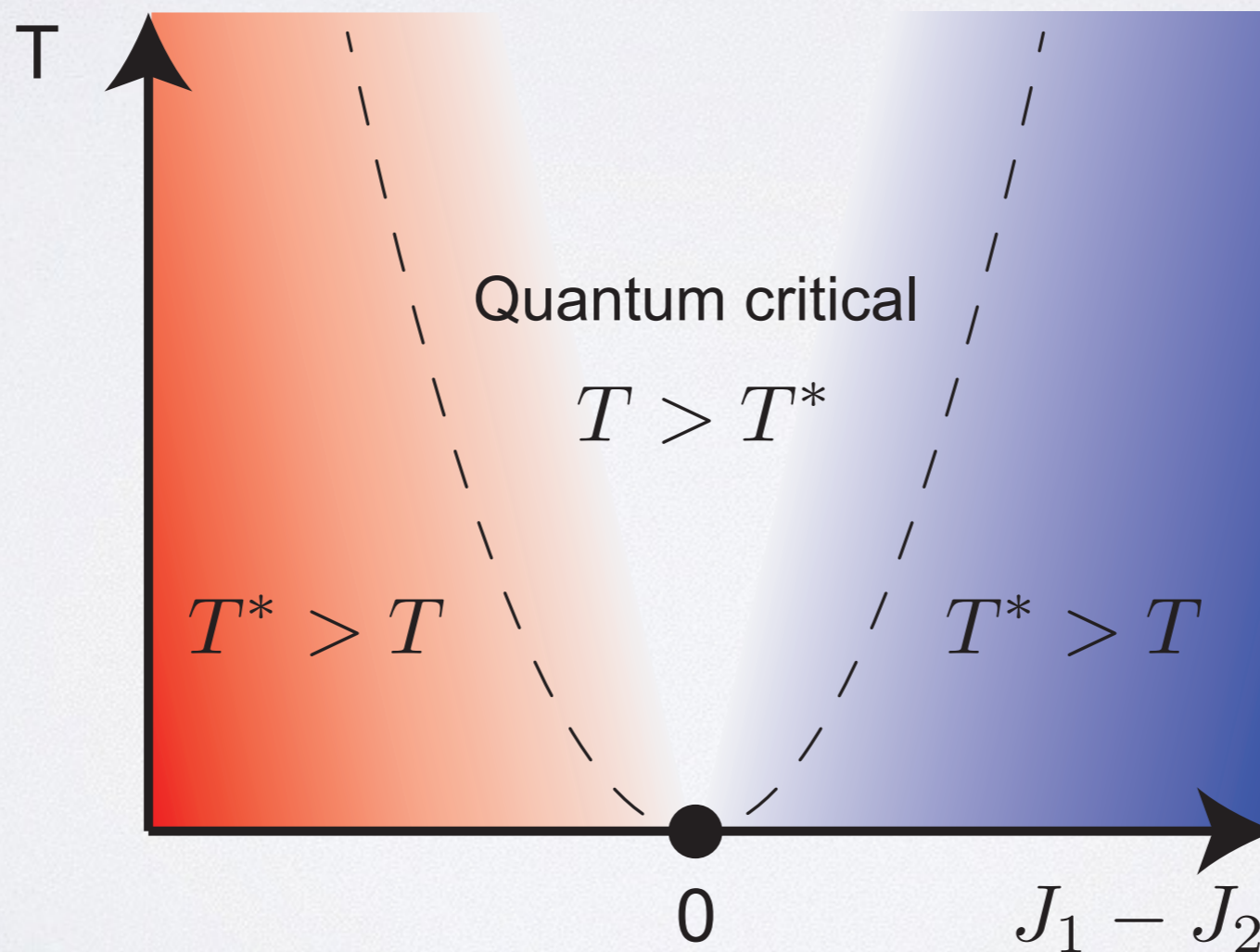
Employee



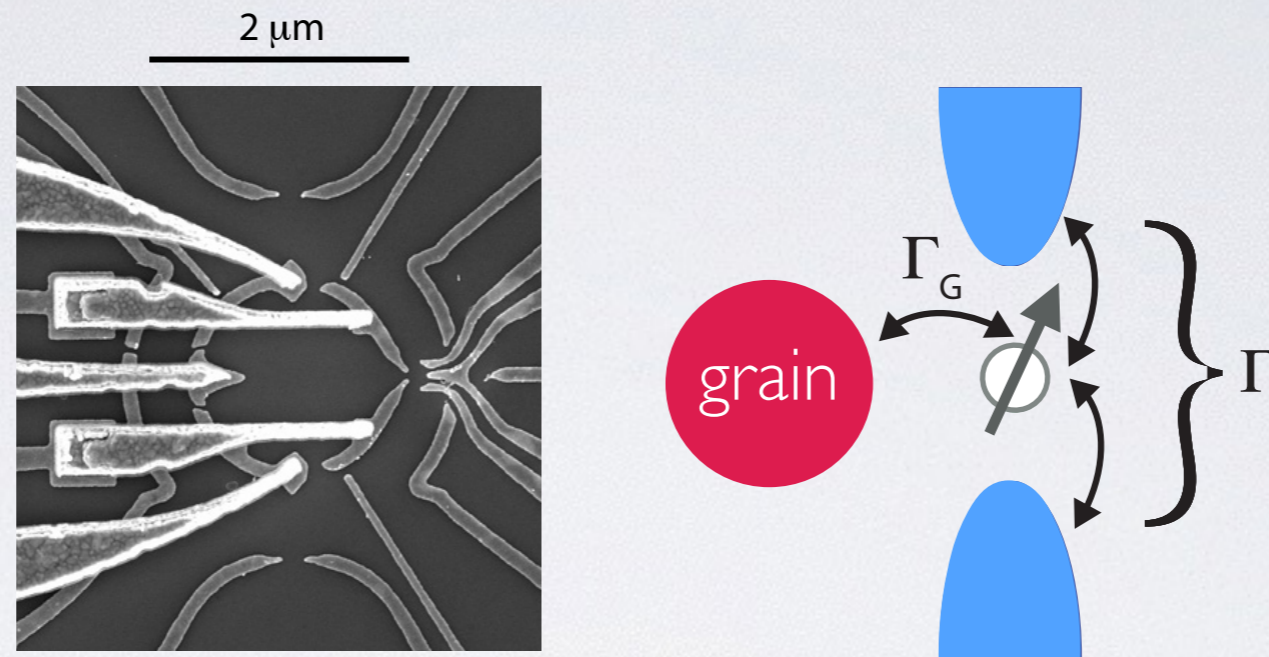
Two-channel Kondo effect

A model quantum phase transition

$$H_{2\text{CK}} = \sum_{\alpha, k} \epsilon_k c_{k\sigma\alpha}^\dagger c_{k\sigma\alpha} + J\vec{S} \cdot (\vec{s}_L + \vec{s}_R) + \delta H_{2\text{CK}}$$



How do we implement this?

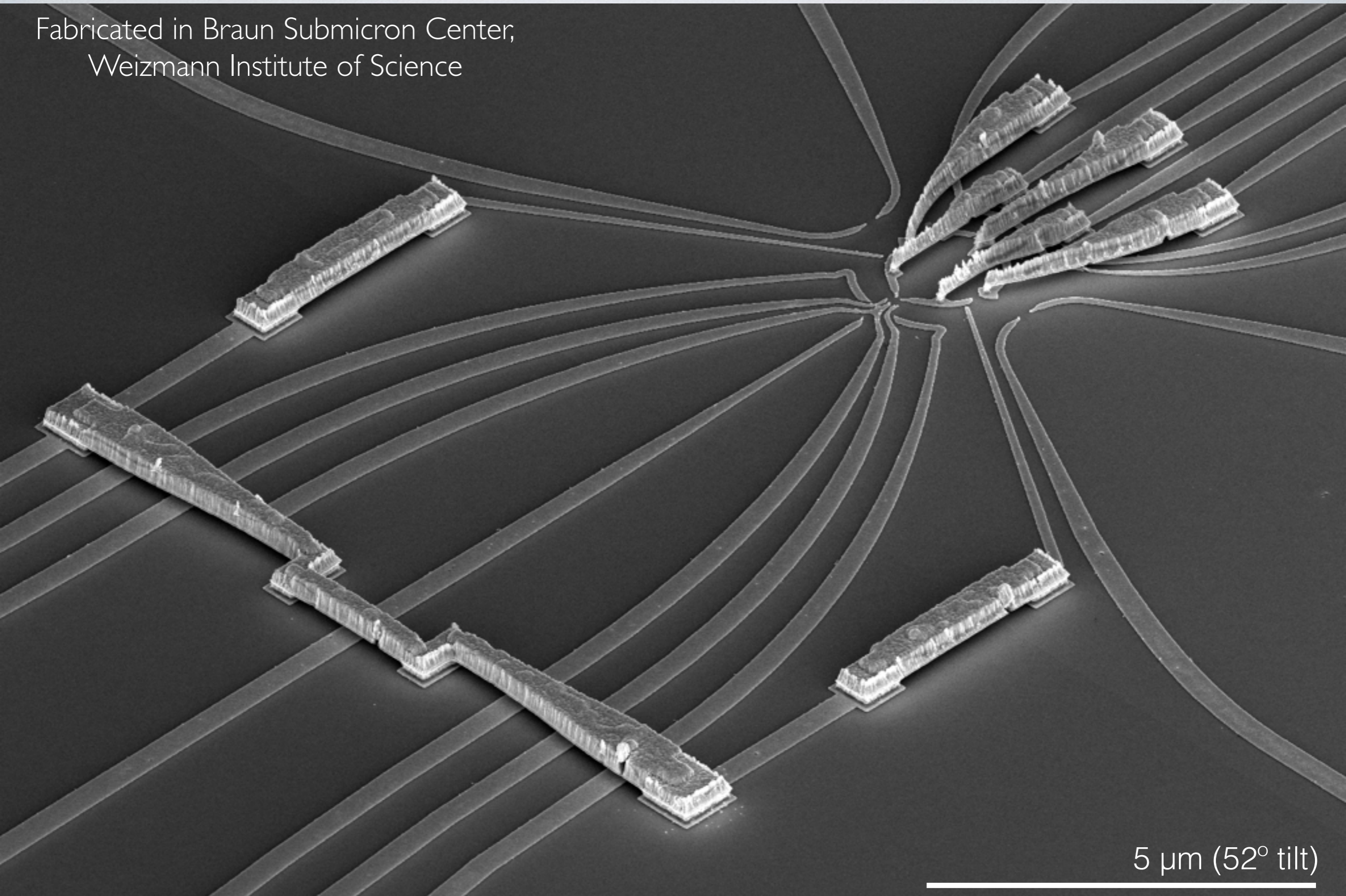


Quantum dot tunnel coupled to a “metallic grain”

Small enough that grain charging energy $E_C \gg k_B T$

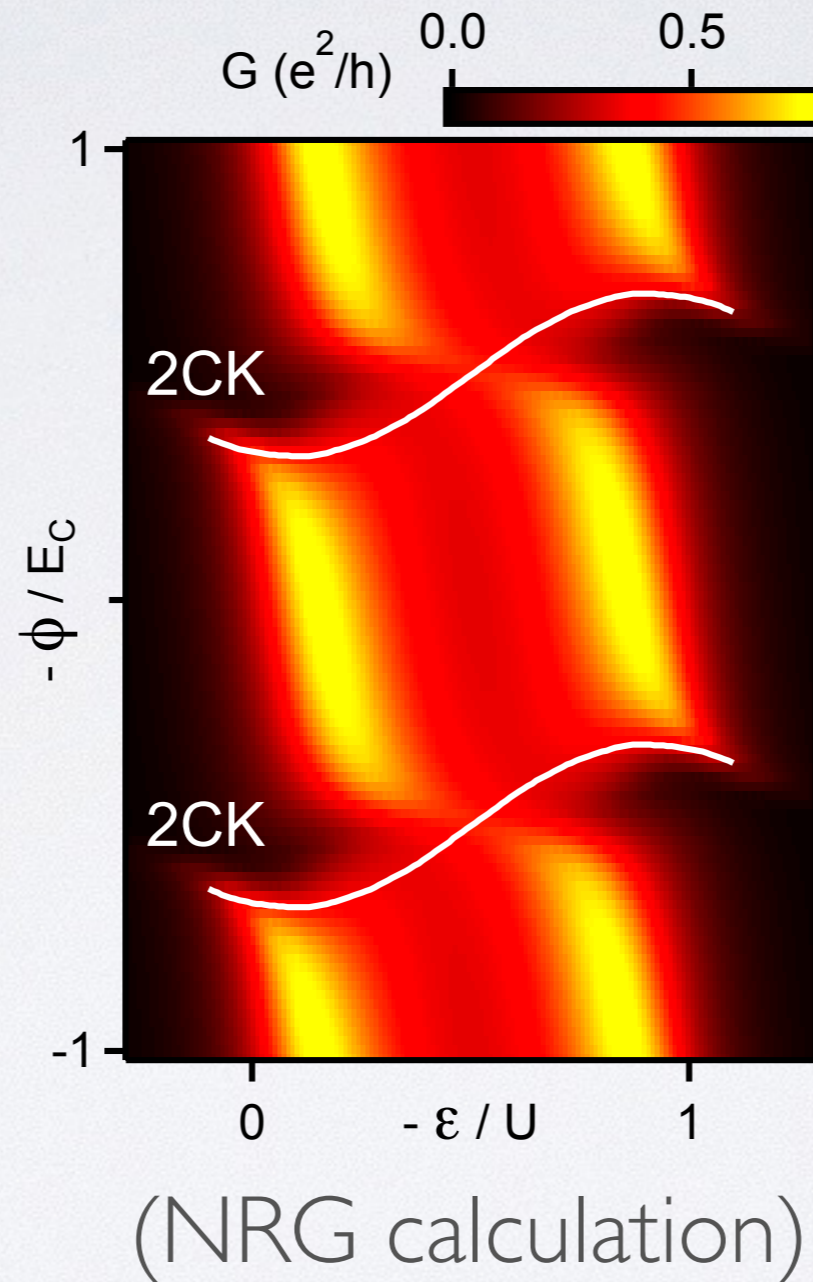
Large enough that the level spacing $\Delta \lesssim k_B T$

Fabricated in Braun Submicron Center,
Weizmann Institute of Science

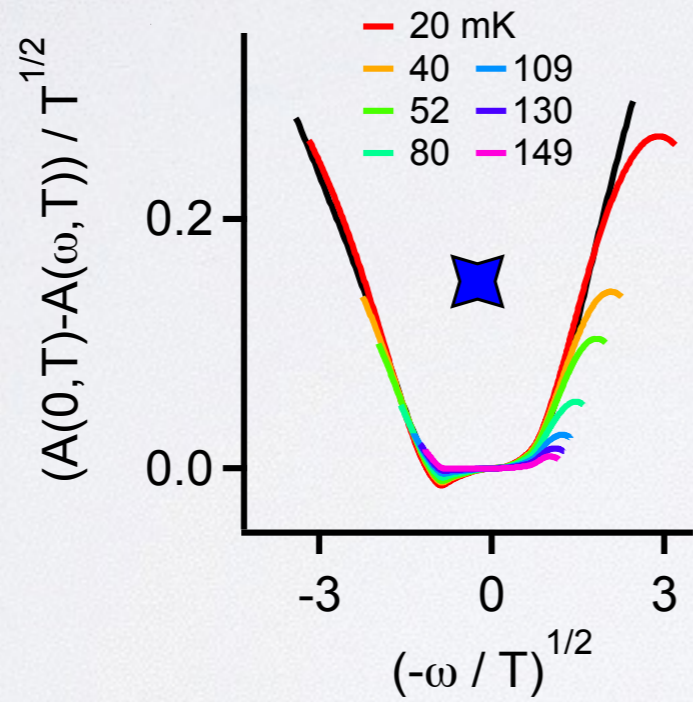
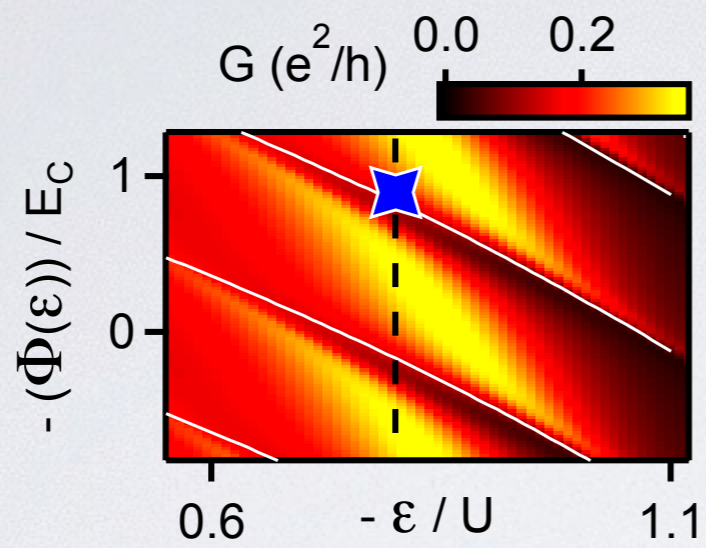


5 μm (52° tilt)

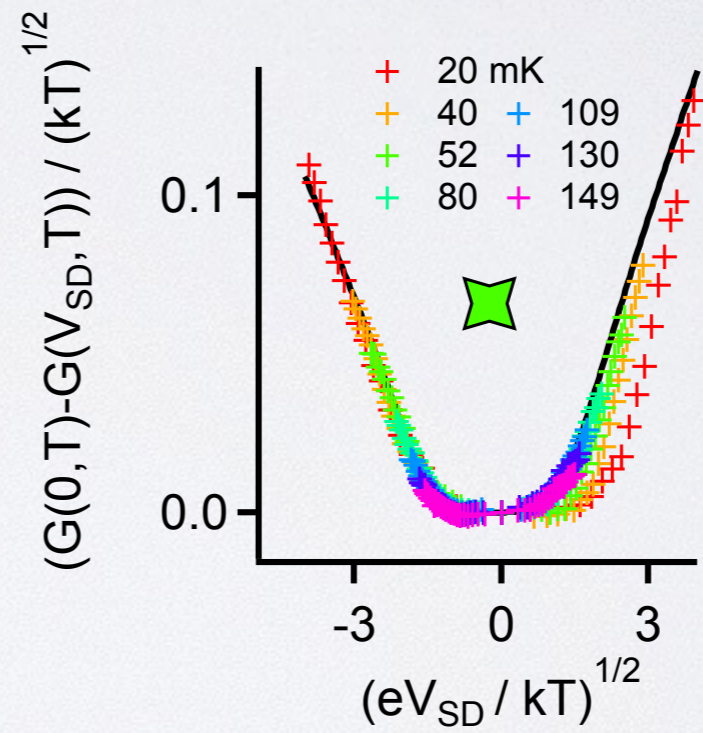
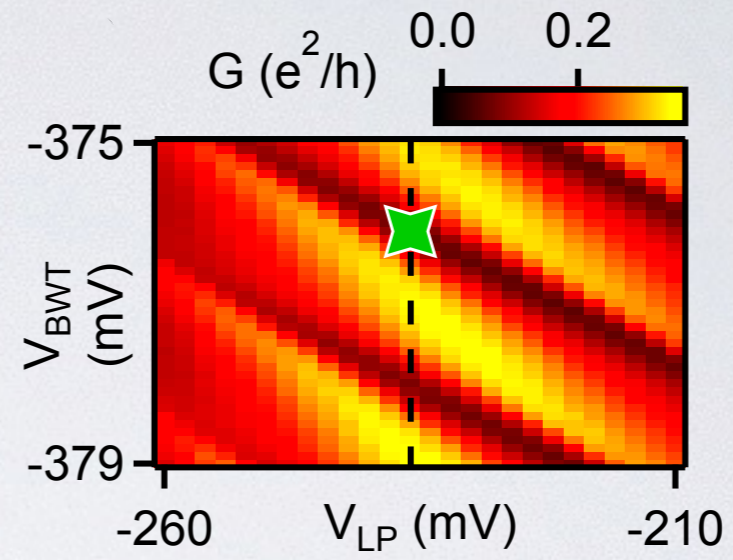
Identifying a 2CK state



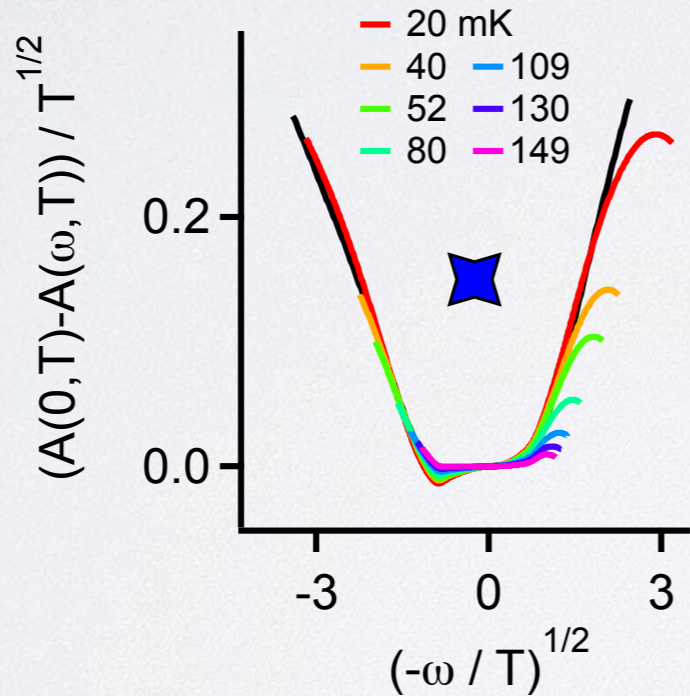
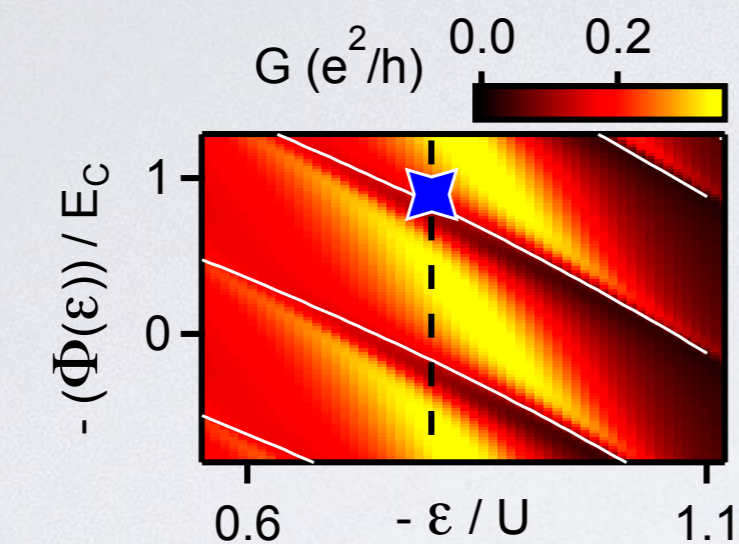
NRG



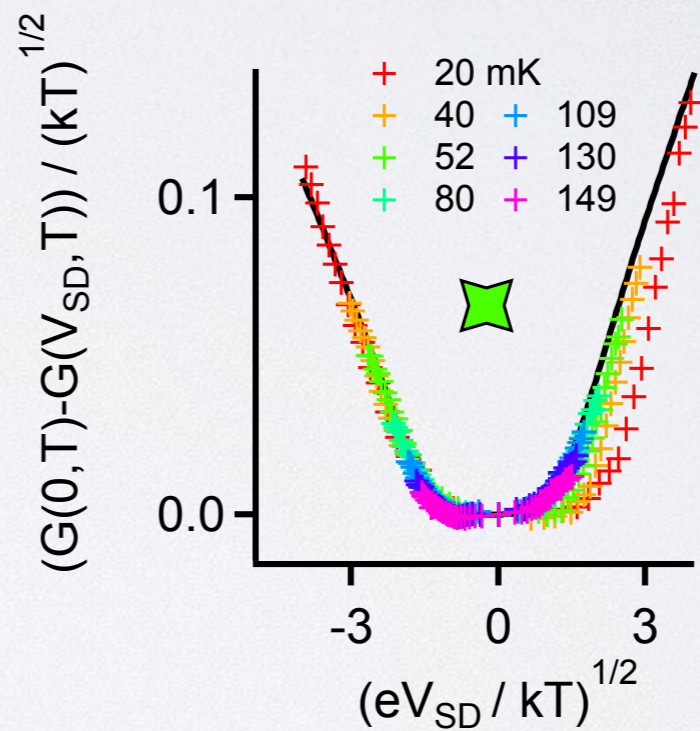
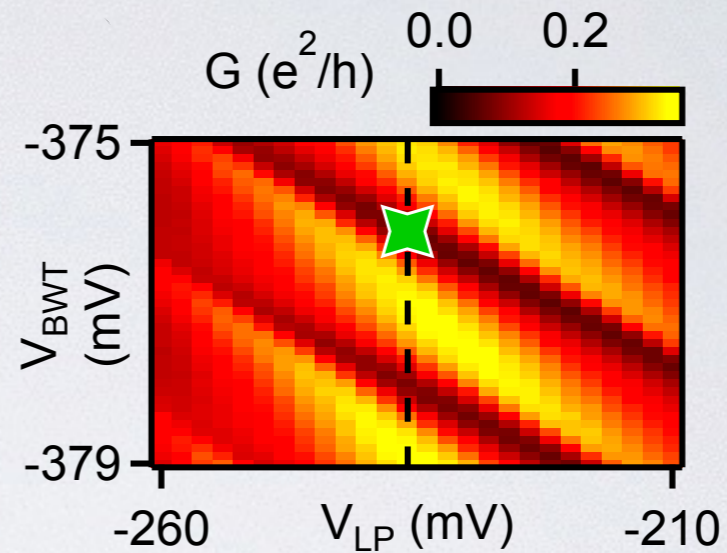
Experiment



NRG



Experiment



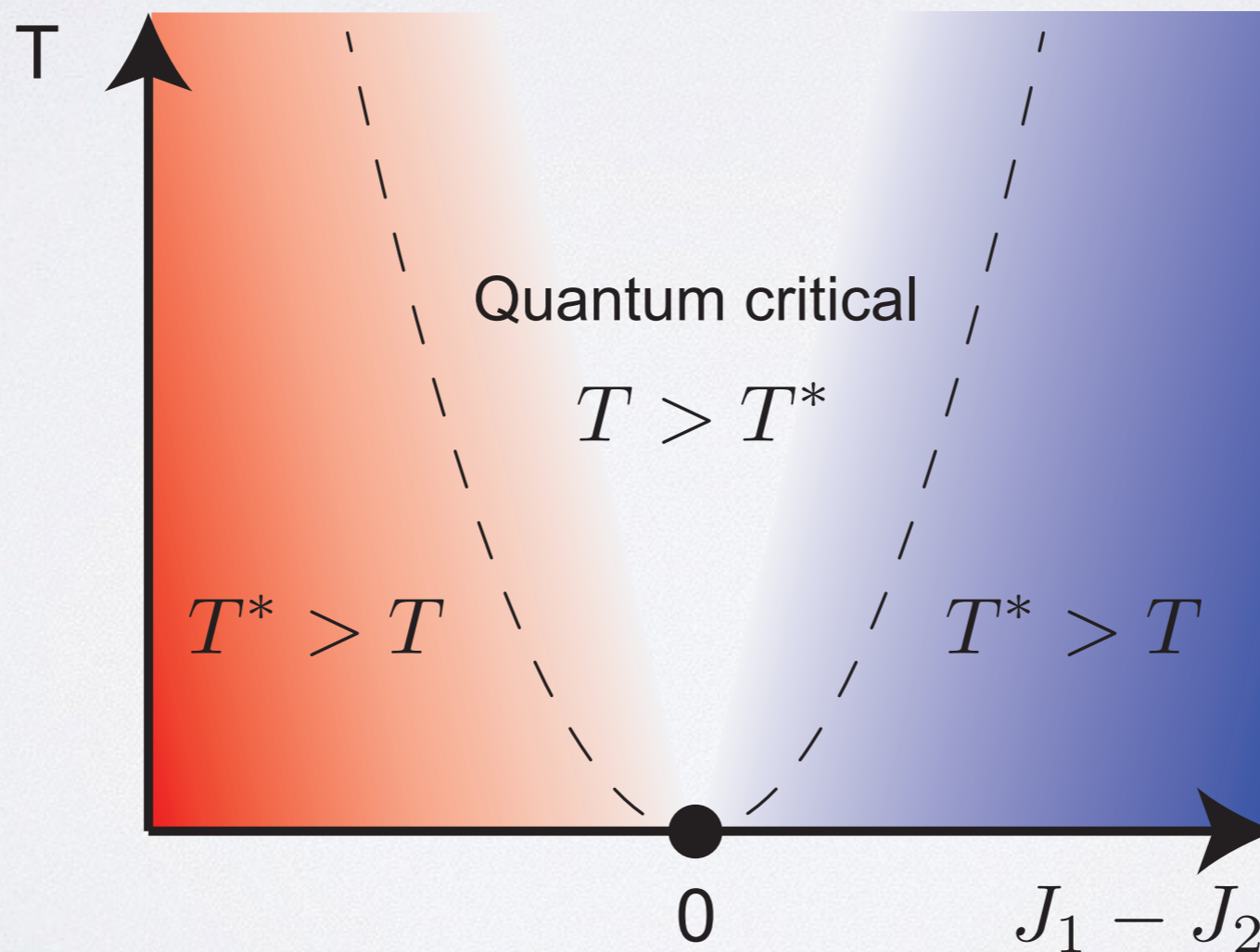
$$\Sigma^R(\omega) = -\frac{i n_i}{2\pi\nu} \left\{ [1 - e^{2i\delta_P} S_{(1)}] - e^{2i\delta_P} N\lambda \left(\frac{2\pi}{\beta}\right)^\Delta 2 \sin(\pi\Delta) \int_0^1 du \left[u^{-i\beta\omega/2\pi} u^{-1/2} (1-u)^\Delta F(u) - \frac{\Gamma(1+2\Delta)}{\Gamma^2(1+\Delta)} u^{(\Delta-1)} (1-u)^{-(1+\Delta)} \right] \right\}.$$

Affleck and Ludwig, PRB 48, 7297 (1993)

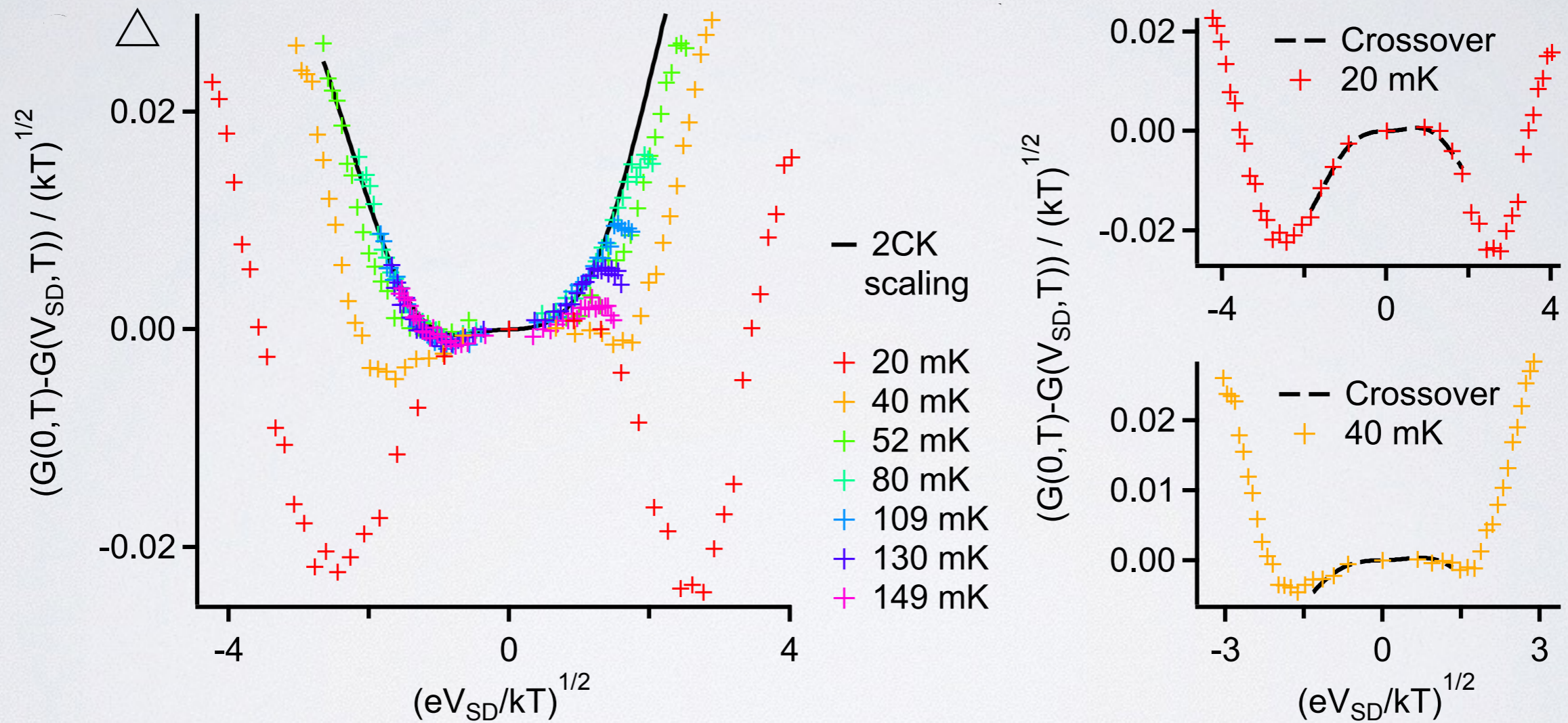
Two-channel Kondo effect

A model quantum phase transition

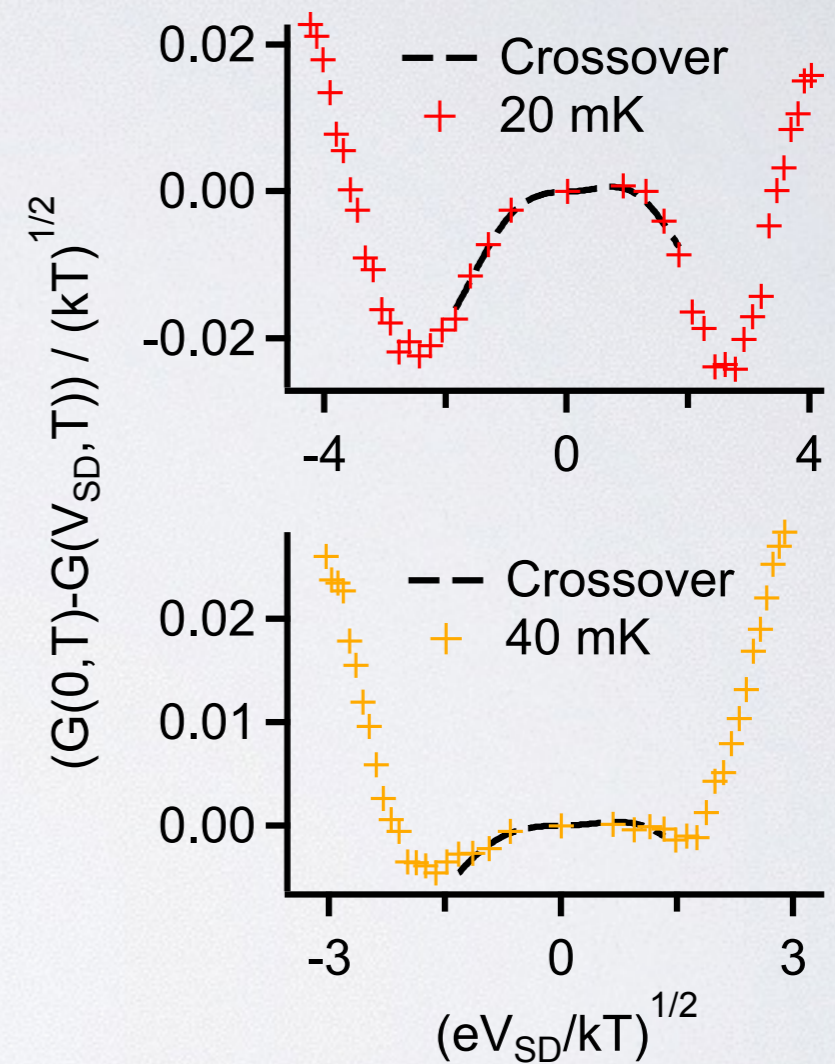
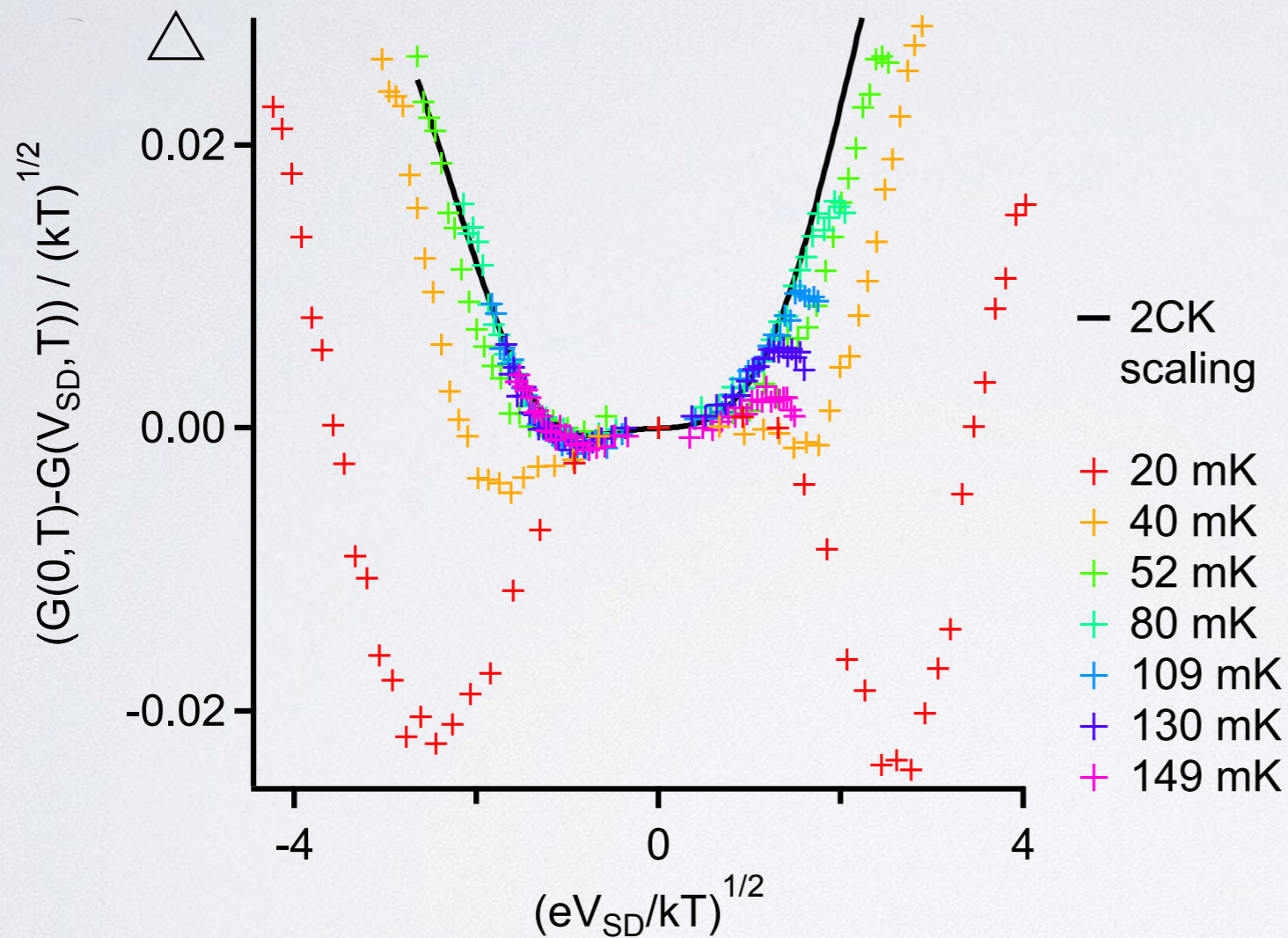
$$H_{2\text{CK}} = \sum_{\alpha, k} \epsilon_k c_{k\sigma\alpha}^\dagger c_{k\sigma\alpha} + J\vec{S} \cdot (\vec{s}_L + \vec{s}_R) + \delta H_{2\text{CK}}$$



The Fermi liquid crossover: near but not at the critical point



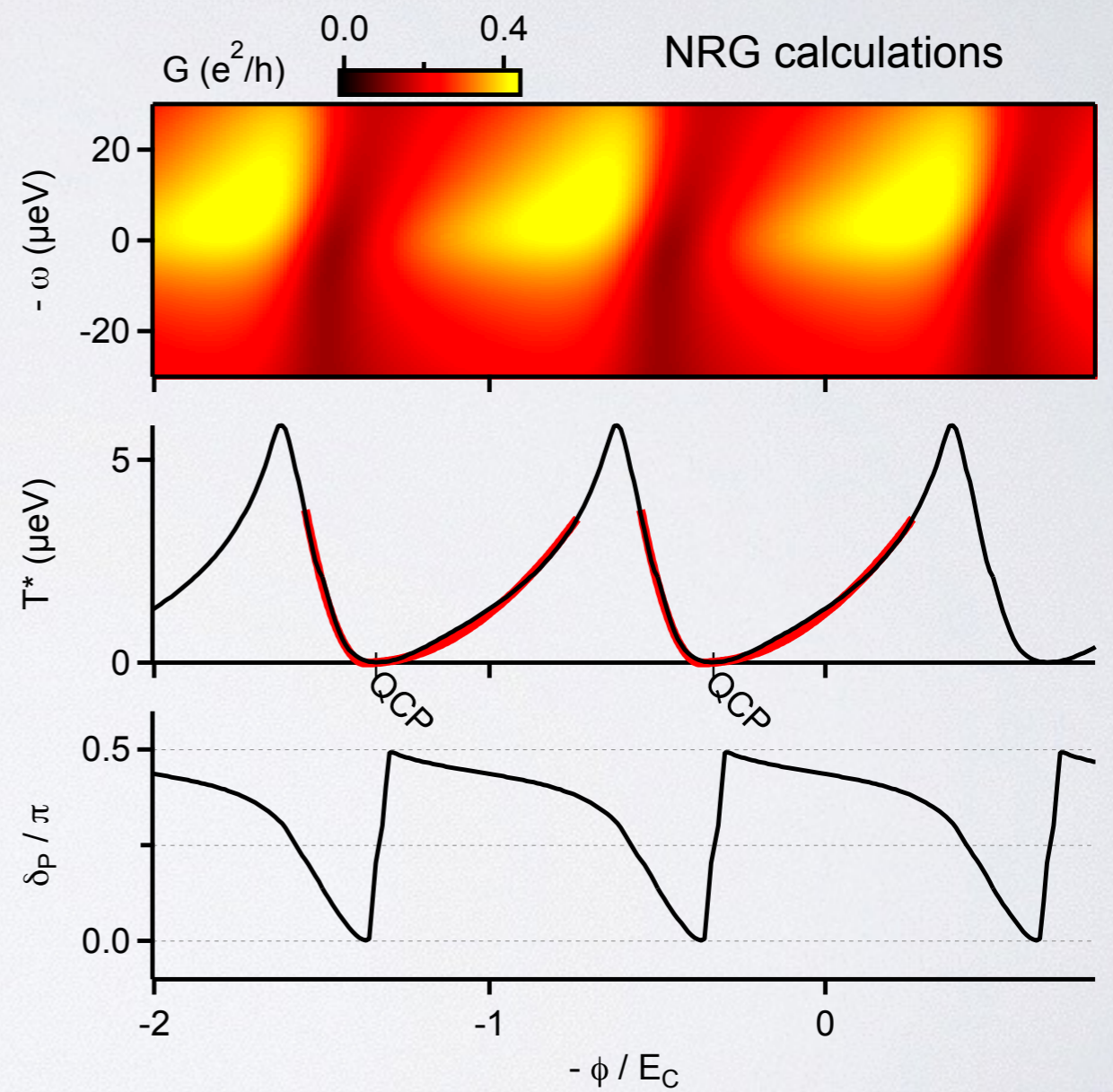
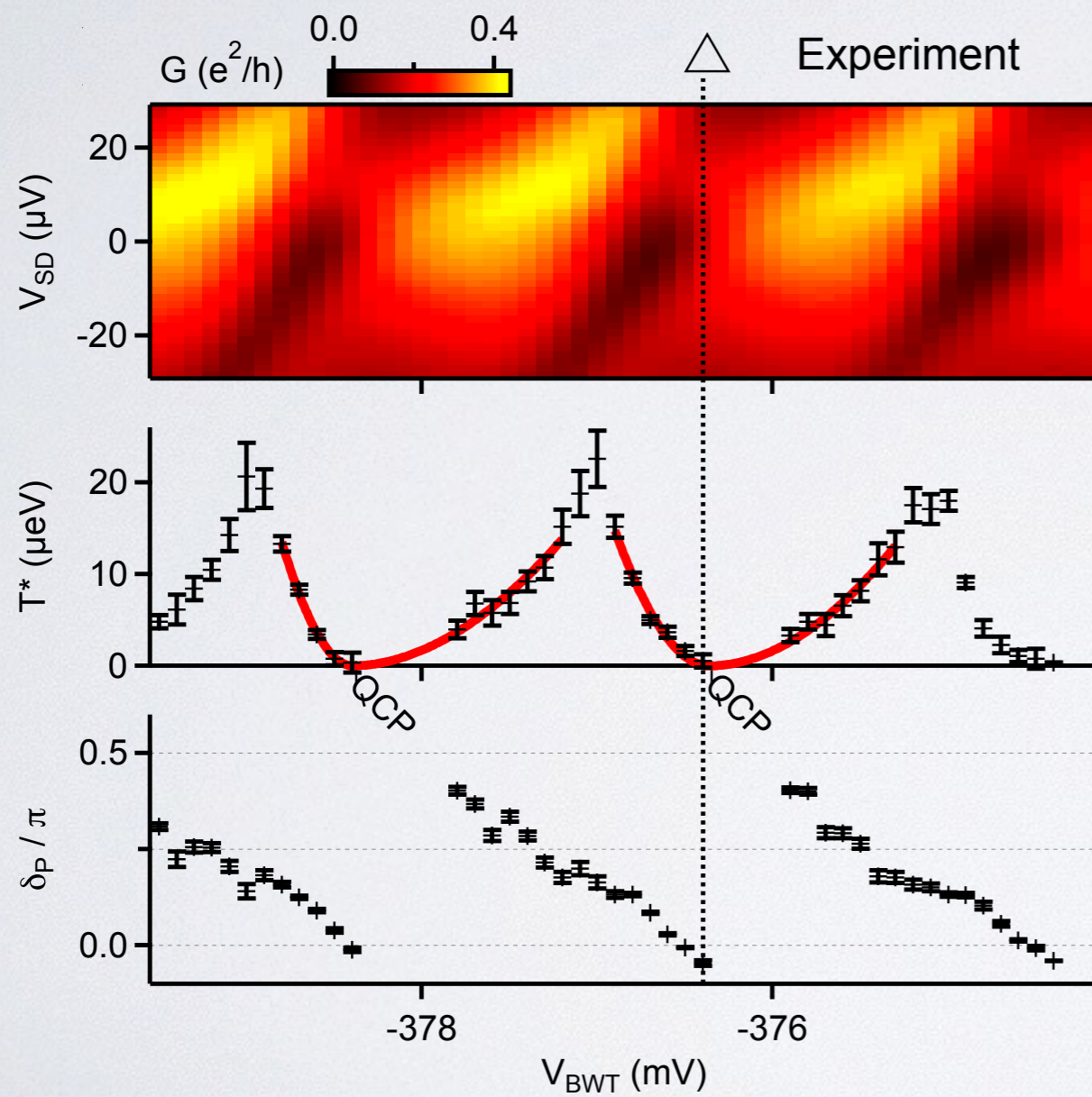
The Fermi liquid crossover



Mitchell and Sela, PRB 85, 235127 (2012)

$$\mathcal{G}(\tilde{\omega}, \tilde{T}) = \frac{-i}{\sqrt{2\pi^3 \tilde{T}}} \frac{\Gamma(\frac{1}{2} + \frac{1}{2\pi \tilde{T}})}{\Gamma(1 + \frac{1}{2\pi \tilde{T}})} \int_{-\infty}^{\infty} dx \frac{e^{ix\tilde{\omega}}}{\sinh x} \times \text{Re} \left[{}_2F_1 \left(\frac{1}{2}, \frac{1}{2}; 1 + \frac{1}{2\pi \tilde{T}}, \frac{1 - \coth x}{2} \right) \right]$$

Boundary of critical regime

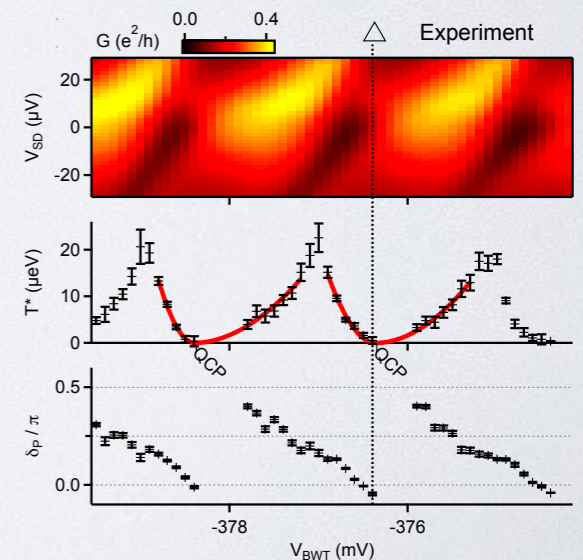
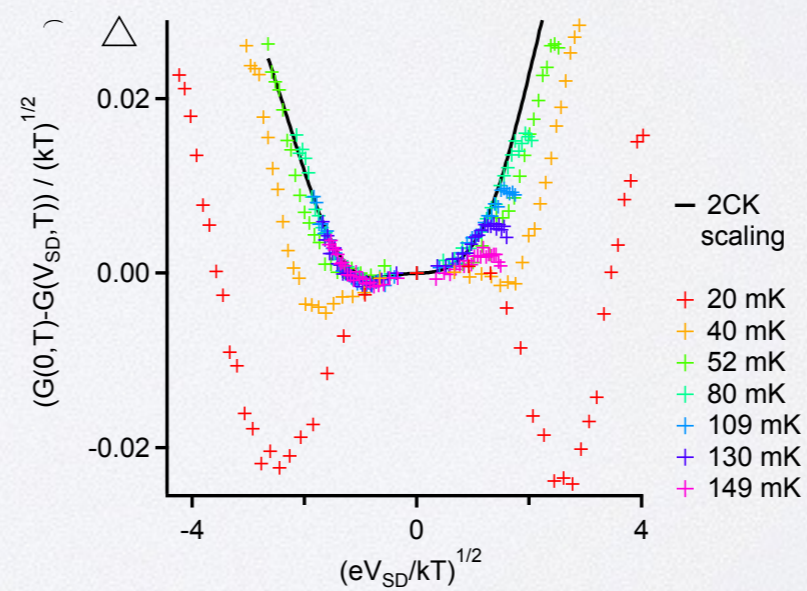
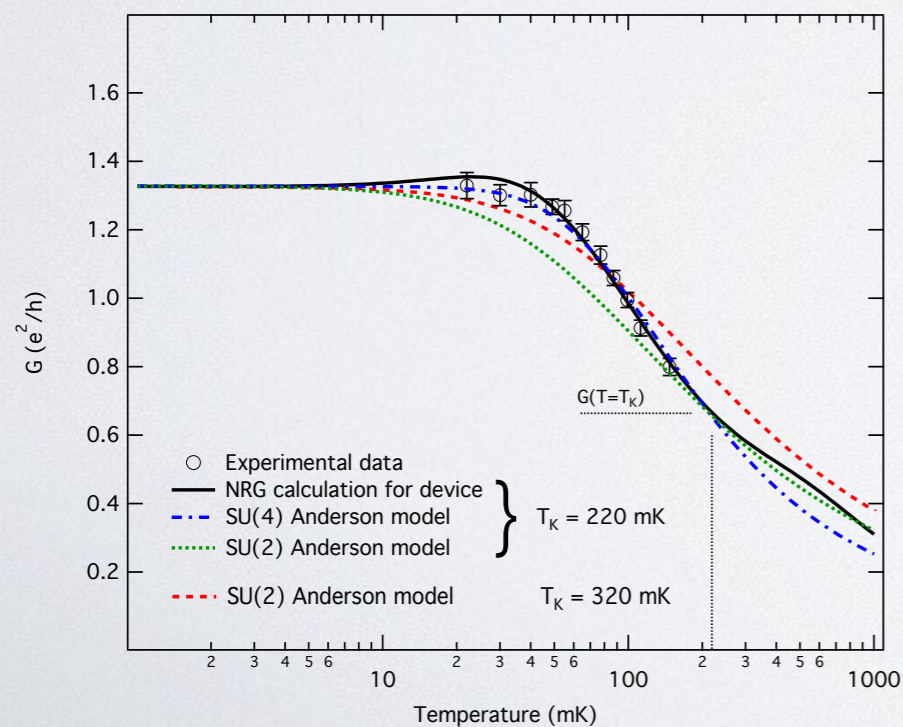
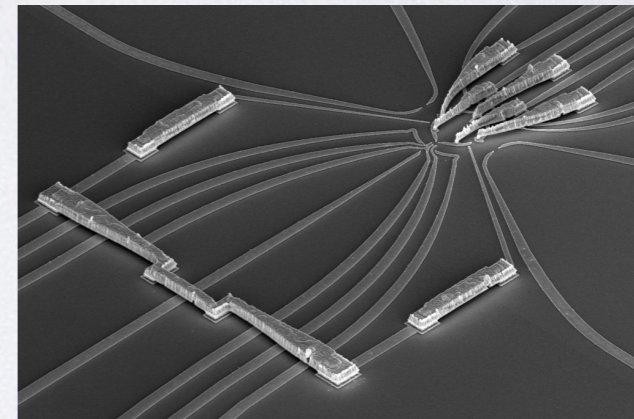


To summarize...

We've seen signatures of emergent symmetry in a double quantum dot



We've closely tested ideas of quantum phase transitions in a model system



Acknowledgments

Quantum dots subgroup:

Andrew Keller (now Caltech), Lucas Peeters, Sami Amasha (now MIT Lincoln Labs), Ileana Rau

Collaborators:

G. Zaránd, C. P. Moca (Budapest University of Technology and Economics), I. Weymann (Adam Mickiewicz University), V. Umansky, H. Shtrikman, D. Mahalu, H.-K. Choi, Y. Oreg, A. Carmi, (Weizmann Institute of Science), J. A. Katine (HGST), J. MacArthur (Harvard), Y. Chung (Pusan National U.)

Funding agencies:



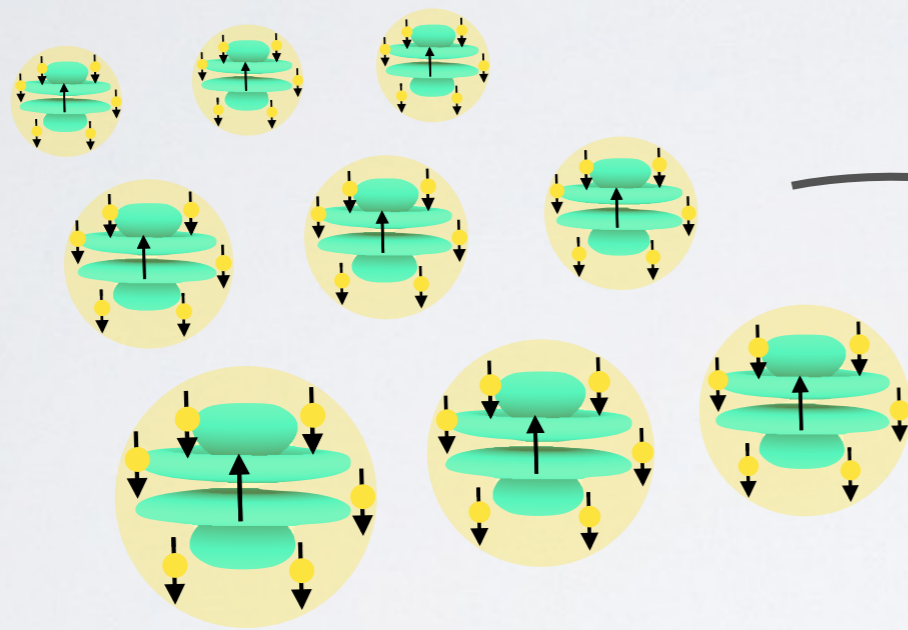
United States - Israel
Binational Science
Foundation



GORDON AND BETTY
MOORE
FOUNDATION

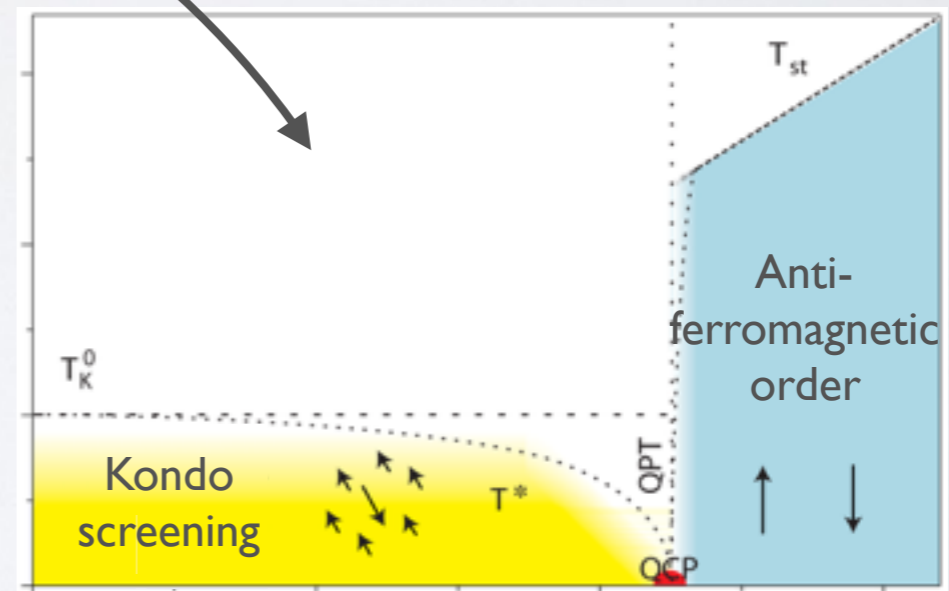
From here?

Coupling of model spins, to understand bulk materials



+ RKKY interaction

Energy



Interaction strength

Adapted from Bork, et al., Nature Physics 7, 11 (2011).

To summarize...

We've seen signatures of emergent symmetry in a double quantum dot



We've closely tested ideas of quantum phase transitions in a model system

