

controlling open quantum systems: tools, achievements, limitations

Christiane P. Koch

U N I K A S S E L
V E R S I T Ä T









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control

steer a dynamical system from an initial to a final state
using an external knob



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using an external knob



navigate spacecraft:

- Newton's eq.
- knob: change acceleration

optimal control

reach desired accuracy with
minimal expenditure of effort

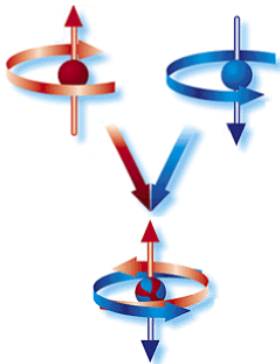


min. time
min. energy



quantum control

matter waves \rightarrow superposition principle



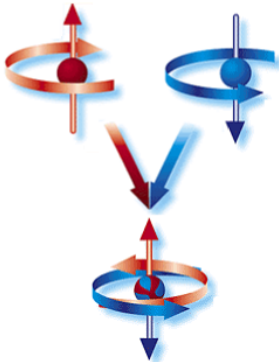
$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle \pm |\downarrow\rangle)$$

$$+1 = e^{2\pi i} \quad -1 = e^{\pi i}$$

variation of relative phases:
interference

quantum control

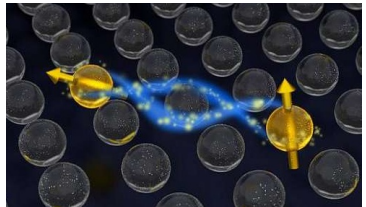
matter waves \rightarrow superposition principle



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$$+1 = e^{2\pi i} \quad -1 = e^{\pi i}$$

variation of relative phases:
interference



we can utilize interference (and entanglement)
to realize desired control
knobs: external fields (electric, magnetic)

our tool: quantum optimal control

define the objective :

$$\text{GOAL} \equiv \|\langle \varphi_{\text{ini}} | \hat{\mathbf{U}}^+(T, 0; \boldsymbol{\epsilon}) | \varphi_{\text{target}} \rangle\|^2 = -J_T$$

as a functional of the external field $\boldsymbol{\epsilon}$

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include additional constraints:

$$J = J_T + \int_0^T J_t(\boldsymbol{\epsilon}, \varphi) dt$$

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optimize J :

$$\partial_{\boldsymbol{\epsilon}} J = 0 \quad \partial_{\varphi(t)} J = 0 \quad \partial_{\boldsymbol{\epsilon}}^2 J > 0$$

$$|\varphi(t)\rangle = \hat{\mathbf{U}}(t, 0; \boldsymbol{\epsilon}) |\varphi_{\text{ini}}\rangle \quad \hat{\mathbf{H}} = \hat{\mathbf{H}}_0 + \boldsymbol{\epsilon}(t) \hat{\mathbf{H}}_1$$

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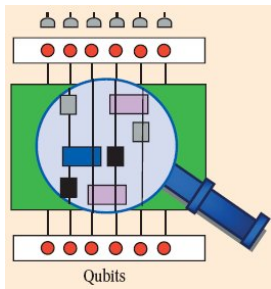
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$$\Delta \boldsymbol{\epsilon}(t) \sim \Im \left[\langle \varphi_{\text{target}} | \hat{\mathbf{U}}^+(T, t; \boldsymbol{\epsilon}^{\text{old}}) \hat{\mathbf{H}}_1 \hat{\mathbf{U}}(t, 0; \boldsymbol{\epsilon}^{\text{new}}) | \varphi_{\text{ini}} \rangle \right]$$

an example: realize a unitary / gate

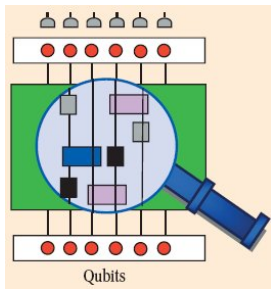


$$J_T = -\Re e \left[\text{Tr} \left\{ \hat{\mathbf{O}}^\dagger \hat{\mathbf{P}}_N \hat{\mathbf{U}}(T, 0; \epsilon) \hat{\mathbf{P}}_N \right\} \right]$$

Palao & Kosloff, Phys Rev A 68, 062308 (2003)

- desired unitary operation : $\hat{\mathbf{O}}$
- desired accuracy/fidelity :
 $1 - \epsilon$ where $\epsilon < 10^{-4}$

an example: realize a unitary / gate



$$J_T = -\Re e \left[\text{Tr} \left\{ \hat{\mathbf{O}}^+ \hat{\mathbf{P}}_N \hat{\mathbf{U}}(T, 0; \epsilon) \hat{\mathbf{P}}_N \right\} \right]$$

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- desired unitary operation : $\hat{\mathbf{O}}$
- desired accuracy / fidelity :
 $1 - \epsilon$ where $\epsilon < 10^{-4}$

$$\Delta \epsilon(t) = \frac{S(t)}{2\alpha} \Im m \left[\sum_{k=1}^N \langle \varphi_{k,ini} | \hat{\mathbf{O}}^+ \hat{\mathbf{U}}^+(T, t; \epsilon^{old}) \hat{\mathbf{H}}_1 \hat{\mathbf{U}}(t, 0; \epsilon^{new}) | \varphi_{k,ini} \rangle \right]$$

- what gate time T needed ?
- best choice of target $\hat{\mathbf{O}}$?

Müller, Reich, Murphy, Yuan, Vala, Whaley, Calarco, CPK, *Phys Rev A* 84, 042315 (2011)

Watts, Vala, Müller, Calarco, Whaley, Reich, Goerz, CPK, *Phys Rev A* 91, 062306 (2015)

Goerz, Gualdi, Reich, CPK, Motzoi, Whaley, Vala, Müller, Montangero, Calarco, *Phys Rev A* 91, 062307 (2015)

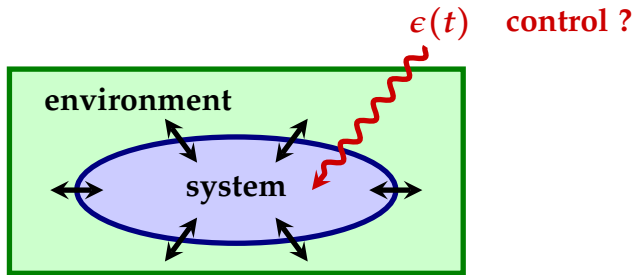
but wait a minute!

open quantum systems !?

decoherence!

control vs decoherence

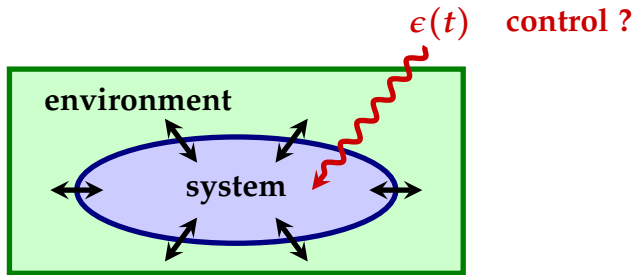
$$\hat{H} = \hat{H}_S(\epsilon) + \hat{H}_{SE} + \hat{H}_E$$



- what are viable quantum control strategies ?
- dependence on properties of environment / coupling ?
- fundamental limits to quantum control ?

control vs decoherence

$$\hat{H} = \hat{H}_S(\epsilon) + \hat{H}_{SE} + \hat{H}_E$$



- what are viable quantum control strategies ?
- dependence on properties of environment / coupling ?
- fundamental limits to quantum control ?

but also: dissipation-enabled control (e.g. cooling)

overview

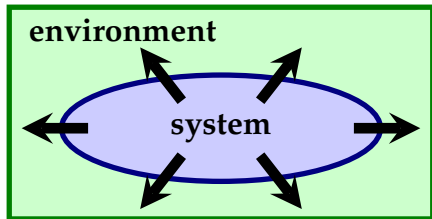
- Markovian vs non-Markovian dynamics
- controllability
- how to measure success of control
in open quantum systems ?
- strategy 1: fighting decoherence
- strategy 2a: utilizing the environment
(Markovian dynamics)
- strategy 2b: utilizing the environment
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- summary & outlook

overview

- **Markovian vs non-Markovian dynamics**
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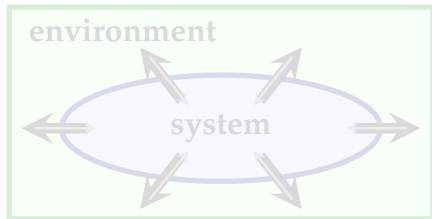
Markovian vs non-Markovian dynamics

Markovian evolution



fully irreversible loss
of energy and phase

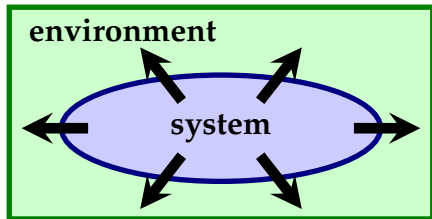
non-Markovian evolution



overall loss
of energy and phase
but some 'back-flow' possible

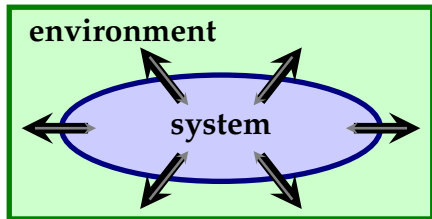
Markovian vs non-Markovian dynamics

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fully irreversible loss
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modelling open system dynamics

- most often employed: Markovian bath model

$$\frac{\partial \rho}{\partial t} = \frac{i}{\hbar} [H, \rho]_- + \mathcal{L}_D(\rho)$$

- non-Markovian variants

- ▶ time-local non-Markovian master eqs.

- ▶ stochastic approaches (under certain assumptions)

e.g. Piilo, Maniscalco, Härkönen, Suominen, Phys. Rev. Lett. 100, 180402 (2008)

Koch, Großmann, Stockburger, Ankerhold, Phys. Rev. Lett. 100, 230402 (2008)

- ▶ poor (wo)man's non-Markovian bath: single TLF

e.g. Rebentrost, Serban, Schulte-Herbrüggen, Wilhelm, Phys. Rev. Lett. 102, 090401 (2009)

- ▶ small-size spin bath / effective modes

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- ▶ ...

small-size bath approximation can be made rigorous:

open system dynamics is quasi-local

open system dynamics quasi-local

Gualdi & CPK, *Phys Rev A* 88, 022122 (2013)

$$\hat{H} = \hat{H}_S + \sum_{i=1}^{N_S^{int}} \sum_{j=1}^{N_B^{int}} \hat{\Phi}_{ij}^{SB} + \sum_{i \leq j=1}^{N_B} \hat{\Phi}_{ij}^B$$

$$\text{with } \hat{\Phi}_{ij} = \sum_{\mu=0}^{\dim(\mathcal{B}(\mathcal{H}_i))-1} \sum_{\nu=0}^{\dim(\mathcal{B}(\mathcal{H}_j))-1} J_{ij}^{\mu\nu} \hat{O}_i^\mu \hat{O}_j^\nu$$

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represent couplings $J_{ij}^{\mu\nu}$ on graph & reorder bath modes according to graph distance:

$$\hat{H} = \sum_{d=0}^{\infty} \left(\hat{h}_d + \hat{h}_{d,d+1} \right)$$

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$$\text{bound: } \left\| \hat{A}_S(t) - \hat{A}_S^{H_n}(t) \right\| \leq 2 \left\| \hat{A}_S \right\| e^{-(n-vt)} \quad \text{with } v = 2\mathcal{O}\bar{c}^2 \|J\| e$$

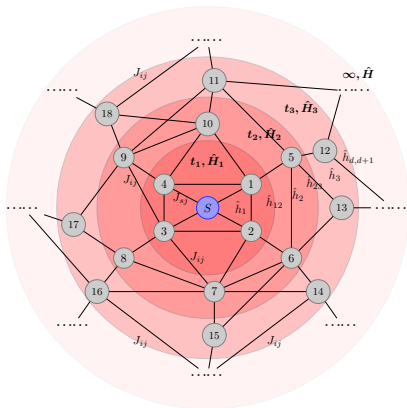
\bar{c} max. connectivity and $\mathcal{O} = \max_{(i,j) \in N; \mu, \nu} \left\| \hat{O}_i^\mu \hat{O}_j^\nu \right\|$

Lieb-Robinson-type bound limits spread of system-bath correlations

dynamical renormalization of SB coupling

Gualdi & CPK, *Phys Rev A* 88, 022122 (2013)

discrete environments

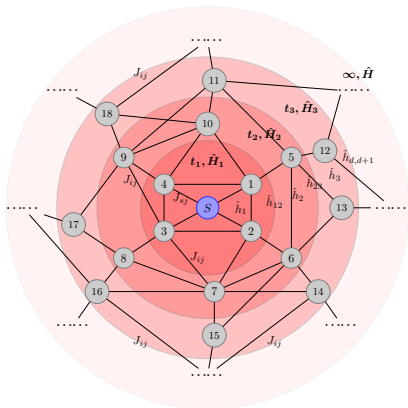


→ given t , size of finite approximate bath known *a priori*

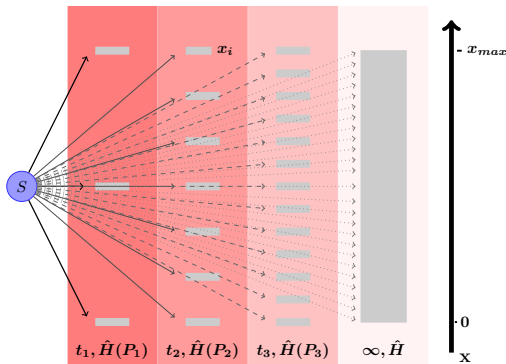
dynamical renormalization of SB coupling

Gualdi & CPK, *Phys Rev A* 88, 022122 (2013)

discrete environments



continuous environments



prerequisite: finite support of SB coupling
discrete approximation by partitioning

→ given t , size of finite approximate bath known *a priori*

overview

- Markovian vs non-Markovian dynamics
- **controllability**
- how to measure success of control
in open quantum systems ?
- control strategy 1: fighting decoherence
- control strategy 2(a): utilizing the environment
(Markovian dynamics)
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controllability: can we reach the target?

– assuming closed system & sufficient resources –

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \left[\hat{\mathbf{H}}_0 + \sum_l \varepsilon_l(t) \hat{\mathbf{H}}_l \right] |\psi(t)\rangle$$

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- \hat{H}_0 has a *finite* spectrum
- state-to-state transitions



It is possible to **completely control** evolution from initial to target state

Huang, Tarn, Clarke, J Math Phys 24, 2608 (1983); Ramakrishna et al., Phys Rev A 51, 960 (1995)

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controllability retained for some systems **w/ infinitely** many states using Galerkin-type methods

Boscain et al., *J Math Phys* 43, 2107 (2002)

intuition: adjust an external field (ion trap)
s.t. system separated into finite and infinite subsystem

Rangan, Bloch, Monroe, Bucksbaum, *Phys Rev Lett* 92, 113004 (2004)

controllability: can we reach the target?

– assuming closed system & sufficient resources –

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \left[\hat{\mathbf{H}}_0 + \sum_l \epsilon_l(t) \hat{\mathbf{V}}_l \right] |\psi(t)\rangle$$

for simplicity: assume piecewise const. controls & single $\epsilon(t) \hat{\mathbf{V}}$

$$\begin{aligned} |\psi(t)\rangle &= e^{-\frac{i}{\hbar}(\hat{\mathbf{H}}_0 + \epsilon \hat{\mathbf{V}})t} |\psi(0)\rangle \\ &= e^{-\frac{i}{\hbar} \hat{\mathbf{H}}_0 t} e^{-\frac{i}{\hbar} \epsilon \hat{\mathbf{V}} t} e^{-\frac{i}{\hbar} \frac{1}{2} [\hat{\mathbf{H}}_0, \epsilon \hat{\mathbf{V}}] t^2} e^{-\frac{i}{\hbar} \frac{1}{12} [\hat{\mathbf{H}}_0, [\hat{\mathbf{H}}_0, \epsilon \hat{\mathbf{V}}]] t^3} \cdot \\ &\quad \cdot e^{\frac{i}{\hbar} \frac{1}{12} [\epsilon \hat{\mathbf{V}}, [\hat{\mathbf{H}}_0, \epsilon \hat{\mathbf{V}}]] t^3} \dots \cdot |\psi(0)\rangle \end{aligned}$$

controllability: can we reach the target?

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generators determine directions in Hilbert space:

$$\hat{H}_0, \hat{V}, [\hat{H}_0, \hat{V}], \dots$$

controllability \iff all directions can be generated

still depends on $|\psi(0)\rangle$

complete (unitary) controllability

– assuming closed system & sufficient resources –

$$i\hbar \frac{\partial}{\partial t} \hat{\mathbf{U}}(t) = \left[\hat{\mathbf{H}}_0 + \sum_l \varepsilon_l(t) \hat{\mathbf{V}}_l \right] \hat{\mathbf{U}}(t) \quad \hat{\mathbf{U}}(0) = \mathbb{1}$$

$$\hat{\mathbf{U}}(t, 0) = e^{Ct} = e^{\sum_{i=1}^{N^2} \alpha_i C_i t}$$

$C, C_i : N \times N$ skew-Hermitian, C_i linearly independent

any $\hat{\mathbf{U}}(t, 0) \iff$ all N^2 generators $\{C_i\}$ available

full rank condition

$\dim(\text{algebra of iterated commutators}) = N^2$

controllability \leftrightarrow connectivity

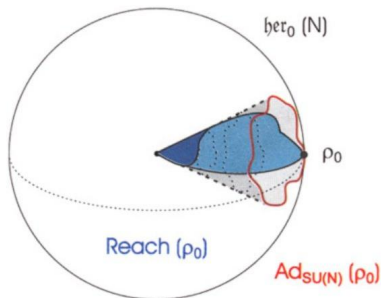
symmetry = enemy of controllability

controllability of open quantum systems

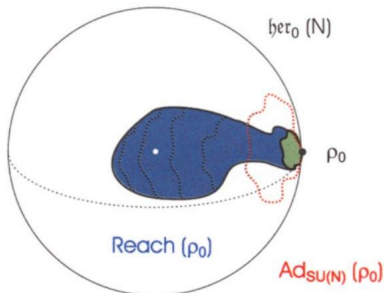
review: Glaser et al., Eur Phys J D 69, 279 (2015)

- for Markovian dynamics
& assuming $\varepsilon(t)$ does not change \mathcal{L}_D
 - ▶ time evolution: Lie group \implies Lie semi-group

$\mathcal{L}_D \sim \mathbb{1}$



generic



Dirr, Helmke, Kurniawan, Schulte-Herbrüggen, Rep Math Phys 64, 93 (2009)

- no rigorous results for non-Markovian dynamics

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our tool: optimal control theory

$$t = 0 \quad |\varphi_{\text{ini}}\rangle \quad \text{~~~~~} \rightsquigarrow \quad |\varphi_{\text{target}}\rangle \quad t = T$$

define the objective :

$$\text{GOAL} \equiv \|\langle \varphi_{\text{ini}} | \hat{\mathbf{U}}^+(T, 0; \boldsymbol{\epsilon}) | \varphi_{\text{target}} \rangle\|^2 = -J_T$$

as a functional of the field $\boldsymbol{\epsilon}$

include additional constraints:

$$J = J_T + \int_0^T J_t(\boldsymbol{\epsilon}, \varphi) dt$$

optimize J :

$$\partial_{\boldsymbol{\epsilon}} J = 0 \quad \partial_{\varphi(t)} J = 0 \quad \partial_{\boldsymbol{\epsilon}}^2 J > 0$$

$$|\varphi(t)\rangle = \hat{\mathbf{U}}(t, 0; \boldsymbol{\epsilon}) |\varphi_{\text{ini}}\rangle$$

$$J_T = -\Re e \left[\text{Tr} \left\{ \hat{\mathbf{O}}^+ \hat{\mathbf{P}}_N \hat{\mathbf{U}}(T, 0; \boldsymbol{\epsilon}) \hat{\mathbf{P}}_N \right\} \right]$$

optimal control for open quantum systems

review: CPK, J Phys Condens Matter 28, 213001 (2016)

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$$\text{GOAL} \equiv \text{Tr} \{ \mathcal{D}_{\epsilon}(\rho_{\text{ini}}) \rho_{\text{target}} \} = -J_T$$

as a functional of the field ϵ

include additional constraints:

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optimize J :

$$\partial_{\epsilon} J = 0 \quad \partial_{\rho(t)} J = 0 \quad \partial_{\epsilon}^2 J > 0$$

$$\rho(t) = \mathcal{D}_{\epsilon}(\rho_{\text{ini}})$$

optimal control for open quantum systems

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$$t = 0 \quad \rho_{\text{ini}} \rightsquigarrow \rho_{\text{target}} \quad t = T$$

define the objective :

$$\text{GOAL} \equiv \text{Tr} \{ \mathcal{D}_{\epsilon}(\rho_{\text{ini}}) \rho_{\text{target}} \} = -J_T$$

as a functional of the field ϵ

include additional constraints:

$$J = J_T + \int_0^T J_t(\epsilon, \rho) dt$$

optimize J :

$$\partial_{\epsilon} J = 0 \quad \partial_{\rho(t)} J = 0 \quad \partial_{\epsilon}^2 J > 0$$

$$\rho(t) = \mathcal{D}_{\epsilon}(\rho_{\text{ini}})$$

$$J_T = -\Re e \left[\text{Tr} \left\{ \hat{\mathbf{O}}^+ \hat{\mathbf{P}}_N \hat{\mathbf{U}}(T, 0; \epsilon) \hat{\mathbf{P}}_N \right\} \right] \quad ???$$

OCT for open quantum systems

$$\rho(T) = \mathcal{D}(\rho(0)) \quad \text{e.g. : } \frac{\partial \rho}{\partial t} = \frac{i}{\hbar} [H, \rho]_- + \mathcal{L}_D(\rho)$$

① state-to-state: $\rho(t=0) \rightarrow \rho(t=T) = \rho_{\text{target}}$

$$\Rightarrow \text{maximize } \text{Tr} \{ \rho(T) \rho_{\text{target}} \}$$

Bartana, Kosloff, Tannor, J Chem Phys 106, 1435 (1997)

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Schmidt, Negretti, Ankerhold, Calarco, Stockburger, Phys Rev Lett 107, 130404 (2011)

② gates: lift $\text{Tr} \{ O^+ P_N U(T, 0; \varepsilon) P_N \}$ to Liouville space

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Kallush & Kosloff, Phys Rev A 73, 032324 (2006)

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- ③ local equivalence class: $J_T = \Delta g_1^2 + \Delta g_2^2 + \Delta g_3^2$ with

$$\Delta g_i^2 = |g_i(O) - g_i(U)|^2$$

but: $U \Leftrightarrow \mathcal{D}(\rho) !?$

characterizing open quantum systems

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using concepts 'commutant space' and 'total rotation'

minimal set of states: total rotation

Reich, Gualdi, CPK, Phys Rev A 88, 042309 (2013)

given a $\rho(T) = U\rho(0)U^+$, we cannot distinguish those U with common eigenbasis with ρ from $\mathbb{1}$

- ① **fix a basis:** basis-complete projectors $\{P_i\}$
 d orthonormal one-dimensional projectors
 $\implies \rho = \sum_{i=1}^d \lambda_i P_i, \lambda_i \neq \lambda_j$

provided time evolution coherent

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$$\implies \rho' = P_{TR} \quad \text{with}$$

$$P_{TR} P_i \neq 0 \quad \forall P_i \in \{P_i\}$$

(note: d P_{TR} 's = mutually unbiased basis)

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(note: d P_{TR} 's = mutually unbiased basis)

$\implies \rho(T), \rho'(T)$ are sufficient to distinguish any two unitaries
(and thus measure success of control)
provided time evolution coherent

gate optimization

third state sufficient to check whether time evolution is unitary

Goerz, Reich, CPK, New J Phys 16, 055012 (2014)

$$J_T = \sum_{j=1}^3 [1 - \text{Tr} \{ O \rho_j O^\dagger \rho_j(T) \}]$$

$$\rho_{1,ij} = \frac{2(d-i+1)}{d(d+1)} \delta_{ij}$$

fix the basis

$$\rho_{2,ij} = \frac{1}{d}$$

totally rotated state

$$\rho_{3,ij} = \frac{1}{d} \delta_{ij}$$

check unitarity
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J_T attains its minimum if and only if

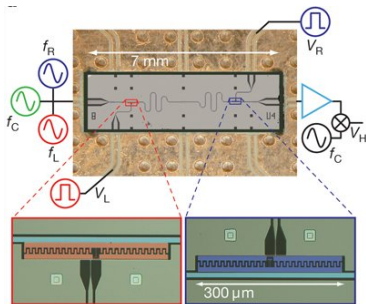
- 1 \mathcal{D} is a unitary dynamical map on the logical subspace
- 2 $\mathcal{D}(\rho_1) = O \rho_1 O^+$ and $\mathcal{D}(\rho_2) = O \rho_2 O^+$

→ propagation of 3 states sufficient, irrespective of $\dim \mathcal{H}$

overview

- Markovian vs non-Markovian dynamics
- how to measure success of control
in open quantum systems ?
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- control strategy 2(a): utilizing the environment
(Markovian dynamics)
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example: superconducting transmon qubits



Koch, Yu, Gambetta, Houck, Schuster, Majer, Blais, Devoret, Girvin, Schoelkopf, Phys. Rev. A 76, 042319 (2007)

DiCarlo, Chow, Gambetta, Bishop, Johnson, Schuster, Majer, Blais, Frunzio, Girvin, Schoelkopf, Nature 460, 240 (2009)

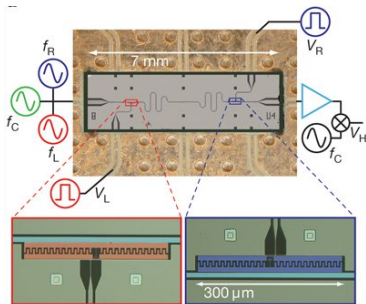
comparatively long decoherence times

$$T_2 = 20 \dots 100 \mu\text{s}$$

gate times $T < 250 \text{ ns}$

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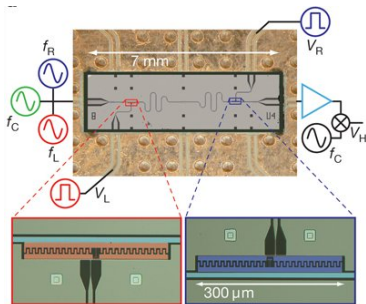
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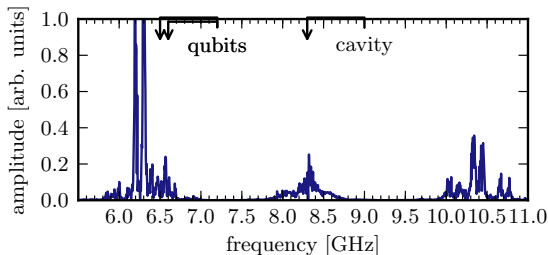
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$$\hat{H} = \sum_{i=1,2} \left[\omega_i \hat{\mathbf{b}}_i^+ \hat{\mathbf{b}}_i - \frac{\alpha_i}{2} \hat{\mathbf{b}}_i^+ \hat{\mathbf{b}}_i^+ \hat{\mathbf{b}}_i \hat{\mathbf{b}}_i \right] + J \left(\hat{\mathbf{b}}_1^+ \hat{\mathbf{b}}_2 + \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_2^+ \right) + \omega_c \hat{\mathbf{a}}^+ \hat{\mathbf{a}} + \epsilon^*(t) \hat{\mathbf{a}} + \epsilon(t) \hat{\mathbf{a}}^+ + \sum_{i=1,2} g_i \left(\hat{\mathbf{b}}_i^+ \hat{\mathbf{a}} + \hat{\mathbf{b}}_i \hat{\mathbf{a}}^+ \right)$$

example: superconducting transmon qubits

200 ns C-phase gate: $F_{av} = 99.72\% \rightsquigarrow 99.06\%$

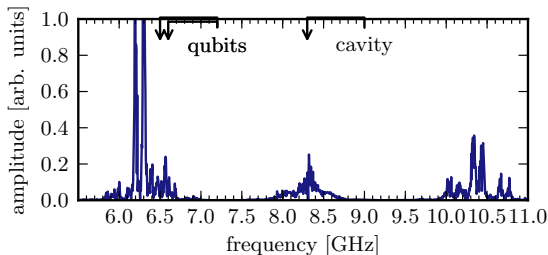


fast gate with additional transitions involving

- qubit $|1\rangle \rightarrow |2\rangle$
- cavity $|0\rangle \rightarrow |2\rangle$ simultaneous with qubit $|1\rangle \rightarrow |0\rangle$

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**→ OCT using full complexity of \hat{H} :
gates fast enough to beat decoherence**

which gate? shortest duration? which system parameters?

charting the circuit QED design landscape with quantum optimal control

Goerz, Motzoi, Whaley, CPK, arXiv:1606.08825

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resonances if qubit-qubit detuning $|\Delta_2| = |\omega_1 - \omega_2| \approx n\alpha_{1,2}$
 $J = 0$

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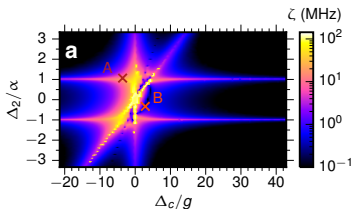
which gates? a universal set!

charting the circuit QED design landscape

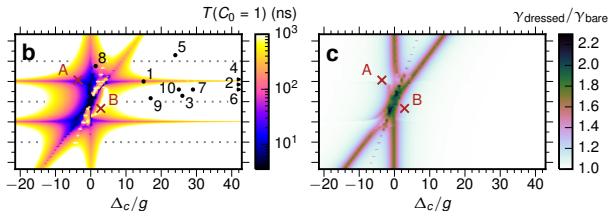
Goerz, Motzoi, Whaley, CPK, arXiv:1606.08825

field-free ($\epsilon(t) = 0$) dynamics

entanglement creation



effective decay



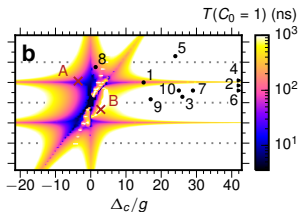
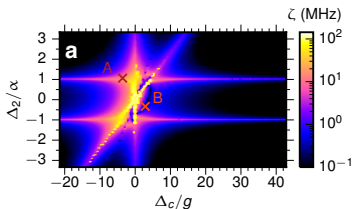
interesting parts of parameter space unexplored !

charting the circuit QED design landscape

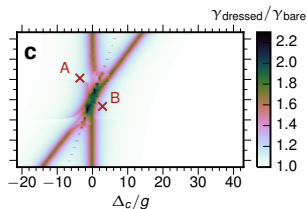
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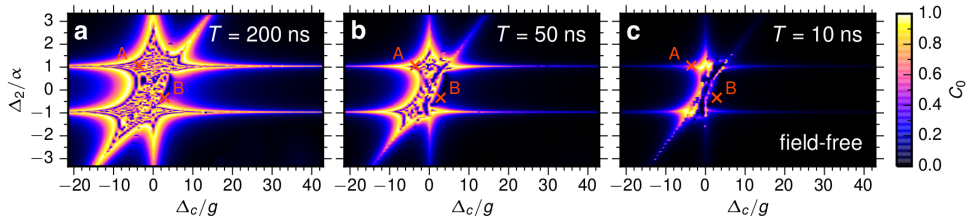
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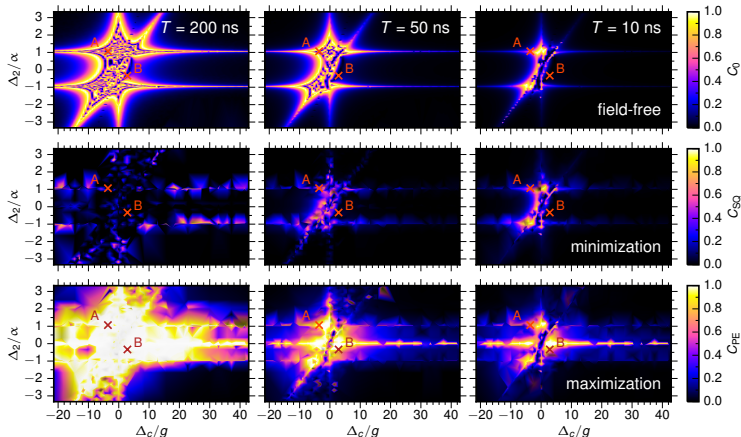
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Goerz, Motzoi, Whaley, CPK, arXiv:1606.08825

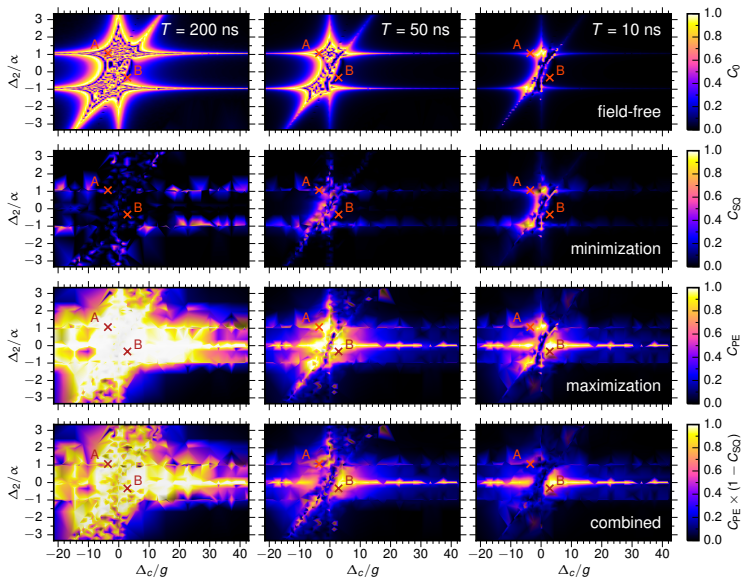
controlling entanglement creation



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Goerz, Motzoi, Whaley, CPK, arXiv:1606.08825

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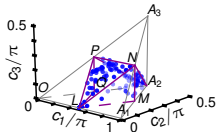


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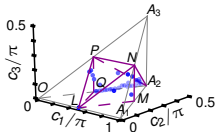
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searching for arbitrary perfect entangler (in universal set)

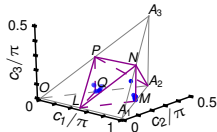
a $T = 200$ ns



b $T = 50$ ns



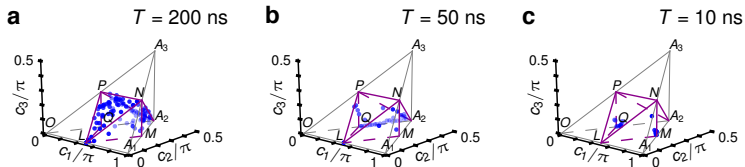
c $T = 10$ ns



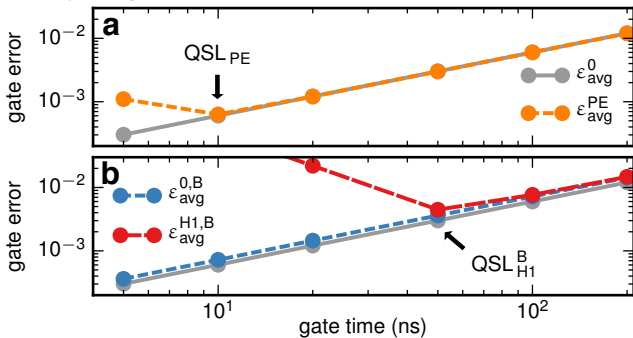
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searching for arbitrary perfect entangler (in universal set)



identifying the quantum speed limit: 50 ns



control strategy 1: fighting decoherence

avoiding decoherence

- our example:

beat decoherence by fastest possible operation

use optimal control theory to identify 'quantum speed limit'

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avoiding decoherence

- our example:

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use optimal control theory to identify 'quantum speed limit'

- steer dynamics through regions of state space less affected by decoherence
 - ▶ relaxation-optimized dynamics

Khaneja, Luy, Glaser, Proc Natl Acad Sci USA 100, 13162 (2003)

Khaneja, Reiss, Luy, Glaser, J Magnet Reson 162, 311 (2005)

- decouple the system from its environment

- ▶ dynamical decoupling
- ▶ spectral engineering

Viola, Knill, Lloyd, Phys Rev Lett 82, 2417 (1999)

Clausen, Bensky, Kurizki, Phys Rev Lett 104, 040401 (2010)

- use symmetry protection (e.g. decoherence-free subspaces)

Lidar, Chuang, Whaley, Phys Rev Lett 81, 2594 (1998)

Knill, Laflamme, Viola, Phys Rev Lett 84, 2525 (2000)

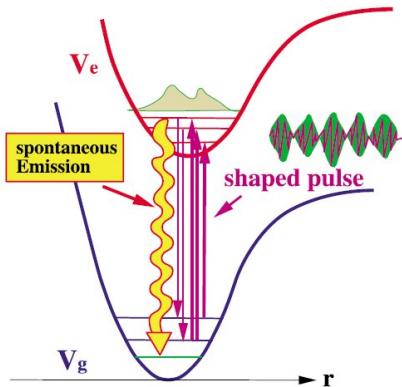
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vibrational cooling – theory

Tannor, Kosloff, Bartana, Faraday Discuss. 113, 365 (1999)

Bartana, Kosloff, Tannor, Chem. Phys. 267, 195 (2001)



cooling = maintaining a dark ground state

many cycles of
shaped laser excitation
and spontaneous emission
without
reexcitation of ground state

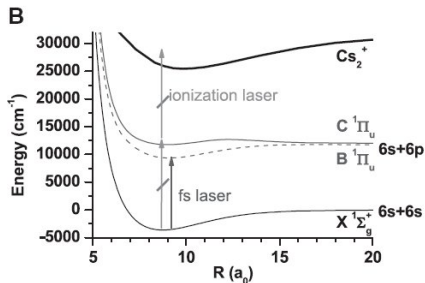
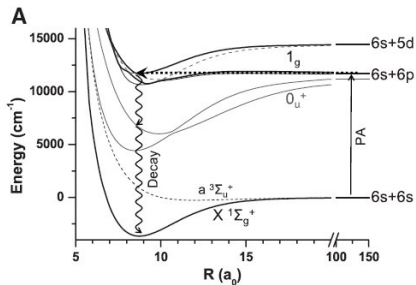
accumulation in ground state

vibrational cooling – experiment

Optical Pumping and Vibrational Cooling of Molecules

Matthieu Viteau,¹ Amdosen Chotia,¹ Maria Allegrini,^{1,2} Nadia Bouloufa,¹ Olivier Dulieu,¹
Daniel Comparat,¹ Pierre Pillet^{1*}

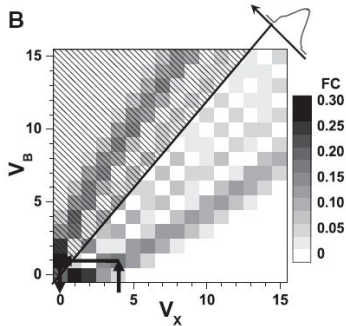
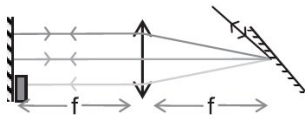
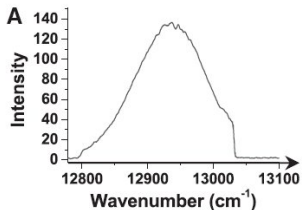
11 JULY 2008 VOL 321 SCIENCE www.sciencemag.org



vibrational cooling – experiment

Viteau et al., *Science* 321, 232 (2008)

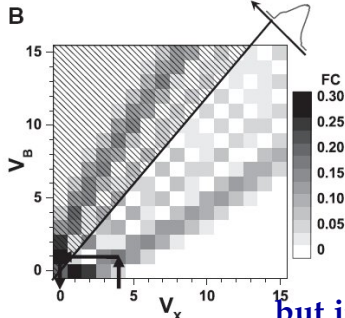
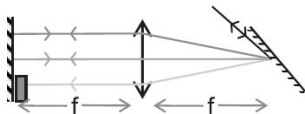
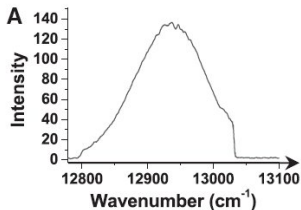
a crude way of maintaining a dark ground state ...



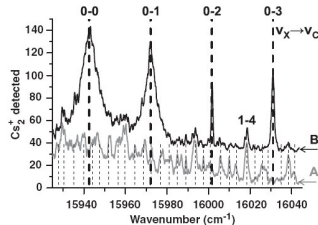
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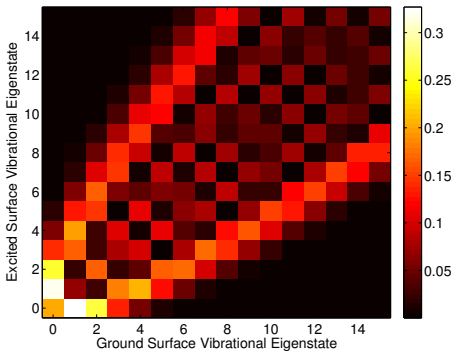
... but it works!



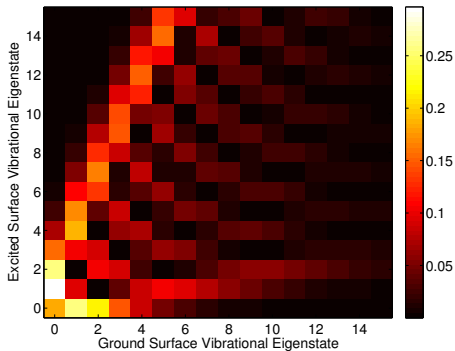
vibrational cooling – the crude way

dependence on molecular structure?

'good' case: Cs_2
cooling



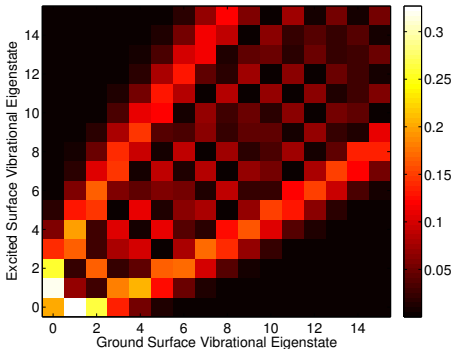
'bad' case: LiCs
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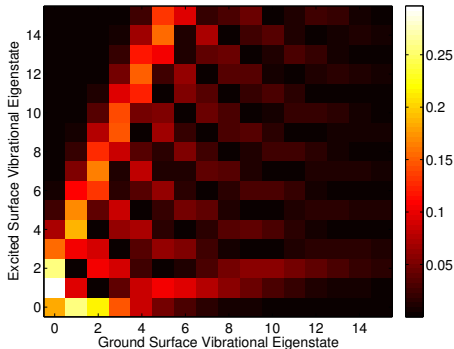
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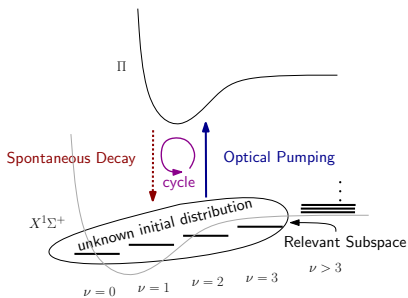
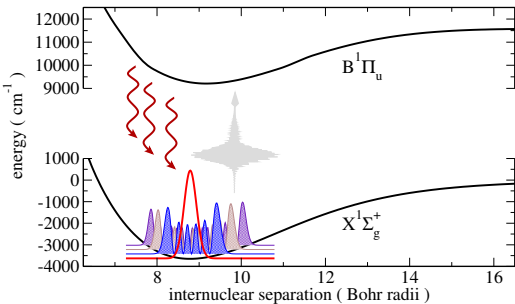
'bad' case: LiCs
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Reich & CPK, New J Phys 15, 125028 (2013)

seek to cool many molecules with the same optimized fs
pulse **irrespective of molecular structure**

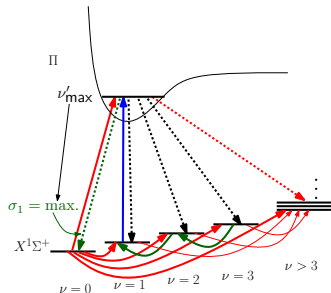
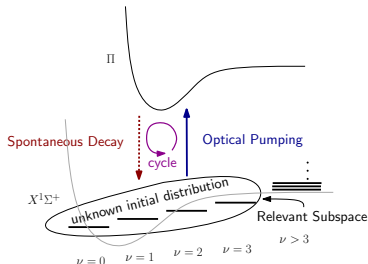
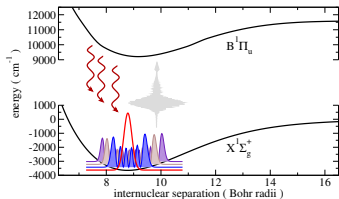
controlling vibrational laser cooling utilizing timescale separation of excitation & spontaneous emission



Reich & CPK, *New J Phys* 15, 125028 (2013)

controlling vibrational laser cooling relying on spontaneous emission

Reich & CPK, *New J Phys* 15, 125028 (2013)



assembly-line cooling

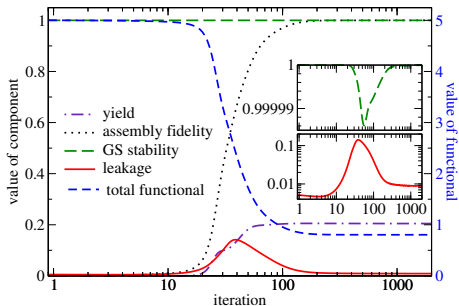
- keep target state dark
 - pump up doorway state
 - reshuffle all other states towards doorway
- ⇒ tailored OC functional

controlling vibrational laser cooling

Reich & CPK, *New J Phys* 15, 125028 (2013)

'bad' case: cold & trapped LiCs molecules

assembly-line cooling

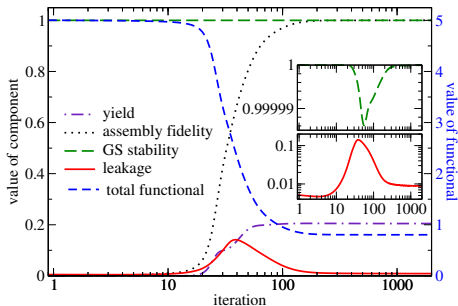


controlling vibrational laser cooling

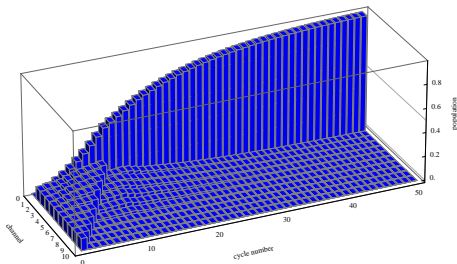
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accumulation in dark state

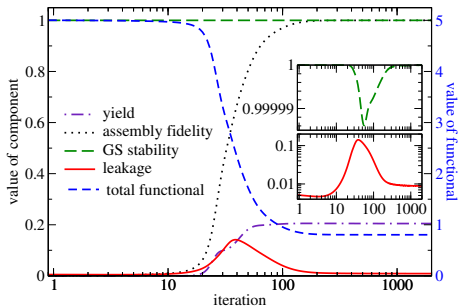


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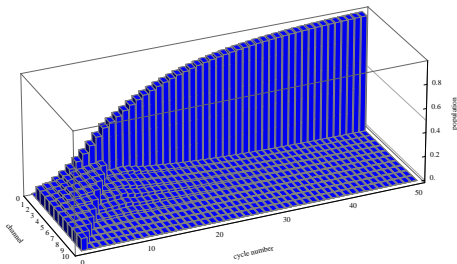
Reich & CPK, *New J Phys* 15, 125028 (2013)

'bad' case: cold & trapped LiCs molecules

assembly-line cooling



accumulation in dark state



minimal requirement on molecular structure for cooling?

- a **single** excited state level favorable to cooling not heating

overview

- Markovian vs non-Markovian dynamics
- how to measure success of control
in open quantum systems ?
- control strategy 1: fighting decoherence
- control strategy 2(a): utilizing the environment
(Markovian dynamics)
- control strategy 2(b): utilizing the environment
(non-Markovian dynamics)
- summary & outlook

Markovian vs non-Markovian evolution

non-Markovian evolution:
loss of energy and phase not monotonic

- distinguishability of two optimal states
(recovery of previously lost information)

Breuer, Laine, Piilo, Phys Rev Lett 103, 210401 (2009)

- divisibility of the dynamical map
(increase of correlations in a bi/multi-partite system)

Rivas, Huelga, Plenio, Phys Rev Lett 105, 050403 (2010)

- ...

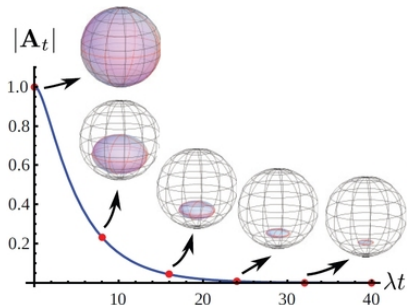
- accessible volume in Liouville (state) space

Lorenzo, Plastina, Paternostro, Phys Rev A 88, 020102 (2013)

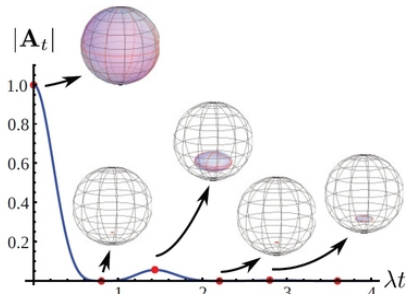
Markovian vs non-Markovian evolution

set of reachable states and Liouville space determinant

Lorenzo, Plastina, Paternostro, Phys Rev A 88, 020102 (2013)



Markovian evolution:
volume always shrinks



non-Markovian evolution:
volume may re-expand

**non-Markovian evolution should allow
for more control than Markovian evolution**

superconducting phase qudit with control

Hamiltonian of a flux-biased phase qudit with **control**

$$\hat{H} = \hbar\omega_0\hat{n} + \frac{\beta}{2}\hat{n}(\hat{n}-1) + \underbrace{\kappa_1 I_c(t)\hat{n}}_{\text{low frequency}} + \underbrace{\kappa_2 I_c(t)(\hat{a} + \hat{a}^+)}_{\substack{\text{high frequency:} \\ \text{all SO(4) operations} \\ \text{(Pythagorean couplings)}}$$

$\beta \ll \omega_0$

Svetitsky et al. Nature Commun. 5, 5617 (2014)

what is missing for full SU(4) controllability?

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Cartan decomposition of $\mathfrak{su}(N)$

$$\forall U \in SU(N) : \quad U = k_1 A k_2 \quad k_1, k_2 \in SO(N)$$

A diagonal, unitary matrix

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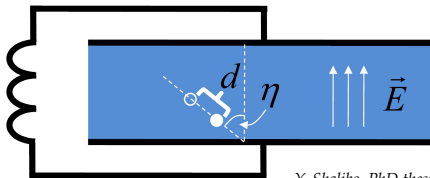
A diagonal, unitary matrix

\curvearrowright **diagonal unitaries (with 'non-local' phases)**

decoherence of the phase qudit

main source of decoherence in Josephson junctions: **charge defects**

- defects switch between two spatial configurations in electric field
- two configurations \rightarrow effective TLS: 'natural' spin bath



Y. Shalibo, PhD thesis (Katz group, Hebrew U Jerusalem)

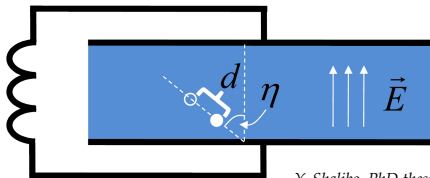
$$\hat{H}^{(i)} = \Delta^{(i)} \sigma_z^{(i)} \quad \hat{H}_{int}^{(i)} = \frac{S^{(i)}}{2} (\hat{\mathbf{a}} \hat{\sigma}_i^+ + \hat{\mathbf{a}}^+ \hat{\sigma}_i^-)$$

- 1 $\Delta^{(i)} \gg \omega_0 \implies$ no effect on qudit
- 2 $\Delta^{(i)} \ll \omega_0 \implies T_2$: pure dephasing of qudit
- 3 $\Delta \simeq \omega_0, S$ weak $\implies T_1$: relaxation of qudit
- 4 $\Delta \simeq \omega_0, S$ strong \implies non-Markovian dynamics

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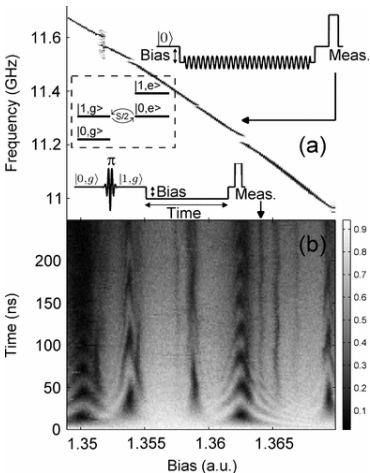
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PRL 105, 177001 (2010)

PHYSICAL REVIEW LETTERS

week ending
22 OCTOBER 2010

Lifetime and Coherence of Two-Level Defects in a Josephson Junction

Yoni Shalibo,¹ Ya'ara Roife,¹ David Shwa,¹ Felix Zeides,¹ Matthew Neeley,² John M. Martinis,² and Nadav Katz¹

¹Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel

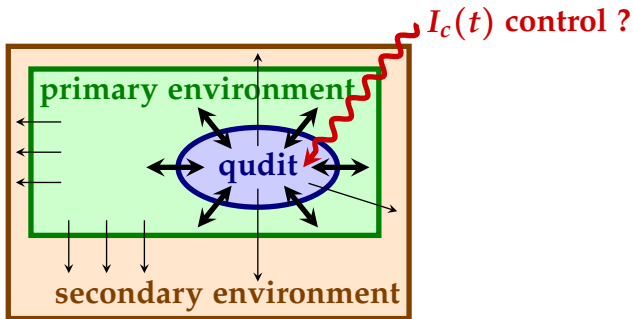
²Department of Physics, University of California, Santa Barbara, California 93106, USA

(Received 14 July 2010; published 19 October 2010)

$$\curvearrowright \Delta^{(i)}, S^{(i)}, T_1^{(i)}, T_2^{(i)}$$

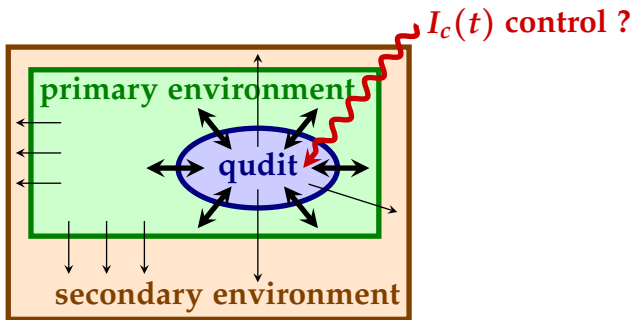
strongly coupled TLS can be characterized experimentally

qudit strongly coupled to a few TLS and weakly coupled to T_1/T_2 background



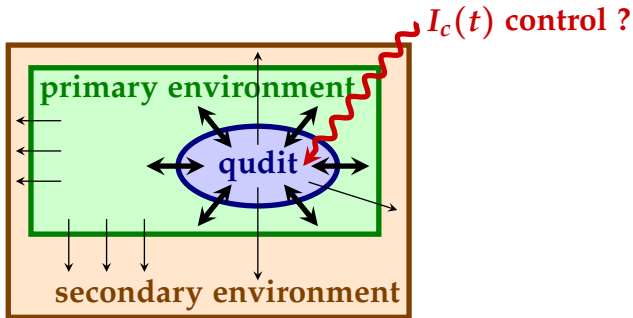
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- } secondary environment
primary env.

qudit strongly coupled to a few TLS and weakly coupled to T_1 background



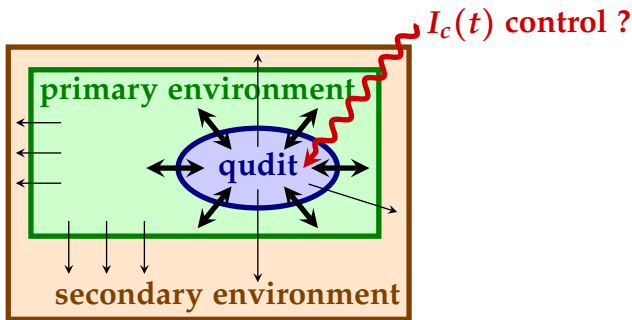
$$\frac{d}{dt}\hat{\rho}_{QP} = -i[\hat{\mathbf{H}}_{QP}, \hat{\rho}_{QP}] + \mathcal{L}_S(\hat{\rho}_{QP})$$

qudit strongly coupled to a few TLS and weakly coupled to T_1 background



$$\frac{d}{dt}\hat{\rho}_{QP} = -i[\hat{\mathbf{H}}_{QP}, \hat{\rho}_{QP}] + \mathcal{L}_S(\hat{\rho}_{QP}) \quad \hat{\mathbf{H}}_{QP} = \hat{\mathbf{H}}_Q + \sum_{i=1}^{n_P} \hat{\mathbf{H}}_P^{(i)} + \sum_{i=1}^{n_P} \hat{\mathbf{H}}_{int}^{(i)}$$

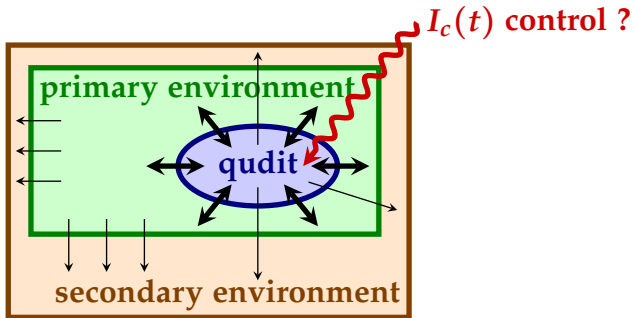
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$$\mathcal{L}_S(\hat{\rho}_{QP}) = \sum_k \left(\hat{\mathbf{A}}_k \hat{\rho}_{QP} \hat{\mathbf{A}}_k^+ - \frac{1}{2} \left[\hat{\mathbf{A}}_k^+ \hat{\mathbf{A}}_k, \hat{\rho}_{QP} \right]_+ \right)$$

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$$\hat{\mathbf{A}}_i = \sqrt{1/T_1^{(i)}} \sigma_i^-$$

SU(4) controllability: phasegates

Reich, Katz, CPK, Sci Rep 5, 12430 (2015)

$$\hat{H}_Q = \hbar\omega_0\hat{n} + \frac{\beta}{2}\hat{n}(\hat{n}-1) + \kappa_1 I_c(t)\hat{n} \quad \hat{H}_P = \Delta\sigma_z \quad \hat{H}_{int} = \frac{S}{2}(\hat{a}\hat{\sigma}^+ + \hat{a}^+\hat{\sigma}^-)$$

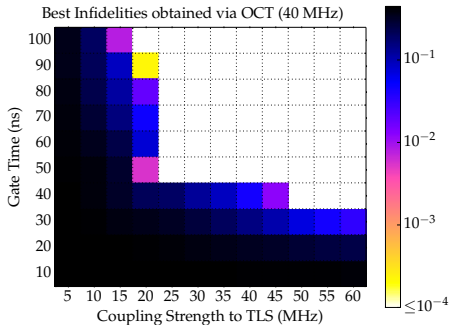
error after optimization for $\text{diag}(1, -1, 1, 1)$ w/ single primary bath TLS

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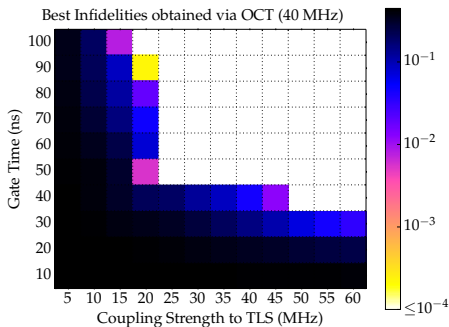
infinite T_1 (no loss)

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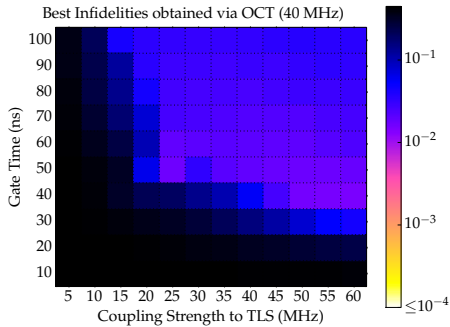
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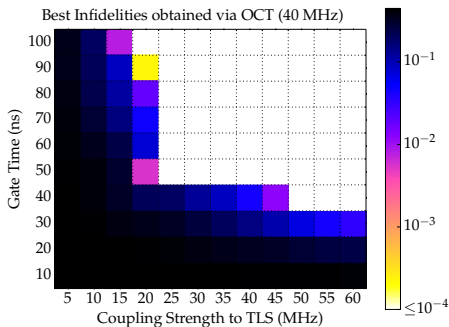
$T_1 = 5\mu s, T_1^{TLS} = 1\mu s$

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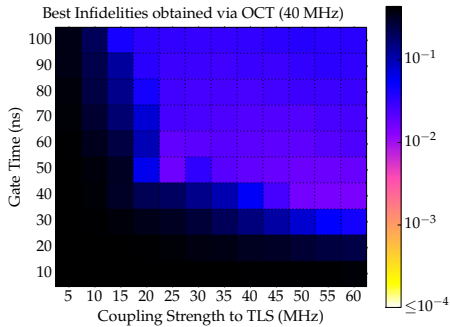
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infinite T_1 (no loss)



$T_1 = 5\mu\text{s}, T_1^{TLS} = 1\mu\text{s}$

→ for strong enough coupling error small even for lossy TLS

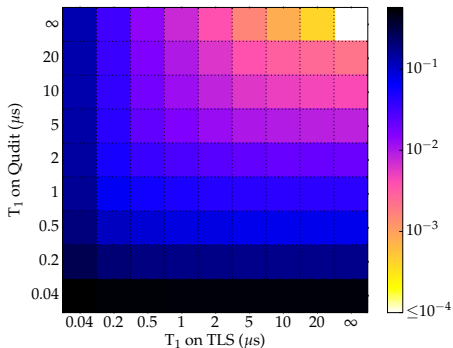
→ for weak coupling no control

role of non-Markovianity

Reich, Katz, CPK, *Sci Rep* 5, 12430 (2015)

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error after optimization for $\text{diag}(1, -1, 1, 1)$ with single TLS

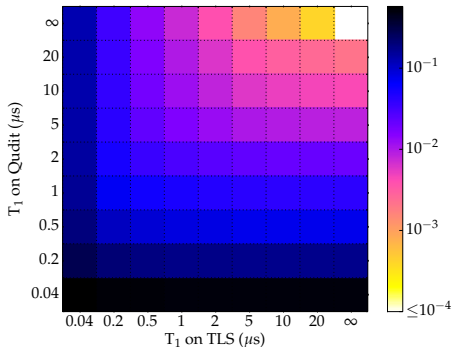


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error after optimization for $\text{diag}(1, -1, 1, 1)$ with single TLS



**control lost once Markovian
decay too fast: $T_1 < 1/S$**

what about additional primary bath TLS?

$\Delta^{(2)}$	$S^{(2)}$	$T_1^{(2)}$	error
50 MHz	40 MHz	2000 ns	$3.076 \cdot 10^{-2}$
50 MHz	40 MHz	200 ns	$4.052 \cdot 10^{-2}$
50 MHz	40 MHz	40 ns	$7.867 \cdot 10^{-2}$
50 MHz	10 MHz	2000 ns	$3.196 \cdot 10^{-2}$
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450 MHz	40 MHz	2000 ns	$1.659 \cdot 10^{-2}$
450 MHz	40 MHz	200 ns	$1.652 \cdot 10^{-2}$
450 MHz	40 MHz	40 ns	$1.758 \cdot 10^{-2}$
450 MHz	10 MHz	2000 ns	$1.663 \cdot 10^{-2}$
450 MHz	10 MHz	200 ns	$1.674 \cdot 10^{-2}$
450 MHz	10 MHz	40 ns	$1.675 \cdot 10^{-2}$

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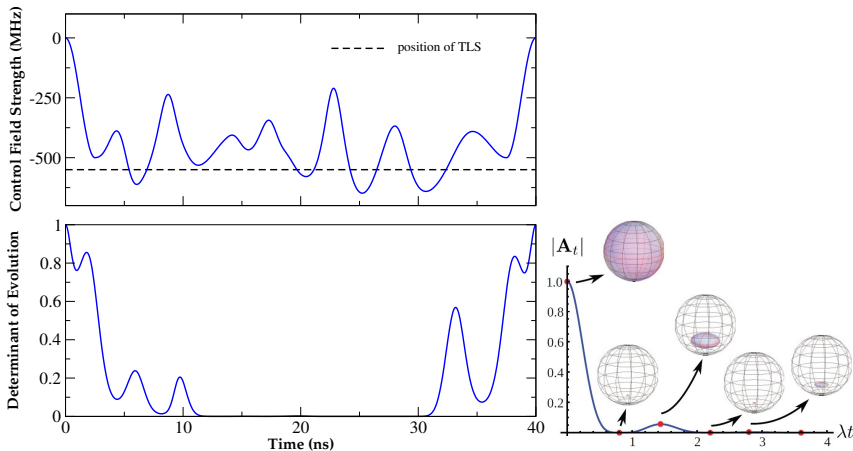
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similar errors for up to 4 TLS
only strongly coupled, very lossy TLS very close by
make life difficult

how does the control work?

Reich, Katz, CPK, *Sci Rep* 5, 12430 (2015)

ramp system into resonance with TLS and back
acquiring non-local phase due to interaction

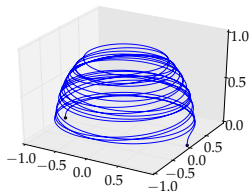


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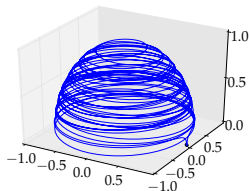
ramp system into resonance with TLS and back
acquiring non-local phase due to interaction

evolution of $|1\rangle$



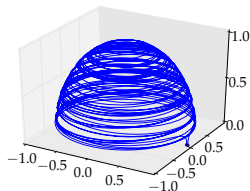
non-local phase
acquired

evolution of $|2\rangle$



original phase
restored

evolution of $|3\rangle$



original phase
restored

proper phase alignment possible thanks to optimal control

opportunities & fundamental limits

3 rules for controlling open quantum systems

review: CPK, J Phys Condens Matter 28, 213001 (2016)

- 1 detrimental case: unwanted Markovian dynamics
 \implies do things as fast as you can (OCT)
- 2 beneficial case: desired Markovian dynamics
target must be fixed state of Liouvillian
 \implies quantum reservoir engineering
- 3 non-Markovian dynamics: beneficial & detrimental effects
beneficial \iff few strongly coupled,
sufficiently isolated modes
directly applicable in current experiments
with superconducting circuits
also applicable to other small 'natural' spin baths
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is it possible to exploit other features of non-Markovianity?

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opportunities & fundamental limits

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