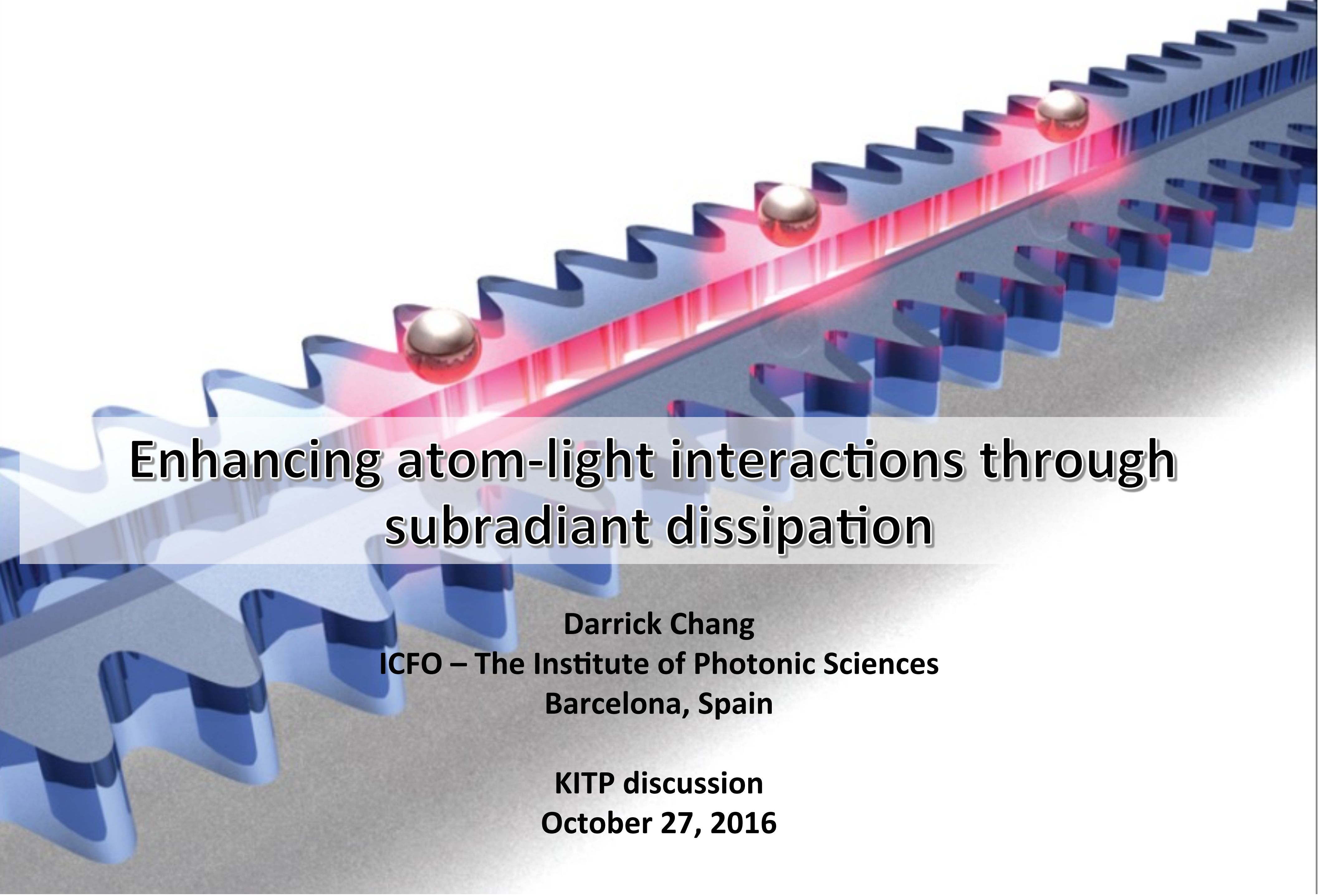


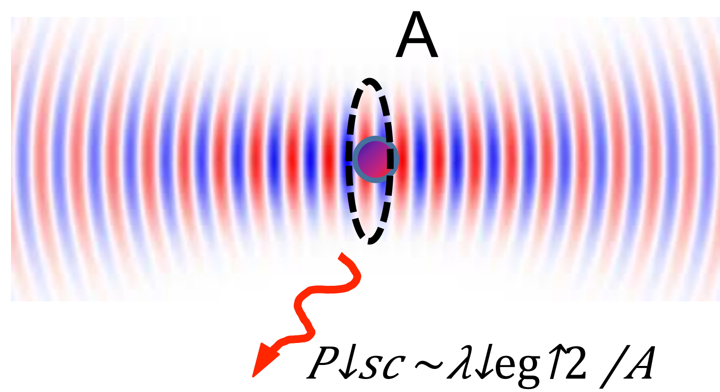
# Enhancing atom-light interactions through subradiant dissipation

**Darrick Chang**
**ICFO – The Institute of Photonic Sciences**
**Barcelona, Spain**

**KITP discussion**
**October 27, 2016**

# Atom-photon interactions

- Problem: single atoms and photons don't like to talk to each other




$P_{sc} \sim \lambda_{eg}^2 / A$

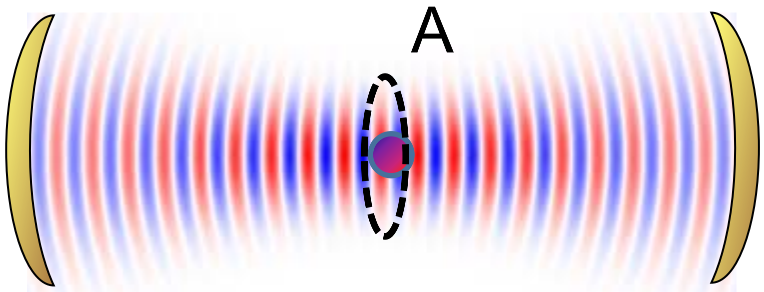
**diffraction limit**

$A > \lambda_{eg}^2$

# Atom-photon interactions

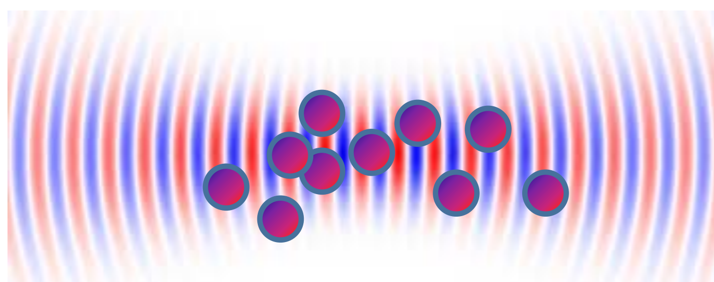
- Some fixes:

- Cavity QED




$P_{sc} \sim \lambda_{eg}^2 / A \; N_{\text{bounces}} \sim g^2 / \kappa\Gamma \sim$ Cooperativity

- Atomic ensembles



$P_{sc} \sim \lambda_{eg}^2 / A \; N_{\text{atoms}} \sim$ Optical depth

# A fundamental limit

- Cooperativity or optical depth widely viewed as a fundamental limit to applications of atom-light interfaces

**A quantum gate between a flying optical photon and a single trapped atom**

Andreas Reiserer[1], Norbert Kalb[1], Gerhard Rempe[1] & Stephan Ritter[1]

In principle, the gate mechanism presented in this work is deterministic. In our experimental implementation, the photon is not back-reflected[8] from the coupled system $|\uparrow^a\uparrow^p\rangle$ with a probability of 34(2)% (due to the finite cooperativity $C=\frac{g^2}{2\kappa\gamma}=3$) and in the uncoupled

**All-Optical Switch and Transistor Gated by One Stored Photon**

Wenlan Chen,[1] Kristin M. Beck,[1] Robert Bücker,[1,2] Michael Gullans,[3] Mikhail D. Lukin,[3] Haruka Tanji-Suzuki,[1,3,4] Vladan Vuletić[1]*

stood in a simple cavity QED model: One atom in state $|s\rangle$ reduces the cavity transmission (*1*, *4*) by a factor of $T=(1+\eta)^{-2}$, where η is the single-atom cooperativity (*30*). In the strong-coupling

**Universal Approach to Optimal Photon Storage in Atomic Media**

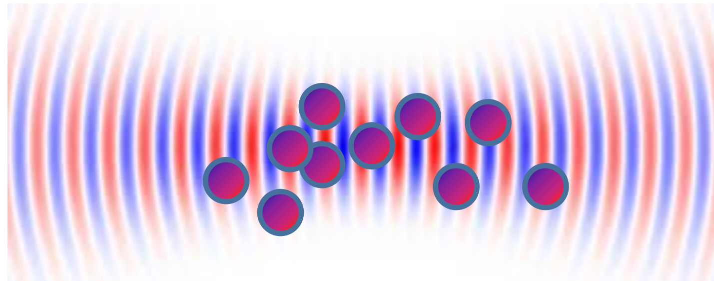
Alexey V. Gorshkov,[1] Axel André,[1] Michael Fleischhauer,[2] Anders S. Sørensen,[3] and Mikhail D. Lukin[1]

off-resonant Raman fields to photon-echo-based techniques. Furthermore, we derive an optimal control strategy for storage and retrieval of a photon wave packet of any given shape. All these approaches, when optimized, yield identical maximum efficiencies, which only depend on the optical depth of the medium.

...

# Is there a loophole?

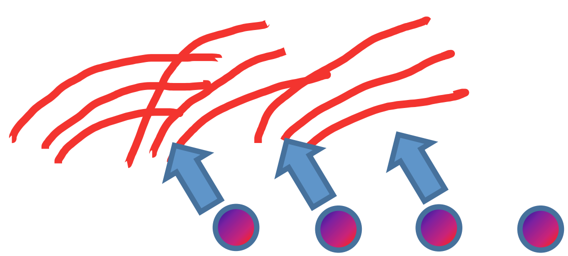
- Atomic ensembles



$P_{sc} \sim \lambda_{eg}^2 / A \; N_{atoms} \sim$ Optical depth

- Why this scaling?
  - Each atom has a probability of talking to the "good" mode (Gaussian) vs. the bad (free space)
  - Effectiveness is multiplied by # of attempts $(\propto N_{atoms})$

- **Important assumption:** coupling to free space assumed to be independent
  - Cannot be true – atoms emit waves, which can interfere



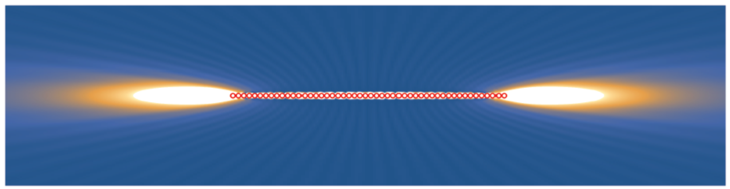
**Physics of super- and sub-radiance**

# Outline

- **Big question:** can we exploit collective emission to enhance atom-light interfaces? **(Yes!)**

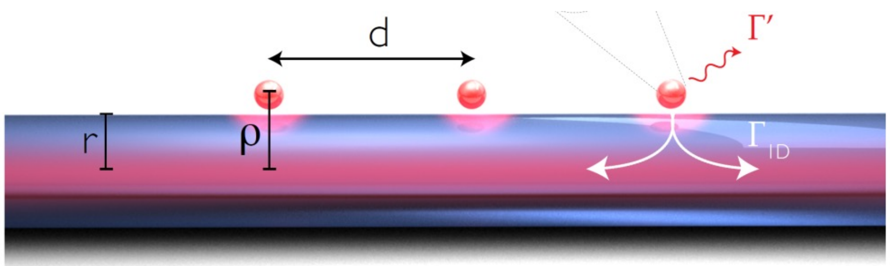
- Formalism to treat atom-light interfaces including subradiance

- Origin of subradiance in atomic arrays in free space



**Subradiance = Guided modes**

- "Selective" subradiance: atoms coupled to a nanofiber



# Light-matter interactions as a spin model

# Electromagnetic Green's function

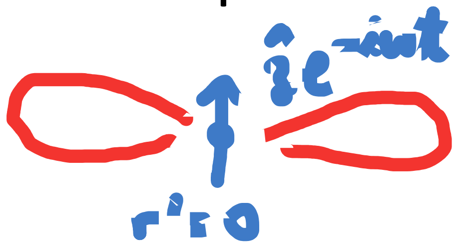
- Formal definition:

$$\left[(\nabla\times\nabla\times) - \frac{\omega^2}{c^2}\varepsilon(r,\omega)\right] G_{\alpha\beta}(r,r',\omega) = \delta(r-r')\, I$$

- G describes the electric field at point r, of a (normalized) oscillating dipole at r'


- Tensor quantity ($\alpha,\beta = x,y,z$): source dipole can have three orientations, and the electric field at r is a vector

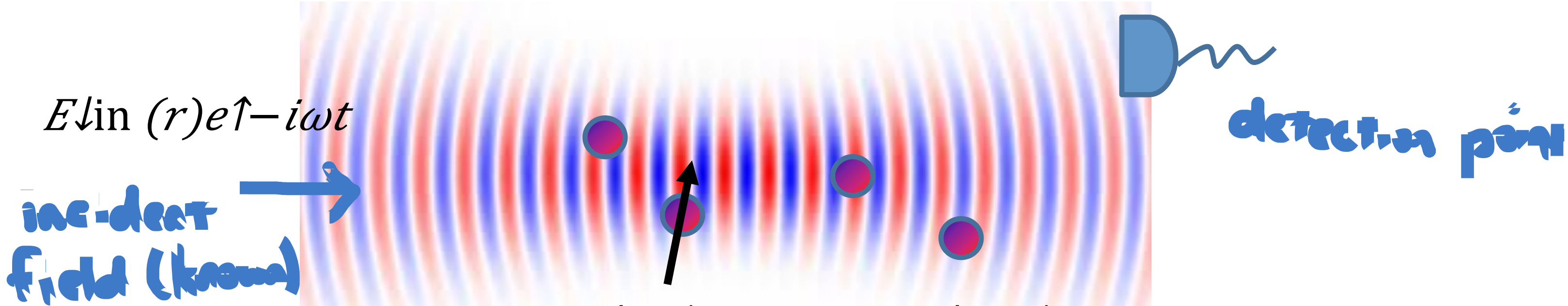
- Simple case: free space




$$G(r,0,\omega) = e^{ikr - i\omega t}\left[\frac{k^2}{r}(\hat{n}\times\hat{p})\times\hat{n} + (3\hat{n}(\hat{n}\cdot\hat{p}) - \hat{p})\left(\frac{1}{r^3} - \frac{ik}{r^2}\right)\right]$$

# Getting rid of the light

- Classical scattering from polarizable particles




$E_{in}(r)e^{-i\omega t}$

$p_i e^{-i\omega t} = \alpha E(r_i) e^{-i\omega t}$

- Know the radiation pattern for a dipole
- Can calculate the total field

$$E(r,\omega) = E_{in}(r,\omega) + \alpha \sum G(r, r_i, \omega) p_i(\omega)$$

Becomes convolution in time domain

  - Classical and quantum fields propagate the same way

- Generalized "input-output" equation *in time* for atoms

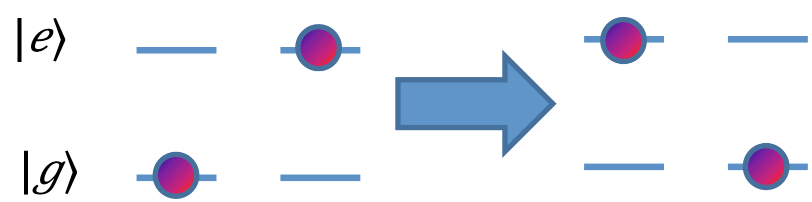
$$E(r,t) = E_{in}(r,t) + (3\pi\hbar c\Gamma_0 / d_0 \omega_{eg}) \sum_i G(r, r_i, \omega_{eg}) \sigma_{ge}^i(t)$$

**Field encoded in atoms!**

# What about the atoms?

- Atoms interact with fields, but fields are dependent on the atoms themselves
- Effective "spin" Hamiltonian involving atoms alone

$$H_{eff} = -\left(\frac{3\pi\hbar\Gamma_0 c}{\omega_{eg}}\right)\sum_{i,j} G(r_j, r_i, \omega_{eg})\, \sigma_{eg}^{i}\, \sigma_{ge}^{j}$$



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- Strength depends on how field propagates from j to i

- Non-Hermitian Hamiltonian – describes both coherent interactions and (collective) spontaneous emission

# A "trivial" example

- Must work for a single atom in free-space too

$$H_{\mathrm{eff}} = -\left(3\pi\hbar\Gamma_0 c/\omega_{eg}\right)\sum_{i,j} G(r_j, r_i, \omega_{eg})\,\sigma_{eg}^{i}\,\sigma_{ge}^{j}$$

$$H_{\mathrm{eff}} = -\left(3\pi\hbar\Gamma_0 c/\omega_{eg}\right) G(r_{atom}, r_{atom}, \omega_{eg})\,\sigma_{ee}$$

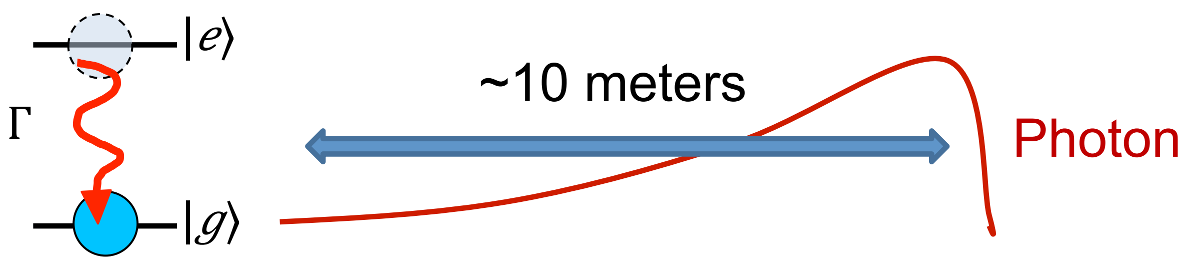
$$H_{\mathrm{eff}} = -\left(3\pi\hbar\Gamma_0 c/\omega_{eg}\right) i\omega_{eg}/6\pi c\;\sigma_{ee} = -i\hbar\Gamma_0/2\;\sigma_{ee}$$

- Describes single-atom spontaneous emission at rate $\Gamma_0$ !

# Spin model

$$H_{eff} = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} G(r_j, r_i, \omega_{eg}) \sigma_{eg}^i \sigma_{ge}^j$$

- Limits of validity
  - No strong coupling effects (e.g. vacuum Rabi oscillations)
  - Ignores time retardation




- Equally captures any system of atoms interacting with light, and treats them on equal footing
  - Cavity QED, free-space atomic ensembles, nanophotonic systems, …

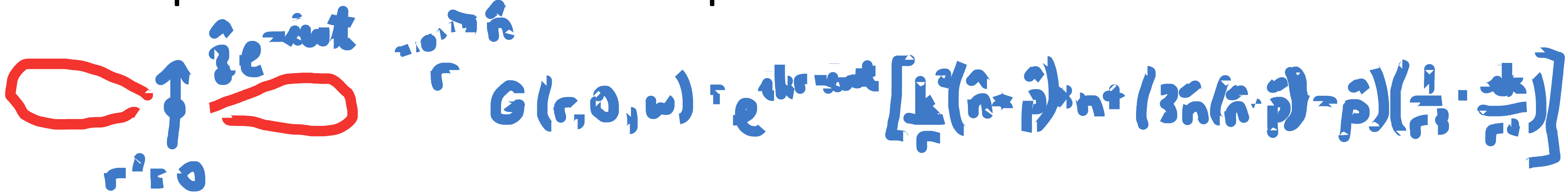
# 1D chain of atoms in free space

# Green's function for 1D atomic chain

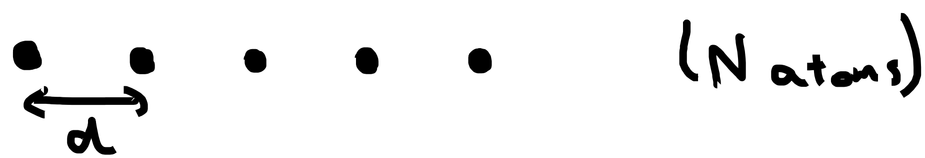
- Spin model in general:

$$H_{eff} = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} G(r_j, r_i, \omega_{eg}) \sigma_{eg}^i \sigma_{ge}^j$$

- Simple case: atoms in free space



- Must specify a geometry: 1D chain of atoms (but living in 3D)




- Considering just one excitation, there are N possible states

$|eggg\ldots\rangle, |gegg\ldots\rangle, |ggeg\ldots\rangle, \ldots$

- Hamiltonian becomes N x N matrix in this subspace, $H_{ij} \propto G(r_j, r_i)$

# Exact diagonalization

- Numerically diagonalize H
  - In general, each eigenstate is a superposition of excitations sitting on different atoms
  - Eigenvalues give the frequency shift and decay rate of each collective state

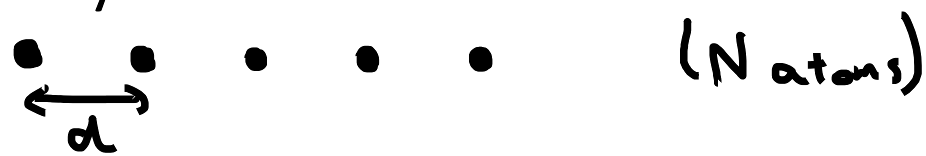
$(j = 1, \ldots, N)$ $\quad \omega_j = (\mathrm{Re}\,\omega_j) + i\,(\mathrm{Im}\,\omega_j)$

real energy shift — decay rate

- Energies and decay rates modified from single-atom values, $\omega_{eg}$ and $\Gamma_0$, due to photon-mediated interactions

# Energy shifts and decay rates

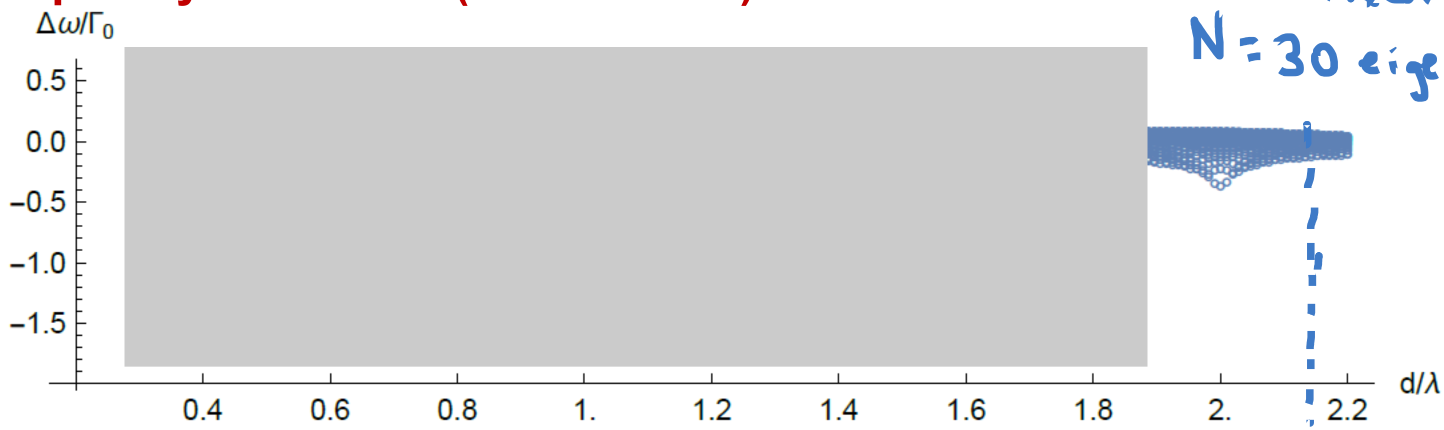
- Shifts and decay rates vs. lattice constant




**Frequency shift vs. d (N=30 atoms)**

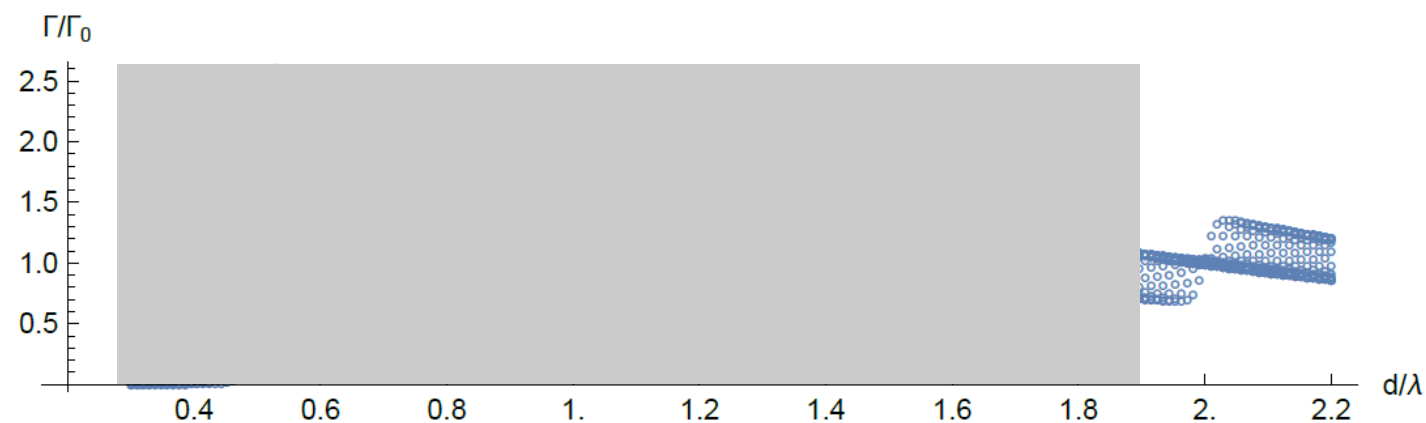
For fixed d, N=30 eigenvalues

Shift normalized by free-space linewidth ($\Delta\omega_j/\Gamma_0$)




**Decay rate vs. d (N=30 atoms)**

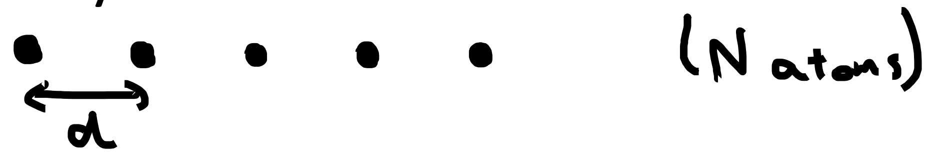
Decay rate normalized by free-space linewidth ($\Gamma_j/\Gamma_0$)




- Minimal effect of interactions at large distances

# Energy shifts and decay rates

- Shifts and decay rates vs. lattice constant

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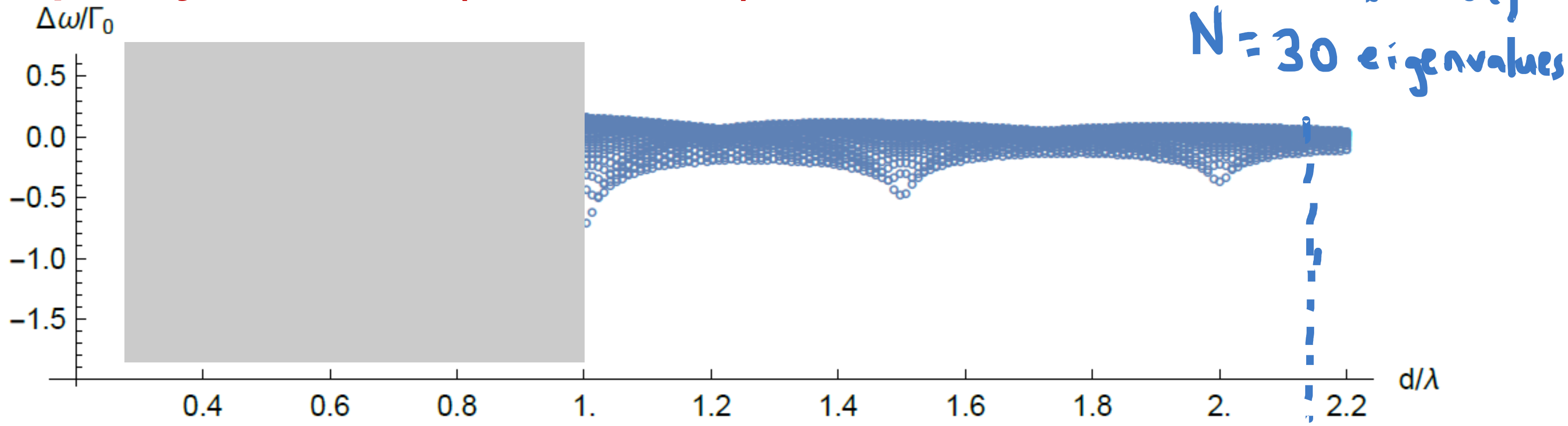


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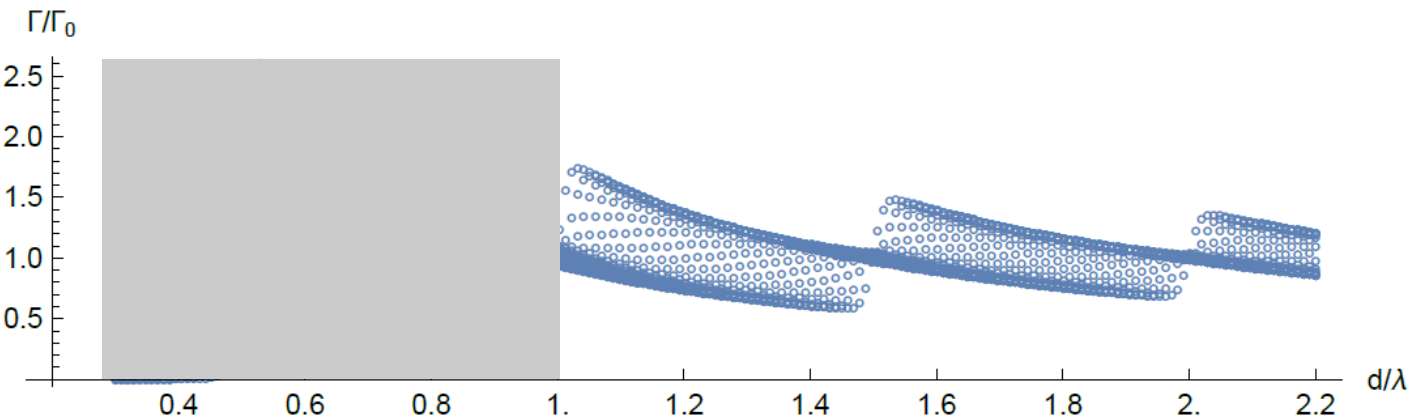
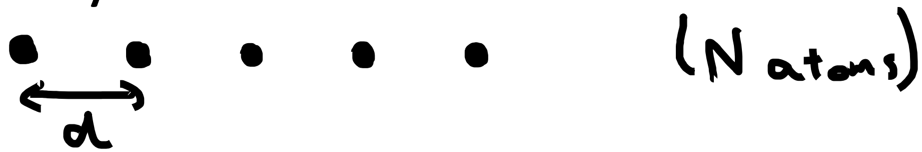



- Noticeable corrections at $d \sim \lambda$ (>50% modification of decay)

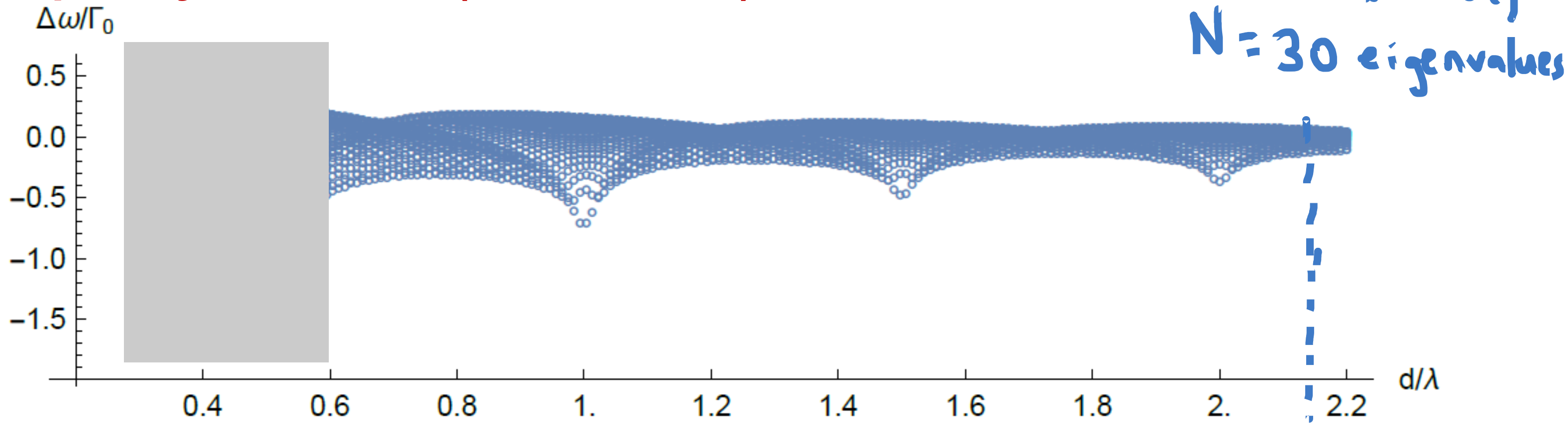
# Energy shifts and decay rates

- Shifts and decay rates vs. lattice constant



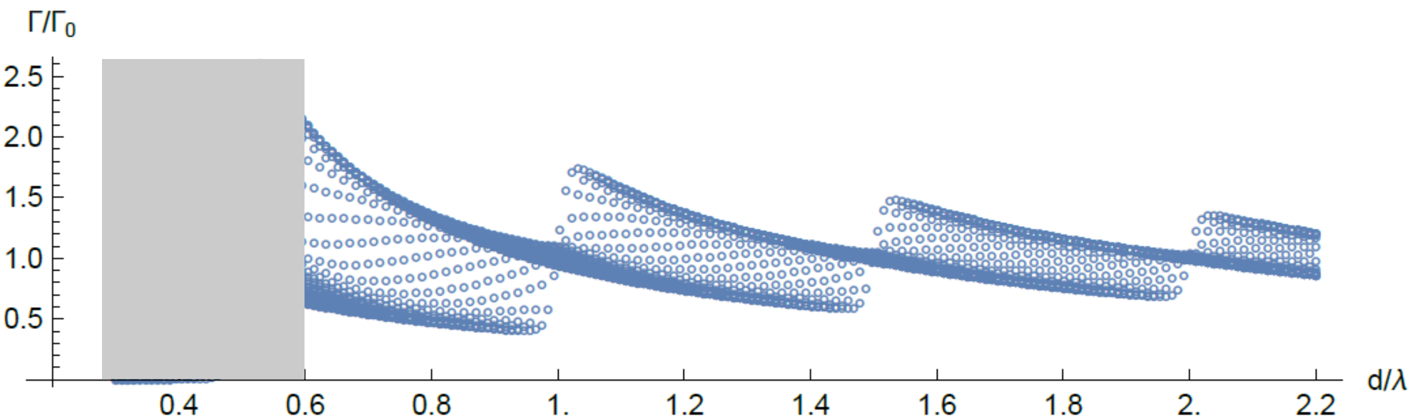

**Frequency shift vs. d (N=30 atoms)**

Shift normalized by free-space linewidth ($\Delta\omega_j/\Gamma_0$)




**Decay rate vs. d (N=30 atoms)**

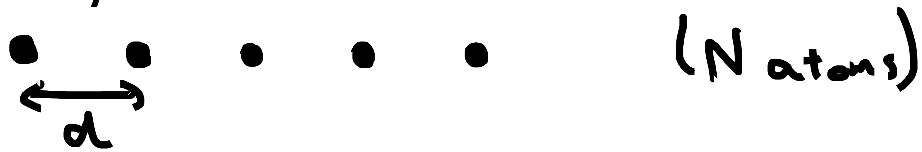
Decay rate normalized by free-space linewidth ($\Gamma_j/\Gamma_0$)




- One state doubles in decay rate as $d \sim \lambda/2$

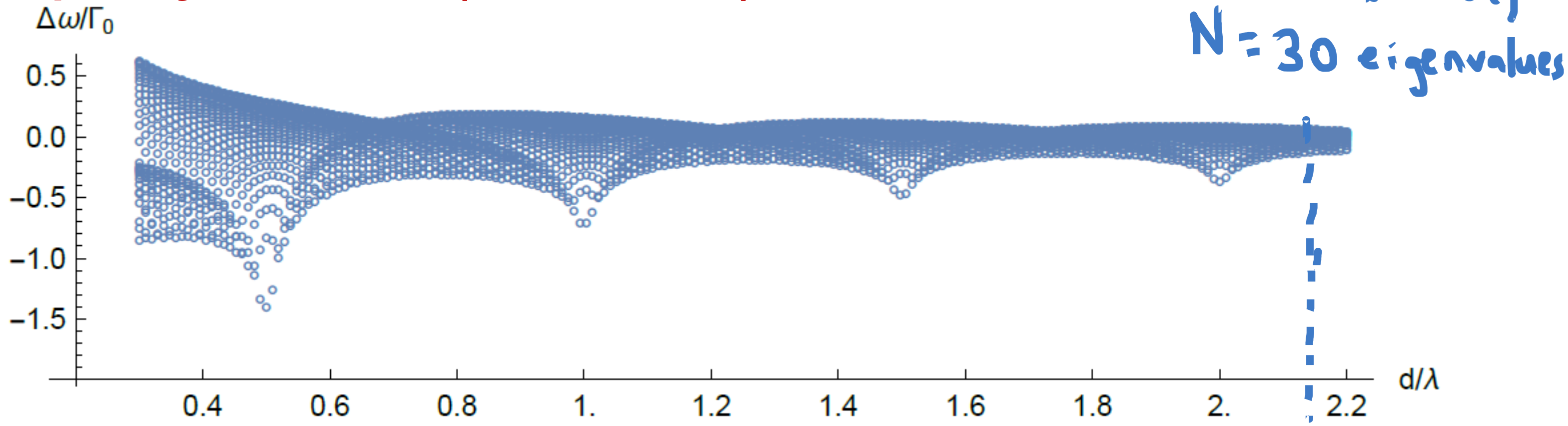
# Energy shifts and decay rates

- Shifts and decay rates vs. lattice constant



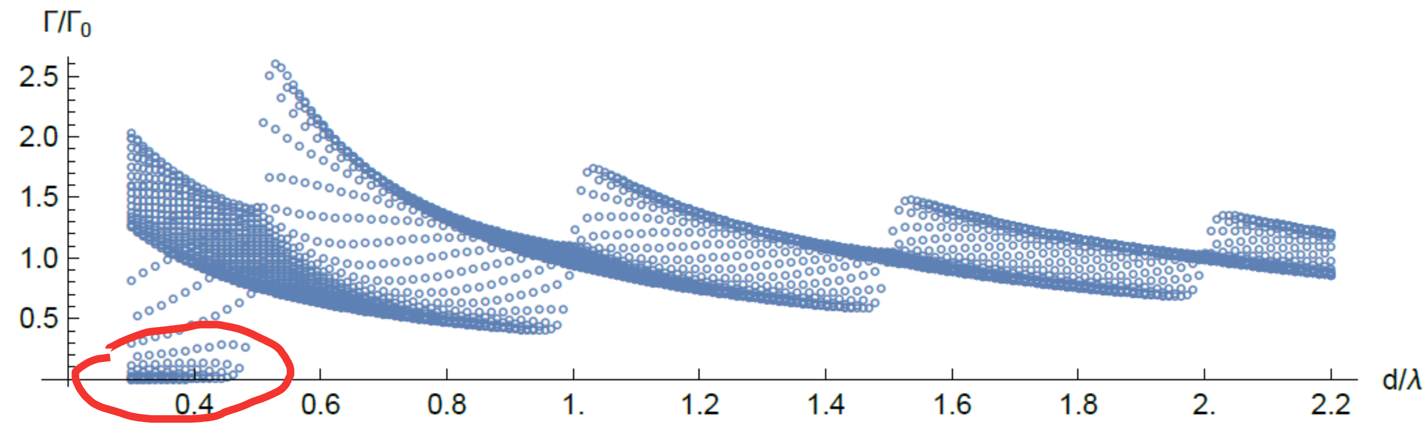

**Frequency shift vs. d (N=30 atoms)**

Shift normalized by free-space linewidth ( $\Delta\omega_j/\Gamma_0$ )




**Decay rate vs. d (N=30 atoms)**

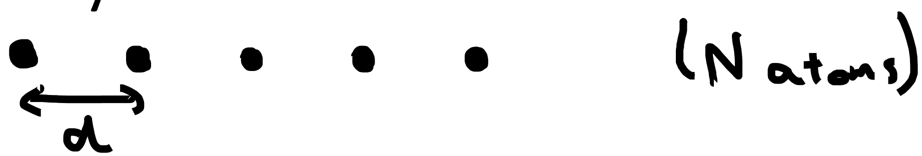
Decay rate normalized by free-space linewidth ( $\Gamma_j/\Gamma_0$ )




- States with nearly zero decay rate for $d<\lambda/2$ !

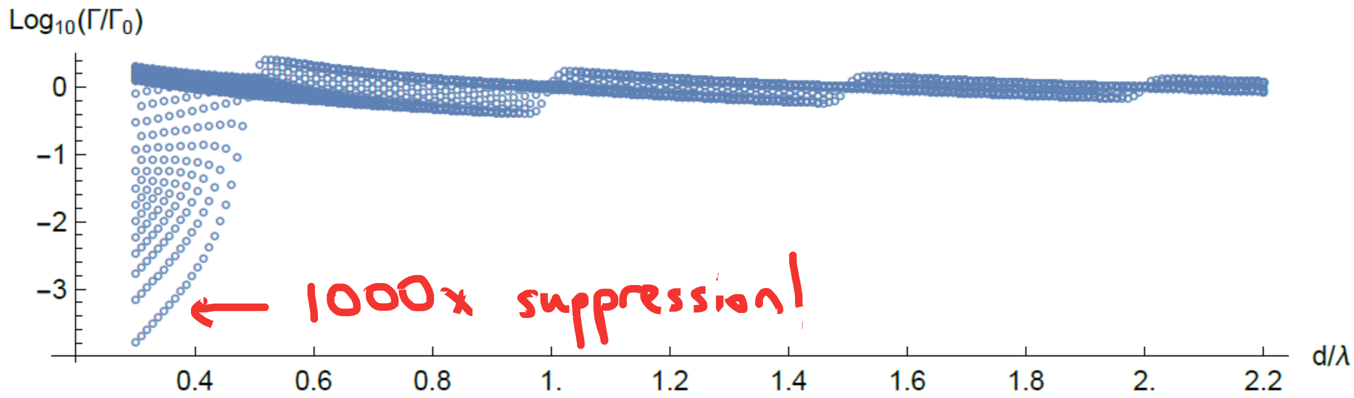
# Energy shifts and decay rates

- Shifts and decay rates vs. lattice constant




**Decay rate (log scale) vs. d (N=30 atoms)**

(Log scale) Decay rate normalized by free-space linewidth $(\Gamma_j/\Gamma_0)$




- Numerical analysis: smallest decay rates go like $\Gamma_m \sim \Gamma_0 m^2 / N^3$

  $m=1,2,3,\ldots$

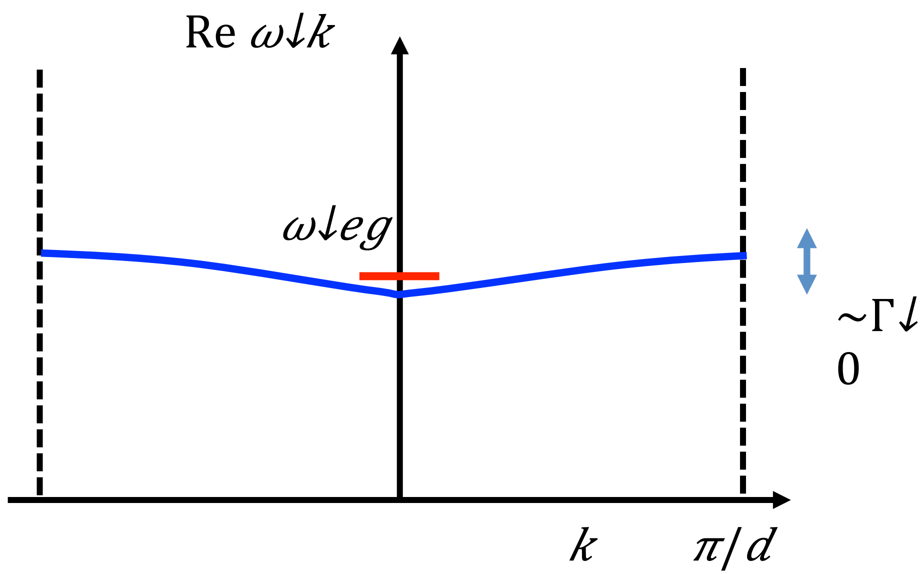
- Interference of wave emission really matters at close distances!

# Band structure of infinite lattice

- Infinite chain: single-excitation eigenstates are Bloch modes

$$|\psi_k\rangle = \sum_j e^{ikz_j} |e_j\rangle$$

- Diagonalize H, represent spectrum by band structure



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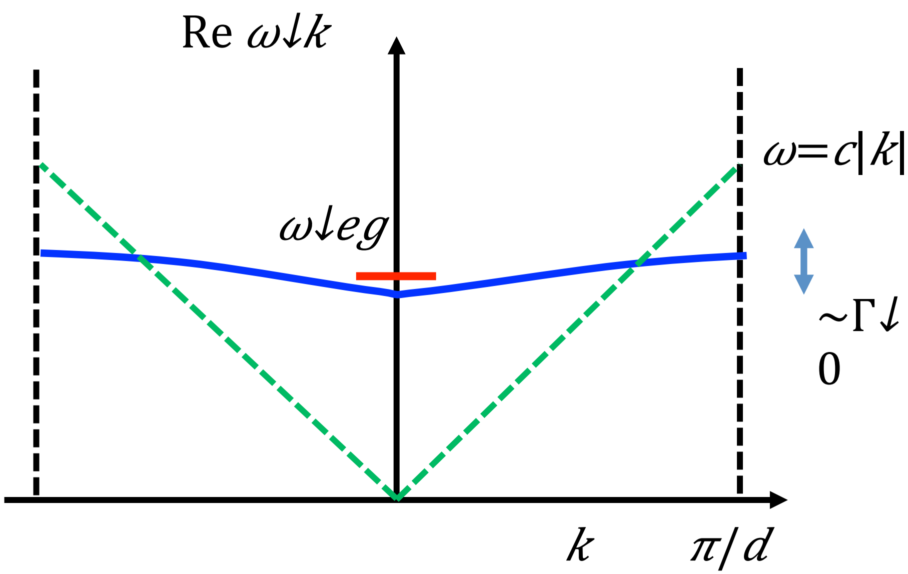


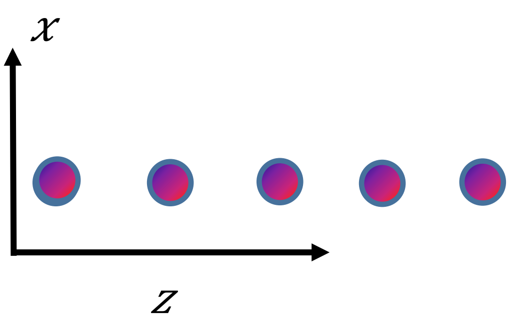


- Emitted light also has same wavevector: $E(r) \sim e^{ikz + ik_\perp x}$
  - Dispersion relation $k^2 + k_\perp^2 = (\omega/c)^2$
  - $|k| > \omega/c$ implies $k_\perp \in \mathrm{Im}$ (evanescent or guided mode)

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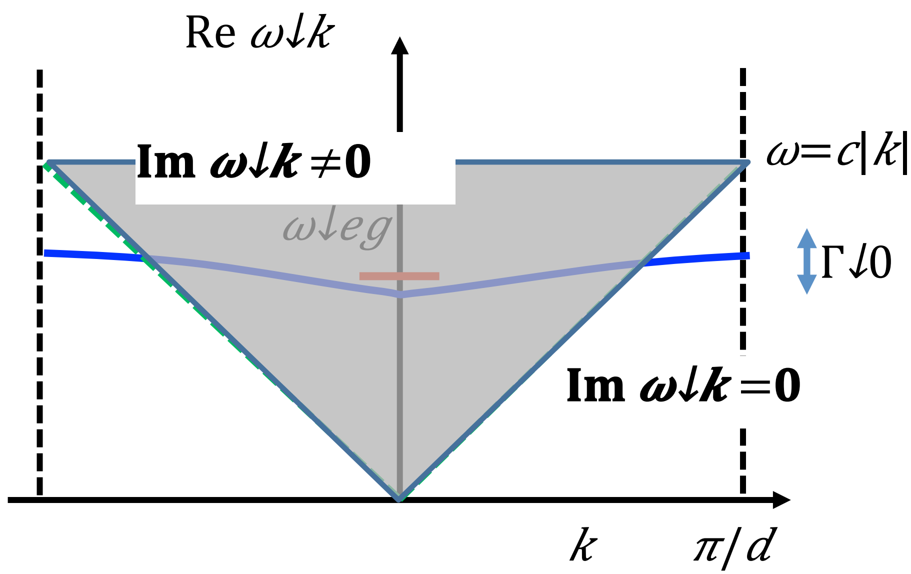


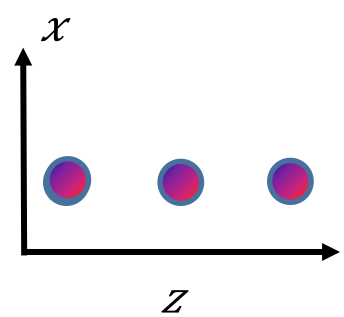


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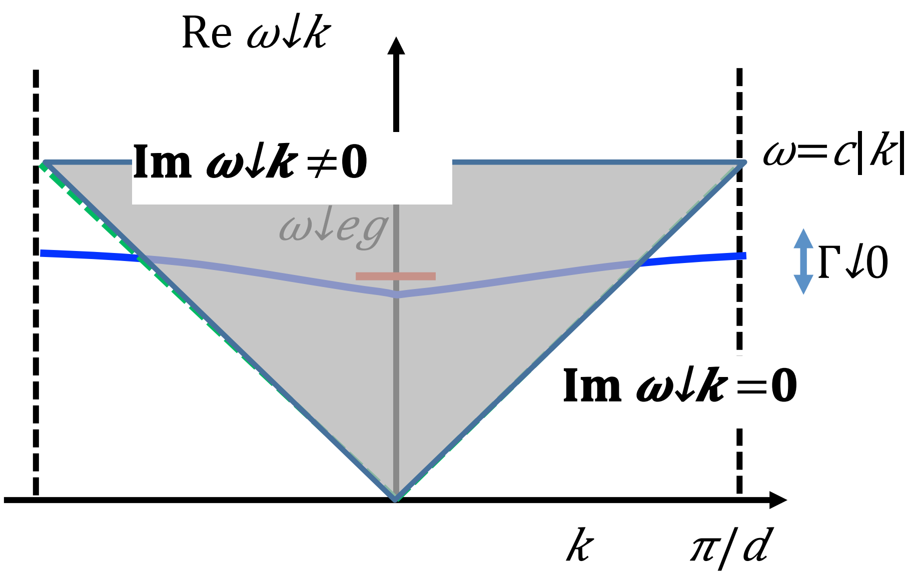
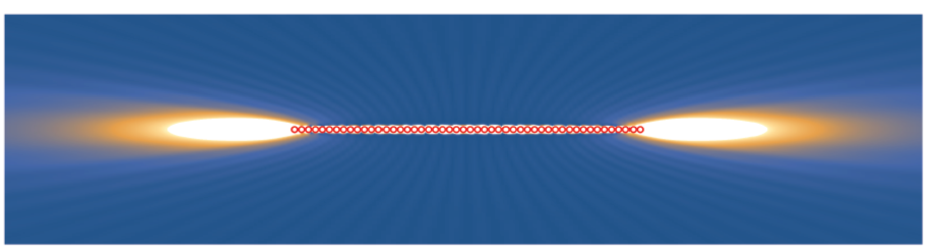
# Subradiant states as guided modes

- An infinite chain supports states with *zero* decay rate, which are simply guided "fiber" modes of the chain!




- Condition: $\omega = c|k|$ intersects Brillouin zone above $\omega_{eg}$
- Equivalent to $d < \lambda_{eg}/2$

- Finite chain: finite decay rate $\Gamma_m \propto \Gamma_0 m^2/N^3$ due to end-fire emission off the "fiber" ends



**Emission pattern for most sub-radiant state, N=30 atoms**

# Multiple excitations

- Single-excitation physics is classical
- Can we encode many excitations in sub-radiant states?

- Most sub-radiant single excitation $\Gamma_1 \propto \Gamma_0 / N^3$

$$|\psi\rangle = S_1^{\dagger} |g\rangle^{\otimes N} = \sum_j c_j^{(1)} |e_j\rangle$$

- Example: two excitations
  - What if we create same excitation twice: $|\psi\rangle = (S_1^{\dagger})^2 |g\rangle^{\otimes N}$

**Eigenstate?**

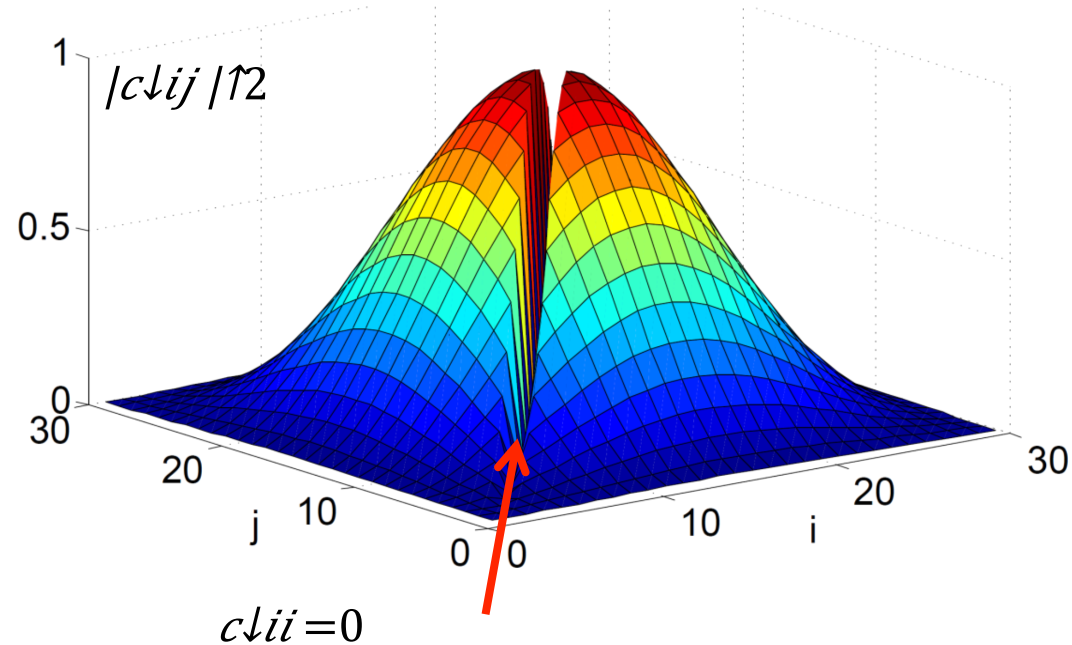
- Bosons: yes! Decay rate $2\Gamma_1$
- Spins: no!

# Two-excitation wavefunction

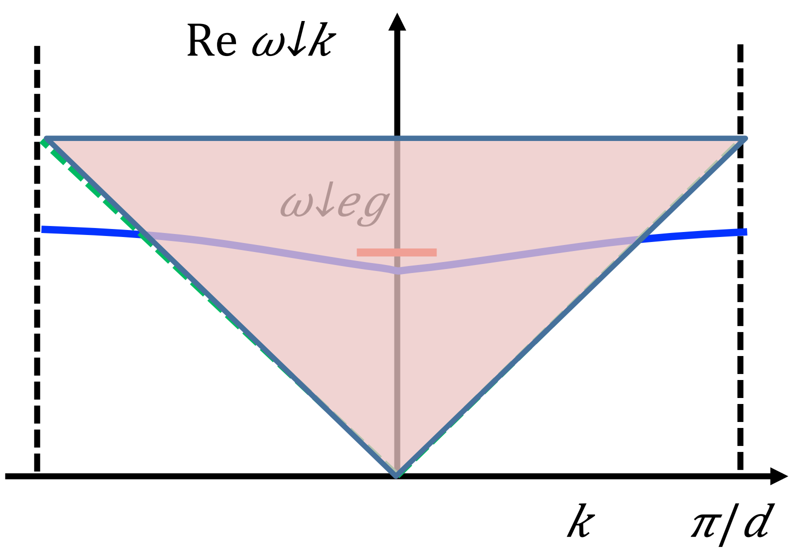
- Wave function:

$$|\psi\rangle = (S_1^\dagger)^2 |g\rangle^{\otimes N}$$

$$= \sum_{i,j} c_{ij} |e_i e_j\rangle$$




- State contains many momentum components within light cone




- Only weakly subradiant

$$\langle \Gamma \rangle \sim \Gamma_0 / N$$

# Multiple excitations

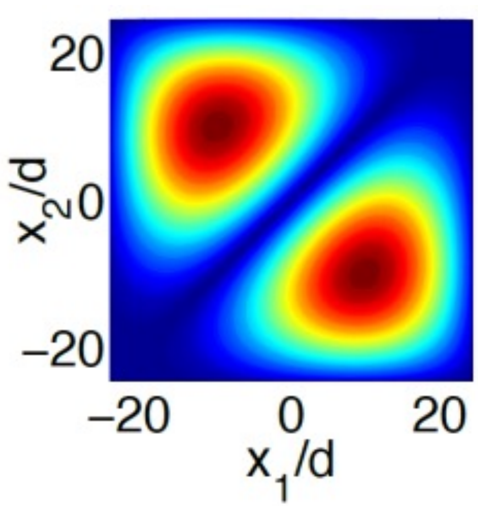
- How to construct many-excitation subradiant states?
- Single-particle eigenstates

  $\Gamma_m \propto m^2 \Gamma_0 / N^3$ ,  $|\psi_m\rangle = \sum_j c_j^{(m)} |e_j\rangle$

- Sub-radiant states should satisfy:
  - Wavevectors beyond the light line, *and* spatially non-overlapping
- Create a *fermionized* wave function
- Example: two excitations

  $|\psi\rangle = \sum_{i>j} c_{ij} |e_i e_j\rangle$

  $c_{ij} = c_i^{(1)} c_j^{(2)} - c_i^{(2)} c_{\downarrow}$




$\Gamma \sim \Gamma_1 + \Gamma_2 \propto 1/N^3$

# Multiple excitations

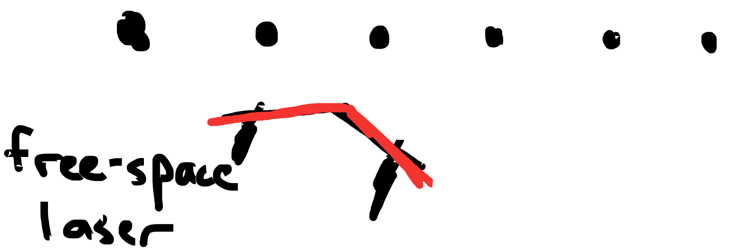
- Can extend to many-body limit
- N atoms, m excitations

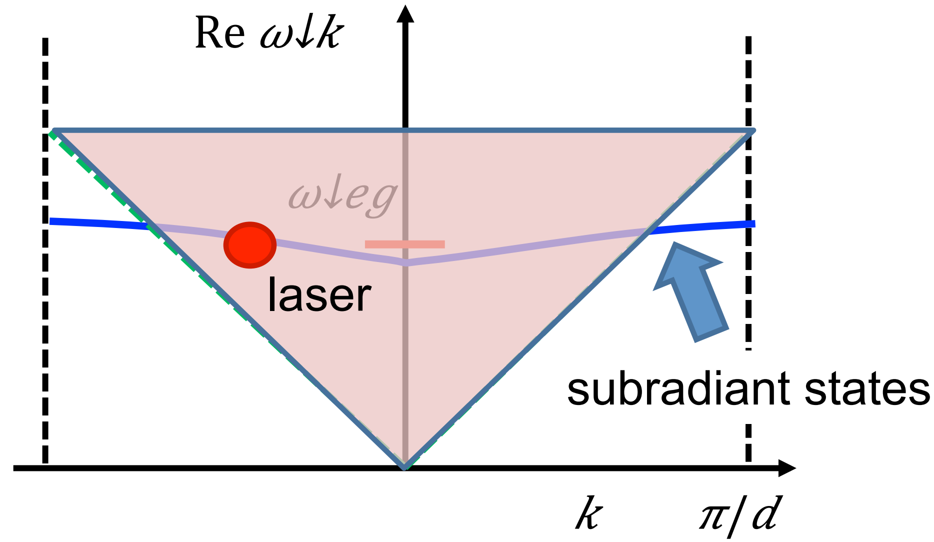
$$\Gamma/\Gamma_0 \sim \sum_{j=1}^{m} j^2 / N^3 \sim (m/N)^3$$

- System can support a low density of excitations in subradiant manifold

# Accessing subradiant states

- Hard to couple to subradiant states from free space



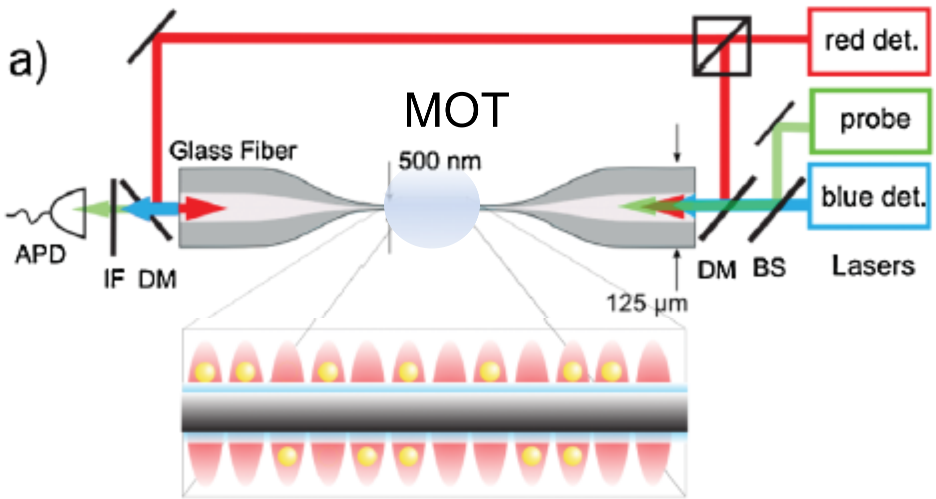




- Excite with another set of guided modes (optical nanofiber!)

# Atom-nanofiber interfaces

# Atom-nanofiber interface

- Trap atoms in lattice with far off-resonant guided modes

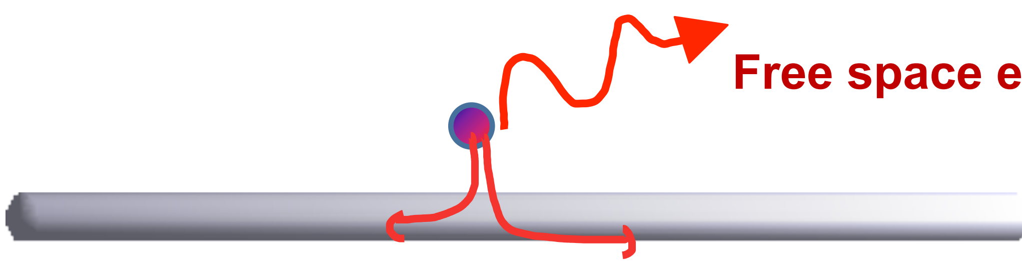



**Vetsch et al, PRL 104, 203603 (2010)**

- Efficient coupling of atoms with near-resonant photons

**Free space emission rate** $\Gamma' \sim \Gamma_0$

**Single atom:**

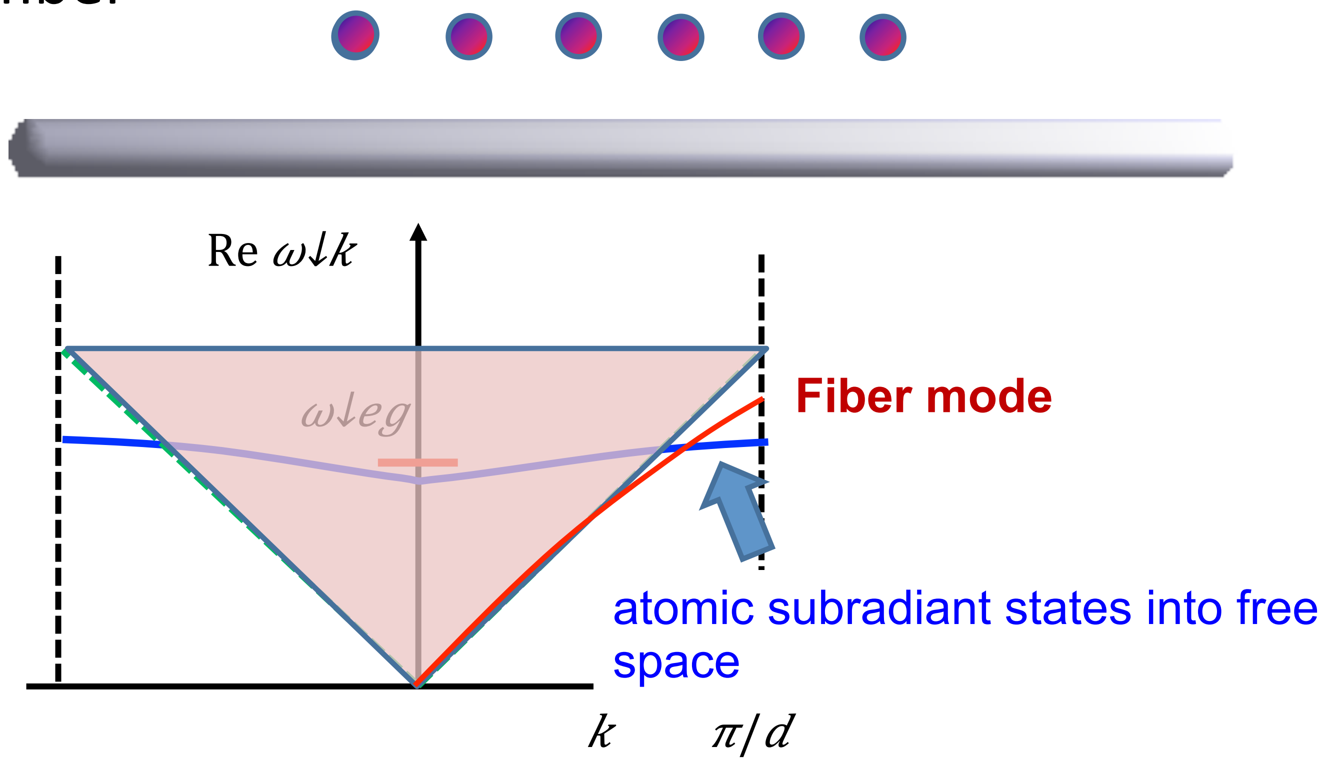


**Emission into guided modes** $\Gamma_{1D} \sim 0.1\Gamma_0$

- Expts: Rauschenbeutel (TU Vienna), Kimble (Caltech), Orozco (JQI), Laurat (Paris), Polzik (Niels Bohr Institute), …

# "Selective" subradiance

- Previously subradiant states can now couple to guided modes of the fiber

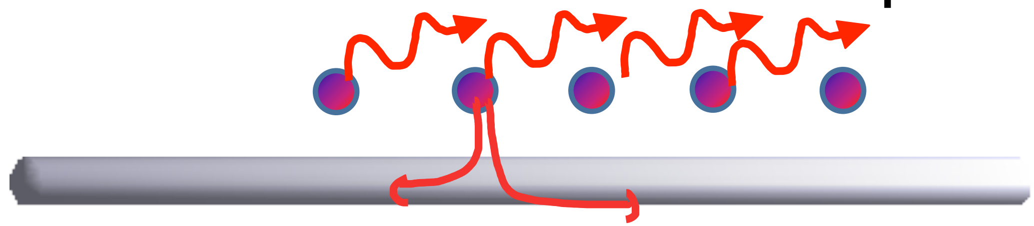



- Not subradiant, but "selectively" subradiant… perfect!
- Efficient atom-light interface: atoms talk very well to modes of interest, rather than undesired modes (free space)
  - Use this to beat the "optical depth" limit

# Model of atom-light interactions

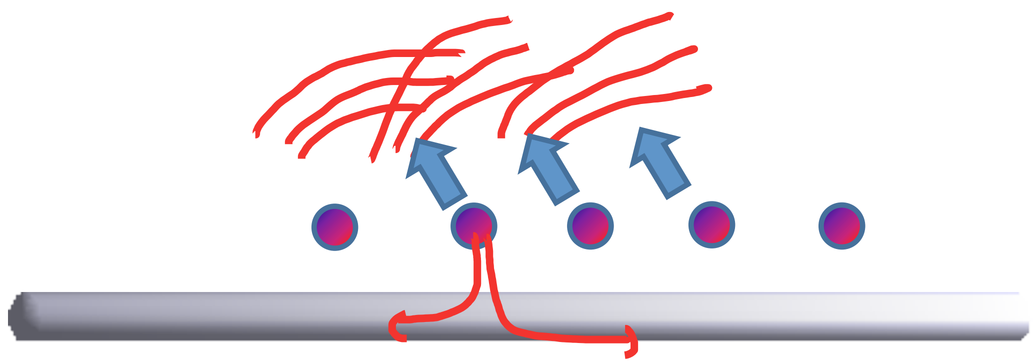
- “Independent emission model”

**Independent emission rate $\Gamma'$**



**Collective emission into guided modes $N\Gamma_{1D}$**

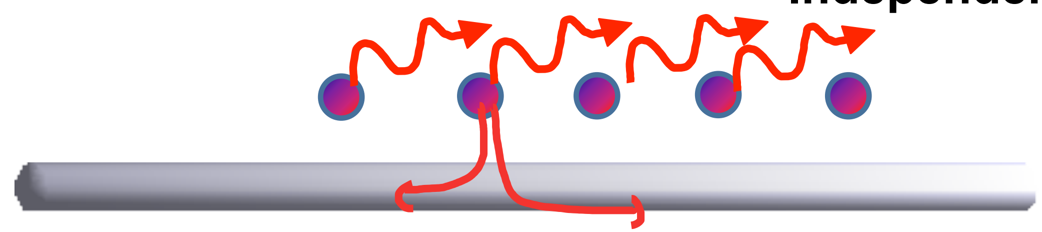
- Collective emission model



**Collective emission into guided modes $N\Gamma_{1D}$**

# Model of atom-light interactions

- “Independent emission model”




- Collective emission model

- Use our “spin model”

$$H_{\text{eff}} = -\mu_0 d_0^2 \omega_{eg}^2 \sum_{i,j} G(r_j, r_i, \omega_{eg}) \sigma_{eg}^i \sigma_{ge}^j$$

Exact for cylindrical fiber

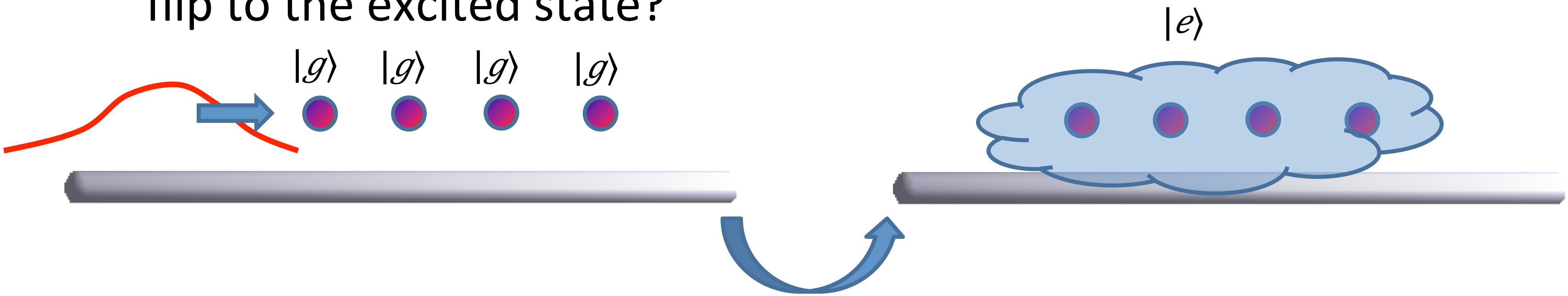
- Field propagation

$$E(r,t) = E_{in}(r,t) + (3\pi\hbar c\Gamma_0 / d_0 \omega_{eg}) \sum_i G(r, r_i, \omega_{eg}) \sigma_{ge}^i(t)$$

Project into guided mode of fiber

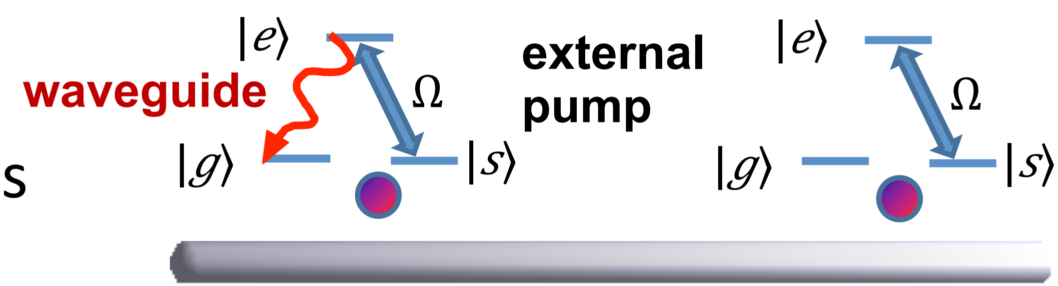
# Photon storage

- How well can one store an incoming photon as a collective spin flip to the excited state?




- By time reversal symmetry, can study the reverse problem of mapping an initial spin excitation into a photon

- Use three-level systems

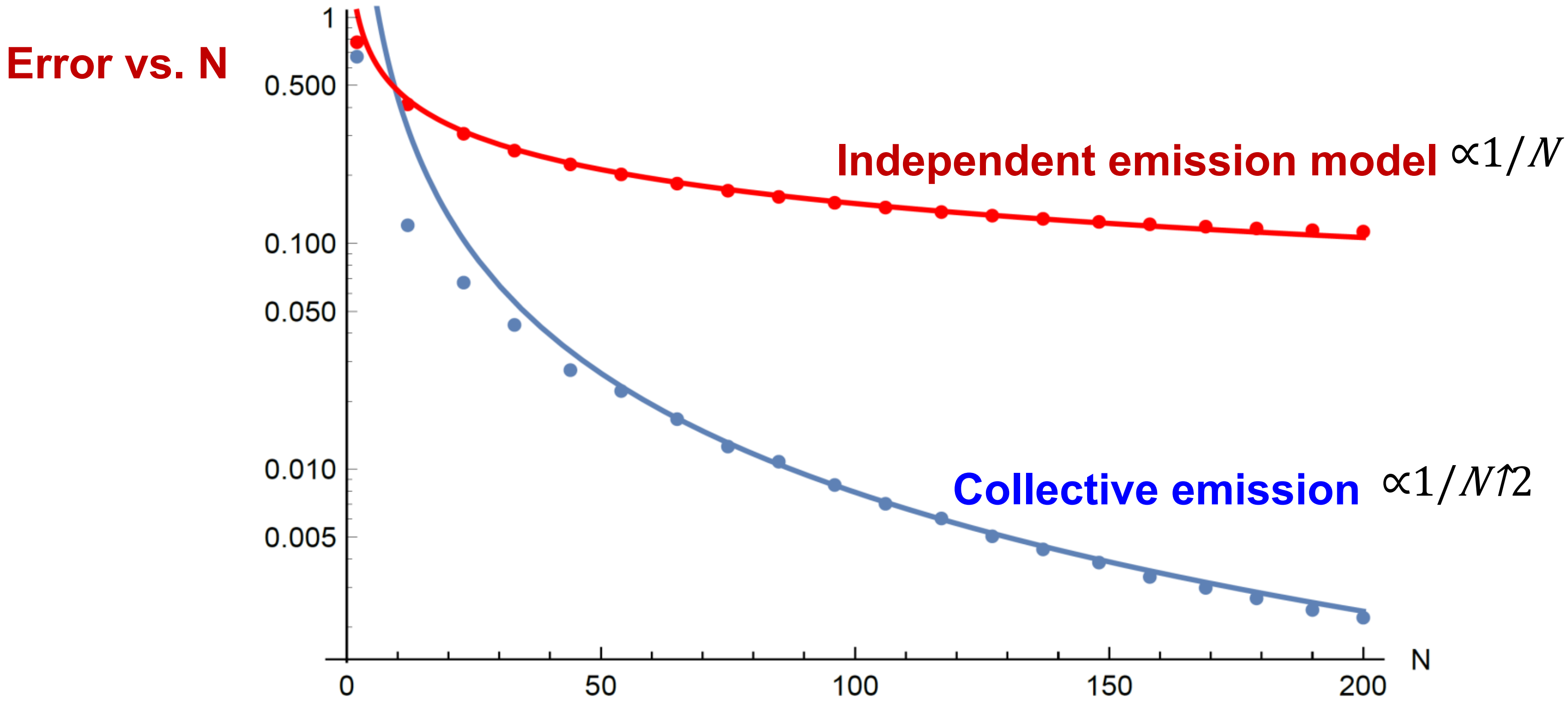



- Independent emission model: error $\sim \Gamma' / N\Gamma_{1D} = 1/\mathrm{OD}$

**Same result as free space: AV Gorshkov et al, PRL 98, 123601 (2007)**

# Photon storage/generation

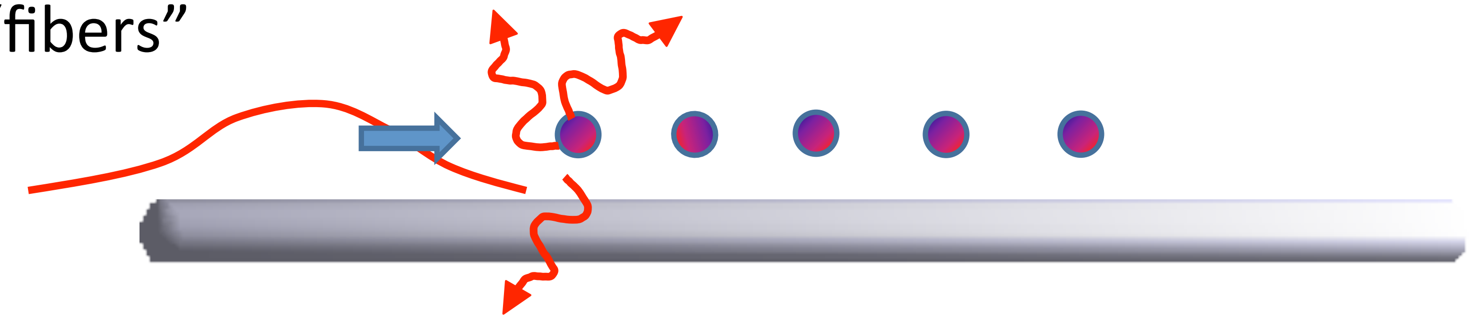
- Collective emission:
  - Numerically diagonalize $H_{eff}$ within manifold of single $|s\rangle$ excitation
  - Look for eigenstate with highest emission probability into waveguide



# What limits the process?

- $1/N^2$ error seems less than ideal
  - $1/N$ from collective enhancement into guided mode
  - "Only" $1/N$ suppression of emission into free space

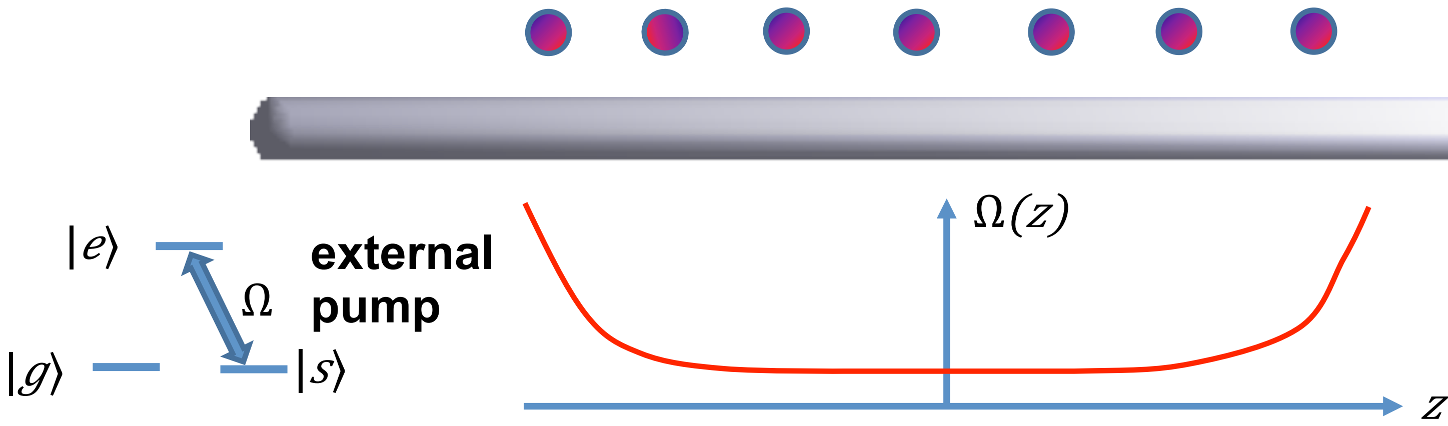
- Origin: scattering losses at interface between two different "fibers"



- Need a fiber "connector!"

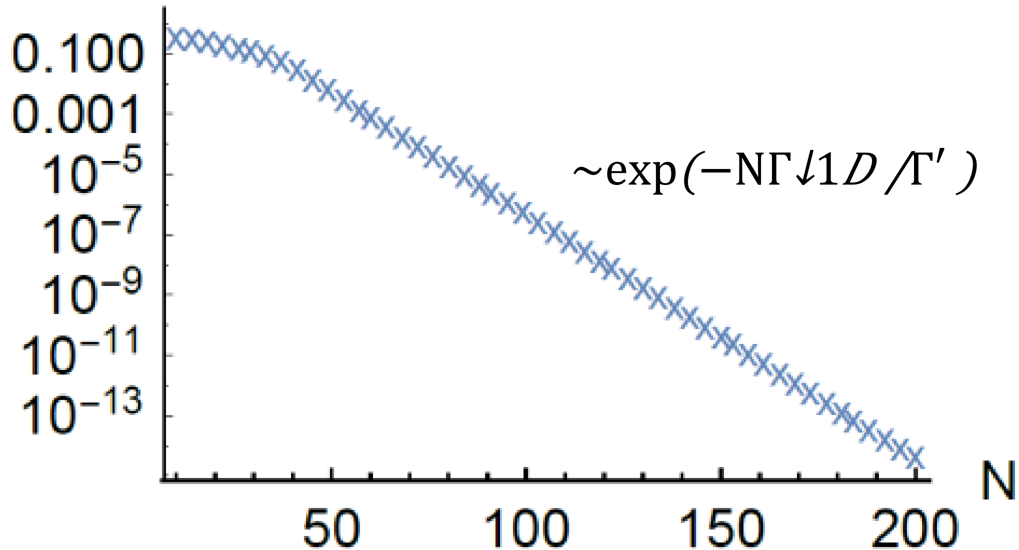
# Photon storage with impedance matching

- Spatially vary the pump field profile, so that atoms at the ends "gradually" couple to fiber



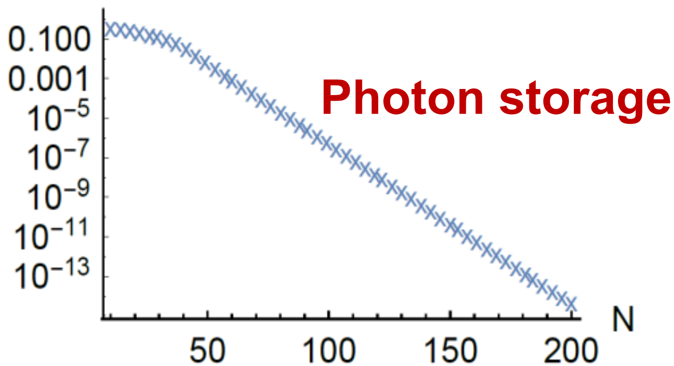

- Exponential suppression of error!

**Error vs. N**



# Outlook

- Spontaneous emission represents a fundamental barrier for atom-light interactions
- What is the new ceiling if one exploits subradiance?




**Nonlinear optics,**
**Photon gates,**
**Spin physics,**
**Lattice clocks, …**

- Atom-light interactions as a quantum spin model

$$H_{\mathrm{eff}} = -\frac{3\pi\hbar\Gamma_0 c}{\omega_{eg}} \sum_{i,j} G(\mathbf{r}_j, \mathbf{r}_i, \omega_{eg})\, \sigma_{eg}^{i} \sigma_{ge}^{j}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_{in}(\mathbf{r},t) + \frac{3\pi\hbar c\Gamma_0}{d_0\, \omega_{eg}} \sum_{i} G(\mathbf{r},\mathbf{r}_i,\omega_{eg})\, \sigma_{ge}^{i}(t)$$

- New insights or numerical tools for AMO physics?

**Matrix product states**