Synthetic Quantum Matter:
Quantum Simulation of …

Lattice Gauge Theories with Cold Atoms & Ions

Peter Zoller

overview / tutorial & recent work

HEP & cond mat

UNIVERSITY OF INNSBRUCK

ÖAW

UQUAM
ERC Synergy Grant

SFB
FoQuS

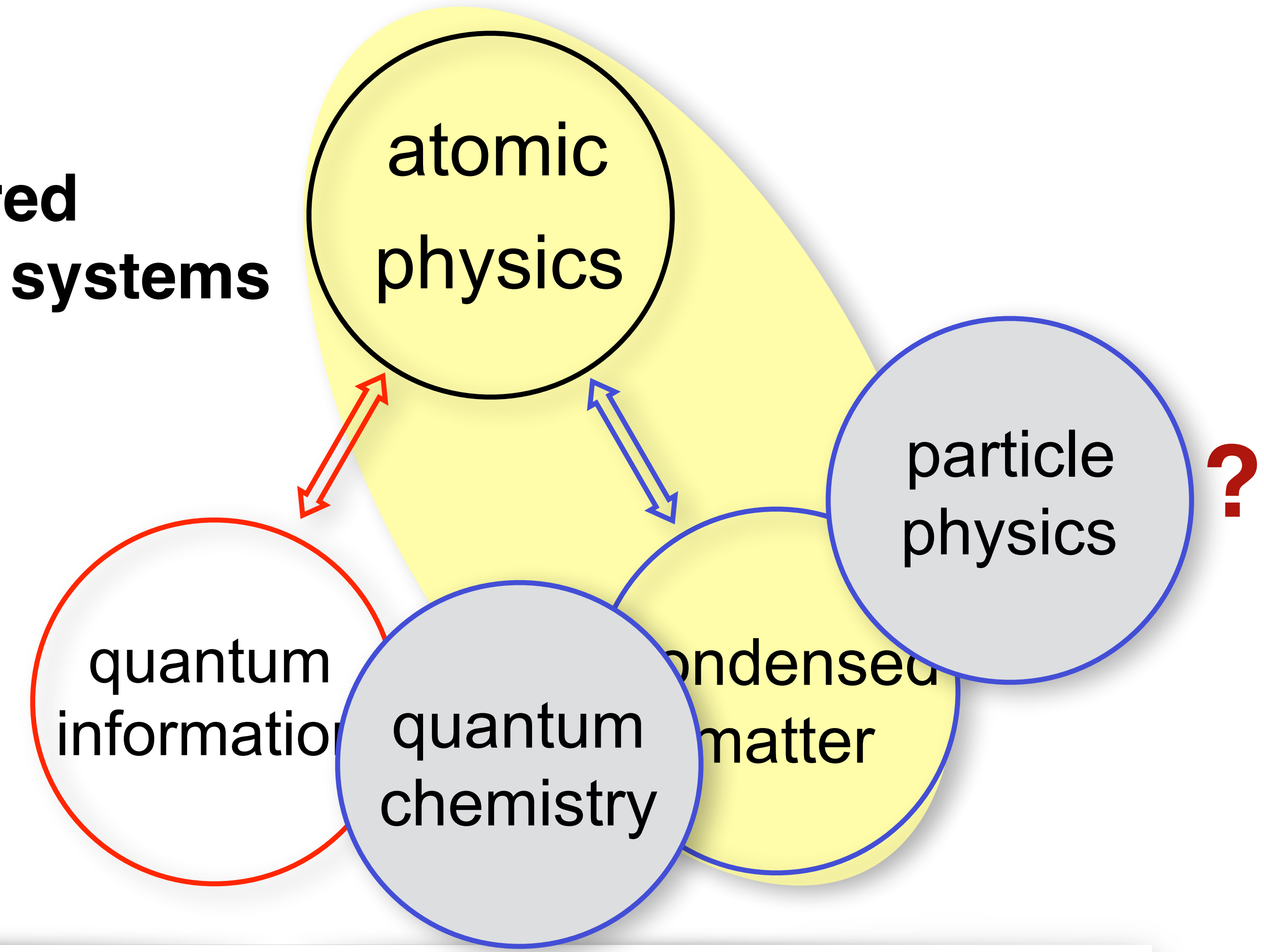
siqs

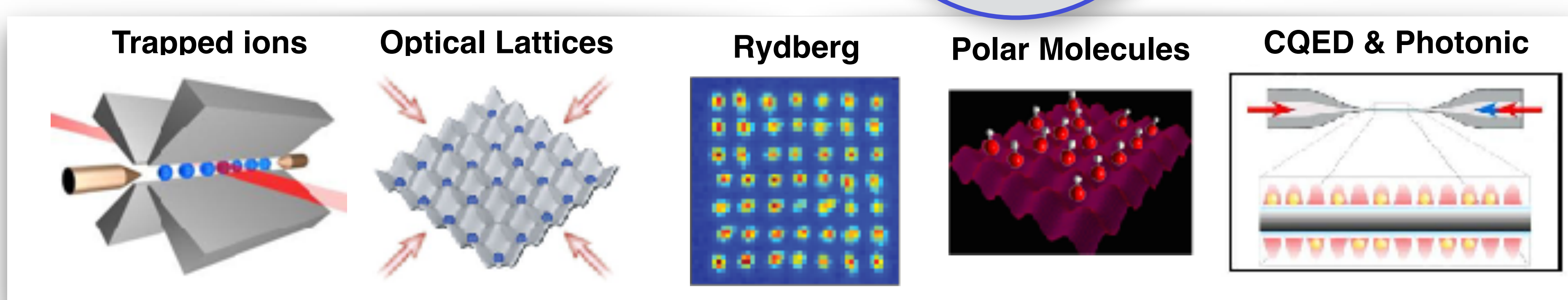
RYQS

Synthetic Quantum Matter

Controlled / engineered quantum many-body systems

- quantum computing
- quantum simulation
  - digital vs. analog

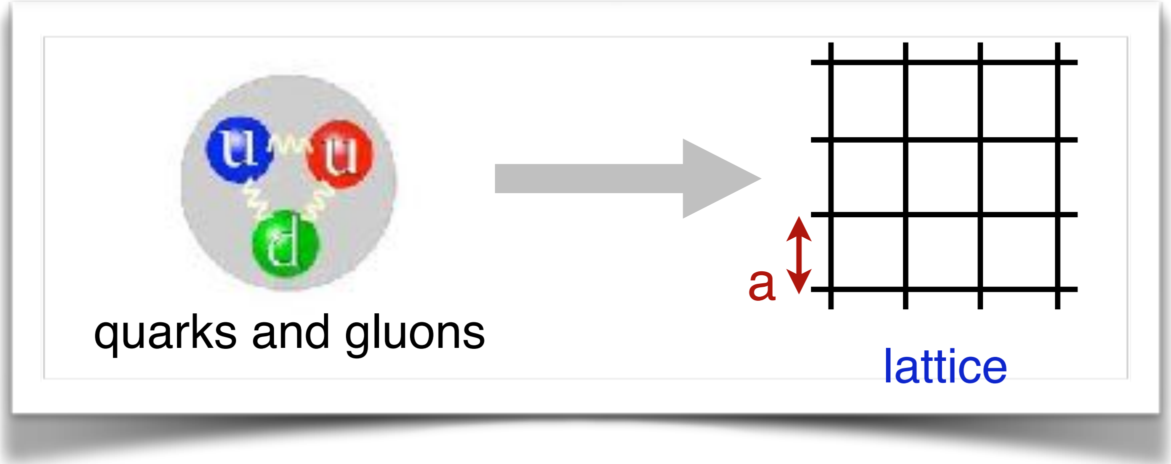




… lattice gauge theories [in particle physics]

K. Wilson 1974

- Gauge theories on a discrete lattice structure
- **Fundamental gauge symmetries:** standard model (every force has a gauge boson)



non-perturbative approach to fundamental theories of matter (e.g. QCD)
→ classical statistical mechanics

Classical Monte Carlo simulations:

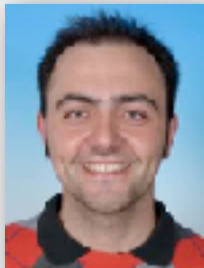
achievements	issues
• first evidence of quark-gluon plasma • ab initio estimate of proton mass • entire hadronic spectrum	Sign problem in its various flavors: • finite density QCD (=fermions) • real time evolution

Quantum simulation (with atoms)? … toy models & simple phenomena

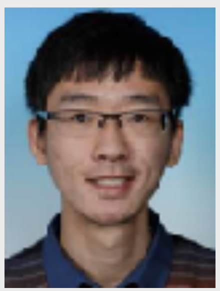
Lattice Gauge Collaborations

Innsbruck - Theory


C. Muschik


M. Dalmonte

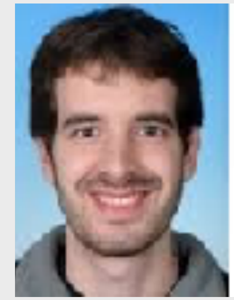

P. Hauke


D. Yang


A. Glätzle


E. Rico

Innsbruck - Ion Experiment

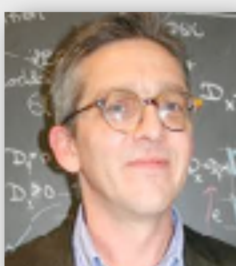

E. Martinez


R. Blatt

Ulm, Dresden, Mainz…

S. Montangero, M. Heyl, R. Mössner,
F. Schmidt-Kaler, …

High Energy Theory @ Bern


U.-J. Wiese


D. Banerjee

related work: Cirac-Reznik, Lewenstein, Berges-Oberthaler, Mueller …

Representative papers and reviews

Zohar, E., Cirac, J. I., & Reznik, B. (Rep. Prog. Phys. 2015).
Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices.

Wiese, U. J. (*Annalen der Physik* 2013). Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories.

Dutta, O., Tagliacozzo, L., Lewenstein, M., & Zakrzewski, J. (*arXiv:1601.03303*).
Toolbox for Abelian lattice gauge theories with synthetic matter.

Innsbruck papers

Büchler, H. P., Hermele, M., Huber, S. D., Fisher, M. P., & PZ. (PRL 2005).
Atomic quantum simulator for lattice gauge theories and ring exchange models..

Banerjee, D., Dalmonte, M., Müller, M., Rico, E., Stebler, P., Wiese, U. J., & PZ. (PRL 2012).
Atomic quantum simulation of dynamical gauge fields coupled to fermionic matter:

Banerjee, D., Bögli, M., Dalmonte, M., Rico, E., Stebler, P., Wiese, U. J., & PZ (PRL 2013).
Atomic quantum simulation of U (N) and SU (N) non-Abelian lattice gauge theories.

Hauke, P., Marcos, D., Dalmonte, M., & PZ. (PRX 2013).
Quantum simulation of a lattice Schwinger model in a chain of trapped ions.

Glaetzle, A. W., Dalmonte, M., Nath, R., Rousochatzakis, I., Moessner, R., & PZ (PRX 2014).
Quantum spin-ice and dimer models with Rydberg atoms.

Glaetzle, A. W., Dalmonte, M., Nath, R., Gross, C., Bloch, I., & PZ (PRL 2015).
Designing frustrated quantum magnets with laser-dressed Rydberg atoms.

Stannigel, K., Hauke, P., Marcos, D., Hafezi, M., Diehl, S., Dalmonte, M., & PZ (PRL 2014).
Constrained dynamics via the Zeno effect in quantum simulation: … non-Abelian lattice gauge theories

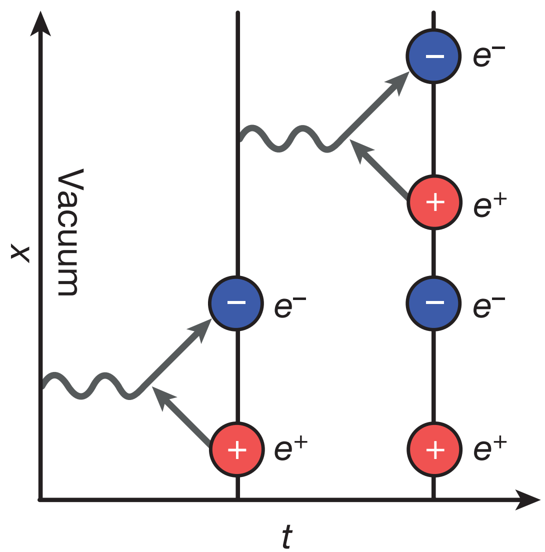
Laflamme, C.,et al., H., Bietenholz, W.iese UJ & PZ. (*Annals of Physics*, 2016).
CP quantum field theories with alkaline-earth atoms in optical lattices

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

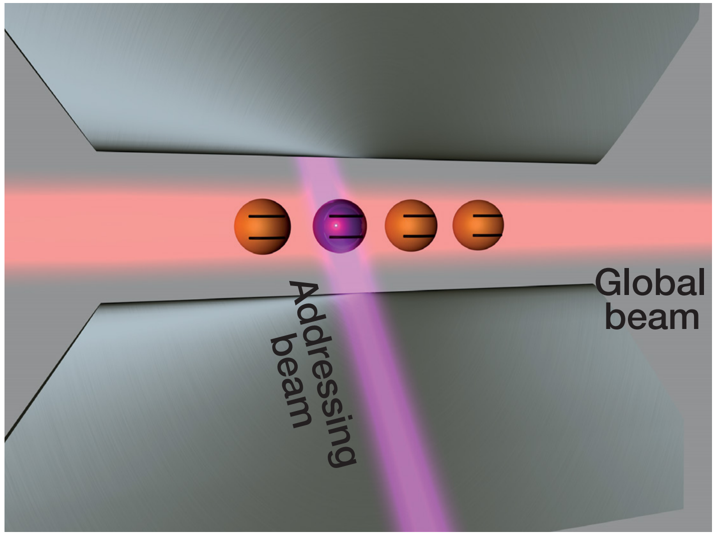
doi:10.1038/nature18318

Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

Schwinger pair production



ion trap quantum computer



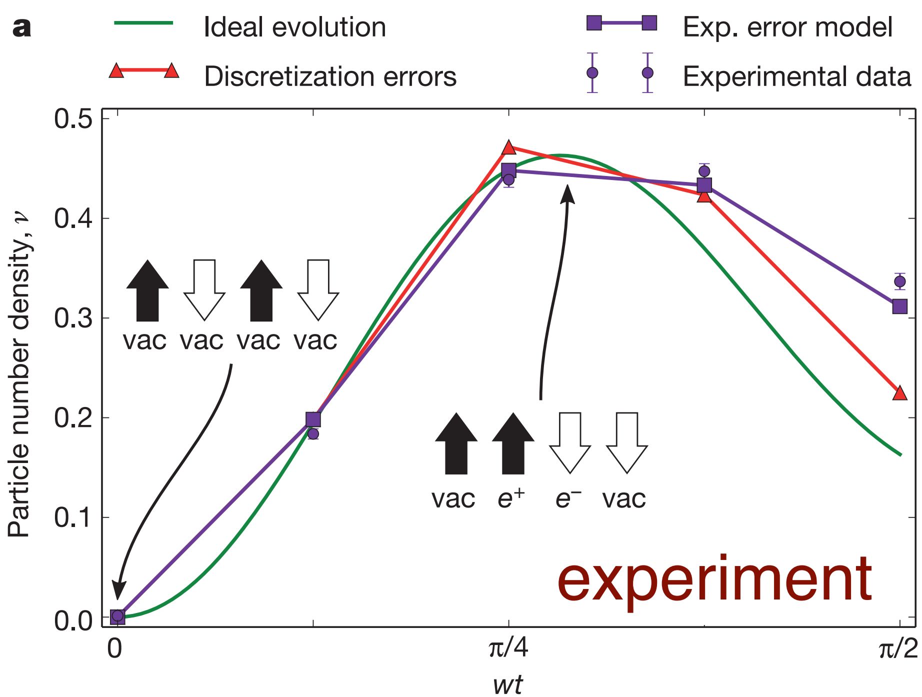
Schwinger Model: 1+1D QED

$$\hat{H}_{\text{lat}} = -i\,w \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{h.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

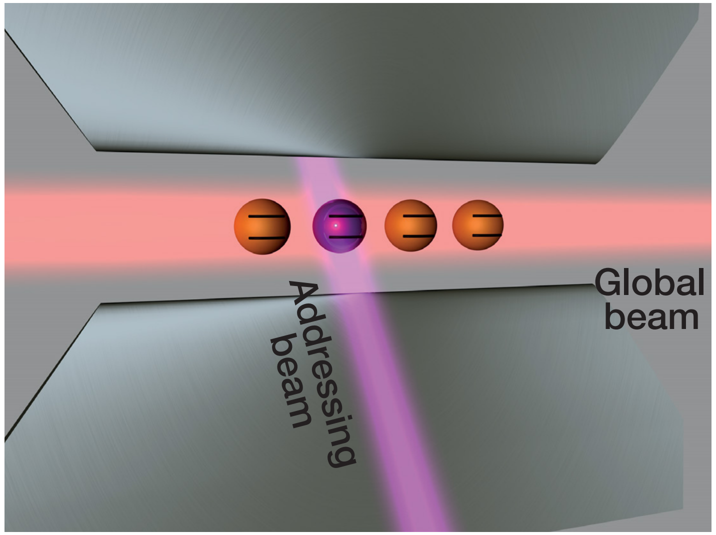
Kogut-Susskind Hamiltonian (Wilson LGT)

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Schwinger pair production



ion trap quantum computer

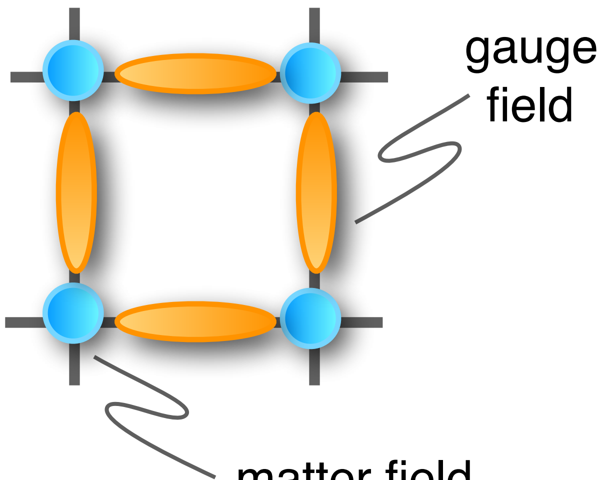


Digital Quantum Simulation of an Exotic Spin Model

- obtained after integrating out gauge field

Atomic Simulation of LGTs

Lattice gauge theory

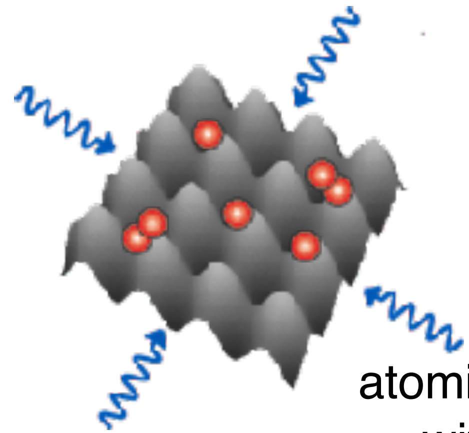


(Non-)Abelian LGT:

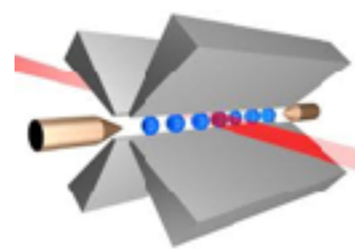
✓QED U(1)

✓QCD SU(N), U(N)

Analog Quantum Simulation



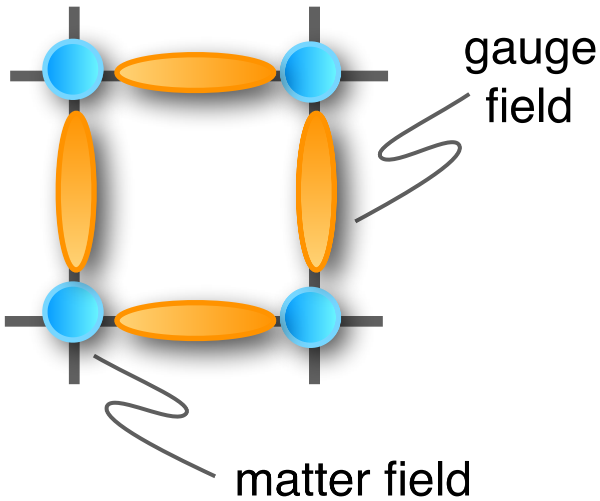
Digital Quantum Simulation



… 'build' / emulate with atomic models

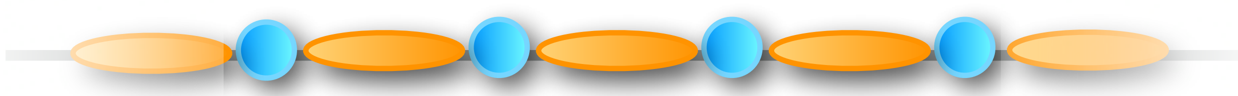
- Kogut-Susskind Hamiltonians [Wilson]
- Quantum Link Models [Wiese]

From STATIC to DYNAMICAL gauge fields



… an AMO perspective

… and the Schwinger Lattice Model

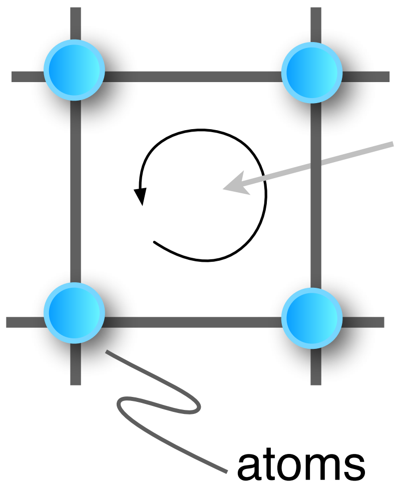


✓ Wilson vs. Quantum Link

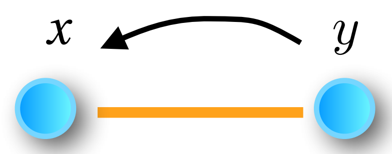
✓ ions / digital vs. atoms / analog

Static vs. Dynamical Gauge Fields

- **c-number / static gauge field**



$$\Phi = \int d^2\vec{f} \cdot \vec{\nabla} \times \vec{A}$$



$$H = -t\psi_x^\dagger e^{i\varphi_{xy}}\psi_y + \mathrm{h.c.}$$

phase $\varphi_{xy} = \int_x^y d\vec{l} \cdot \vec{A}$

U(1) (abelian)

Peierls substitution

"synthetic gauge fields"

- Hofstadter butterfly
- Chern insulators
- Fractional Chern insulators

Review: N. Goldman, J. Budich & PZ, Nature Physics (2016)

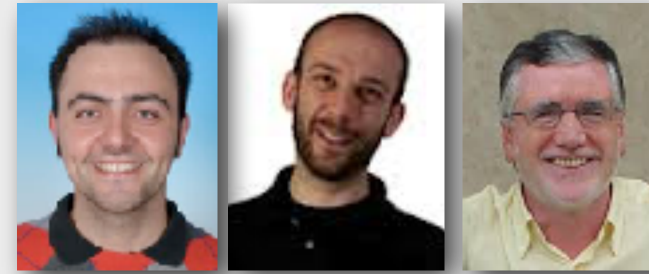
REPORTS

SCIENCE
25 SEPTEMBER 2015 • VOL 349 ISSUE 6255

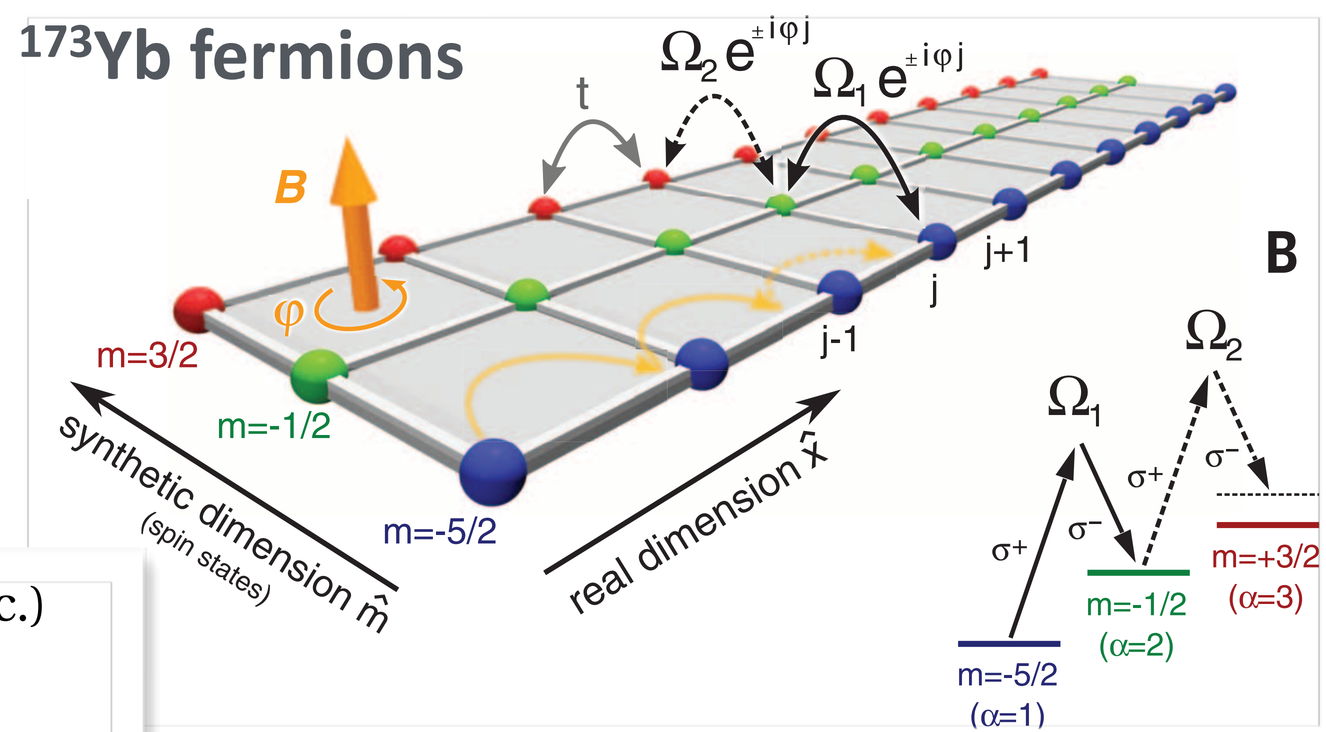
QUANTUM SIMULATION

Observation of chiral edge states with neutral fermions in synthetic Hall ribbons

M. Mancini,[1] G. Pagano,[1] G. Cappellini,[2] L. Livi,[2] M. Rider,[3,4] J. Catani,[5,2] C. Sias,[6,2] P. Zoller,[3,4] M. Inguscio,[6,1,2] M. Dalmonte,[3,4] L. Fallani[1,2]*



M. Dalmonte — UIBK
L. Fallani, M. Inguscio — LENS, Florence

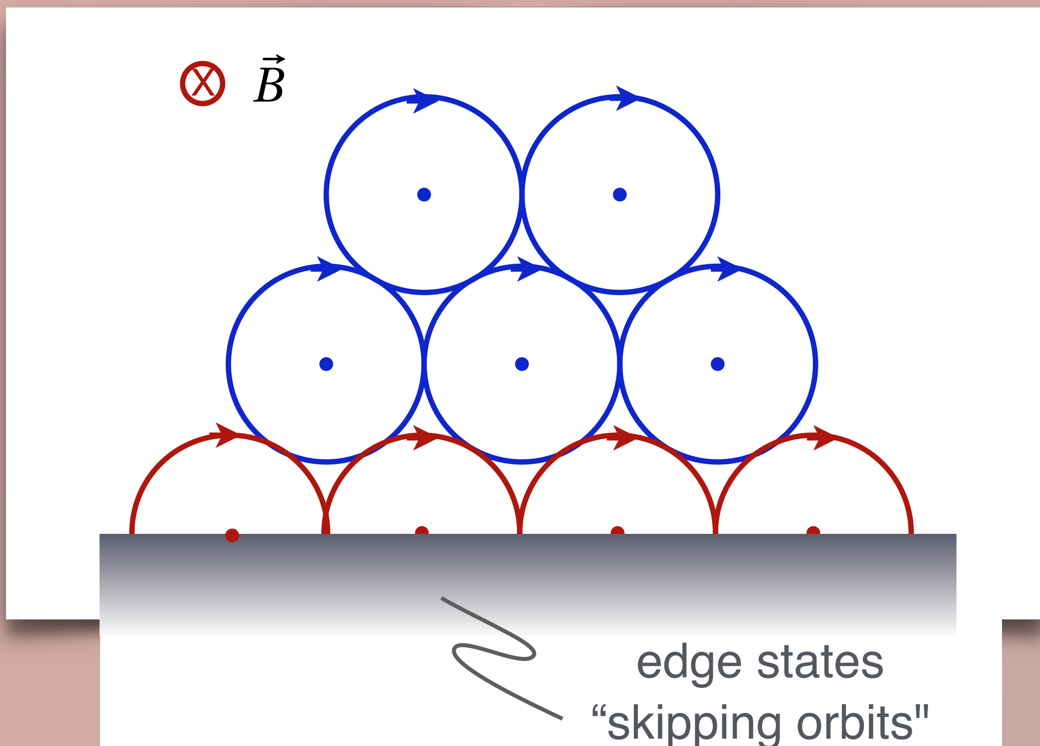


$$
\begin{aligned}
H &= \sum_j \sum_m -t\,(c^\dagger_{j,m} c_{j+1,m} + \text{h.c.}) \\
&+ \sum_j \sum_m \frac{\Omega}{2} (e^{i\varphi j} c^\dagger_{j,m} c_{j,m+1} + \text{h.c.})
\end{aligned}
$$

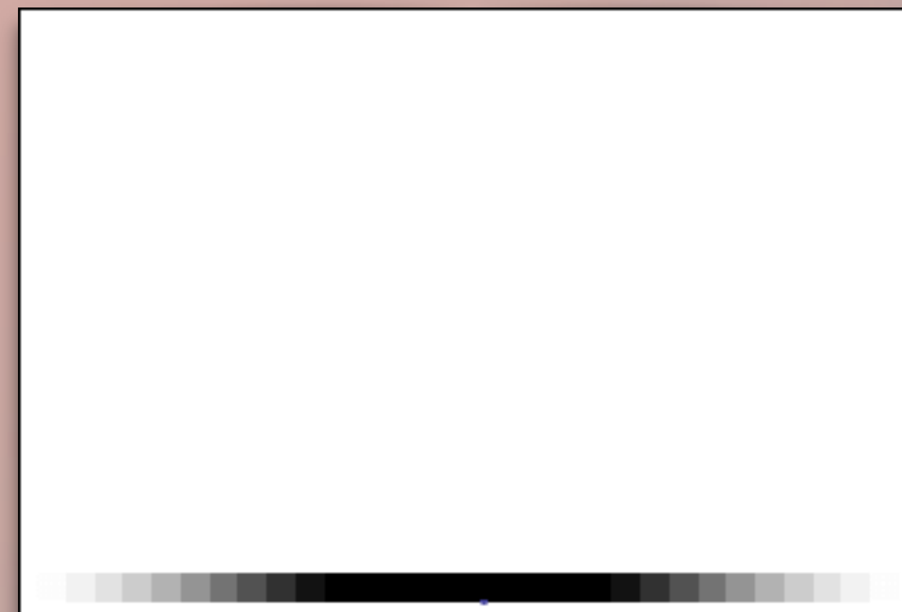
See also: I. Spielman et al., *ibid.* (bosons)

Particle in a [synthetic] magnetic field

cyclotron orbit in B-field

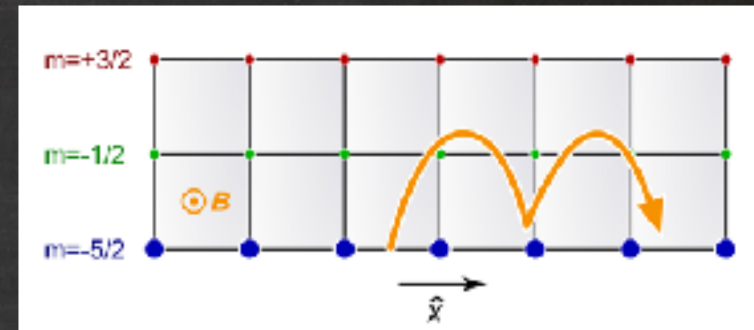


quantum wave packet



M. Mancini et al., Science ***349****, 1510 (2015)*

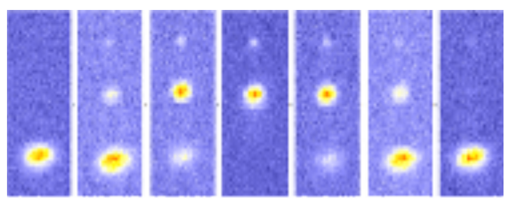
Edge-cyclotron orbits

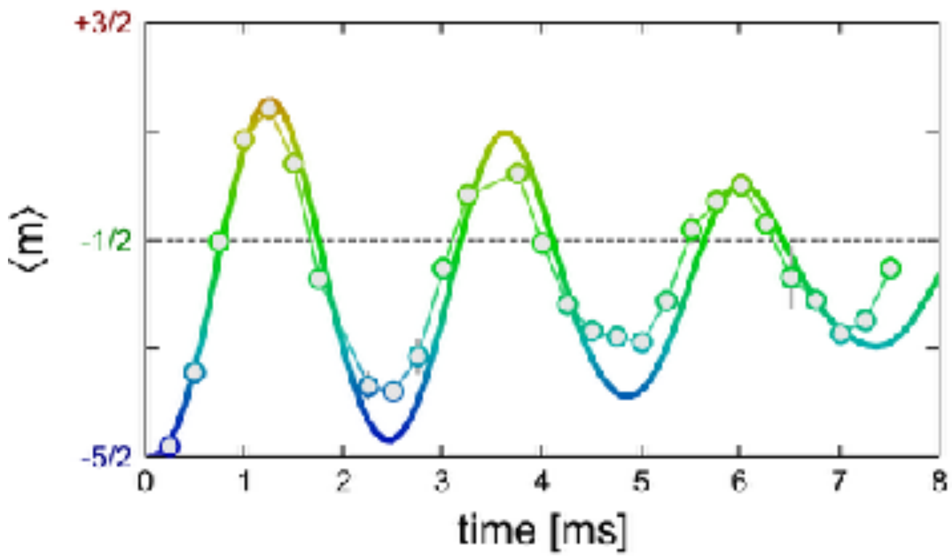


Initial state with <k>=0 on the m=-5/2 leg

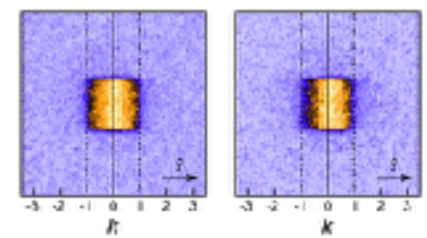
Quenched dynamics after activation of synthetic tunneling

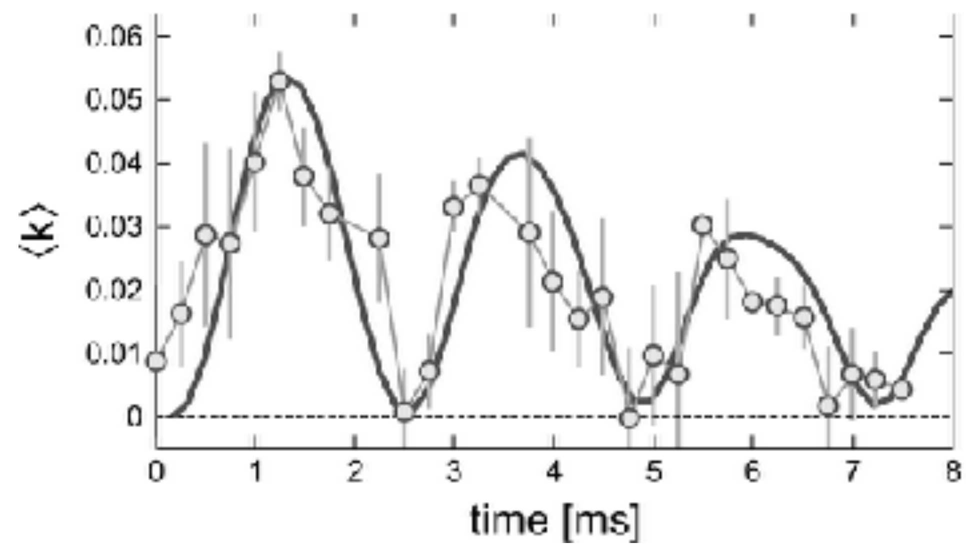
Magnetization:





Momentum:

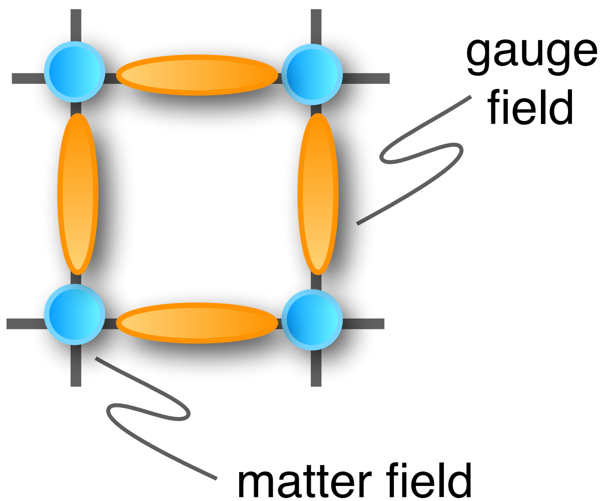


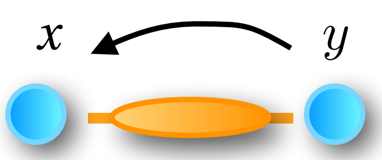


Static vs. *Dynamical* Gauge Fields: U(1)

- **dynamical gauge fields**

  particles hopping around a plaquette assisted by link degrees of freedom





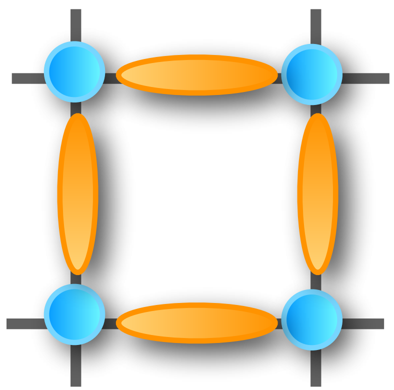
$$H = -t\psi_x^\dagger U_{xy}\psi_y + \text{h.c.} + \ldots$$

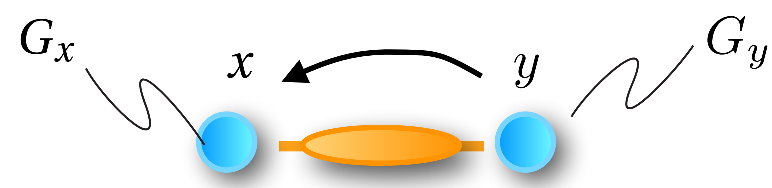
link operator

Static vs. *Dynamical* Gauge Fields: U(1)

- **dynamical gauge fields**

particles hopping around a plaquette assisted by link degrees of freedom





$$H = -t\psi_x^\dagger U_{xy}\psi_y + \text{h.c.} + \dots$$

gauge *(local)* symmetry U(1)

gauge bosons $\quad U_{xy} \xrightarrow{V} e^{i\alpha_x} U_{xy} e^{-i\alpha_y},$

fermions $\quad \psi_x \xrightarrow{V} e^{i\alpha_x}\psi_x$

unitary trafo: $\quad V = \prod_x e^{i\alpha_x G_x}$

generator

***local* conserved quantity**

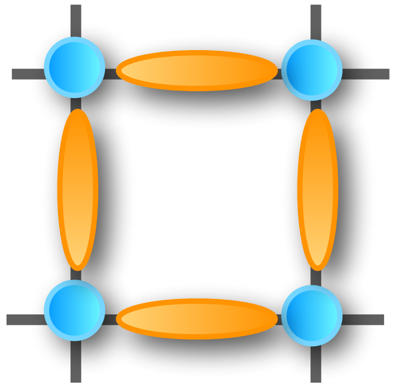
$$[H, G_x] = 0 \ \forall x$$

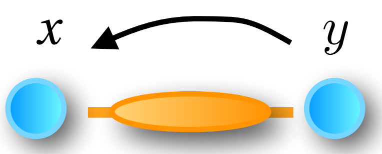
generator of gauge transformation

Static vs. *Dynamical* Gauge Fields: U(1)

- **dynamical gauge fields**

particles hopping around a plaquette assisted by link degrees of freedom





$$H = -t\psi_x^\dagger U_{xy}\psi_y + \text{h.c.} + \dots$$

Gauss Law

$$G_x = \psi_x^\dagger\psi_x - \sum_i \left(E_{x,x+\hat{i}} - E_{x-\hat{i},x}\right)$$

matter — electric field operator

$$\rho - \nabla\cdot E = 0$$

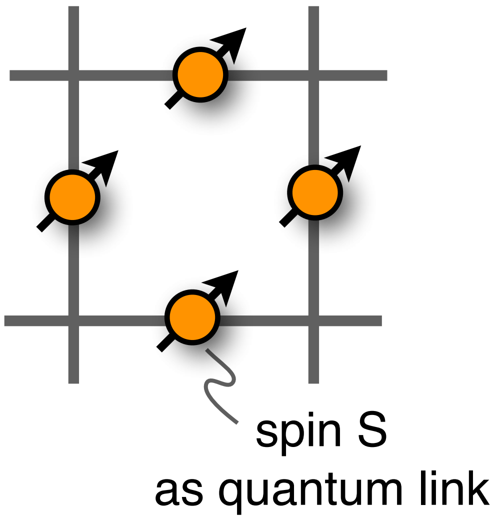
***local* conserved quantity**

$$[H, G_x] = 0 \;\forall x$$

generator of gauge transformation

QED with Spins [Quantum Link Model]

quantum spin ice

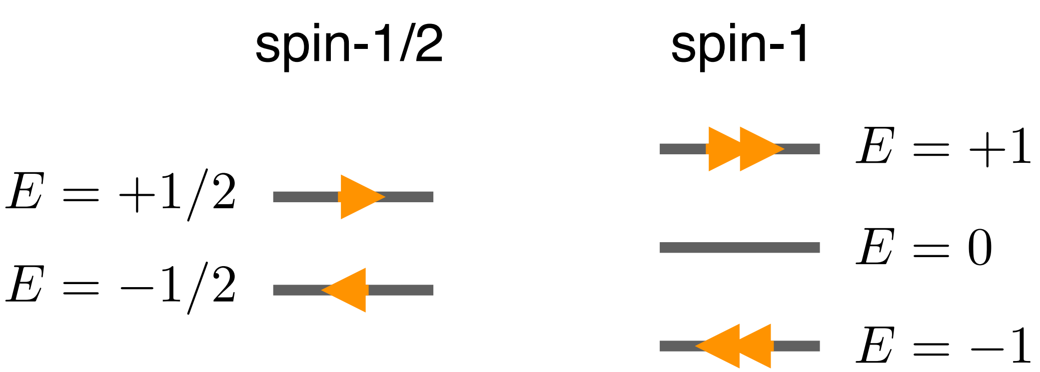


$$U_{x,x+1} \rightarrow S^{+}_{x,x+1} \qquad E_{x,x+1} \rightarrow S^{z}_{x,x+1}$$

electric flux

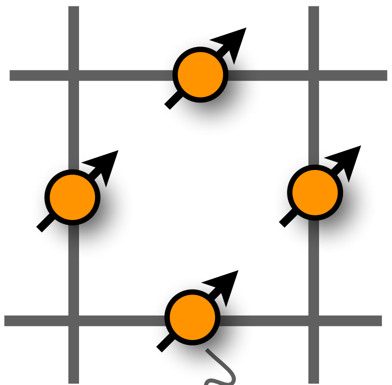
Spin S=½,1,...

quantum link carrying an electric flux



D. Horn (1981), P. Orland & D. Rohrlich (1990), S. Chandrasekharan and U.J. Wiese (1996).

QED with Spins [Quantum Link Model]



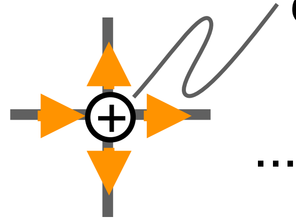
spin S
as quantum link

$$U_{x,x+1} \rightarrow S^{+}_{x,x+1} \qquad E_{x,x+1} \rightarrow S^{z}_{x,x+1}$$

electric flux

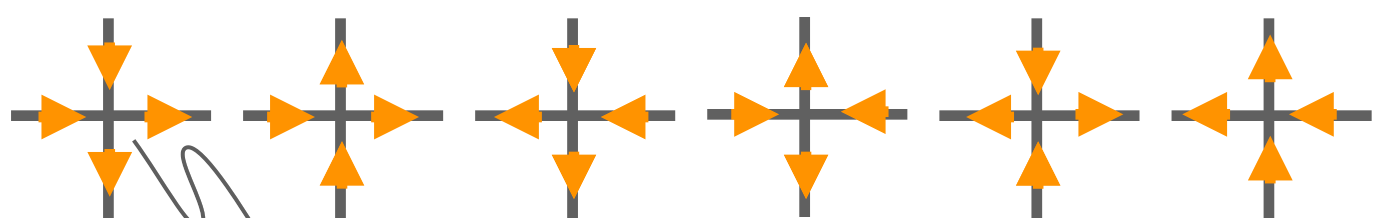
Spin S=½,1,...

configurations: spin-1/2

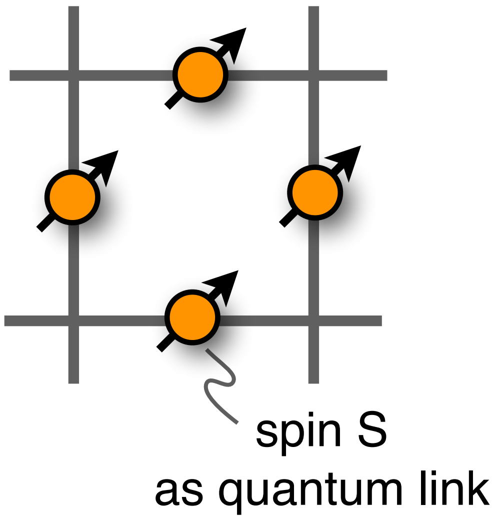


Gauss Law

$$\rho - \nabla \cdot E = 0$$



QED with Spins [Quantum Link Model]



$$U_{x,x+1} \rightarrow S^{+}_{x,x+1} \qquad E_{x,x+1} \rightarrow S^{z}_{x,x+1}$$

electric flux

Spin S=½,1,...

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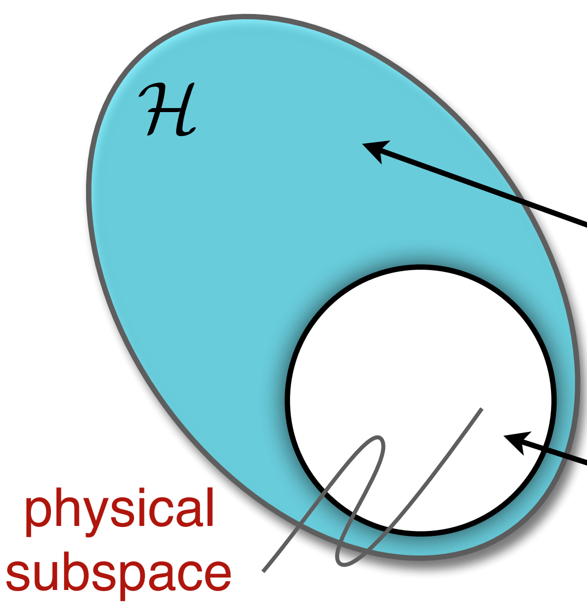
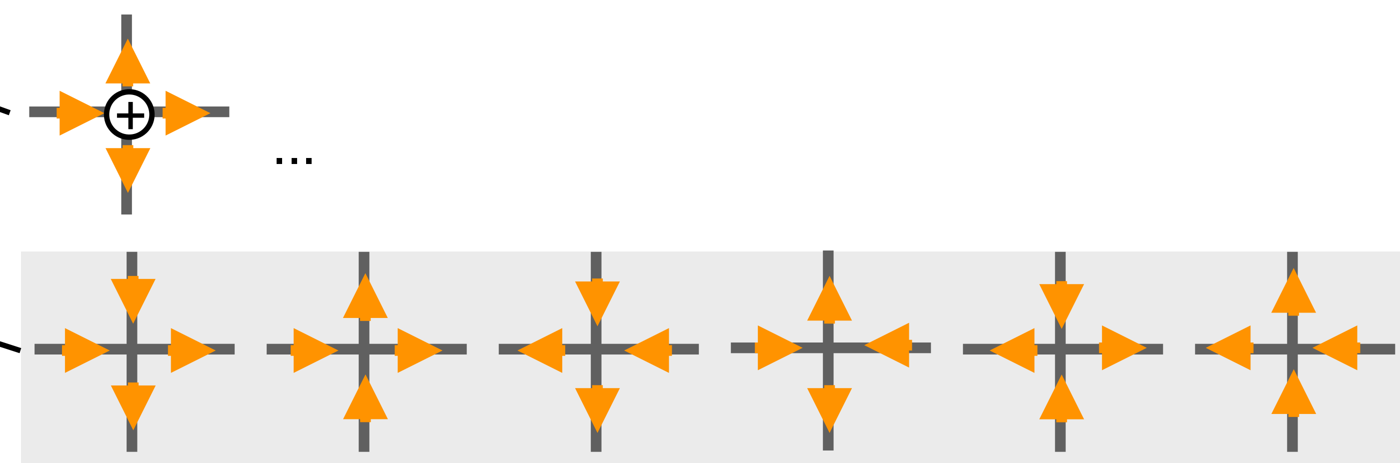
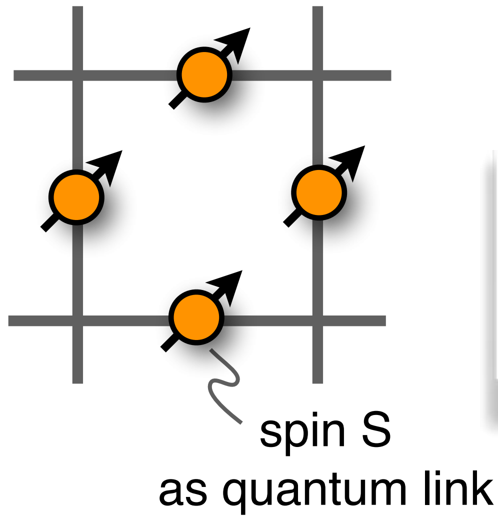


$$G_x\,|\psi\rangle = 0 \qquad \forall x$$



"ice rule" $G_x = \Sigma_\mu \left(S^3_{x-\hat{\mu},\mu} - S^3_{x,\mu} \right) \leftrightarrow \nabla \cdot E = 0$

QED with Spins [Quantum Link Model]



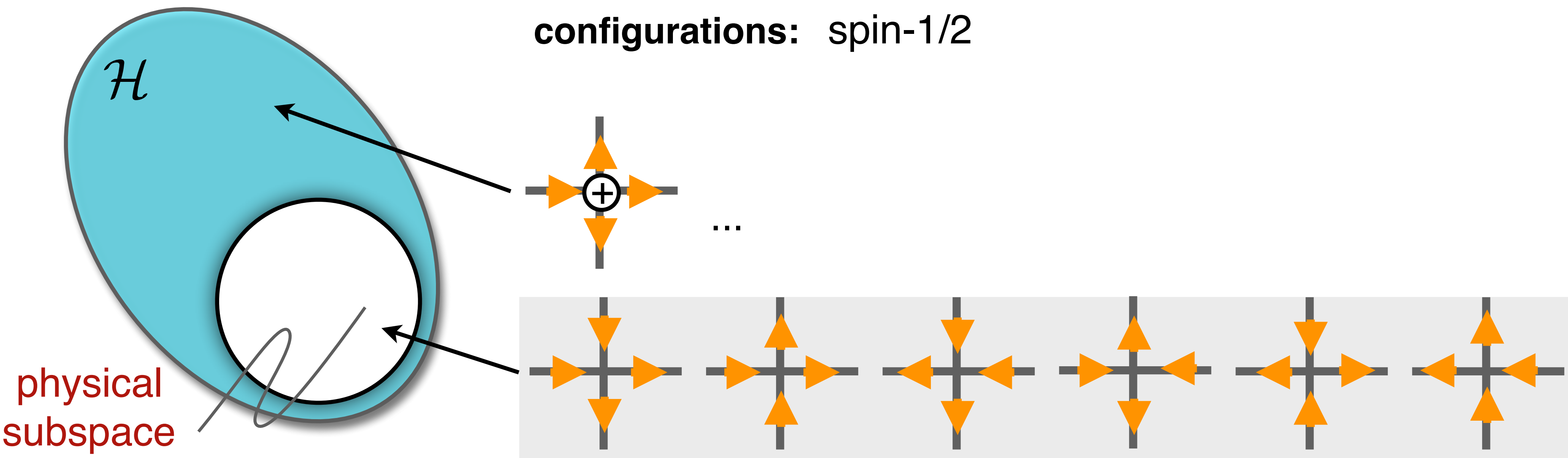
Hamiltonian

$$H = \frac{g^2}{2}\sum_{\langle xy \rangle} E_{xy}^2 - \frac{1}{4g^2}\sum_{\langle wxyz \rangle}\left(U_{wx}U_{xy}U_{yz}U_{zw} + \text{h.c.}\right),$$

E_{xy}^2: S_{xy}^3 — electric energy

U_{wx}: S_{wx}^+ — magnetic energy

configurations: spin-1/2

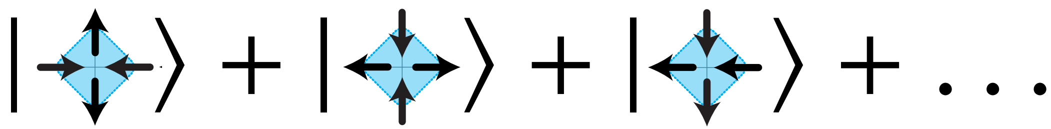


$G_x \, |\psi\rangle = 0 \quad \forall x$

"ice rule" $G_x = \Sigma_\mu \left(S^3_{x-\hat{\mu},\mu} - S^3_{x,\mu} \right) \leftrightarrow \nabla \cdot E = 0$

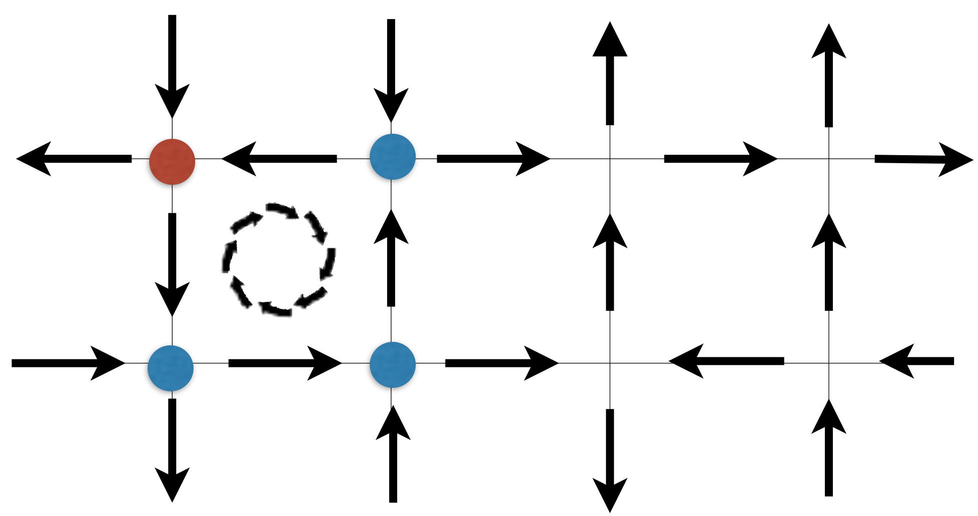
Hamiltonian: Ring Exchange

tunneling between Ice-rule configurations



Quantum Spin Liquids (3D)
Resonating Valence bonds solid (2D)

Non-trivial dynamics of Quantum Spin Ice models

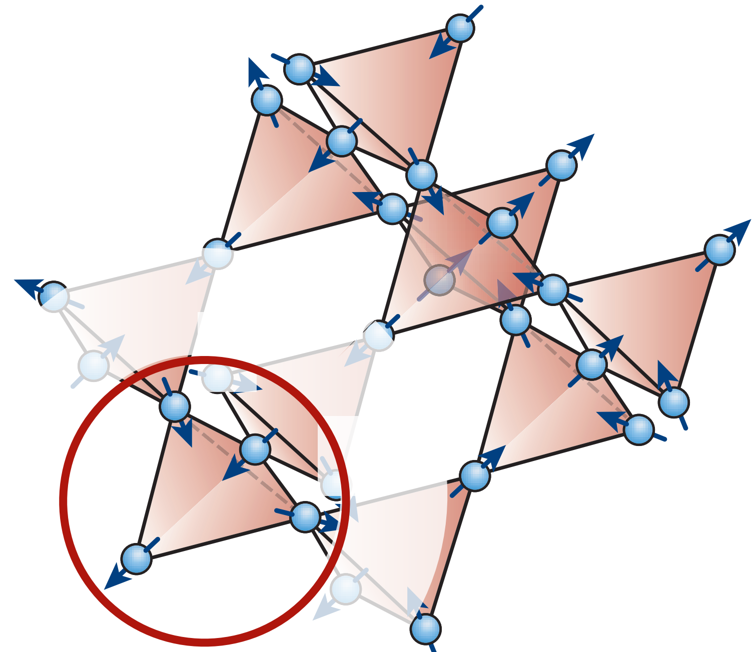


Minimal Hamiltonian for *quantum ice*

$$H = J \sum_{j,i,k,\ell \in \square} S_j^+ S_i^- S_k^+ S_\ell^-$$

Cond Mat: From *Classical* to *Quantum* Spin Ice

- **classical spin ice**
  - magnetic rare earth atoms with large electron spins $Dy_2Ti_2O_7$, $Ho_2Ti_2O_7$, $Ho_2Sn_2O_7$
  - pyrochlore lattice

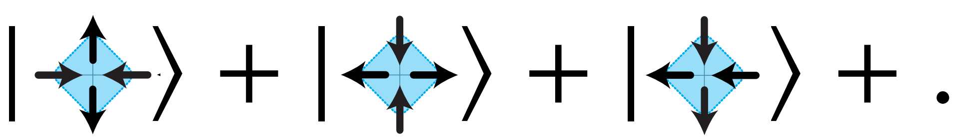


Balents 2008

“ice rule” = strong correlations

analogy to electromagnetism

- ***quantum* spin ice**



quantum superpositions

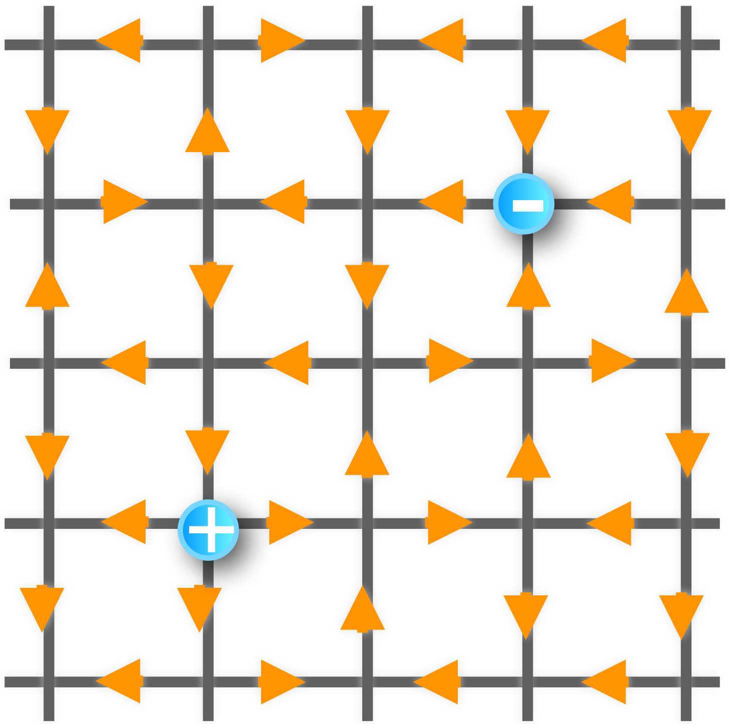
Quantum Spin Liquids (3D)
Resonating Valence bonds solid (2D)

L. Balents, Insight Article in Nature 464, 199 (2010)
C. L. Henley, Annual Review of Condensed Matter Physics 1, 179 (2010)
C. Castelnovo, R. Moessner, and S.L. Sondhi, Annual Review of Condensed Matter Physics (2011)

QED with Spins [Quantum Link Model]

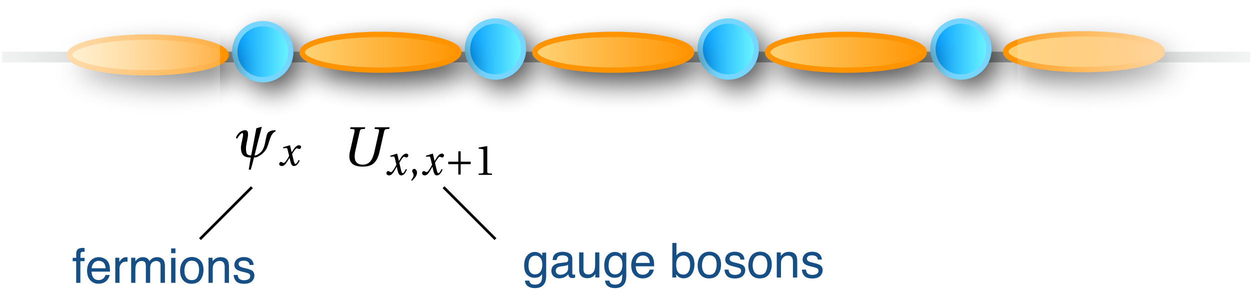
- **… putting matter back**

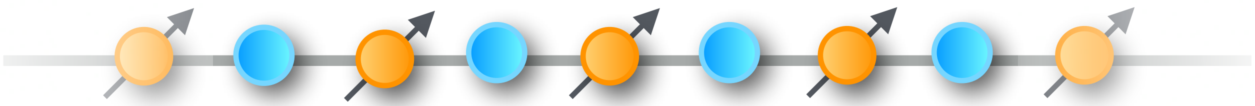
superpositions of configurations
satisfying ice rule



“two in & two out"

Schwinger Lattice Model: (1+1)D QED





Wilson & Quantum Link

Schwinger Model

- **QED in 1+1D: U(1) gauge theory**

$$S^M = \int d^2x\, \mathscr{L}^M \qquad \text{Minkowski action (real time)}$$

$$\mathscr{L}^M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left\{ i\sigma^{\mu M}\left(\partial_\mu + igA_\mu\right) - m_0 \right\}\psi \qquad (\mu = 0, 1)$$

Dirac spinor $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

EM field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\gamma^0 \equiv \sigma^{0M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 \equiv \sigma^{1M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \gamma^5 \equiv \sigma^{3M} = -\sigma^{0M}\sigma^{1M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

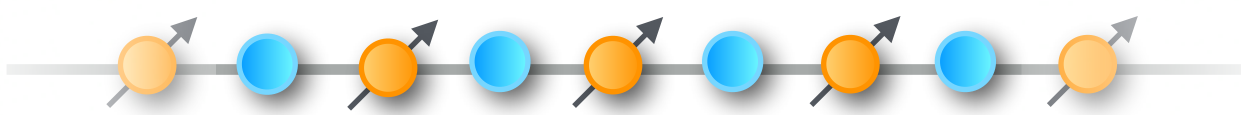
Euclidean …

1. Schwinger: Quantum Link Model

- **Hamiltonian**

$$H = -t\sum_x \left[\psi^\dagger_{x+1} S^+_{x,x+1}\psi_x + \text{h.c.}\right] + m\sum_x (-1)^x \psi^\dagger_x \psi_x + \frac{g^2}{2}\sum_x \left(S^z_{x,x+1}\right)^2$$

hopping | staggered fermions | electric field energy



ψ_x $U_{x,x+1}$

fermions

gauge bosons

$$U_{x,x+1} = S^+_{x,x+1}$$

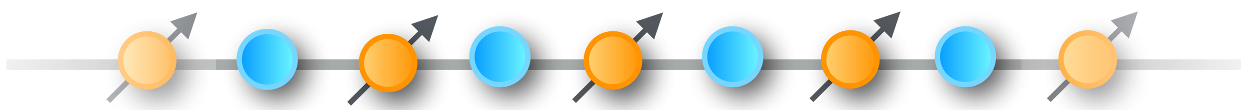
$$E_{x,x+1} = S^z_{x,x+1}$$

Schwinger: Quantum Link Model

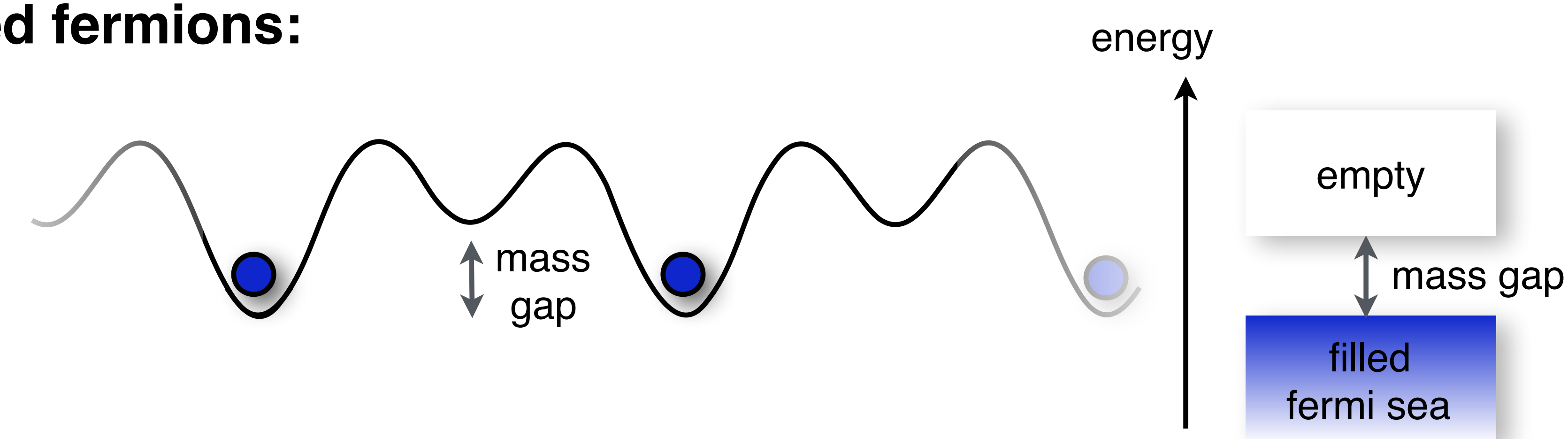
- **Hamiltonian**

$$H = -t \sum_x \left[\psi^\dagger_{x+1} S^+_{x,x+1} \psi_x + \text{h.c.} \right] + m \sum_x (-1)^x \psi^\dagger_x \psi_x + \frac{g^2}{2} \sum_x \left(S^z_{x,x+1} \right)^2$$

hopping — staggered fermions — electric field energy



Staggered fermions:

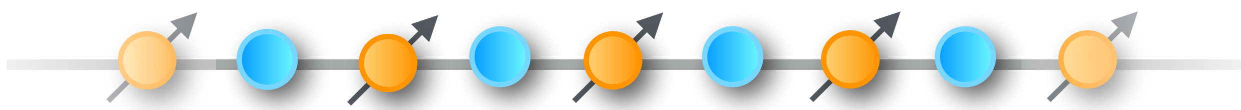


Schwinger: Quantum Link Model

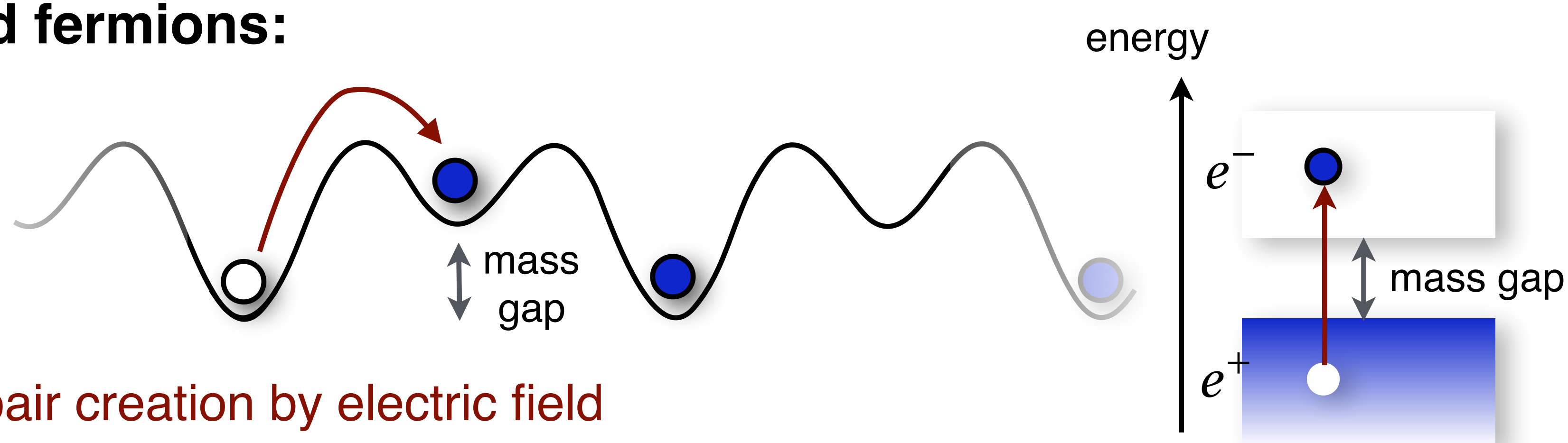
- **Hamiltonian**

$$H = -t\sum_x \left[\psi^\dagger_{x+1} S^+_{x,x+1} \psi_x + \text{h.c.}\right] + m\sum_x (-1)^x \psi^\dagger_x \psi_x + \frac{g^2}{2}\sum_x \left(S^z_{x,x+1}\right)^2$$

hopping — staggered fermions — electric field energy

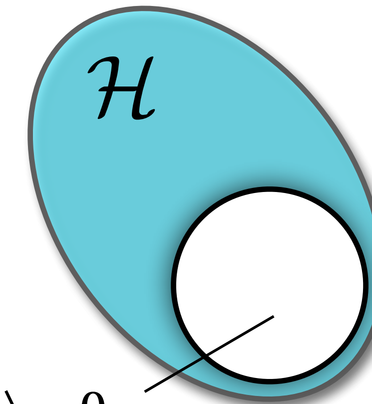


Staggered fermions:



pair creation by electric field

Schwinger: Quantum Link Model



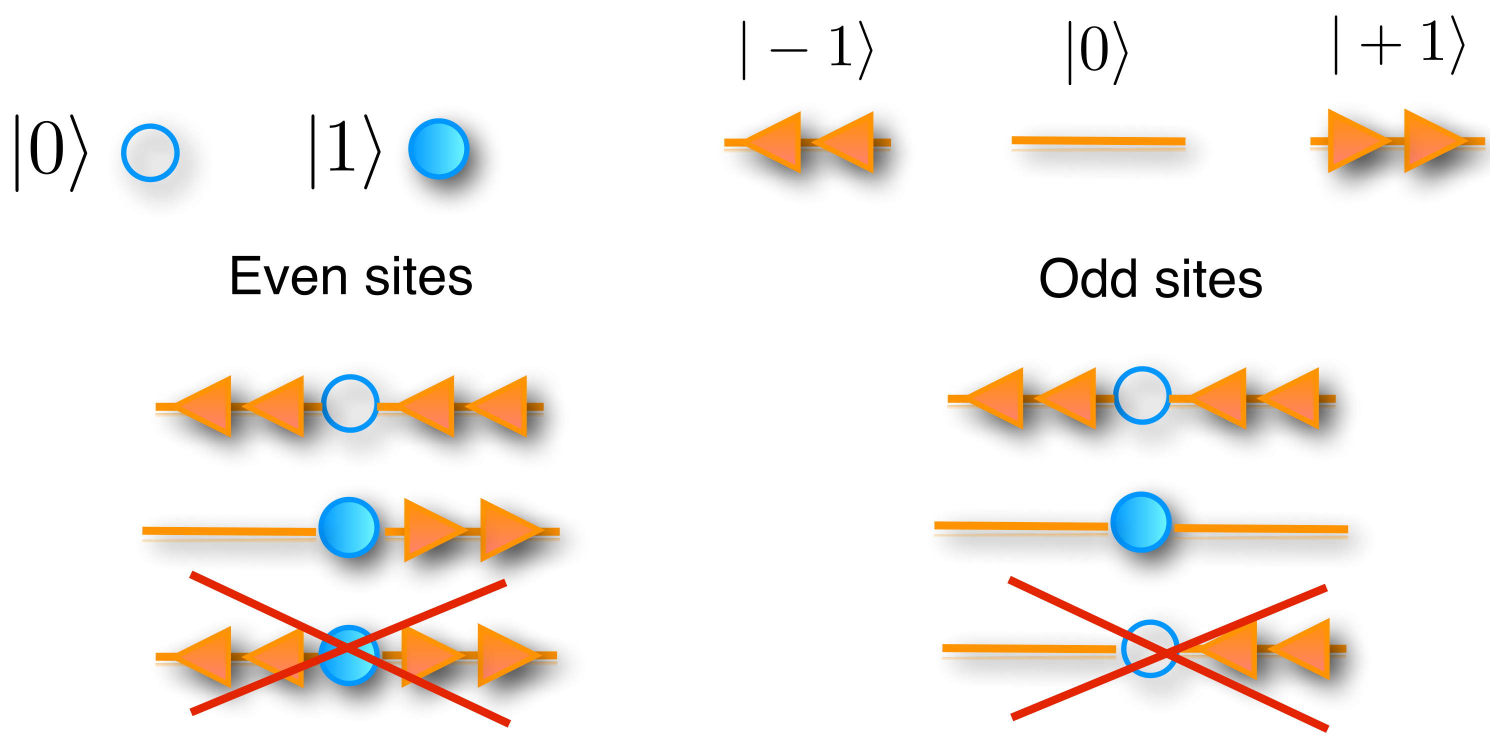
- **Gauss Law**

$$\tilde{G}_x = \psi_x^\dagger \psi_x + E_{x,x+1} - E_{x-1,x} + \frac{(-1)^x - 1}{2} \quad \leftrightarrow \quad \nabla \cdot E = \rho$$

$[H, \tilde{G}_x] = 0$

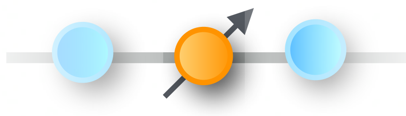
$\tilde{G}_x |\text{physical states}\rangle = 0$

- **Example: Spin 1**



Schwinger: Quantum Link Model

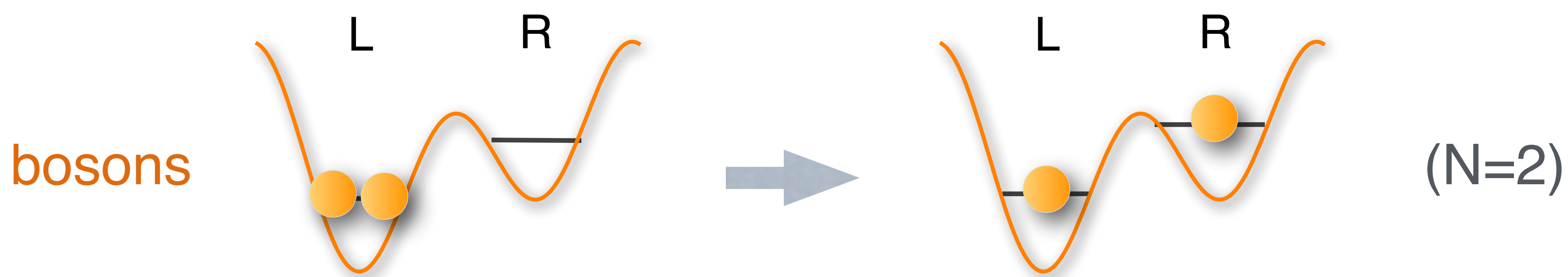
- **Spin as Schwinger boson**



spin S=½,1,...

Schwinger: Quantum Link Model

- **Spin as Schwinger boson**



N bosons in double well (S=N/2)

$$S^{+}_{x,y} = b^{\dagger}_{L} b_{R}$$

$$S^{z}_{x,y} = \frac{1}{2}\left(b^{\dagger}_{R} b_{R} - b^{\dagger}_{L} b_{L}\right)$$

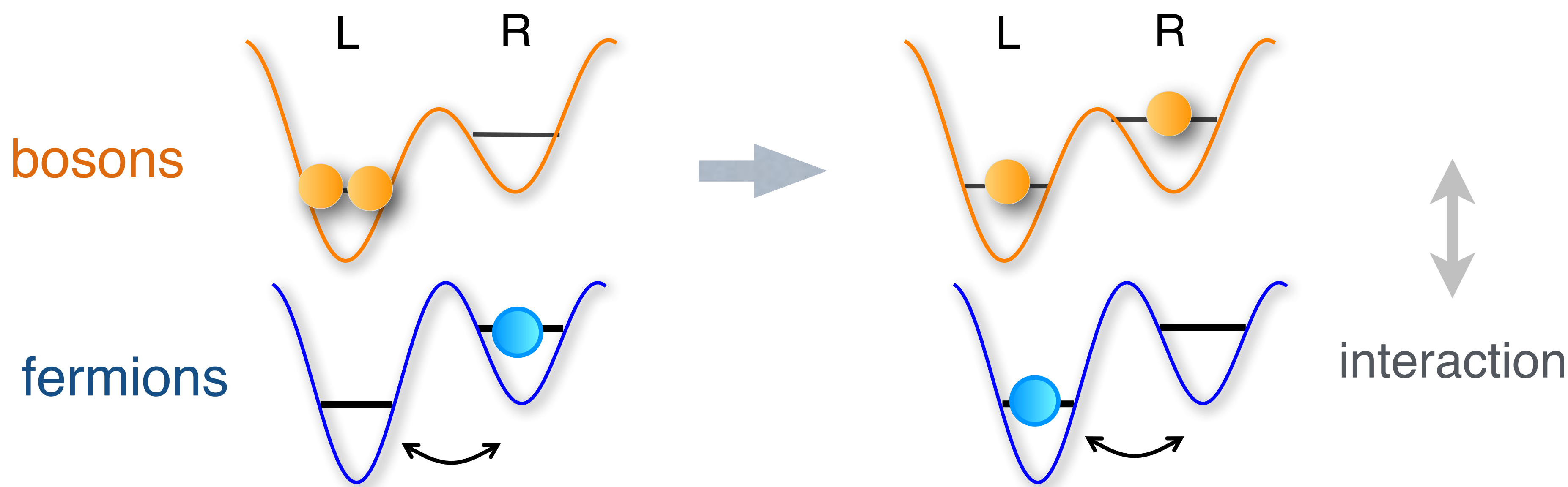
Hamiltonian

$$h_B = -t_B (S^{+}_{x,y} + \text{h.c.}) + U_B (S^{z}_{x,y})^2$$

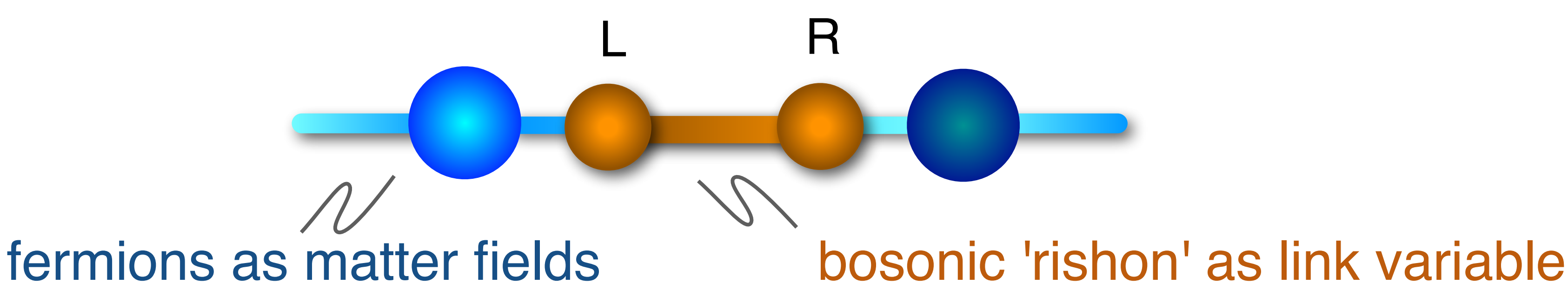
hopping   electric energy

Schwinger: Quantum Link Model

- **Correlated hopping**

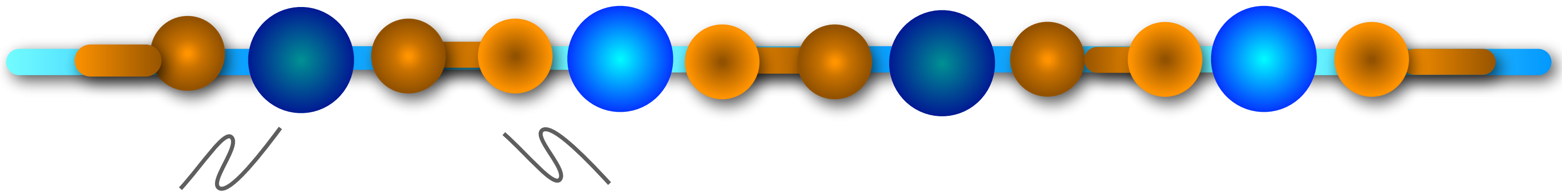


$$H = -\frac{t_B t_F}{U}\,\psi_x^\dagger\, b_R^\dagger b_L\,\psi_y + \mathrm{h.c.}$$



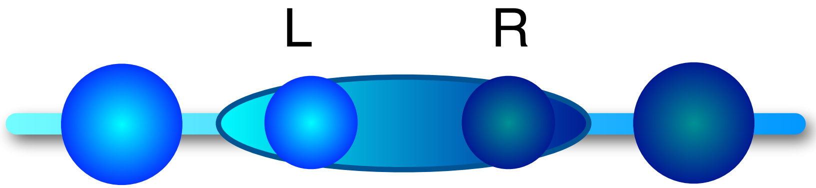
Schwinger: Quantum Link Model

- **Bosonic 'Rishons'**



$$H = -\frac{t_B t_F}{U}\, \psi_x^\dagger\, b_R^\dagger b_L\, \psi_y + \text{h.c.}$$

Non-Abelian Quantum Link Model with Fermionic Rishons



$$U^{ij} = c_R^i c_L^{j\dagger}$$

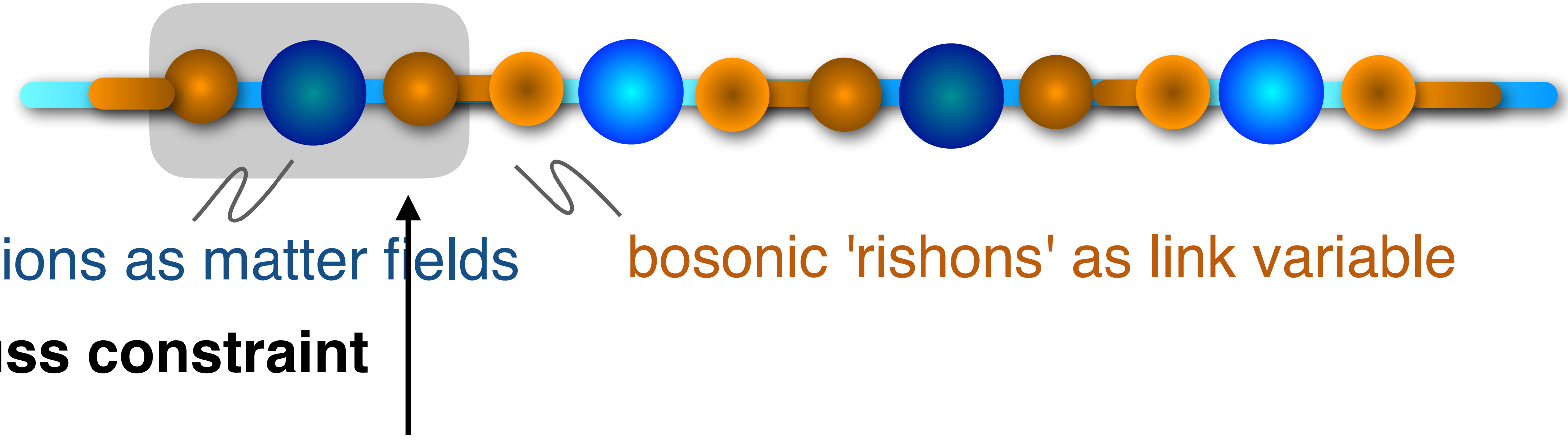
representation as *fermionic* rishons

$(i = 1, \ldots, N)$

Banerjee, D., Bögli, M., Dalmonte, M., Rico, E., Stebler, P., Wiese, U. J., & Zoller, P. PRL (2013)

Schwinger: Quantum Link Model

- **Bosonic 'Rishons'**



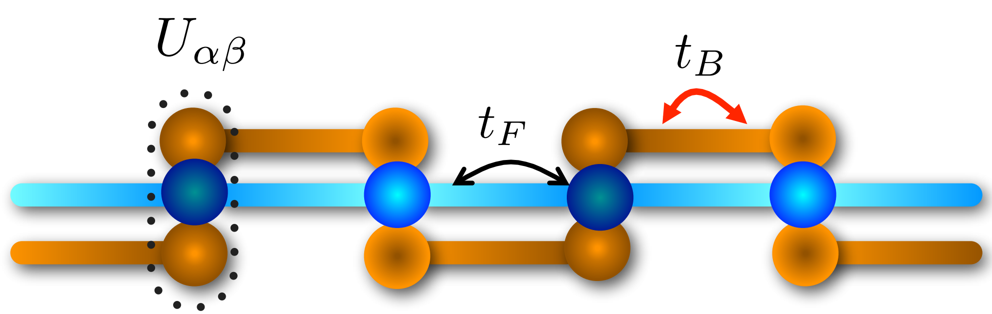
Gauss constraint

$$\tilde{G}_x = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}\left[(-1)^x - 1\right]$$ "super-Mott insulator"

~ total number of atoms on site *x* fixed:

Schwinger: Quantum Link Model

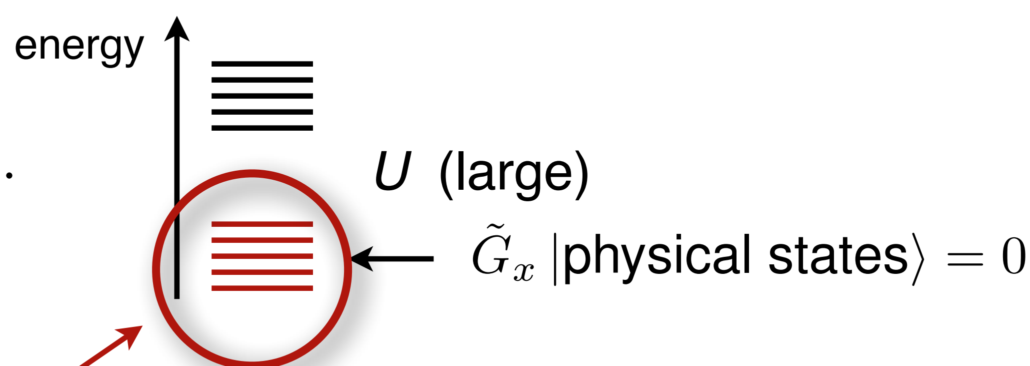
- **Bose-Fermi mixtures in superlattices … implementation**



- **enforcing the Gauss Law as an *energy constraint***

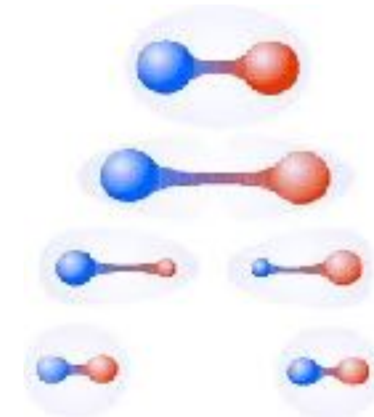
$$H_{\text{microscopic}} = U \sum_x \tilde{G}_x^2 + \ldots$$

Bose + Fermi Hubbard model



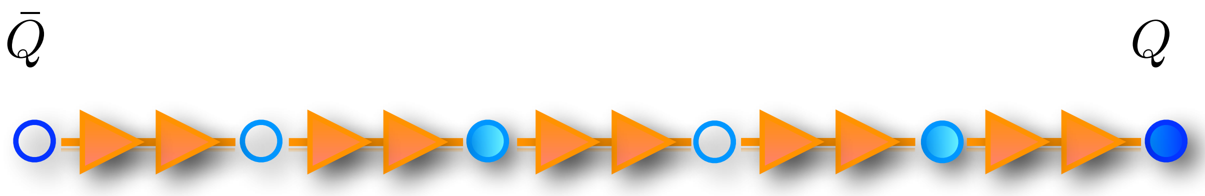
- emergent lattice gauge theory

String Breaking and Confinement



string breaking

Unbroken String



$$E_{\mathrm{string}} - E_0 = \frac{g^2}{2}(L-1)$$

Mesons + vacuum

$$E_{\mathrm{mesons}} - E_0 = 2\left(\frac{g^2}{2} + m\right)$$

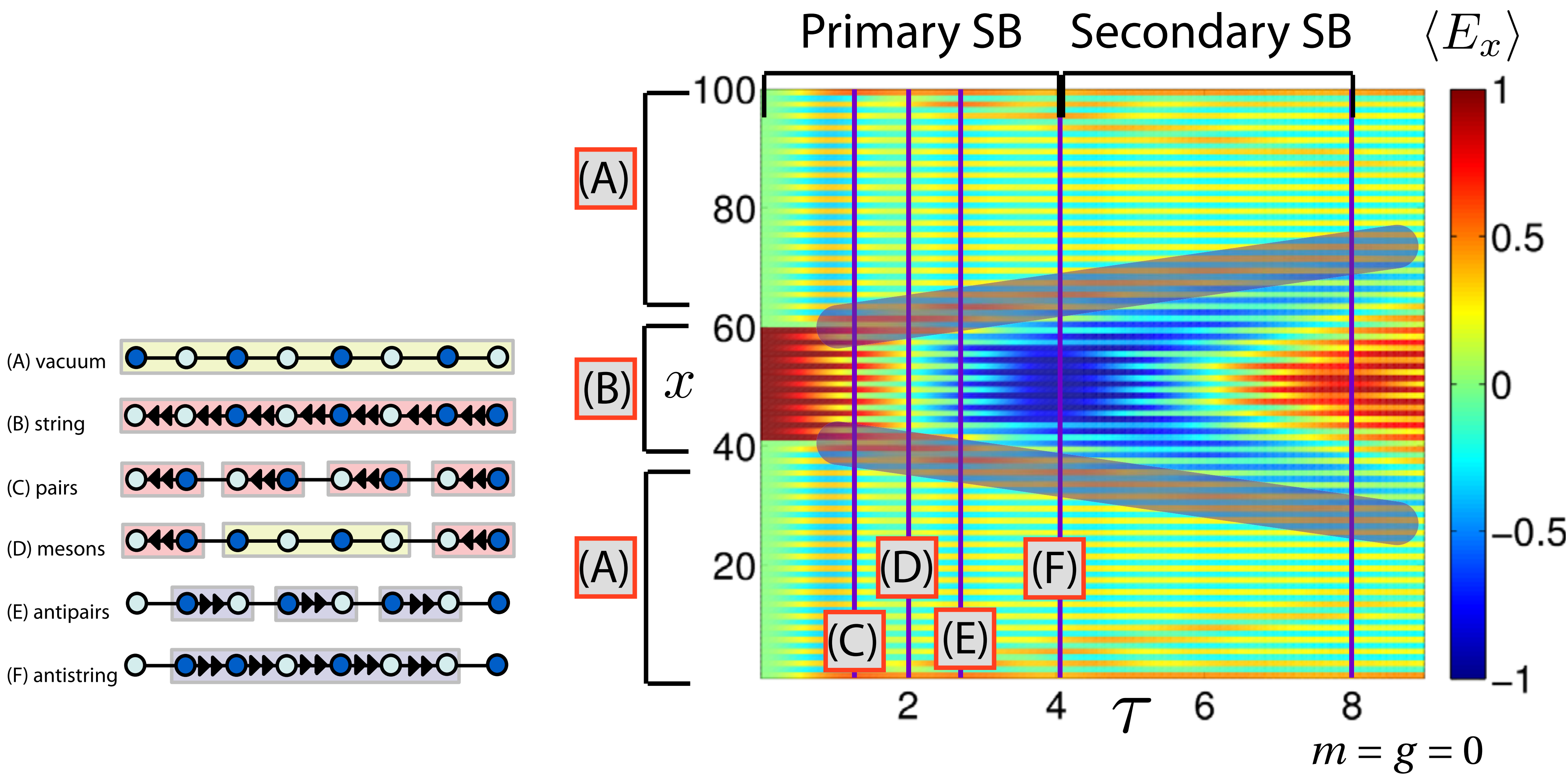
Critical string length $L^{(c)} = 2 + 2m/g^2$

Rem.: string breaking 1D vs. 3D / non-Abelian

PHYSICAL REVIEW X 6, 011023 (2016)

Real-Time Dynamics in U(1) Lattice Gauge Theories with Tensor Networks

T. Pichler,[1] M. Dalmonte,[2,3] E. Rico,[4,5,6] P. Zoller,[2,3] and S. Montangero[1]



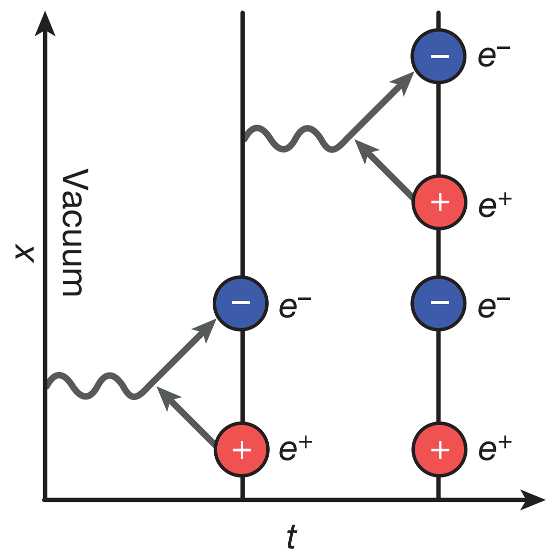
MPS for LGT by Cirac, Verstraete, …

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

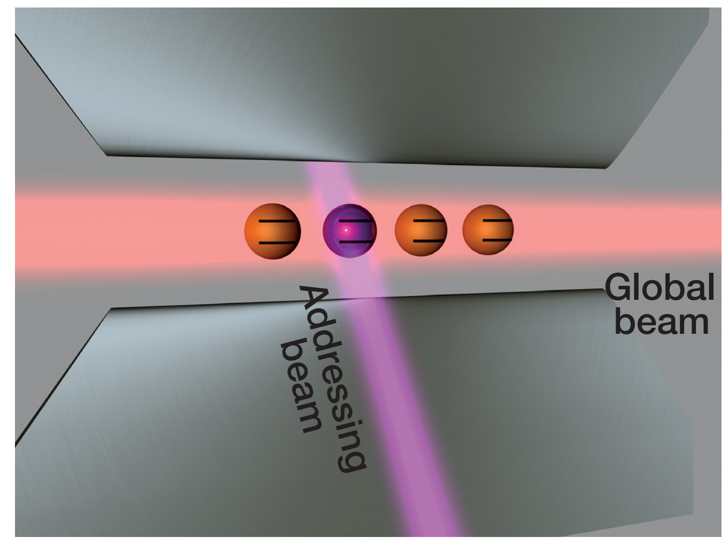
doi:10.1038/nature18318

Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

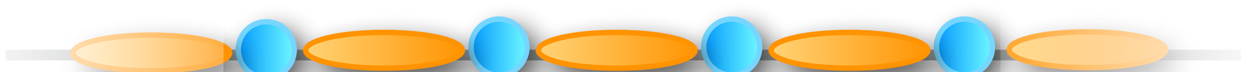
Schwinger pair production



ion trap quantum computer



Schwinger Model: 1+1D QED



$$\hat{H}_{\text{lat}} = -i\,w \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{h.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

Kogut-Susskind Hamiltonian (Wilson LGT)

2. Schwinger Model on a Lattice [Wilson]

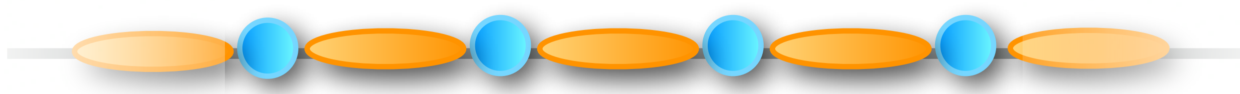
- **Hamiltonian: Kogut-Susskind (1975)**

$$\hat{H} = -i w \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{h.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

$$w = \frac{1}{2a} \qquad J = \frac{g^2 a}{2}$$

Peierls substitution

electric energy ~rigid rotor $\sim E^2$



$\hat{\Phi}_n$ fermionic matter field

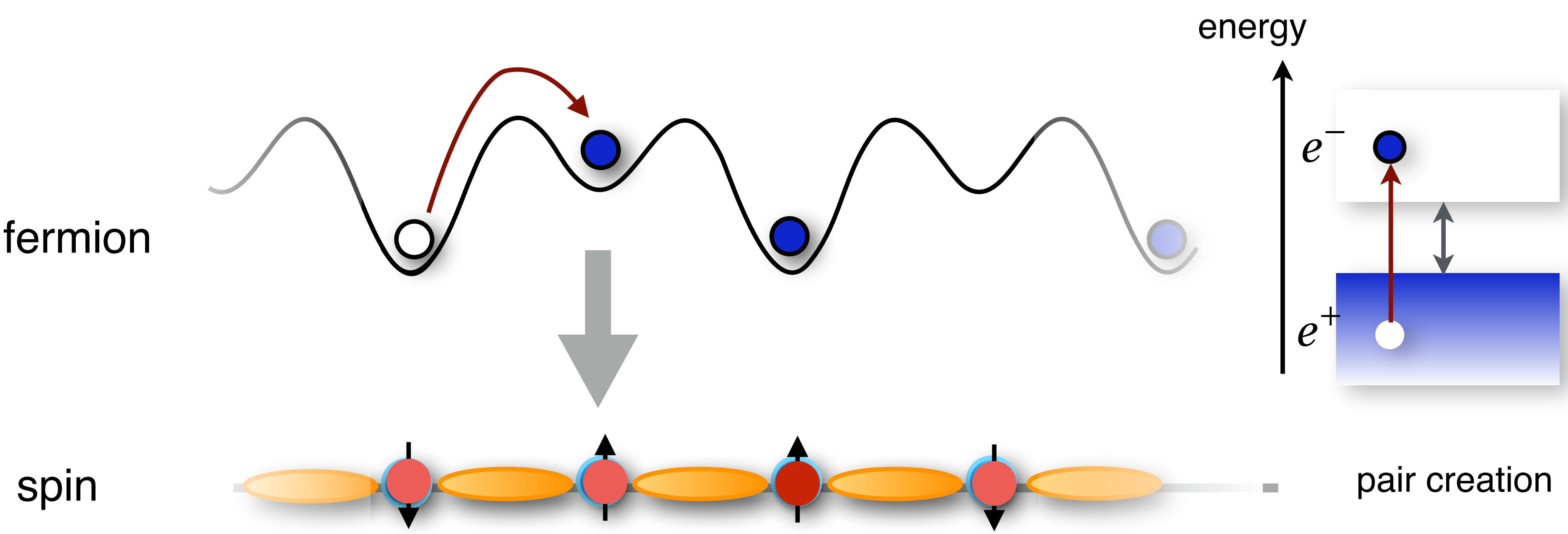
$$[\hat{\theta}_n, \hat{L}_m] = i\delta_{n,m} \longrightarrow [\theta_n, -i\frac{\partial}{\partial\theta_n}] = i\delta_{n,m}$$

vector potential electric field

Schwinger Model on a Lattice

- **Jordan-Wigner transformation:** $\hat{\Phi}_n = \prod_{l<n} [i\hat{\sigma}_l^z]\,\hat{\sigma}_n^-, \quad \hat{\Phi}_n^\dagger = \prod_{l<n} [-i\hat{\sigma}_l^z]\,\hat{\sigma}_n^+$

$$\hat{H}_{\text{spin}} \quad = \quad +w\sum_{n=1}^{N-1}\left[\hat{\sigma}_n^+ e^{i\hat{\theta}_n}\hat{\sigma}_{n+1}^- + \text{h.c.}\right] + J\sum_{n=1}^{N-1}\hat{L}_n^2 + \frac{m}{2}\sum_{n=1}^{N}(-1)^n\hat{\sigma}_n^z$$



$$|0000\rangle = |\uparrow\downarrow\uparrow\downarrow\rangle \qquad |\mathrm{e^-e^+00}\rangle = |\downarrow\uparrow\uparrow\downarrow\rangle \qquad |\mathrm{0e^+e^-0}\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle$$

$$|\mathrm{00e^-e^+}\rangle = |\uparrow\downarrow\downarrow\uparrow\rangle \qquad |\mathrm{e^-00e^+}\rangle = |\downarrow\downarrow\uparrow\uparrow\rangle \qquad |\mathrm{e^-e^+e^-e^+}\rangle = |\downarrow\uparrow\downarrow\uparrow\rangle$$

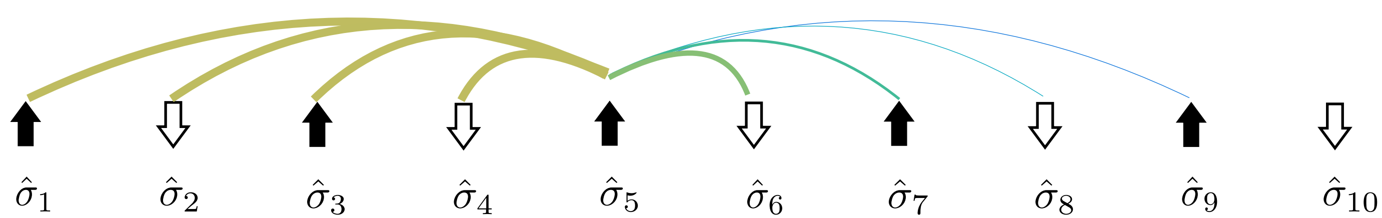
Schwinger Model as 'Exotic Spin Model'

- **gauge transformation:** we eliminate gauge field $\hat{\sigma}_n^- \to \prod_{l<n} \left[e^{-i\hat{\theta}_l} \right] \hat{\sigma}_n^-$

$$\hat{H}_S = +\frac{m}{2}\sum_{n=1}^{N}(-1)^n \hat{\sigma}_n^z + w\sum_{n=1}^{N-1}\left[\hat{\sigma}_n^+\hat{\sigma}_{n+1}^- + \text{h.c.}\right] + J\sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left[\hat{\sigma}_m^z + (-1)^m\right]\right]^2$$

$$\epsilon_0 = 0$$

$$\hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$



Schwinger Model as 'Exotic Spin Model'

- **Spin model with exotic long range interactions**

$$\hat{H}_S = J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

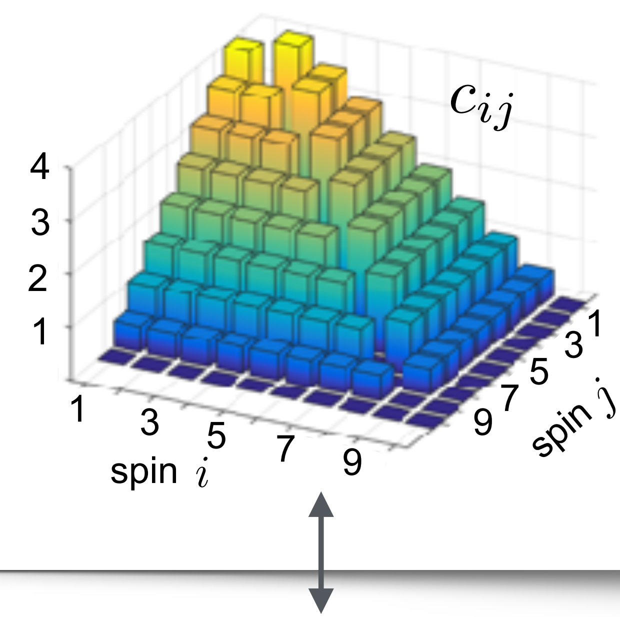
long - range interaction

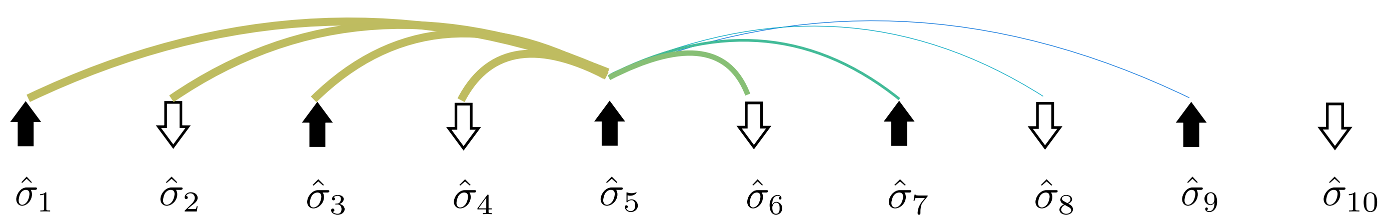
$$+ \, w \sum_i \left(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^- \right)$$

particle - antiparticle creation/annihilation

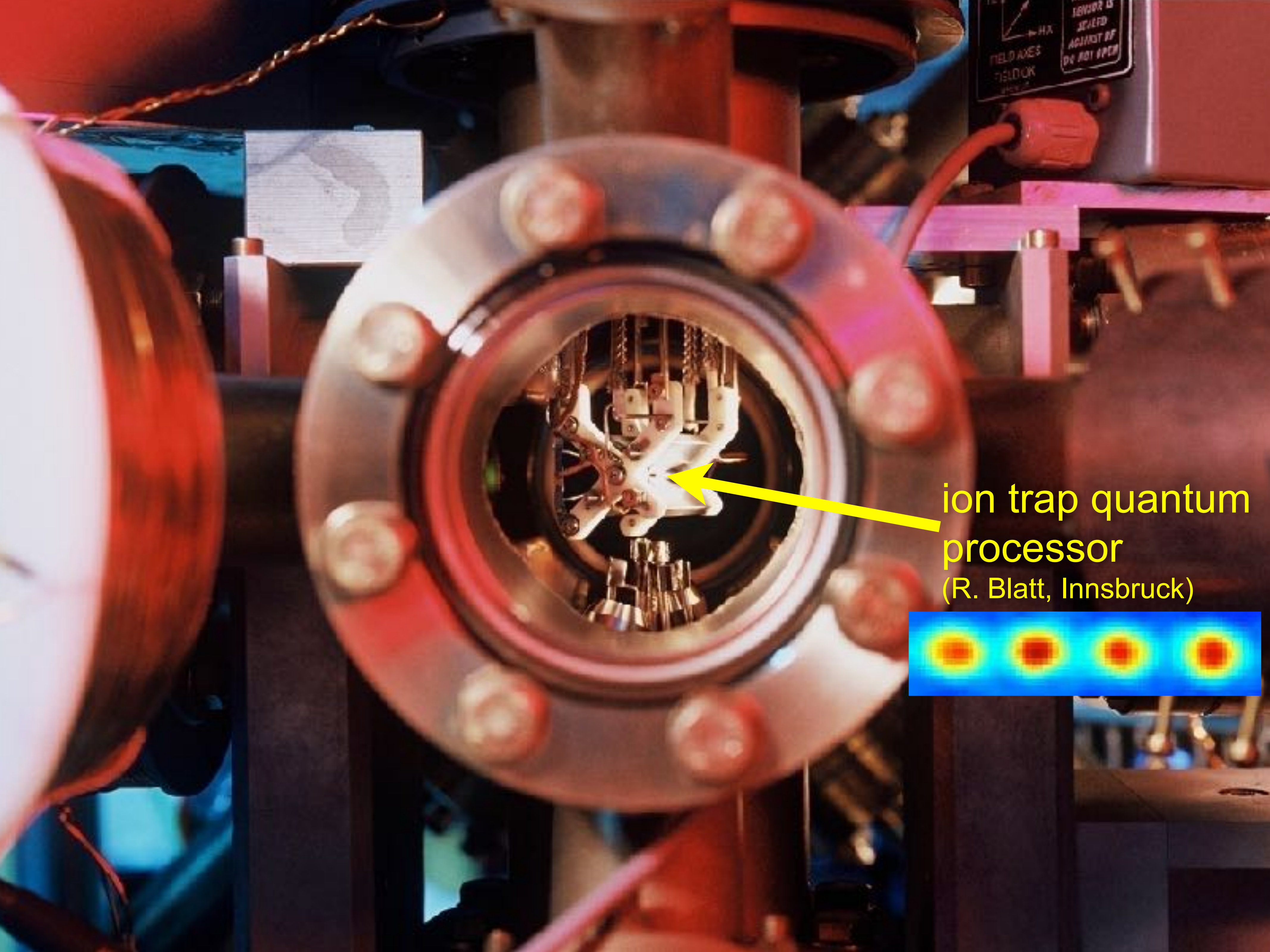
$$+ \, m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses





… efficient to program on an ion trap quantum computer

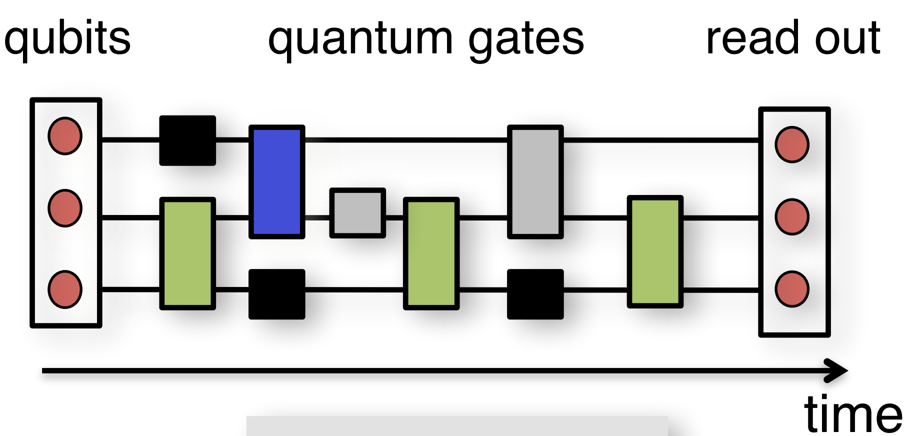

ion trap quantum processor
(R. Blatt, Innsbruck)

Quantum Info

Quantum Optics

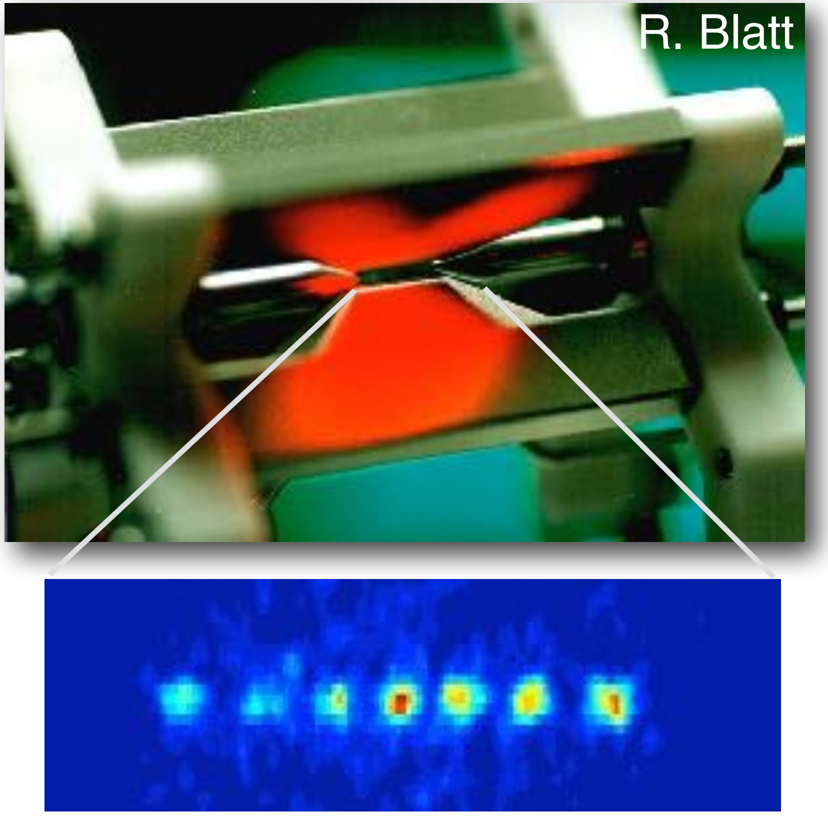
- **universal quantum computing**

quantum logic network model



$$|\psi\rangle \to U_t\,|\psi\rangle$$

- **ion trap quantum computer**

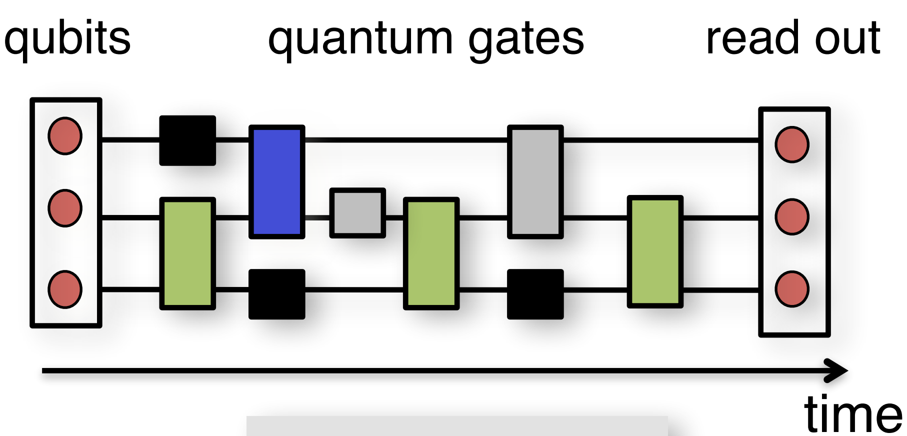


Innsbruck, NIST, JQI, Oxford, Mainz ...

Quantum Info

- **universal quantum computing**

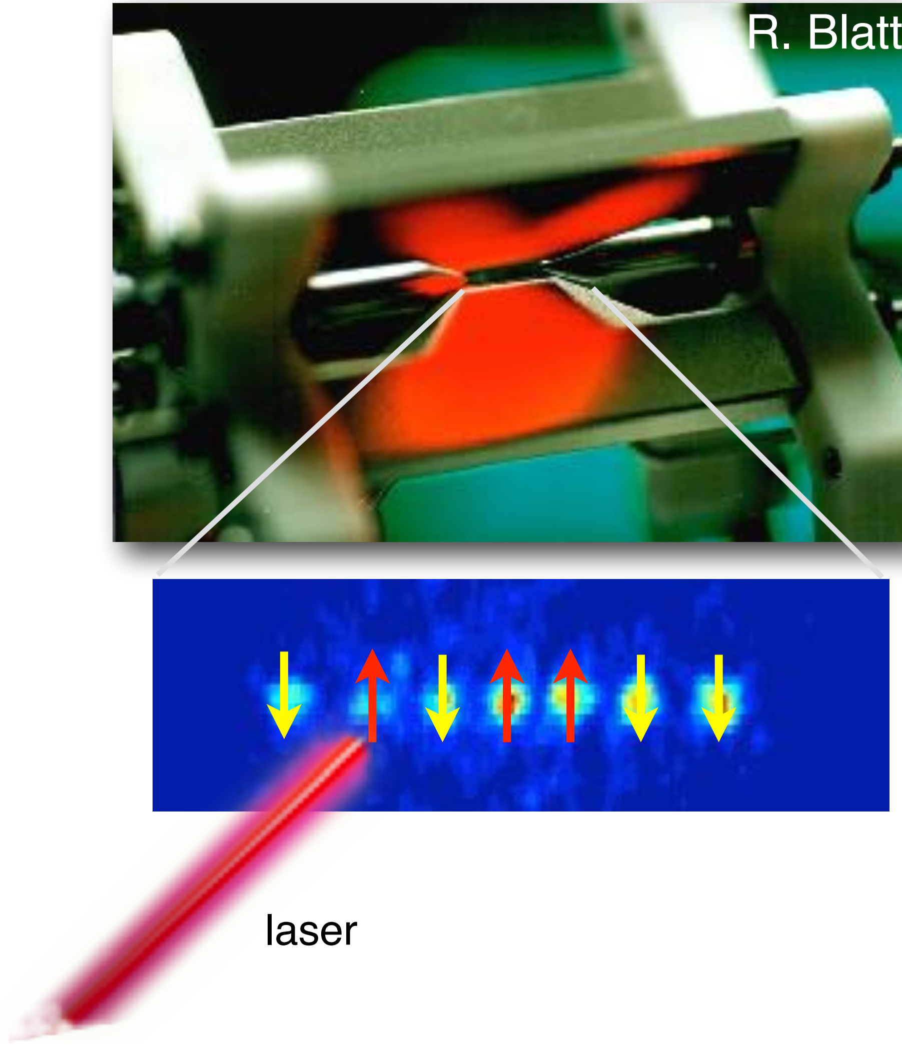
quantum logic network model



$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

Quantum Optics

- **ion trap quantum computer**

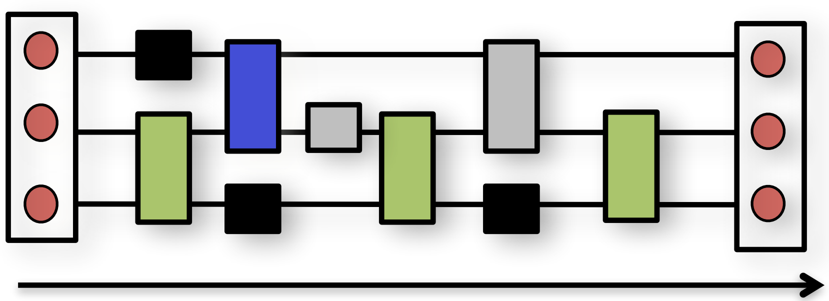


Quantum Info

- **universal quantum computing**

quantum logic network model

qubits quantum gates read out

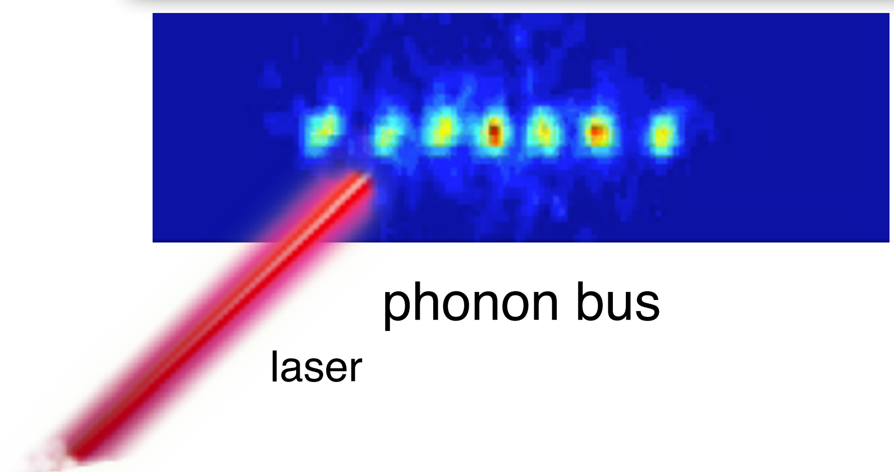


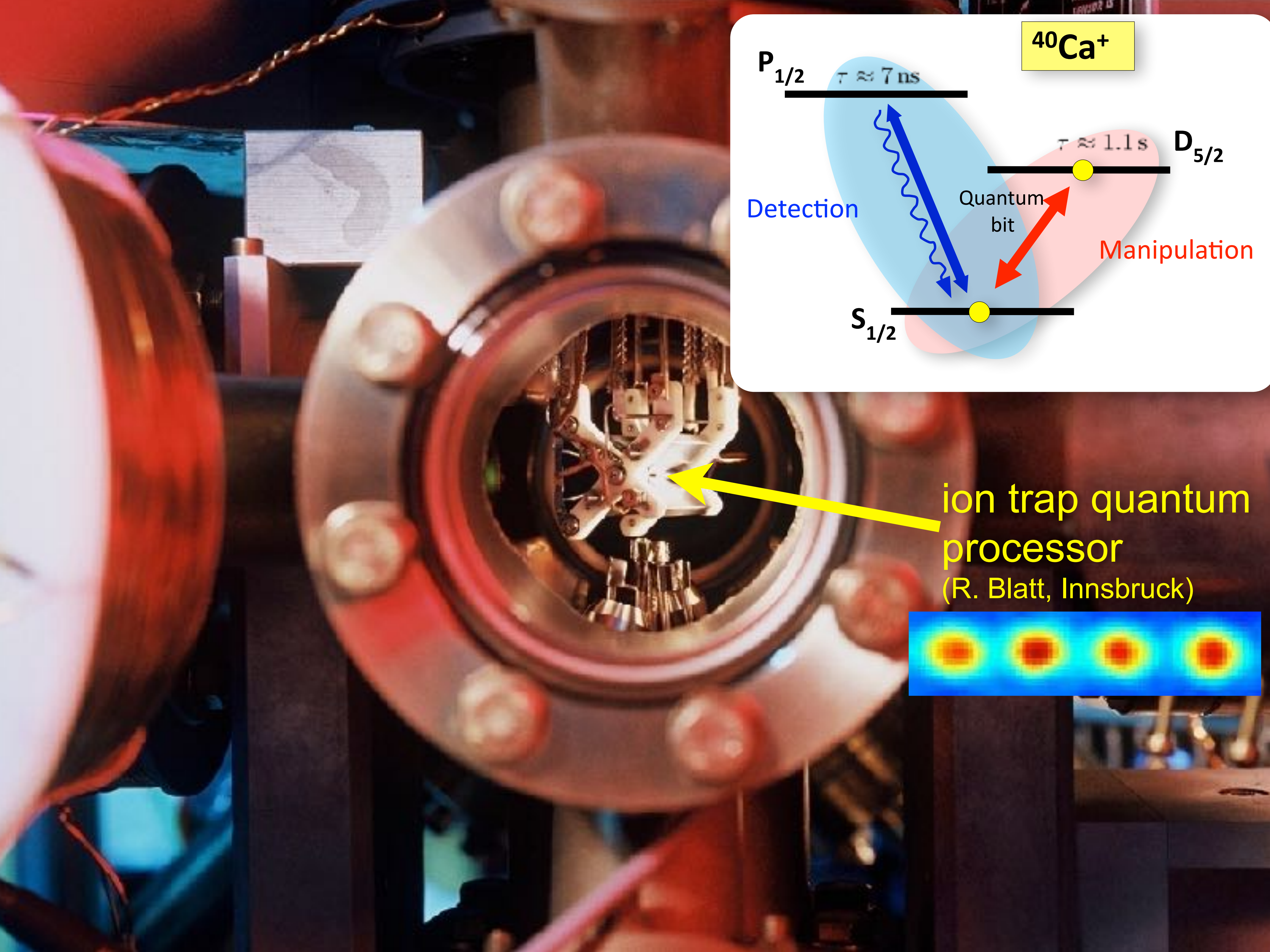
time

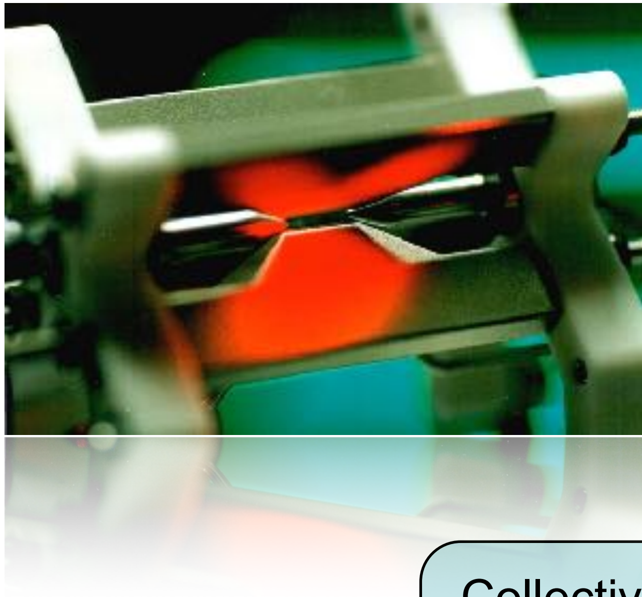
$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

Quantum Optics

- **ion trap quantum computer**



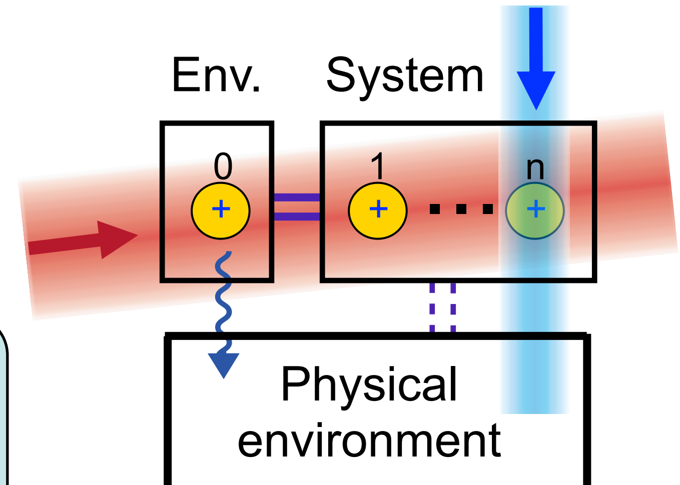

$^{40}Ca^+$
$P_{1/2}$
$\tau \approx 7$ ns
$D_{5/2}$
$\tau \approx 1.1$ s
Detection
Quantum bit
Manipulation
$S_{1/2}$
ion trap quantum processor
(R. Blatt, Innsbruck)



Quantum operations & compiler: Innsbruck ion-trap quantum computer

Individual light-shift gates

$\sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)}$

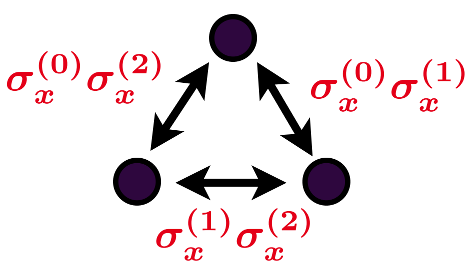


Collective spin flips

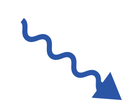
S_x, S_y

Mølmer-Sørensen gate

$$S_x^2 = \sigma_x^{(0)}\sigma_x^{(1)} + \sigma_x^{(1)}\sigma_x^{(2)} + \sigma_x^{(0)}\sigma_x^{(2)}$$



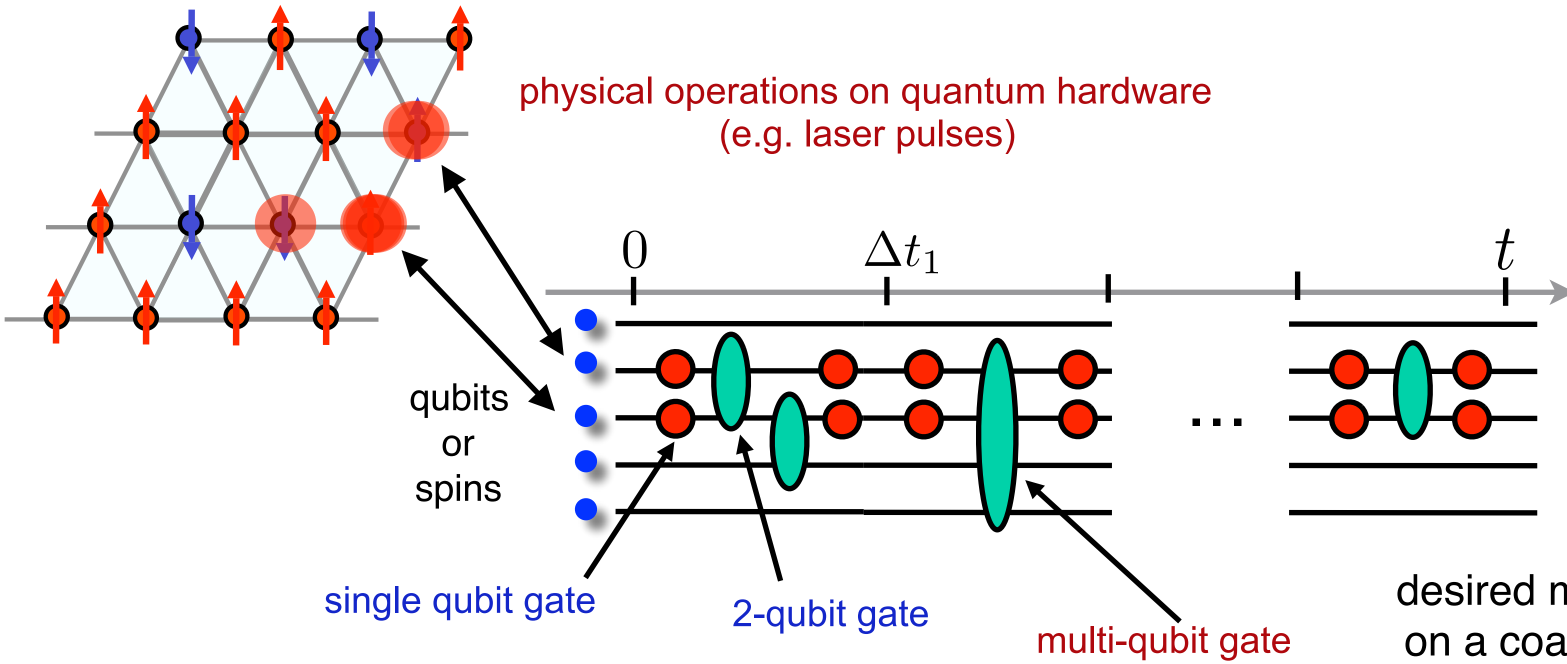
Coupling of environment with physical environment

Optical pumping of „environmental“ ion 

J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Digital Quantum Simulation

S. Lloyd 1996



$\simeq$ $e^{-iH_{\text{eff}}t}$

desired many-body Hamiltonian on a coarse-grained time scale

idea: approximate time evolution by a stroboscopic sequence of gates

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots {}^{-iH\Delta t_1/\hbar}$$

Trotter expansion:

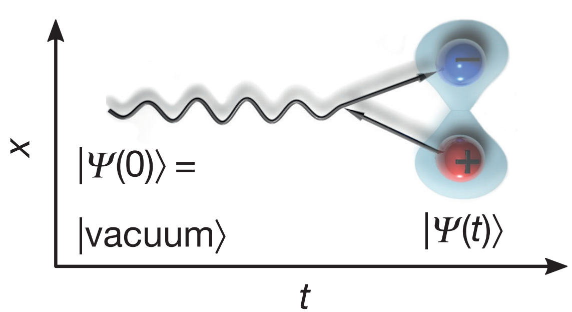
$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\text{Trotter errors for non-commuting terms}}$$

$$H = \boxed{-J\sigma_1^z\sigma_2^z} + \boxed{B(\sigma_1^x + \sigma_2^x)}$$

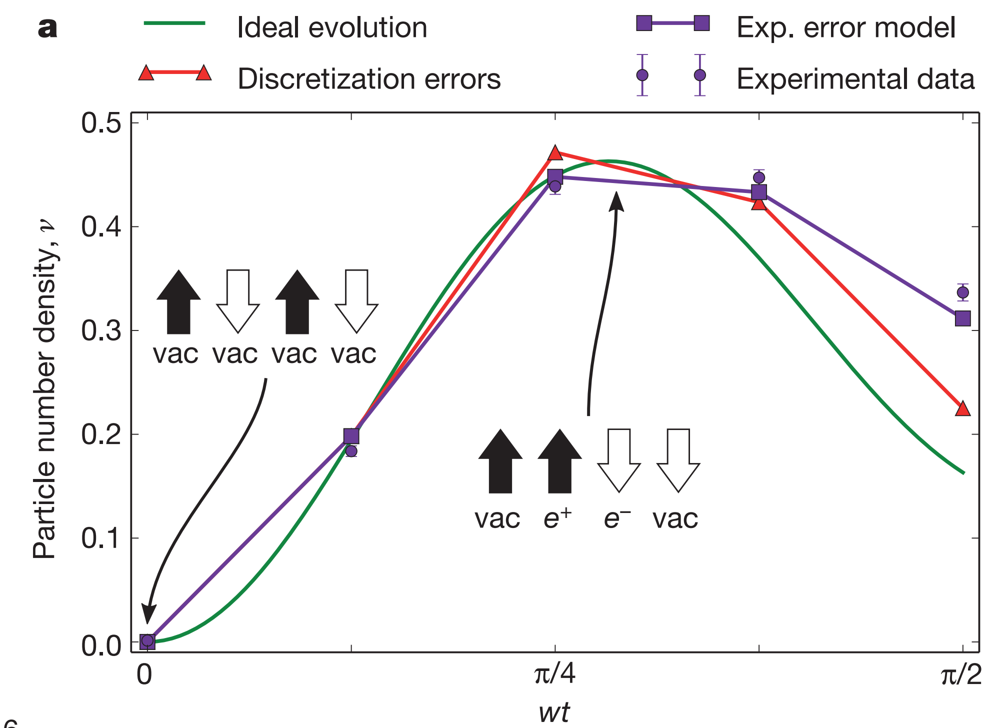
... not restricted to unitary dynamics

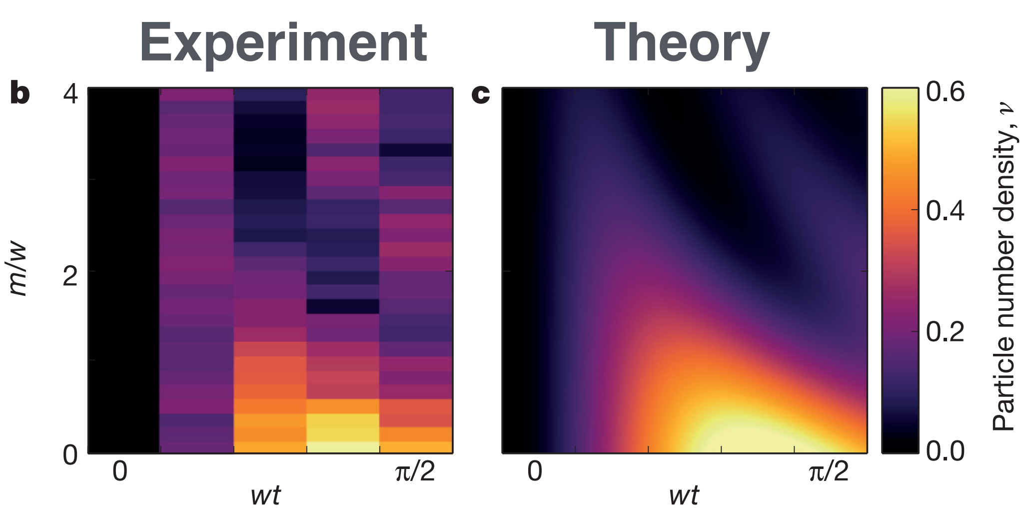
Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Schwinger Mechanism



Time evolution particle density



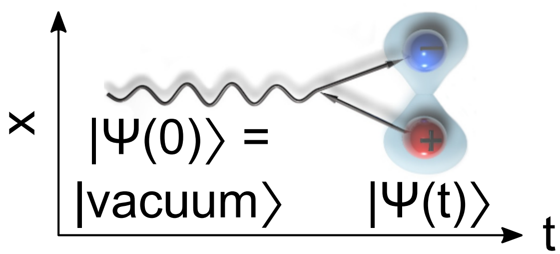


Particle number density:

$$\nu(t) = \frac{1}{N} \sum_{n=1}^{N} \langle (-1)^n \sigma_n^z(t) + 1 \rangle$$

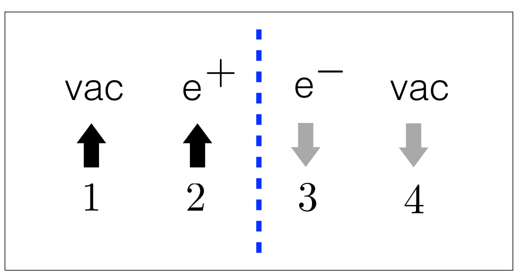
Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Entanglement

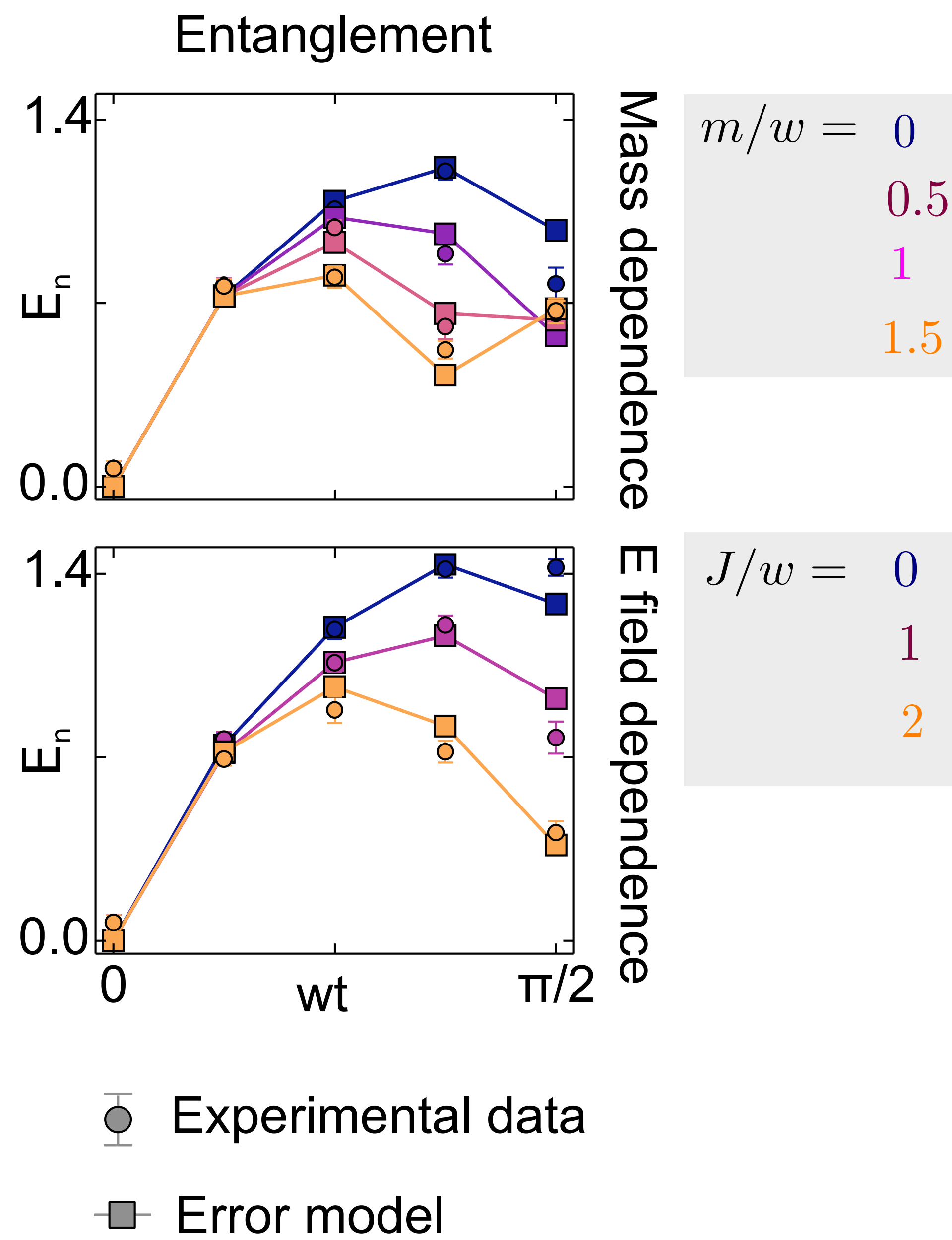


State tomography:
access to the full density matrix

E_n : logarithmic negativity
evaluated with respect to this bipartition:



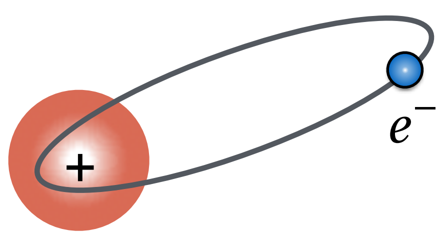
Entanglement between the two halves of the system.



Quantum Spin Ice with Cold Rydberg Atoms

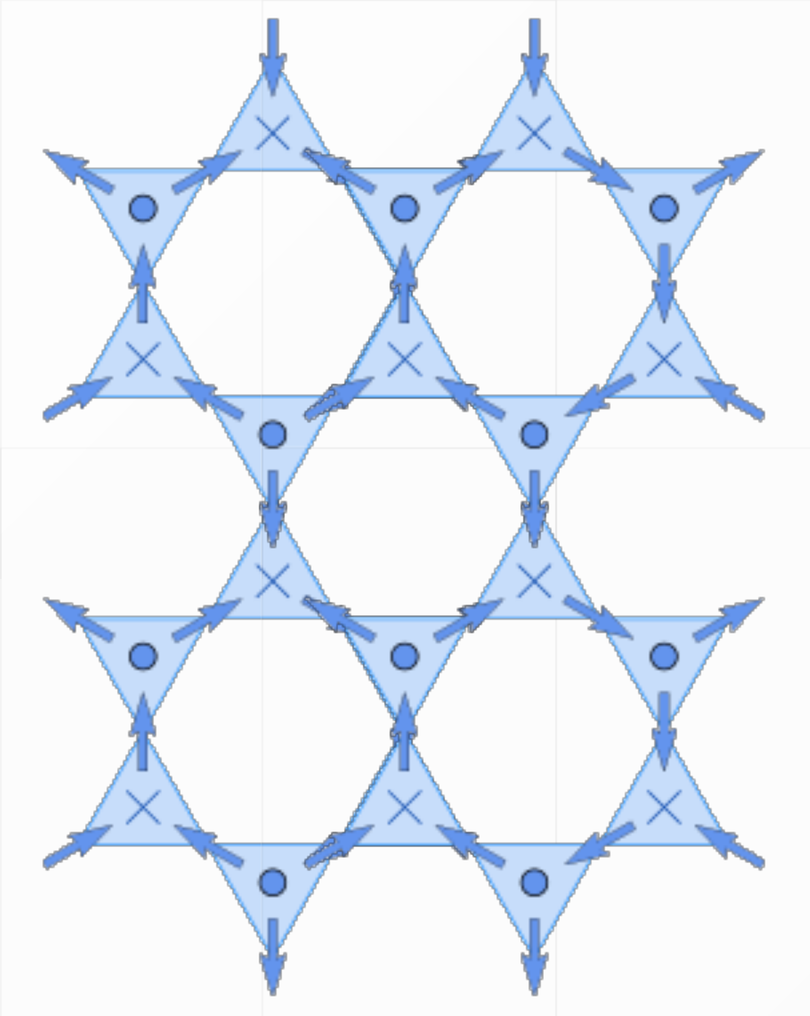
A. Glätzle M. Dalmonte R. Nath I Bloch & C Gross@MPQ

Laser excited Rydberg atoms



condensed matter physics

quantum spin ice, spin liquids, frustrated magnetism, …



Anisotropic Heisenberg Model

$$H = \sum_{\substack{i,j=1 \\ i<j}}^{N} \left[J_z(\mathbf{r}_{ij}) S_z^{(i)} S_z^{(j)} + \frac{J_{+-}(\mathbf{r}_{ij})}{2} \left(S_+^{(i)} S_-^{(j)} + \text{h.c.} \right) + \frac{J_{++}(\mathbf{r}_{ij})}{2} \left(S_+^{(i)} S_+^{(j)} + \text{h.c.} \right) \right]$$

Quantum Simulation of Exotic Quantum Magnetism with Atoms or Ions?

knowledge of the microscopic mechanisms that inhibit magnetic order in materials. Here we

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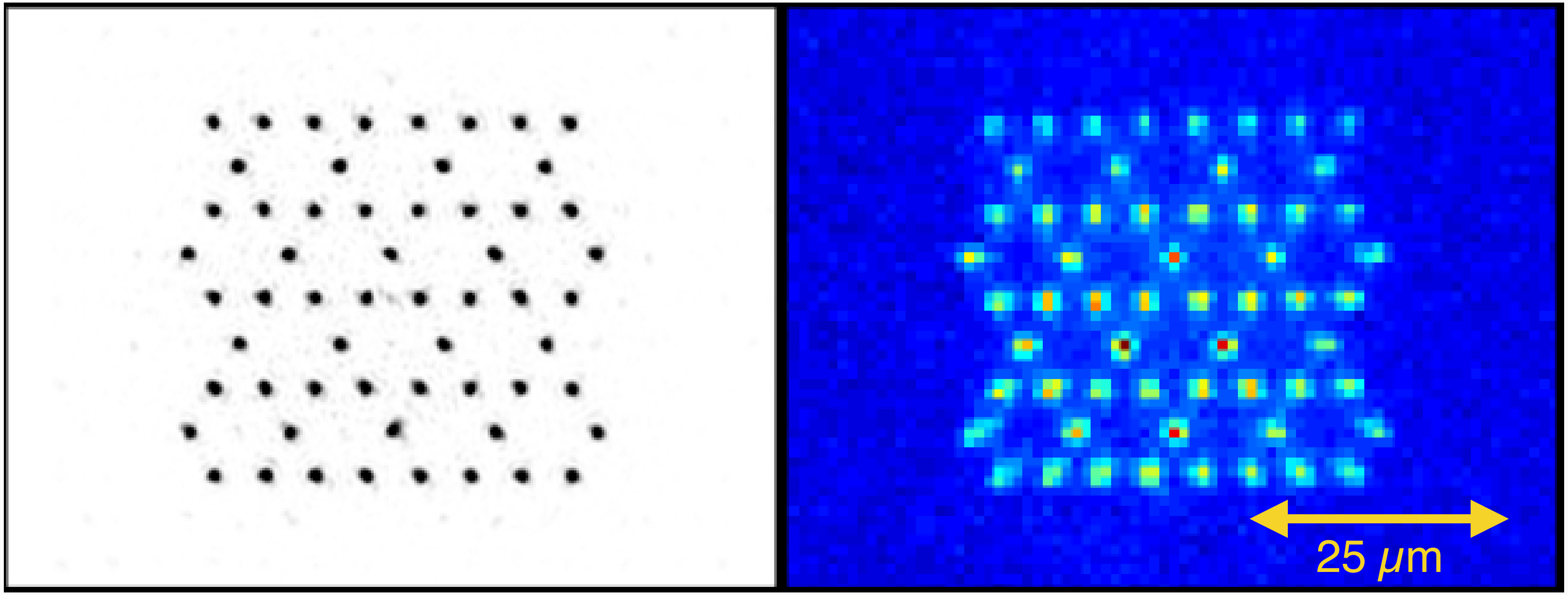
Single-Atom Trapping in Holographic 2D Arrays of Microtraps with Arbitrary Geometries

F. Nogrette, H. Labuhn, S. Ravets, D. Barredo, L. Béguin, A. Vernier, T. Lahaye, and A. Browaeys
Laboratoire Charles Fabry, UMR 8501, Institut d'Optique, CNRS, Univ Paris Sud 11, 2 avenue Augustin Fresnel, 91127 Palaiseau cedex, France



large spacing **optical lattices:**

Palaiseau, Madison, Harvard, Caltech, …

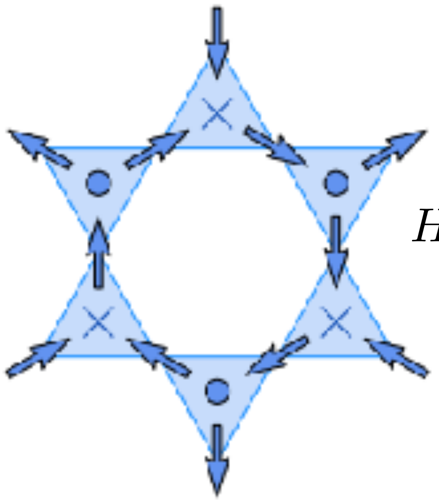
Q.: How design spin-spin interactions?

- *arbitrary* interaction patterns
- *large* interactions (~ temperature)

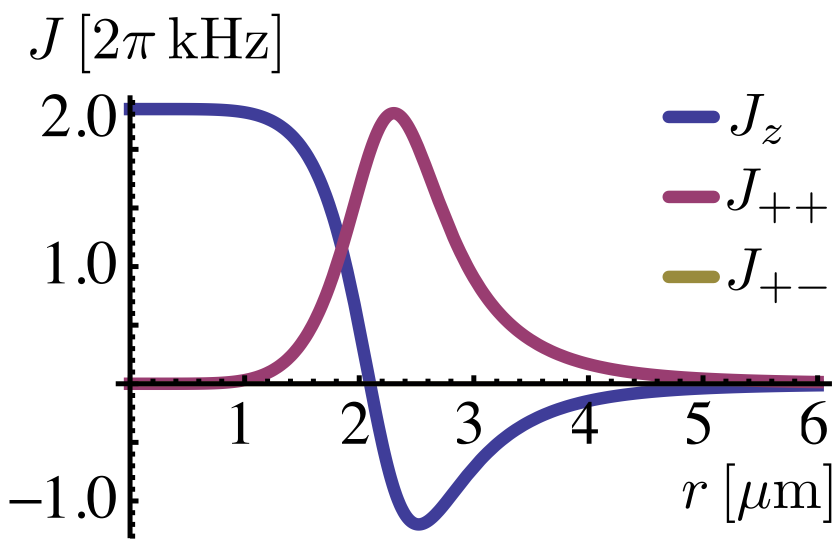
Quantum Kagome Ice [Frustrated Magnetism]

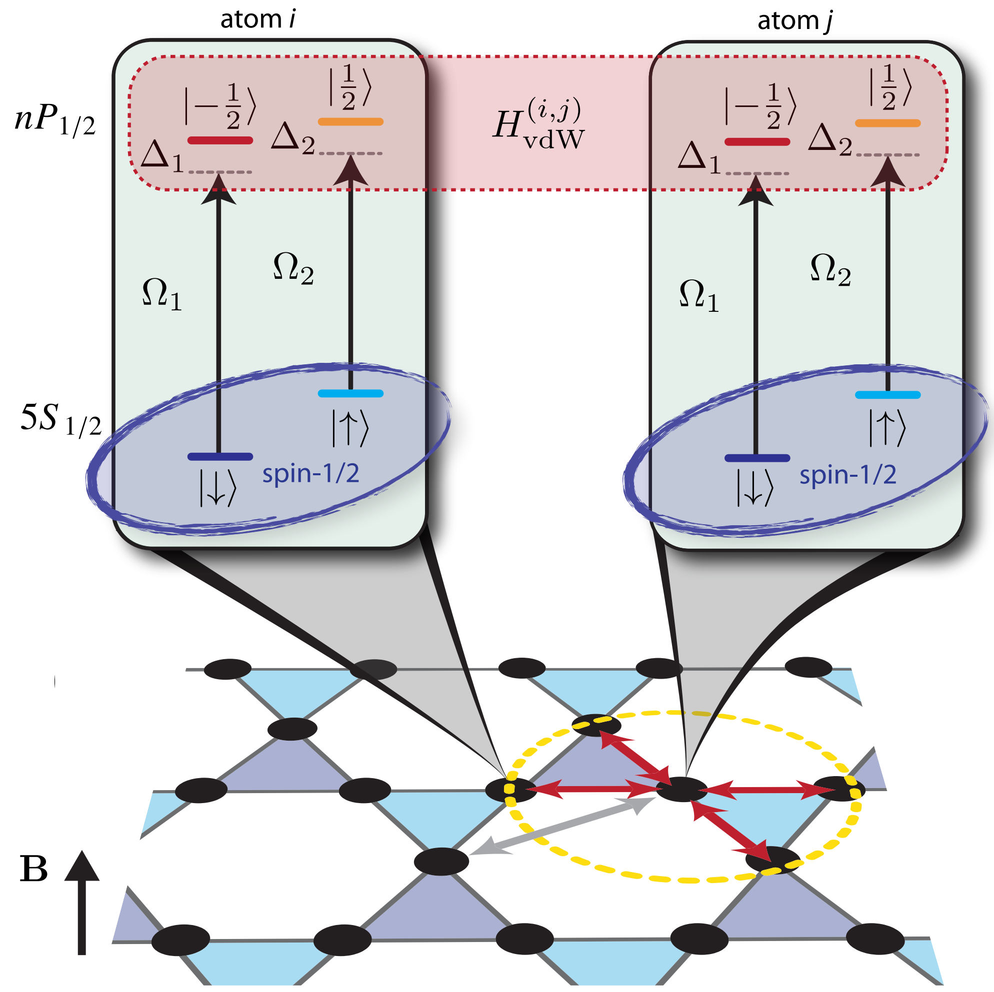
2D spin liquid in quantum kagome ice

J Carrasquilla, Z Hao & RG Melko, Nature Comm 2015



$$H_{ij} \simeq J^{++}(S_i^+ S_j^+ + \text{h.c.}) + J^z S_i^z S_j^z$$





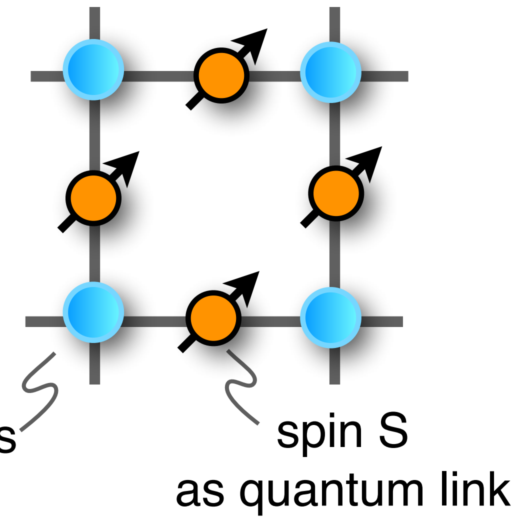
Conclusion & Outlook

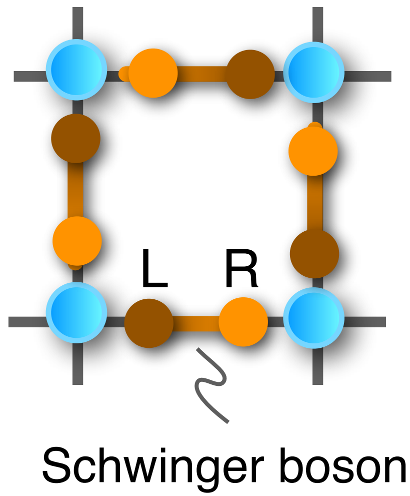
Quantum Simulation of Lattice Gauge Theories:

U(1) Abelian LGT

D. Banerjee et al., PRL 2012

"spin ice" QED + matter





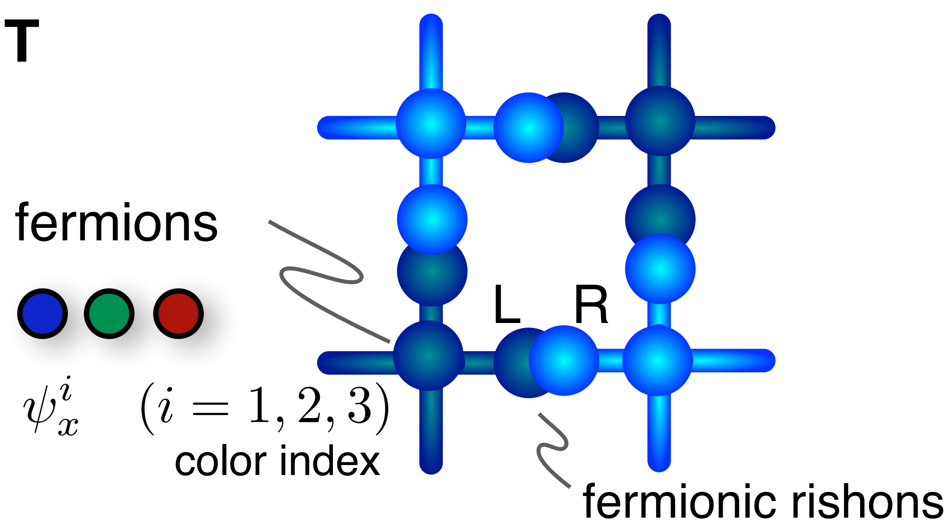
atomic boson-fermi mixtures in optical lattices

U(N),SU(N) Non-Abelian LGT

D. Banerjee et al., PRL 2013

K. Stannigel et al., PRL 2014

Gauss law by particle number conservation



multi-species fermi gases