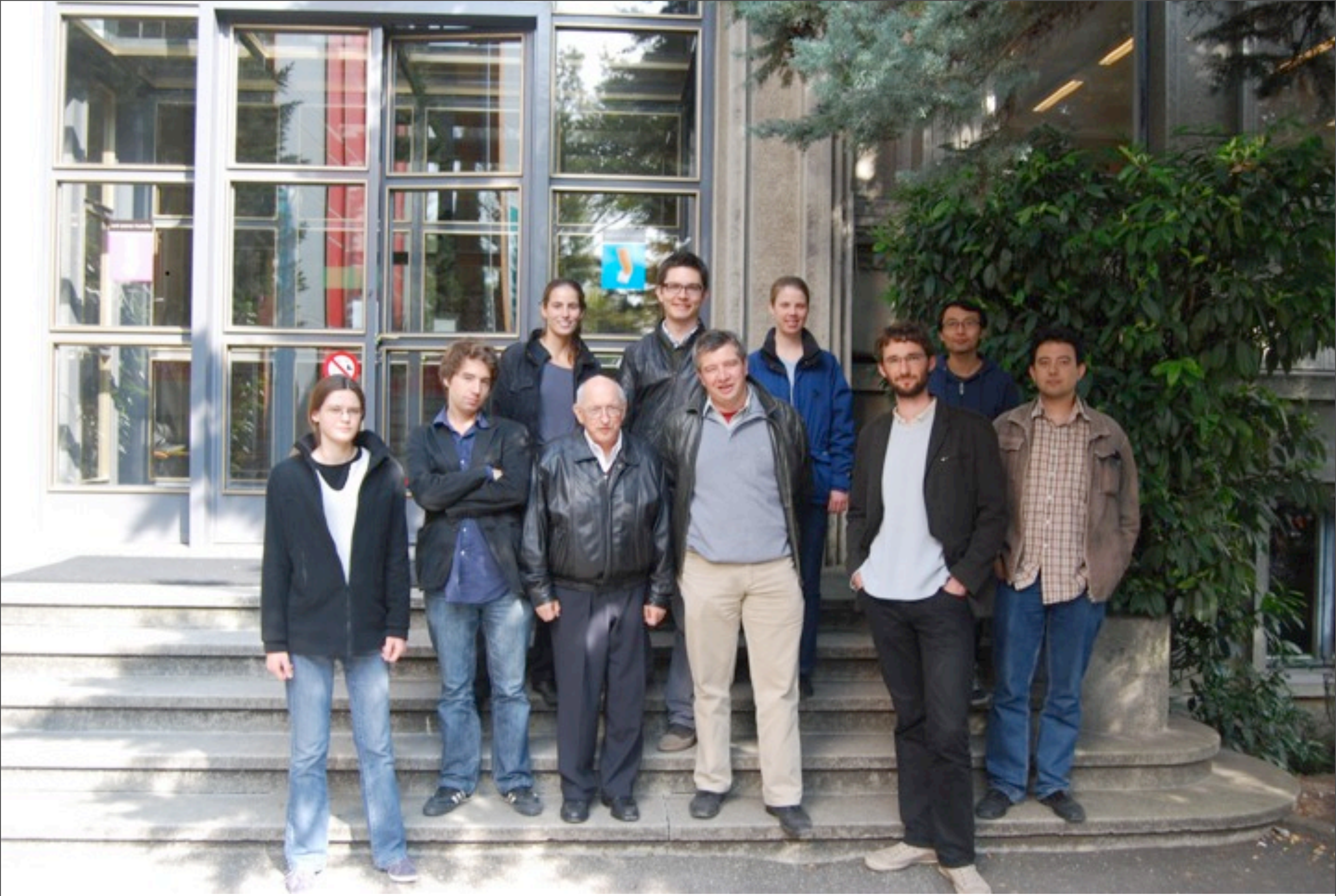
NANO-CTM

Markus Buttiker

with

Geneviève Fleury , Saclay/Paris
Jian Li, University of Geneva

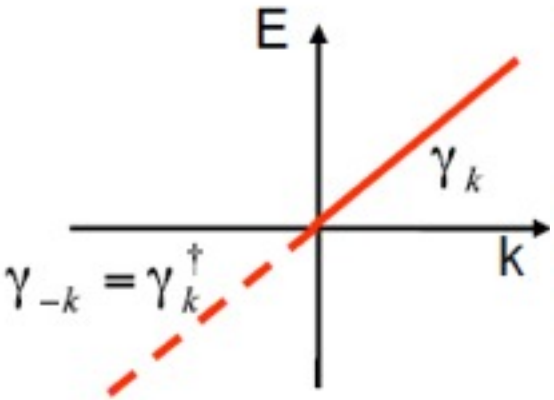
KITP Program: "Topological Insulators and Superconductors"
Sep. 19 – Dec. 16 (2011)

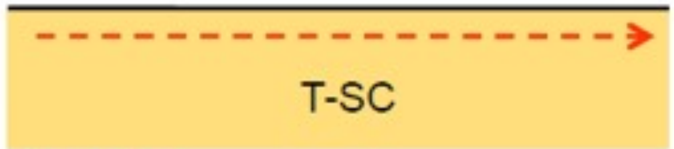


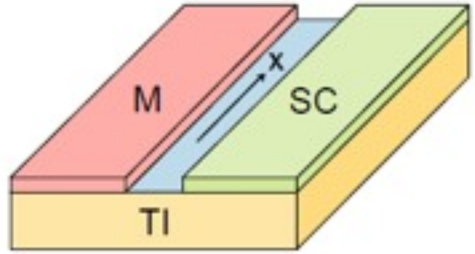
Dec 2009

Chiral Majorana Fermion modes

@Kane







Examples

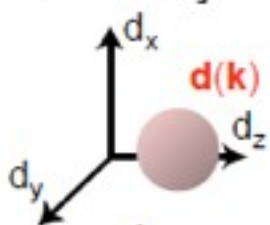
- Spinless p_x+ip_y superconductor (n=1)
- Chiral triplet p wave superconductor (eg Sr_2RuO_4) (n=2)

Class D in d =2

Read Green model : $H = \sum_{\mathbf{k}} \left(\frac{\mathbf{k}^2}{2m} - \mu \right) c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \left(\Delta(\mathbf{k}) c_{\mathbf{k}} c_{-\mathbf{k}} + c.c. \right)$ $\quad \Delta(\mathbf{k}) = \Delta_0 \left(k_x + i k_y \right)$

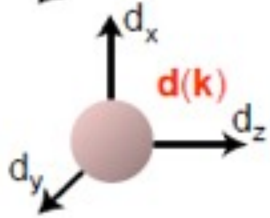
Lattice BdG model : $H_{BdG}(\mathbf{k}) = \sigma_z \left(2t \left[\cos k_x + \cos k_y \right] - \mu \right) + \Delta \left(\sigma_x \sin k_x + \sigma_y \sin k_y \right) = \mathbf{d}(k) \cdot \boldsymbol{\sigma}$

$|\mu|>4t$: Strong pairing phase trivial superconductor



Chern number 0

$|\mu|<4t$: Weak pairing phase topological superconductor



Chern number 1

Chiral Majorana Fermion modes

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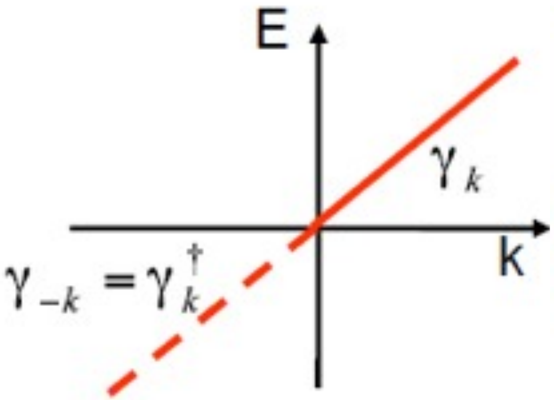

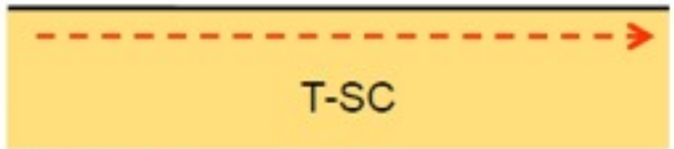

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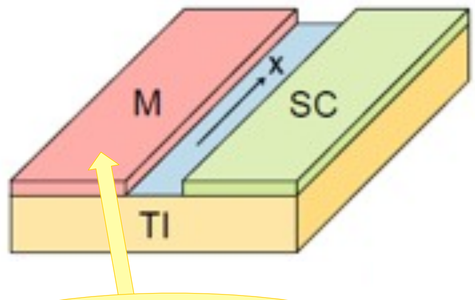

chirality controllable

Class D in d =2

Read Green model : $H = \sum_{\mathbf{k}} \left(\frac{\mathbf{k}^2}{2m} - \mu \right) c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \left(\Delta(\mathbf{k}) c_{\mathbf{k}} c_{-\mathbf{k}} + c.c. \right)$ $\quad \Delta(\mathbf{k}) = \Delta_0 \left(k_x + i k_y \right)$

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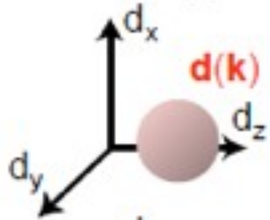
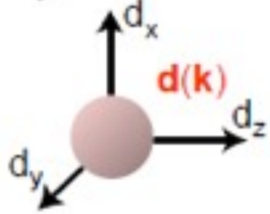
$|\mu| > 4t$: Strong pairing phase
trivial superconductor



Chern number 0

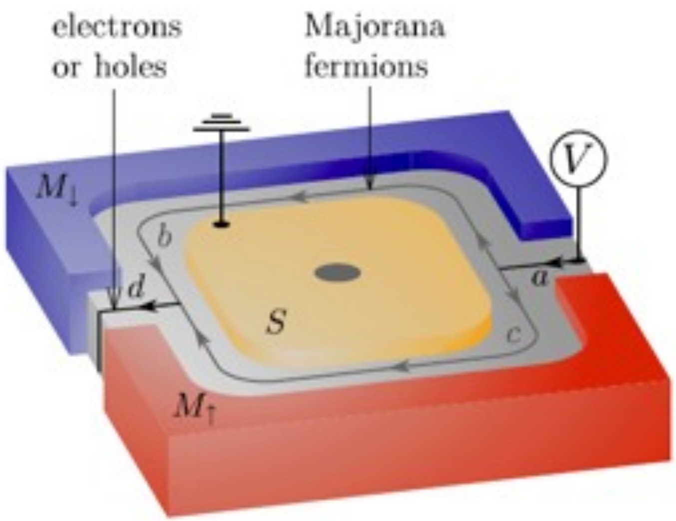
$|\mu| < 4t$: Weak pairing phase
topological superconductor



Chern number 1

Z_2 – Interferometers

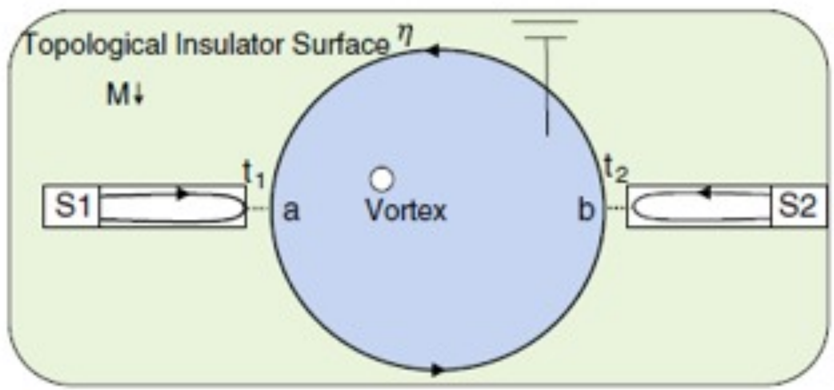
Two prototypes



Mach-Zehnder

Fu & Kane, PRL, 2009

Akhmerov, Nilsson & Beenakker, PRL, 2009



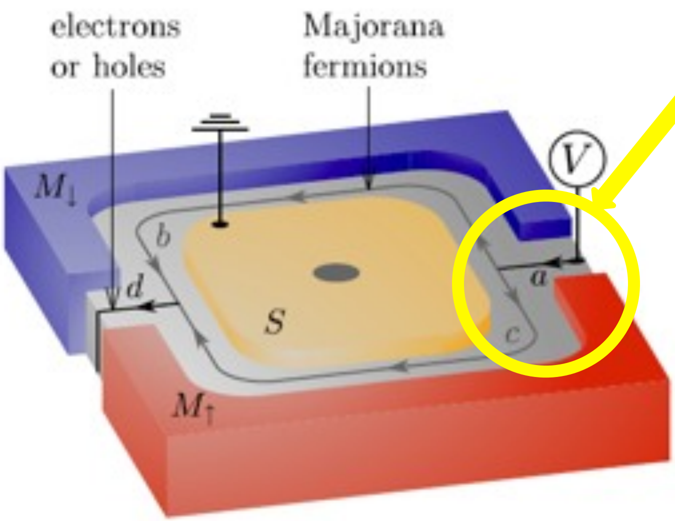
Fabry-Perot

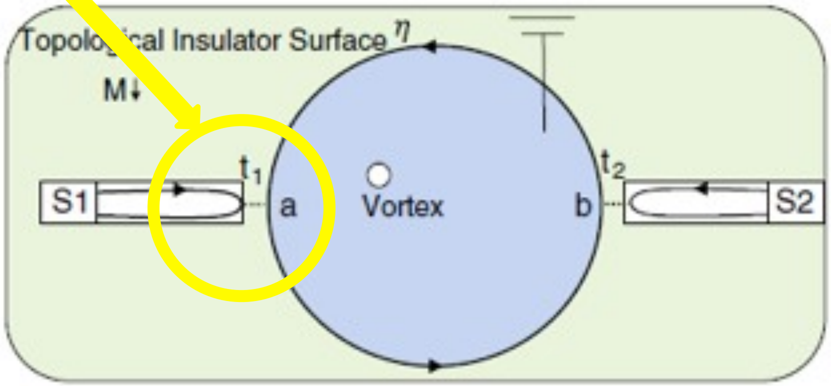
Law, Lee & Ng, PRL, 2009

Z_2 – Interferometers

Two prototypes

contact with
normal part
?





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Fabry-Perot

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Motivation

Novel type of particles:

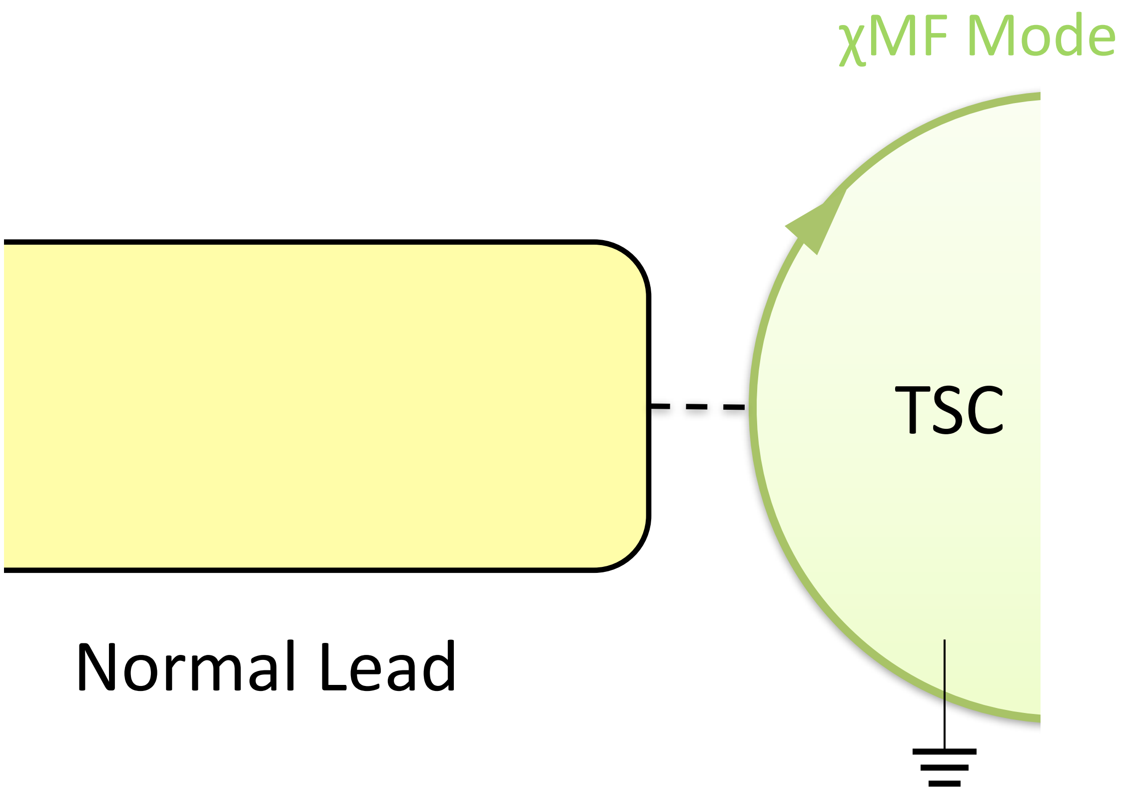
Neutral Majorana modes

Formulation of the scattering problem in a Majorana basis

Theory in which charge and current are not diagonal operators

Charge and current are a manfestation of interference

Scattering theory – the building block



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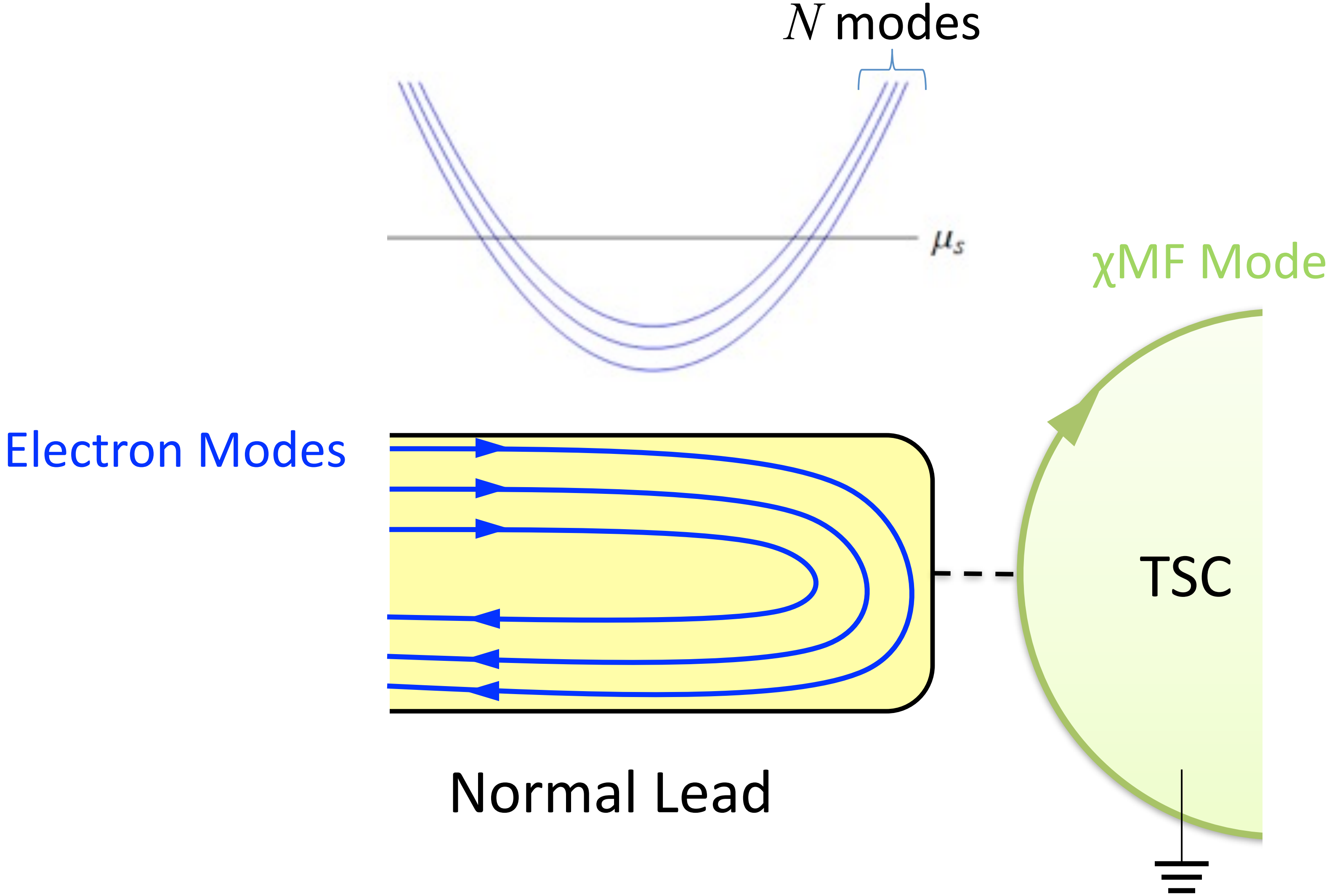

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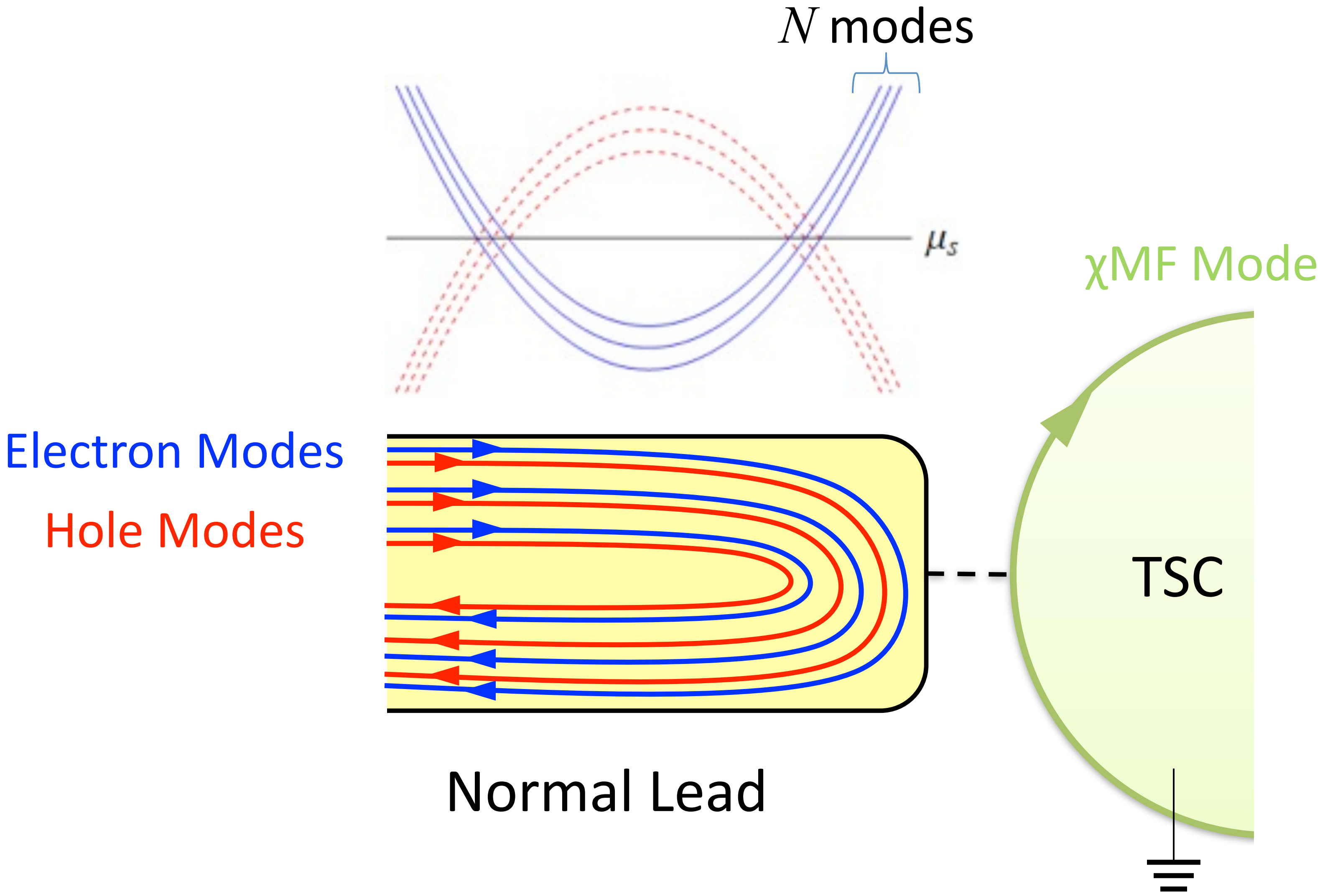
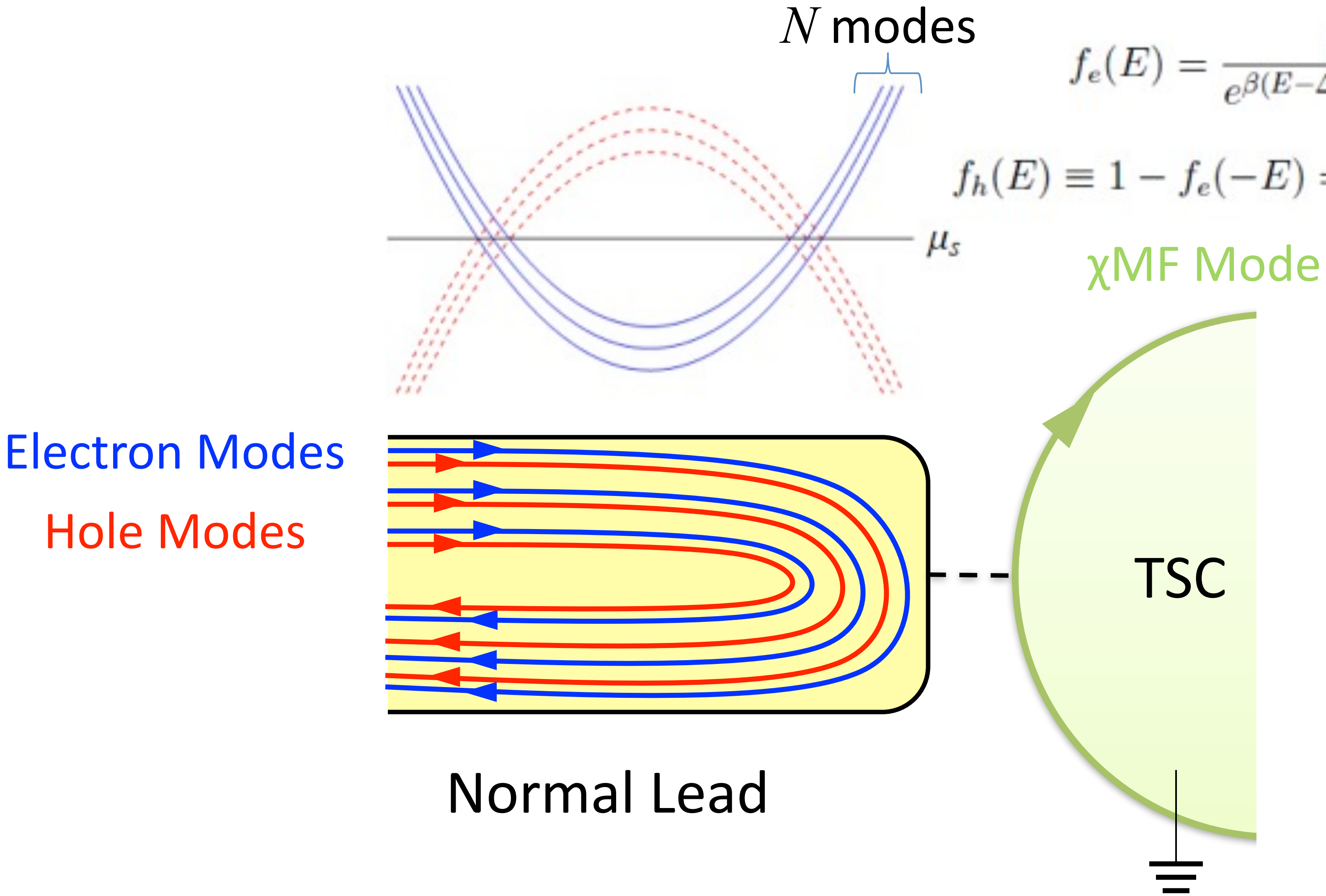


Scattering theory – the building block

$$f_e(E) = \frac{1}{e^{\beta(E-\Delta\mu_n)}+1}$$

$$f_h(E) \equiv 1 - f_e(-E) = \frac{1}{e^{\beta(E+\Delta\mu_n)}+1}$$



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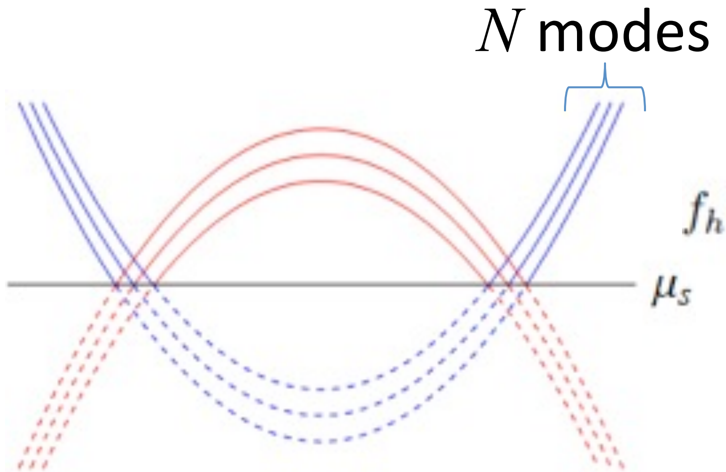

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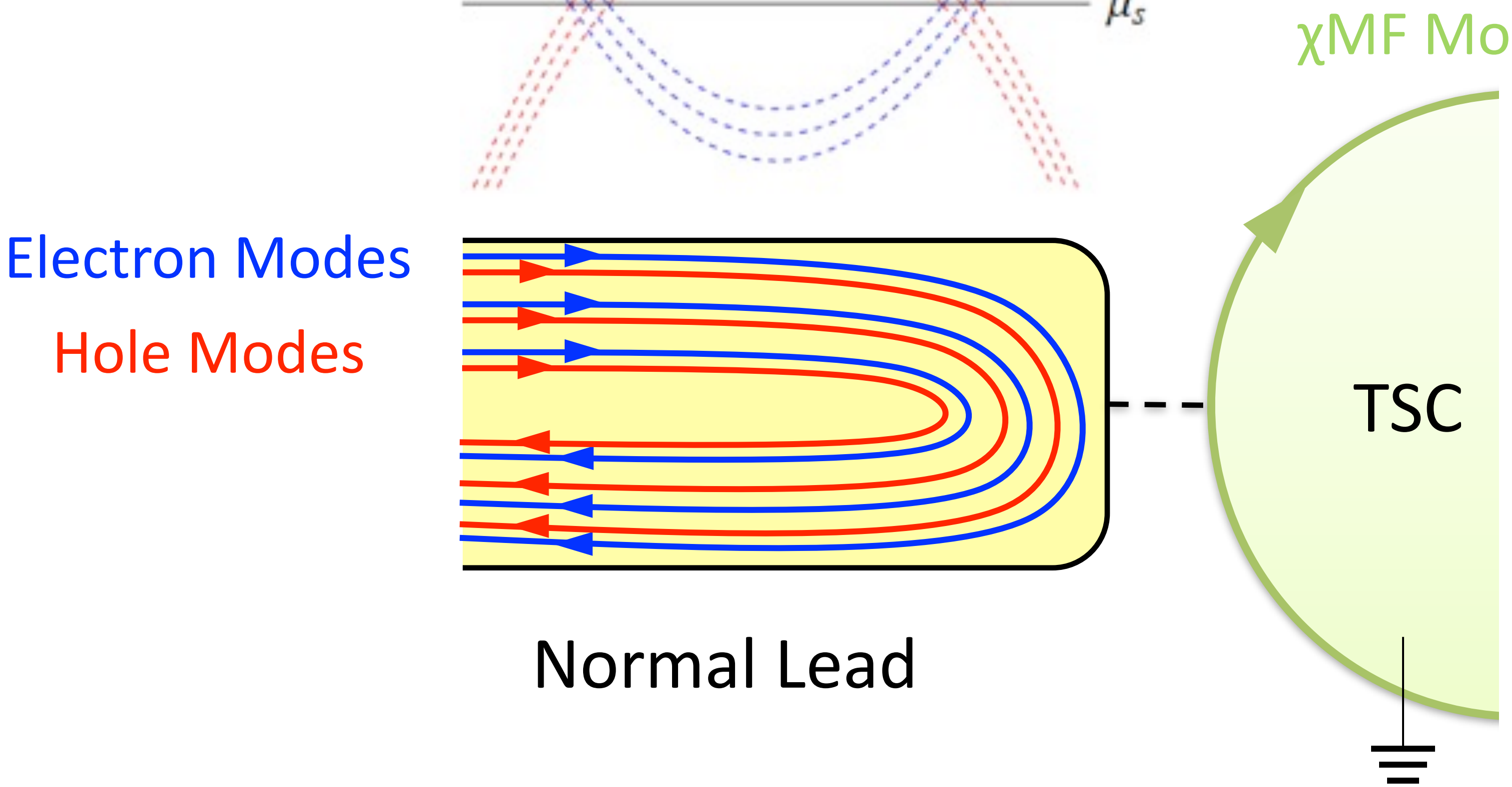

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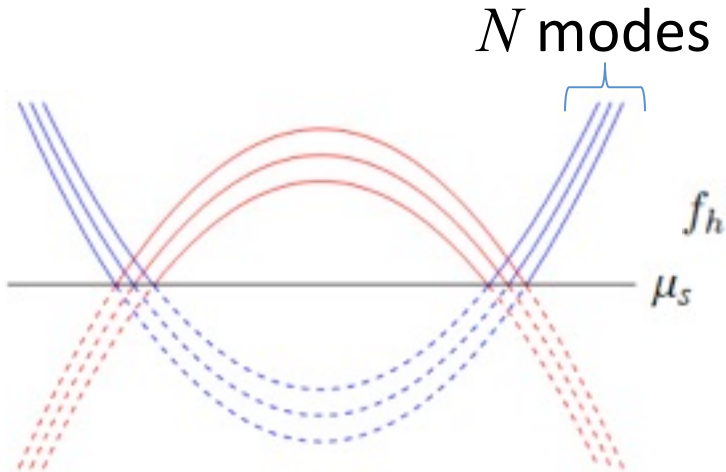

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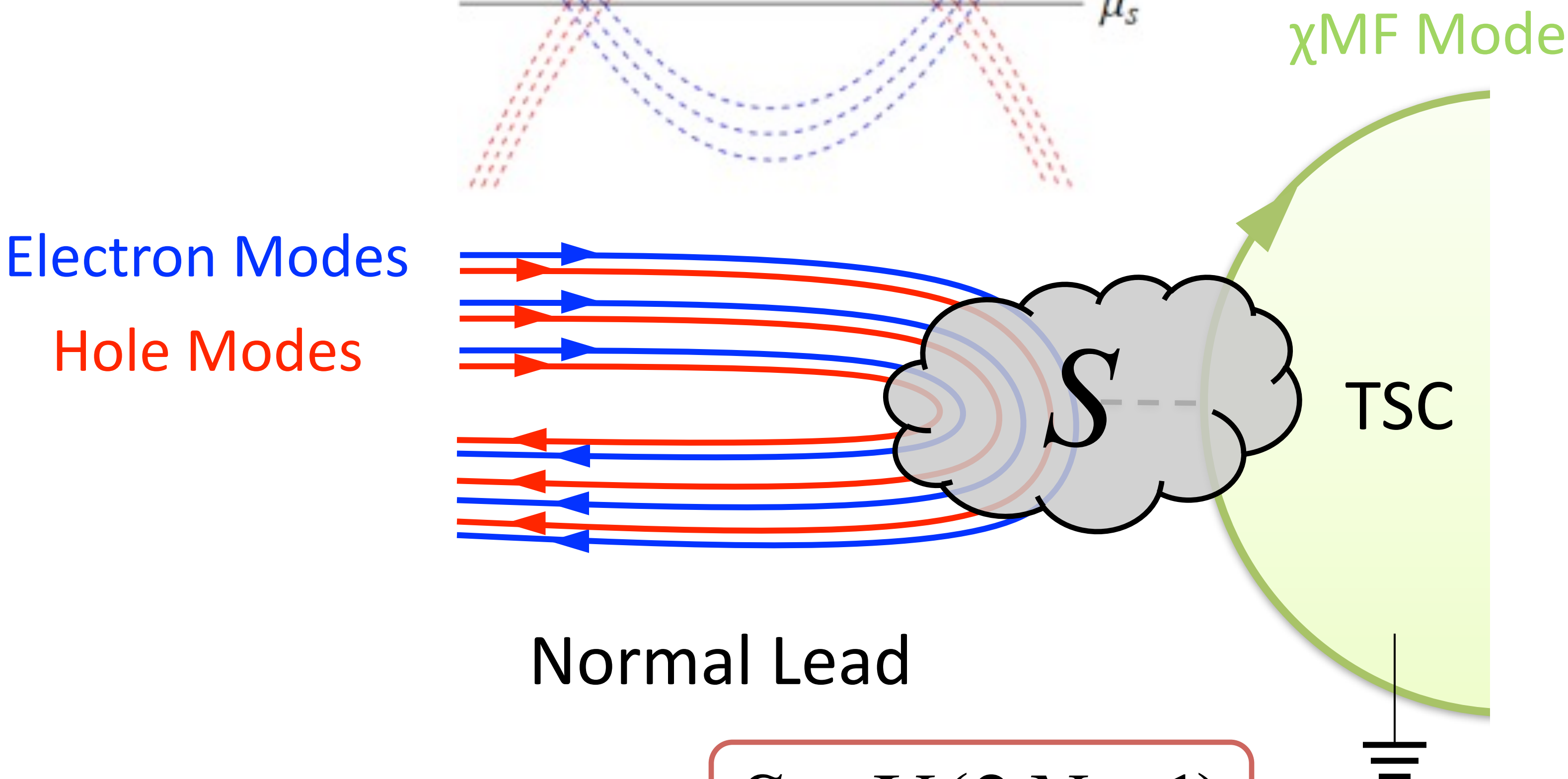

$$\boxed{S \in U(2N+1)}$$

PHS & scattering matrix electron-hole basis

$$\boxed{\mathcal{P} = \sigma_x \mathcal{K}}$$

- PHS on electron/hole states

$$\mathcal{P}(a|k,E;e\rangle) = a^*|-k,-E;h\rangle$$

Group velocity is unchanged!

$$S_{\alpha\beta}(E) = S^*_{\bar{\alpha}\bar{\beta}}(-E) \quad (\alpha,\beta = e,h; \bar{e} = h, \bar{h} = e)$$

- Simplest example (2-by-2 matrix; E = 0)

$$S = \begin{pmatrix} b & a \\ a^* & b^* \end{pmatrix}, \quad \begin{matrix} b^*a + a^*b = 0 \\ 2ab = 2a^*b^* = 0 \end{matrix}$$

$$\Rightarrow S = \begin{pmatrix} e^{i\varphi_b} & 0 \\ 0 & e^{-i\varphi_b} \end{pmatrix} \quad \text{or} \quad S = \begin{pmatrix} 0 & e^{i\varphi_a} \\ e^{-i\varphi_a} & 0 \end{pmatrix}$$

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Only total normal reflection or total Andreev reflection is allowed!

Det(S) = +1 Det(S) = -1

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PHS & scattering matrix Majorana basis

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$$\mathcal{P}(a|E;m\rangle) = \mathcal{P}[\frac{a}{\sqrt{2}}(e^{i\varphi}|E;e\rangle + e^{-i\varphi}|E;h\rangle)]$$
$$= \frac{a^*}{\sqrt{2}}(e^{-i\varphi}|-E;h\rangle + e^{i\varphi}|-E;e\rangle)$$
$$= a^*|-E;m\rangle$$
$$S_{m_1m_2}(E) = S^*_{m_1m_2}(-E)$$

- Simple example (2-by-2 matrix; E = 0)

$$S = \begin{pmatrix} \cos\varphi_b & -\sin\varphi_b \\ \sin\varphi_b & \cos\varphi_b \end{pmatrix} \quad \text{or} \quad S = \begin{pmatrix} \cos\varphi_a & \sin\varphi_a \\ \sin\varphi_a & -\cos\varphi_a \end{pmatrix}$$

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$$E \to 0$$

$$S_{m_1m_2} = S^*_{m_1m_2}$$

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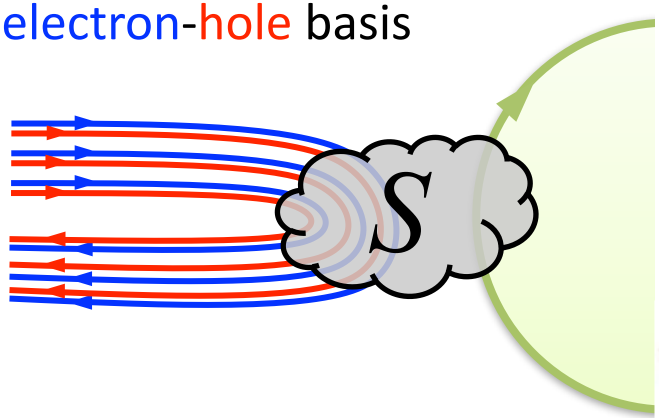
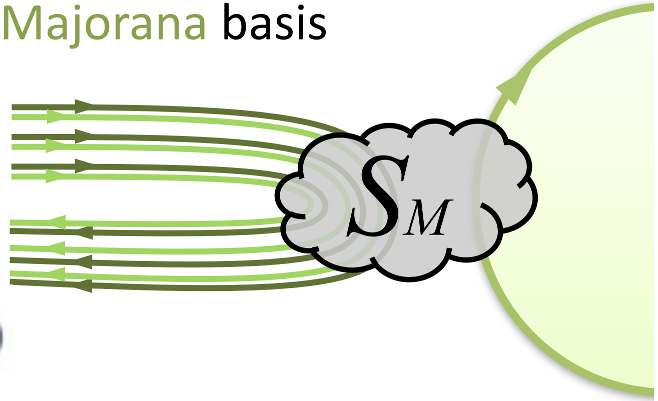
Changing basis

electron-hole basis



Majorana basis



$$\eta_{2i-1}(E) = \frac{1}{\sqrt{2}}(\psi_{ie}(E) + \psi_{ih}(E))$$

$$\eta_{2i}(E) = \frac{i}{\sqrt{2}}(\psi_{ie}(E) - \psi_{ih}(E))$$

$$(i = 1, \ldots, N)$$

$$\begin{pmatrix} \gamma^{(+)}(E) \\ \psi^{(+)}_{1e}(E) \\ \psi^{(+)}_{1h}(E) \\ \vdots \\ \psi^{(+)}_{Ne}(E) \\ \psi^{(+)}_{Nh}(E) \end{pmatrix} = S(E) \begin{pmatrix} \gamma^{(-)}(E) \\ \psi^{(-)}_{1e}(E) \\ \psi^{(-)}_{1h}(E) \\ \vdots \\ \psi^{(-)}_{Ne}(E) \\ \psi^{(-)}_{Nh}(E) \end{pmatrix} \Longrightarrow \begin{pmatrix} \gamma^{(+)}(E) \\ \eta^{(+)}_{1}(E) \\ \eta^{(+)}_{2}(E) \\ \vdots \\ \eta^{(+)}_{2N-1}(E) \\ \eta^{(+)}_{2N}(E) \end{pmatrix} = S_M(E) \begin{pmatrix} \gamma^{(-)}(E) \\ \eta^{(-)}_{1}(E) \\ \eta^{(-)}_{2}(E) \\ \vdots \\ \eta^{(-)}_{2N-1}(E) \\ \eta^{(-)}_{2N}(E) \end{pmatrix}$$

Changing basis

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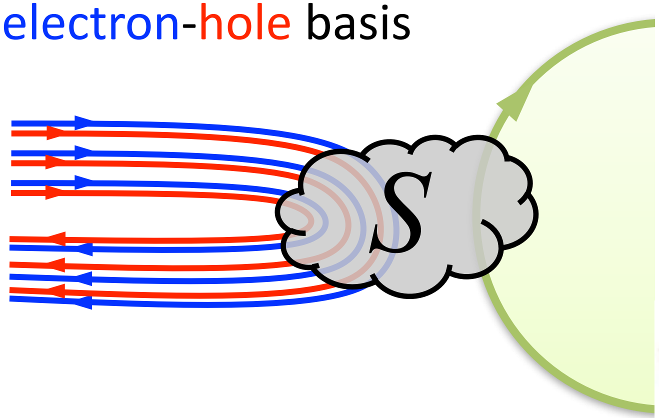

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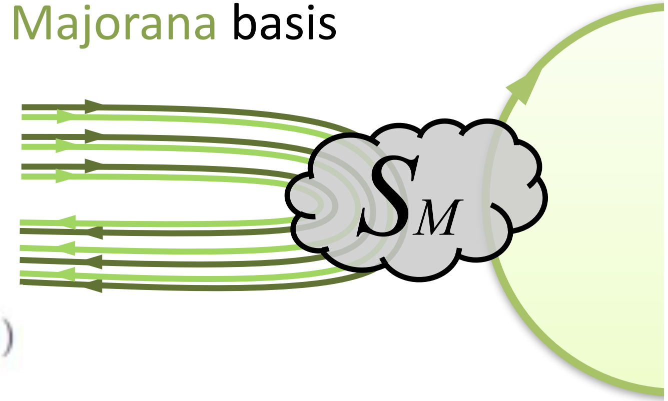

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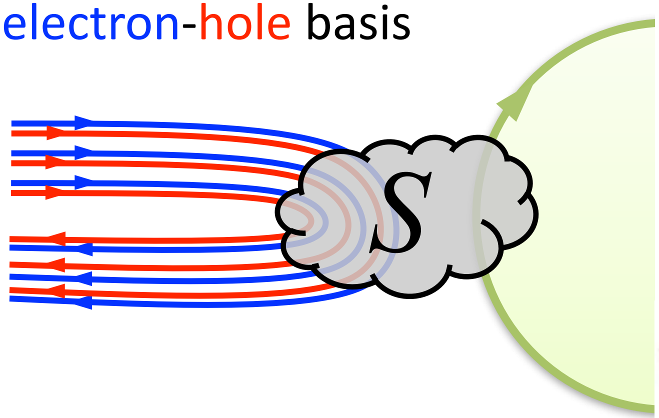
$$\mathcal{U} = \bigoplus_{i=1}^{N} U_0, \quad U_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

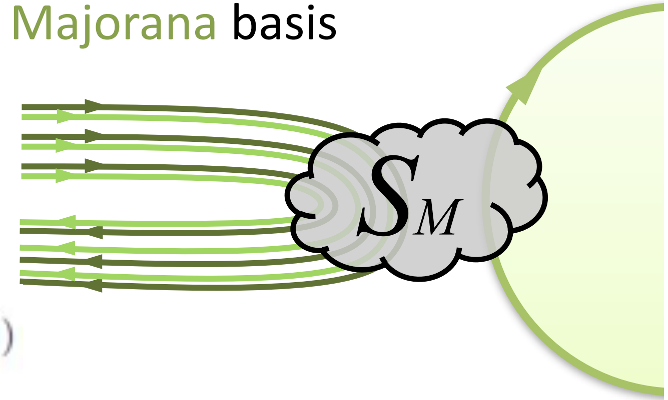
$$\boxed{S = \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{U}^\dagger \end{pmatrix} S_M \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{U} \end{pmatrix}}$$

Changing basis

electron-hole basis



Majorana basis



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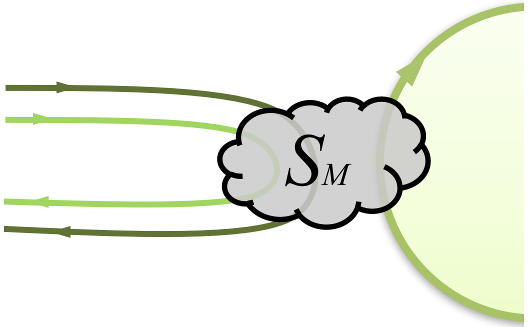
$$\boxed{S \in U(2N+1)}$$

$$\boxed{S = \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{U}^\dagger \end{pmatrix} S_M \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{U} \end{pmatrix}}$$

$$\boxed{S_M \in O(2N+1)}$$

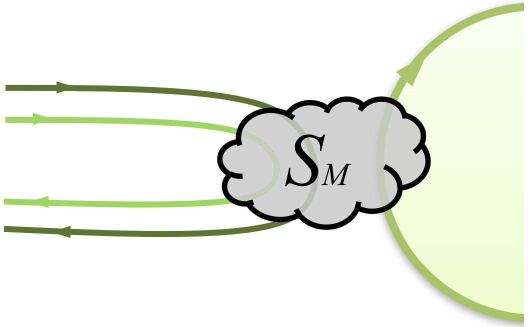
$$(E \approx 0)$$



$\underline{N=1}$ $S_M \in O(3)$

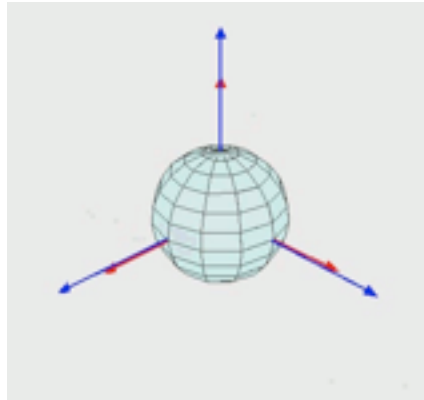
$+1$

$+1$



$\underline{N=1}$ $\quad S_M \in O(3)$

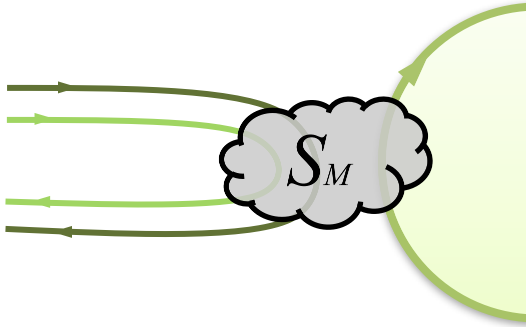
- Euler decomposition



$$S_M = \begin{pmatrix} 1 & 0 \\ 0 & R_0(\alpha) \end{pmatrix} \begin{pmatrix} R_0(\theta) & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & R_0(\beta) \end{pmatrix}$$

$$R_0(\xi) = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \quad (\xi = \alpha, \beta, \theta)$$

$+1$



$\mathcal{N}$=1 $S_M \in O(3)$

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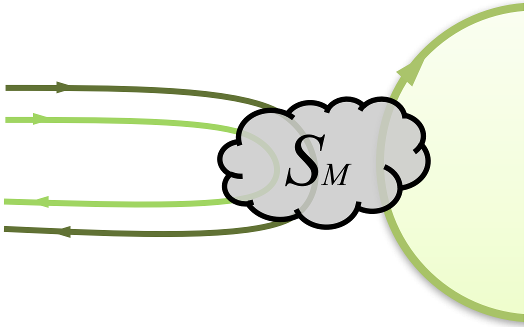
$$R_0(\xi) = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \quad (\xi = \alpha, \beta, \theta)$$

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & R_0(-\alpha) \end{pmatrix} \begin{pmatrix} \gamma^{(+)} \\ \eta_1^{(+)} \\ \eta_2^{(+)} \end{pmatrix} \right] = \begin{pmatrix} R_0(\theta) & 0 \\ 0 & +1 \end{pmatrix} \left[\begin{pmatrix} 1 & 0 \\ 0 & R_0(\beta) \end{pmatrix} \begin{pmatrix} \gamma^{(-)} \\ \eta_1^{(-)} \\ \eta_2^{(-)} \end{pmatrix} \right]$$



$\underline{N=1}$ $\quad S_M \in O(3)$

- Euler decomposition

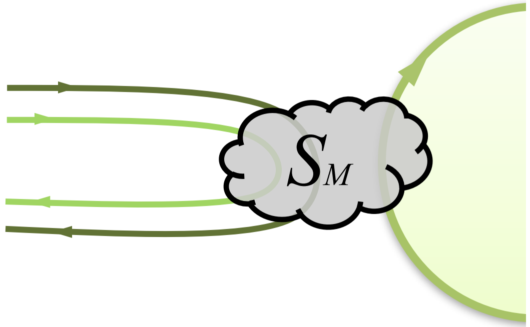
$$S_M = \begin{pmatrix} 1 & 0 \\ 0 & R_0(\alpha) \end{pmatrix} \begin{pmatrix} R_0(\theta) & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & R_0(\beta) \end{pmatrix}$$

$$R_0(\xi) = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \qquad (\xi = \alpha, \beta, \theta)$$

$$\begin{pmatrix} \gamma^{(+)} \\ \tilde{\eta}_1^{(+)} \\ \tilde{\eta}_2^{(+)} \end{pmatrix} = \begin{pmatrix} R_0(\theta) & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} \gamma^{(-)} \\ \tilde{\eta}_1^{(-)} \\ \tilde{\eta}_2^{(-)} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\eta}_1^{(+)} \\ \tilde{\eta}_2^{(+)} \end{pmatrix} = R_0(-\alpha) \begin{pmatrix} \eta_1^{(+)} \\ \eta_2^{(+)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(e^{i\alpha}\psi_e^{(+)} + e^{-i\alpha}\psi_h^{(+)}) \\ \frac{i}{\sqrt{2}}(e^{i\alpha}\psi_e^{(+)} - e^{-i\alpha}\psi_h^{(+)}) \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\eta}_1^{(-)} \\ \tilde{\eta}_2^{(-)} \end{pmatrix} = R_0(\beta) \begin{pmatrix} \eta_1^{(-)} \\ \eta_2^{(-)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(e^{-i\beta}\psi_e^{(-)} + e^{i\beta}\psi_h^{(-)}) \\ \frac{i}{\sqrt{2}}(e^{-i\beta}\psi_e^{(-)} - e^{i\beta}\psi_h^{(-)}) \end{pmatrix}$$



$\underline{N=1}$ $\quad S_M \in O(3)$

- Euler decomposition

$$S_M = \begin{pmatrix} 1 & 0 \\ 0 & R_0(\alpha) \end{pmatrix} \begin{pmatrix} R_0(\theta) & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & R_0(\beta) \end{pmatrix}$$

physically relevant S_M

$$R_0(\xi) = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \qquad (\xi = \alpha, \beta, \theta)$$

$$\begin{pmatrix} \gamma^{(+)} \\ \tilde{\eta}_1^{(+)} \\ \tilde{\eta}_2^{(+)} \end{pmatrix} = \boxed{\begin{pmatrix} R_0(\theta) & 0 \\ 0 & +1 \end{pmatrix}} \begin{pmatrix} \gamma^{(-)} \\ \tilde{\eta}_1^{(-)} \\ \tilde{\eta}_2^{(-)} \end{pmatrix}$$

$U(1)$ gauge transformation

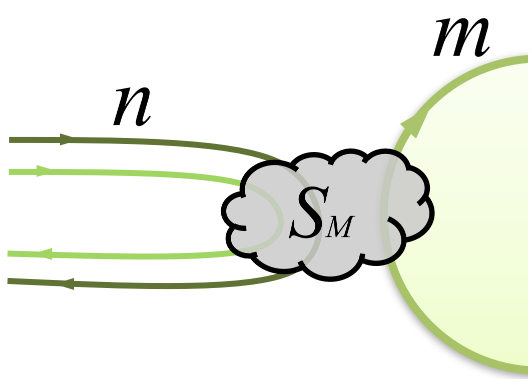
$$\begin{pmatrix} \tilde{\eta}_1^{(+)} \\ \tilde{\eta}_2^{(+)} \end{pmatrix} = \boxed{R_0(-\alpha)} \begin{pmatrix} \eta_1^{(+)} \\ \eta_2^{(+)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(e^{i\alpha}\psi_e^{(+)} + e^{-i\alpha}\psi_h^{(+)}) \\ \frac{i}{\sqrt{2}}(e^{i\alpha}\psi_e^{(+)} - e^{-i\alpha}\psi_h^{(+)}) \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\eta}_1^{(-)} \\ \tilde{\eta}_2^{(-)} \end{pmatrix} = \boxed{R_0(\beta)} \begin{pmatrix} \eta_1^{(-)} \\ \eta_2^{(-)} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(e^{-i\beta}\psi_e^{(-)} + e^{i\beta}\psi_h^{(-)}) \\ \frac{i}{\sqrt{2}}(e^{-i\beta}\psi_e^{(-)} - e^{i\beta}\psi_h^{(-)}) \end{pmatrix}$$

N=1

- Physics w/ reduced S_M

$$S_M = \begin{pmatrix} R_0(\theta) & 0 \\ 0 & \pm 1 \end{pmatrix} \quad r_2 = +1$$

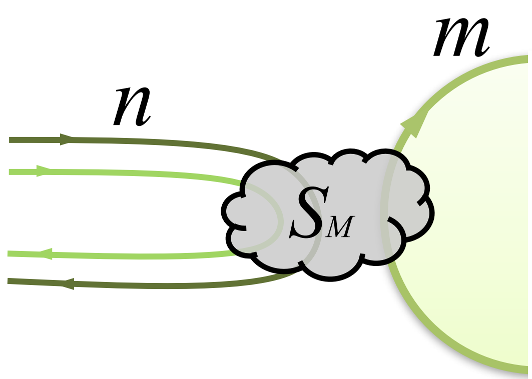


Current is determined only by reflection part of S matrix

N=1

- Physics w/ reduced S_M

$$S_M = \begin{pmatrix} r_1 & -t_1 & \\ t_1 & r_1 & \\ & & r_2 \end{pmatrix}, \quad r_2 = +1$$

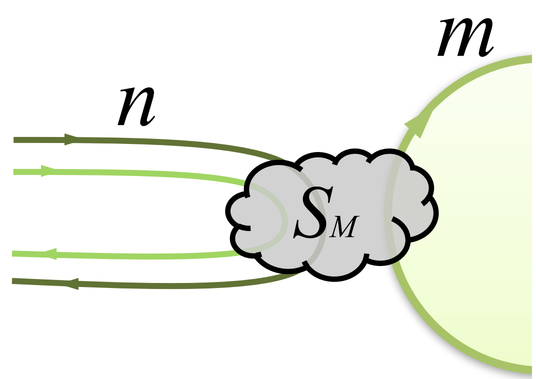


Current is determined only by reflection part of S matrix

N=1

- Physics w/ reduced S_M

$$S_M = \begin{array}{c} \begin{pmatrix} r_1 & -t_1 & \\ t_1 & r_1 & \\ & & r_2 \end{pmatrix} \\ \begin{array}{ccc} \gamma & \eta_1 & \eta_2 \end{array} \end{array}, \quad r_2 = +1$$

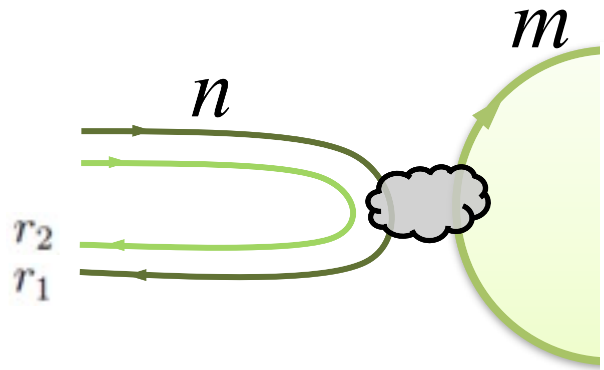


Current is determined only by reflection part of S matrix

N=1

- Physics w/ reduced S_M

$$S_M = \begin{array}{c} \begin{pmatrix} r_1 & -t_1 & \\ t_1 & r_1 & \\ & & r_2 \end{pmatrix} \\ \begin{array}{ccc} \gamma & \eta_1 & \eta_2 \end{array} \end{array}, \quad r_2 = +1$$

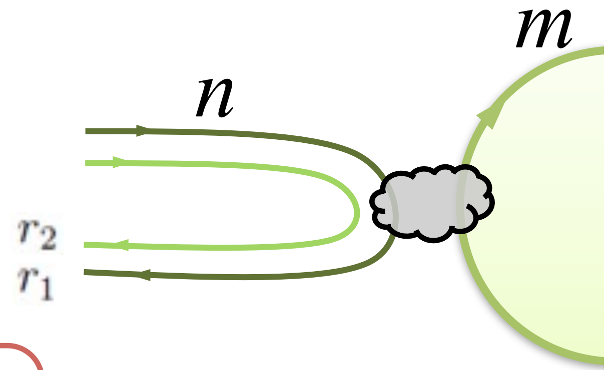


Current is determined only by reflection part of S matrix

N=1

- Physics w/ reduced S_M

$$
\begin{array}{c}
S_M = \left(\begin{array}{c|cc} r_1 & -t_1 & \\ \hline t_1 & r_1 & \\ & & r_2 \end{array}\right), \quad r_2 = +1 \\
\begin{array}{ccc} \Uparrow & \Uparrow & \Uparrow \\ \gamma & \eta_1 & \eta_2 \end{array}
\end{array}
$$



How about N>1 ?
later…

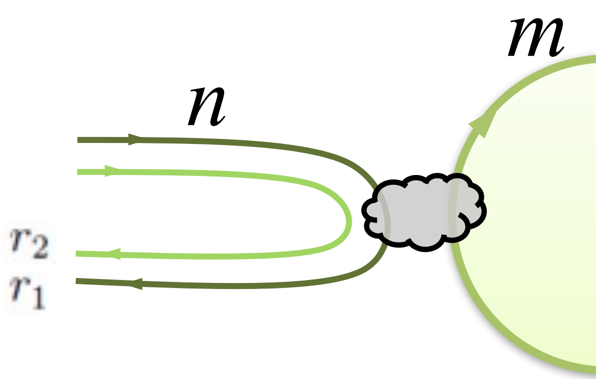
Current is determined only by reflection part of S matrix

N=1

- Physics w/ reduced S_M

$$S_M = \begin{pmatrix} r_1 & -t_1 & \\ t_1 & r_1 & \\ & & r_2 \end{pmatrix}, \quad r_2 = +1$$

$$\gamma \quad \eta_1 \quad \eta_2$$



- Current

$$I_n = I_{nm} + I_{nn}$$

$$I_{nm} = -\frac{e}{h}\int_{E>0} dE\, S^\dagger_{nm}\sigma_z S_{nm} f_m(E) = 0$$

$$I_{nn} = \frac{e}{h}\int_{E\geq 0} dE\, \mathrm{Tr}[(\sigma_z - S^\dagger_{nn}\sigma_z S_{nn})F_n(E)]$$

$$= \frac{e}{h}\int_{E\geq 0} dE\, [1 - \mathrm{Re}(r_1^* r_2)][f_e(E) - f_h(E)]$$

$$S_{nm} = U_0^\dagger \begin{pmatrix} t_1 \\ 0 \end{pmatrix}$$

$$S_{nn} = U_0^\dagger \begin{pmatrix} r_1 & \\ & r_2 \end{pmatrix} U_0$$

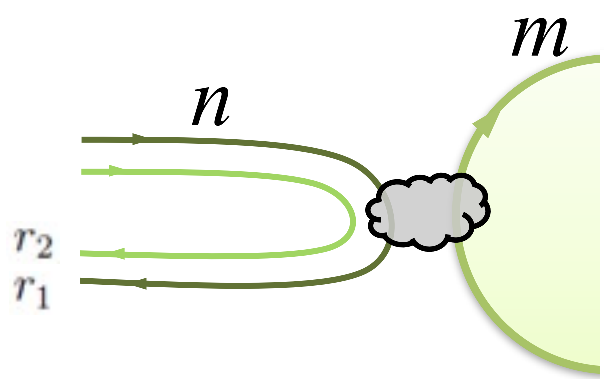
$$F_n(E) = \begin{pmatrix} f_e(E) & 0 \\ 0 & f_h(E) \end{pmatrix}$$

Current is determined only by reflection part of S matrix

N=1

- Physics w/ reduced S_M

$$\begin{array}{c} S_M = \begin{pmatrix} r_1 & -t_1 & \\ t_1 & r_1 & \\ & & r_2 \end{pmatrix}, \quad r_2 = +1 \\ \gamma \quad \eta_1 \quad \eta_2 \end{array}$$



- Current

$$I_n = I_{nm} + I_{nn}$$

$$I_{nm} = -\frac{e}{h}\int_{E>0} dE\, S^\dagger_{nm}\sigma_z S_{nm} f_m(E) = 0$$

$$I_{nn} = \frac{e}{h}\int_{E\geq 0} dE\, \mathrm{Tr}[(\sigma_z - S^\dagger_{nn}\sigma_z S_{nn})F_n(E)]$$

$$= \frac{e}{h}\int_{E\geq 0} dE\, [1 - \mathrm{Re}(r_1^* r_2)][f_e(E) - f_h(E)]$$

$$S_{nm} = U_0^\dagger \begin{pmatrix} t_1 \\ 0 \end{pmatrix}$$

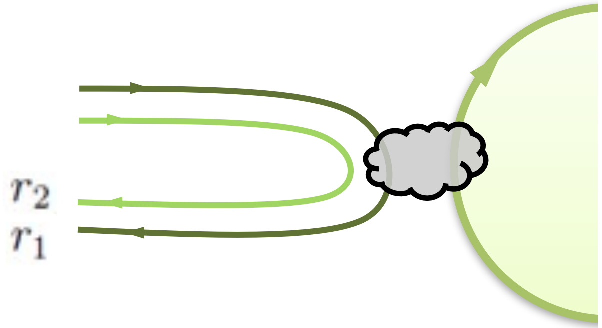
$$S_{nn} = U_0^\dagger \begin{pmatrix} r_1 & \\ & r_2 \end{pmatrix} U_0$$

$$F_n(E) = \begin{pmatrix} f_e(E) & 0 \\ 0 & f_h(E) \end{pmatrix}$$

Current is determined only by reflection part of S matrix

N = 1

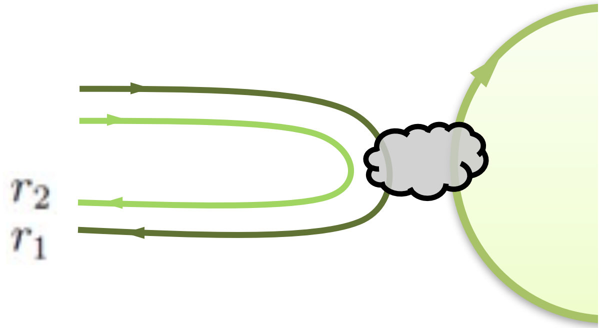
- Current in terms of Majorana fermions



$$I_n = \frac{e}{h}\int_{E\geq 0} dE\ [1 - \mathrm{Re}(r_1^* r_2)][f_e(E) - f_h(E)]$$

N = 1

- Current in terms of Majorana fermions



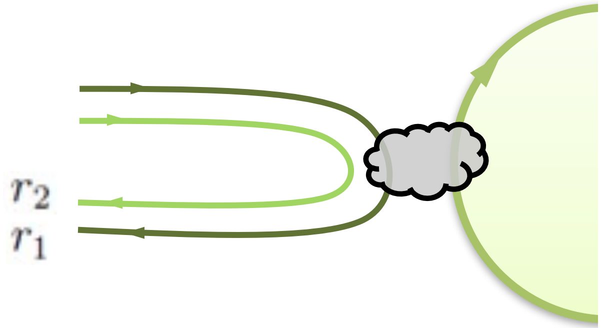
$$I_n = \frac{e}{h}\int_{E\geq 0} dE\,[1 - \mathrm{Re}(r_1^* r_2)][f_e(E) - f_h(E)]$$

incoming

outgoing

N = 1

- Current in terms of Majorana fermions



$$I_n = \frac{e}{h}\int_{E\geq 0} dE\,[1 - \mathrm{Re}(r_1^* r_2)][f_e(E) - f_h(E)]$$

incoming

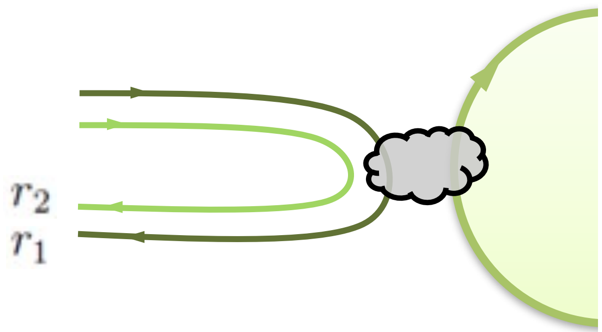
outgoing

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(r_1|\eta_1\rangle + ir_2|\eta_2\rangle) = \frac{1}{2}\left[(r_1 + r_2)|\psi_e\rangle + (r_1 - r_2)|\psi_h\rangle\right]$$

$$I(E) = |\frac{1}{2}(r_1 + r_2)|^2 - |\frac{1}{2}(r_1 - r_2)|^2 = \mathrm{Re}(r_1^* r_2)$$

N = 1

- Current in terms of Majorana fermions



$$I_n = \frac{e}{h}\int_{E\geq 0} dE\,[1 - \mathrm{Re}(r_1^* r_2)][f_e(E) - f_h(E)]$$

incoming

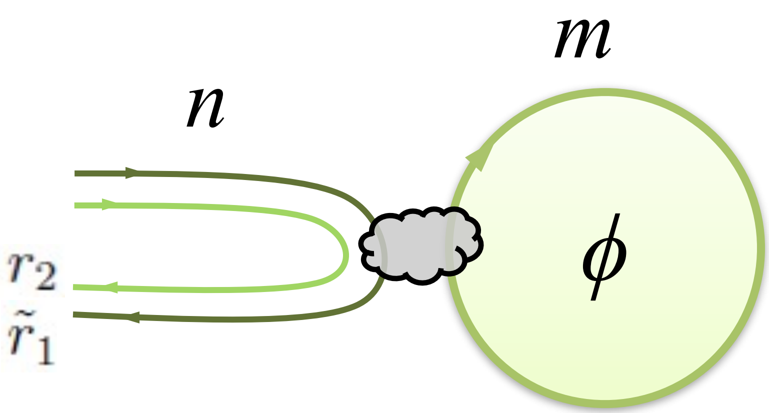
outgoing

current is given by the interference of (a pair of) Majorana fermions

(even though each individual of them does not carry charge)

N = 1

- Interferometer-case 0



$$\phi(E) = EL/\hbar v_m + \pi + n_v \pi$$

π Berry phase

# of vortices

$$S_M = \begin{pmatrix} \tilde{r}_1 & \\ & r_2 \end{pmatrix}$$

$$\tilde{r}_1 = \frac{r_1 - e^{i\phi}}{1 - r_1 e^{i\phi}}$$

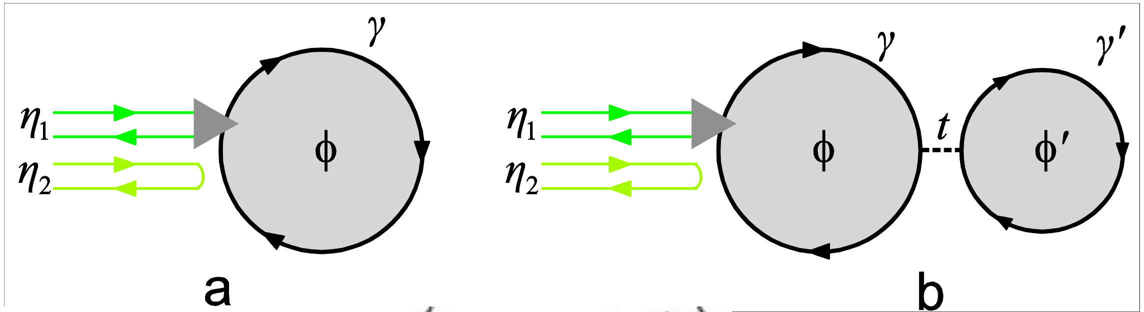
$$I_n = \frac{e}{h} \int_{E \geq 0} dE \, [1 - \mathrm{Re}(\tilde{r}_1^* r_2)][f_e(E) - f_h(E)]$$

$E = 0, \ \phi = (n_v + 1)\pi; \ r_2 = +1$

n_v even $\Longrightarrow \tilde{r}_1 = 1, \Longrightarrow G = 0,$ normal reflection

n_v odd $\Longrightarrow \tilde{r}_1 = -1, \Longrightarrow G = 2e^2/h,$ perfect Andreev reflection

N = 1-doublet



$$\varphi_{ext} = \varphi + \arg\left(\frac{r - e^{i\varphi'}}{1 - re^{i\varphi'}}\right)$$

$$\varphi = EL/\hbar v_M + \pi + \phi$$
$$\varphi' = EL'/\hbar v_M + \pi + \phi'$$

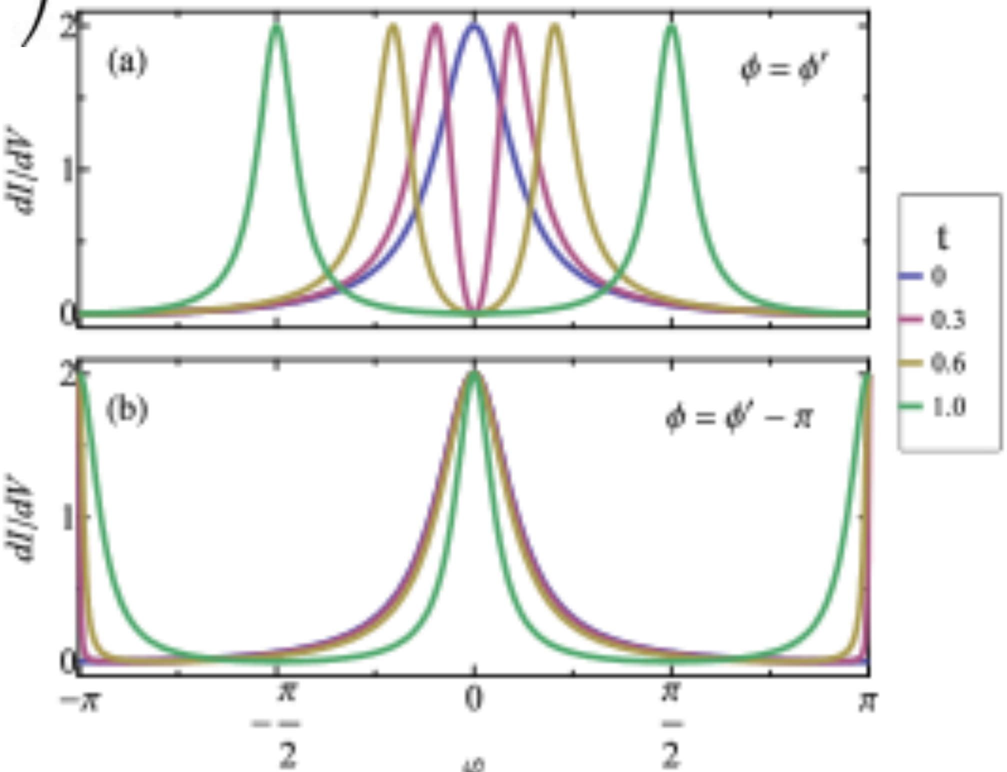
Upper panel:

$$L = L'\ ,\ \ \phi = \phi' = \pi$$

(analog of two-coupled MBS)

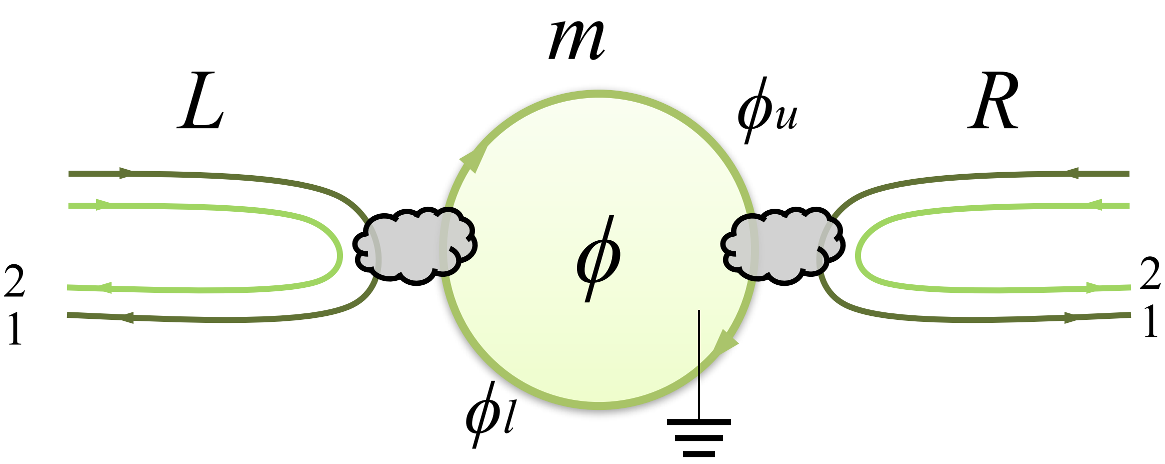
Lower panel:

$$L = L'\ ,\ \ \phi = \phi' - \pi$$



N=1

- Build up interferometers – case 1



$$S_M = \left(\begin{array}{cc|cc} \tilde{r}_{L1} & & t_{LR} & \\ & r_{L2} & & 0 \\ \hline t_{RL} & & \tilde{r}_{R1} & \\ & 0 & & r_{R2} \end{array}\right)$$

$$I_L = I_{LL} + I_{LR}$$

$$I_{LR} = 0$$

$$I_{LL} = \frac{e}{h}\int_{E\geq 0} dE\,[1 - \mathrm{Re}(\tilde{r}^*_{L1} r_{L2})][f_{Le}(E) - f_{Lh}(E)]$$

$$\tilde{r}_{L1} = \frac{r_{L1} - r_{R1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

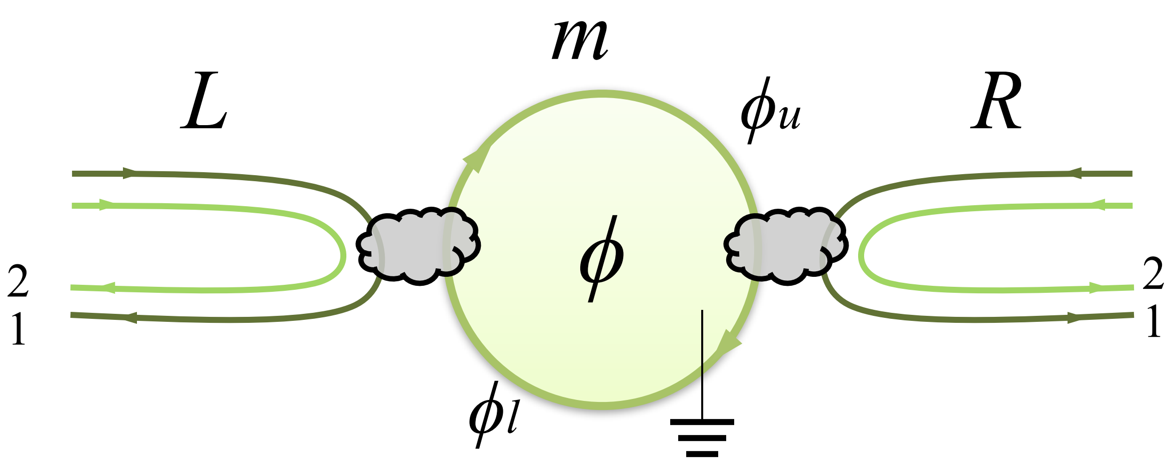
$$\tilde{r}_{R1} = \frac{r_{R1} - r_{L1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

N=1

- Build up interferometers – case 1



$$S_M = \left(\begin{array}{cc|cc} \tilde{r}_{L1} & & t_{LR} & \\ & r_{L2} & & 0 \\ \hline t_{RL} & & \tilde{r}_{R1} & \\ & 0 & & r_{R2} \end{array}\right)$$

$$\tilde{r}_{L1} = \frac{r_{L1} - r_{R1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$\tilde{r}_{R1} = \frac{r_{R1} - r_{L1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$I_L = I_{LL} + I_{LR}$$

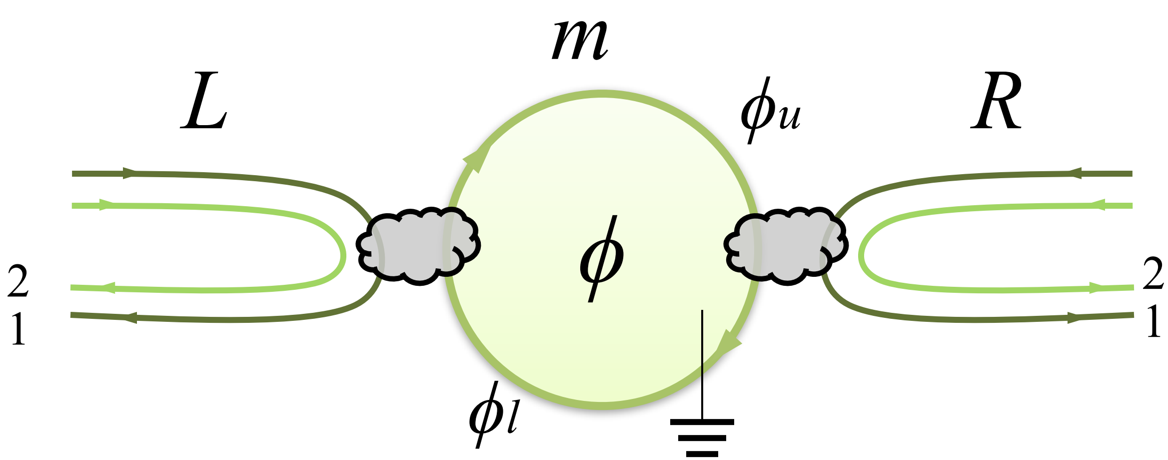
$$I_{LR} = 0$$

$$I_{LL} = \frac{e}{h}\int_{E\geq 0} dE\,[1 - \mathrm{Re}(\tilde{r}^*_{L1}r_{L2})][f_{Le}(E) - f_{Lh}(E)]$$

Left (Right) contact contributes no current to Right (Left) lead

N=1

- Build up interferometers – case 1



$$S_M = \left(\begin{array}{cc|cc} \tilde{r}_{L1} & & t_{LR} & \\ & r_{L2} & & 0 \\ \hline t_{RL} & & \tilde{r}_{R1} & \\ & 0 & & r_{R2} \end{array}\right)$$

$$\tilde{r}_{L1} = \frac{r_{L1} - r_{R1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$\tilde{r}_{R1} = \frac{r_{R1} - r_{L1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$I_L = I_{LL} + I_{LR}$$

$$I_{LR} = 0$$

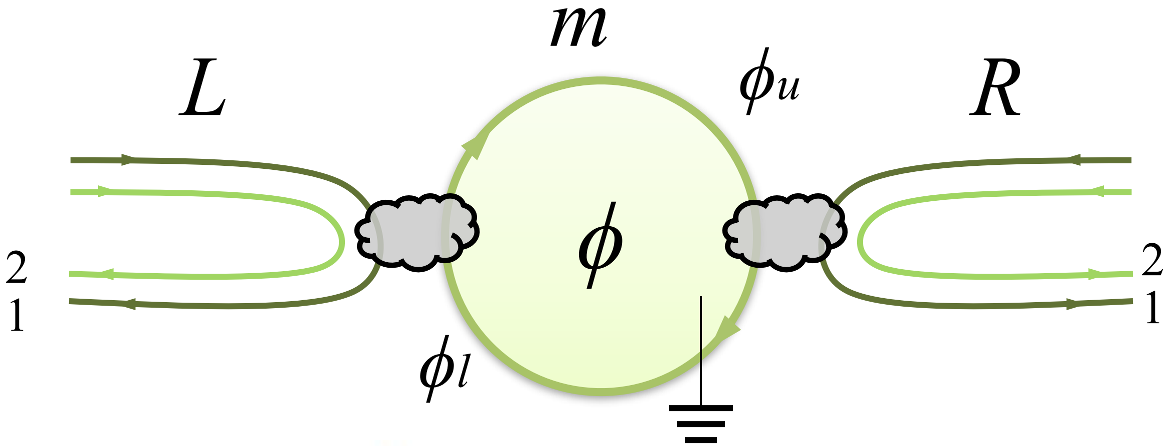
$$I_{LL} = \frac{e}{h}\int_{E\geq 0} dE\,[1 - \mathrm{Re}(\tilde{r}^*_{L1}r_{L2})][f_{Le}(E) - f_{Lh}(E)]$$

Absence of phase coherence on the average:

Left (Right) contact contributes no current to Right (Left) lead

N=1

- Cross-correlator (zero-frequency noise power)



$$S_M = \left(\begin{array}{cc|cc} \tilde{r}_{L1} & & t_{LR} & \\ & r_{L2} & & 0 \\ \hline t_{RL} & & \tilde{r}_{R1} & \\ & 0 & & r_{R2} \end{array}\right)$$

$$\tilde{r}_{L1} = \frac{r_{L1} - r_{R1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$\tilde{r}_{R1} = \frac{r_{R1} - r_{L1}e^{i\phi}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u}\frac{t_{L1}t_{R1}}{1 - r_{L1}r_{R1}e^{i\phi}}$$

$$P_{LR}(t-t') \equiv \frac{1}{2}\langle \Delta\hat{I}_L(t)\Delta\hat{I}_R(t') + \Delta\hat{I}_R(t')\Delta\hat{I}_L(t)\rangle$$

$$P_{LR}(\omega = 0) = P_{RL}(\omega = 0)$$

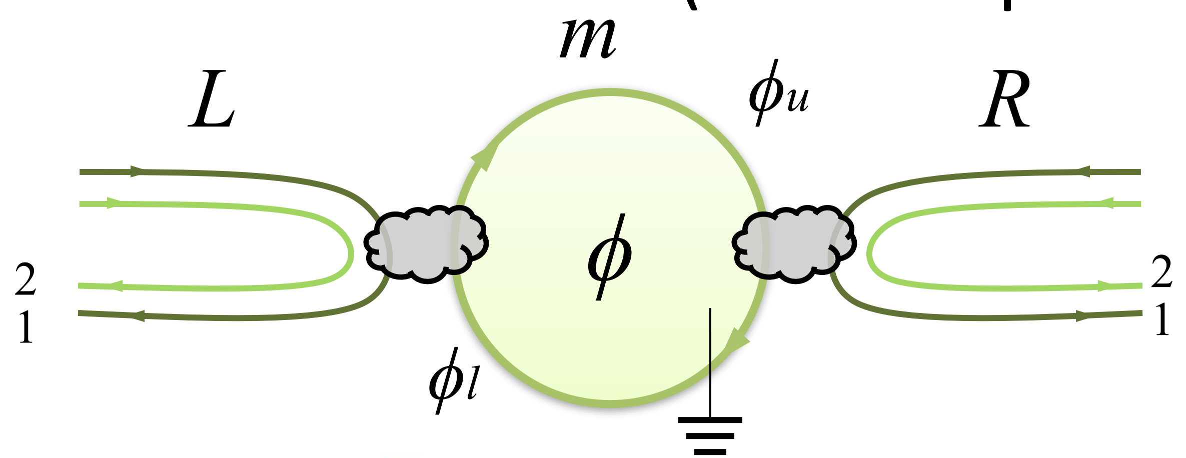
$$= \frac{e^2}{h}\int_{E\geq 0} dE\ \mathrm{Tr}\left[F(E)A_L[\mathbb{1} - F(E)]A_R\right]$$

$$= -\frac{e^2}{h}\int_{E\geq 0} dE\ [2\mathrm{Re}(r^*_{L2}t_{LR}r^*_{R2}t_{RL}/4)]$$

$$\cdot [f_{Le}(E) - f_{Lh}(E)][f_{Re}(E) - f_{Rh}(E)]$$

N=1

- Cross-correlator (zero-frequency noise power)



$$S_M = \begin{pmatrix} \tilde{r}_{L1} & & t_{LR} & \\ & r_{L2} & & 0 \\ t_{RL} & & \tilde{r}_{R1} & \\ & 0 & & r_{R2} \end{pmatrix}$$

$$\tilde{r}_{L1} = \frac{r_{L1} - r_{R1} e^{i\phi}}{1 - r_{L1} r_{R1} e^{i\phi}}$$

$$\tilde{r}_{R1} = \frac{r_{R1} - r_{L1} e^{i\phi}}{1 - r_{L1} r_{R1} e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l} \frac{t_{L1} t_{R1}}{1 - r_{L1} r_{R1} e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u} \frac{t_{L1} t_{R1}}{1 - r_{L1} r_{R1} e^{i\phi}}$$

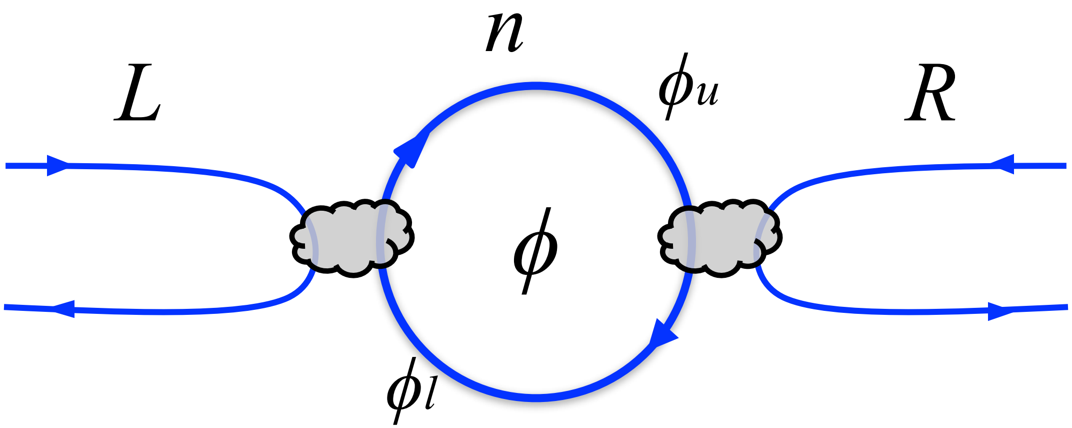
$$P_{LR}(t - t') \equiv \frac{1}{2} \langle \Delta \hat{I}_L(t) \Delta \hat{I}_R(t') + \Delta \hat{I}_R(t') \Delta \hat{I}_L(t) \rangle$$

$$P_{LR}(\omega = 0) = P_{RL}(\omega = 0)$$

$$= \frac{e^2}{h} \int_{E \geq 0} dE \, \mathrm{Tr}\, [F(E) A_L [\mathbb{1} - F(E)] A_R]$$

$$= - \frac{e^2}{h} \int_{E \geq 0} dE \, [2\mathrm{Re}(r^*_{L2} t_{LR} r^*_{R2} t_{RL} / 4)] \cdot [f_{Le}(E) - f_{Lh}(E)][f_{Re}(E) - f_{Rh}(E)]$$

N=1 "normal" Fabry-Perot interferometry



$$S = \begin{pmatrix} \tilde{r}_L & t_{LR} \\ t_{RL} & \tilde{r}_R \end{pmatrix}$$

$$\tilde{r}_L = \frac{r_L - r_R e^{i\phi}}{1 - r_L r_R e^{i\phi}}$$

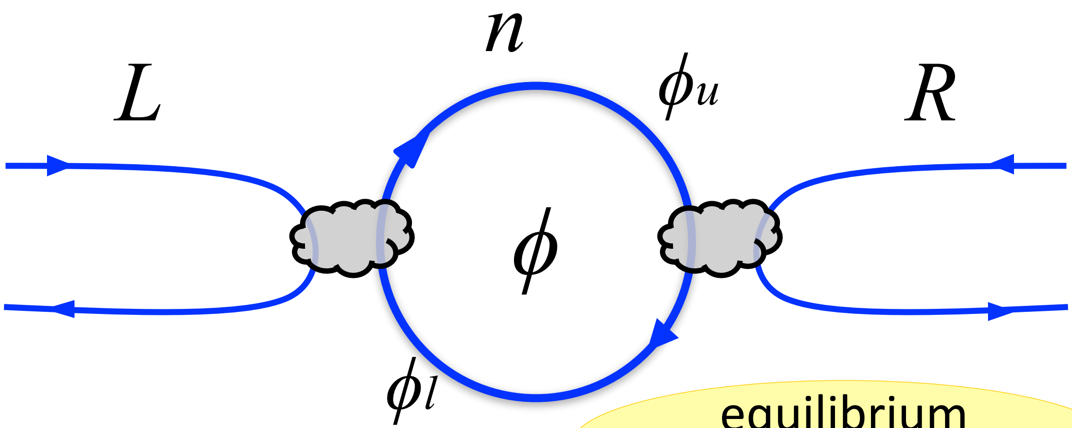
$$\tilde{r}_R = \frac{r_R - r_L e^{i\phi}}{1 - r_L r_R e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l}\frac{t_L t_R}{1 - r_L r_R e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u}\frac{t_L t_R}{1 - r_L r_R e^{i\phi}}$$

$$\begin{aligned} P_{LR}(\omega=0) &= P_{RL}(\omega=0) \\ &= -\frac{e^2}{h}\int dE\,\Big\{\left[|t_{RL}|^2 f_L(1-f_L) + |t_{LR}|^2 f_R(1-f_R)\right] \\ &\quad + \left(|r_L|^2|t_{RL}|^2 f_L^2 + |r_R|^2|t_{LR}|^2 f_R^2\right) \\ &\quad + \left[2\mathrm{Re}(r_L^* t_{LR} r_R^* t_{RL}) f_L f_R\right]\Big\} \end{aligned}$$

N=1 "normal" Fabry-Perot interferometry



$$S = \begin{pmatrix} \tilde{r}_L & t_{LR} \\ t_{RL} & \tilde{r}_R \end{pmatrix}$$

$$\tilde{r}_L = \frac{r_L - r_R e^{i\phi}}{1 - r_L r_R e^{i\phi}}$$

$$\tilde{r}_R = \frac{r_R - r_L e^{i\phi}}{1 - r_L r_R e^{i\phi}}$$

$$t_{LR} = -e^{i\phi_l} \frac{t_L t_R}{1 - r_L r_R e^{i\phi}}$$

$$t_{RL} = -e^{i\phi_u} \frac{t_L t_R}{1 - r_L r_R e^{i\phi}}$$

$$P_{LR}(\omega = 0) = P_{RL}(\omega = 0)$$
$$= -\frac{e^2}{h} \int dE \, \Big\{ \boxed{\left[|t_{RL}|^2 f_L (1 - f_L) + |t_{LR}|^2 f_R (1 - f_R)\right]}$$
$$+ \boxed{\left(|r_L|^2 |t_{RL}|^2 f_L^2 + |r_R|^2 |t_{LR}|^2 f_R^2\right)}$$
$$+ \boxed{\left[2\mathrm{Re}(r_L^* t_{LR} r_R^* t_{RL}) f_L f_R\right]} \Big\}$$

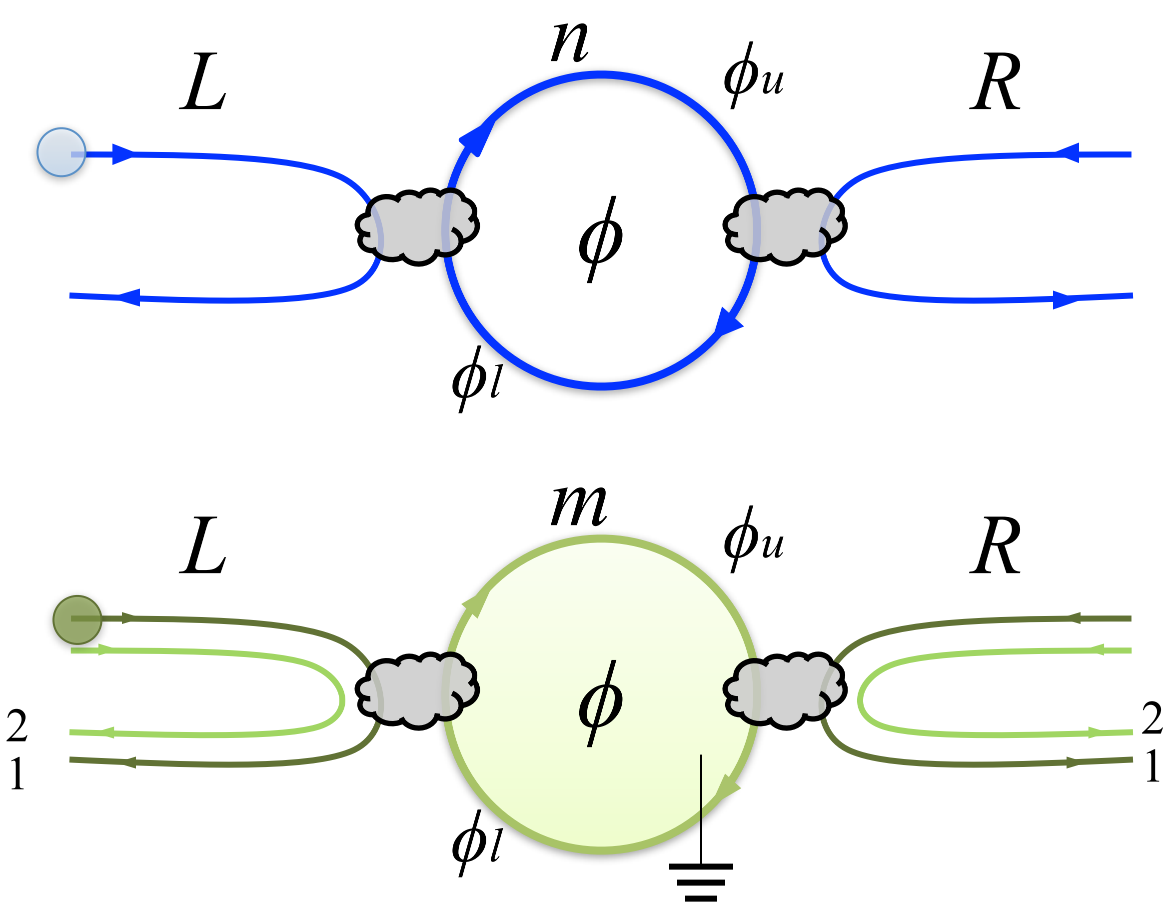
equilibrium (thermal) noise

partition noise

exchange noise!

N=1

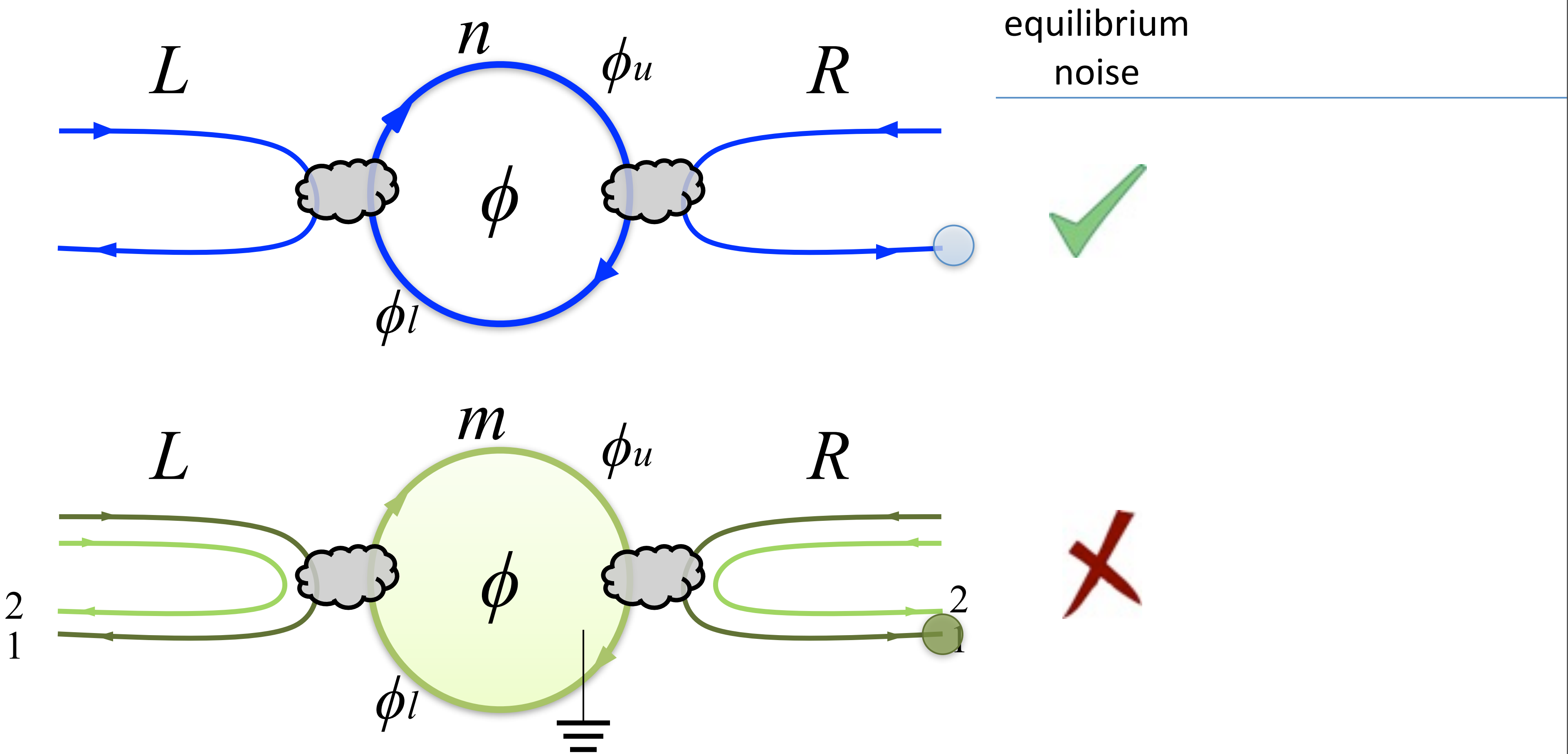
- Cross-correlator -- comparing Majorana FPI and "Normal" FPI



equilibrium noise

N=1

- Cross-correlator -- comparing Majorana FPI and “Normal” FPI



N=1

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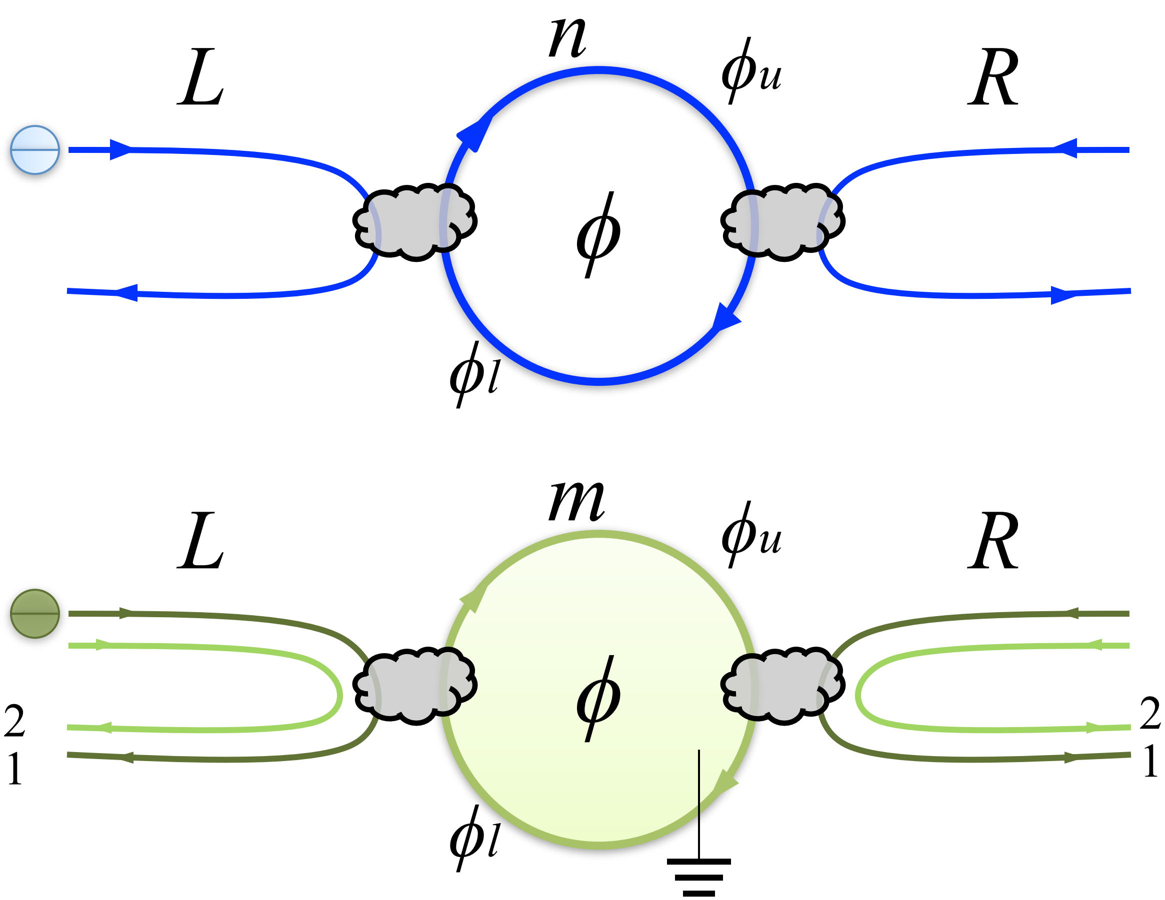
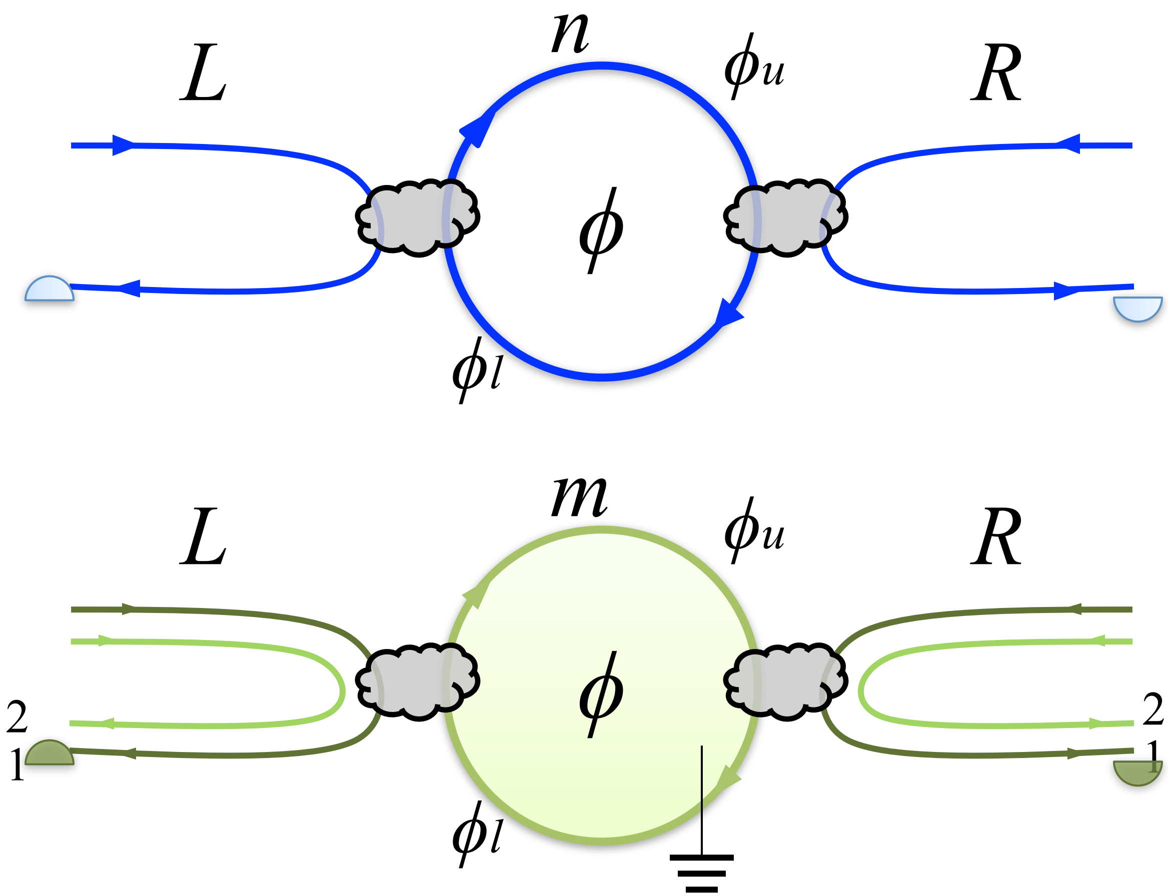


equilibrium noise	partition noise
✓	
✗	

N=1

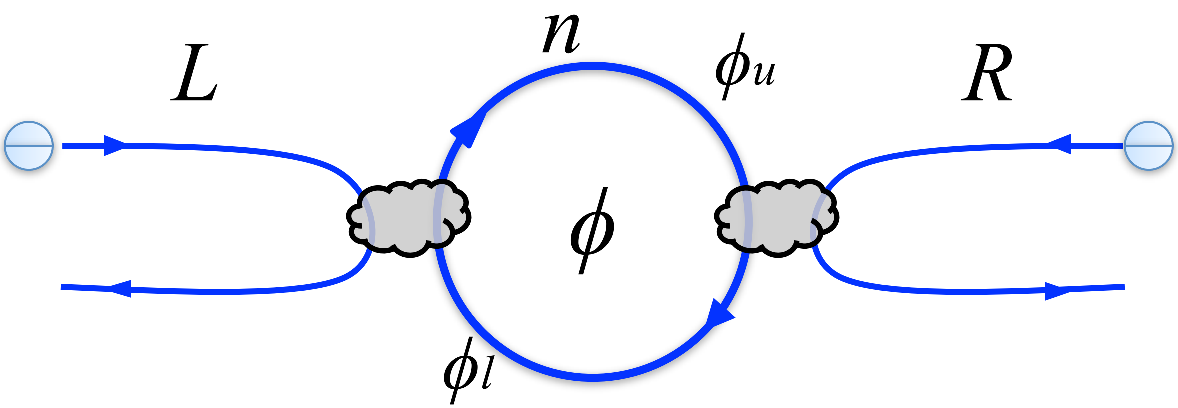
- Cross-correlator -- comparing Majorana FPI and "Normal" FPI

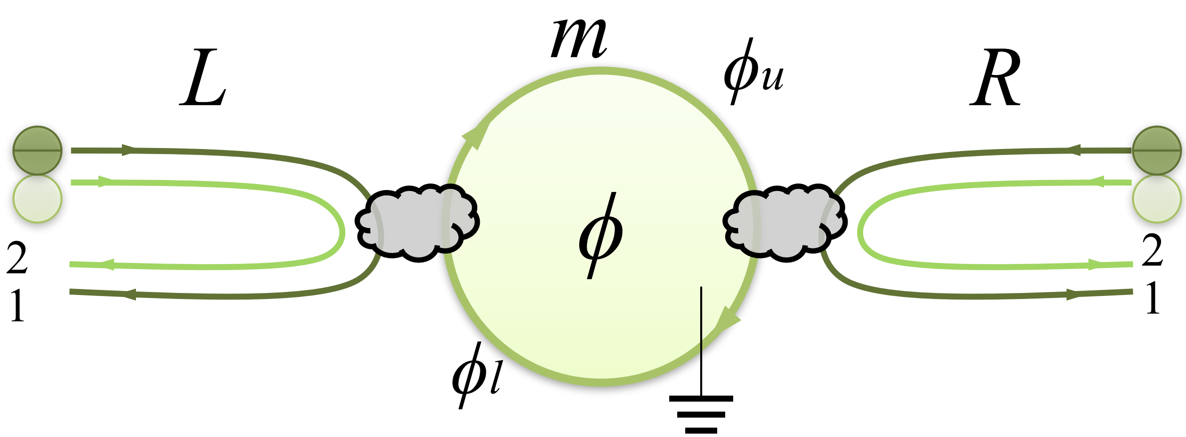


equilibrium noise	partition noise
	
	

N=1

- Cross-correlator -- comparing Majorana FPI and “Normal” FPI





equilibrium noise	partition noise	exchange noise
✓	✓	
✗	✗	

N=1

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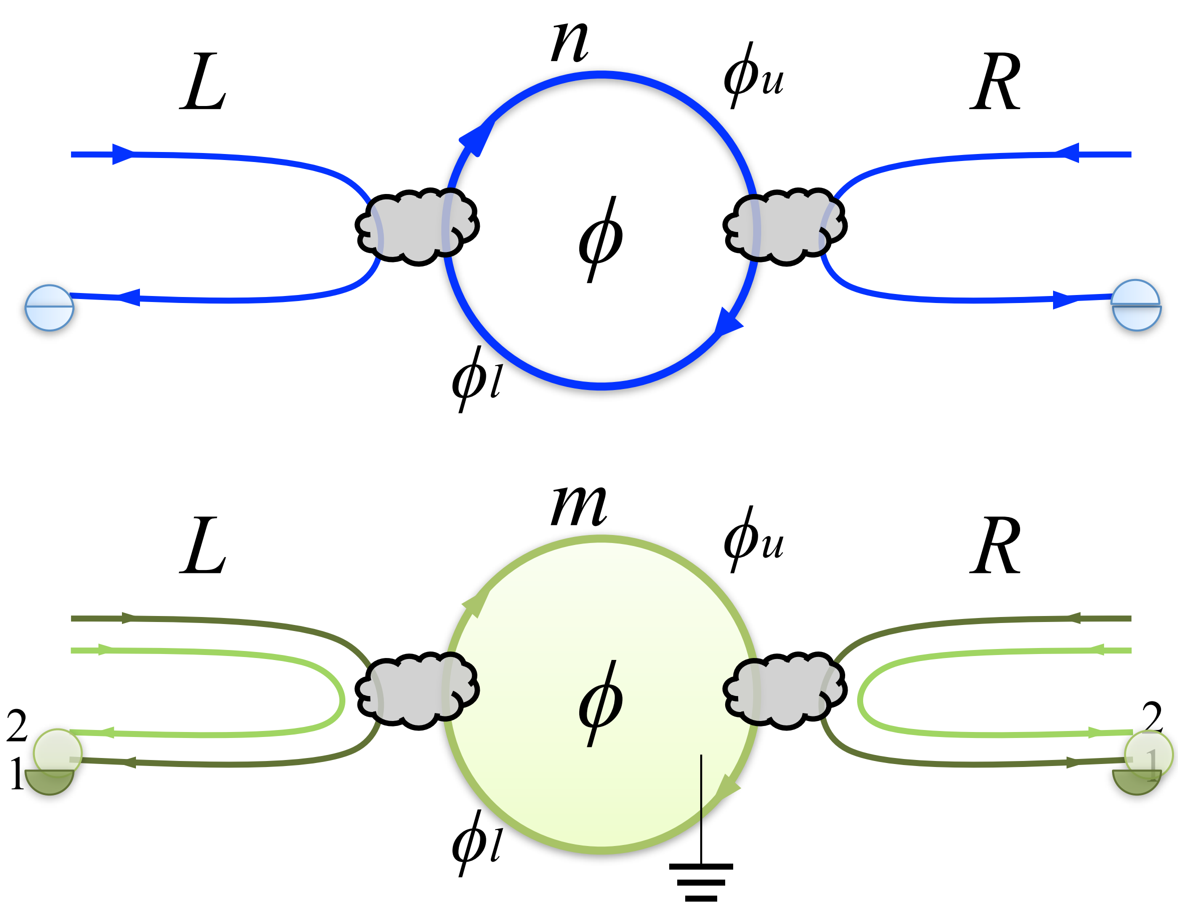
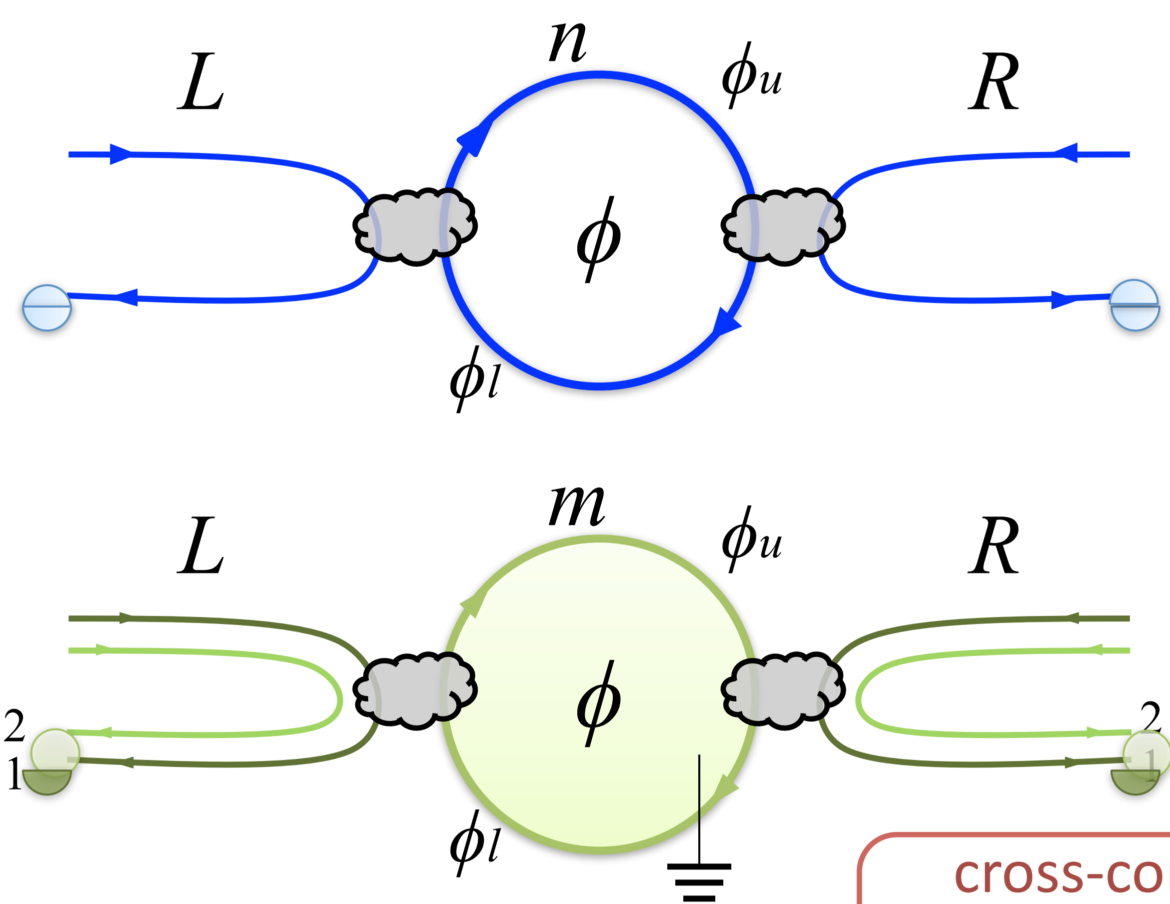


equilibrium noise	partition noise	exchange noise
✓	✓	✓
✗	✗	✓

N=1

- Cross-correlator -- comparing Majorana FPI and “Normal” FPI



equilibrium noise	partition noise	exchange noise
✓	✓	✓
✗	✗	✓

cross-correlation is given by the interference of two pairs of Majorana fermions only

N>1

- Physics w/ reduced S_M

$$\tilde{S}_M = \begin{pmatrix} R_0(\theta_1) & & & & \\ & R_0(\theta_2) & & 0 & \\ & & \ddots & & \\ & 0 & & R_0(\theta_N) & \\ & & & & +1 \end{pmatrix}$$

[B. Béri, Phys. Rev. B 79, 245315 (2009)]

N>1

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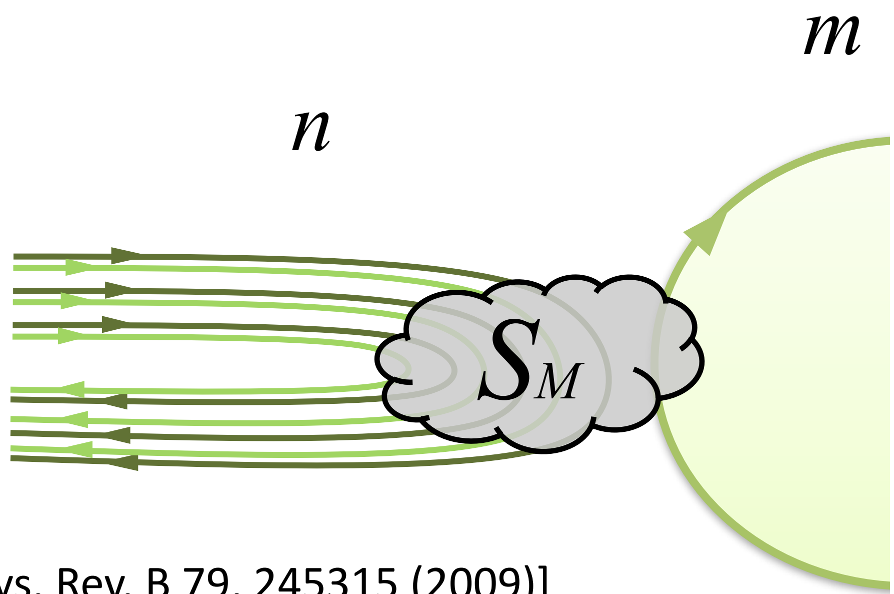

[B. Béri, Phys. Rev. B 79, 245315 (2009)]

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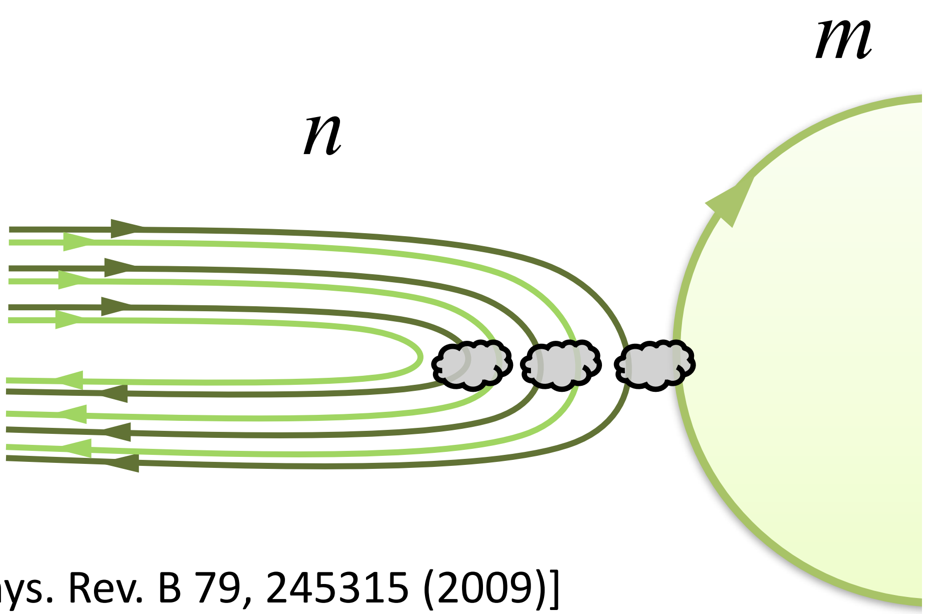
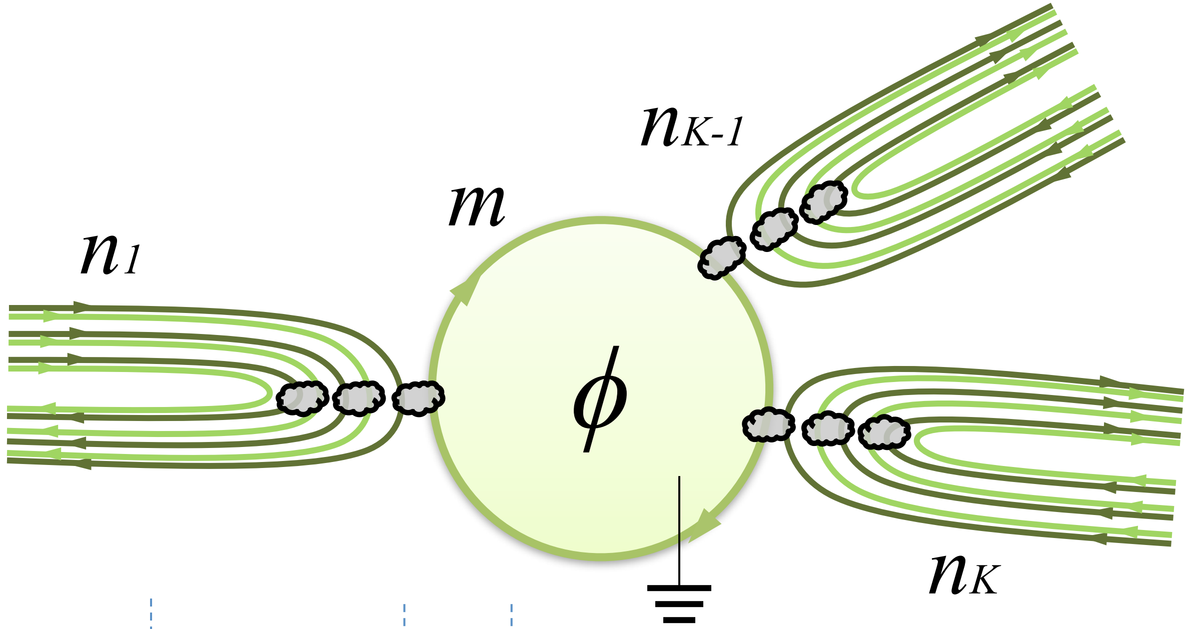


[B. Béri, Phys. Rev. B 79, 245315 (2009)]

K terminals * N_k modes



$$S_M = \left(\begin{array}{cc|cc|c|cc}
\tilde{r}_1^{(1)} & & t^{(12)} & & & t^{(1K)} & \\
 & R^{(1)} & & 0 & & & 0 \\
\hline
t^{(21)} & & \tilde{r}_1^{(2)} & & & t^{(2K)} & \\
 & 0 & & R^{(2)} & & & 0 \\
\hline
 & & & & \ddots & & \\
\hline
t^{(K1)} & & t^{(K2)} & & & \tilde{r}_1^{(K)} & \\
 & 0 & & 0 & & & R^{(K)}
\end{array}\right)$$

$$R^{(k)} \equiv \begin{pmatrix}
r_2^{(k)} & -t_2^{(k)} & & & & \\
t_2^{(k)} & r_2^{(k)} & & & & \\
 & & \ddots & & & \\
 & & & r_{N_k}^{(k)} & -t_{N_k}^{(k)} & \\
 & & & t_{N_k}^{(k)} & r_{N_k}^{(k)} & \\
 & & & & & r_{N_k+1}^{(k)}
\end{pmatrix}$$

$k = 1, 2, \ldots K$

K terminals * N_k modes

$$I_k = I_{kk} \qquad (I_{kj} = 0 \text{ if } j \neq k)$$

$$= \frac{e}{h} \int_{E \geq 0} dE \left(N_k - \sum_{i=1}^{N_k} \mathrm{Re}\big[(r_i^{(k)})^* r_{i+1}^{(k)} \big] \right) [f_{ke}(E) - f_{kh}(E)]$$

$$P_{kj}(\omega = 0) = P_{kj}(\omega = 0)$$

$$= -\frac{e^2}{h} \int_{E \geq 0} dE \left[2\mathrm{Re}(r_2^{(k)^*} t^{(kj)} r_2^{(j)^*} t^{(jk)}/4) \right] \cdot [f_{ke}(E) - f_{kh}(E)][f_{je}(E) - f_{jh}(E)]$$

K terminals * N_k modes

interference
of MF pairs

$$I_k = I_{kk} \qquad (I_{kj} = 0 \text{ if } j \neq k)$$

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exchange contribution

$$P_{kj}(\omega = 0) = P_{kj}(\omega = 0)$$

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K terminals * N_k modes

interference
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In many-mode case,
resonance at r1 will be
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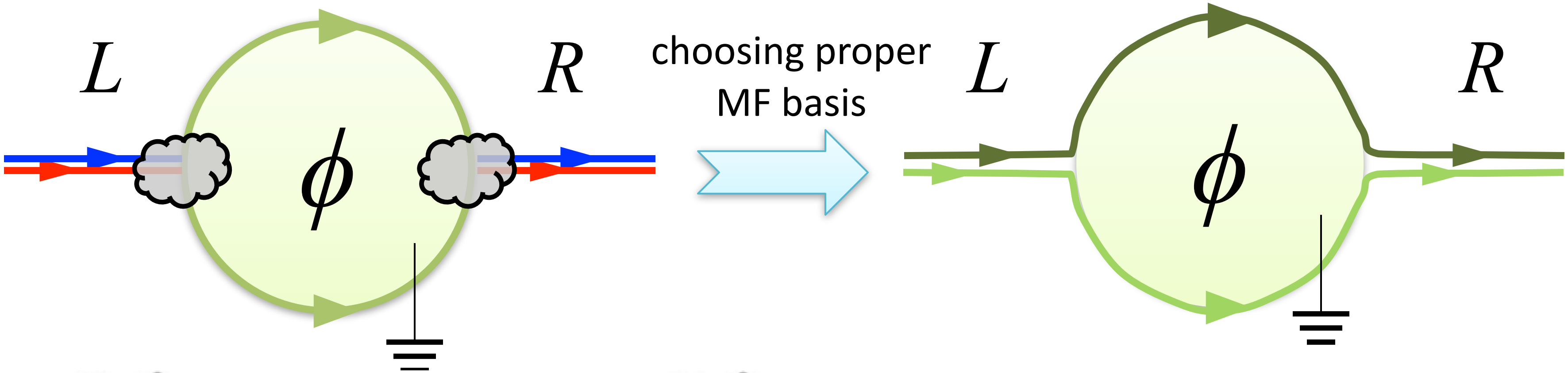
Noise keeps track of MF feature

Z_2-Mach-Zehnder Interferometer

Fu, Kane, PRL 2009; Akhmerov, Nilsson and Beenakker, PRL 2009



$$I_L = \frac{e}{h}\int_{E\geq 0} dE\, \Delta f_L, \quad I_R = -\frac{e}{h}\int_{E\geq 0} dE\, \Re(t_1 t_2^*)\, \Delta f_L,$$

$$I_S = -(I_R + I_L) = \frac{e}{h}\int_{E\geq 0} dE\, [1 - \Re(t_1 t_2^*)]\, \Delta f_L,$$

$$P_{LL} = \frac{e^2}{h}\int_{E\geq 0} dE\, \Delta\theta_L, \quad P_{RR} = \frac{e^2}{h}\int_{E\geq 0} dE\, [\Delta\theta_L + \Im(t_1 t_2^*)^2\, \Delta f_L^2]$$

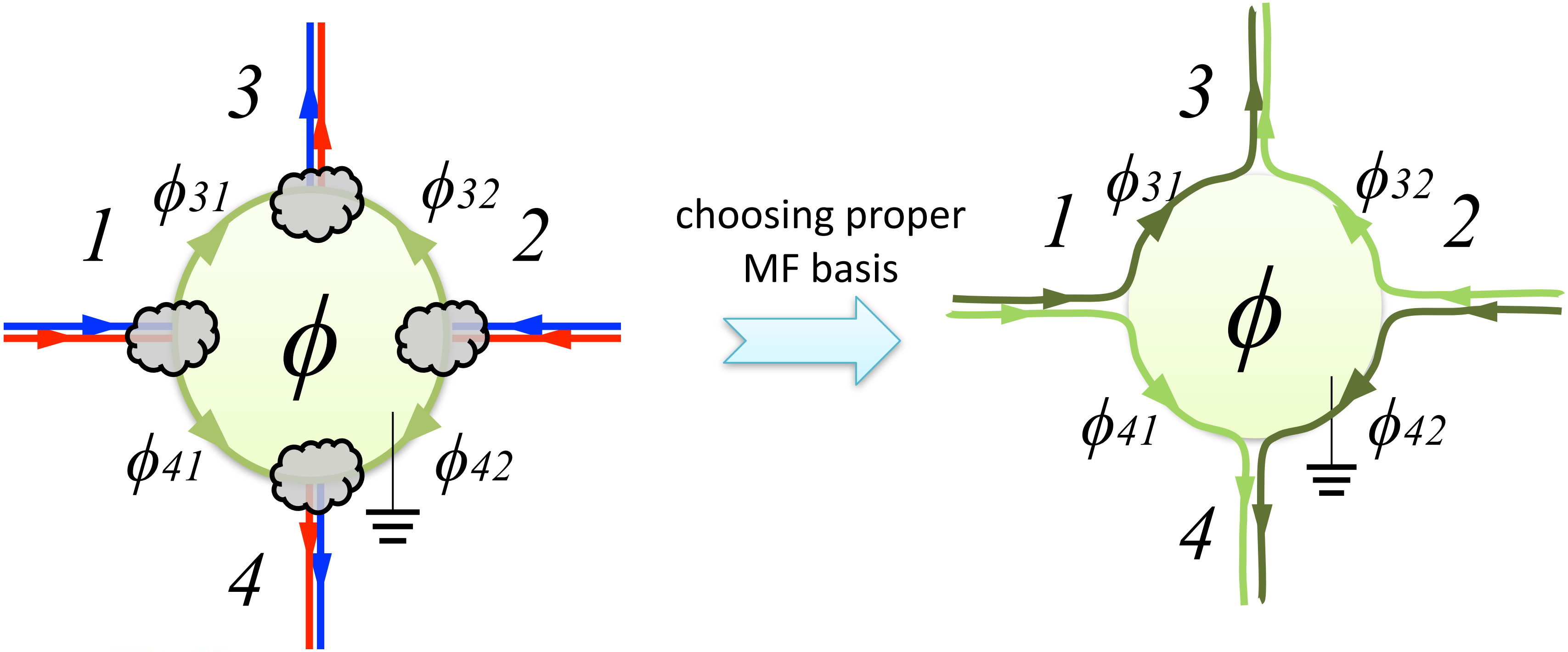
$$P_{LR} = P_{RL} = -\frac{e^2}{h}\int_{E\geq 0} dE\, \Re(t_1 t_2^*)\, \Theta_L$$

$$\Re(t_1 t_2^*) = \cos\varphi \;,\; \Im(t_1 t_2^*) = \sin\varphi$$

$Z_2 \implies \phi = 0, \pi$ odd and even number of Majoranas, thermal noise only

Z_2 – Two –particle interferometer

Strübi, Belzig, Choi and Bruder, Phys. Rev. Lett. 107, 136403 (2011)



$$I_i = \frac{e}{h}\int_{E>0} dE\,[f_{ie}(E) - f_{ih}(E)] \quad (i = 1,2) \qquad I_3 = I_4 = 0$$

$$P_{34} = P_{43} = \frac{e^2}{h}\int_{E\geq 0} dE\,(-\frac{1}{2})\,\Re(t_{31}t^*_{32}t_{42}t^*_{41})\,\Delta f_1\,\Delta f_2$$

$$P_{34}(\omega = 0) = -\frac{e^2}{h}\int_{E\geq 0} dE\,\left(\frac{1}{2}\cos\phi\right)[f_{1e}(E) - f_{1h}(E)][f_{2e}(E) - f_{2h}(E)]$$

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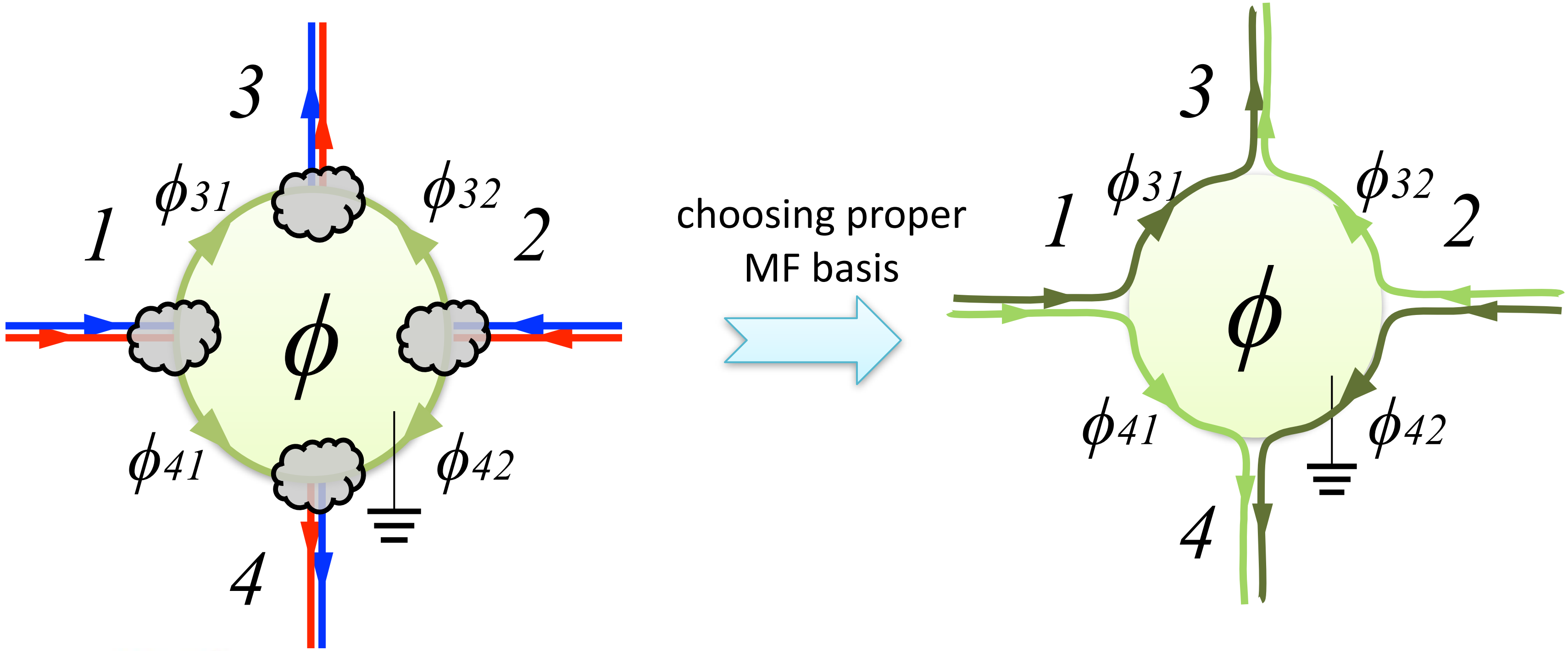

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$$P_{34}(\omega = 0) = -\frac{e^2}{h}\int_{E\geq 0} dE\,\left(\underline{\frac{1}{2}\cos\phi}\right)[f_{1e}(E) - f_{1h}(E)][f_{2e}(E) - f_{2h}(E)]$$

$$\boxed{2\mathrm{Re}(e^{i(\phi_{31}-\phi_{32}+\phi_{42}-\phi_{41})}/4)}$$

Summary

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Scattering problem in Majorana basis leads to a decompostion of the s-matrix with a minimal number of parameters directly related to the current and noise

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The current is determined by the interference of (a pair of) Majorana fermions.

Exchange interference of two pairs of Majorana fermions. Exchange is sensitive to transmission even in geometries where current is only determined by the reflection matrix.