How to ‘get’ Majorana fermions: candidate material and detection

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Stanford

KITP Program on
“Topological Insulator and Superconductor”
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Collaborators

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Works: **SBC**, H-J Zhang, X-L Qi, and S-C Zhang, PRB (2011) **84**, 060510
SBC and S-C Zhang, PRL (2009) **105**, 235301

Acknowledgments

H Weng, R Martin, P A Lee, S Kivelson, S Raghu, K Kono, W Halperin, M Stone

Outline

- Majorana fermions and Topological superconductor (TSC)
- What system to find Majorana fermions?
  - ▶ TSC from half-metal / s-wave SC proximity effect in 2D
  - ▶ 'right' half-metal band structure - Ex: NaCoO2
- How do we see Majorana nature in 3D TSC ^{3}He-B?
  - ▶ Majorana surface state
  - ▶ spin relaxation $1/T_1 \sim \sin^2 \vartheta \Rightarrow$ Majorana detection

What is Majorana fermions?

(Stern, Ann Phys '08; Wilczek, Nat Phys '09; Franz, Physics '10)

E Majorana (1906 - 1938?)

TEORIA SIMMETRICA DELL' ELETTRONE E DEL POSITRONE

Nota di ETTORE MAJORANA

Il Nouvo Cimento **14** 171 (1937)

- A fermion identical to its own anti-particle
  (a *real* fermion, w/ degrees freedom halved)

- Is the neutrino Majorana?
  $\Rightarrow$ Don't know! (neutrinoless double β decay?)

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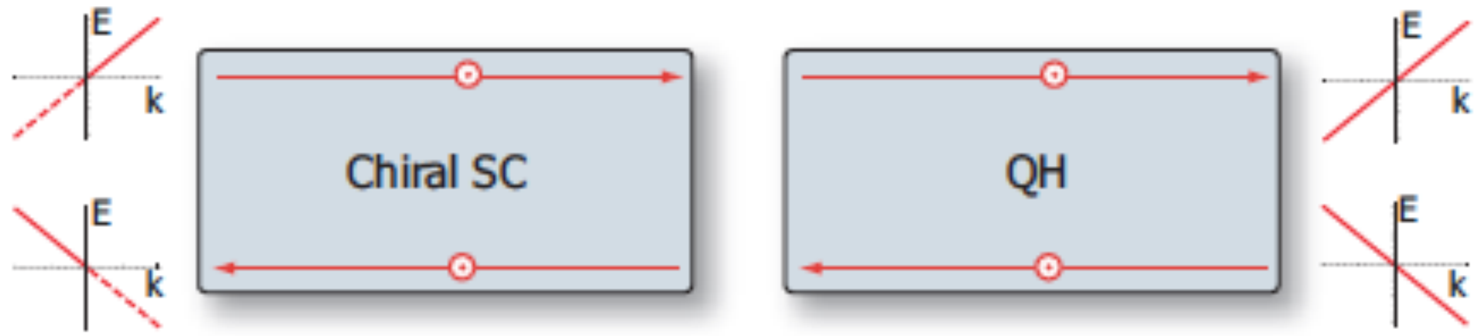
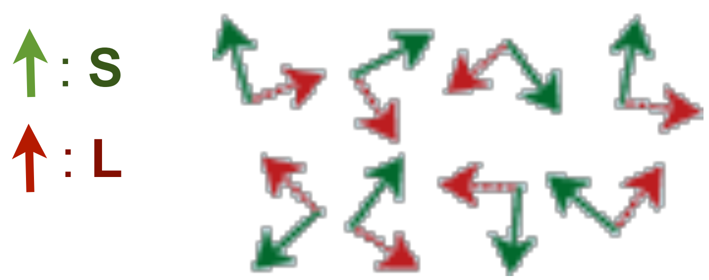
- We should see $\psi=\psi^\dagger$ in topological superconductor!

Why in Topological Superconductor (TSC)?



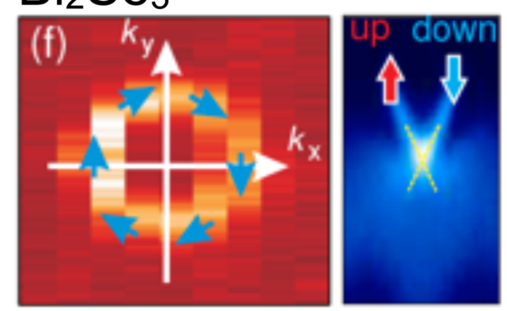
(Qi, Hughes, Raghu and Zhang PRL ’09)

^{3}He superfluid Phase B



(Osheroff, Richardson and Lee, PRL ’72)

Bi_2Se_3



(Hsieh et al, Nature ’09)

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  & gapless boundary state topologically protected

  ⇐ analogous to topological insulator (TI)

  but unlike conventional s-wave or cuprate superconductor

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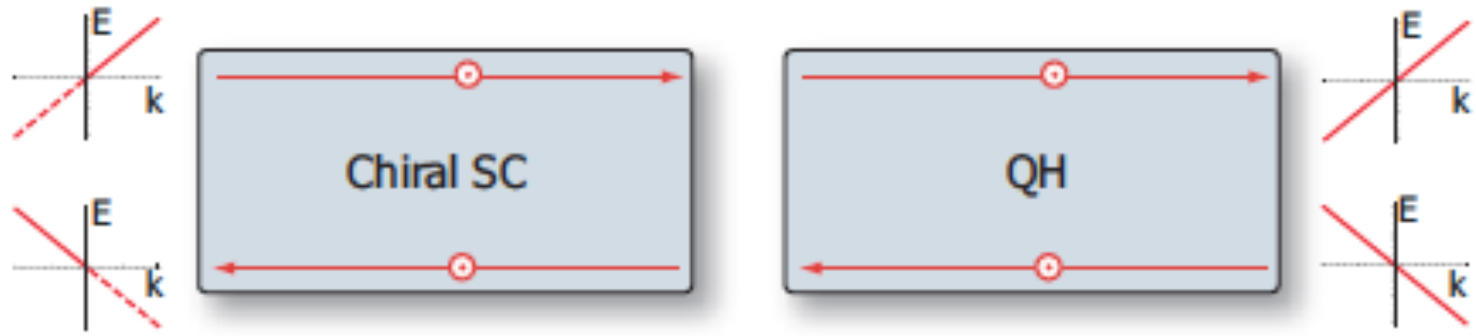

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- Particle-hole redundancy in TSC $\Rightarrow$

  TI boundary : free electrons = TSC boundary : Majorana fermions

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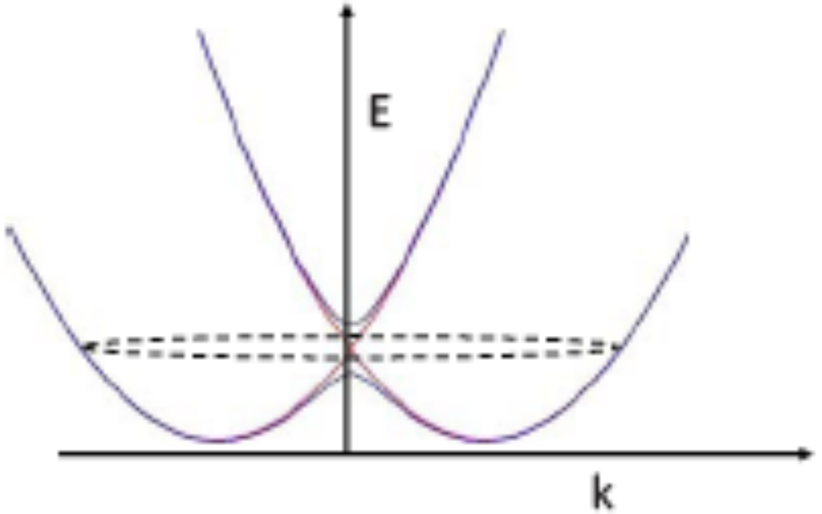

(Sau, Tewari, Lutchyn, Stanescu, Das Sarma PRB '10)

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  $\Rightarrow$ (SC proximity effect) +

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- Need to induce SC on material satisfying
  - ▶ no E_F fine-tuning
  - ▶ better SC contact
  - ▶ 2D description justified

What is half-metal?

- $\Rightarrow$ metal for majority spin
  insulator for minority spin
  (ex: CrO_2, NiMnSb, $La_{1-x}Ca_xMnO_2$...) (de Groot et al. PRL '83)

- Sufficient to have no spin degeneracy at Fermi level
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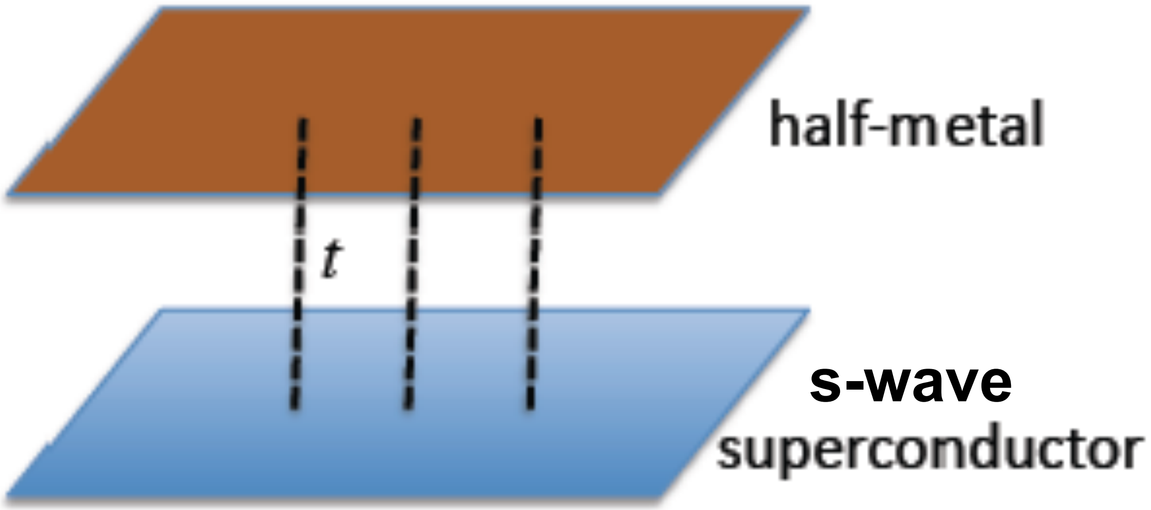
- Sufficient to have no spin degeneracy at Fermi level
  $\Rightarrow$ OK to have spin-orbit coupling

- Metallic $\Rightarrow$ good SC contact
  Minority spin gap $\Rightarrow$ no E_F fine-tuning

Half-metal / SC proximity effect

(**SBC**, H-J Zhang, X-L Qi, and S-C Zhang PRB ’11)



$$\mathcal{H} = \mathcal{H}_{SC} + \mathcal{H}_{HM} + \mathcal{H}_t$$

$$\mathcal{H}_{SC} = \sum_{\mathbf{k},\sigma}(\epsilon_{\mathbf{k}} - \mu)c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}}(\Delta_{\mathbf{k}}c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow} + \mathrm{h.c.})$$

$$\mathcal{H}_{HM} = \sum_{\mathbf{k}}(\epsilon^{HM}_{\mathbf{k}} - \mu^{HM})f^\dagger_{\mathbf{k}\uparrow}f_{\mathbf{k}\uparrow},$$

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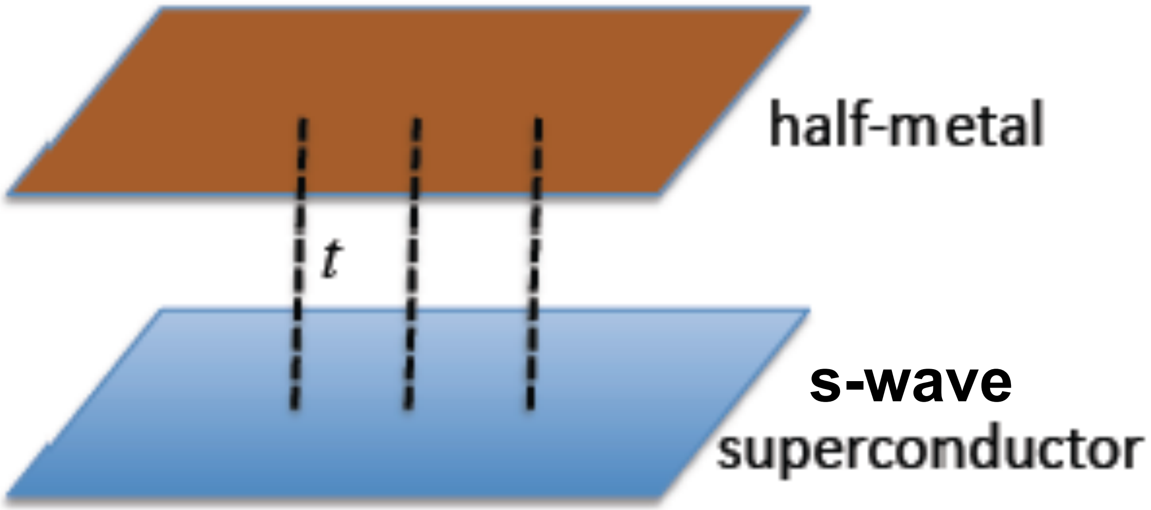

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- Half-metal and s-wave superconductor coupled by hopping across the interface

- Half-metal electrons can pair up only by hopping to SC & one of the pair need to flip spin!

$$\langle f_{-\mathbf{k}\uparrow}f_{\mathbf{k}\uparrow}\rangle \propto (t_{\mathbf{k},\uparrow\uparrow}t_{-\mathbf{k},\uparrow\downarrow} - t_{-\mathbf{k},\uparrow\uparrow}t_{\mathbf{k},\uparrow\downarrow})\Delta_{\mathbf{k}}$$

p+ip pairing from interface spin-orbit coupling

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- At interface, Rashba spin-orbit coupling due to broken inversion:

$$\mathcal{H}_{SOC} = \hbar\alpha(\boldsymbol{\sigma} \times \mathbf{k}) \cdot \hat{\mathbf{n}}\delta(\hat{\mathbf{n}} \cdot \mathbf{r})$$

(Gorkov and Rashba, PRL ’01; Edelstein, PRB ’03)

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- Hopping across interface should have the same symmetry

$$\begin{aligned} t_{\mathbf{k},\uparrow\uparrow} &= t_0 & \Leftarrow \quad & t_0 \sum_i (f^\dagger_{i\uparrow} c_{i\uparrow} + \text{h.c.}) \\ t_{\mathbf{k},\uparrow\downarrow} &= t_{SOC}(i \sin k_x a + \sin k_y a) & \Leftarrow \quad & t_{SOC} \sum_{\langle ij \rangle} [\exp(i\theta_{ij}) f^\dagger_{i\uparrow} c_{j\downarrow} + \text{h.c.}] \end{aligned}$$

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$$\begin{aligned}\langle f_{-\mathbf{k}\uparrow} f_{\mathbf{k}\uparrow}\rangle &\propto (t_{\mathbf{k},\uparrow\uparrow} t_{-\mathbf{k},\uparrow\downarrow} - t_{-\mathbf{k},\uparrow\uparrow} t_{\mathbf{k},\uparrow\downarrow})\Delta \\ &= -2it_0 t_{SOC}(\sin k_x a - i \sin k_y a)\Delta\end{aligned}$$

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- $\Rightarrow$ p+ip pairing from hopping
  $\Rightarrow$ HM domain boundary becomes p+ip / p-ip domain boundary

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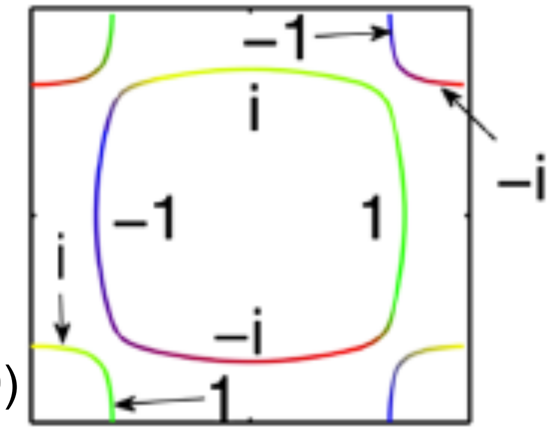
‘Right’ band structure required for half-metal

- Spinless p+ip SC may NOT be TSC!
  ex: one electron pocket around Γ
  + one hole pocket around M

(Qi, Hughes, Zhang PRB ’10; Raghu, Kapitulnik, Kivelson PRL ’10)



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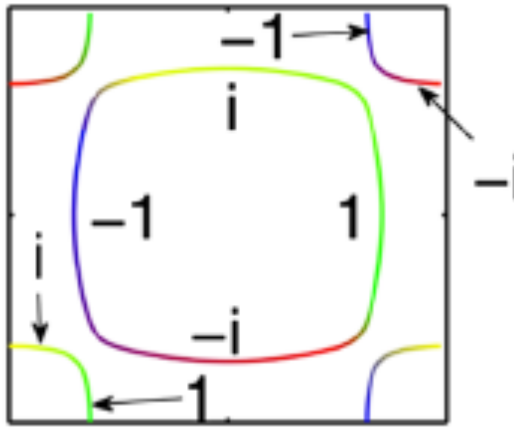


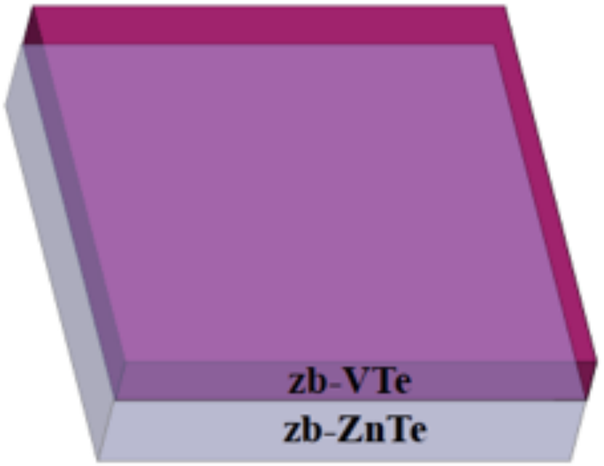
- By itself, half-metal CANNOT have a single Fermi pocket!
  ⇐ integer no. of e^- per unit cell without spin degeneracy

- For a single Fermi pocket, need fractional no. of e^- per unit cell through charge transfer mechanism
  ⇒ surface half-metal with insulating bulk
  ⇒ possible to realize without any gating!

‘Right’ half-metal: heterostructure

(**SBC**, H-J Zhang, X-L Qi, and S-C Zhang PRB ’11)

- Two atomic layers of half-metal zb-VTe as surface of insulating zb-ZnTe



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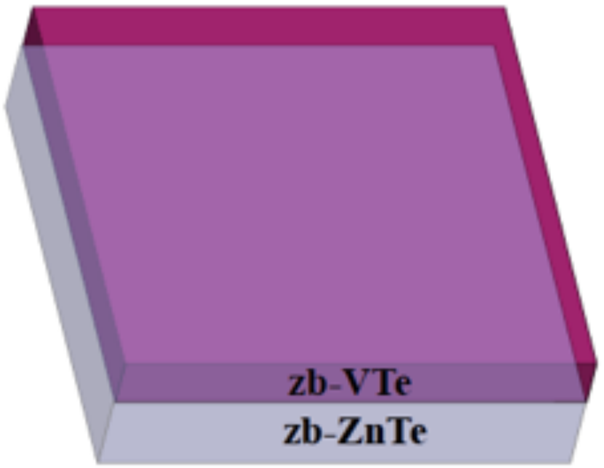
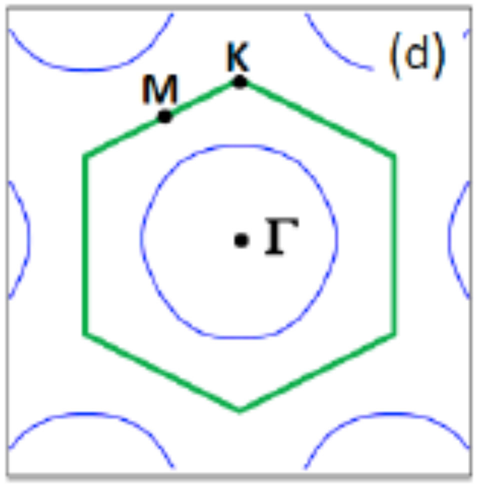
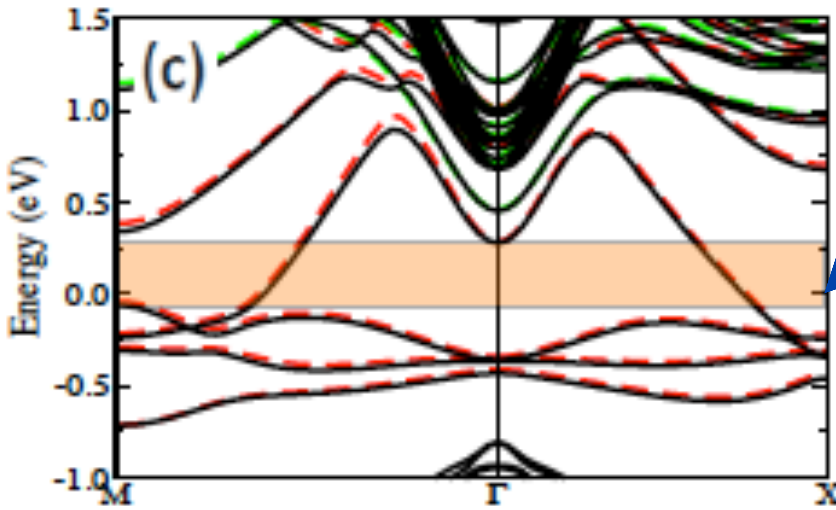




- Charge transfer to ZnTe due to covalent bonding gives 1/2 electrons per unit cell in VTe (‘polarization catastrophe’ as in oxide interface)
  $\Longrightarrow$ a single Fermi pocket for the VTe surface band

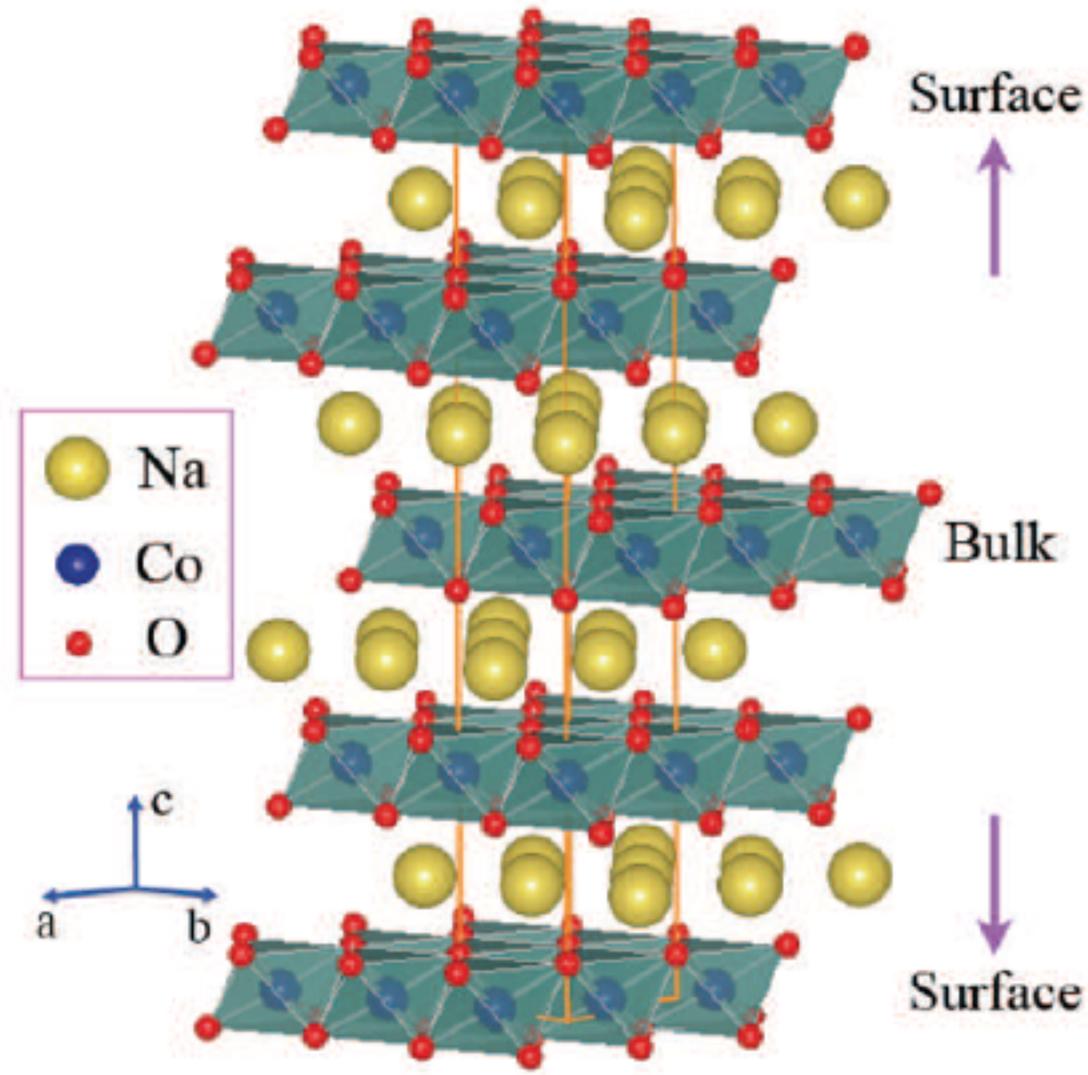
- No fine tuning of the Fermi level required
  $\Longleftarrow$ the single Fermi pocket maintained even when Fermi level changes by ~300meV!

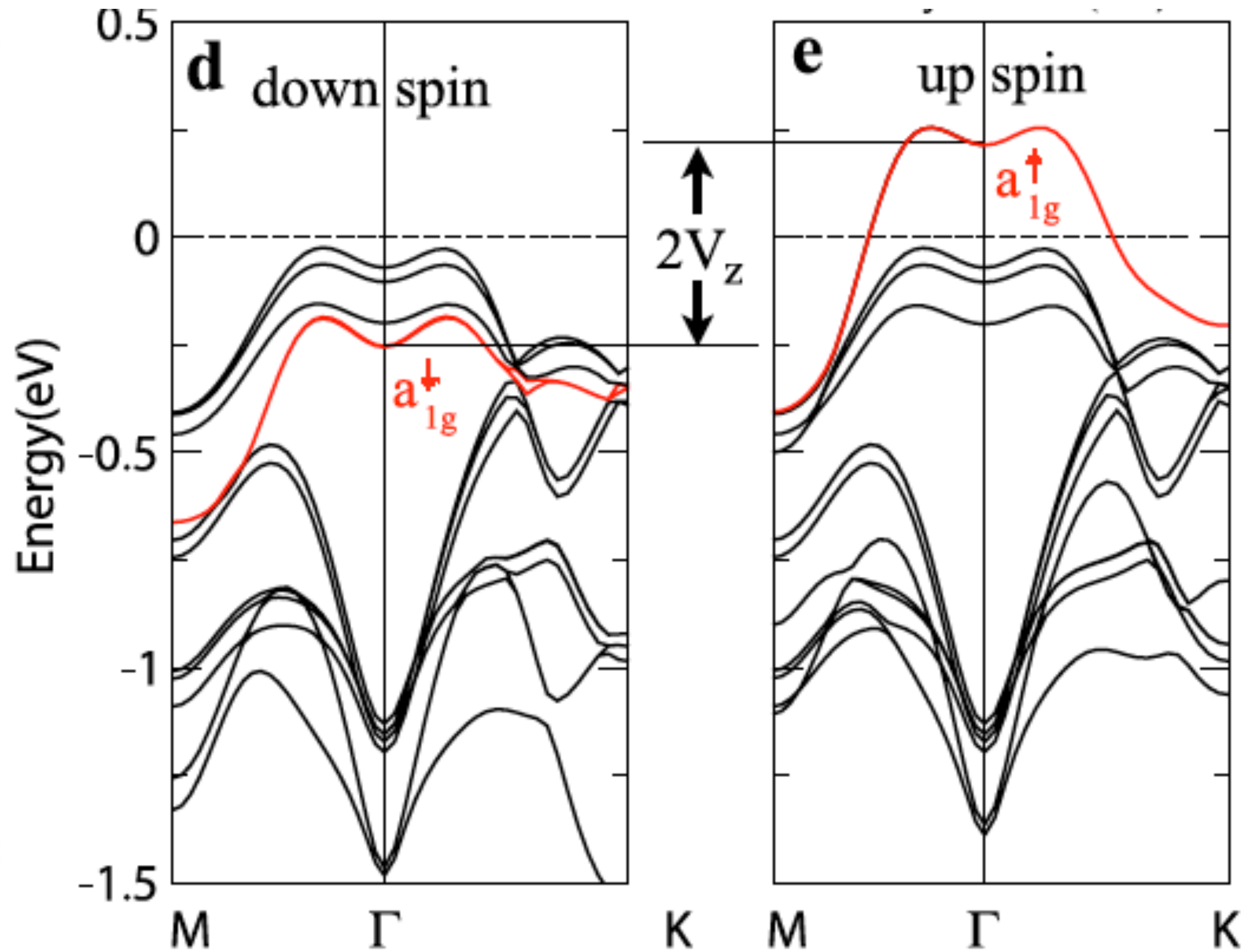


single Fermi pockets for Fermi level in this range

‘Right’ 2D half-metal can be stochiometric!

(H Weng et al. PRB ’11)





$\Rightarrow$ $NaCoO_2$ can have a surface half-metal with a single Fermi pocket!

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Some relevant 3He-B characteristic

- Known to be topological paired superfluid
  - fully gapped bulk (Balian and Werthamer PR ’63)
  - gapless surface (Buchholtz and Zwicknagl PRB ’81; Salomaa and Volovik PRB ’88)

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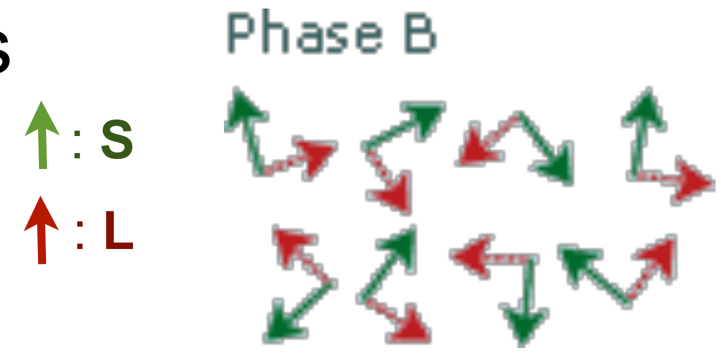
  $\Longrightarrow$ spontaneous $SO(3)_L \times SO(3)_L \rightarrow SO(3)_{L+S}$ symmetry breaking
  locks spin & orbital states of Cooper pairs



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$$
\begin{aligned}
|\Psi\rangle &= |S_x = +1, L_x = -1\rangle - |S_x = 0, L_x = 0\rangle + |S_x = -1, L_x = +1\rangle \\
&= (-p_y + ip_z)|\rightarrow\rightarrow\rangle - (-p_x)|\rightarrow\leftarrow + \leftarrow\rightarrow\rangle + (p_y + ip_z)|\leftarrow\leftarrow\rangle
\end{aligned}
$$

- Momentum dependence of the pairing gap has the same symmetry as the spin-orbit coupling in Bi_2Se_3

$$
\hat{\Delta}(\mathbf{p}) = \begin{bmatrix} \Delta_{\rightarrow\rightarrow} & \Delta_{\rightarrow\leftarrow} \\ \Delta_{\leftarrow\rightarrow} & \Delta_{\leftarrow\leftarrow} \end{bmatrix} \propto \begin{bmatrix} -p_y + ip_z & p_x \\ p_x & p_y + ip_z \end{bmatrix}
$$

3He-B compared to 3D TI

$$\hat{\mathcal{H}} = \begin{bmatrix} \epsilon_{\mathbf{k}} - E_F & 0 & -\frac{\Delta}{k_F}(\hat{p}_y - i\hat{p}_z) & \frac{\Delta}{k_F}\hat{p}_x \\ 0 & \epsilon_{\mathbf{k}} - E_F & \frac{\Delta}{k_F}\hat{p}_x & \frac{\Delta}{k_F}(\hat{p}_y + i\hat{p}_z) \\ -\frac{\Delta}{k_F}(\hat{p}_y + i\hat{p}_z) & \frac{\Delta}{k_F}\hat{p}_x & -\epsilon_{\mathbf{k}} + E_F & 0 \\ \frac{\Delta}{k_F}\hat{p}_x & \frac{\Delta}{k_F}(\hat{p}_y - i\hat{p}_z) & 0 & -\epsilon_{\mathbf{k}} + E_F \end{bmatrix}$$

- The same 1st quantized Hamiltonian, yet different bases!

  Bi_2Se_3: $(\psi_{A\rightarrow}, \psi_{A\leftarrow}, \psi_{B\rightarrow}, \psi_{B\leftarrow})$ $\Leftrightarrow$ ^{3}He-B: $(\psi_{\rightarrow}, \psi_{\leftarrow}, \psi^{\dagger}_{\rightarrow}, \psi^{\dagger}_{\leftarrow})$

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- The same spectra, including linearly dispersing surface

  $\Longrightarrow$ How is Majorana (one half degrees of freedom) different?

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- The same effective surface Hamiltonian: $\hat{\mathcal{H}}_{surf} = v\hat{\mathbf{z}} \cdot (\mathbf{p} \times \sigma)$

  $\Longrightarrow$ What does spin mean in ^{3}He-B?

Bi2Se3 Dirac surface state

$$\begin{bmatrix} \hat{\psi}_{A\rightarrow}(\mathbf{r}) \\ \hat{\psi}_{A\leftarrow}(\mathbf{r}) \\ \hat{\psi}_{B\rightarrow}(\mathbf{r}) \\ \hat{\psi}_{B\leftarrow}(\mathbf{r}) \end{bmatrix} = \sum_{\mathbf{k}} (\hat{\gamma}_{\mathbf{k}c} e^{i\mathbf{k}_\parallel \cdot \mathbf{r}_\parallel} + \hat{\gamma}_{-\mathbf{k}v} e^{-i\mathbf{k}_\parallel \cdot \mathbf{r}_\parallel}) \begin{bmatrix} \cos\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \cos\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\mathbf{k}}+\pi/2}{2} \end{bmatrix}$$

$$\times \quad u_{\mathbf{k}} \sin(k_\perp z) e^{\Delta z/\hbar v_F} + (\text{gapped modes})$$

3He-B Majorana surface state

(**SBC** and Zhang PRL '09)

$$\begin{bmatrix} \hat{\psi}_{\rightarrow}(\mathbf{r}) \\ \hat{\psi}_{\leftarrow}(\mathbf{r}) \\ \hat{\psi}^{\dagger}_{\rightarrow}(\mathbf{r}) \\ \hat{\psi}^{\dagger}_{\leftarrow}(\mathbf{r}) \end{bmatrix} = \sum_{\mathbf{k}} (\hat{\gamma}_{\mathbf{k}} e^{i\mathbf{k}_{\parallel}\cdot\mathbf{r}_{\parallel}} + \hat{\gamma}^{\dagger}_{\mathbf{k}} e^{-i\mathbf{k}_{\parallel}\cdot\mathbf{r}_{\parallel}}) \begin{bmatrix} \cos\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \cos\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\mathbf{k}}+\pi/2}{2} \end{bmatrix}$$

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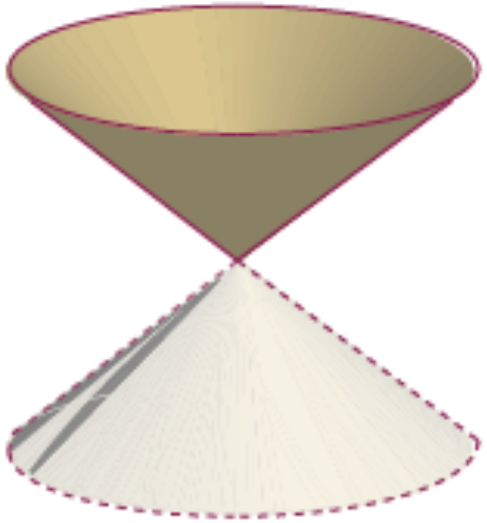

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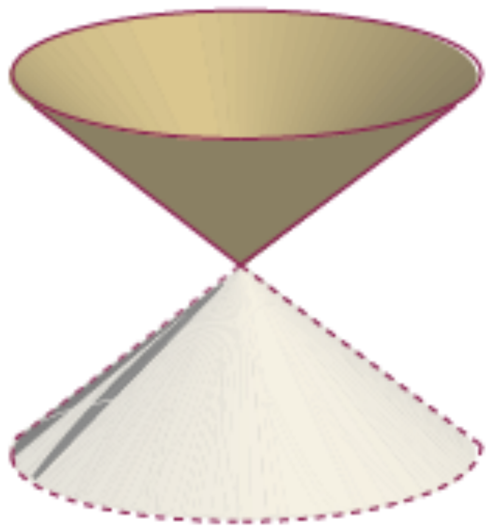

- momentum eigenstates NOT particle number or spin eigenstates!

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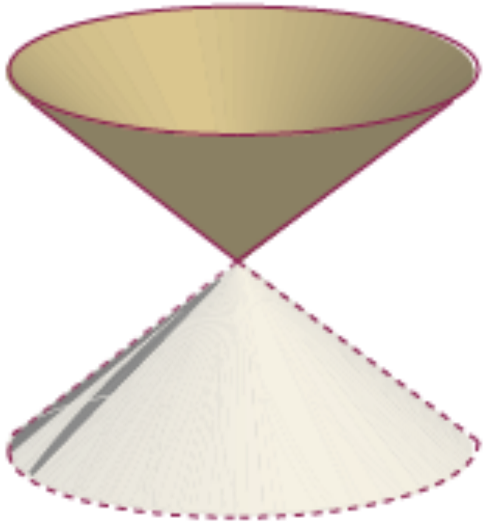

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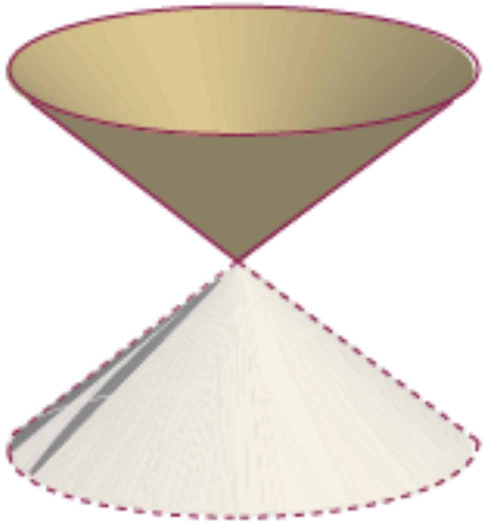

- momentum eigenstates NOT particle number or spin eigenstates!
- For the surface state, $\psi_{\sigma}(\mathbf{r}) = \psi^{\dagger}_{\sigma}(\mathbf{r})$
- Surface excitation change

  $I_z = i\psi_{\rightarrow}\psi_{\leftarrow}$

  Surface excitation do not change

  $\rho = \sum_{\sigma} \psi^{\dagger}_{\sigma}\psi_{\sigma}$ $\quad I_x = \psi^{\dagger}_{\rightarrow}\psi_{\rightarrow} - \psi^{\dagger}_{\leftarrow}\psi_{\leftarrow}$ $\quad I_y = \psi^{\dagger}_{\rightarrow}\psi_{\leftarrow} + \psi^{\dagger}_{\leftarrow}\psi_{\rightarrow}$

Constraint on dissipation channels

(**SBC** and Zhang PRL ’09)

- ^{3}He-B surface spin fluctuation Ising

Constraint on dissipation channels

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- ^{3}He-B surface spin fluctuation Ising

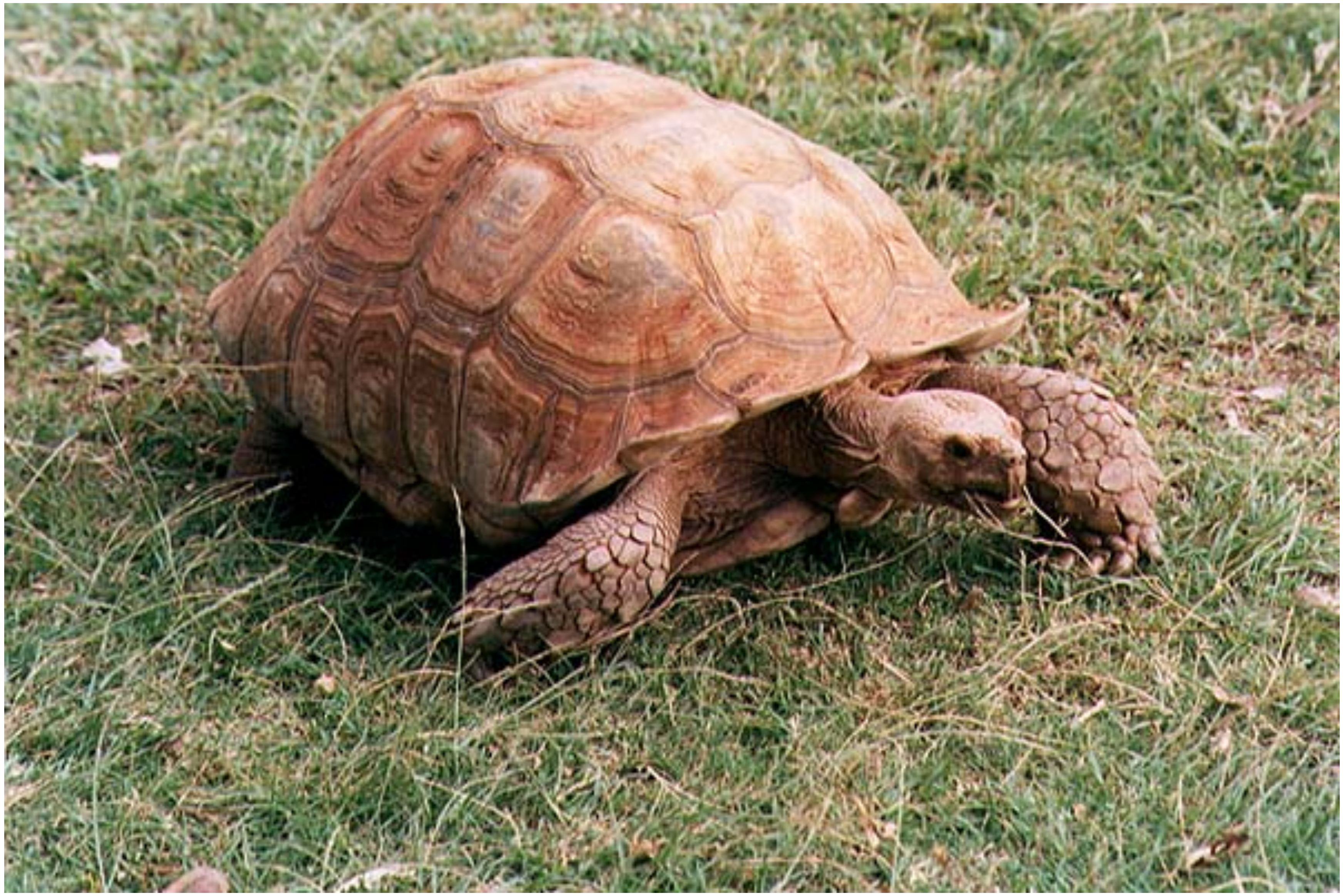
- Some dissipation channel BLOCKED!
  $\Longrightarrow$ from fluctuation-dissipation theorem
  Im $\chi_{zz} \neq 0$ but Im χ_{xx} = Im χ_{yy} = 0

- Resonant spin spectroscopy into ^{3}He-B surface state
  $\Longrightarrow$ Anisotropy due Majorana nature

Outline

- Majorana fermions and Topological superconductor (TSC)
- What system to find Majorana fermions?
  - ▶ TSC from half-metal / s-wave SC proximity effect in 2D
  - ▶ ‘right’ half-metal band structure - Ex: NaCoO2
- How do we see Majorana nature in 3D TSC ^{3}He-B?
  - ▶ Majorana surface state
  - ▶ spin relaxation $1/T_1 \sim \sin^2 \vartheta \Rightarrow$ Majorana detection

^{3}He-B surface state is slow!



v ~ 5cm/s (Tc = 1mK)

$\Rightarrow$ difficult to do inelastic scattering experiment!

Anisotropic spin relaxation

(**SBC** and Zhang PRL '09)

- Relaxation rate for external spin near surface:

$$\frac{1}{T_1} = \frac{k_B T}{\hbar} \quad \sum_q \quad \int dz dz' A_+(\mathbf{q}, z) A_-(-\mathbf{q}, z') \times \quad \frac{\mathrm{Im}\chi^{zz}(q, \omega_L; z, z')}{\omega_L}$$

$\Rightarrow A_\pm = A_{x'} \pm iA_{y'}$ defined from Zeeman field direction

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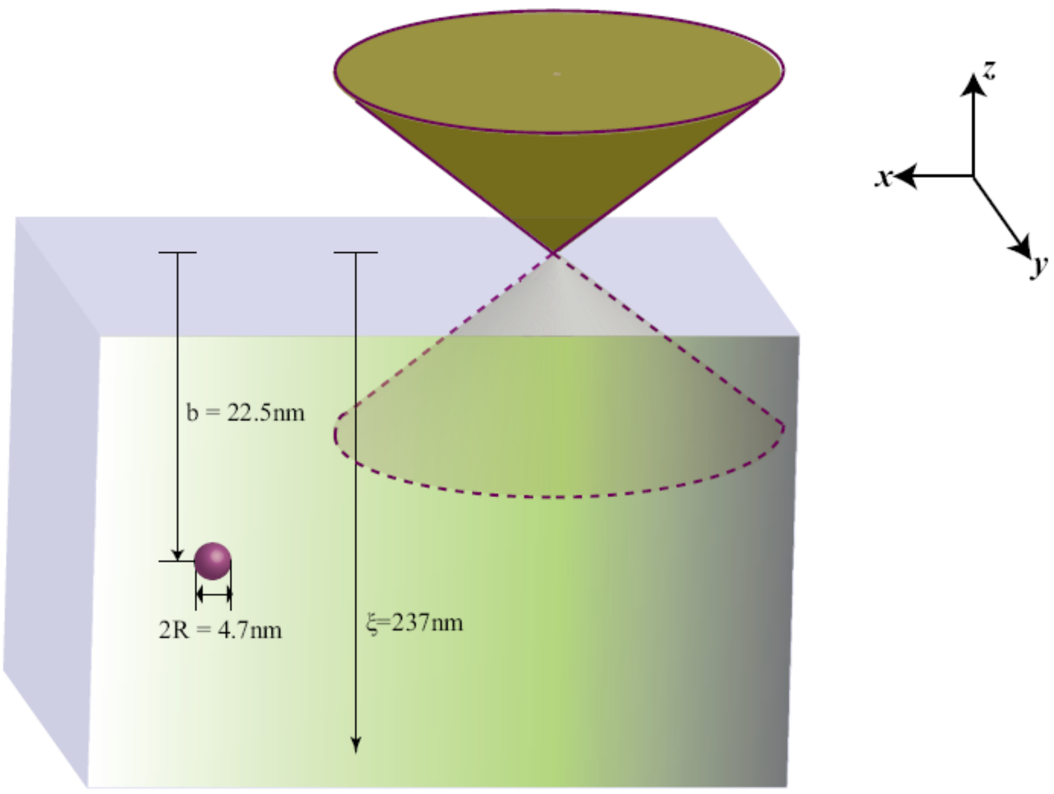
- Contact interaction $H_{\text{int}} = -A S_z I_z$ gives
  $A_+ = A \sin \vartheta$

- Relaxation rate $1/T_1 \sim \sin^2 \vartheta$
  $\Rightarrow$ $1/T_1 = 0$ for field along surface normal!
  $\Rightarrow$ also true for dipole-dipole interaction between external spin and surface state

Experimental proposal for 3He-B

(**SBC** and Zhang PRL '09)



- Surface repel bubble containing external spin as dielectricity of 3He

- Depth control through perpendicular electric field (E_z = 150V/cm)

(H Ikegami, K Kono et al. under preparation)

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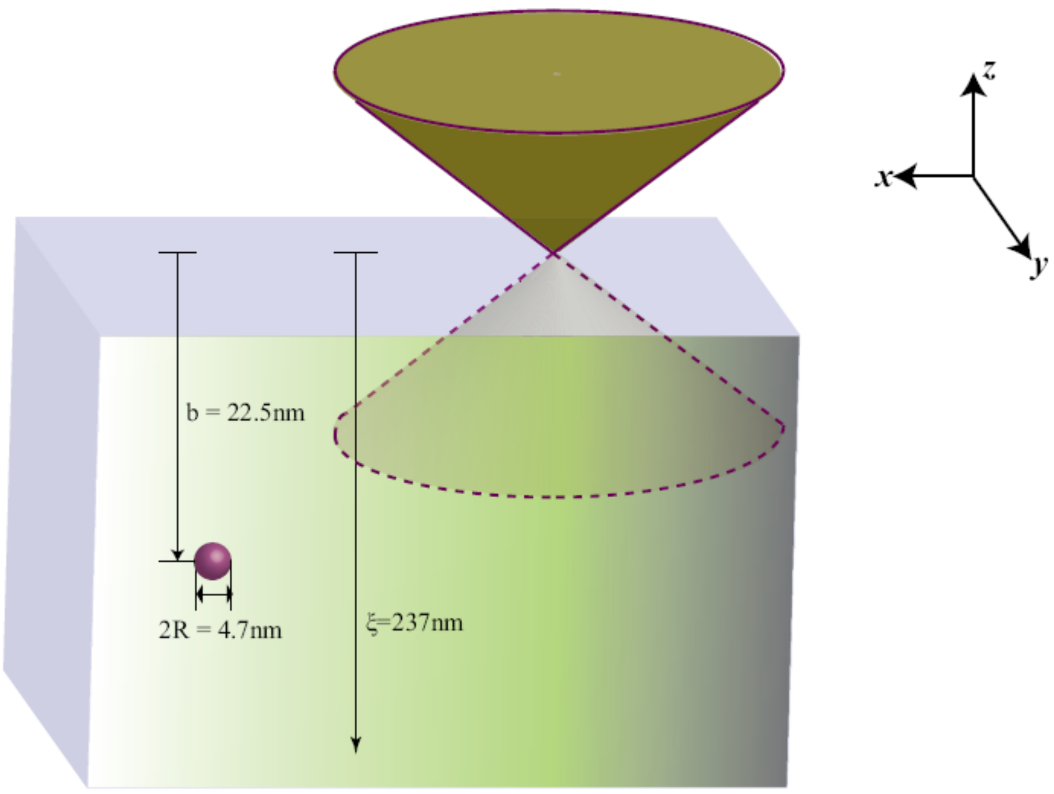

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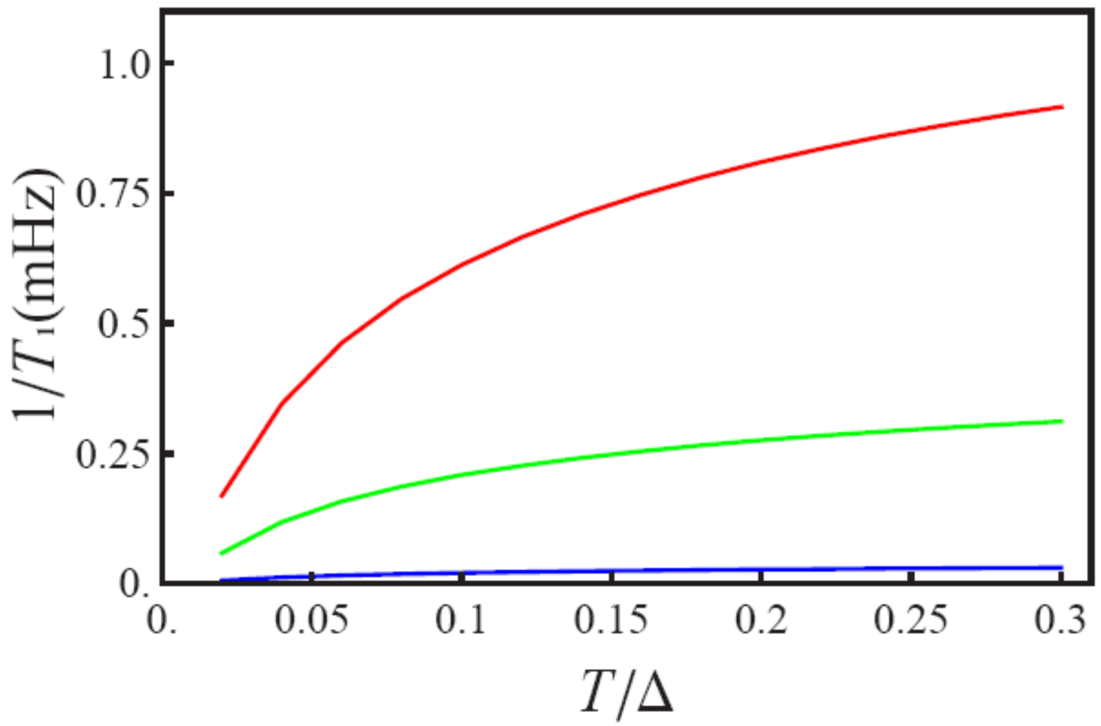
- With Zeeman field along surface normal, surface state do NOT enhance spin relaxation rate
- Relaxation rate enhancement with Zeeman field along surface:

(b = 22.5, 87.4, 225.2nm)



Conclusion

- TSC has fully gapped bulk but protected gapless edge state similar to TI
- TSC edge state has one half the degree of freedom as TI edge state and therefore is Majorana
- Spinful Majorana boundary has Ising spin fluctuation because its degree of freedom is one-half
- Spinless p+ip can be obtained from half-metal in proximity to s-wave superconductor

Dependence on Zeeman field direction

SBC and Zhang PRL '09

- Contact interaction H_{int} = - $A S_z\, I_z$ gives
  $A_+ = A \sin \vartheta$

- Relaxation rate $1/T_1 \sim \sin^2 \vartheta$
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- Qualitatively holds even with dipole-dipole interaction:

$$\begin{aligned} H^{(D)} &= -\frac{\mu_0}{4\pi}\frac{r^2 \boldsymbol{\mu}_e \cdot \boldsymbol{\mu}_{\mathrm{He}} - 3(\boldsymbol{\mu}_e \cdot \mathbf{r})(\boldsymbol{\mu}_{\mathrm{He}} \cdot \mathbf{r})}{r^5} \\ &= \frac{\mu_0 g \mu_B \gamma \hbar}{4\pi (r_{\parallel}^2 + z^2)^{5/2}} I_z [(r_{\parallel}^2 - 2z^2) S_z - 3xz S_x - 3yz S_y]. \end{aligned}$$

$$A_+^{(D)}(\mathbf{q}, z) = \frac{\mu_0 g \mu_B \gamma \hbar}{2} e^{-q|z|} [(i q_x \cos\theta - q_y)\mathrm{sgn}(z) - q \sin\theta].$$

For electrons below surface, cancelation between ^{3}He atoms above (z>0) and below (z<0)