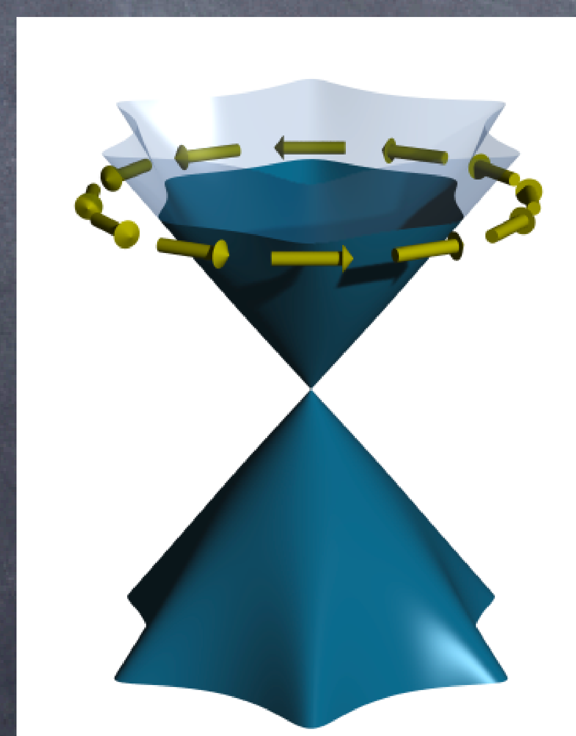
Majorana Fermions in proximity-coupled TI nanowiress

M. Franz and A. Cook
University of British Columbia



NATURE PHYSICS | VOL 5 | SEPTEMBER 2009 | www.nature.com/naturephysics

perspective

Majorana returns

Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

Enrico Fermi had to cajole his friend Ettore Majorana into publishing his big idea: a modification of the Dirac equation that would have profound ramifications for particle physics. Shortly afterwards, in 1938, Majorana mysteriously disappeared, and for 70 years his modified equation remained a rather obscure footnote in theoretical physics (Box 1). Now suddenly, it seems, Majorana's concept is ubiquitous, and his equation is central to recent work not only in neutrino physics, supersymmetry and dark matter, but also on some exotic states of ordinary matter.

Indeed, when, in 1928, Paul Dirac discovered[1] the theoretical framework for describing spin-½ particles, it seemed that complex numbers were unavoidable (Box 2). Dirac's original equation contained both real and imaginary numbers, and therefore it can only pertain to complex fields. For Dirac, who was concerned with describing electrons, this feature posed no problem, and even came to seem an advantage because it 'explained' why positrons, the antiparticles of electrons, exist.

Enter Ettore Majorana. In his 1937 paper[2], Majorana posed, and answered, the

number of electrons minus the number of antielectrons, plus the number of electron neutrinos minus the number of antielectron neutrinos is a constant (call it L_e). These laws lead to many successful selection rules. For example, the particles (muon neutrinos, ν_μ) emitted in positive pion (π) decay, $\pi^+ \rightarrow \mu^+ + \nu_\mu$, will induce neutron-to-proton conversion $\nu_\mu + n \rightarrow \mu^- + p$, but not proton-to-neutron conversion $\nu_\mu + p \rightarrow \mu^+ + n$; the particles (muon antineutrinos, $\bar{\nu}_\mu$) emitted in the negative pion decay $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ obey the opposite pattern. Indeed, it was through studies of this kind that the existence of different

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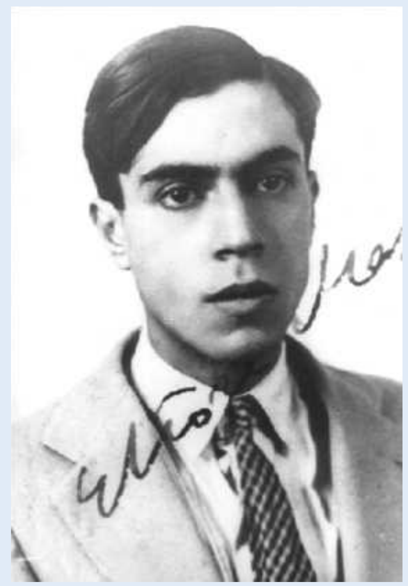
Box 1 | The romance of Ettore Majorana

"There are many categories of scientists: people of second and third rank, who do their best, but do not go very far; there are also people of first-class rank, who make great discoveries, fundamental to the development of science. But then there are the geniuses, like Galileo and Newton. Well Ettore Majorana was one of them." Enrico Fermi, not known for flightiness or overstatement, is the source of these much-quoted lines.

The bare facts of Majorana's life are briefly told. Born in Catania, Italy, on 5 August 1906, into an accomplished family, he rose rapidly through the academic ranks, became a friend and scientific collaborator of Fermi, Werner Heisenberg and other luminaries, and produced a stream of high-quality papers. Then, beginning in 1933, things started to go terribly wrong. He complained of gastritis, became reclusive, with no official position, and published nothing for several years. In 1937, he allowed Fermi to write-up and submit, under his (Majorana's) name, his last and most profound paper — the point of departure of this article — containing results he had derived some years before. At Fermi's urging, Majorana applied for professorships and was awarded the Chair in Theoretical Physics at Naples,



which he took up in January 1938. Two months later, he embarked on a mysterious trip to Palermo, arrived, then boarded a ship straight back to Naples and disappeared without a trace.

Majorana published only nine papers in his lifetime, none very lengthy. They are collected, with commentaries, all in both Italian and English versions, in a slim volume[30]. Each is a substantial contribution to quantum physics. At least two are masterpieces: the last, as mentioned, and another on the quantum theory of spins in magnetic fields, which anticipates the later brilliant development of molecular-beam and magnetic resonance techniques.

In recent years, a small industry has developed, bringing Majorana's unpublished notebooks into print (see for example ref. 31). They are impressive documents, full of original calculations and expositions covering a wide range of physical problems. They leave an overwhelming impression of gathering strength; physics might have advanced more rapidly on several fronts had Majorana pulled this material together and shared it with the world.

How did he vanish? There are two leading theories. According to one, he retired to a monastery, to escape a spiritual crisis and accept the embrace of his deep Catholic faith (not unlike another tortured scientific genius, Blaise Pascal). According to another, he jumped overboard, an act of suicide recalling the alienated supermind of fiction, Odd John[32]. Fermi's appreciation had a wistful conclusion, which is less well known: "Majorana had greater gifts than anyone else in the world. Unfortunately he lacked one quality which other men generally have: plain common sense."

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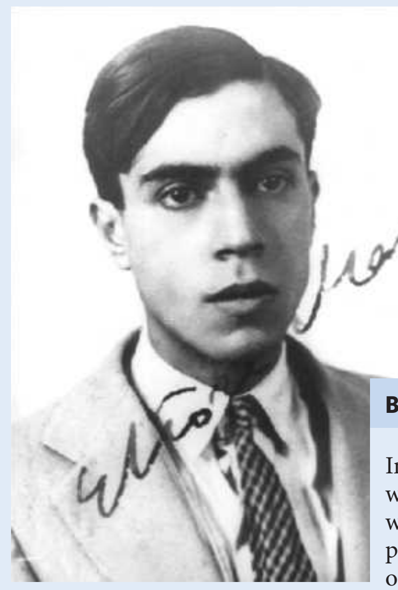

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Box 2 | The Majorana equation

In 1928, Dirac proposed his relativistic wave equation for electrons[33]. This was a watershed event in theoretical physics, leading to a new understanding of spin, predicting the existence of antimatter, and impelling — for its adequate interpretation — the creation of quantum field theory. It also inaugurated a new method in theoretical physics, emphasizing mathematical aesthetics as a source of inspiration. Majorana's most influential work is especially poetic, in that it applies Dirac's method to Dirac's equation itself, to distill from it an equation both elegant and new. For many years, Majorana's idea seemed to be an ingenious but unfulfilled speculation. Recently, however, it has come into its own, and now occupies a central place in several of the most vibrant frontiers of modern physics.

Dirac's equation connects the four components of a field ψ. In modern (covariant) notation it reads

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

The γ matrices are required to obey the rules of Clifford algebra, that is

$$\{\gamma^{\mu}\gamma^{\nu}\} \equiv \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}$$

where $\eta^{\mu\nu}$ is the metric tensor of flat space. Spelling it out, we have

$$(\gamma^0)^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = -(\gamma^3)^2 = 1$$
$$\gamma^j\gamma^k = -\gamma^k\gamma^j \text{ for } i \neq j$$

(in which I have adopted units such that $\hbar = c = 1$). Furthermore, we require that γ^0 be Hermitian, and the remaining marices anti-Hermitian. These conditions ensure that the equation properly describes the wavefunction of a spin-½ particle with mass m.

Dirac found a suitable set of 4 × 4 γ matrices, whose entries contain both real and imaginary numbers. For the equation to make sense, ψ must then be a complex field. Dirac and most other physicists regarded this consequence as a good feature, because electrons are electrically charged, and the description of charged particles requires complex fields, even at the level of the Schrödinger equation. This is also true in the language of quantum field theory. In quantum field theory, if a given field φ creates the particle A (and destroys its antiparticle $\bar{A}$), the complex conjugate φ^* will create $\bar{A}$ and destroy A. Particles that are their own antiparticles must be associated with fields obeying $\varphi = \varphi^*$, that is, real fields. Because electrons and positrons are distinct, the associated fields ψ and ψ^* and must therefore be different; this feature appeared naturally in Dirac's equation.

Majorana inquired whether it might be possible for a spin-½ particle to be its own antiparticle, by attempting to find the equation that such an object would satisfy. To get an equation of Dirac's type (that is, suitable for spin-½) but capable of governing a real field, requires γ matrices that satisfy the Clifford algebra and are purely imaginary. Majorana found such matrices. Written as tensor products of the usual Pauli matrices σ, they take the form:

$$\tilde{\gamma}^0 = \sigma_2 \otimes \sigma_1$$
$$\tilde{\gamma}^1 = i\sigma_1 \otimes 1$$
$$\tilde{\gamma}^2 = i\sigma_3 \otimes 1$$
$$\tilde{\gamma}^3 = i\sigma_2 \otimes \sigma_2$$

or alternatively, as ordinary matrices:

$$\tilde{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^2 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$$\tilde{\gamma}^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

Majorana's equation, then, is simply

$$(i\tilde{\gamma}^{\mu}\partial_{\mu} - m)\tilde{\psi} = 0)$$

Because the $\tilde{\gamma}^{\mu}$ matrices are purely imaginary, the matrices $i\tilde{\gamma}^{\mu}$ are real, and consequently this equation can govern a real field $\tilde{\psi}$.

NATURE|Vol 464|11 March 2010|doi:10.1038/nature08915 REVIEW INSIGHT

Non-Abelian states of matter

Ady Stern[1]

Quantum mechanics classifies all elementary particles as either fermions or bosons, and this classification is crucial to the understanding of a variety of physical systems, such as lasers, metals and superconductors. In certain two-dimensional systems, interactions between electrons or atoms lead to the formation of quasiparticles that break the fermion–boson dichotomy. A particularly interesting alternative is offered by 'non-Abelian' states of matter, in which the presence of quasiparticles makes the ground state degenerate, and interchanges of identical quasiparticles shift the system between different ground states. Present experimental studies attempt to identify non-Abelian states in systems that manifest the fractional quantum Hall effect. If such states can be identified, they may become useful for quantum computation.

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Physics

Physics **3**, 24 (2010)

Viewpoint

Race for Majorana fermions

Marcel Franz
Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1

Published March 15, 2010

The race for realizing Majorana fermions—elusive particles that act as their own antiparticles—heats up, but we still await ideal materials to work with.

Subject Areas: **Semiconductor Physics, Mesoscopics, Particles and Fields**

A Viewpoint on:
Majorana fermions in a tunable semiconductor device
Jason Alicea
Phys. Rev. B **81**, 125318 (2010) – Published March 15, 2010


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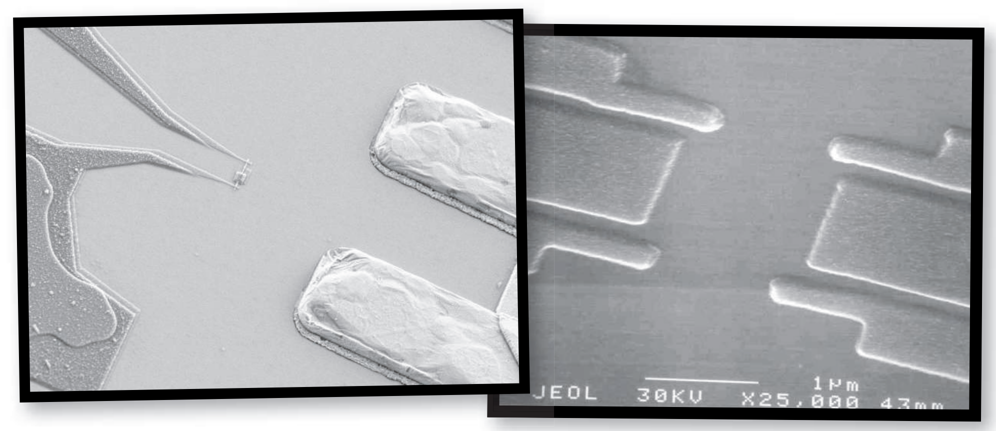

Majorana detectors? Those in use include tiny transistors (*far left*) and quantum interferometers.

NEWS

Search for Majorana Fermions Nearing Success at Last?

Researchers think they are on the verge of discovering weird new particles that borrow a trick from superconductors and could give a big boost to quantum computers

IT HAPPENS OVER AND OVER AGAIN IN particle physics: Theorists predict the existence of a particle and then, sometime later, experimenters find it. Neutrons, positrons, neutrinos, pions, W and Z bosons, and other subatomic denizens all existed on paper

Alto, California. Adds Michael Freedman, a mathematician turned theoretical physicist at Station Q, a collaborative research center between Microsoft and the University of California (UC), Santa Barbara: "This is the decade for Majorana fermions. I am

tary particles come in two families: bosons, such as photons, and fermions, such as electrons, that have different groupings of spin.

In 1926, Austrian physicist Erwin Schrödinger came up with an equation that describes how quantum matter changes over time. Two years later, a young English physicist named Paul Dirac tweaked Schrödinger's equation to make it apply to fermions, such as electrons, that move at speeds near that of light. The expansion integrated quantum mechanics for the first time with Einstein's special theory of relativity.

Dirac's new equations also implied the existence of antimatter, matching each fundamental particle with an antiparticle that would annihilate it if the two should ever meet. To their surprise, physicists realized that certain particles, including some photons, could serve as their own antiparticles and annihilate themselves. But fermions weren't thought to be among them.

Then the story took a twist. In some cases, Dirac's equations produced results involving imaginary numbers, which some physicists considered inelegant. That's where a young, gifted Italian physicist named Ettore Majorana

● Majorana fermions - `half fermions' - can occur as collective excitations in solids with unconventional SC pairing.

● Obey non-abelian exchange statistics, can serve as a platform for fault-tolerant quantum computation.

Ordinary fermions $\{c_i^\dagger, c_j\} = \delta_{ij}$

Write in terms of Majorana fermions: $c_j = (\gamma_{j1} + i\gamma_{j2})/2$

$$\{\gamma_{i\alpha}, \gamma_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \gamma_{i\alpha}^\dagger = \gamma_{i\alpha}$$

Canonical transformation: can be used to recast ANY fermionic Hamiltonian in terms of Majorana operators

Certain Hamiltonians can support solutions with isolated localized Majorana fermions

Example: `Kitaev 1D model' [Phys. Usp. 44, 131 (2001)]

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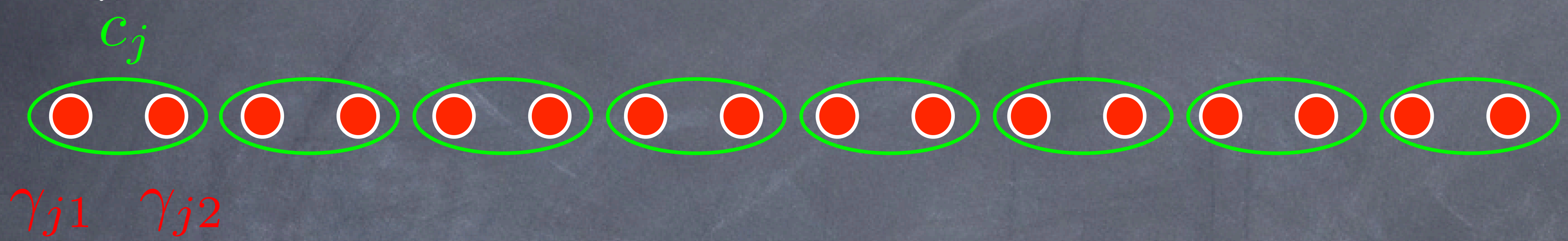

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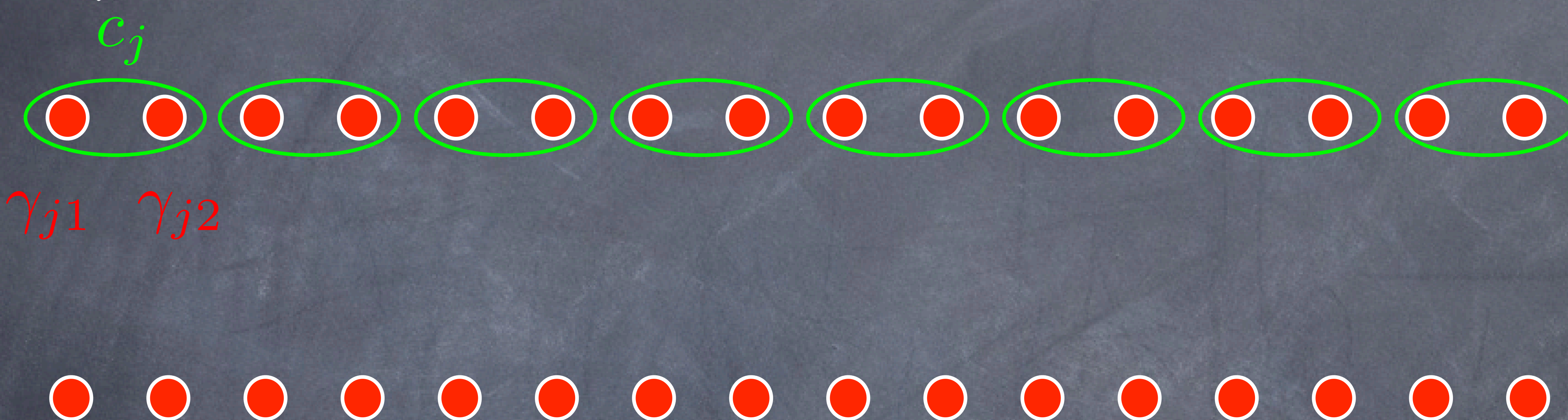

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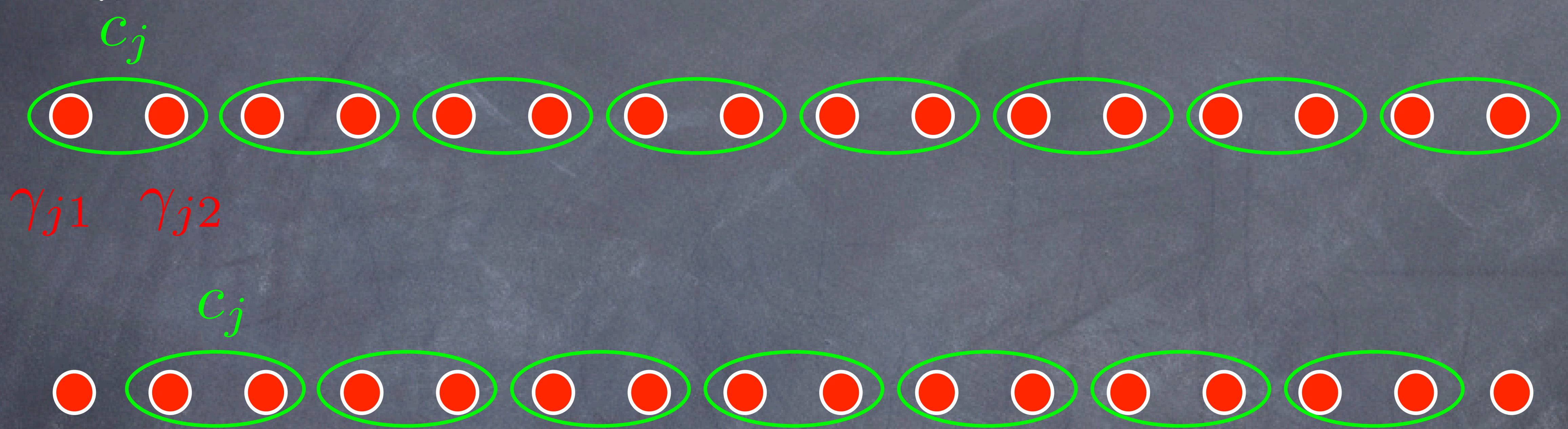

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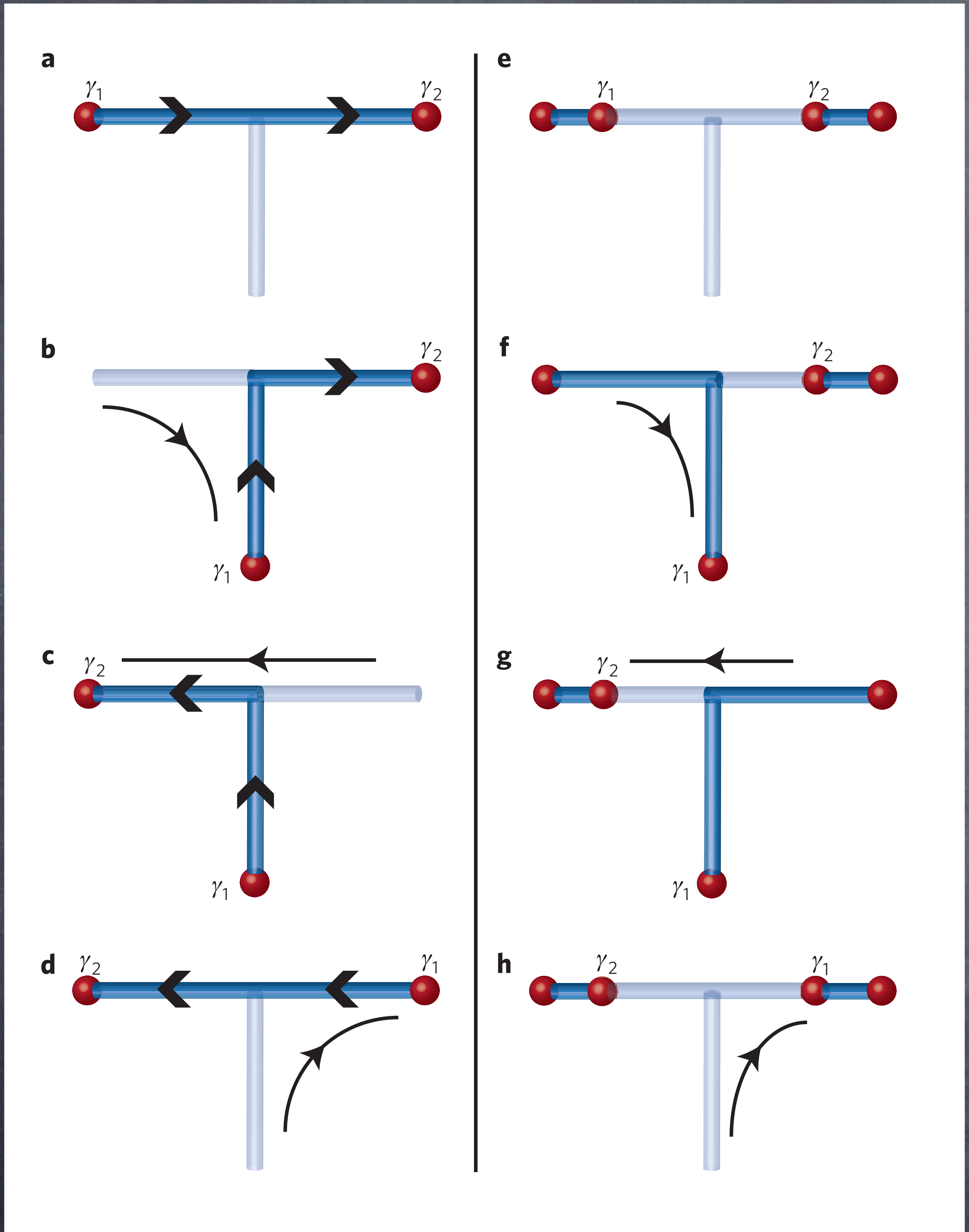
These also encode one complex fermion but in a way that is robust to any local perturbation --> ideal quantum bit.



`Braiding' of Majoranas in T-junctions shows non-abelian exchange statistics.

[Alicea et al. Nat Phys 2010]

- Proposed realizations:
  a. Moore-Read FQHE
  b. Spin-polarized p+ip superconductor
  c. TI/SC interface
  d. Rashba-coupled semicond. + SC + magnetic insulator
  e. 1D quantum wires

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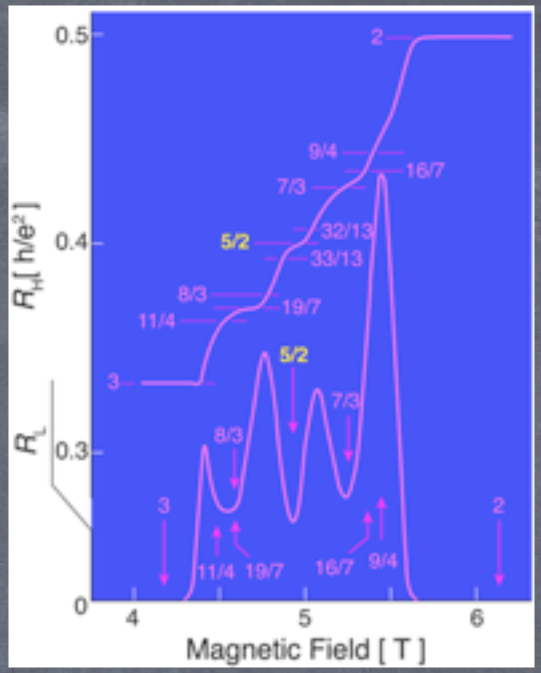

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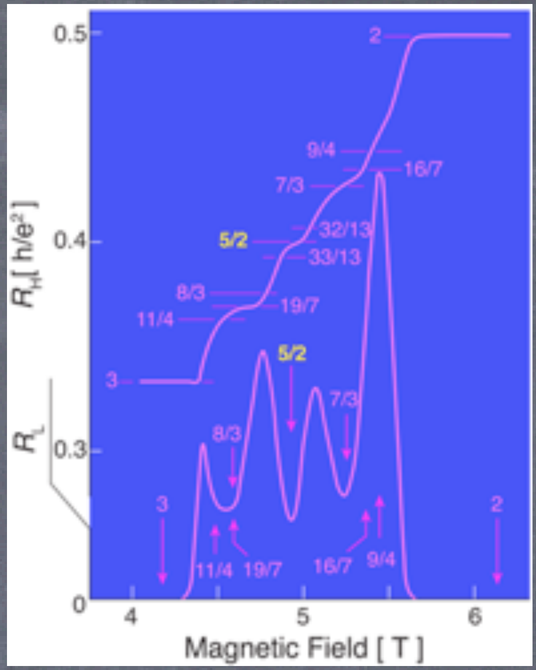

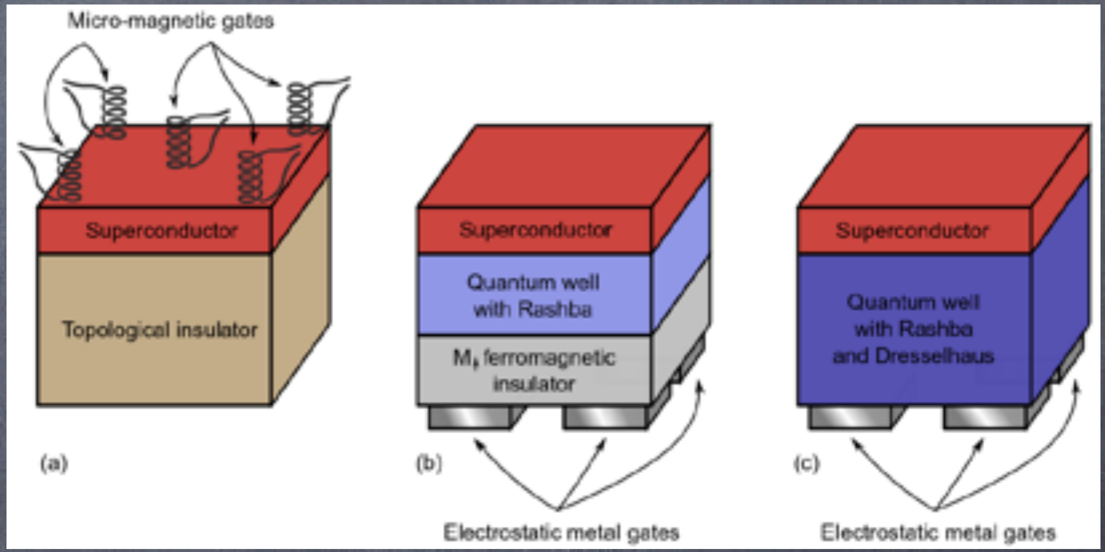

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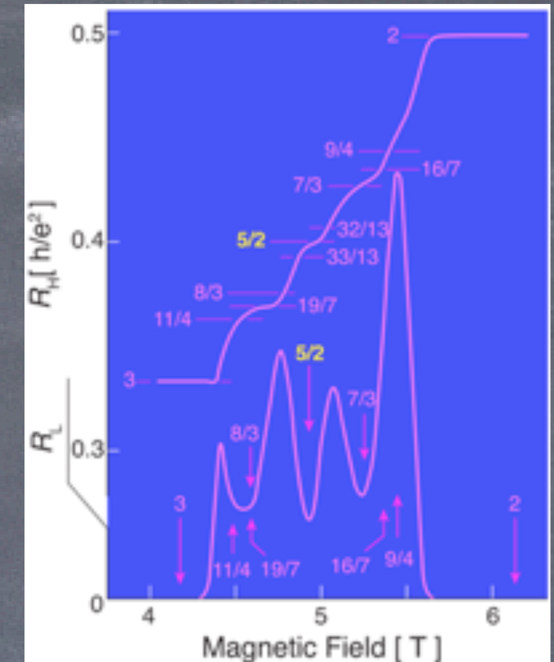

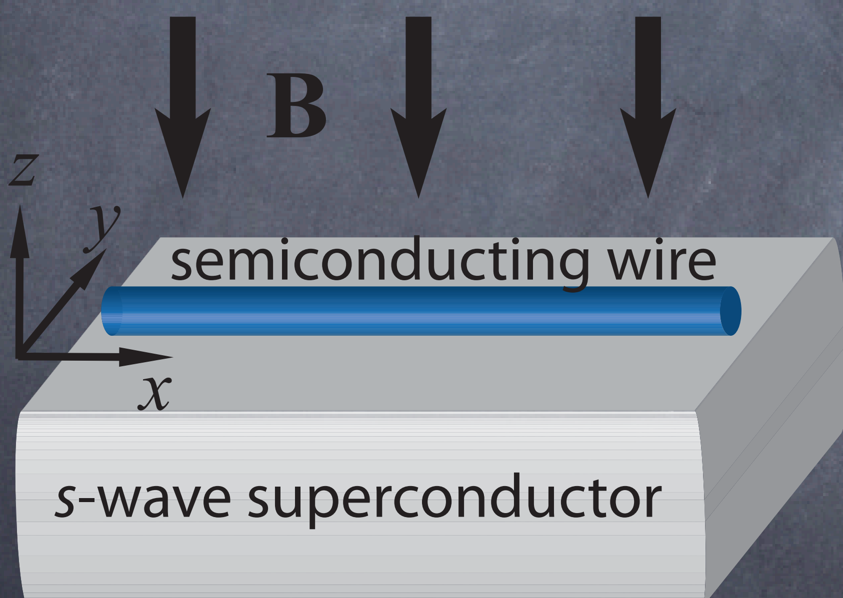

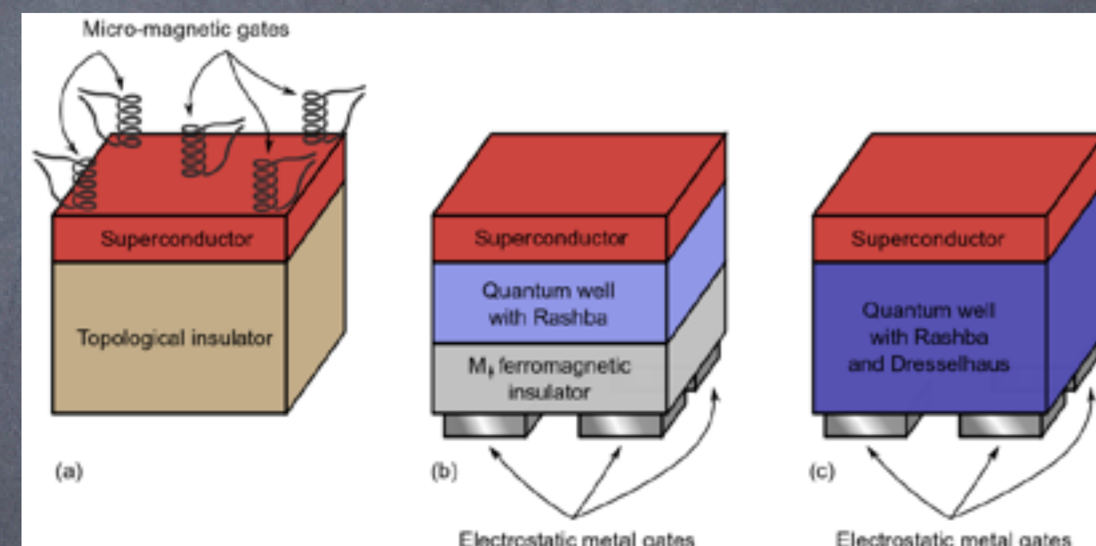

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Experimental realizations

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Experimental realizations

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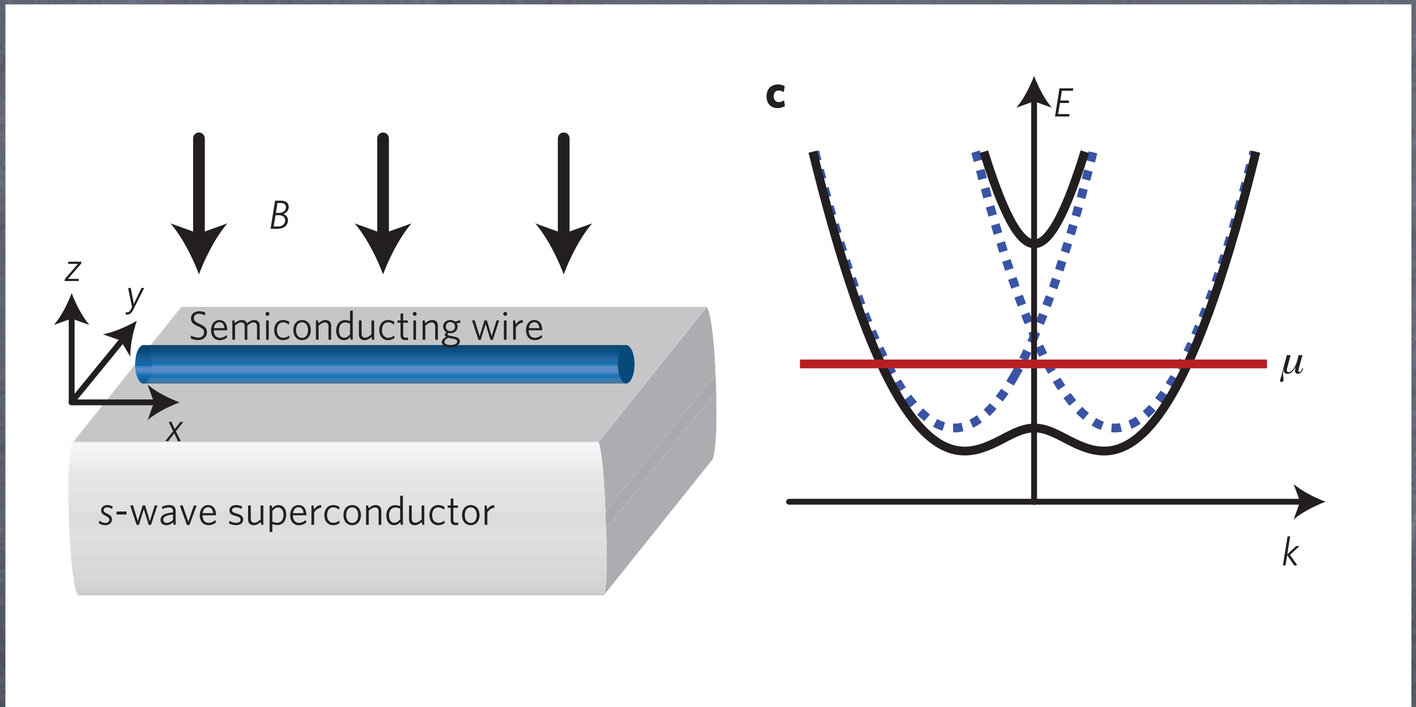
The race is on...

Rashba-coupled semiconductor quantum wire

(a brief review)

Lutchyn et al. PRL 2010, Oreg et al. PRL 2010

$$H_0 = \int_{-\infty}^{\infty} dx \psi_\sigma^\dagger(x) \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'}(x),$$



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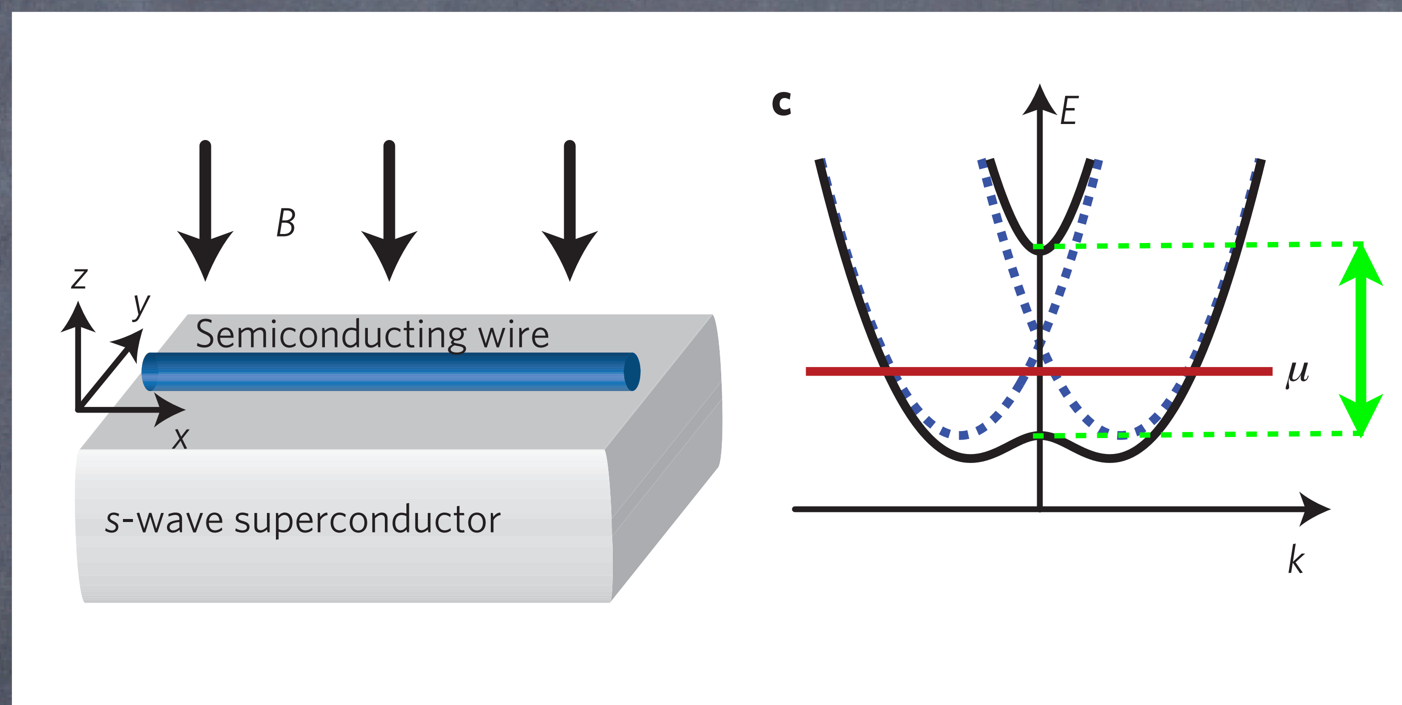

1-10K

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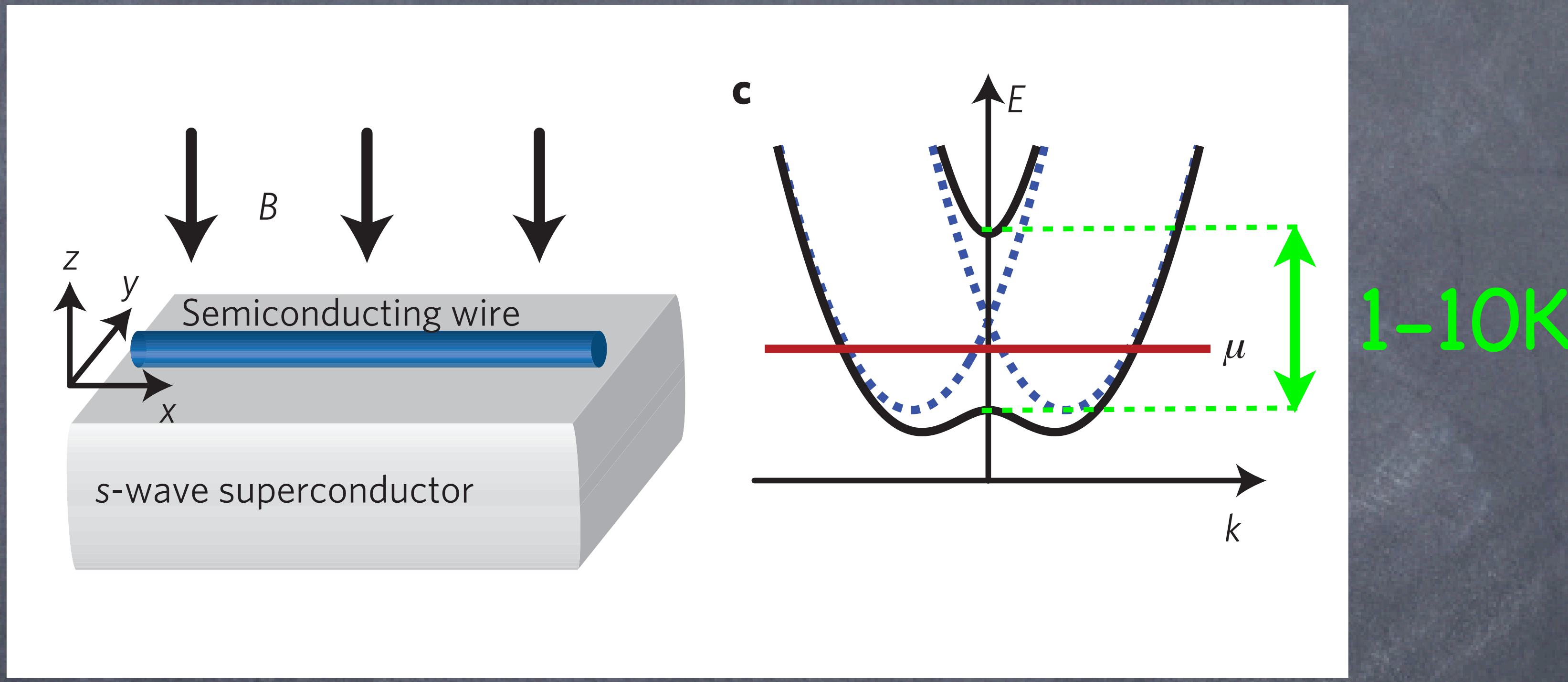

Potential issues:

Rashba-coupled semiconductor quantum wire

(a brief review)

Lutchyn et al. PRL 2010, Oreg et al. PRL 2010

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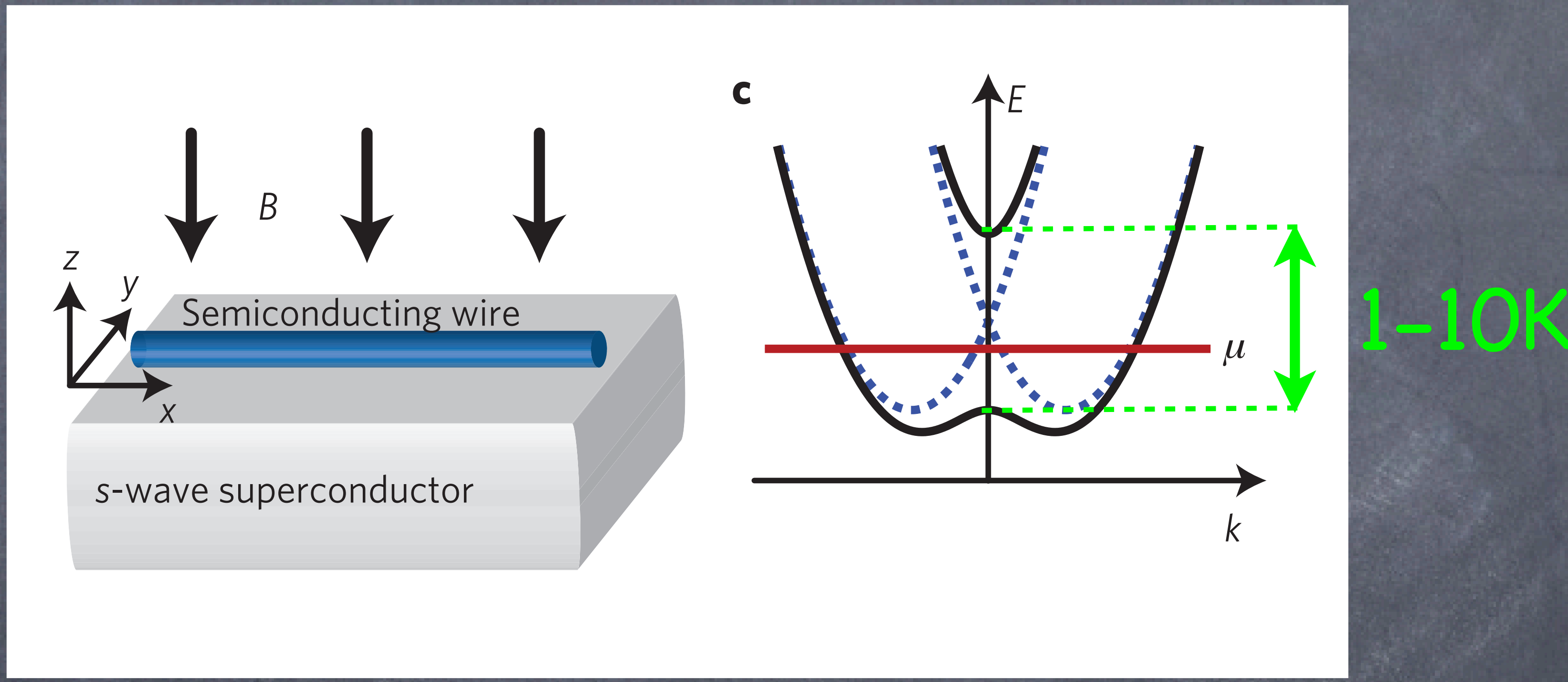
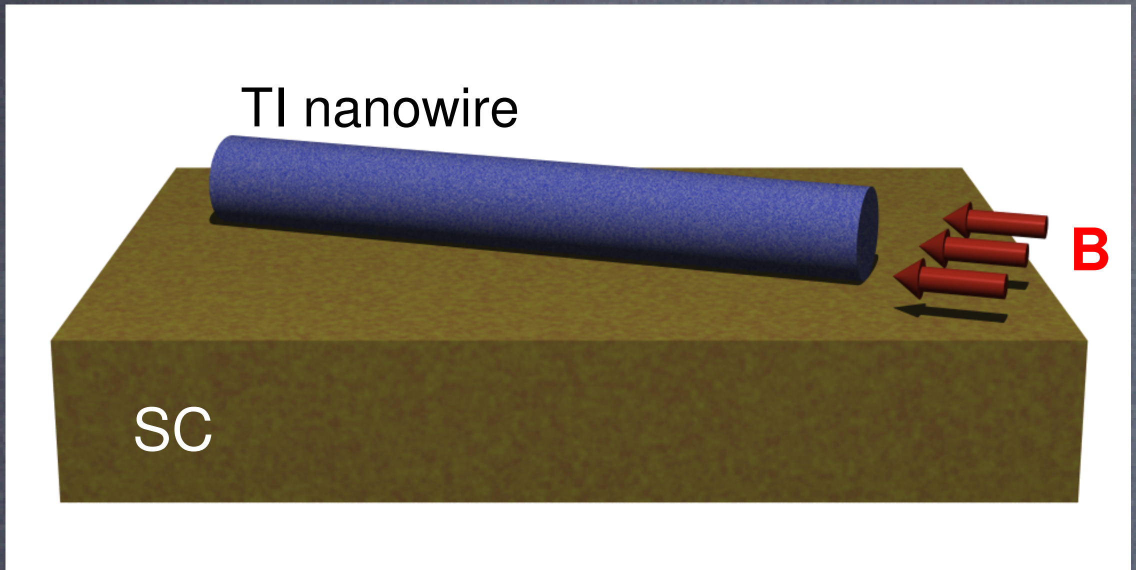


Potential issues:

- Chemical potential tuning
- Effects of disorder
- Detection

New proposal

[A. Cook and M. Franz, Phys. Rev. B 84, 201105R (2011)]



TI nanowire placed on top of ordinary s-wave SC in longitudinal applied magnetic field.

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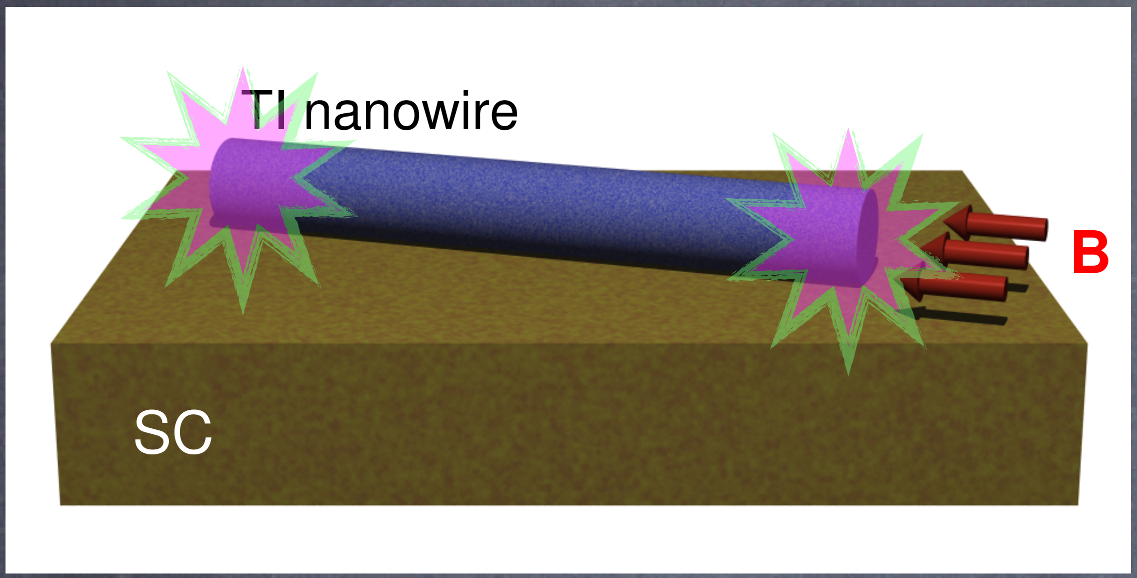

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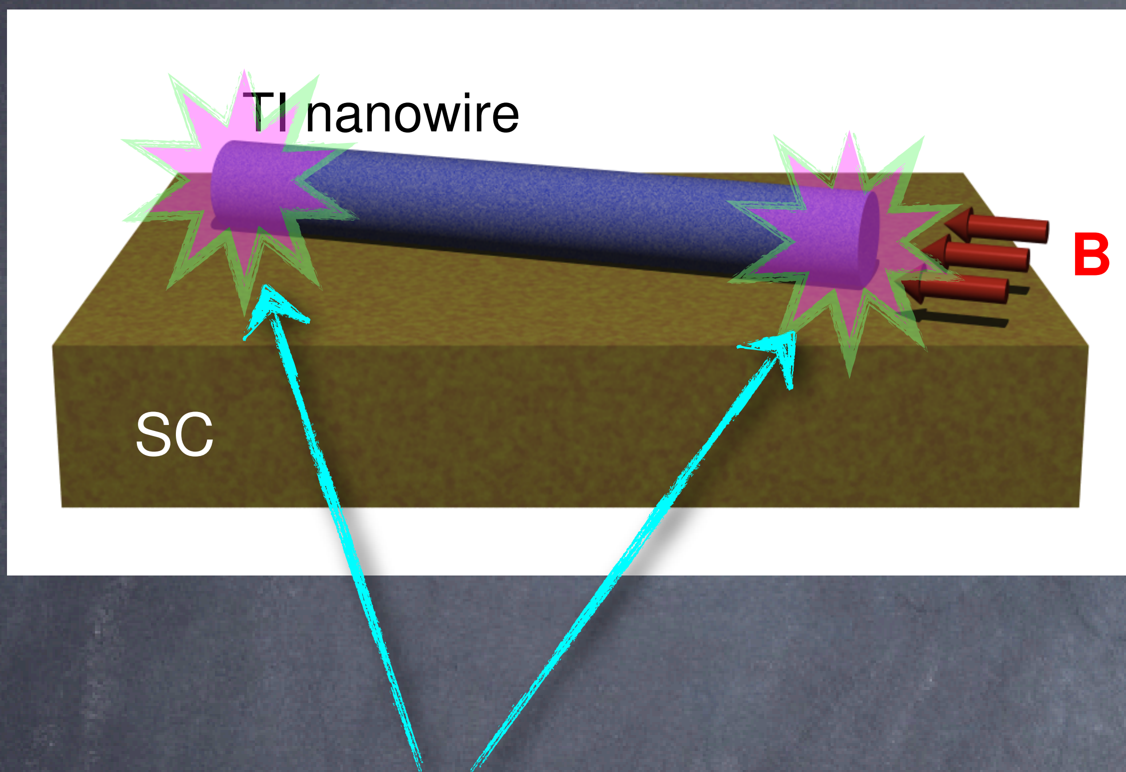

Majorana fermions

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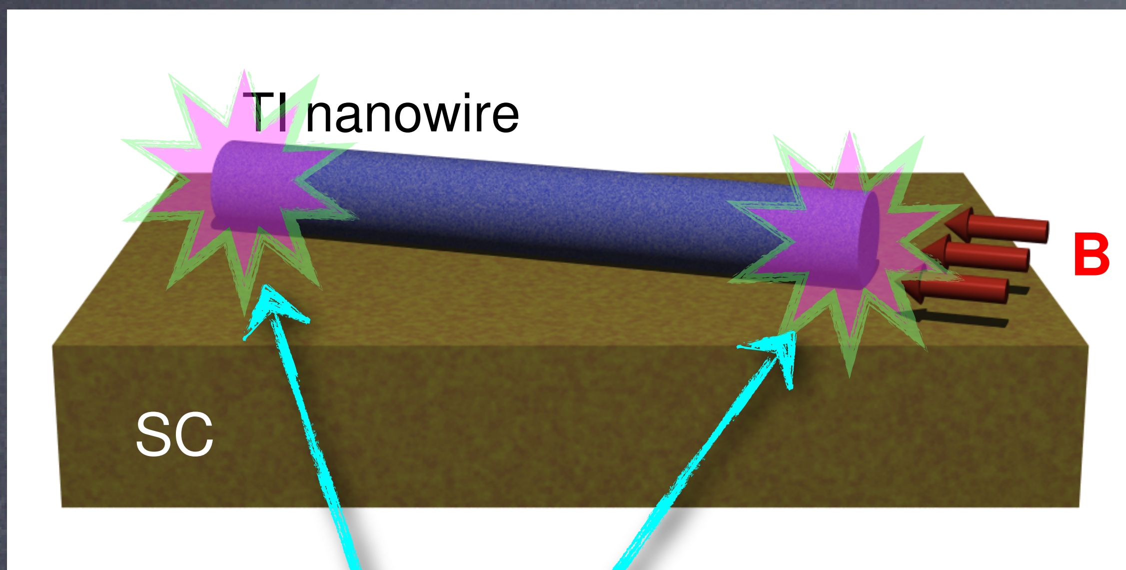


TI nanowire placed on top of ordinary s-wave SC in longitudinal applied magnetic field.

Majorana fermions

No fine tuning:

1. Chemical potential inside the bulk gap (~300meV in Bi_2Se_3).

2. Flux close to 1/2 flux quantum.

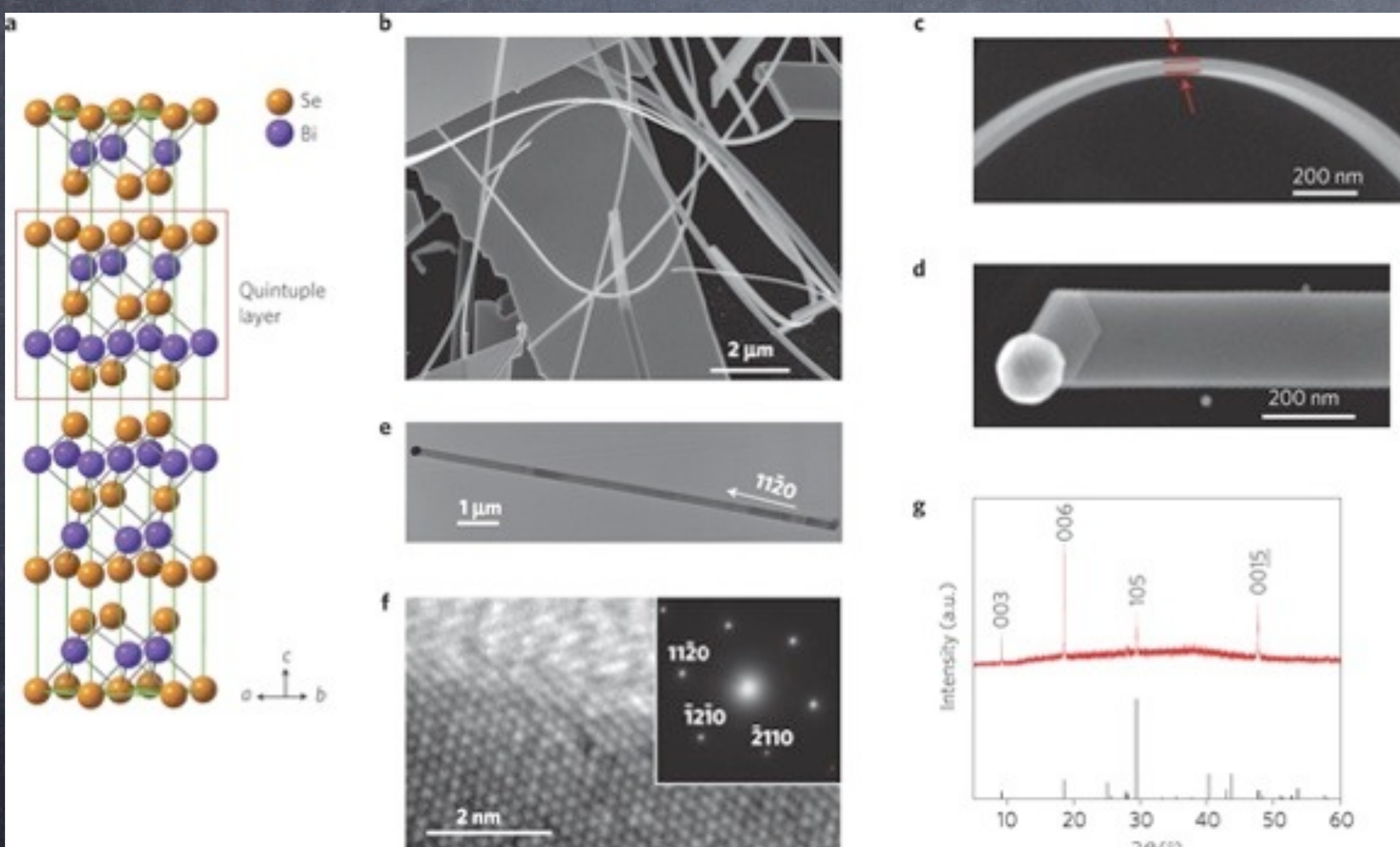
3. Robust against non-magnetic didisorder.

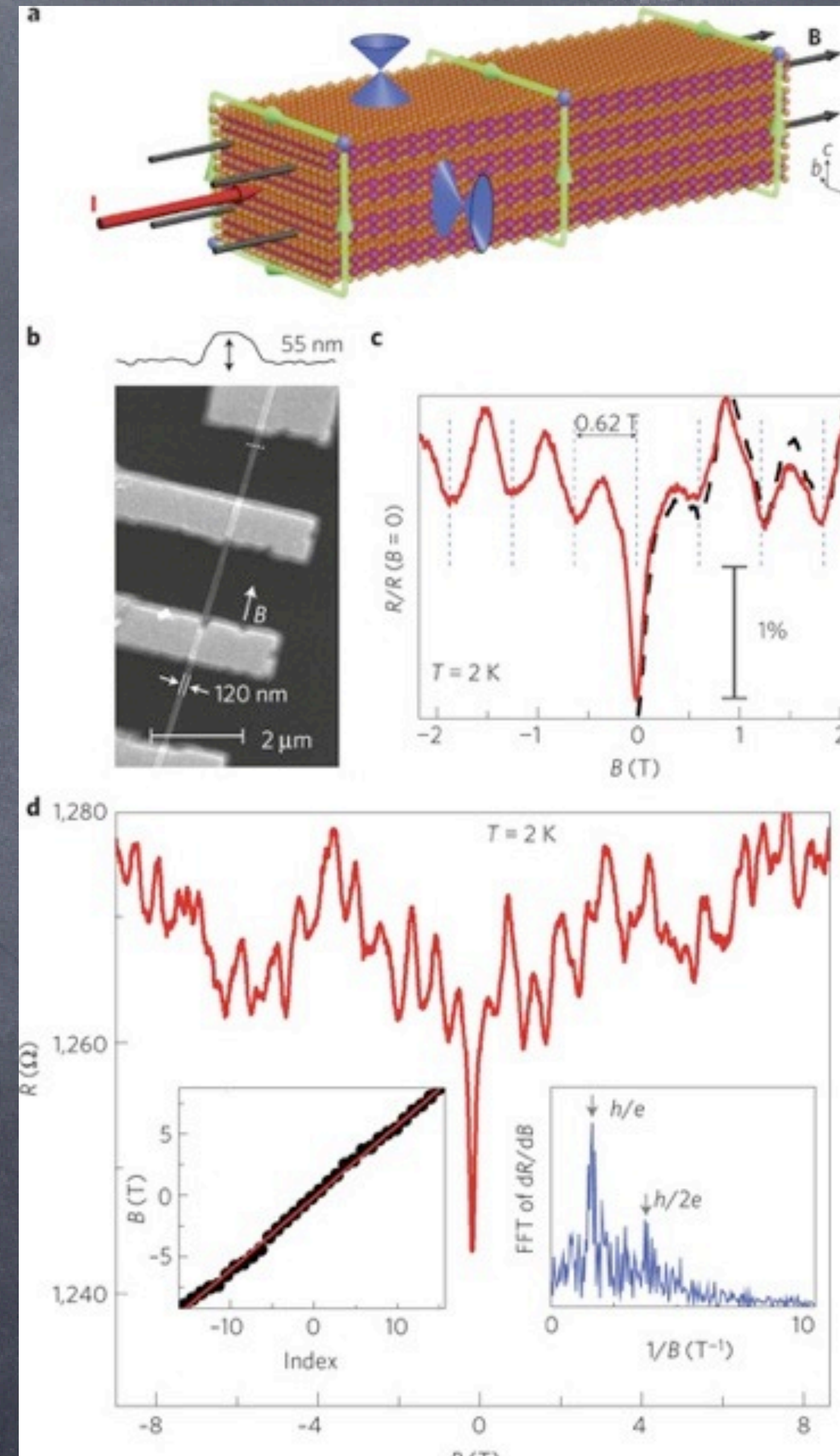
TI nanowires ("nanoribbons")

nature materials
LETTERS
PUBLISHED ONLINE: 13 DECEMBER 2009 | DOI: 10.1038/NMAT2609

Aharonov–Bohm interference in topological insulator nanoribbons

Hailin Peng[1,2]★, Keji Lai[3,4]★, Desheng Kong[1], Stefan Meister[1], Yulin Chen[3,4,5], Xiao-Liang Qi[4,5], Shou-Cheng Zhang[4,5], Zhi-Xun Shen[3,4,5] and Yi Cui[1]†





PHYSICAL REVIEW B **84**, 165120 (2011)

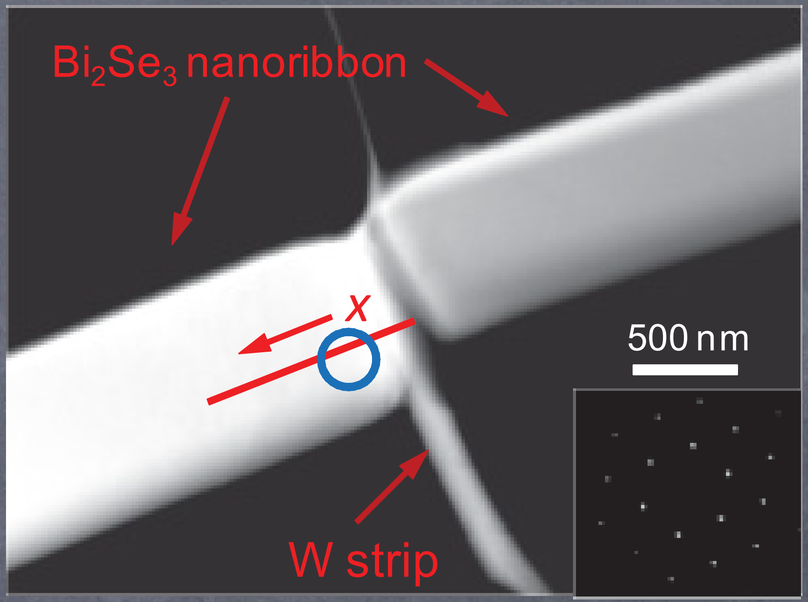
Superconducting proximity effect and possible evidence for Pearl vortices in a candidate topological insulator

Duming Zhang, Jian Wang, Ashley M. DaSilva, Joon Sue Lee, Humberto R. Gutierrez, Moses H. W. Chan, Jainendra Jain, and Nitin Samarth*

The Center for Nanoscale Science and Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802-6300, USA

(Received 17 June 2011; revised manuscript received 21 August 2011; published 24 October 2011)

We report the observation of the superconducting proximity effect in nanoribbons of a candidate topological insulator (Bi_2Se_3), which is interfaced with superconducting (tungsten) contacts. We observe a supercurrent and multiple Andreev reflections for channel lengths that are much longer than the inelastic and diffusive thermal lengths deduced from normal-state transport. This suggests that the proximity effect couples preferentially to a ballistic surface transport channel, even in the presence of a coexisting diffusive bulk channel. When a magnetic field is applied perpendicular to the plane of the nanoribbon, we observe magnetoresistance oscillations that are periodic in magnetic field. Quantitative comparison with a model of vortex blockade relates the occurrence of these oscillations to the formation of Pearl vortices in the region of proximity-induced superconductivity.



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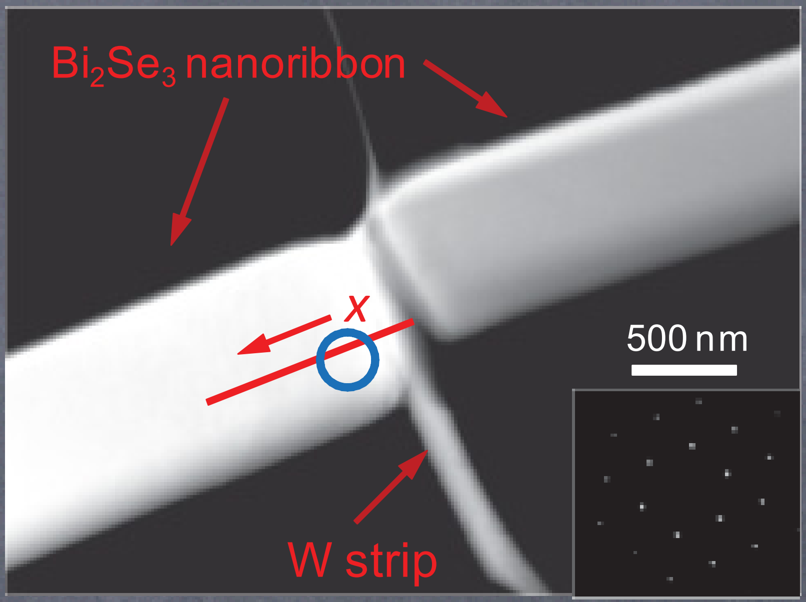
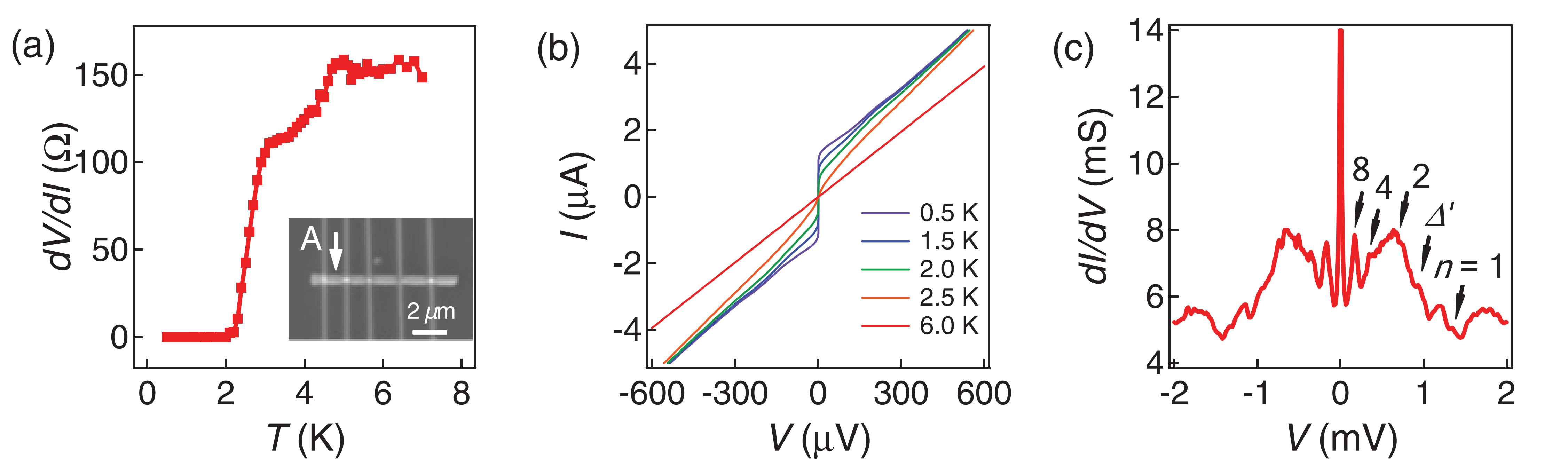
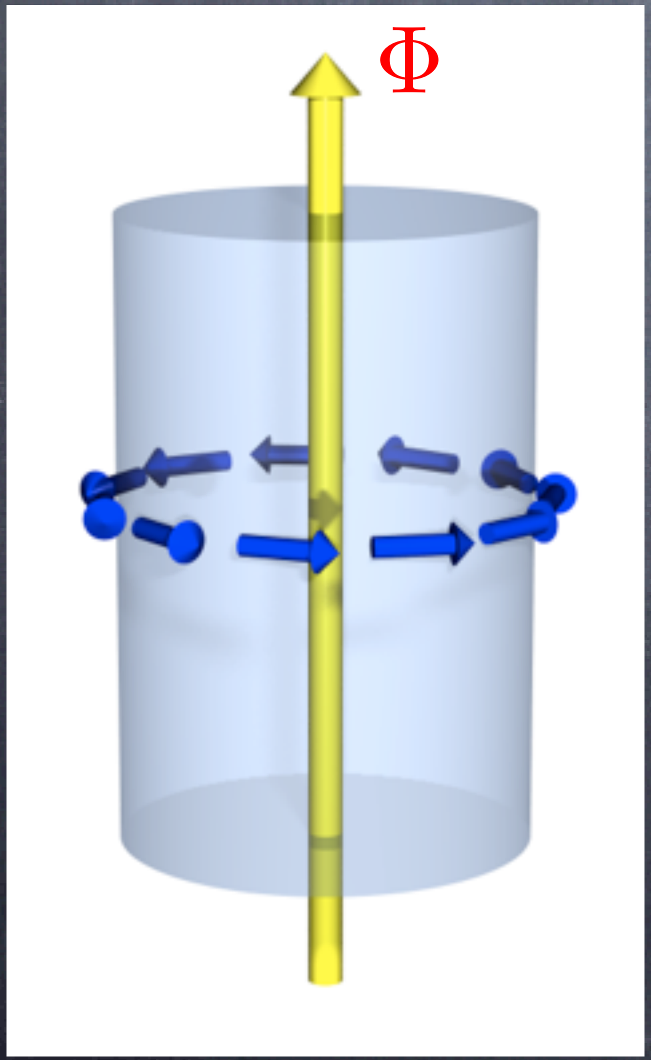




Theory: solve Dirac equation for the surface states of a TI cylinder threaded by magnetic flux

$$h = \frac{1}{2}v\left[\hbar\nabla\cdot\mathbf{n} + \mathbf{n}\cdot(\mathbf{p}\times\sigma) + (\mathbf{p}\times\sigma)\cdot\mathbf{n}\right] + \mathbf{m}\cdot\sigma$$

[see Ostrovsky et al. PRL **105**, 036803 (2010)]



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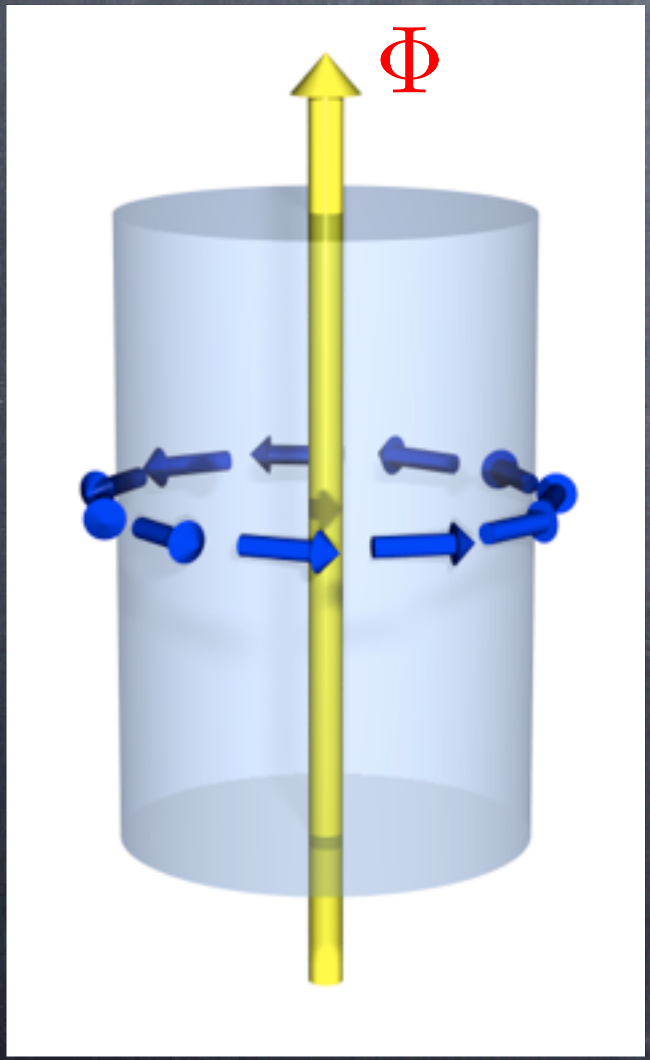


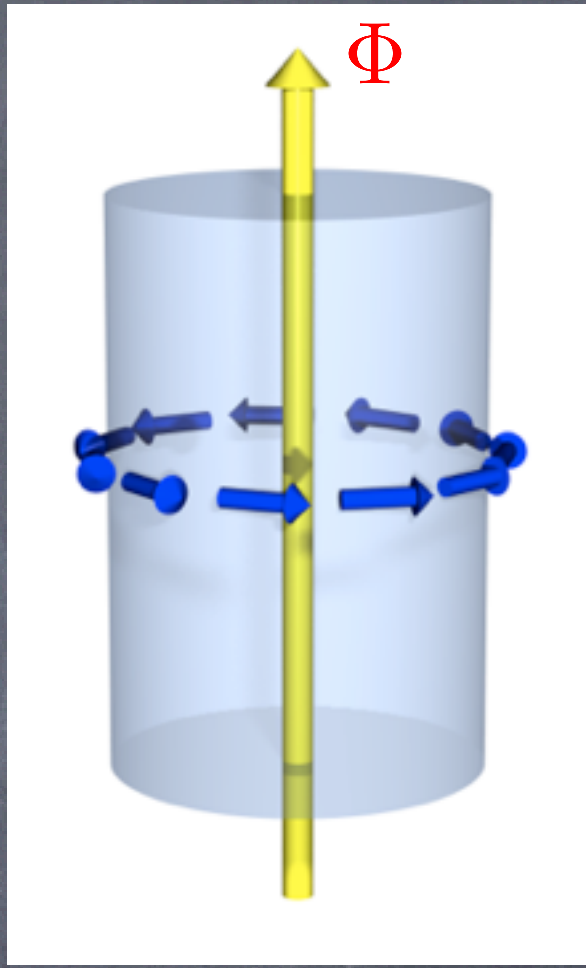
Include flux through minimal substitution

$$\mathbf{p} \to \mathbf{p} - (e/c)\mathbf{A}$$

and solve using cylindrical symmetry

Assuming **m** along **z** the solution is of the form

$$\psi_{kl}(z,\varphi) = e^{i\varphi l} e^{-ikz} \begin{pmatrix} f_{kl} \\ e^{i\varphi} g_{kl} \end{pmatrix}$$

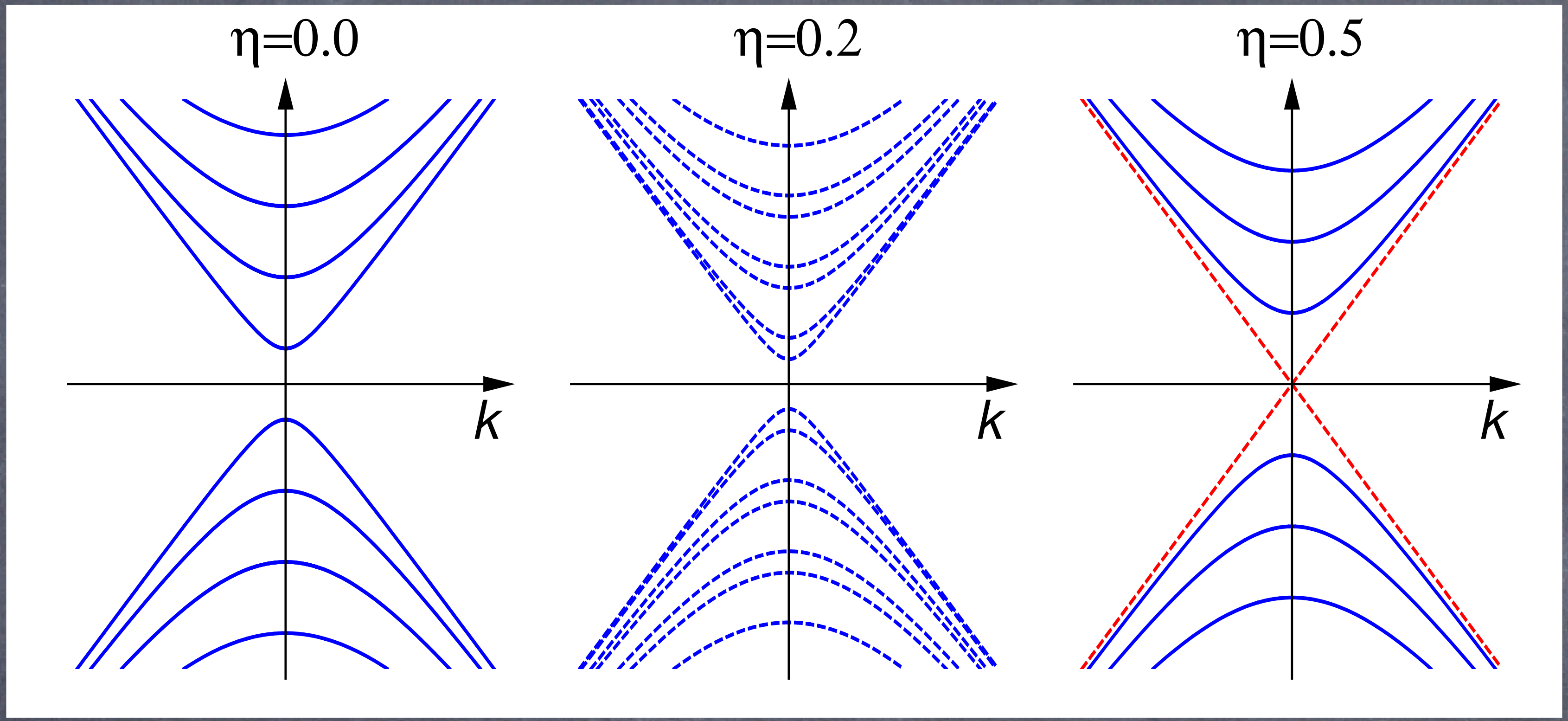


The spinor $\tilde{\psi}_{kl} = \begin{pmatrix} f_{kl} \\ g_{kl} \end{pmatrix}$ is an eigenstate of

$$h_{kl} = \sigma_2 k + \sigma_3 [(l + \frac{1}{2} - \eta)/R + m_z]$$

The spectrum is [Rosenberg et al. PRB **82**, 041104R (2010)]

$$E_{kl} = \pm v\hbar\sqrt{k^2 + \frac{(l+\frac{1}{2}-\eta)^2}{R^2}}; \quad \eta = \frac{\Phi}{\Phi_0}$$



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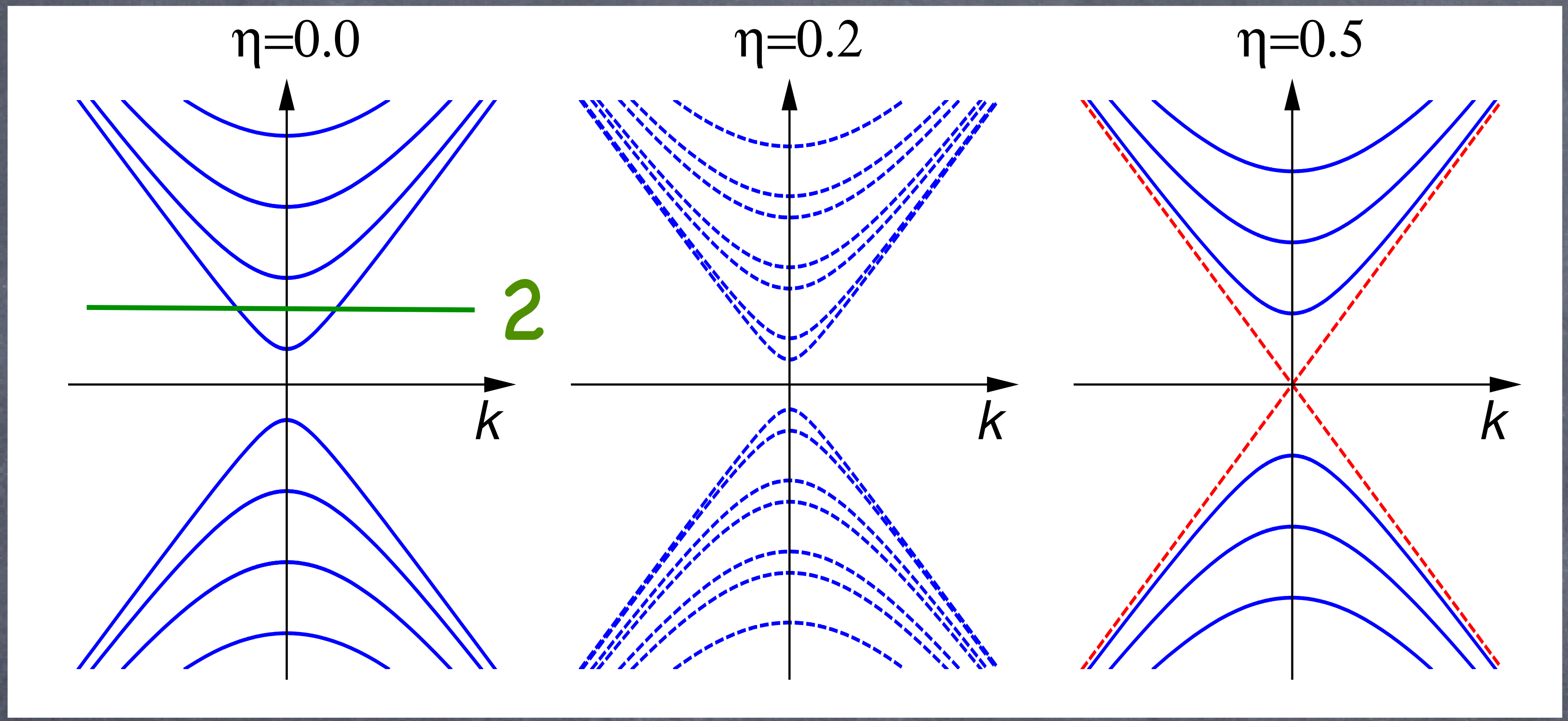

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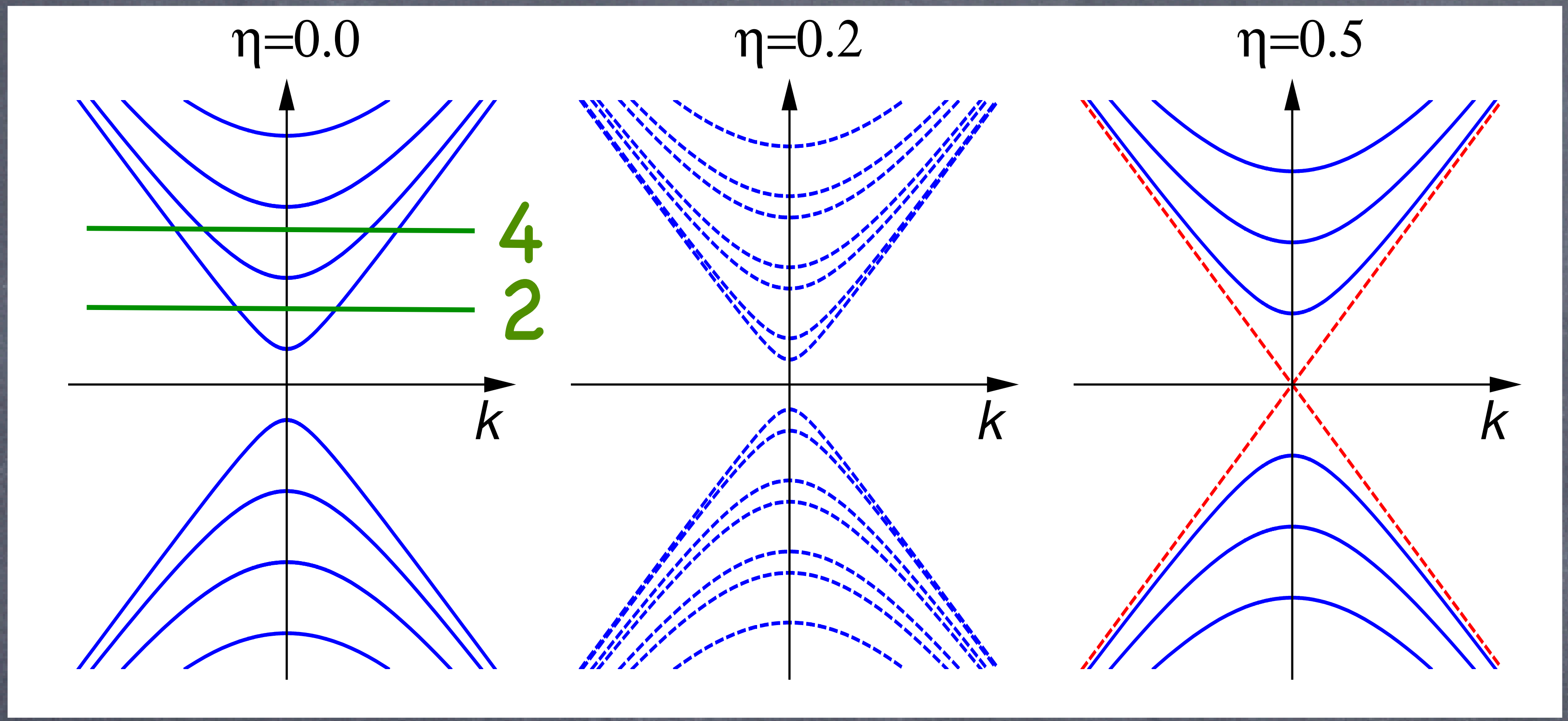

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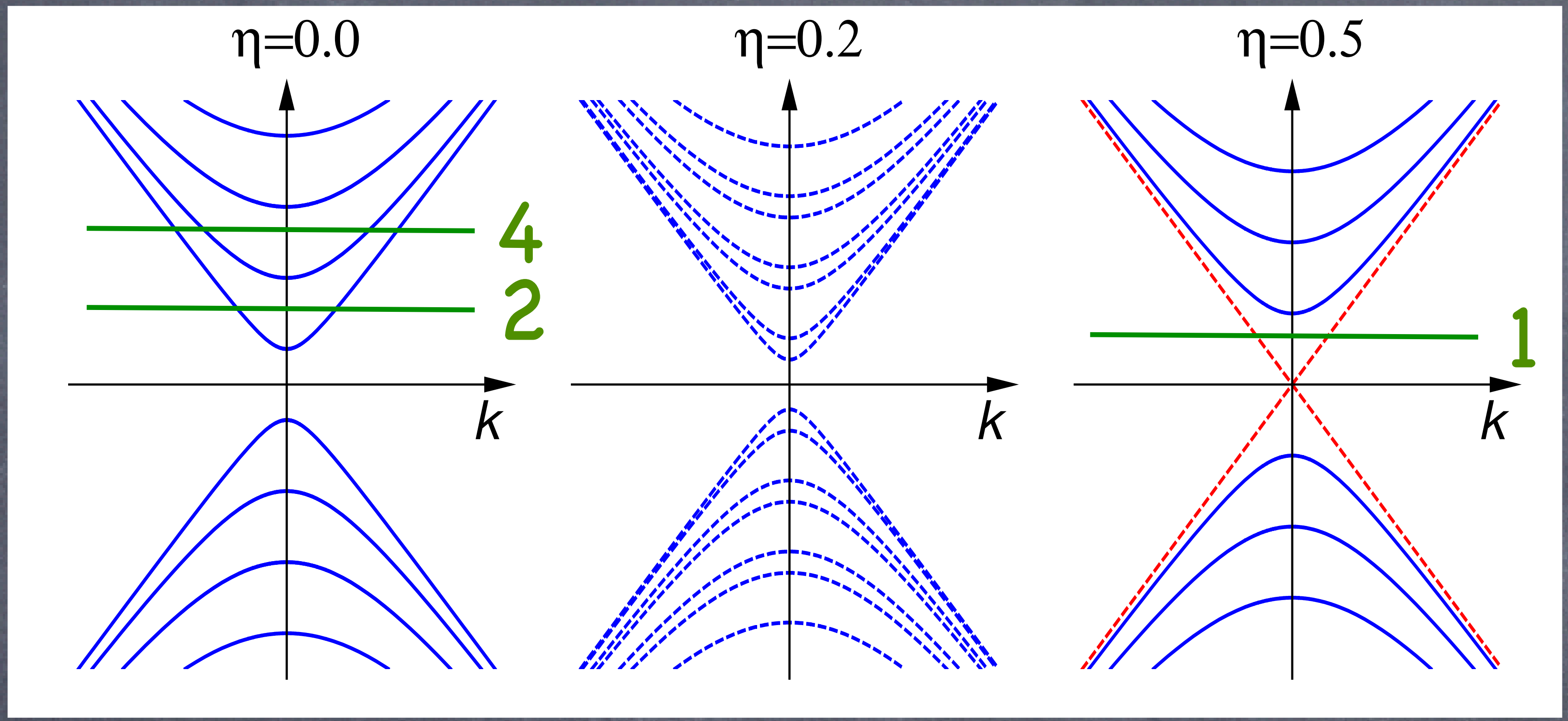

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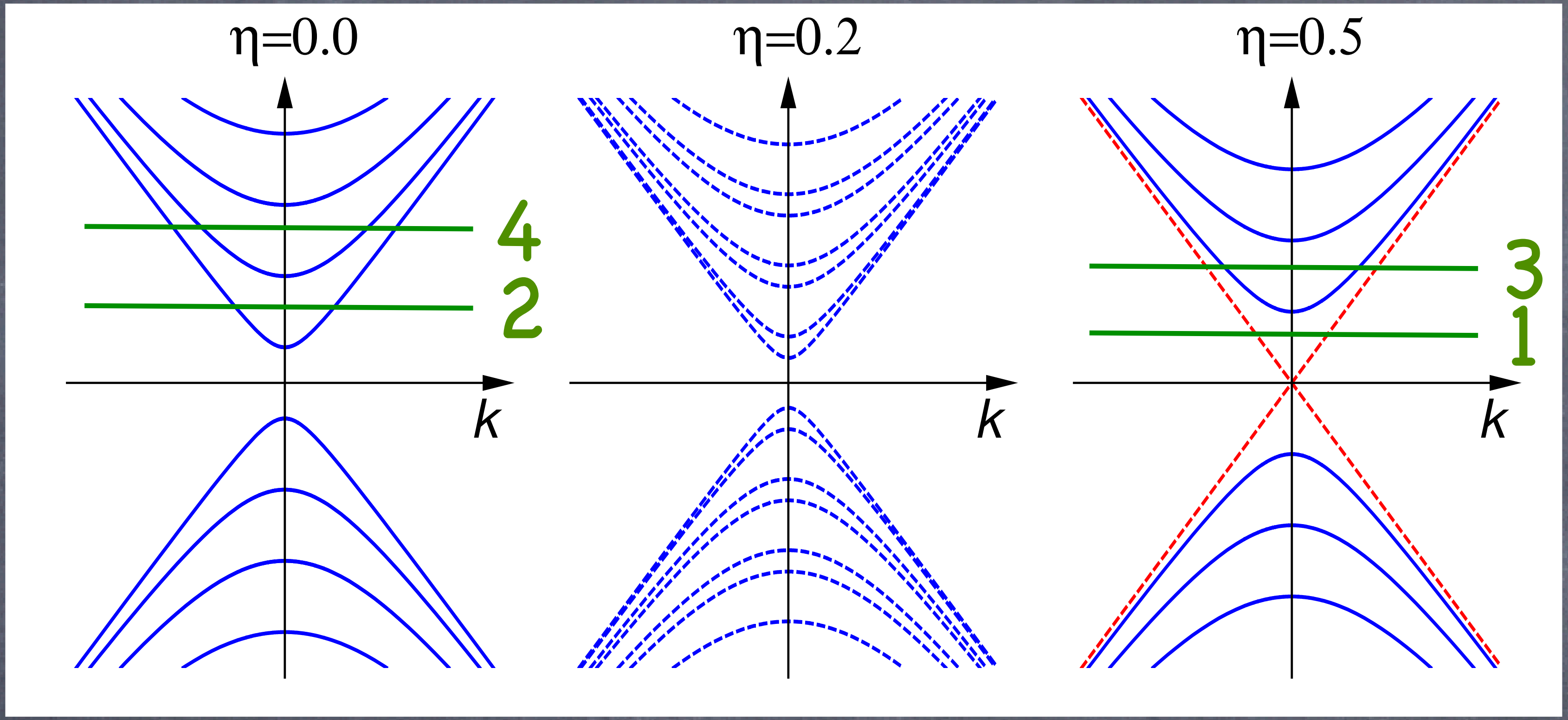

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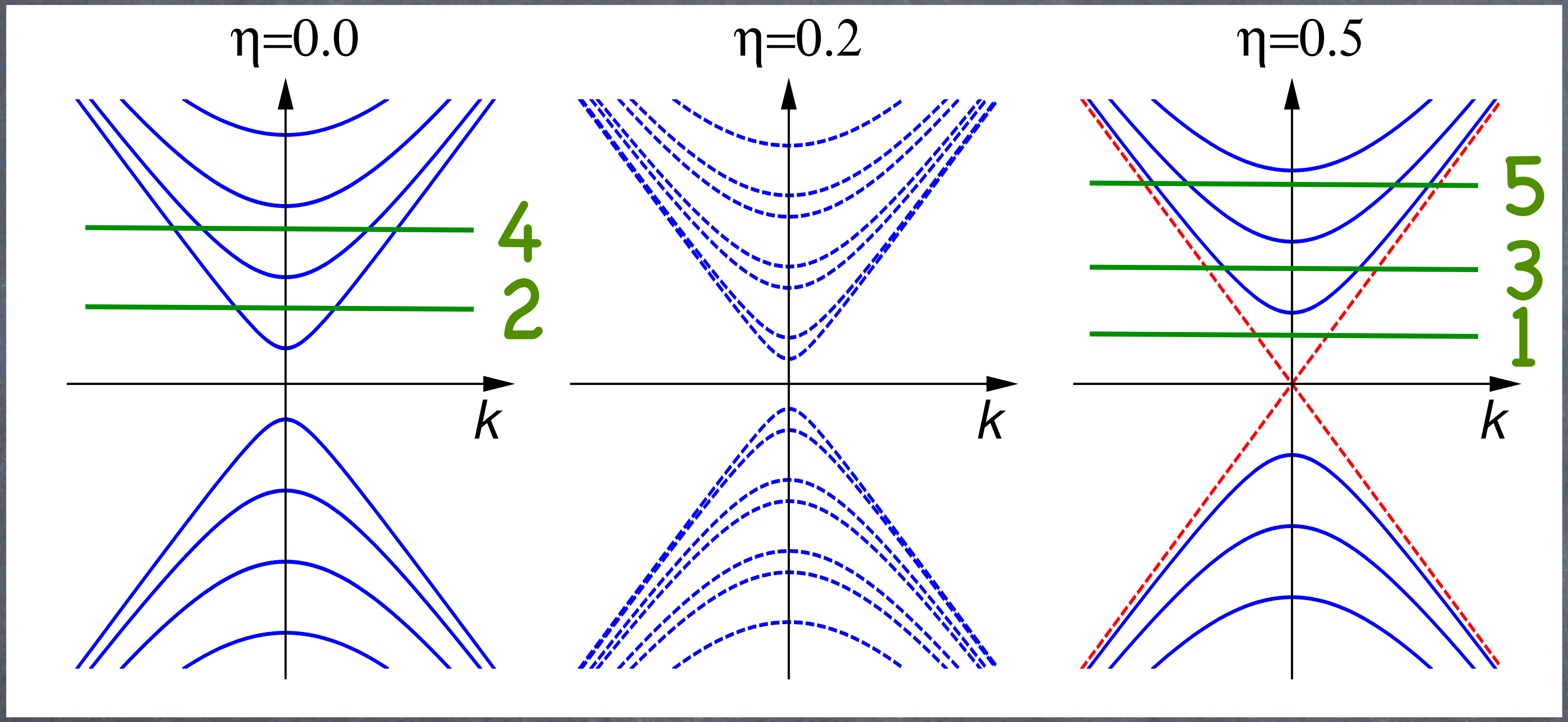

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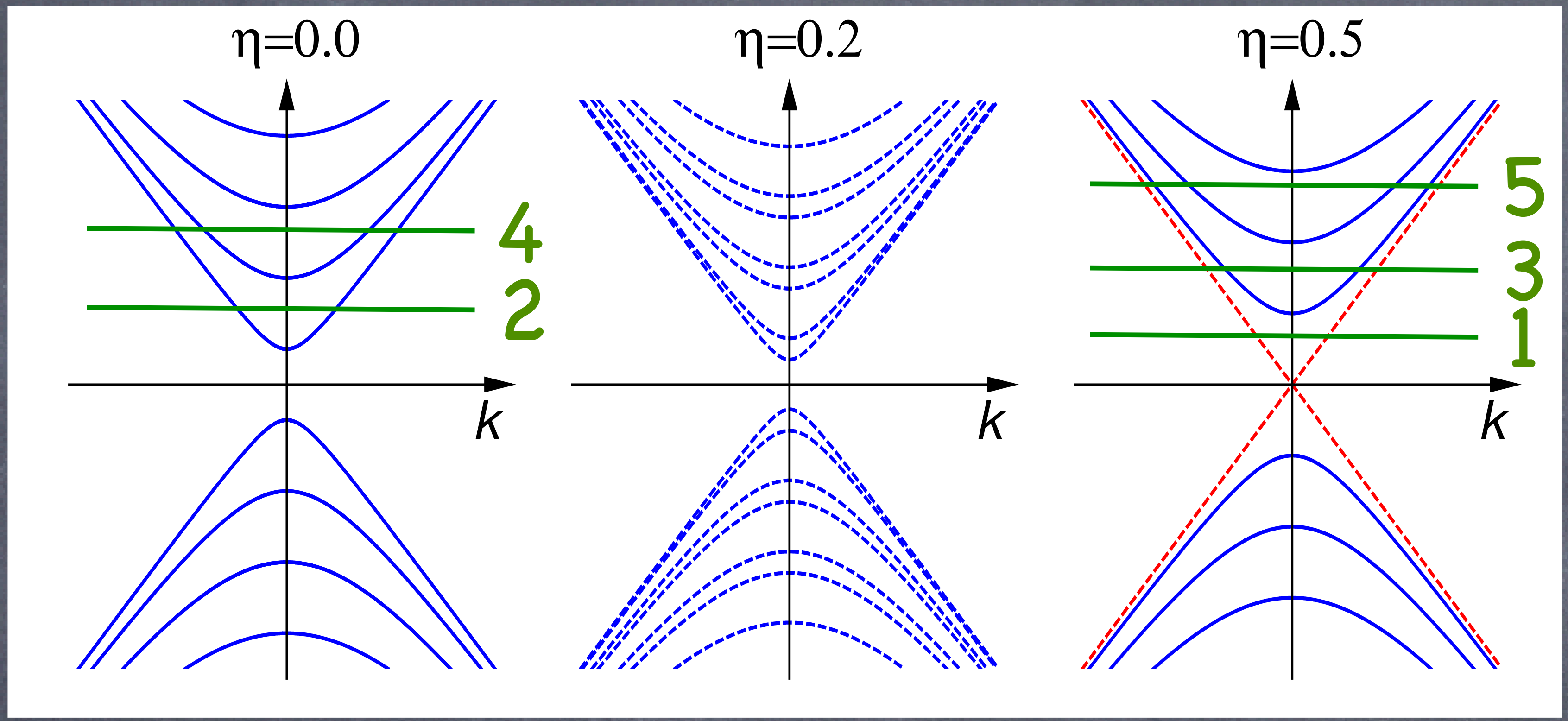

For $\eta = \frac{1}{2}$ there is always an **odd # of bands** crossing the Fermi surface.

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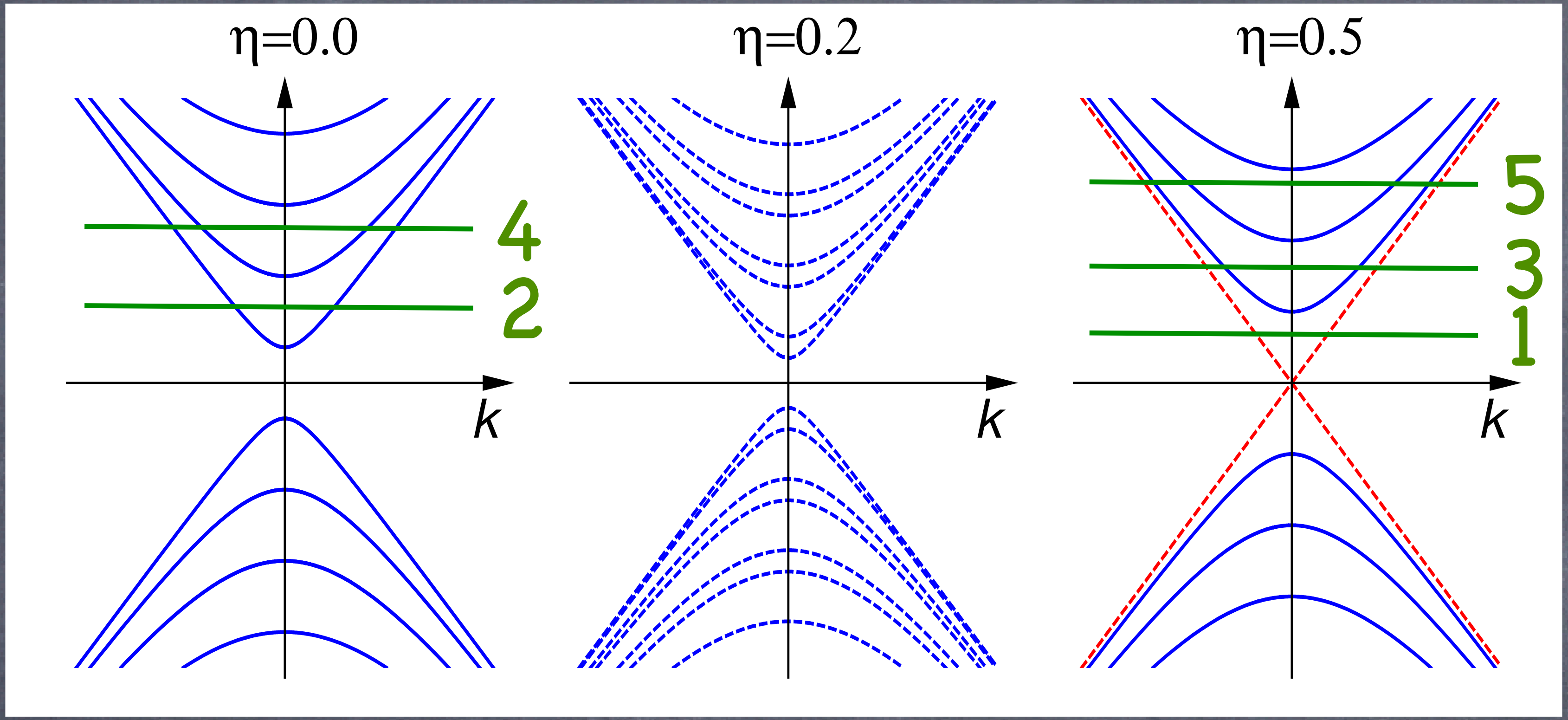

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Topological superconductivity

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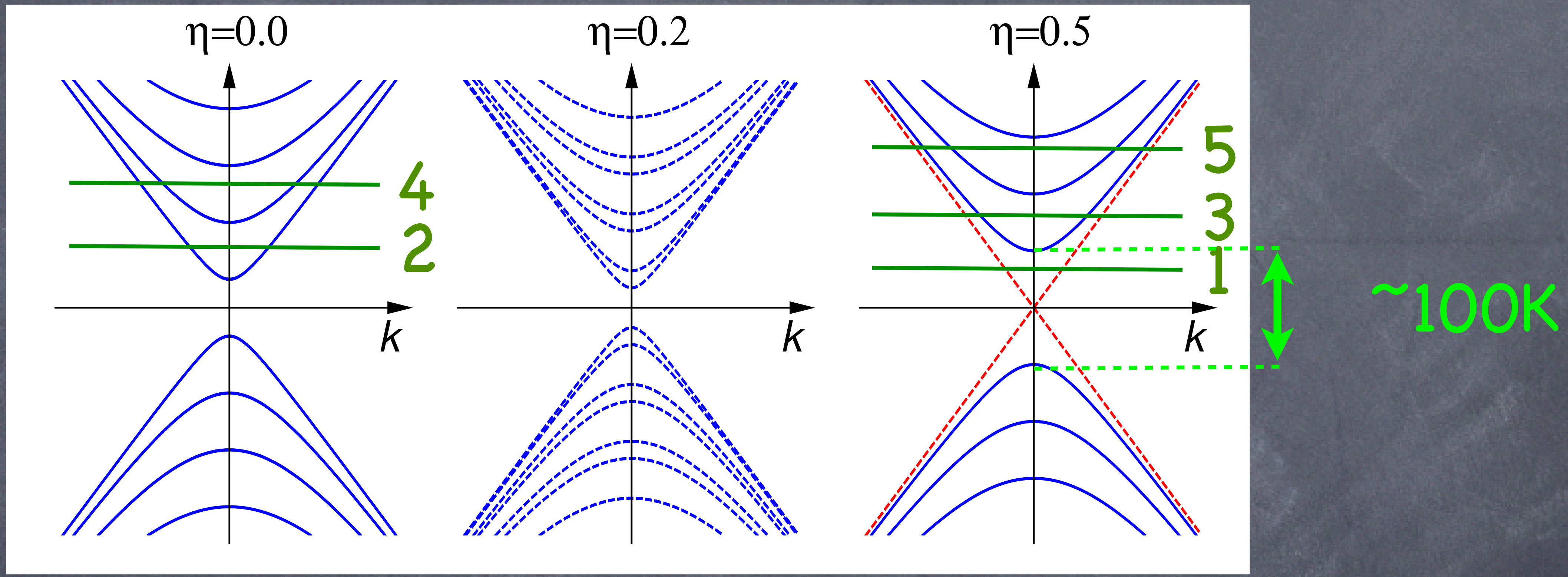

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Topological superconductivity

Kitaev's Majorana number

Chernogolovka 2000: Mesoscopic and strongly correlated electron systems *Usp. Fiz. Nauk* (Suppl.) **171** (10)

Unpaired Majorana fermions in quantum wires

A Yu Kitaev

3. A general condition for Majorana fermions

Let us consider a general translationally invariant one-dimensional Hamiltonian with short-range interactions. It has been mentioned that the necessary conditions for unpaired Majorana fermions are superconductivity and a gap in the bulk excitation spectrum. The latter is equivalent to the quasi-particle tunneling amplitude vanishing as $\exp(-L/l_0)$. Besides that, it is clear that there should be some parity condition. Indeed, Majorana fermions at the ends of parallel weakly interacting chains may pair up and cancel each other (i.e. the ground state will be non-degenerate). So, provided the energy gap, each one-dimensional Hamiltonian H is characterized by a 'Majorana number' $\mathcal{M} = \mathcal{M}(H) = \pm 1$: the existence of unpaired Majorana fermions is indicated as $\mathcal{M} = -1$. The Majorana number should satisfy $\mathcal{M}(H' \oplus H'') = \mathcal{M}(H')\mathcal{M}(H'')$, where $\oplus$ means taking two non-interacting chains.

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$$\mathcal{M} = (-1)^{\nu}$$

number of Fermi points in the right half of the Brillouin zone

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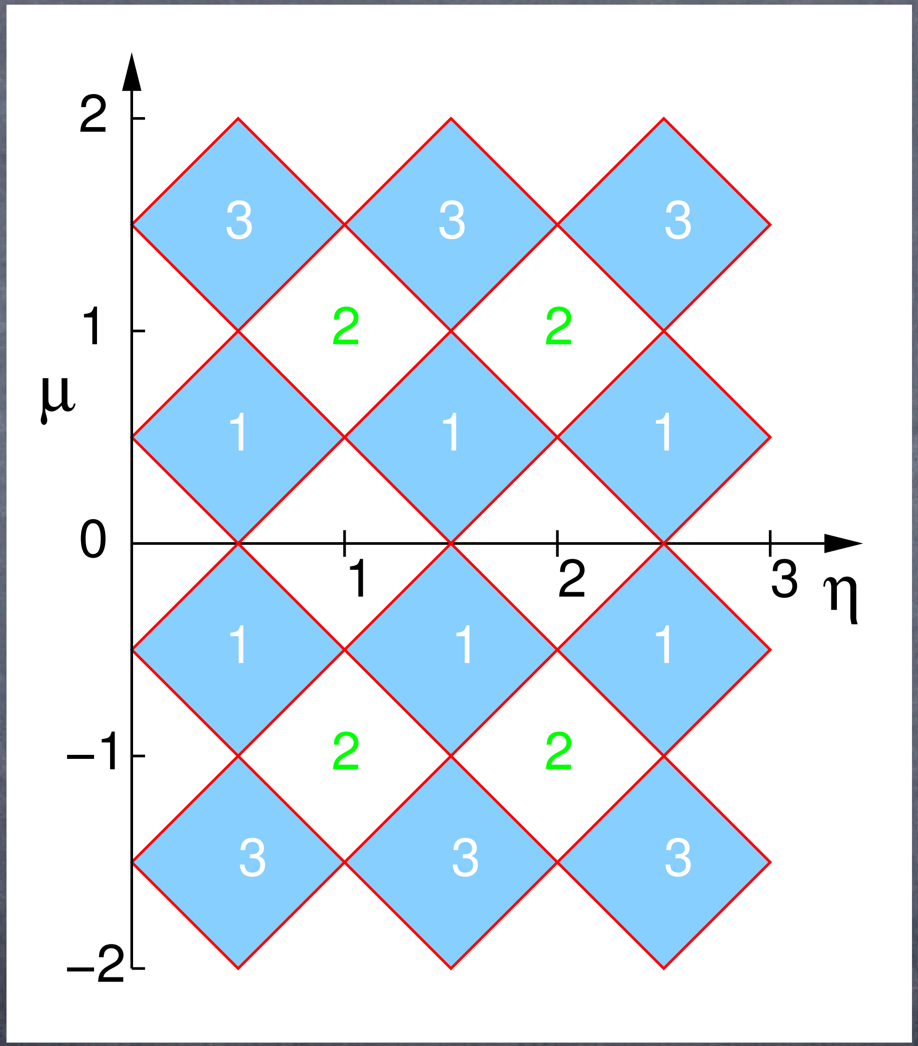
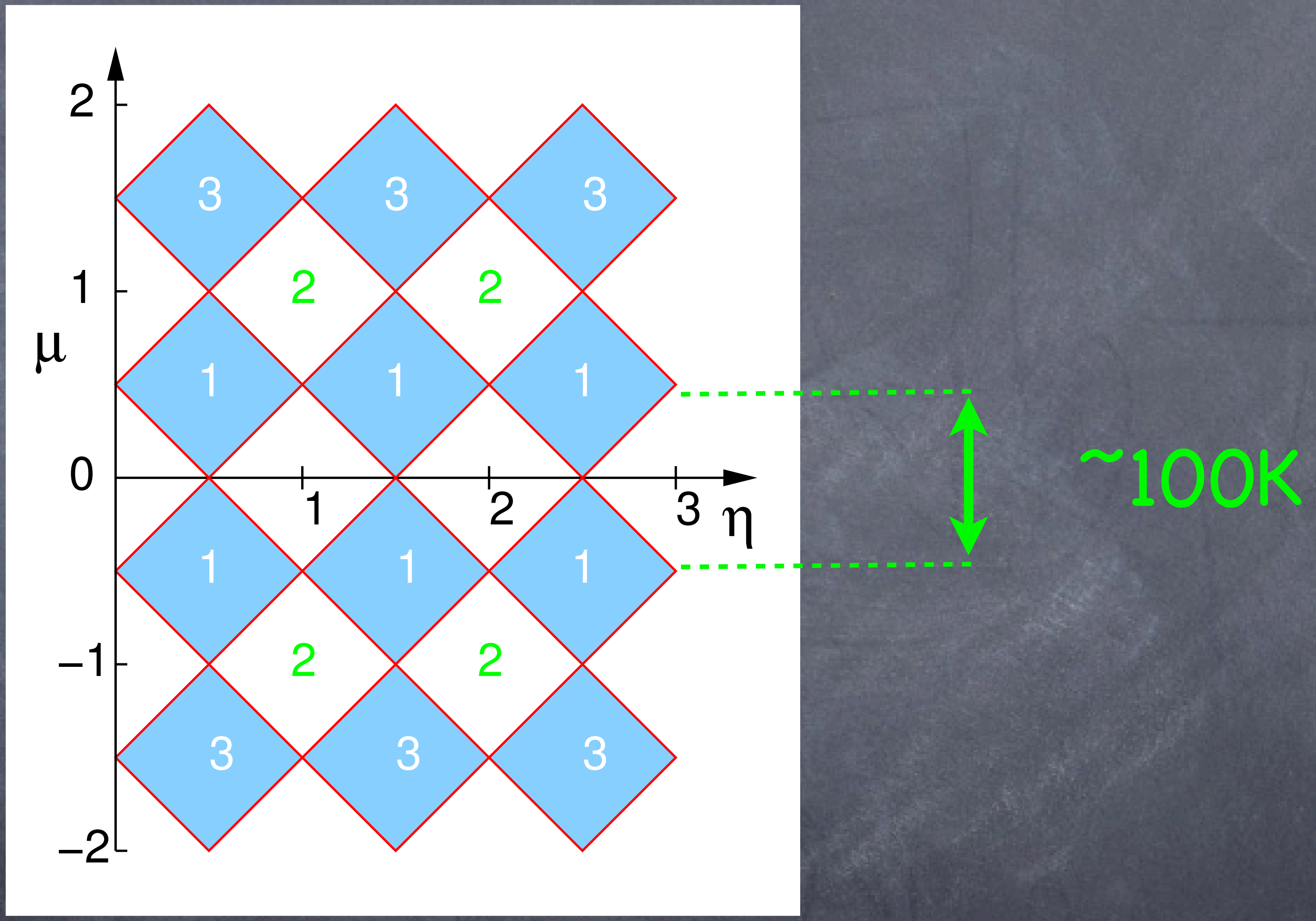
Phase diagram for
$\mathcal{M}(\mu,\eta)$



TI nanoribbon

Phase diagram for $\mathcal{M}(\mu, \eta)$



TI nanoribbon

Explicit solution for the Majorana zero mode

Bogoliubov-de Gennes Hamiltonian

$$\mathcal{H}_k = \begin{pmatrix} h_k & \Delta_k \\ -\Delta^*_{-k} & -h^*_{-k} \end{pmatrix}$$

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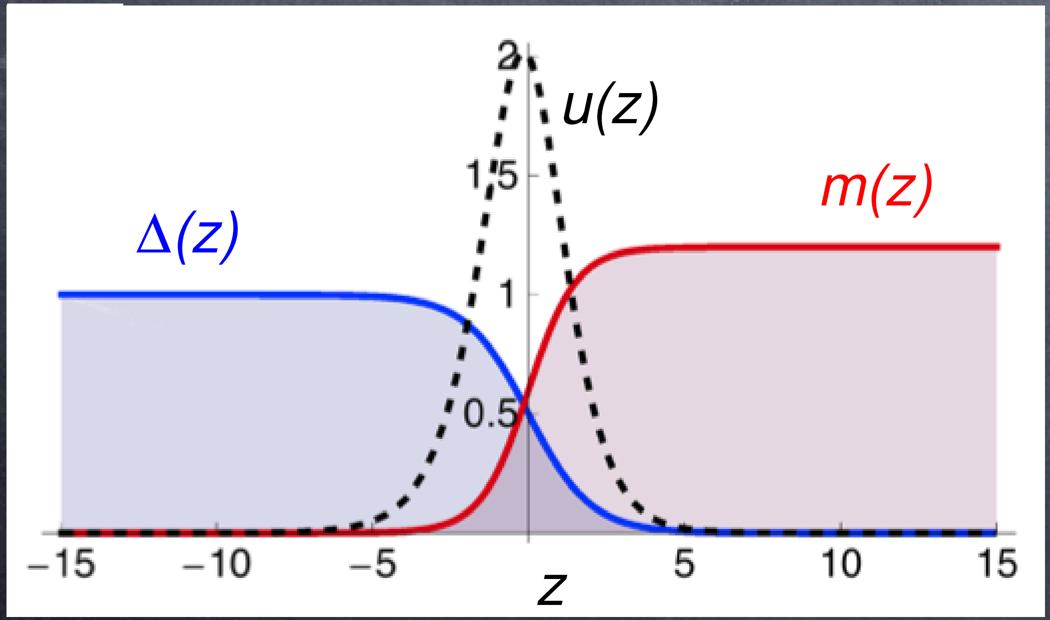
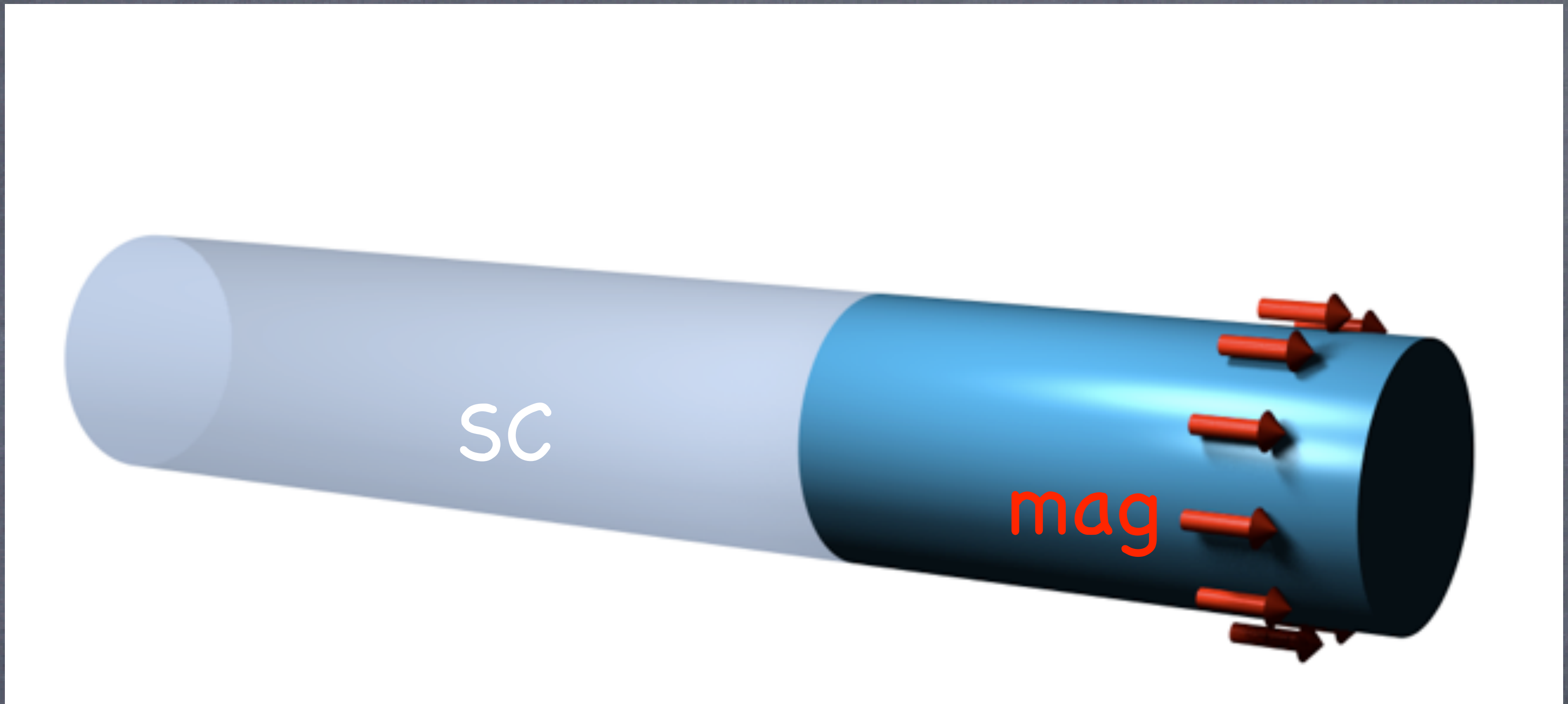
$$\mathcal{H} = \tau_3[-\sigma_2 i\partial_z + \sigma_3 m(z)] - \tau_2\sigma_2\Delta(z)$$



Majorana bound state

$$\Psi_0(z) = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} u_0 \exp \int_0^z dz'[\Delta(z') - m(z')]$$

Irrelevance of magnetic order



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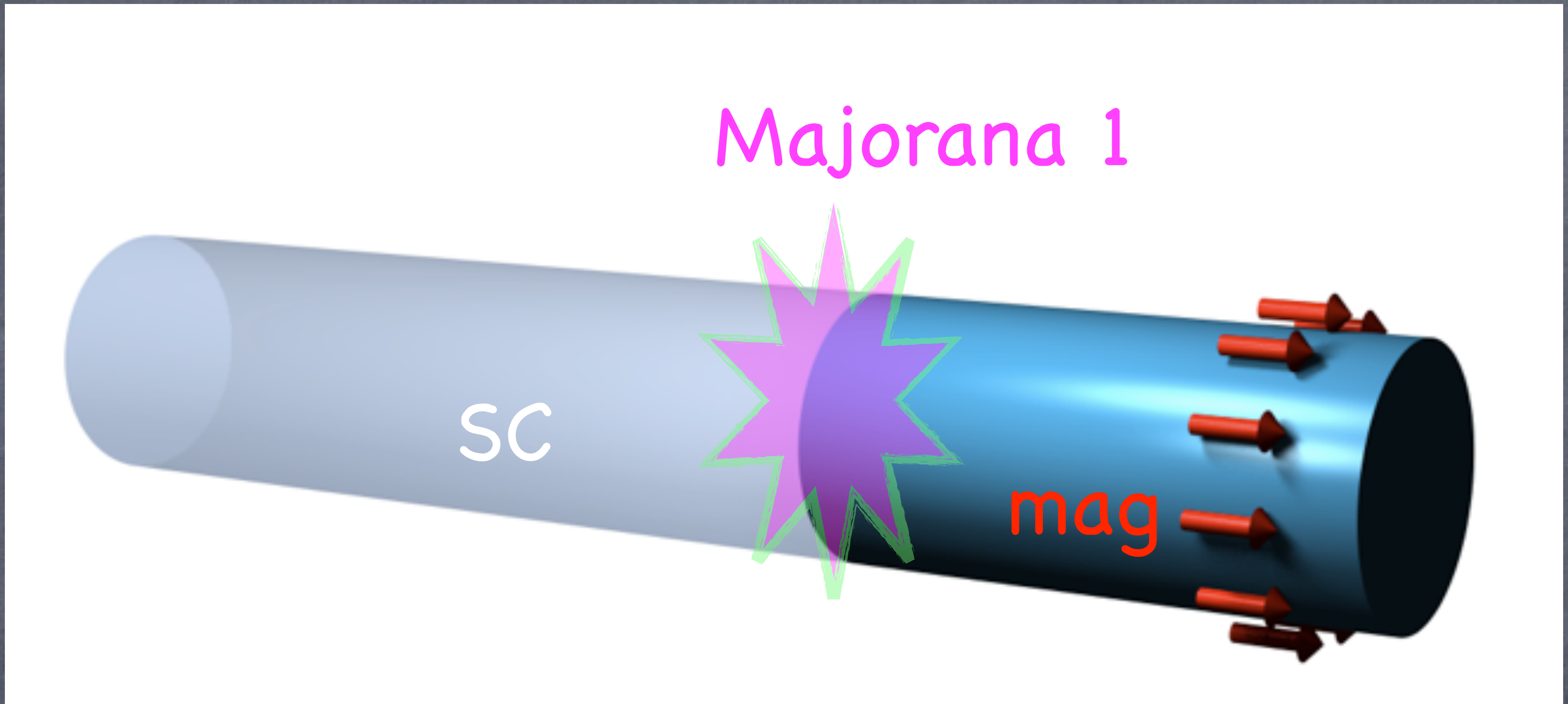

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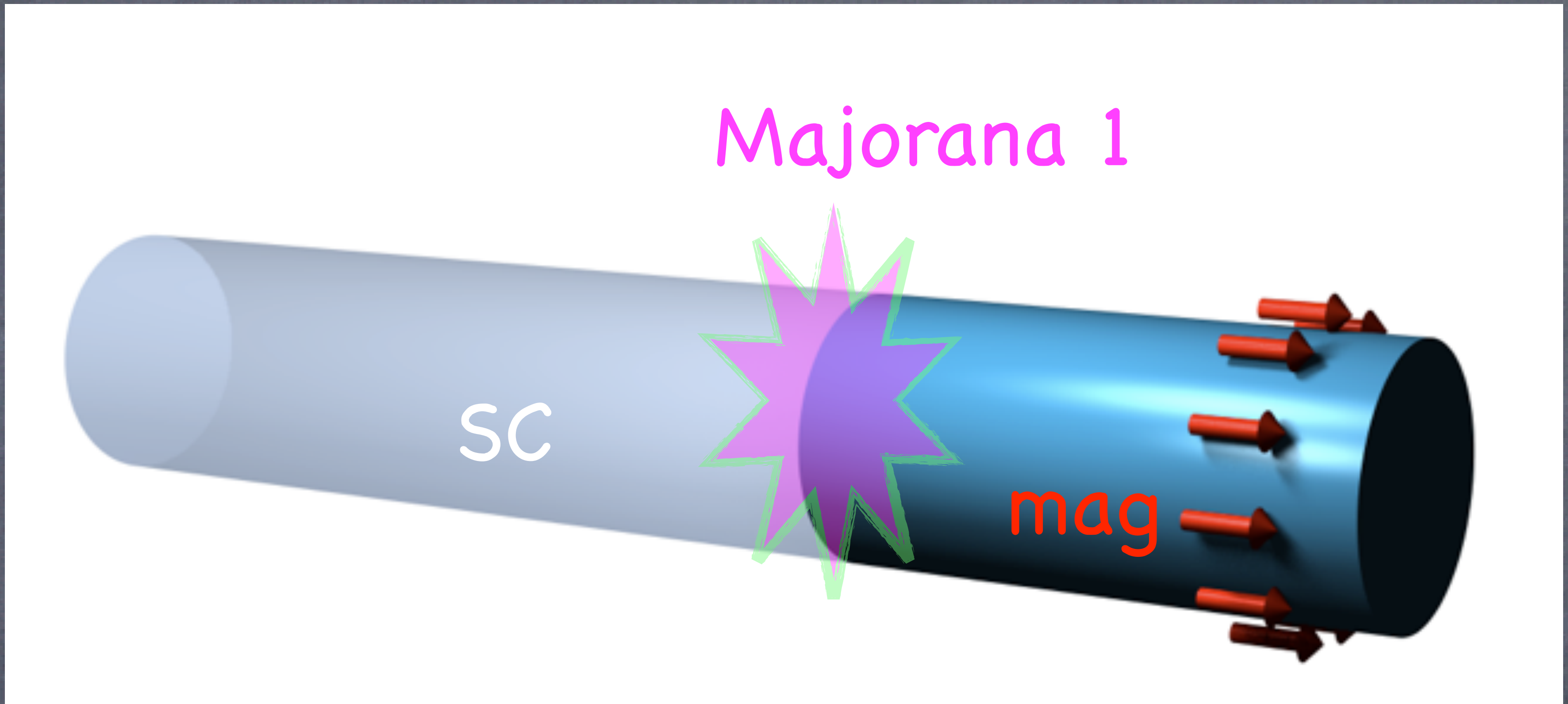
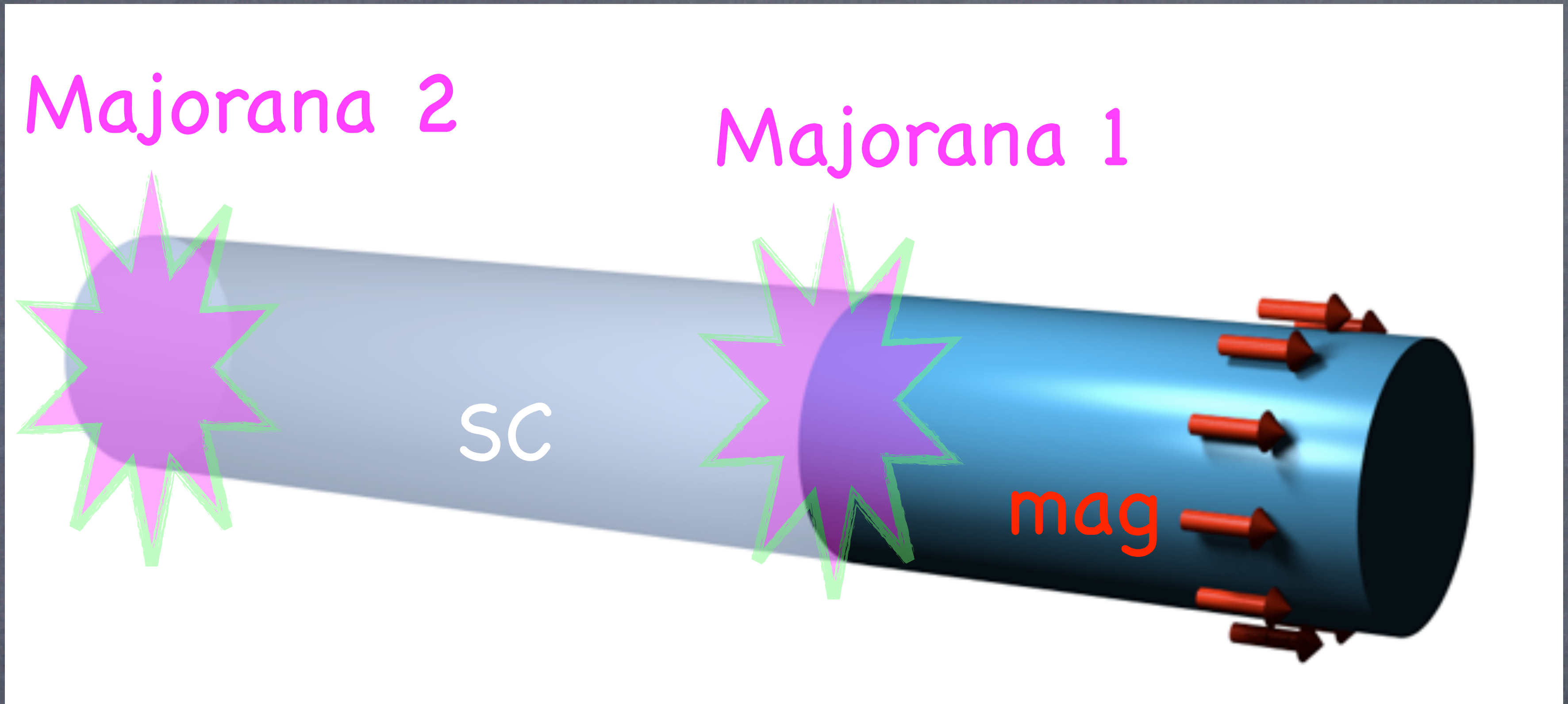


Where is the second Majorana?

Irrelevance of magnetic order



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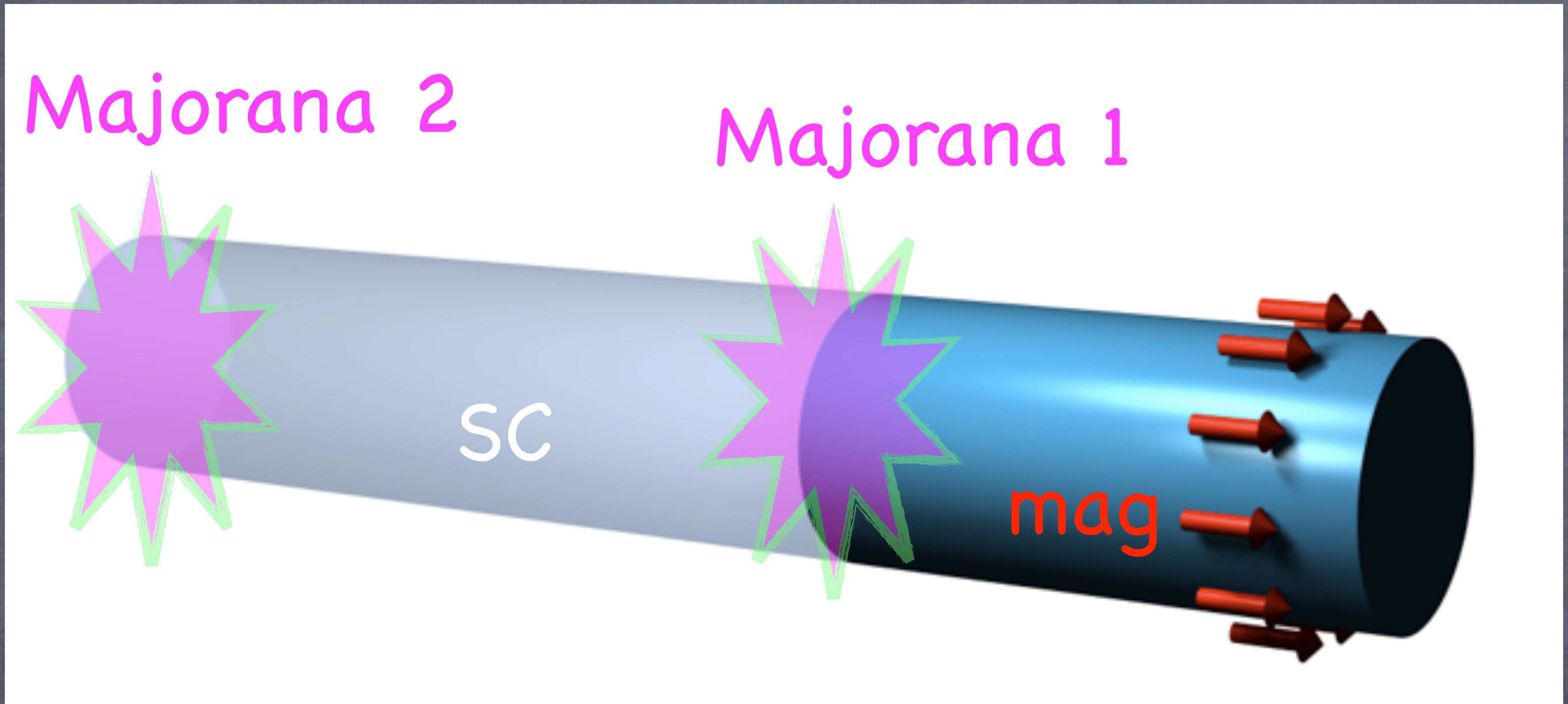


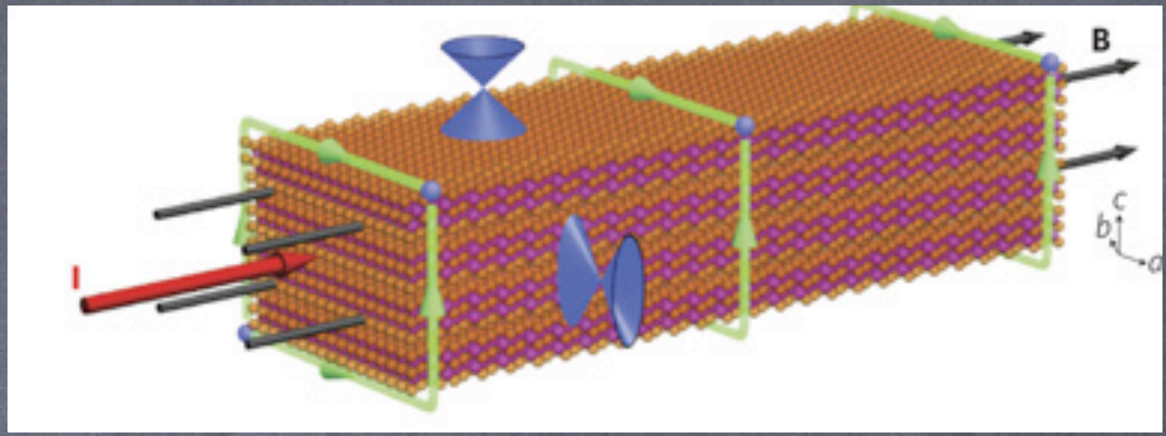
Where is the second Majorana?

Majorana fermion existence at the ends of a SC wire is independent of local details

Lattice model - numerical results

Exact numerical diagonalization for a Bi_2Se_3 model regularized on a simple cubic lattice in a wire geometry:

[L. Fu and E. Berg, PRL. **105**, 097001 (2010)]



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$$M_{\mathbf{k}} = \epsilon - 2t \sum_\alpha \cos k_\alpha$$

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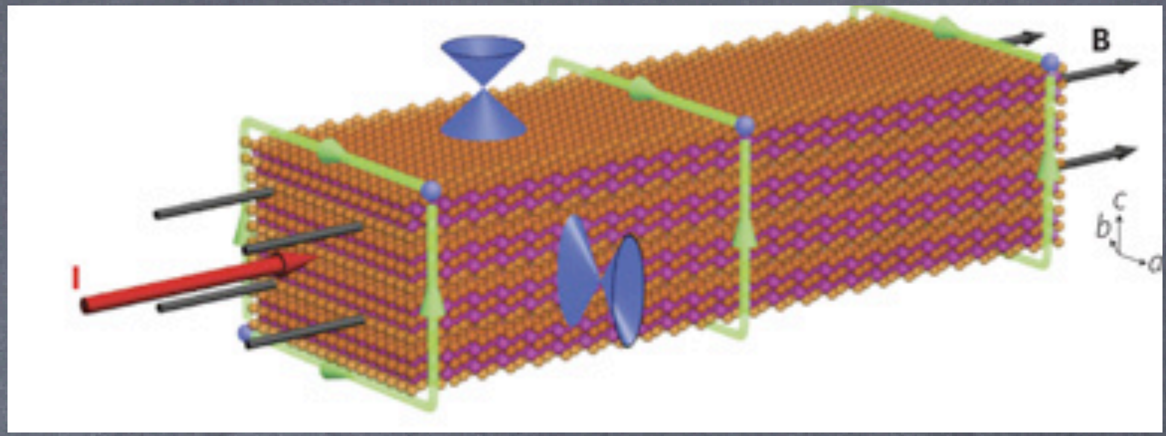

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$$M_{\mathbf{k}} = \epsilon - 2t\sum_\alpha \cos k_\alpha$$

For $2t < \epsilon < 6t$ this model describes strong TI in Z_2 class (1;000)

Include magnetic field by Peierls substitution and Zeemann coupling

$$t_{ij} \quad \rightarrow \quad t_{ij} \exp\left[\frac{2\pi i}{\Phi_0}\int_i^j \mathbf{A}\cdot d\mathbf{l}\right]$$

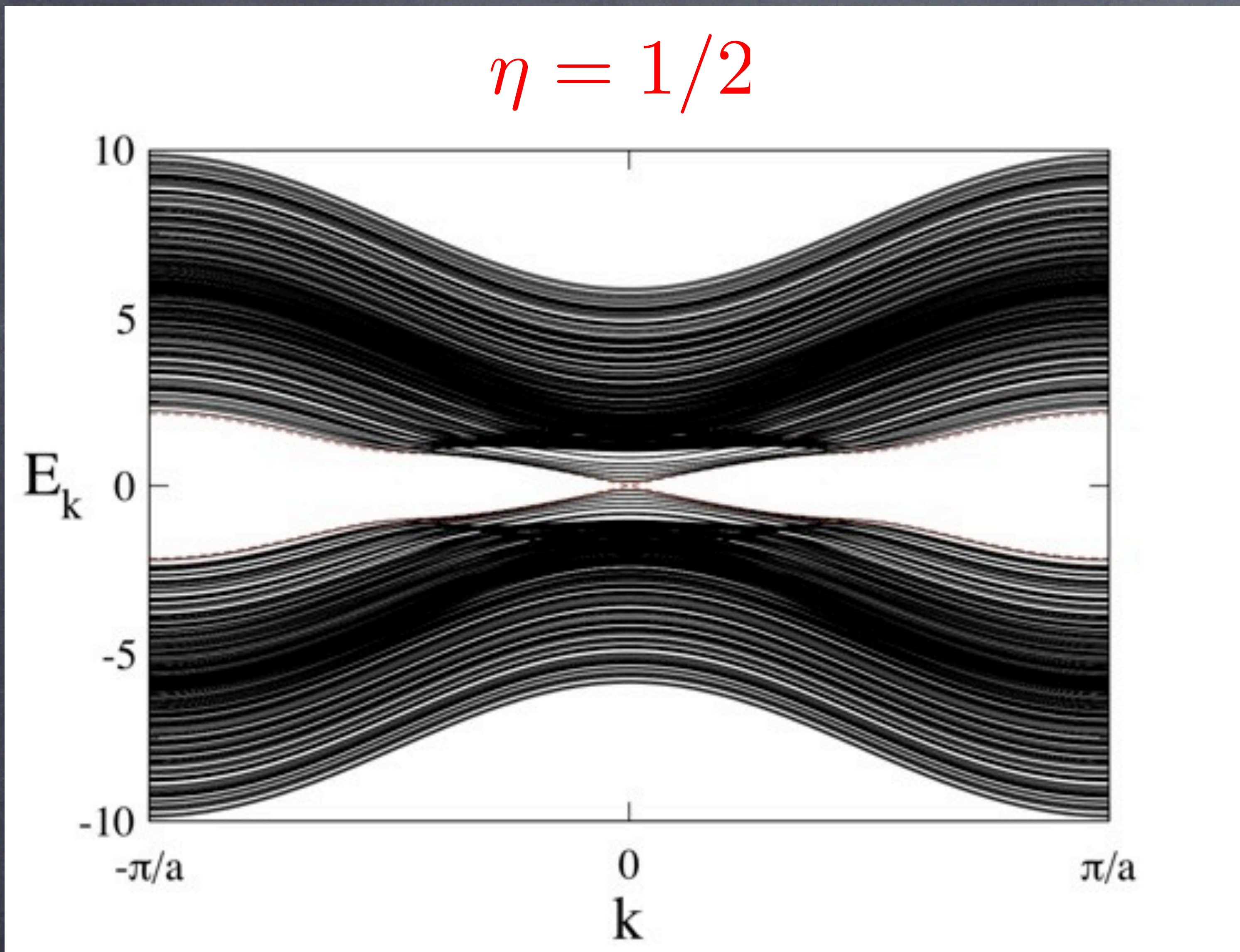
$$H_Z \quad = \quad -g\mu_B \mathbf{B}\cdot\sigma$$

Include magnetic field by Peierls substitution and Zeemann coupling

$$
\begin{aligned}
t_{ij} &\rightarrow t_{ij} \exp\left[\frac{2\pi i}{\Phi_0}\int_i^j \mathbf{A}\cdot d\mathbf{l}\right] \\
H_Z &= -g\mu_B \mathbf{B}\cdot\sigma
\end{aligned}
$$

Solve by exact numerical diagonalization and sparse matrix techniques

20x20 wire, infinite length, normal state



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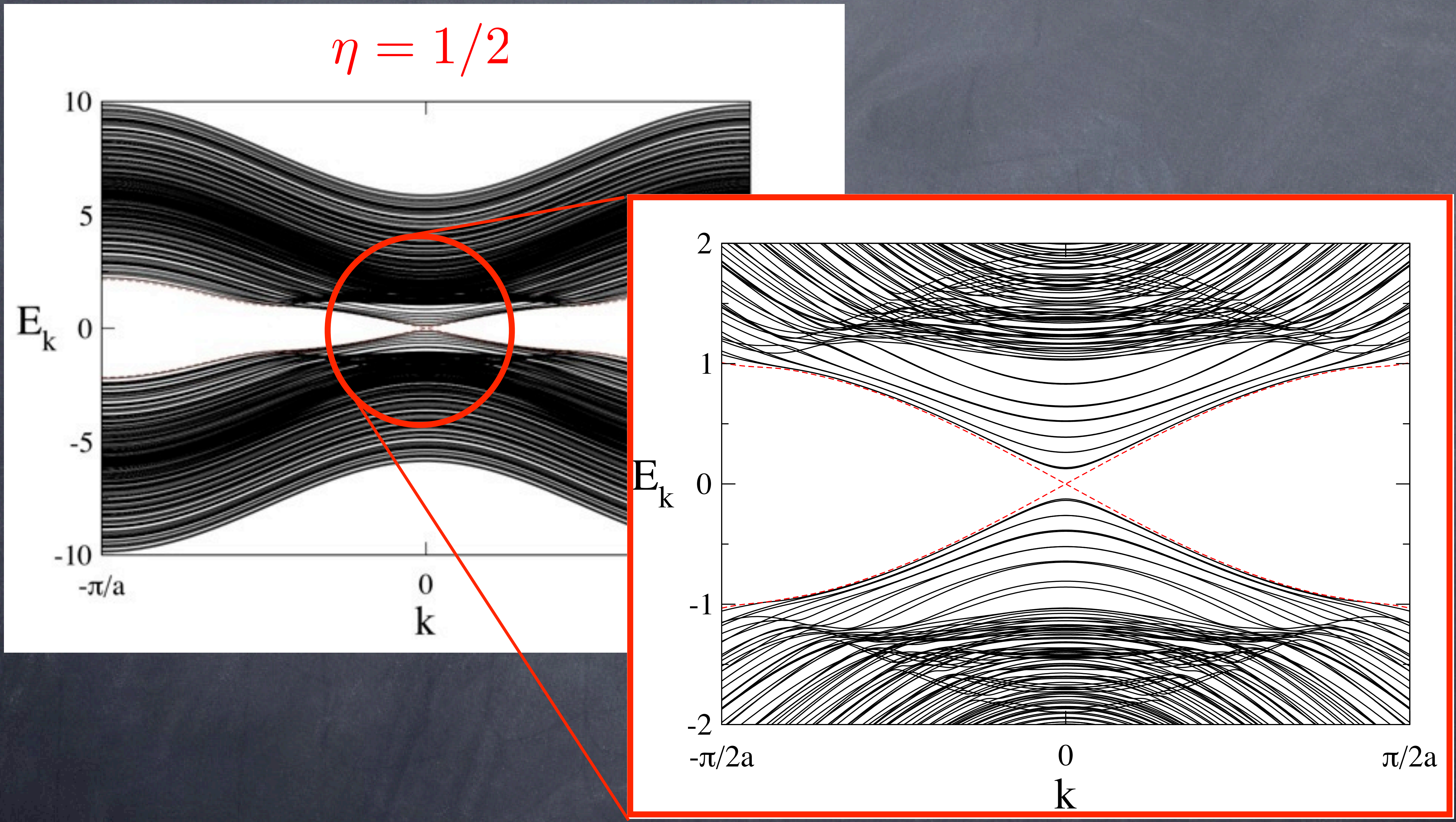

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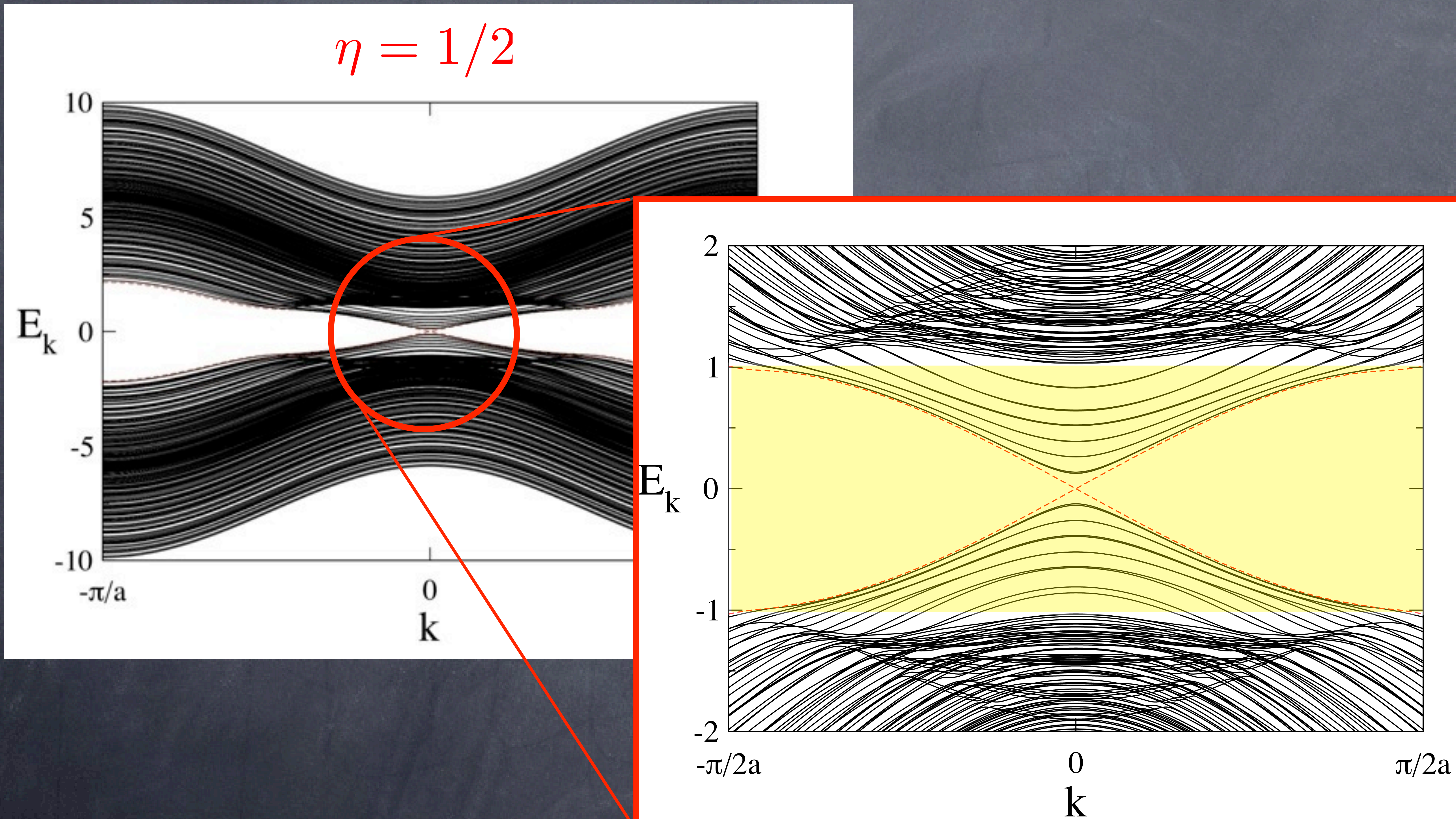
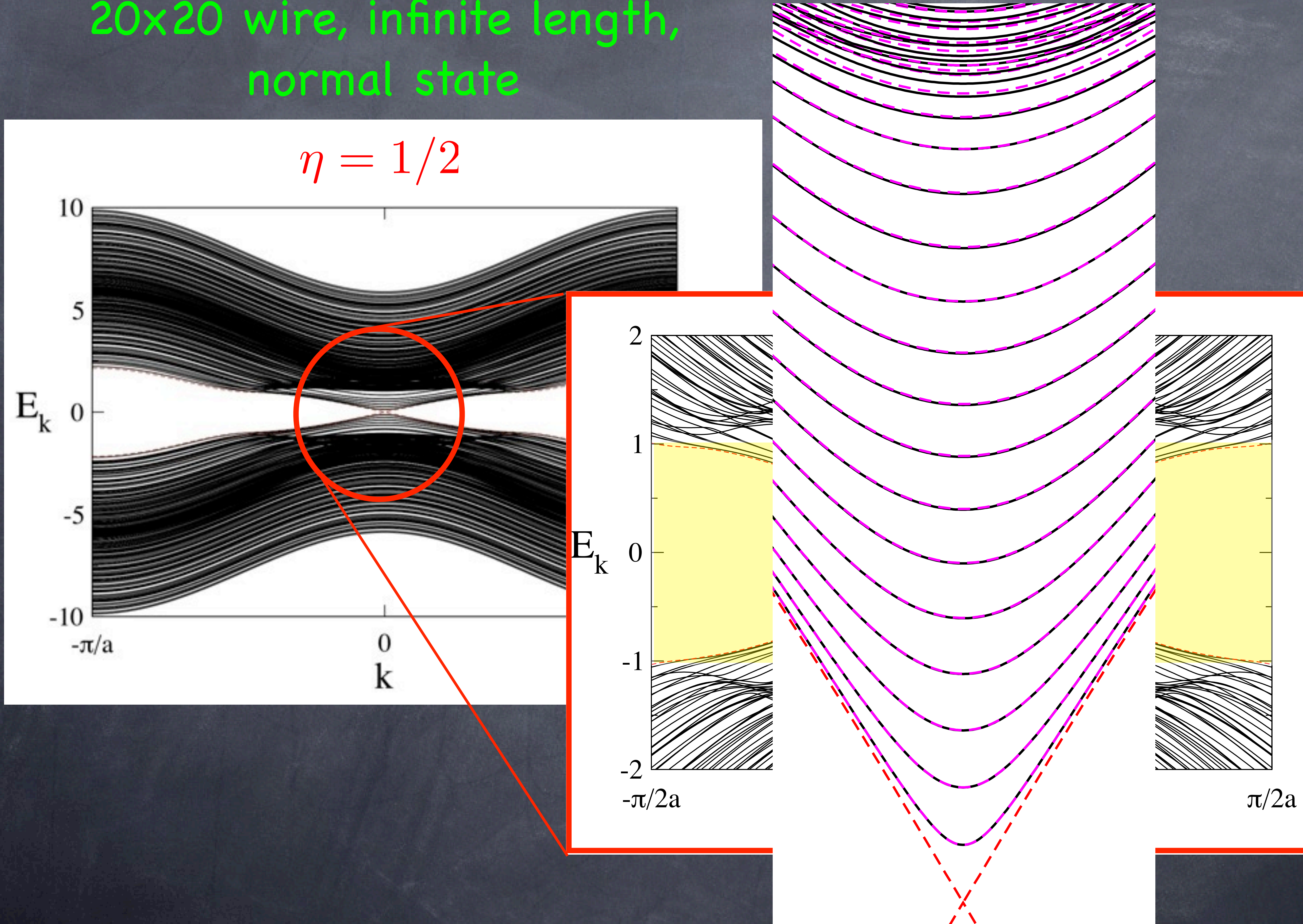
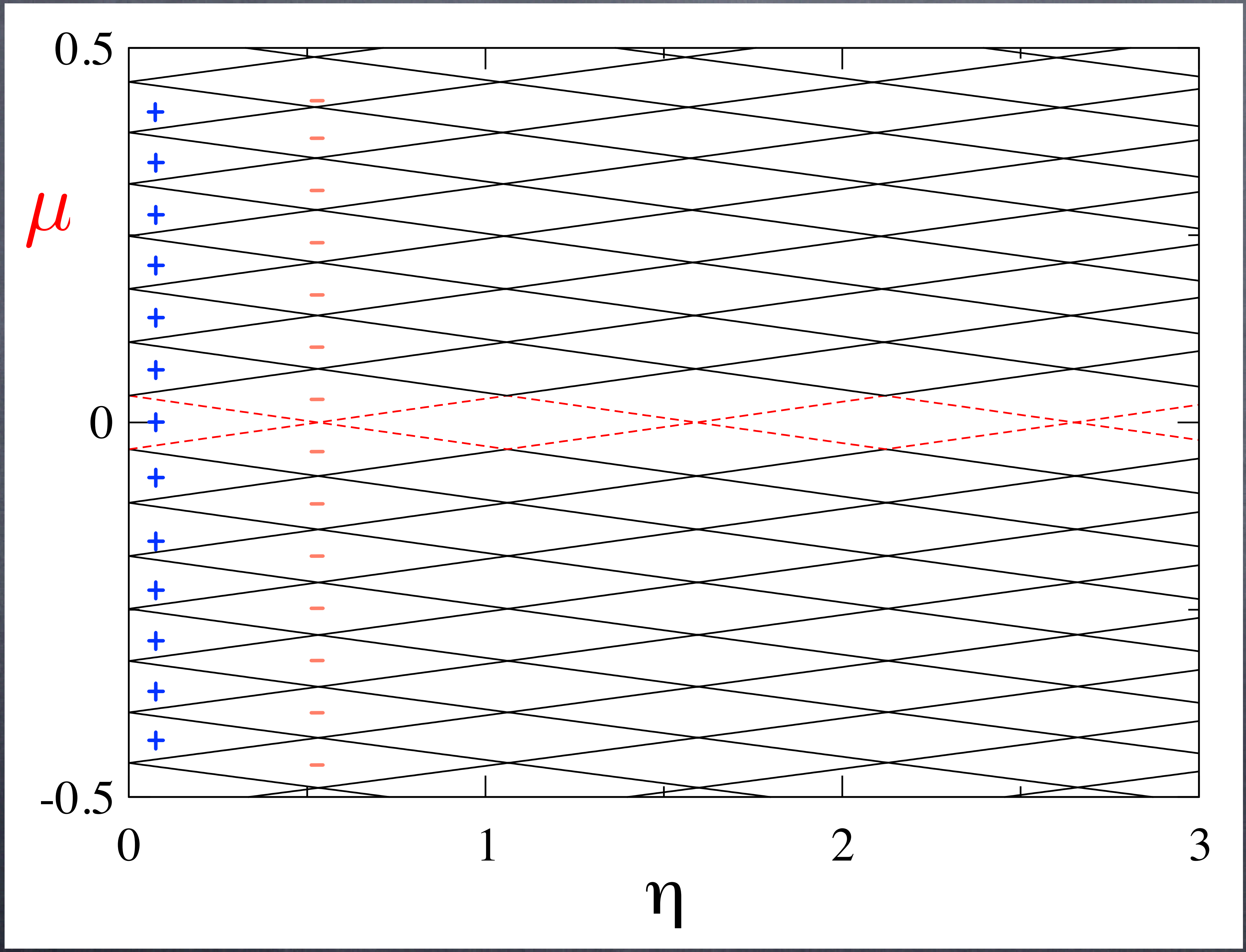


20x20 wire, infinite length, normal state



Majorana number



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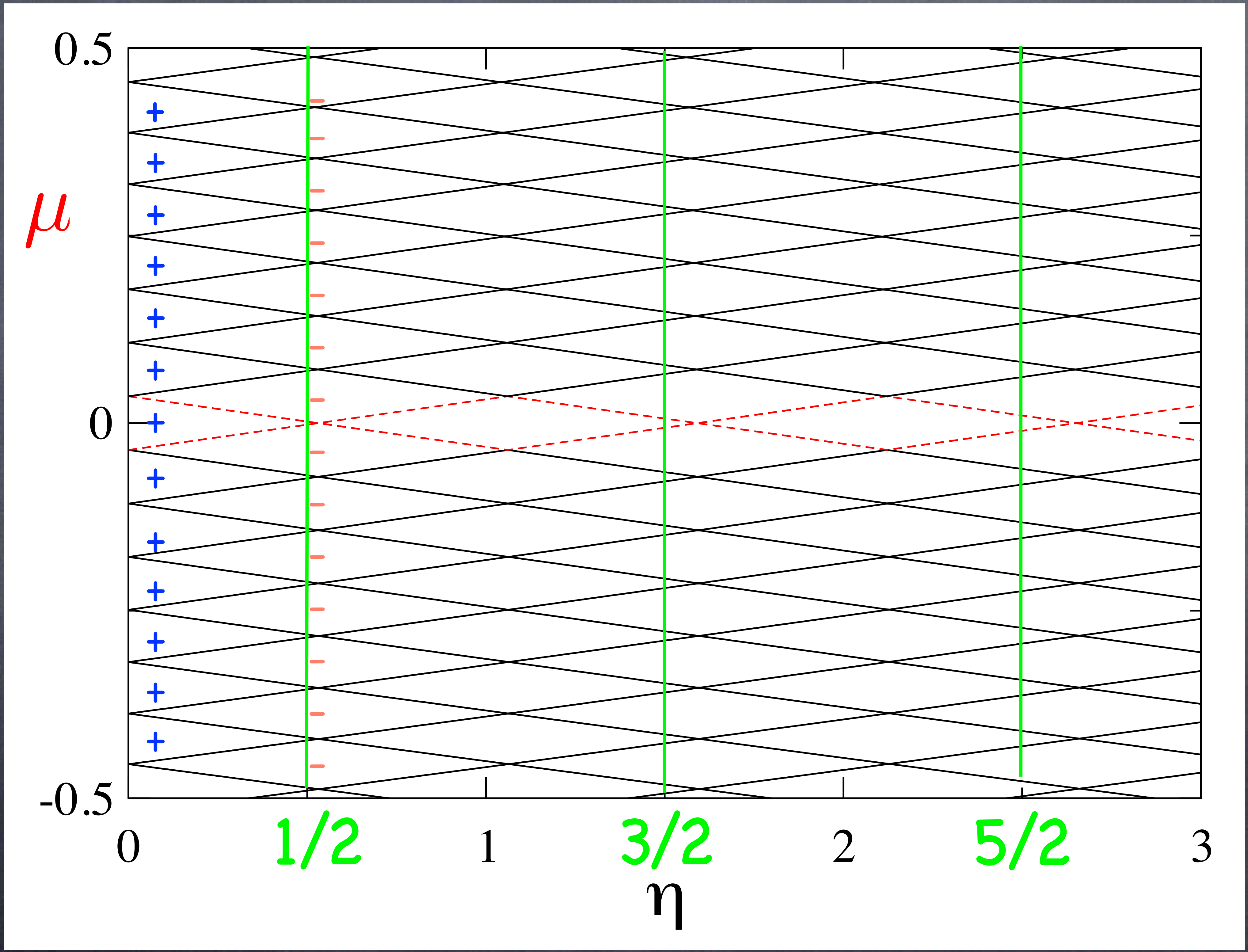

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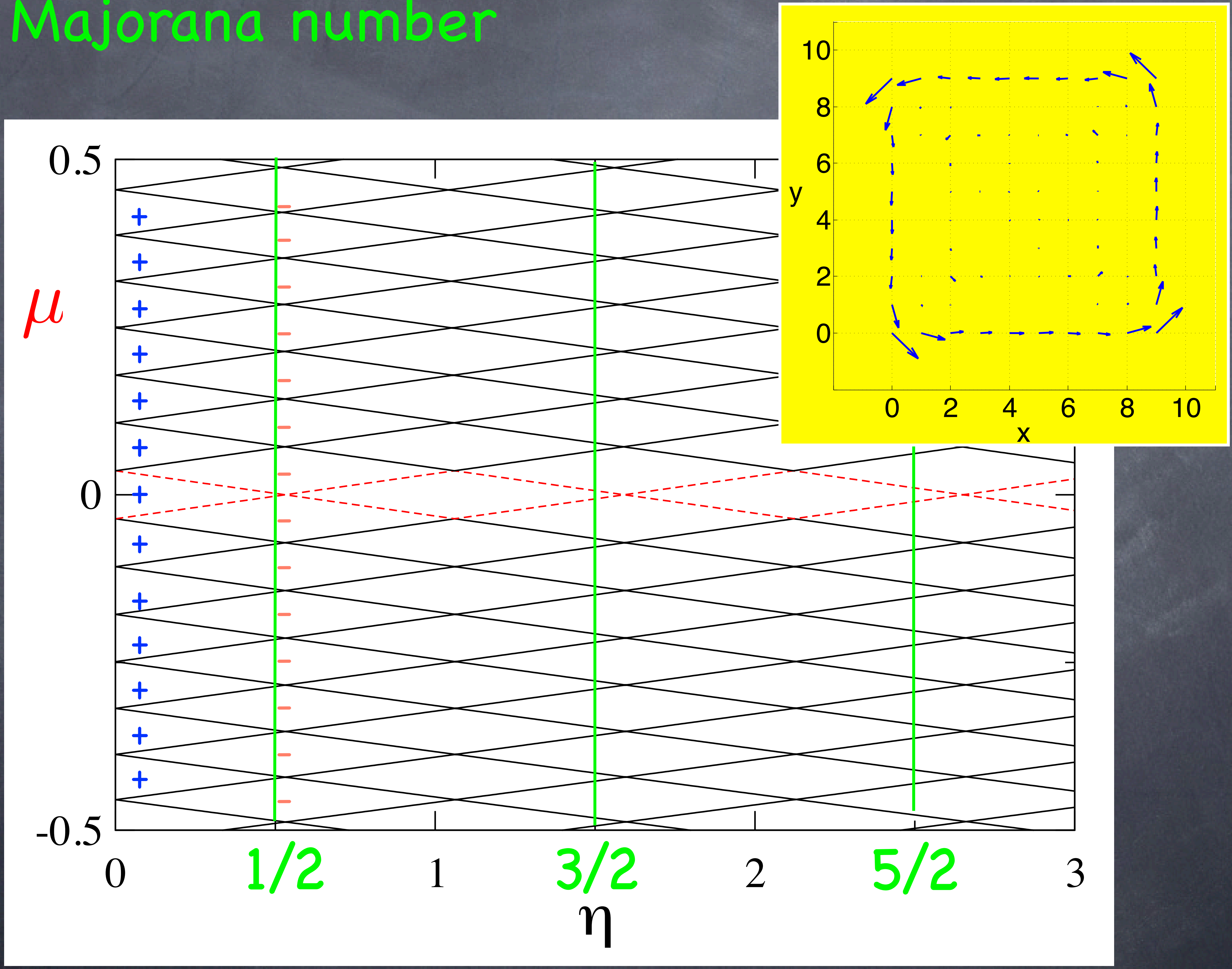
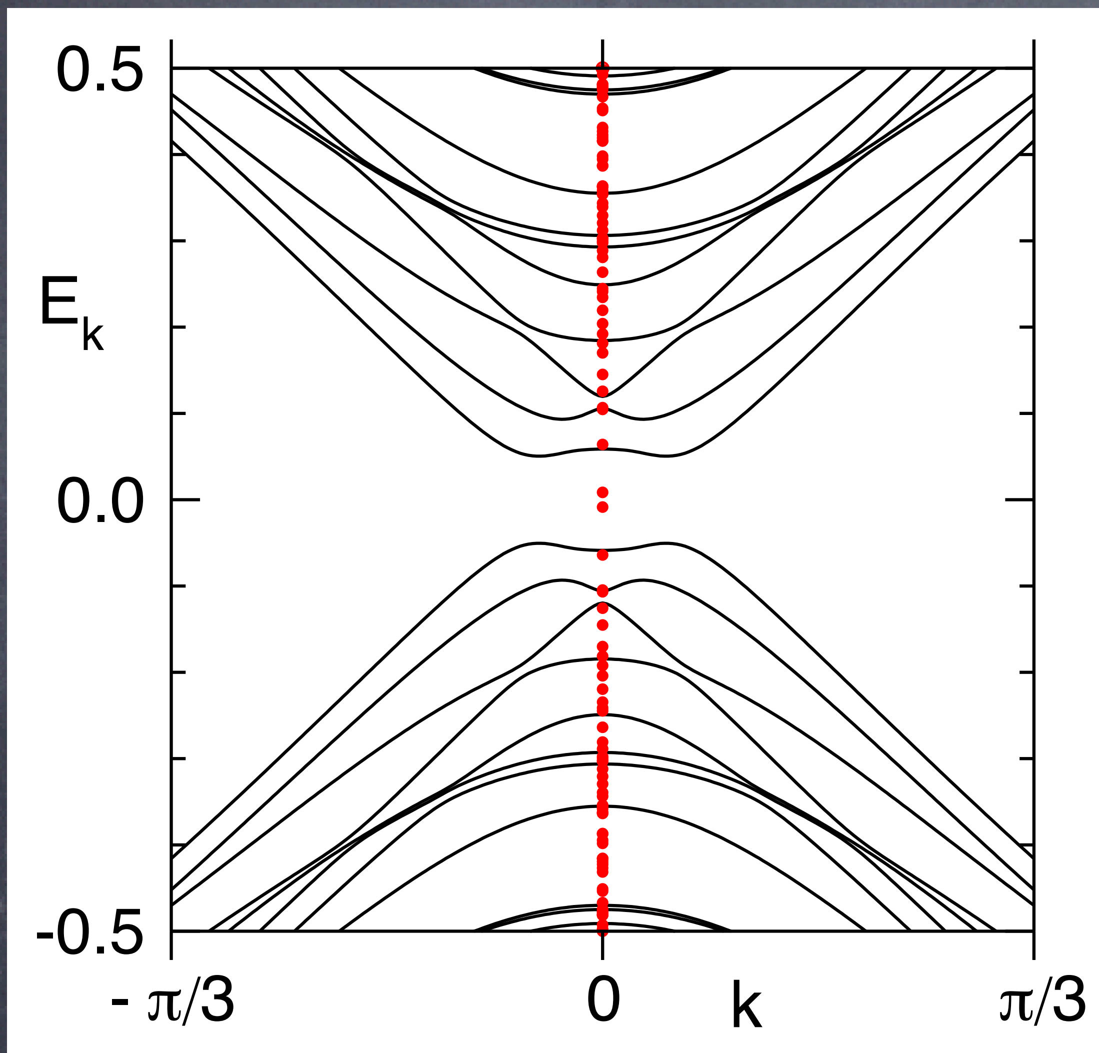


Superconducting state



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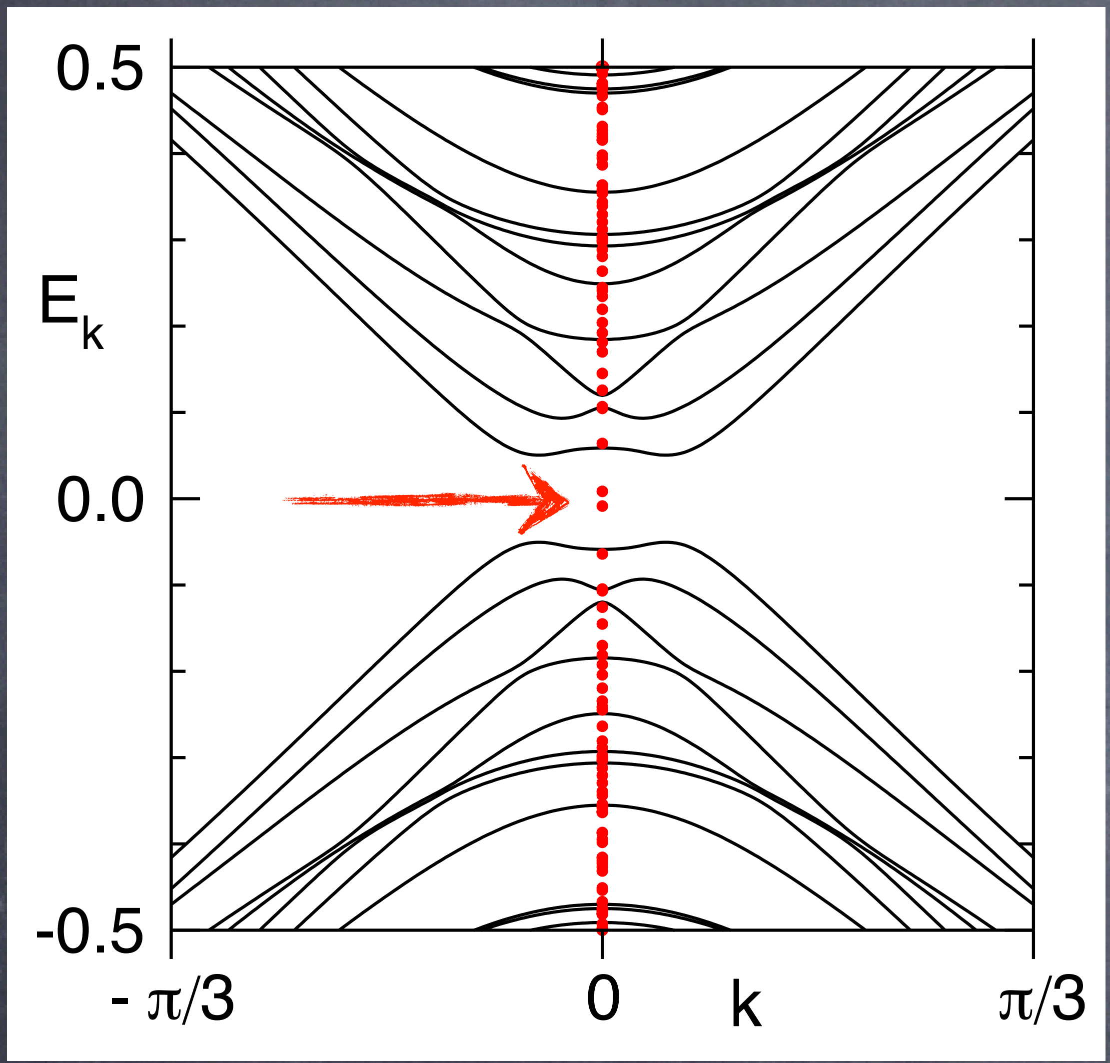

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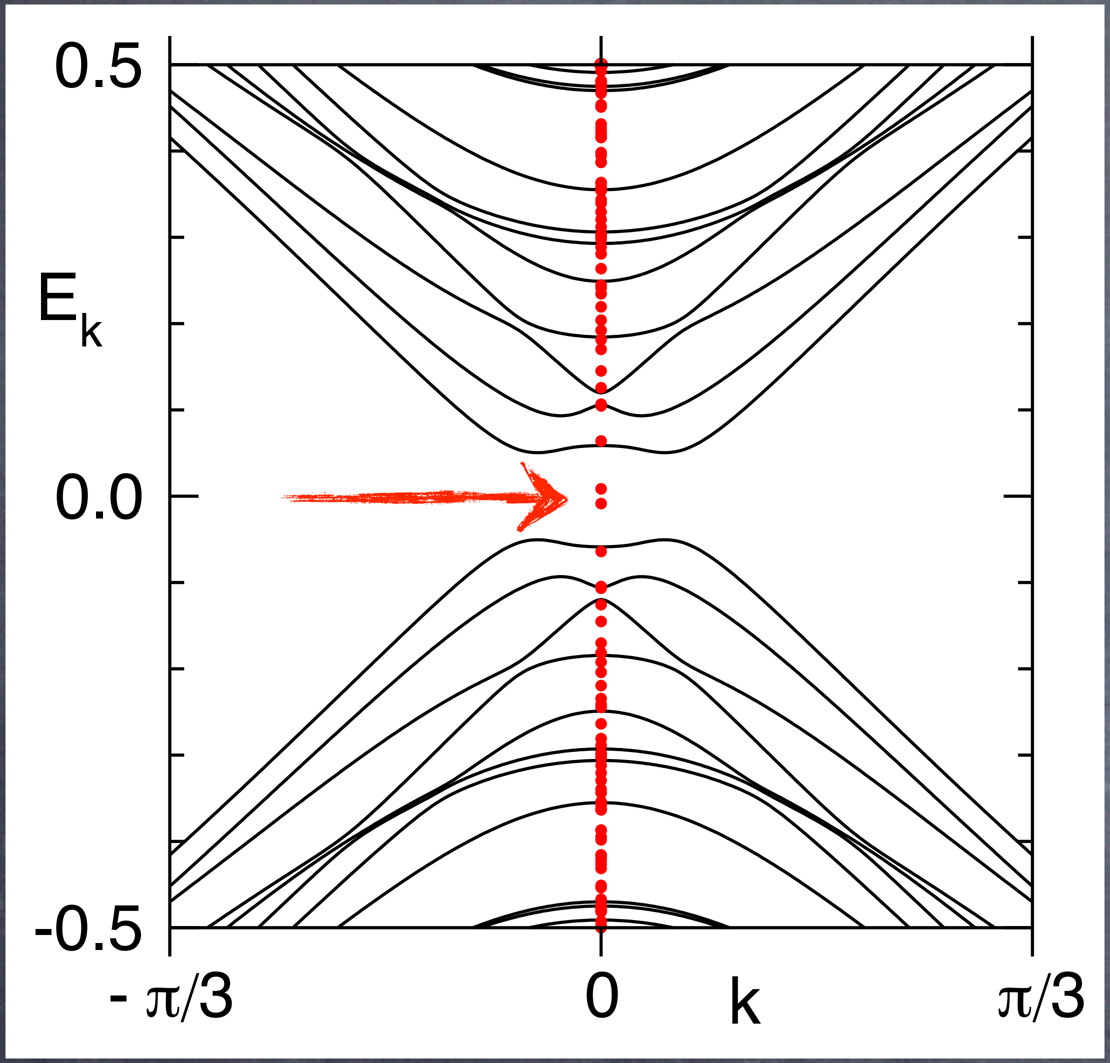

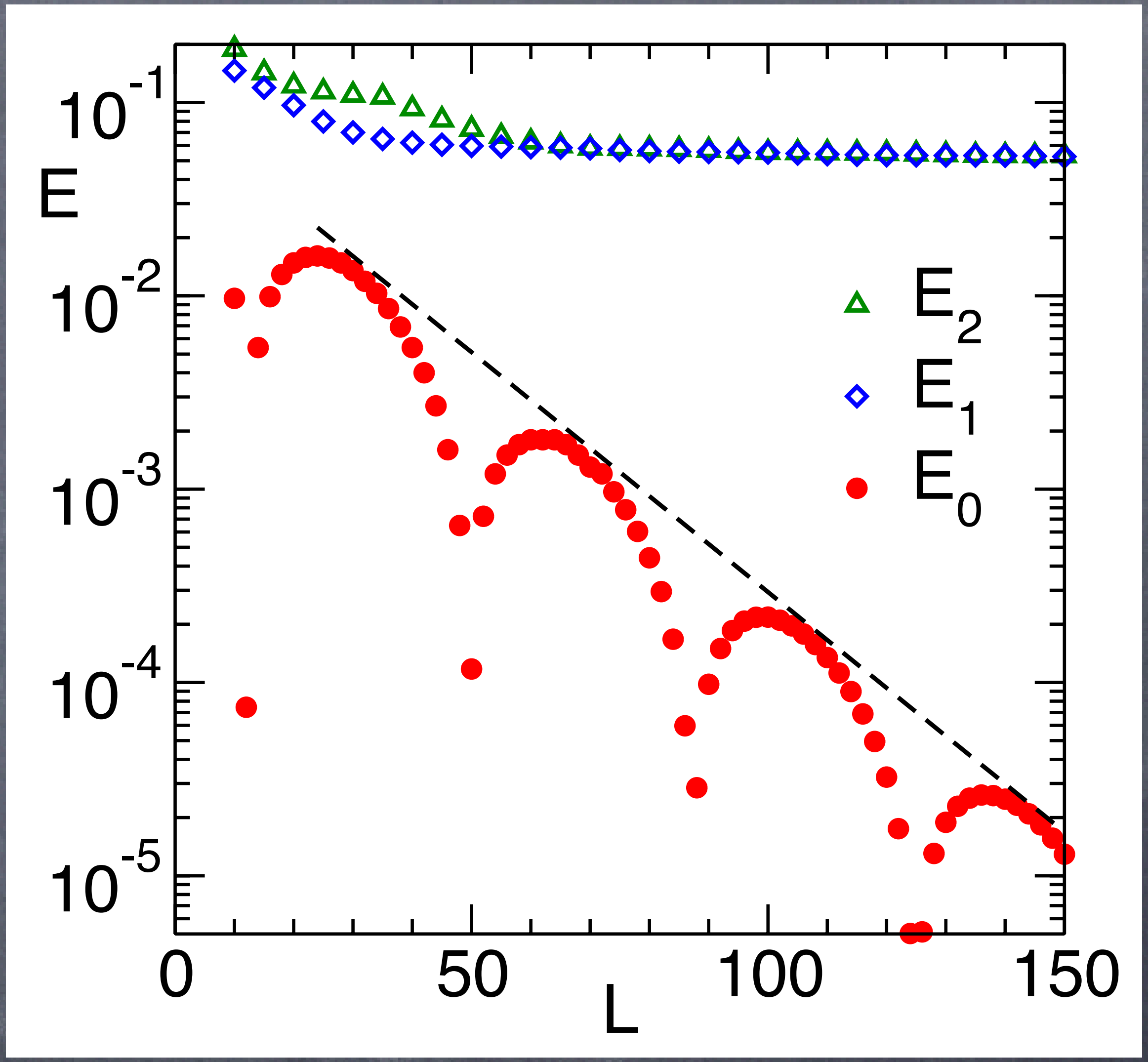

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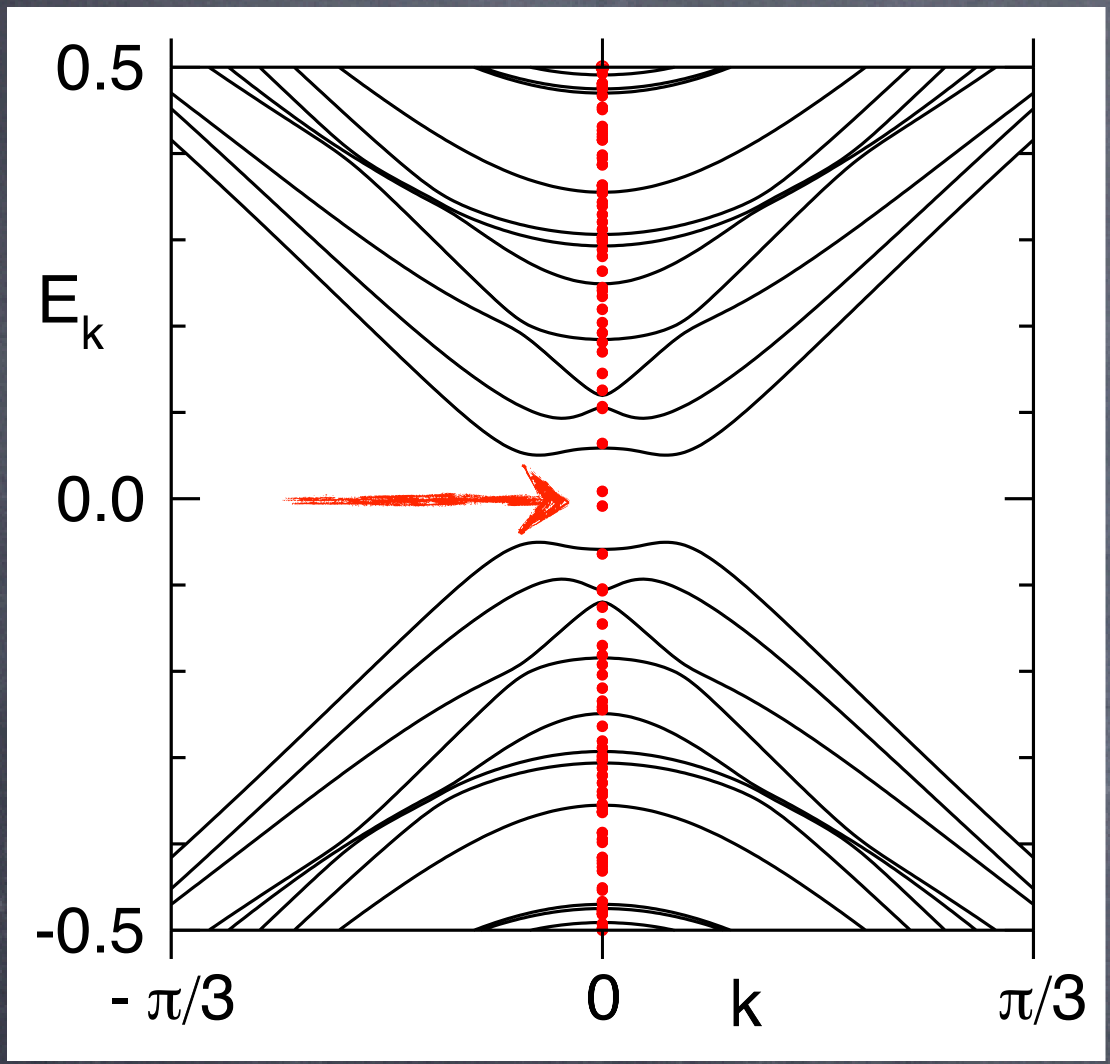
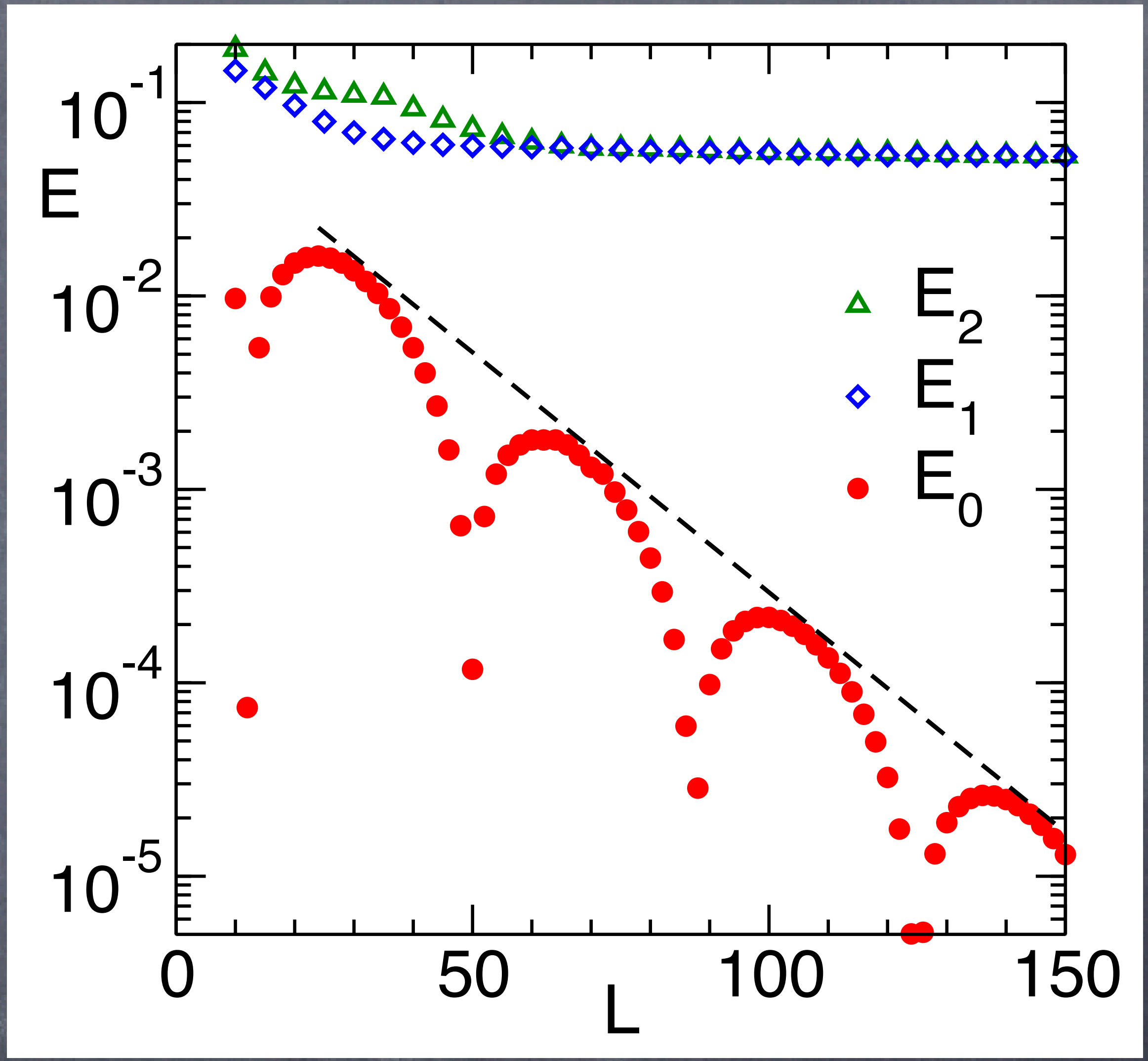
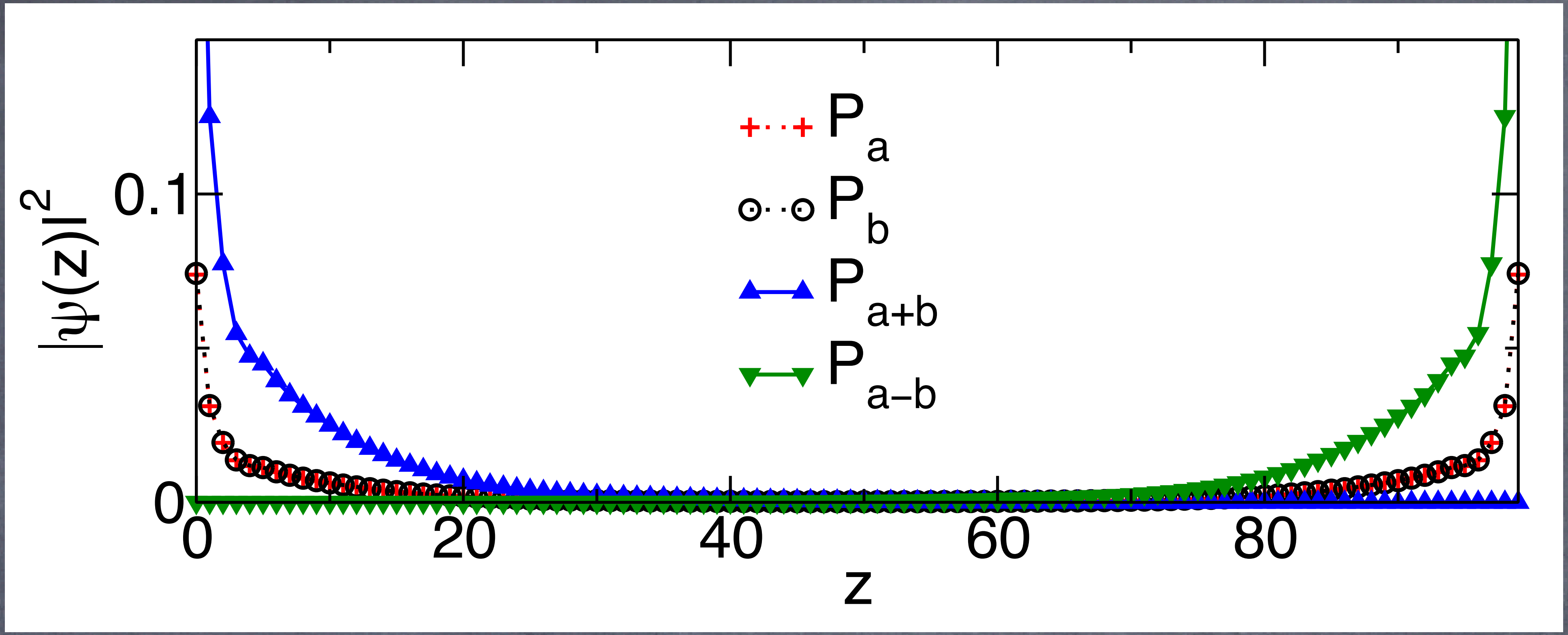




For large wire length L we observe isolated energy eigenvalue exponetially approaching zero.

Zero-mode eigenfunctions are localized near wire ends



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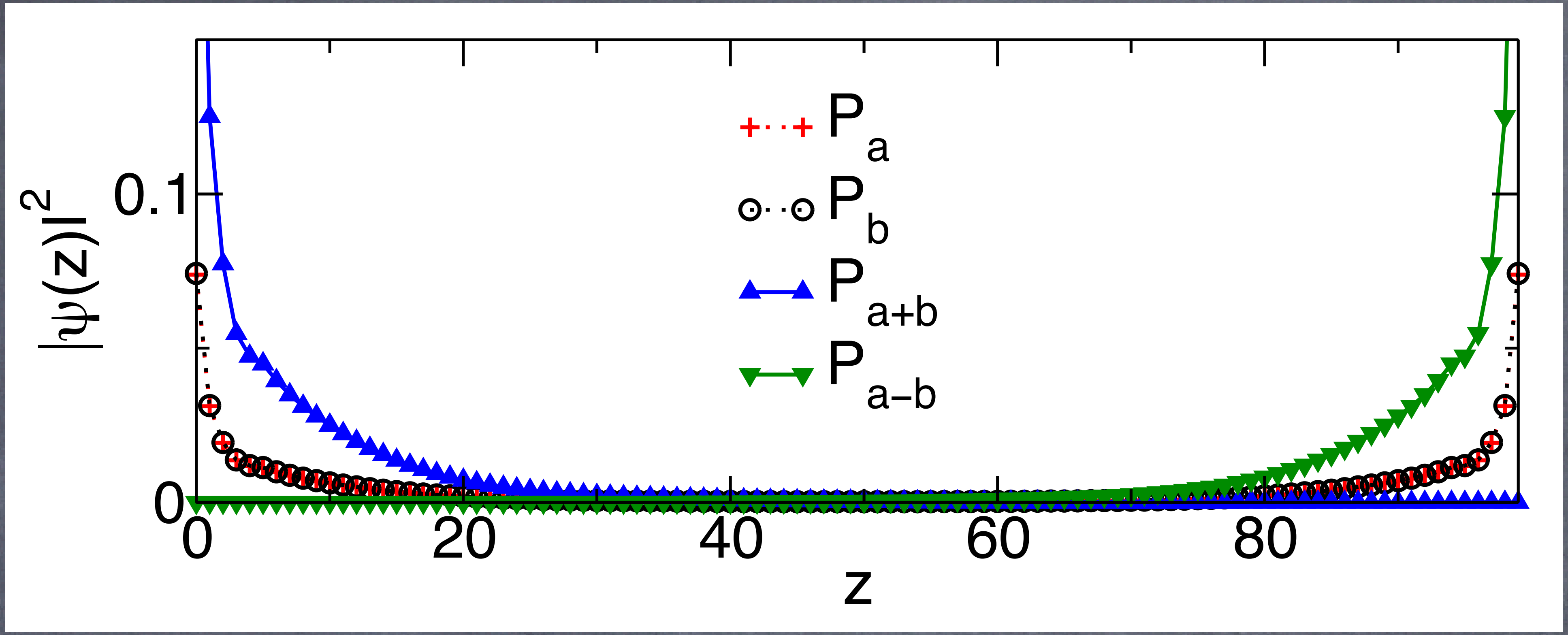

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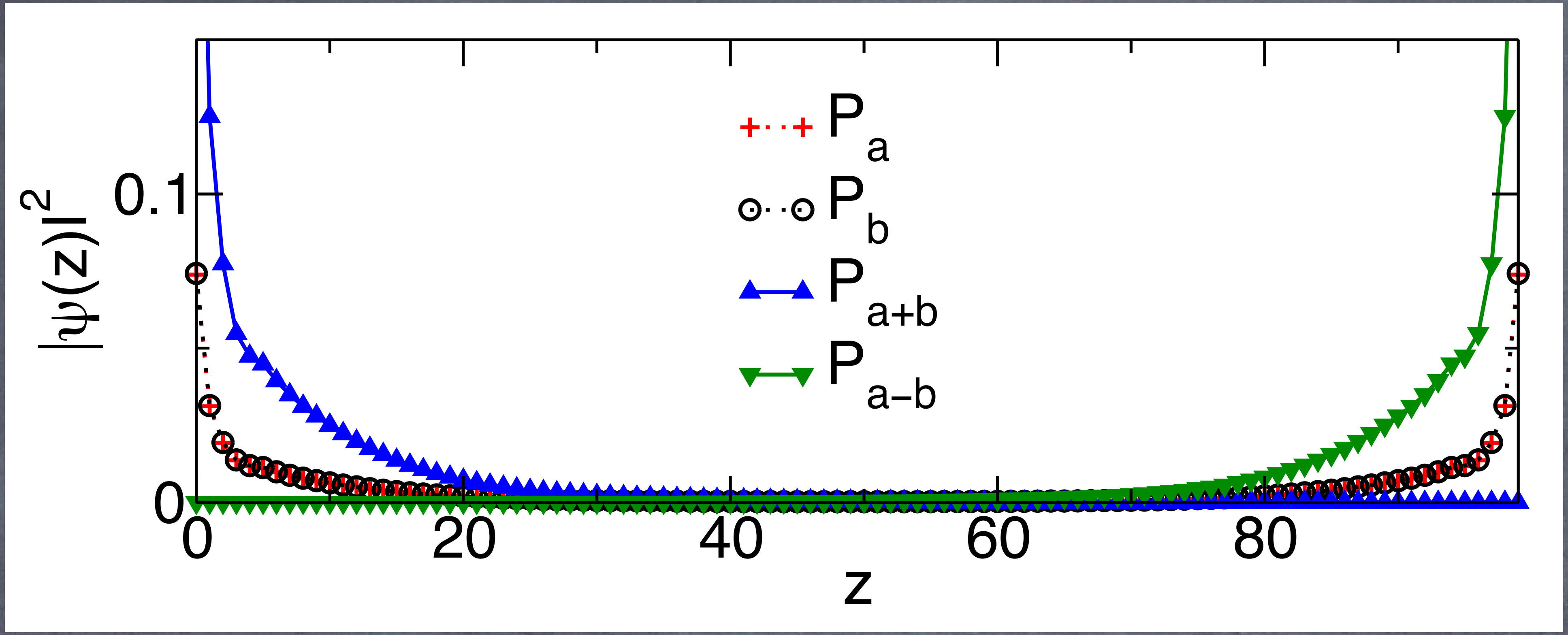

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Near-zero modes found in numerical calculation provide strong evidence for the expected Majorana end states

Effects of disorder

- Robustness of SC gap with respect to non-magnetic disorder: expect on the basis of Anderson's theorem
- Robustness of Majorana end states with respect to disorder

Study on-site disorder described by Hamiltonian

$$H_{\rm dis} = H_0 + \sum_{i\alpha} U_i c^\dagger_{i\alpha} c_{i\alpha}, \qquad U_i \in (-U/2, U/2)$$

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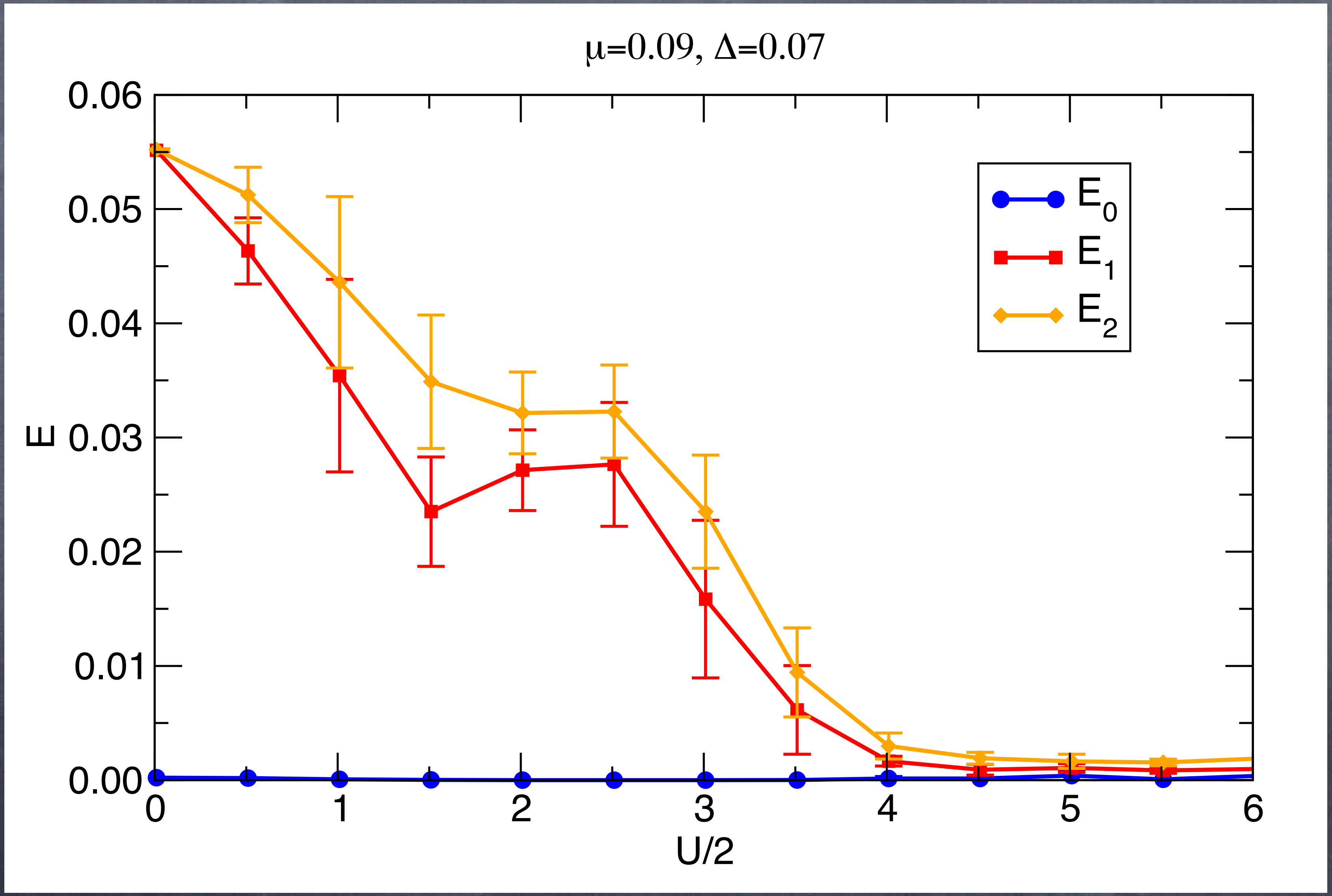

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$$\mathcal{M}(H) = \mathrm{sgn}[\mathrm{Pf}(\tilde{H}(k=0))\mathrm{Pf}(\tilde{H}(k=\pi))]$$

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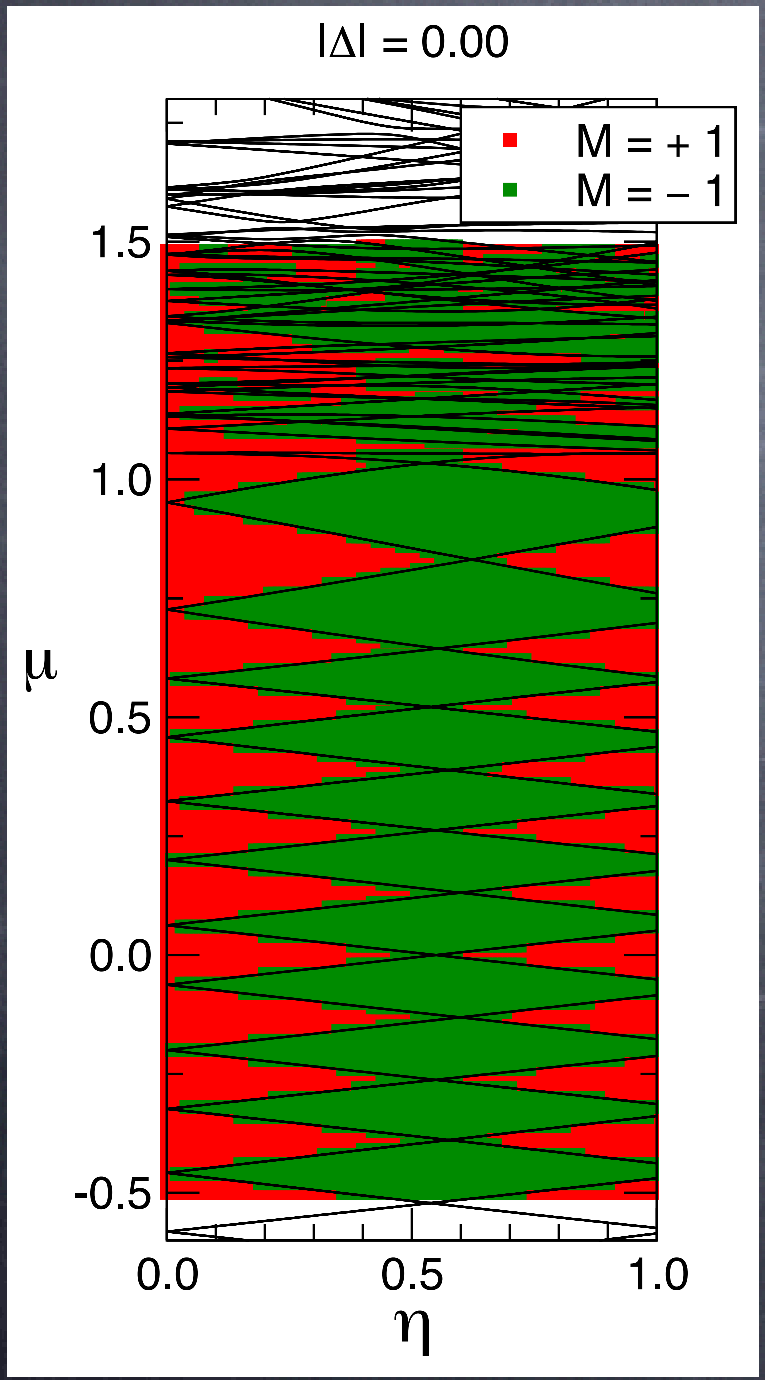

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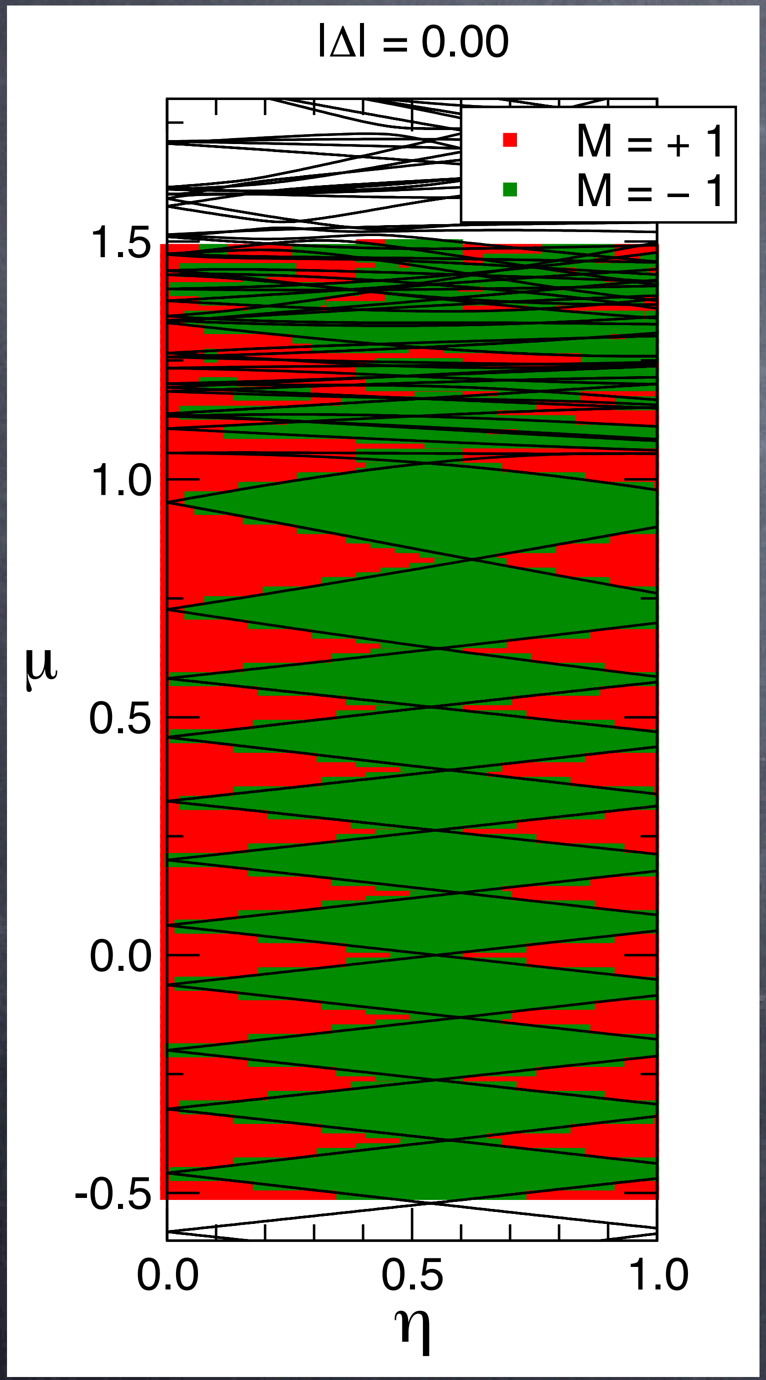

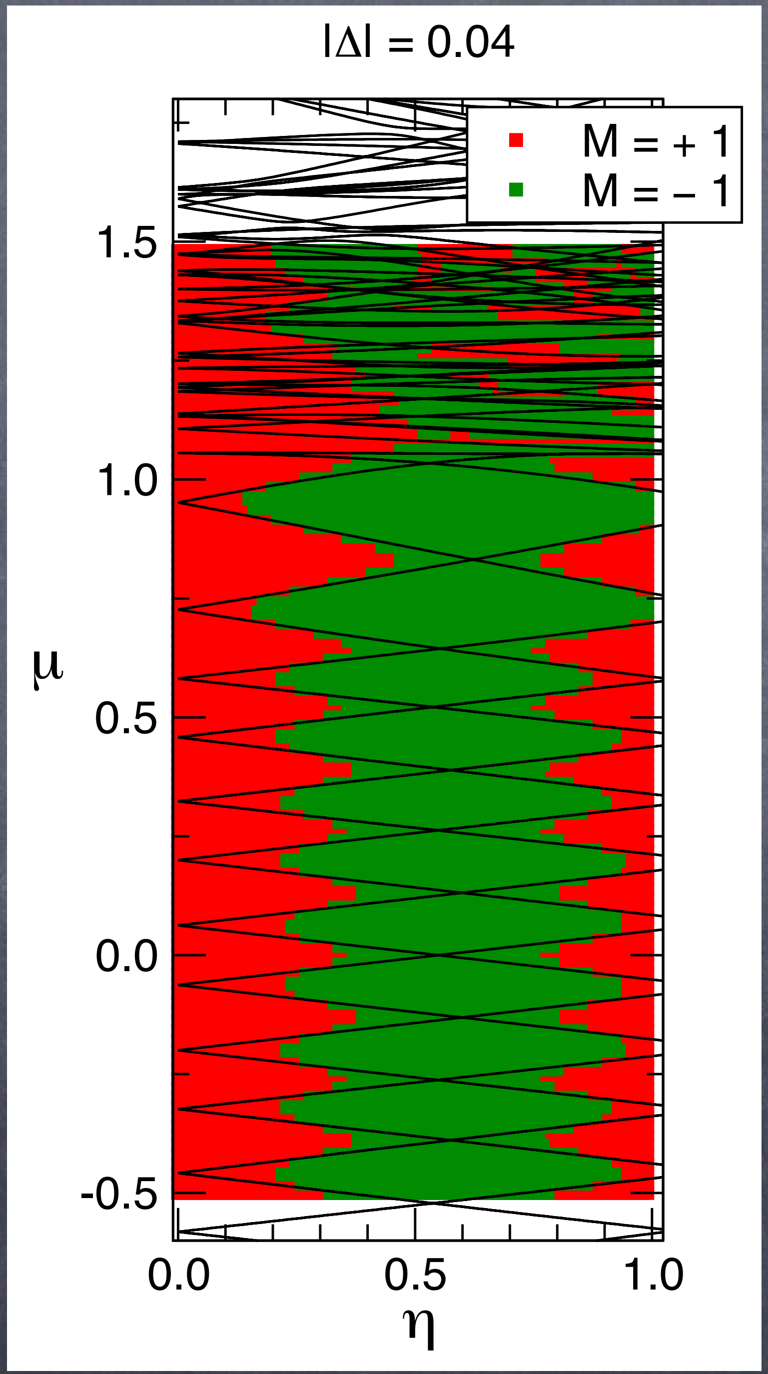

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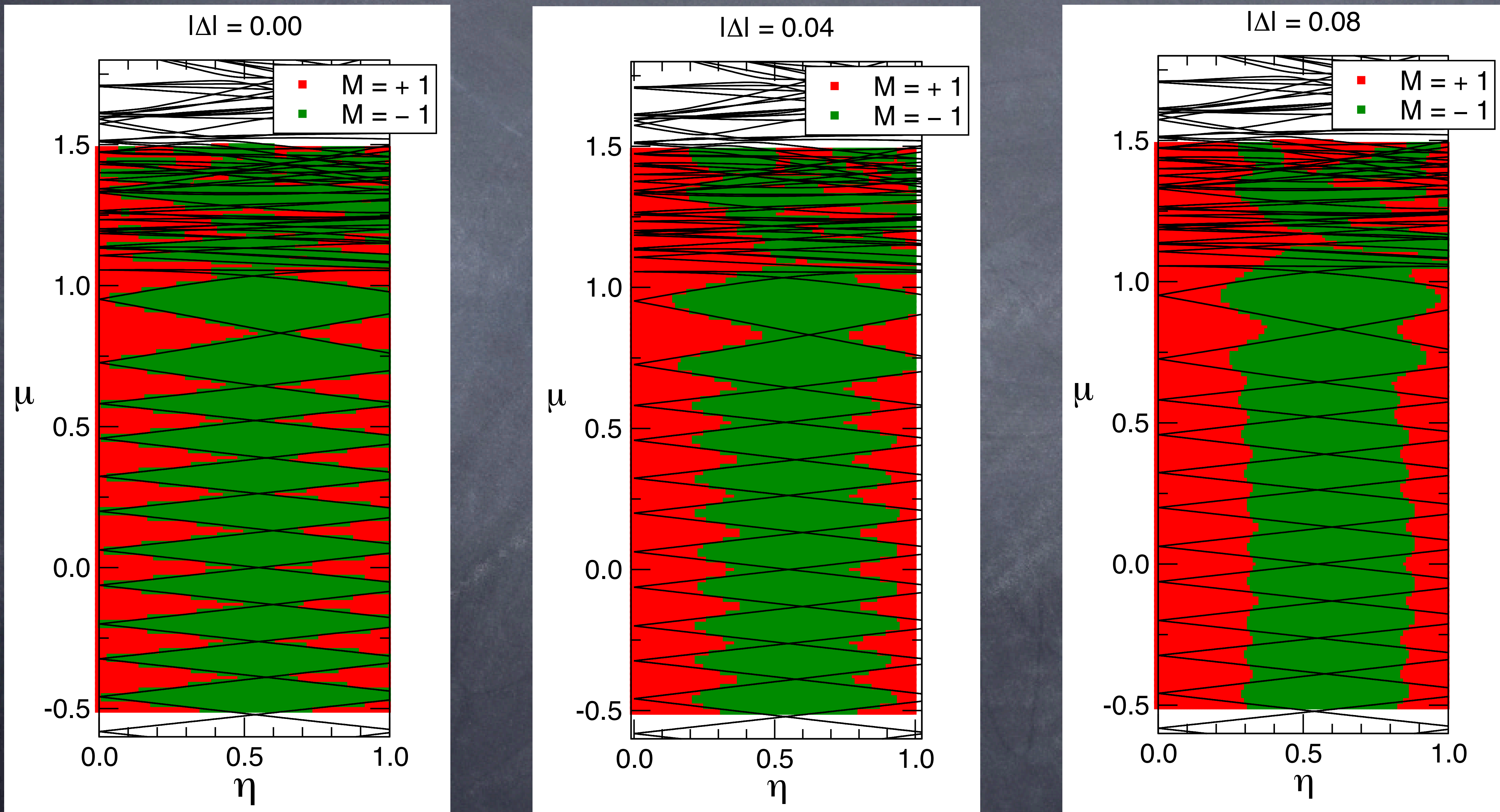
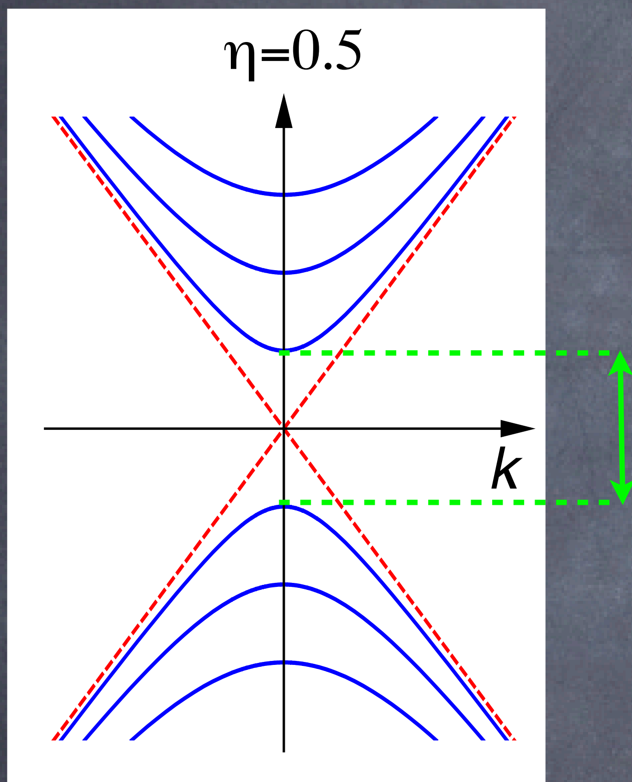


Experimental considerations



The existing Bi_2Se_3 nanoribbons have typical cross section area $S \approx 6 \times 10^{-15} \mathrm{m}^2$

$$\delta E_S \simeq 2v\hbar\sqrt{\pi/S} \simeq 14\mathrm{meV}$$

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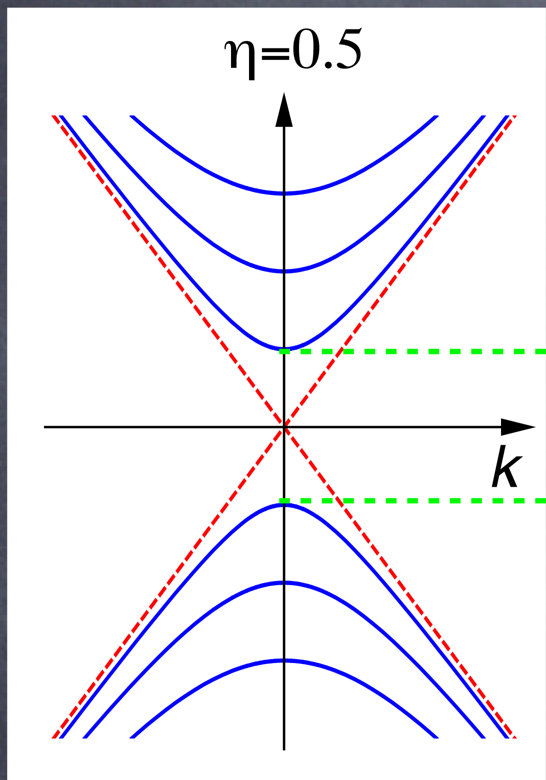

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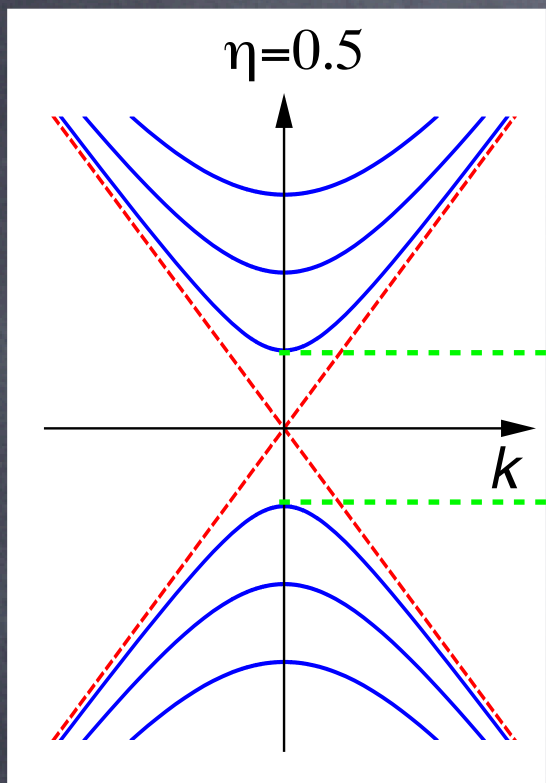

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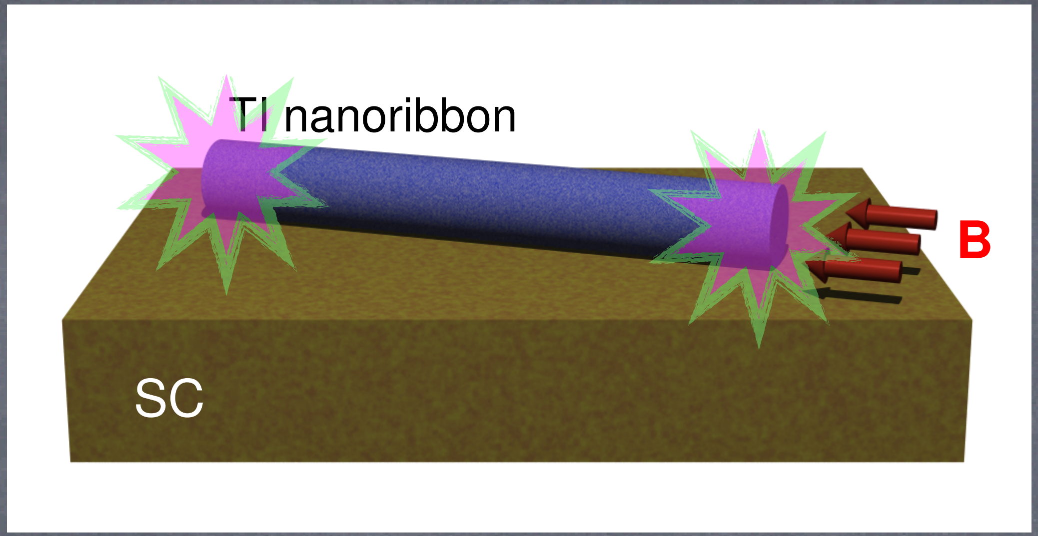
The Zeemann energy scale $\delta E_Z \simeq g\pi\hbar^2/2m_e S \simeq 0.6\mathrm{meV}$ is negligible.

Conclusions



- The proposed device, a TI nanoribbon proximity-coupled to an ordinary superconductor, hosts Majorana end states under wide range of conditions.
- Relevant energy scales are about order of magnitude larger than in Rashba-coupled semicond wires and no significant fine-tuning is required
- At half flux quantum the bulk SC gap is protected by time-reversal symmetry, Majorana modes remarkably stable against non-magnetic disorder