Half quantum spin Hall effect on the surface of weak topological insulators

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Thanks to:

X.L. Qi, S.C. Zhang (Stanford)

Outline

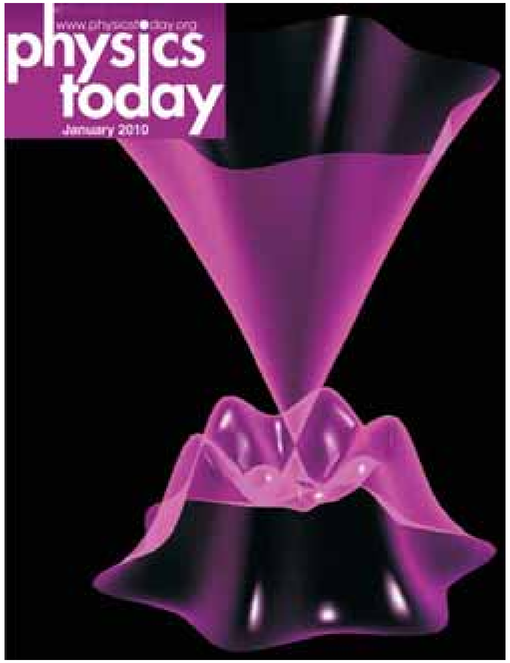
- Introduction
- A concrete model for surface half quantum spin Hall effect of weak topological insulators
- General surface case and interaction effect
- Summary

Topological insulator

- 3D strong topological insulator (TI)

  ➢ Odd number of Dirac cones, eg. a single Dirac cone in Bi_2Se_3 family of materials, ¼ of graphene.

  ➢ Spin-momentum locking



L. Fu, *et al,* PRB (2007)

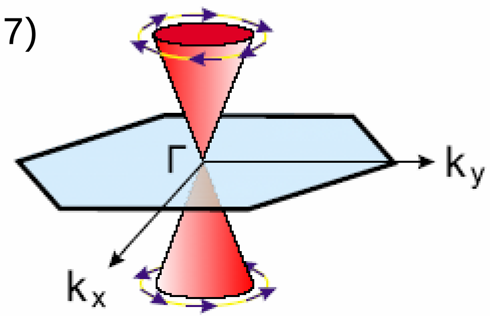
J. E. Moore and L. Balents, PRB (2007)

Hsieh, *et al,* Nature (2007)

H.J. Zhang *et al*, Nat Phys (2009)

Y. Xia *et al*, Nat Phys (2009).

Y. L. Chen *et al*, Science (2009).



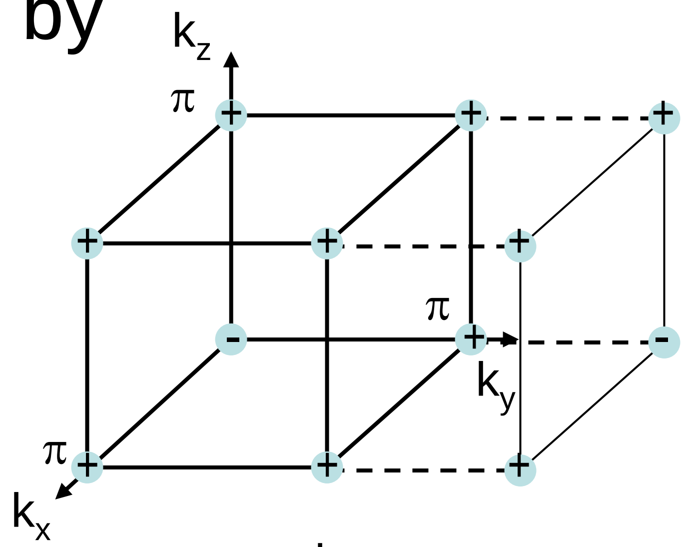
Strong and weak TI

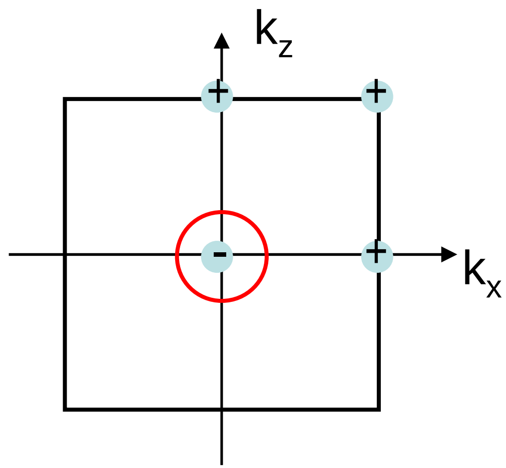
- 3D TI can be characterized by 4 Z_2 invariants $(\nu_0;\nu_1\nu_2\nu_3)$

  - ν_0: the index for strong TI

$$(-1)^{\nu_0} = \prod_{i=1}^{8} \delta_i. \qquad \delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i).$$

ν_0=1, strong TI with odd number of Dirac cone at the surface





L. Fu, *et al,* PRB (2007)

J. E. Moore and L. Balents, PRB (2007)

Roy, cond-mat (2006)

Strong and weak TI

- $(\nu_1\nu_2\nu_3)$: the indices for weak TI

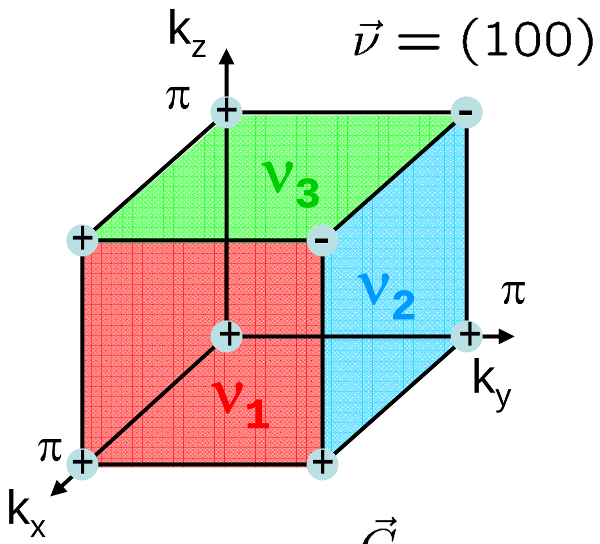
$$(-1)^{\nu_k} = \prod_{n_k=1;\, n_{j\neq k}=0,1} \delta_{i=(n_1 n_2 n_3)}. \qquad \delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i).$$

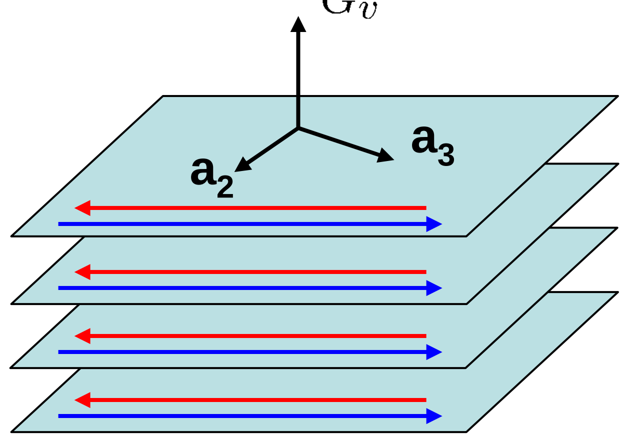
• Three weak TI indices are determined by three $k_i=\pi$ plane (i=x,y,z).

• Weak TI can be viewed as stacked quantum spin Hall layers. The stacking direction is determined by the vector G_v which are formed by three weak TI indices.

$$\vec{G}_\nu = \nu_1\vec{b}_1 + \nu_2\vec{b}_2 + \nu_3\vec{b}_3$$

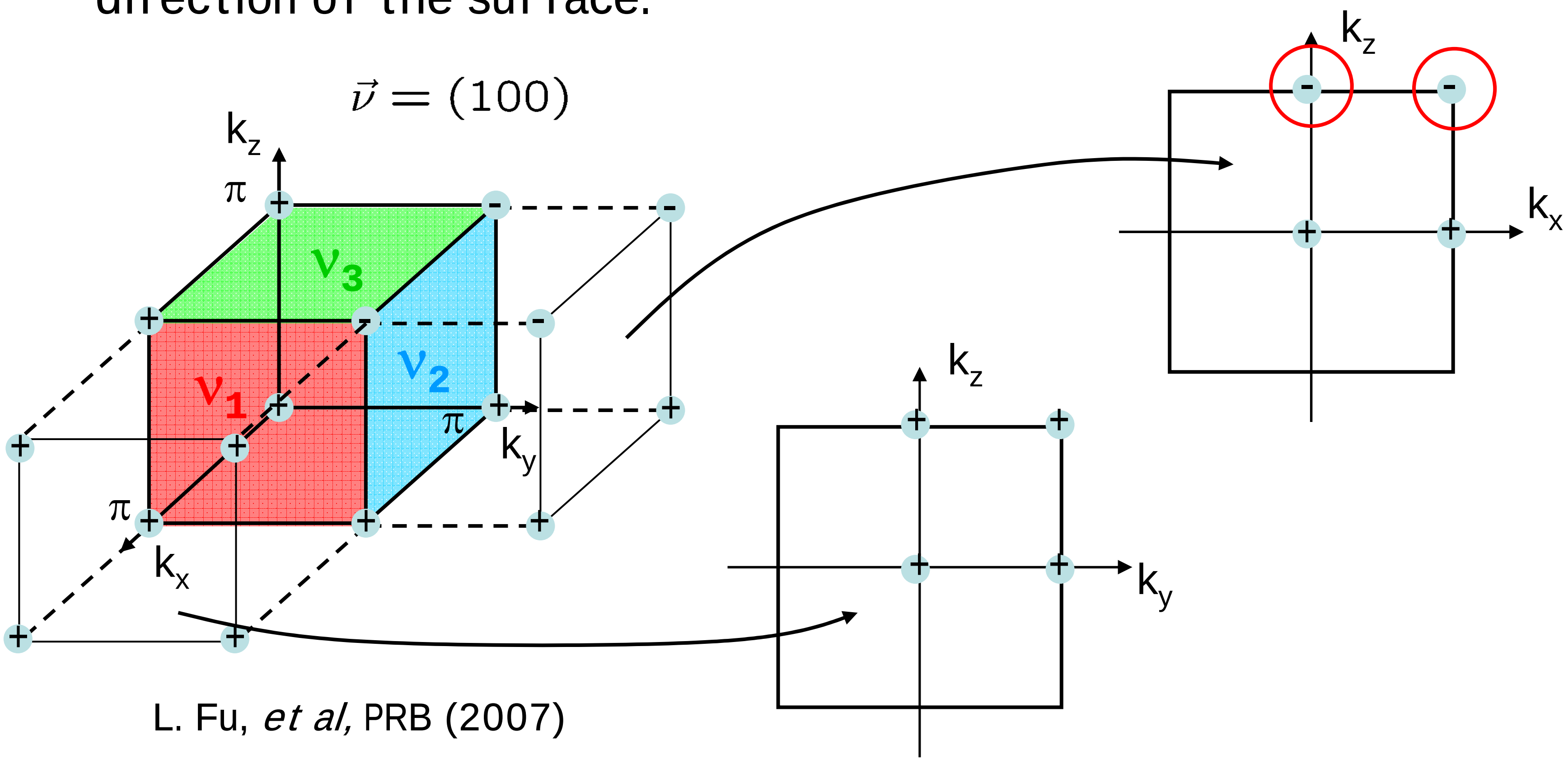
L. Fu, *et al*, PRB (2007)





Strong and weak TI

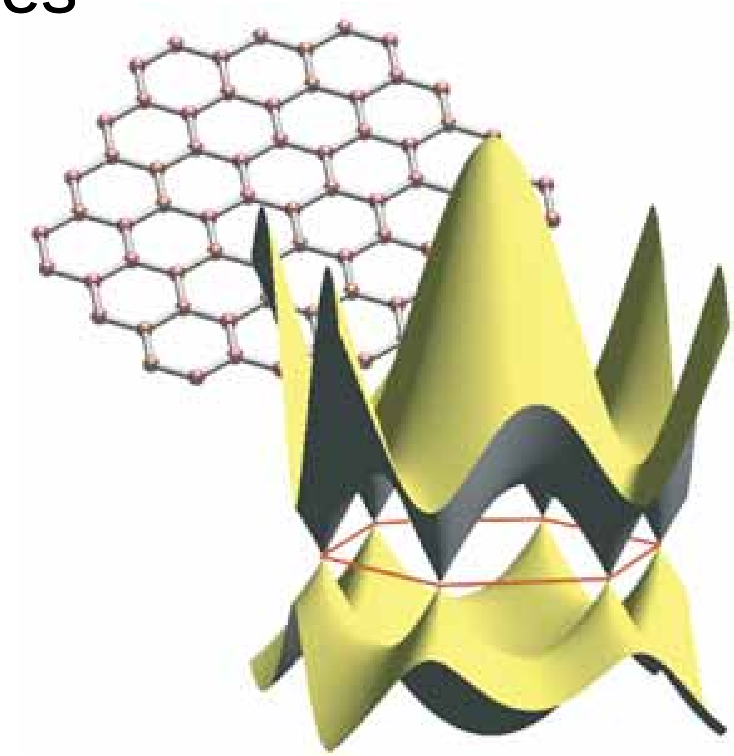
- Weak TI has even number of Dirac cones at the surface, but the detailed number of the Dirac cones depends on the direction of the surface.



L. Fu, *et al,* PRB (2007)

Is weak TI boring?

- Graphene also has even number of massless Dirac cones



What is interesting for weak TI?

- Dislocation line in weak TI

Y. Ran, *et al,* Nature physics (2009)

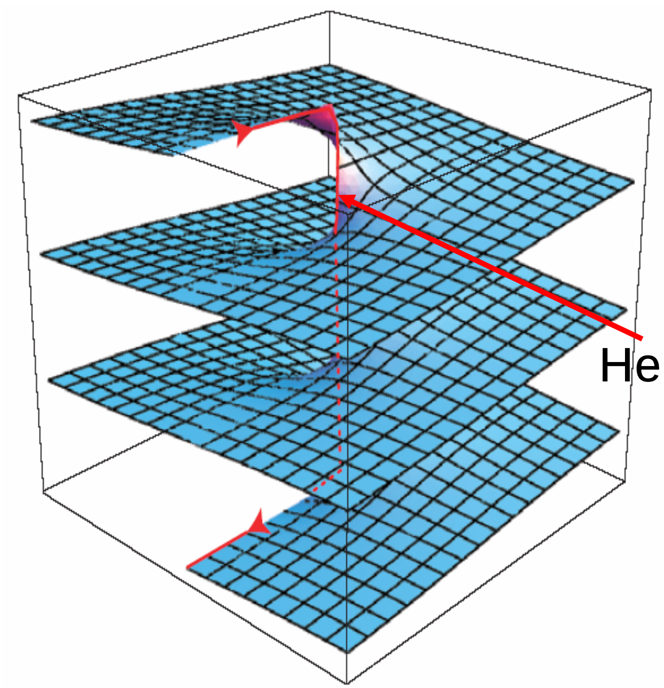
Teo and Kane, PRB (2010)

$$\mathbf{B} \cdot \mathbf{M}_\nu = \pi \ (\mathrm{mod}\ 2\pi)$$

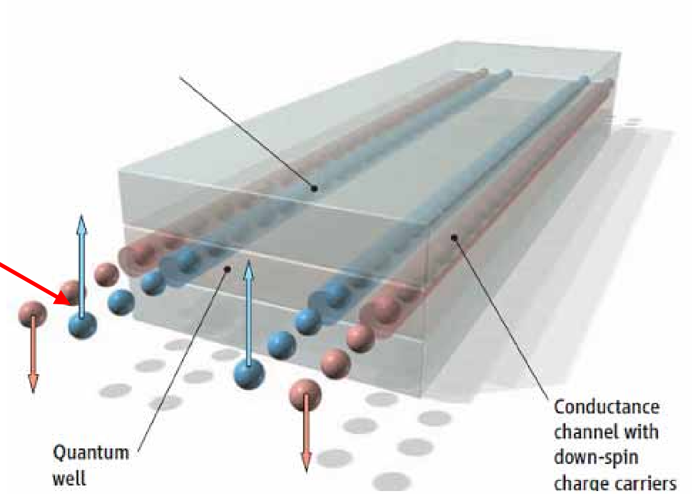
B: burger vector;

M_ν: weak TI index vector

$$\vec{M}_\nu = \frac{1}{2}(\nu_1 \vec{b}_1 + \nu_2 \vec{b}_2 + \nu_3 \vec{b}_3)$$



Helical liquid



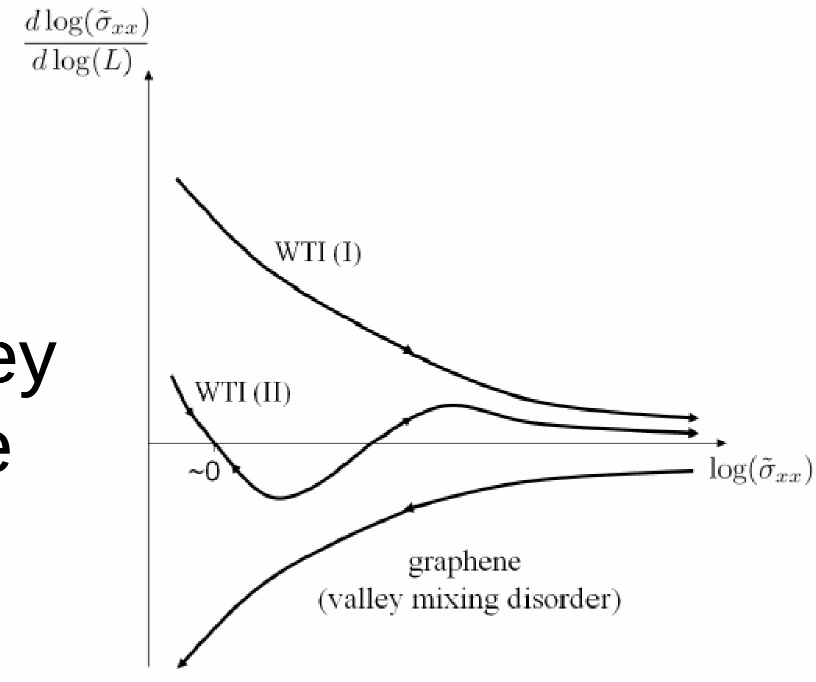
HgTe/CdTe QWs, Koenig, *et al,* (2007)

What is interesting for weak TI?

- The surface state of weak TI always shows anti-localization effect for the impurities scattering which preserves time reversal.

The strong side of weak topological insulator

Z. Ringel, *et al,* (2011)

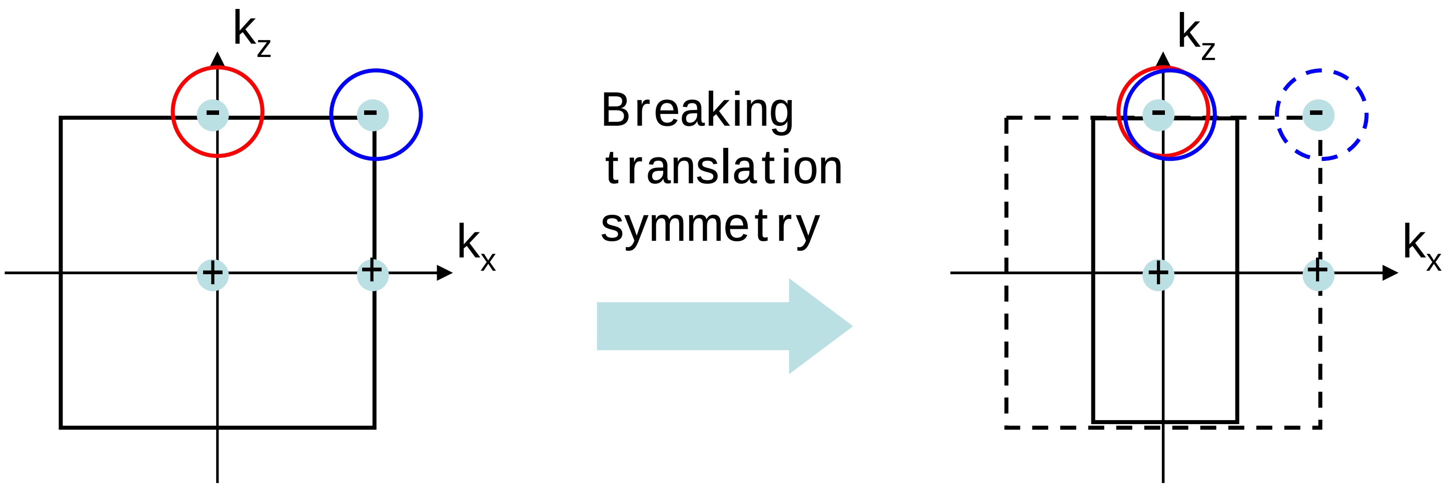


- In graphene, inter-valley scattering will change the anti-localization to localization.

What is interesting for weak TI?

- “Weak” topological insulator relies on translation symmetry.

L. Fu, *et al*, PRB (2007)



- The scattering between two Dirac cones is similar to the inter-valley scattering in graphene.

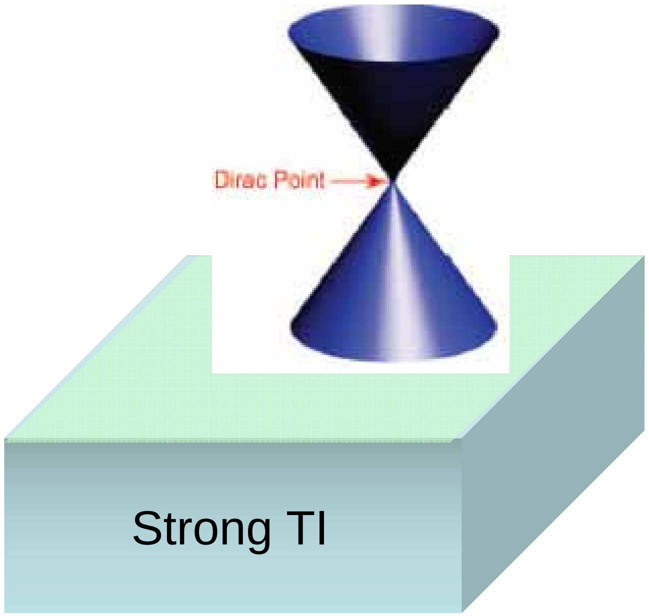
How to understand Ringel's results? How to understand the difference between the surface states of weak TI and graphene?

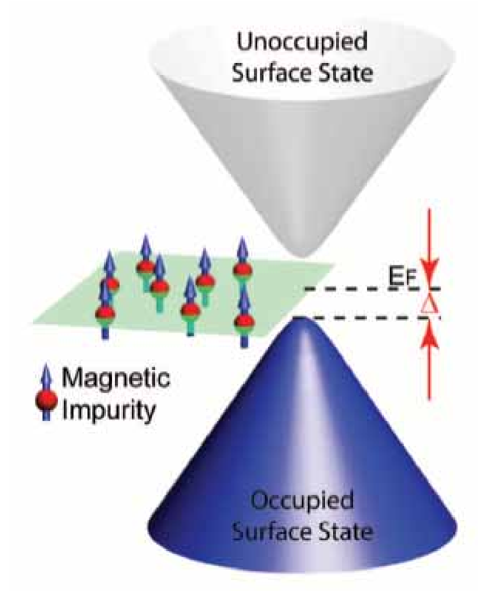
Indication from strong TI

- The surface state of Strong TI relies on the protection of time reversal symmetry and can open a gap when T is broken.

L. Fu, *et al,* PRB (2007)

Qi, *et al,* PRB (2008)





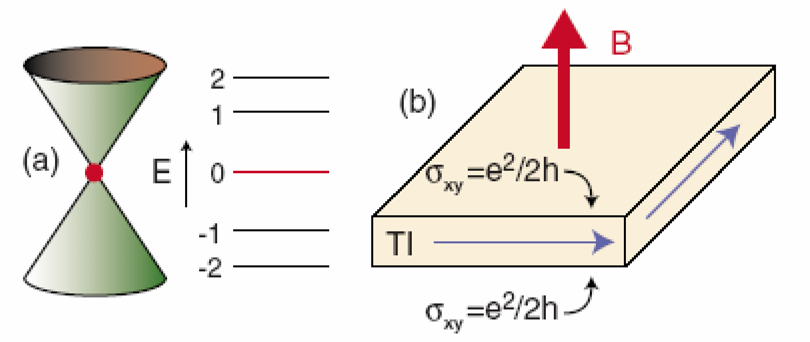
Indication from strong TI

- Surface half quantum Hall effect

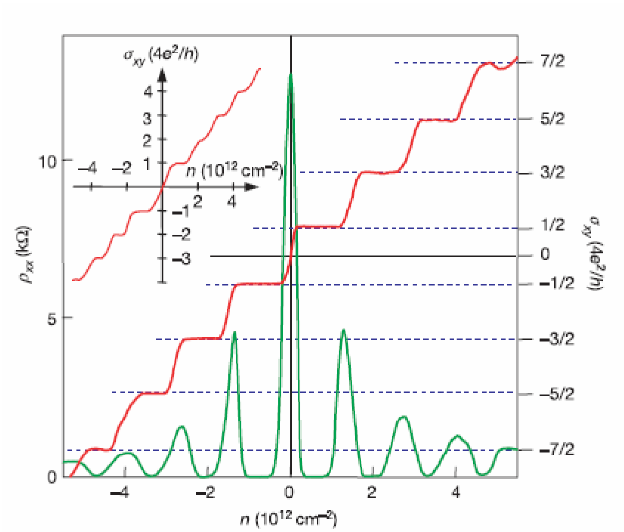
Once gaped, the surface states carry a half-quantized Hall conductance

$$\sigma_H = (2n-1)e^2/2h$$

((2+1)d parity anomaly)



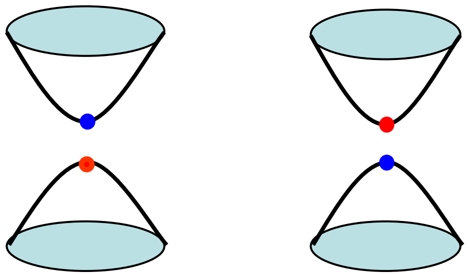
L. Fu, *et al,* PRB (2007) Qi, *et al,* PRB (2008)

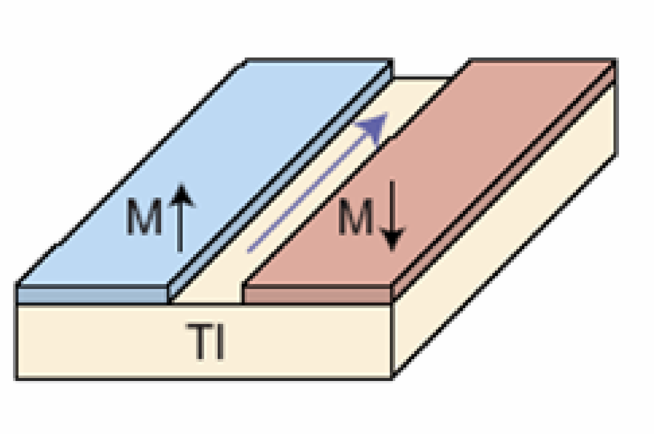


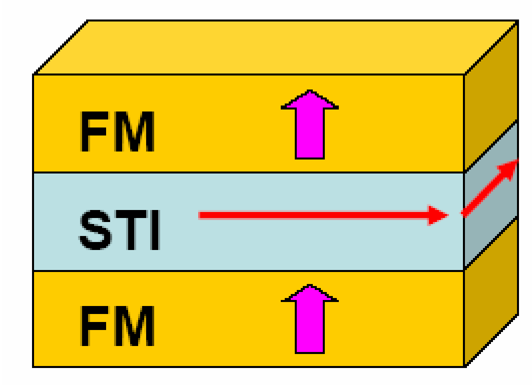
Graphene QHE: No zero plateau
(Novoselov et al, Nat. 2005)

Indication from strong TI

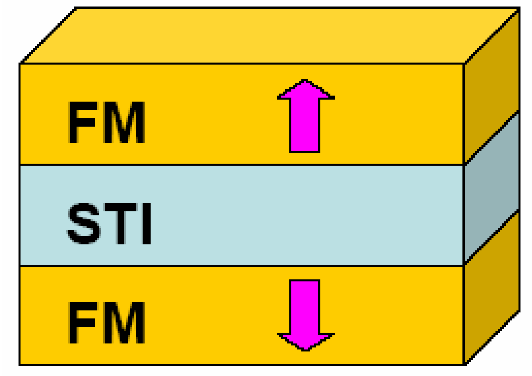
- Surface half quantum Hall effect of strong TI







$$\frac{e^2}{2h} + \frac{e^2}{2h} = \frac{e^2}{h}$$



$$\frac{e^2}{2h} - \frac{e^2}{2h} = 0$$

Indication from Strong TI

- For strong TI, the gapless surface state is protected by time reversal symmetry, then when time reversal is broken at the surface, surface half quantum Hall effect appears as a non-trivial physical consequence.

- Similarly, for weak TI, the gapless surface state is protected by translation symmetry, and what is physical consequence when the translation symmetry is broken at the surface?

Minimal model for weak TI

$$\hat{H} = \sum_{\mathbf{k}} \hat{\psi}^{\dagger}(\mathbf{k}) H(\mathbf{k}) \hat{\psi}(\mathbf{k})$$

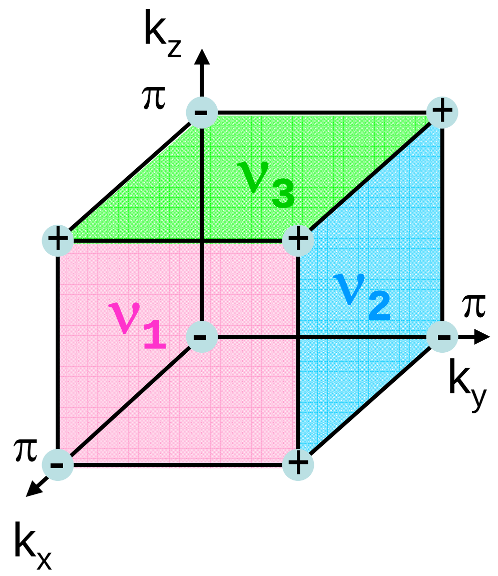
$$\mathcal{M}(\mathbf{k}) = M_0 + 6B - 2B \sum_i \cos k_i,$$

$$H(\mathbf{k}) = \mathcal{M}(\mathbf{k})\Gamma_5 + A \sum_{i=1,2,3} \Gamma_i \sin k_i$$

Five Γ matrices anti-commutate with each other

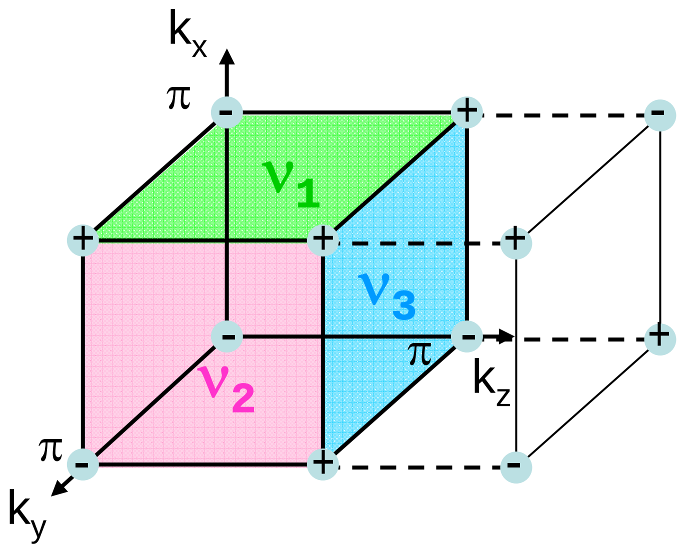
parameter	$(\nu_0;\nu_1\nu_2\nu_3)$	class
$M_0>0$ or $M_0<-2B$	(0;000)	Normal Insulator
$0>M_0>-4B$	(1;000)	Strong TI
$-8B>M_0>-12B$	(1;111)	Strong TI
$-4B>M_0>-8B$	(0;111)	Weak TI

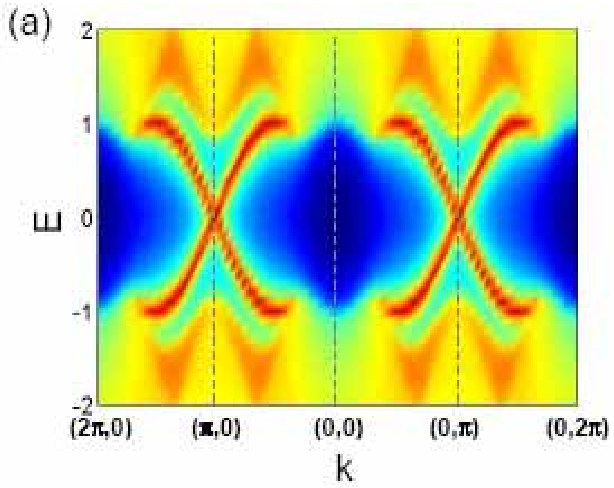
$$(-1)^{\nu_k} = \prod_{n_k=1;n_{j\neq k}=0,1} \delta_{i=(n_1 n_2 n_3)}. \qquad \delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i).$$

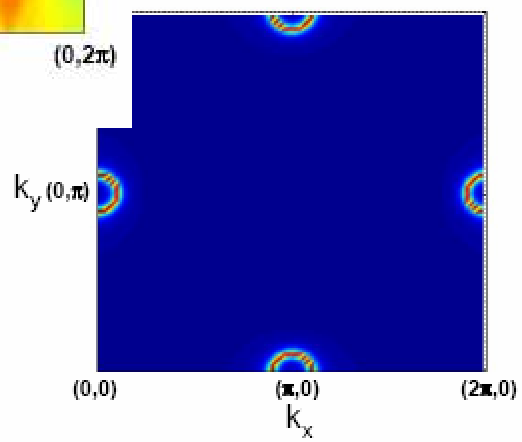


Minimal model for weak TI

- For the (001) surface, there are two Dirac cones at $(\pi,0)$ and $(0,\pi)$ respectively.







Minimal model for weak TI

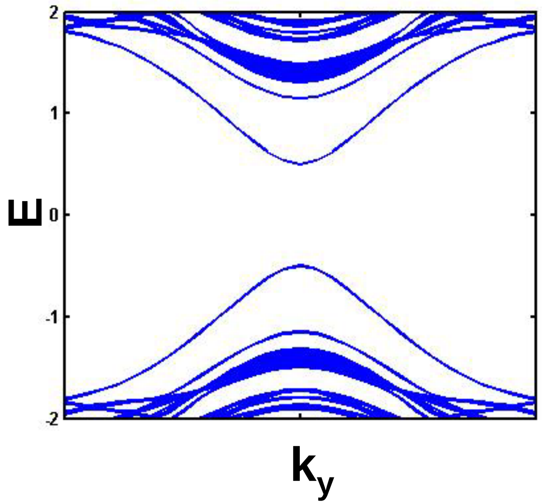
➢ (π,π) charge density wave or staggered potential can open a gap for two Dirac cones without breaking T. (Similar to the inter-valley scattering in graphene.)

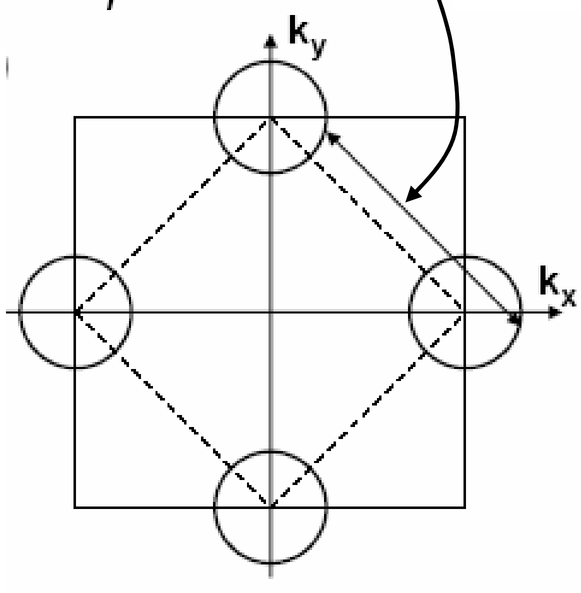
$$\hat{H}_{sur} = A \int d\vec{\rho}\,\Psi^\dagger(\vec{\rho})\left(k_x\sigma_2 \otimes \tau_3 + k_y\sigma_1 \otimes \tau_3\right)\Psi(\vec{\rho})$$

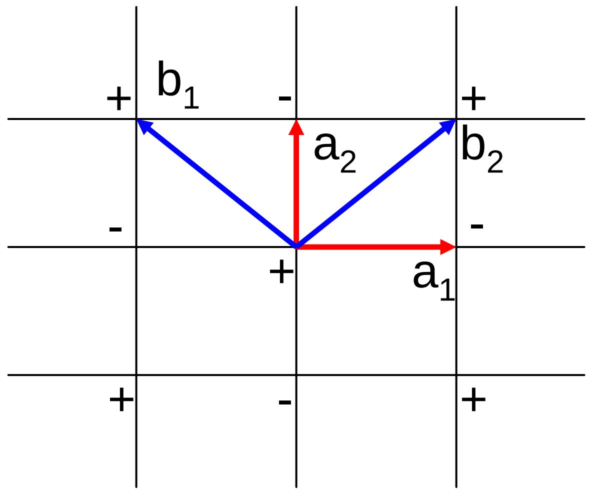
$$H_1 = D \int d^2 r \Psi^\dagger 1 \otimes \tau_1 \Psi.$$

σ: spin

τ: different valleys



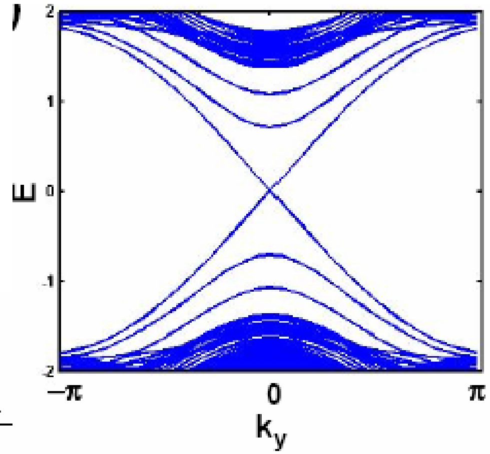


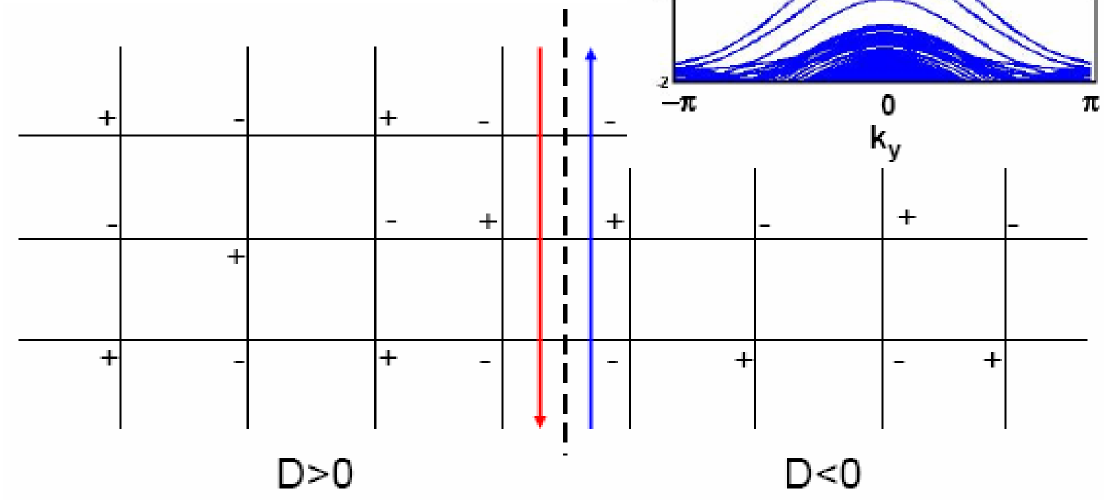


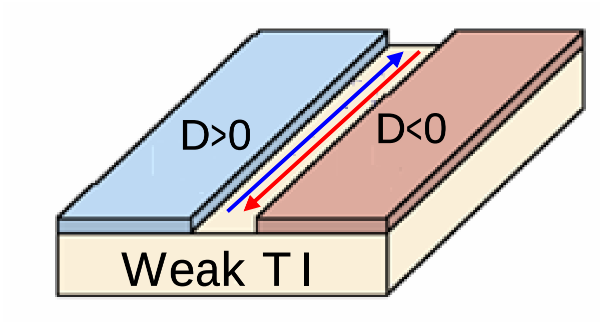
Minimal model for weak TI

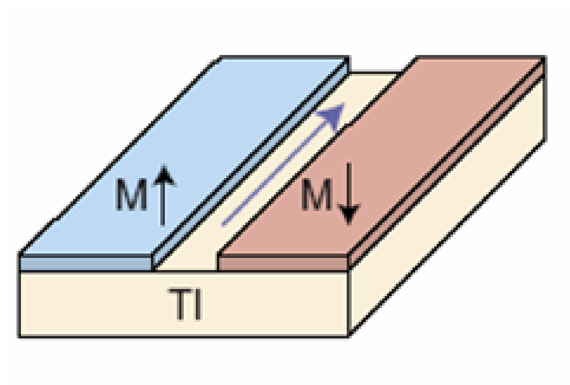
➢ Helical liquid exists at the domain wall of CDW when T is preserved. (Graphene with CDW is just trivial insulator).

$$H_1 = D \int d^2 r \Psi^\dagger 1 \otimes \tau_1 \Psi.$$

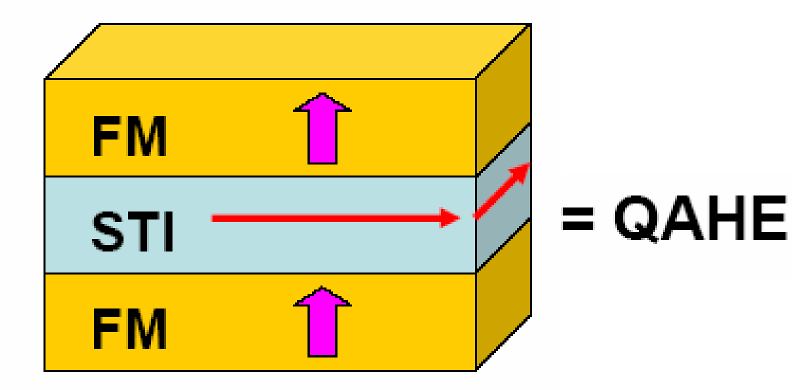




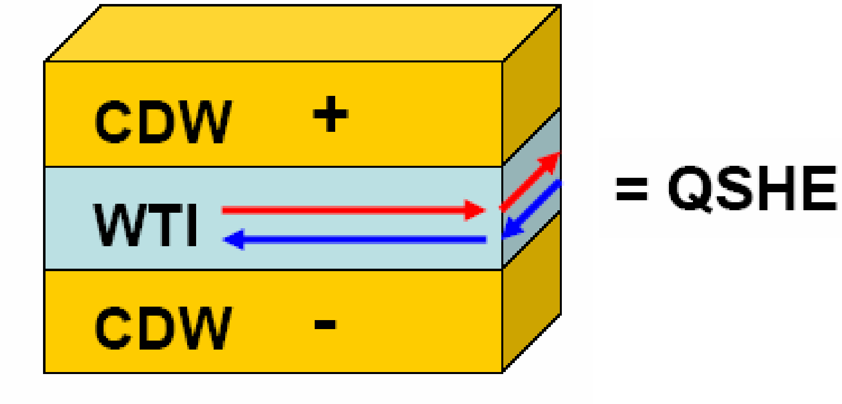


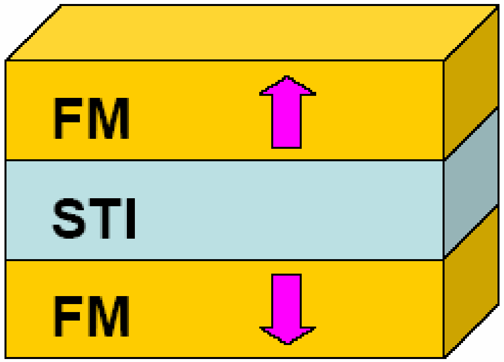


Surface half quantum spin Hall effect

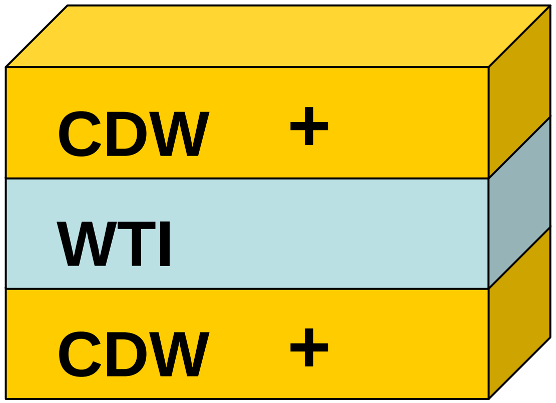


$$\frac{e^2}{2h} + \frac{e^2}{2h} = \frac{e^2}{h}$$



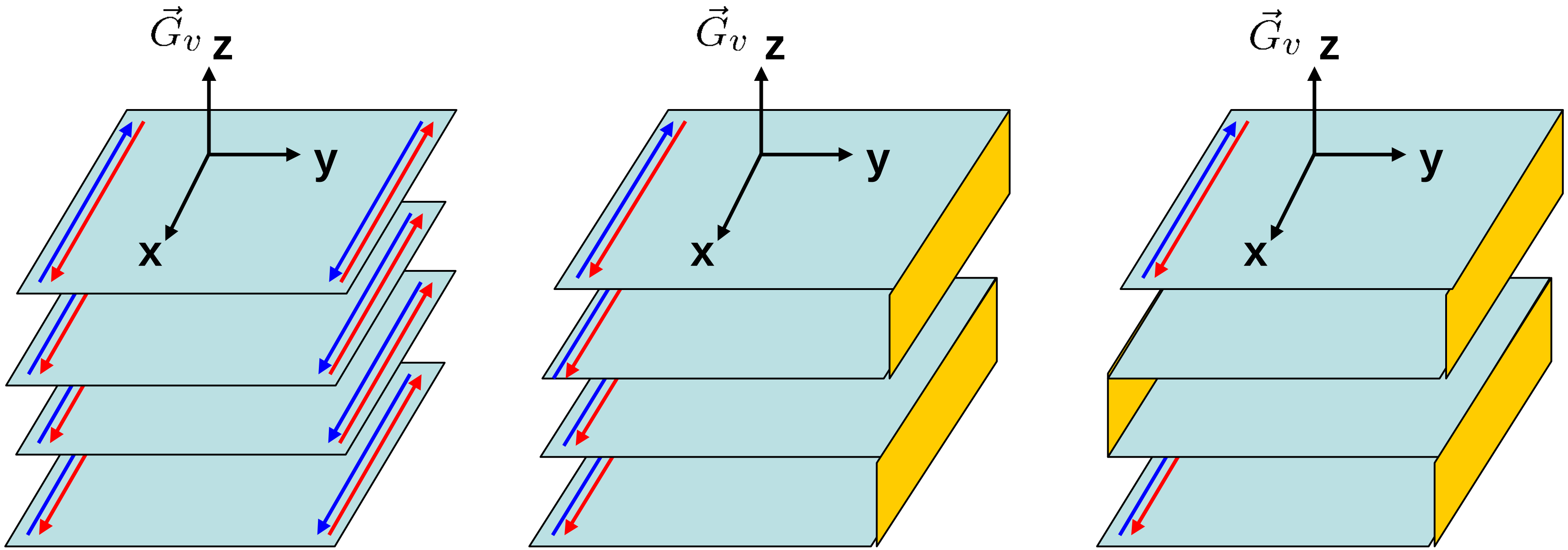


$$\frac{e^2}{2h} - \frac{e^2}{2h} = 0$$



Stacked QSH layer picture

- The weak TI with opposite CDW at opposite surfaces can be viewed as one folded quantum spin Hall layer, so one surface is just half of the whole quantum spin Hall layer.



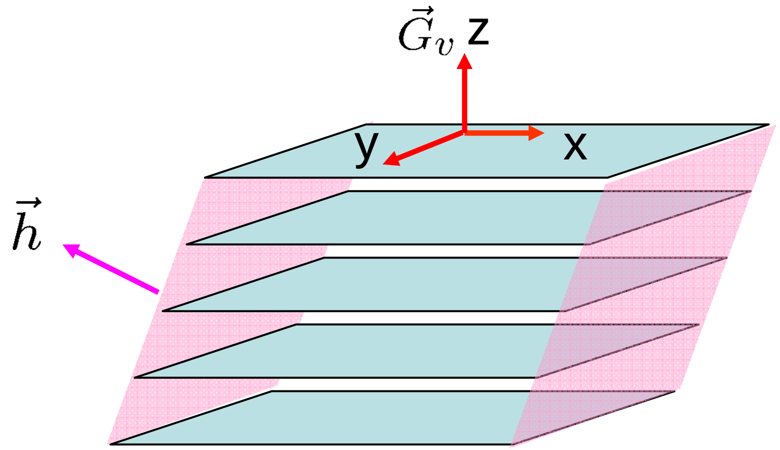
Arbitrary direction surface

- The surface is gapped, when the condition (h_i-ν_i) mod 2=0 is satisfied.

  Miller indices $\vec{h} = h_1\vec{b}_1 + h_2\vec{b}_2 + h_3\vec{b}_3$

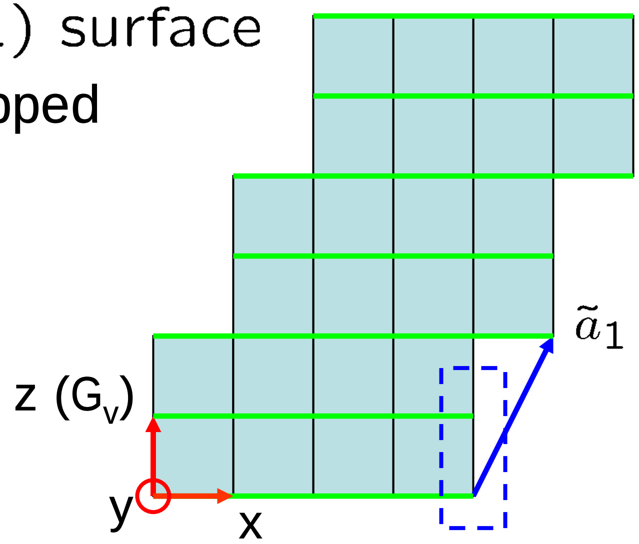
L. Fu, *et al,* PRB (2007)   Z. Ringel, *et al,* (2011)

Example: cubic lattice, ν=(001)

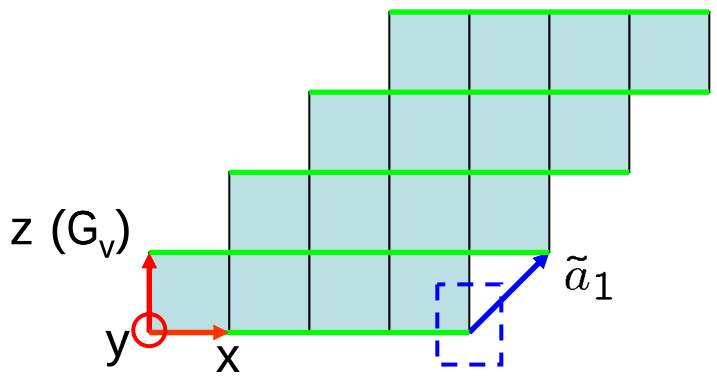


($\bar{2}$01) surface

gapped



($\bar{1}$01) surface gapless



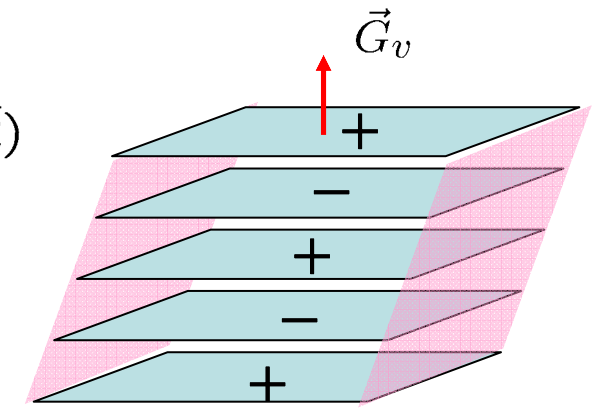
Arbitrary direction surface

Example: cubic lattice, ν=(001)

Charge density wave $H_{CDW} = \eta\,\cos(\vec{Q}_{CDW}\cdot\vec{R})$

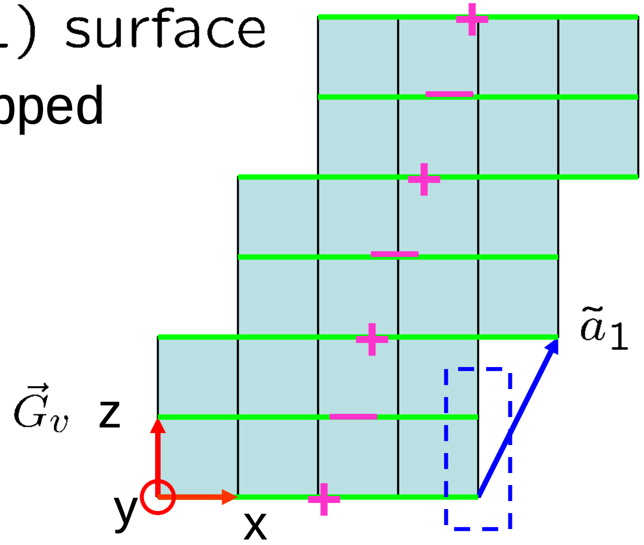
$\vec{Q}_{CDW} = \frac{1}{2}\vec{G}_v = \frac{1}{2}\sum_i \nu_i \vec{b}_i$ and $\vec{R} = \sum_i n_i \vec{a}_i$

Here $H_{CDW} = \eta\,\cos(\pi n_3)$



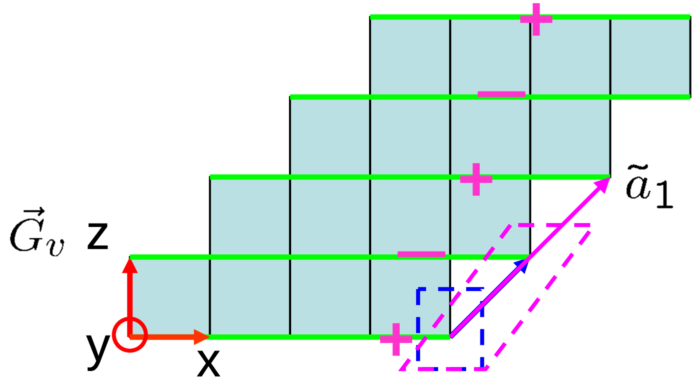
($\bar{2}$01) surface

gapped



($\bar{1}$01) surface

Gapped due to CDW

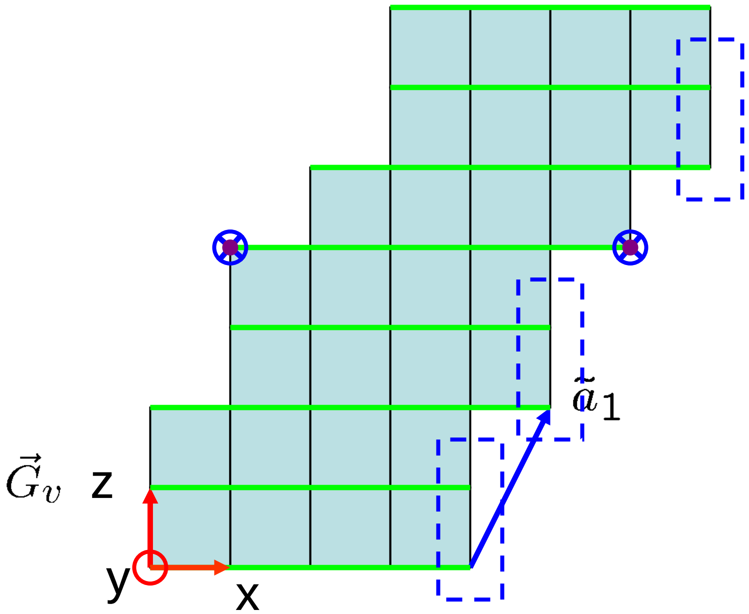


Arbitrary direction surface

- Helical state exists at the domain wall

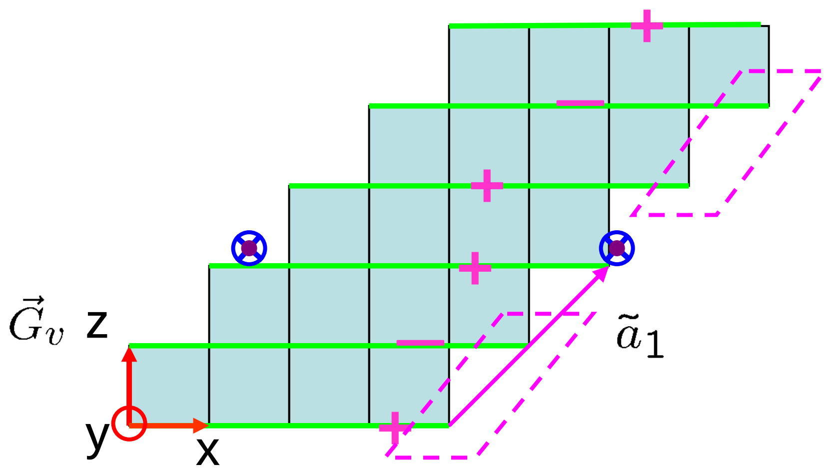
$(\bar{2}01)$ surface

gapped



$(\bar{1}01)$ surface

Gapped due to CDW

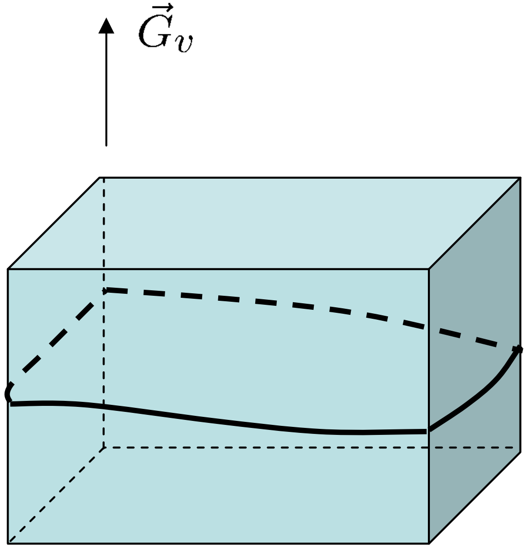


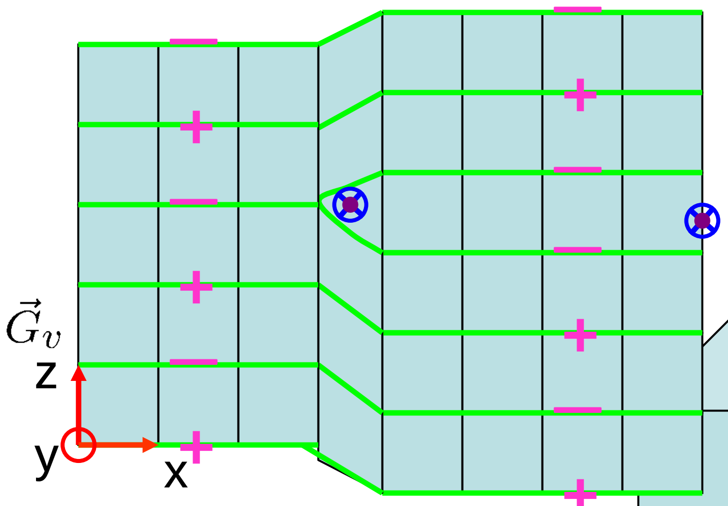
Relation to Dislocation line

- CDW domain wall can be terminated by surfaces or by dislocation lines, and both possess helical liquid.

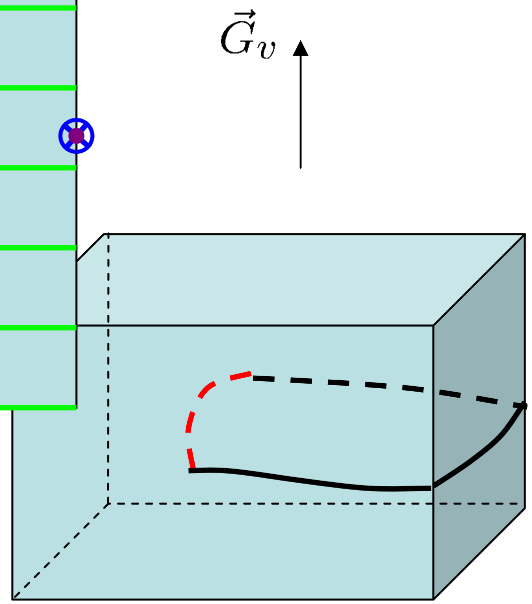
Y. Ran, *et al*, Nat phys (2009)

$$\vec{Q}_{CDW} = \frac{1}{2}\vec{G}_v = \frac{1}{2}\sum_i \nu_i \vec{b}_i$$





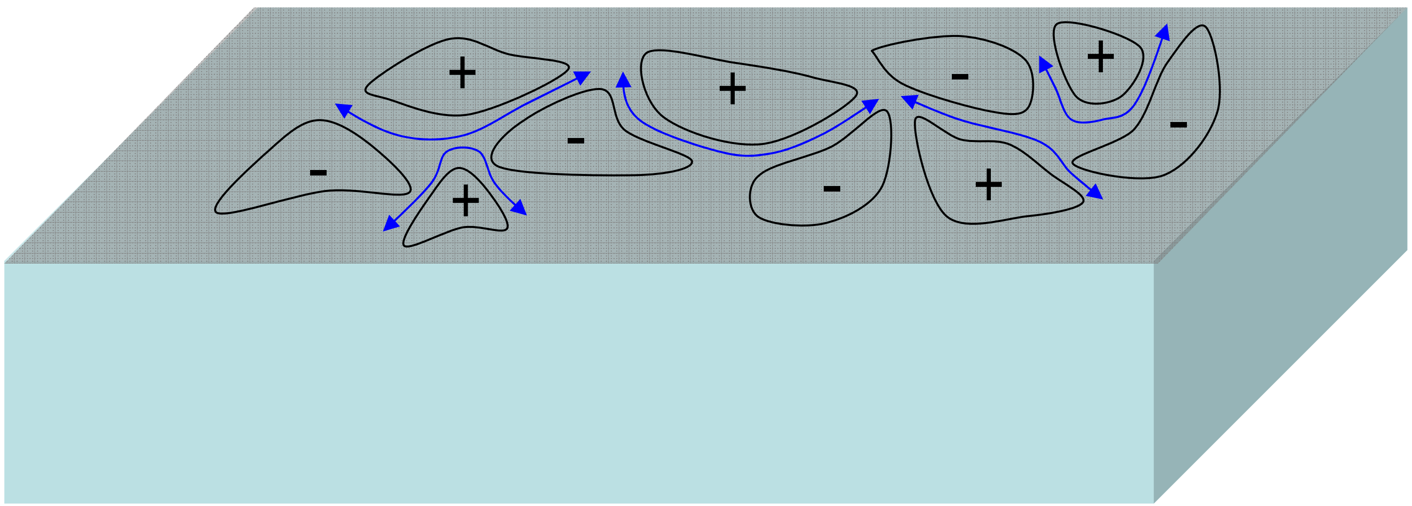
Edge dislocation



Relation to Disorder effect

➢ Disorder which preserves time reversal symmetry can be regarded as random CDW domains. The helical liquids between these random CDW domain walls can conduct electrons.

Z. Ringel, *et al,* (2011)



Interaction effect

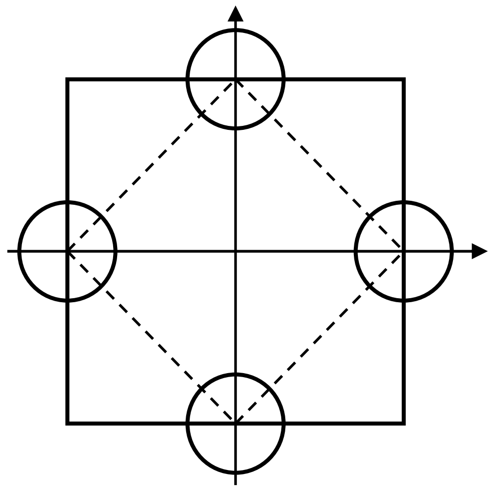
- We include the on-site interaction U and nearest neighbor V in the above four band model.

$$\hat{H}_U = U \sum_{\vec{n},\eta} \hat{c}^\dagger_{\eta\uparrow}(\vec{n})\hat{c}_{\eta\uparrow}(\vec{n})\hat{c}^\dagger_{\eta\downarrow}(\vec{n})\hat{c}_{\eta\downarrow}(\vec{n}) \qquad \hat{H}_V = V \sum_{\langle i,j\rangle} \hat{n}_i \hat{n}_j$$

- We project the interaction on the surface sub-space and treat the interaction Hamiltonian in the mean field level.

Charge density wave	$D = \langle \hat{\Psi}^\dagger \sigma_0 \otimes \tau_1 \hat{\Psi} \rangle$
ferromagnetic	$\vec{m} = \langle \hat{\Psi}^\dagger \vec{\sigma} \otimes \tau_0 \hat{\Psi} \rangle$
Anti-ferromagnetic	$\vec{S} = \langle \hat{\Psi}^\dagger \vec{\sigma} \otimes \tau_1 \hat{\Psi} \rangle.$

σ: spin; τ: valley



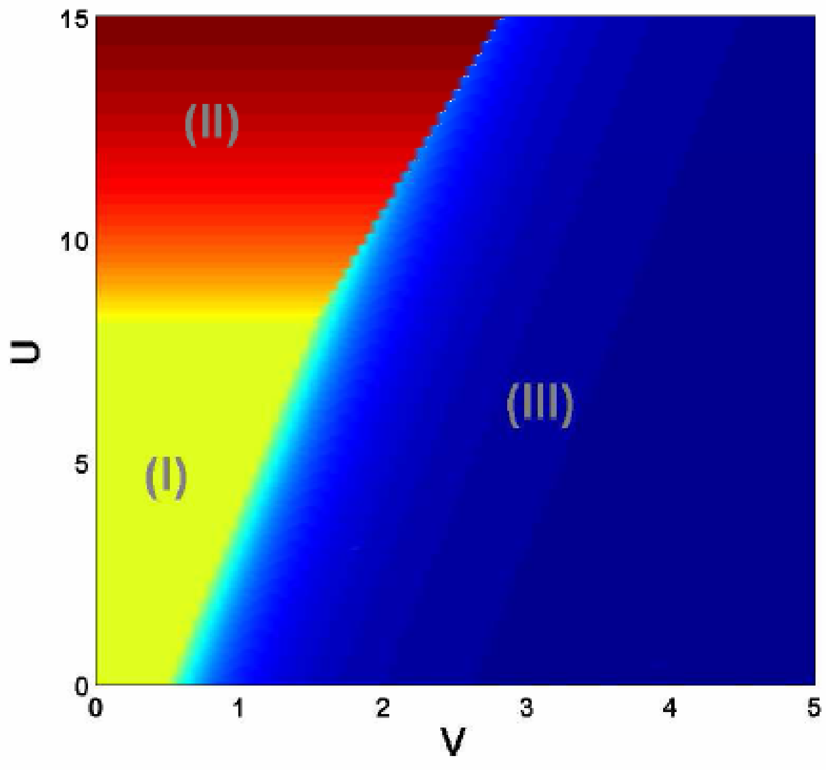
Interaction effect

- Phase diagram

(I) Semi-metal regime

(II) Ferromagnetic regime with quantum anomalous Hall effect. Ferromagnetic order parameter m can open a gap but anti-ferromagnetic order parameter S can not.

(III) Charge density wave regime with half quantum spin Hall effect.



Summary

- The weak TI is not so boring and its surface states are different from graphene because of the surface half quantum spin Hall effect.
- One example for the symmetry breaking inducing non-trivial phenomena.
- $Bi_{1-x}Sb_x$, with both strong and weak indices.
  Other materials only with weak indices?

Thank you for your attention!