The search for elusive Majorana particles in semiconductor-superconductor structures

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KITP Program: Topological Insulators and Superconductors

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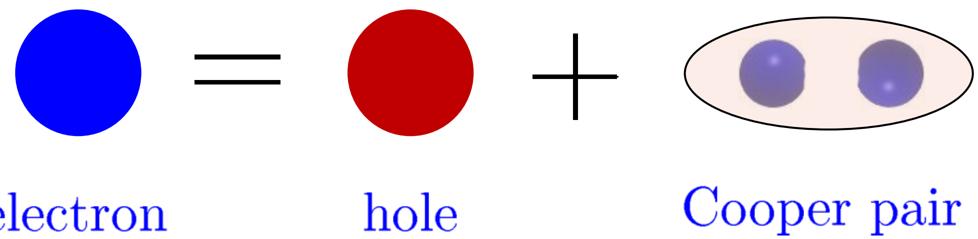
Outline

- Brief history of Majorana fermions
- Proposal to realize topological superconductivity in semiconductor/superconductor heterostructures
- Effect disorder & chemical potential fluctuations
- Beyond mean field theory: effect of quantum fluctuations

Superconductors are natural hosts for Majorana

Bogoliubov quasiparticle $\gamma = u\psi + v\psi^\dagger$

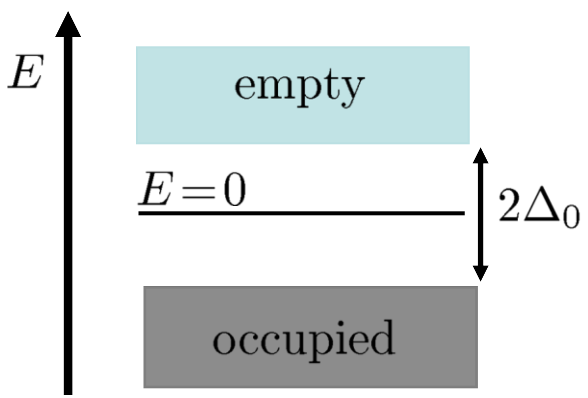
$u = v^*$ → equal superposition of a particle and a hole



Majorana fermion

$$\gamma = \gamma^\dagger$$

Look for ZERO energy states !



Bound states in vortices

Midgap states at the interfaces

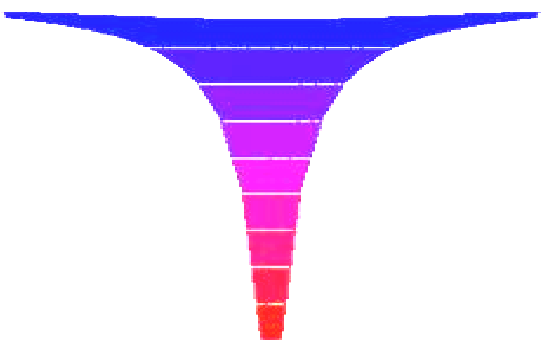
Example: 2D chiral p-wave superconductors

Zero-energy states appear in chiral superfluids

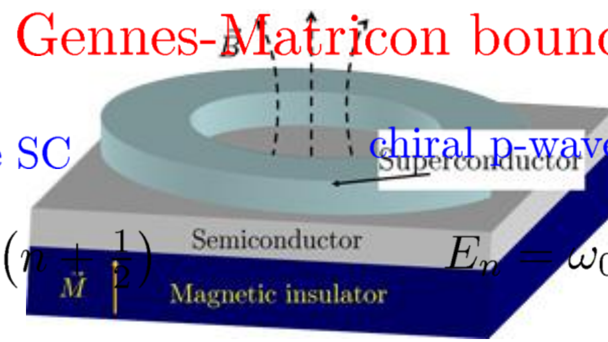
He-3: Kopnin and Salomaa PRB'91;

Chiral superfluids/superconductors, Volovik (1999), Read & Green(2000)

Chirality may originate from the order parameter or band structure



Caroli-de Gennes-Matricon bound states



s-wave SC

$$E_n = \omega_0 \left(n + \frac{1}{2}\right)$$

$$\Psi(r, \theta + 2\pi) = -\Psi(r, \theta)$$

chiral p-wave SC

$$E_n = \omega_0 n$$

$$\Psi(r, \theta + 2\pi) = \Psi(r, \theta)$$

Chiral superconductors:

- strontium ruthenate

Rice & Sigrist, 1995

Heterostructures:

- topological insulator/s-wave superconductor
- semiconductor/s-wave superconductor

... among others

1D Majorana chain - Kitaev's toy model

Spinless fermion with p-wave pairing

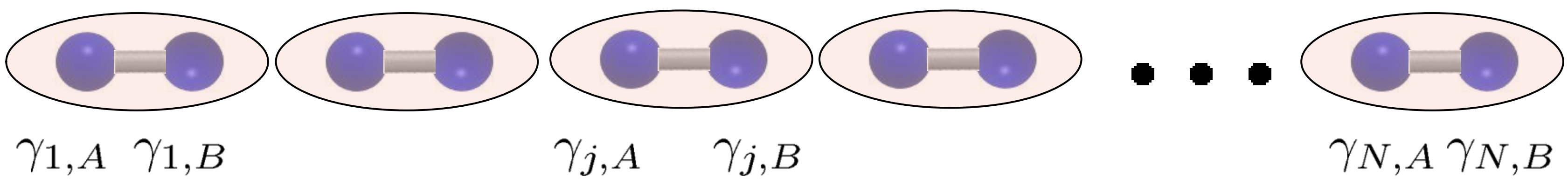
$$H = -\mu \sum_{j=1}^{N} c_j^\dagger c_j - \sum_{j=1}^{N-1} (t c_j^\dagger c_{j+1} + |\Delta| e^{i\phi} c_j c_{j+1} + h.c.)$$

Kitaev, arXiv'00

Two topologically distinct phases:

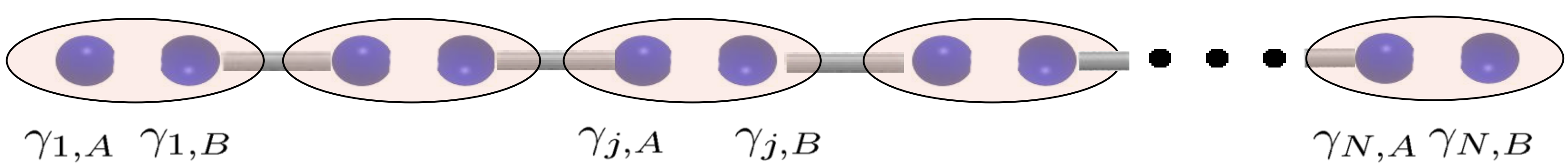
trivial: $t = 0$ and $\Delta = 0$ and $\mu < 0$

$c_j = \gamma_{jA} + i\gamma_{jB}$



non-trivial: $\mu = 0$ and $t = \Delta$

$H = it \sum_{j=1}^{N-1} \gamma_{B,j}\gamma_{A,j+1}$



GS degeneracy: $i\gamma_{1,A}\gamma_{N,B} \left|\Psi_{e/o}\right\rangle = \pm \left|\Psi_{e/o}\right\rangle$

Topological protection of zero-energy mode

Bogoliubov-de-Gennes equations

$$\begin{pmatrix} h_0 & \Delta \\ \Delta^\dagger & -h_0^T \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}$$

Particle-hole symmetry:

If $\begin{pmatrix} u \\ v \end{pmatrix}$ is a solution with E

If $\begin{pmatrix} v^* \\ u^* \end{pmatrix}$ is a solution with $-E$

For spinless fermions particle-hole symmetry guarantees Majorana mode at $E=0$

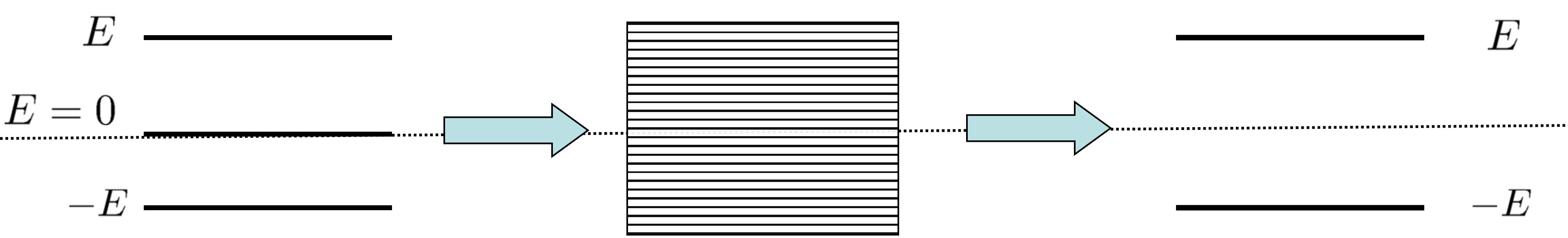
Two topological classes of BdG Hamiltonians

$2N+1$ solutions

Topological reconstruction of the spectrum
requires closing of the bulk gap

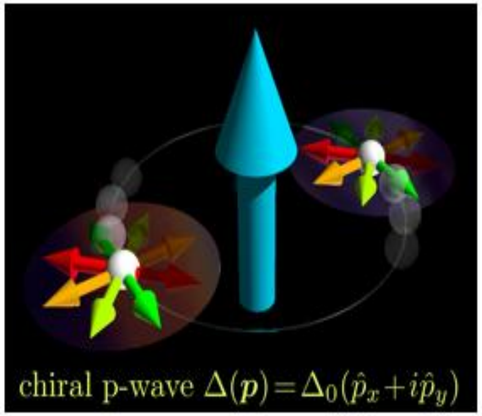
Bound states in the vortex core

$2N$ solutions



Read and Green, PRB'00

How do we find chiral p-wave superconductor ?

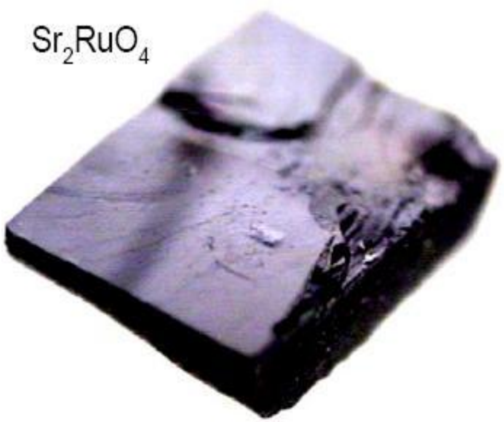


Order parameter: $\Delta(p) = \Delta_0(p_x + ip_y)$

Time-reversal symmetry Θ: $p \to -p$ and $i \to -i$

$$\Theta\Delta(p) \propto \Delta_0(p_x - ip_y)$$

Order parameter breaks T (and P) symmetry !



$T_c \sim 1$ K, strongly varies with disorder: unconventional SC

- spin-triplet pairing [spin susceptibility, Josephson effect (Penn State'04), observation of HQV (UIUC'10)]

- T breaking: Kerr effect (Stanford'06), Josephson interferometry(UIUC'06)

Engineering spinless p+ip superconductor

Rather than looking for $p_x + ip_y$ SC in nature, we could try to engineer suitable Hamiltonians via proximity effect

Chirality has to come from the bandstructure

Strong spin-orbit interaction is necessary
to avoid fermion doubling

Superconducting heterostructures

2D: Majoranas "live" in vortices

Fu and Kane, PRL'08

Sau, Lutchyn, Tewari, Das Sarma, PRL'10

Alicea, PRB'10

1D: Majoranas "live" at the ends of wires

Lutchyn, Sau, Das Sarma, PRL(2010)

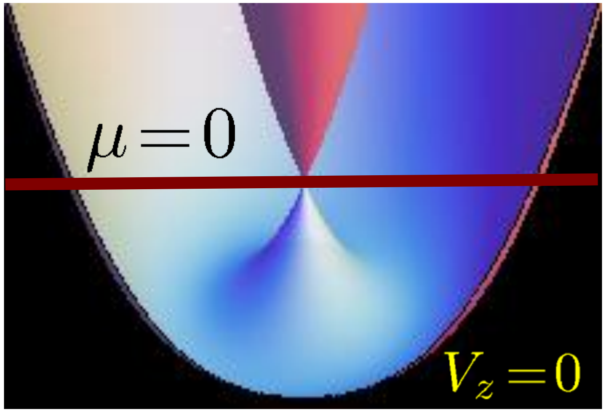
Oreg, Refael, von Oppen, PRL(2010)

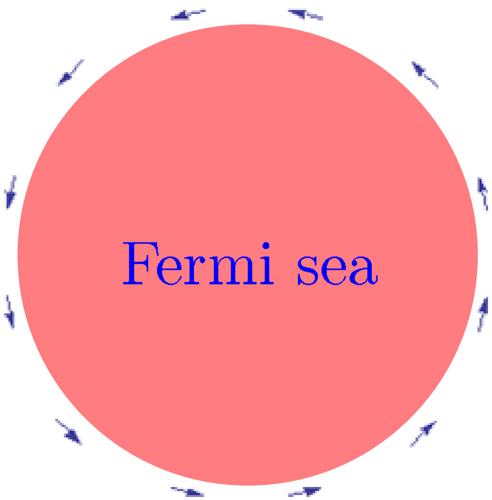
among others

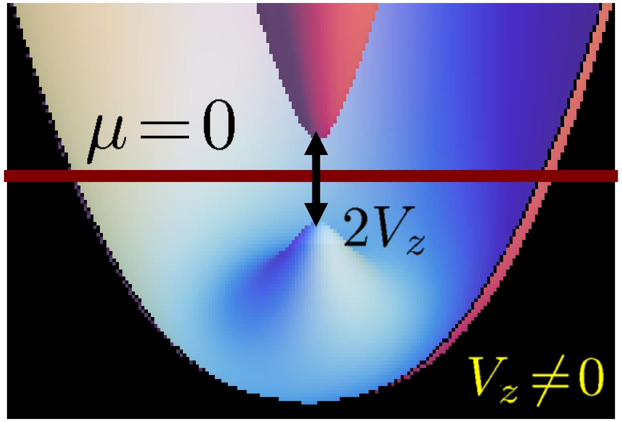
Semiconductor with spin-orbit interaction

Semiconductor with Rashba interaction

$$H_0 = \begin{pmatrix} \frac{p^2}{2m} - \mu & \alpha i(p_x - ip_y) \\ -\alpha i(p_x + ip_y) & \frac{p^2}{2m} - \mu \end{pmatrix}$$





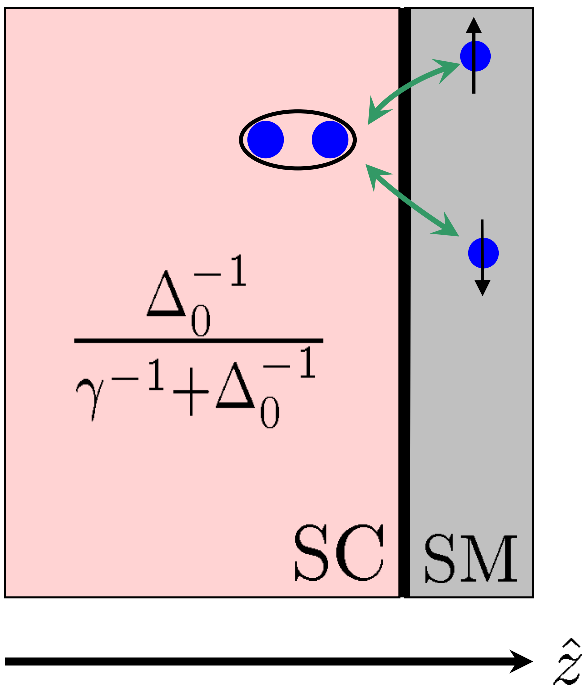


spin orientation changes around Fermi surface

Single Fermi surface !

Sau, Lutchyn, Tewari, Das Sarma, PRL'10;

Superconducting proximity effect



tunneling Hamiltonian approach

$$H = H_{\mathrm{SM}} + H_{\mathrm{SC}} + H_t$$

Integrate out SC degrees of freedom

$$\hat{\Sigma}(\omega) = -\frac{\gamma}{\sqrt{\Delta_0^2-\omega^2}}(\omega\tau_0 + \Delta_0\tau_x)$$

γ is tunneling rate

$$G^{-1}(\omega) = \omega\left(1+\frac{\gamma}{\Delta_0}\right) - \hat{H}_{\mathrm{SM}} + \gamma\tau_x$$

Changes effective Hamiltonian for semiconductor

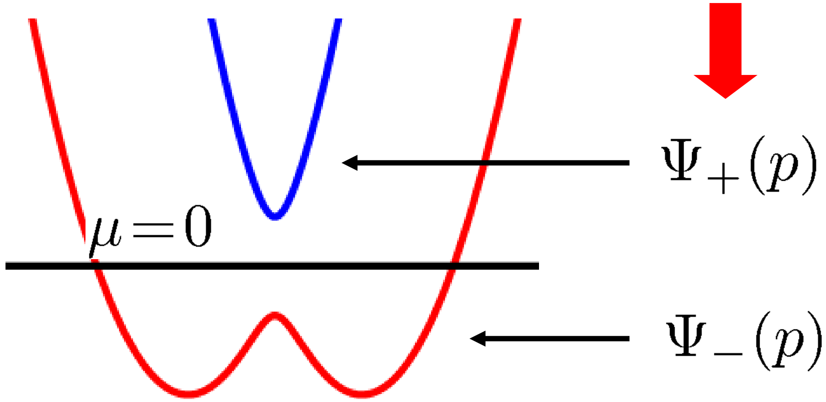
$$H = \Psi_\lambda^\dagger\left(\frac{p^2}{2m} - \underline{\tilde{\mu}} + \underline{\tilde{V}_z}\sigma_z + \underline{\tilde{\alpha}}\hat{z}(\sigma\times p)\right)_{\lambda\lambda'}\Psi_{\lambda'} + \Delta_{\mathrm{ind}}\Psi_\uparrow^\dagger\Psi_\downarrow^\dagger + h.c.$$

Sau, Lutchyn, Tewari, Das Sarma, PRB’10

Spinless p+ip superconductivity in semiconductor/superconductor heterostructures

$$H = \Psi^\dagger_\lambda \left(\frac{p^2}{2m} - \mu + V_z \sigma_z + \alpha \hat{z}(\sigma \times p) \right)_{\lambda\lambda'} \Psi_{\lambda'} + \Delta_{\text{ind}} \Psi^\dagger_\uparrow \Psi^\dagger_\downarrow + h.c.$$

Diagonalize H_0



$$H_{\text{SC}} = \begin{cases} \Delta_{--}(p)\Psi^\dagger_-(p)\Psi^\dagger_-(-p) \\ \cancel{\Delta_{+-}(p)\Psi^\dagger_+(p)\Psi^\dagger_-(-p)} \\ \cancel{\Delta_{++}(p)\Psi^\dagger_+(p)\Psi^\dagger_+(-p)} \end{cases}$$

$$\Delta_{--}(p) \propto \Delta_0 \frac{p_x + ip_y}{|p|}$$

Rigorous proof: calculate topological index (first Chern number)

$$C_1 = \frac{1}{24\pi^2} \int d^2\boldsymbol{p} d\omega \, \mathrm{Tr} \left[\varepsilon^{\mu\nu\lambda} G \partial_\mu G^{-1} G \partial_\nu G^{-1} G \partial_\lambda G^{-1} \right]$$

$C_1 = 1$ for $|V_z| > \sqrt{\mu^2 + \Delta^2}$

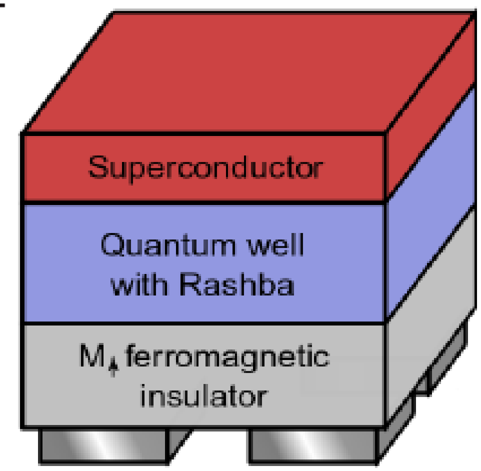
$C_1 = 0$ for $|V_z| < \sqrt{\mu^2 + \Delta^2}$

Practical route to spinless p+ip superconductivity

PRL **104**, 040502 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

Generic New Platform for Topological Quantum Computation Using Semiconductor Heterostructures

Jay D. Sau,[1] Roman M. Lutchyn,[1] Sumanta Tewari,[1,2] and S. Das Sarma[1]



Proximity-induced Δ_{ind}

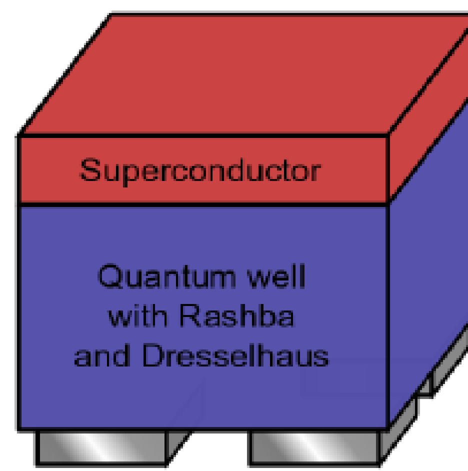
Proximity-induced V_z

Challenge: creating two interfaces

PHYSICAL REVIEW B **81**, 125318 (2010)

Majorana fermions in a tunable semiconductor device

Jason Alicea
Department of Physics, California Institute of Technology, Pasadena, California 91125, USA



Proximity-induced Δ_{ind}

In-plane magnetic field

Challenge: low electron density, effects of disorder

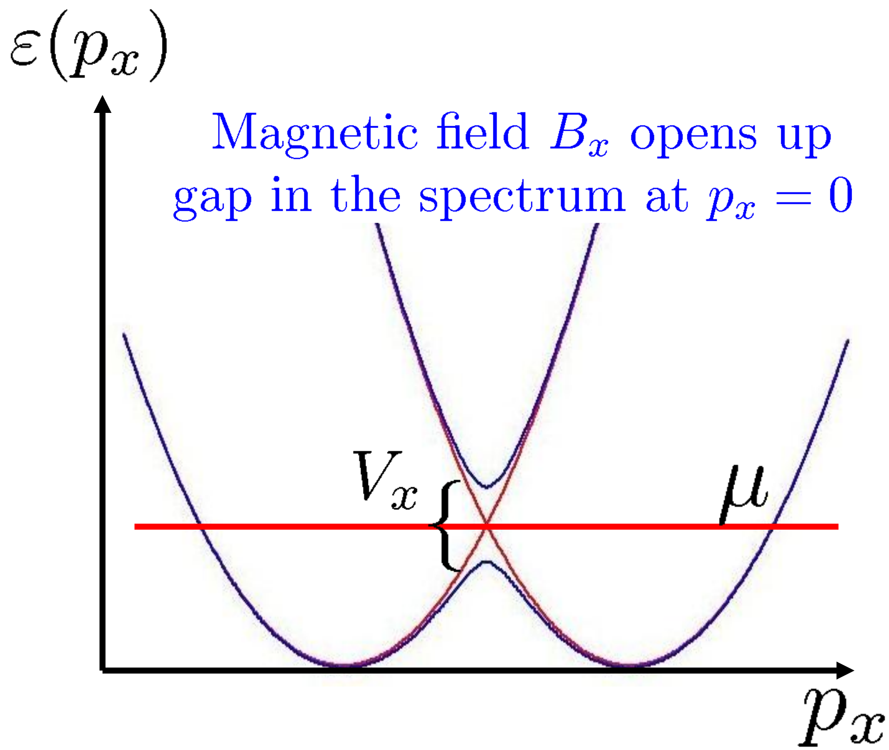
1D semiconductor nanowires

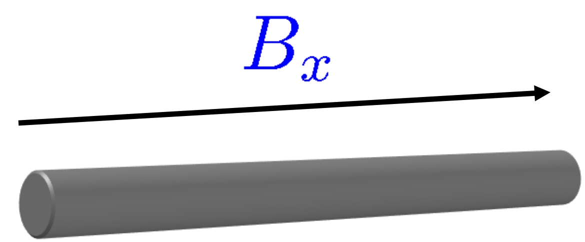
$$H_0 = \int_{-L}^{L} dx\, \psi^\dagger_\sigma(x) \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'}(x)$$

single channel nanowire

spin-orbit coupling

Zeeman splitting





InAs, InSb nanowires

large spin-orbit ($\alpha \sim 0.1 eV\AA$)

large g-factor ($g \sim 10 - 50$)

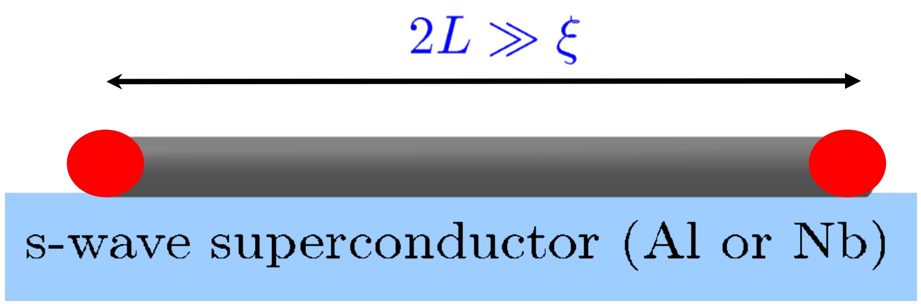
good contacts with metals

Majorana quantum wires

$$H_{\mathrm{MW}} = \int_{-L}^{L} dx \left[\psi_\sigma^\dagger \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'} + \Delta_0^* \psi_\uparrow \psi_\downarrow + \Delta_0 \psi_\downarrow^\dagger \psi_\uparrow^\dagger \right]$$

Rashba spin-orbit+in-plane field

Proximity-induced superconductivity



topologically non-trivial

$$|V_x| > \sqrt{\mu^2 + \Delta_0^2}$$

topologically trivial

$$|V_x| < \sqrt{\mu^2 + \Delta_0^2}$$

Lutchyn, Sau, Das Sarma, PRL(2010)
Oreg, Refael, von Oppen, PRL(2010)

Drive topological phase transition by changing V_x or μ

Summary

- Model for semiconductor nanowires
- Proximity-induced superconductivity
- Majorana zero modes detection schemes

- How important is one-dimensionality (single band)

Lutchyn, Stanescu, Das Sarma, PRL (2011); Lutchyn & Fisher, arXiv(2011)

- Disorder and chemical potential fluctuations

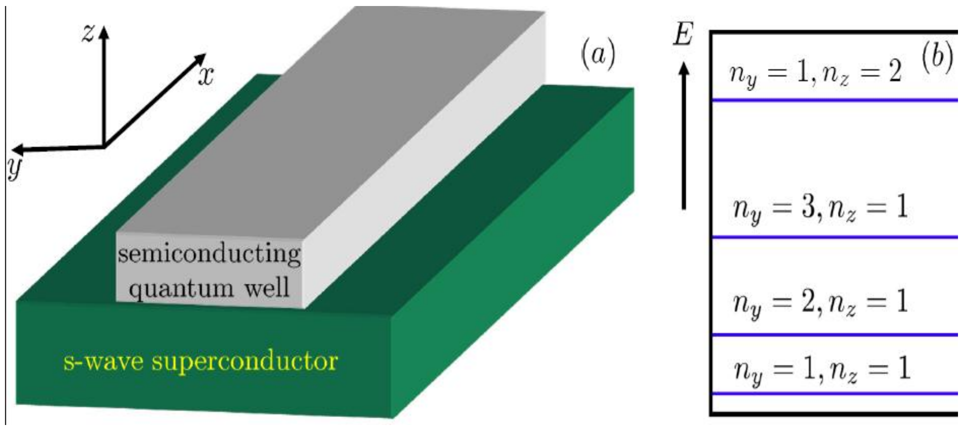
Stanescu, Lutchyn, Das Sarma, PRB (2011); Lutchyn, Stanescu, Das Sarma, arXiv(2011)

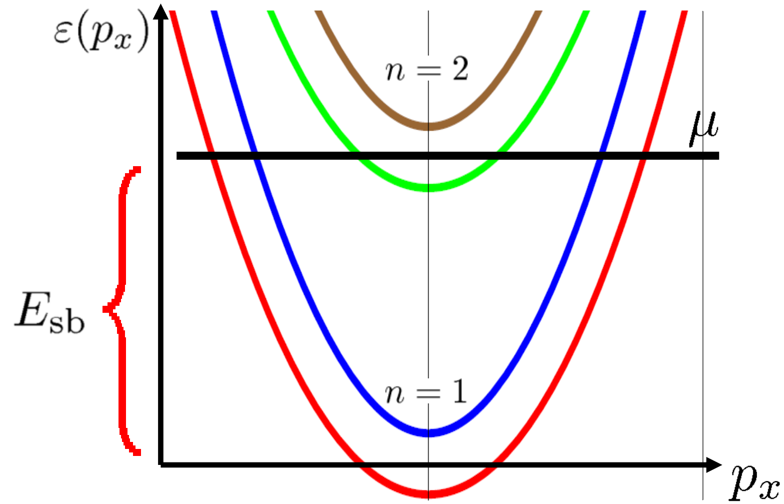
- Majorana fermions without long range SC order

Fidkowski, Lutchyn, Nayak, Fisher, arXiv(2011)

Experimental efforts: Delft, Harvard, UCSB ...

Multi-band semiconductor nanowires





Weak coupling analysis $\Delta \to 0$

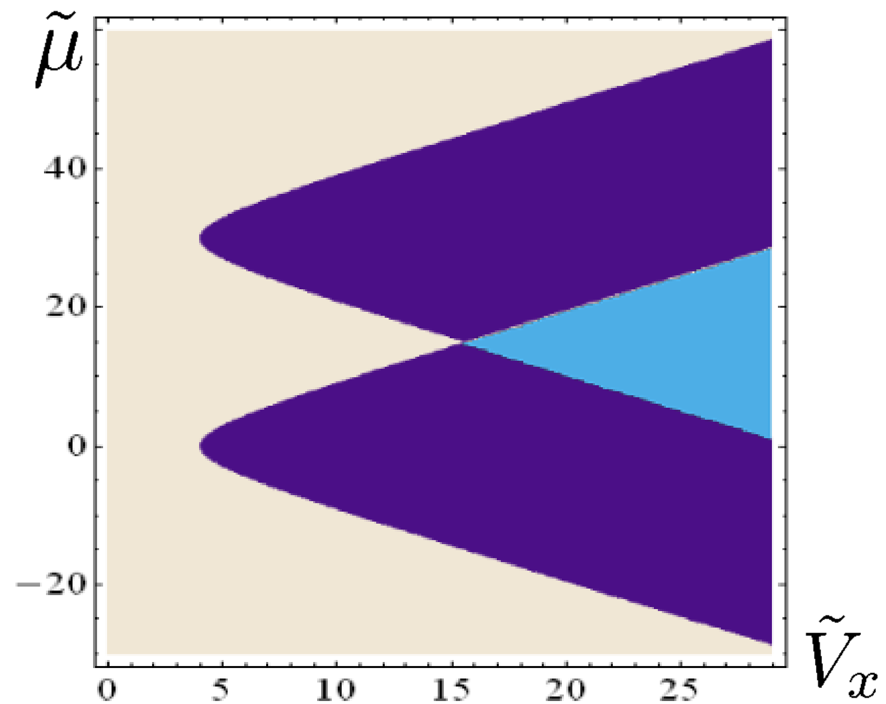
$\mathcal{M} = (-1)^{\nu(0)-\nu(\Lambda)}$ Kitaev, arXiv'00

Topological phase exists when

Second band $|V_x| > \sqrt{(\mu - E_{\mathrm{sb}})^2 + \Delta_0^2}$

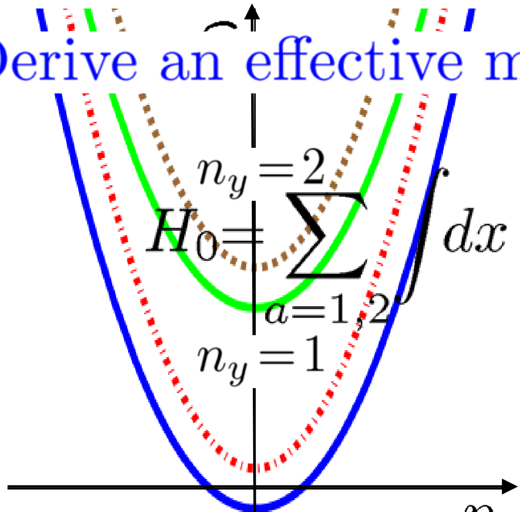
First band $|V_x| > \sqrt{\mu^2 + \Delta_0^2}$

Lutchyn, Stanescu, Das Sarma, PRL'11



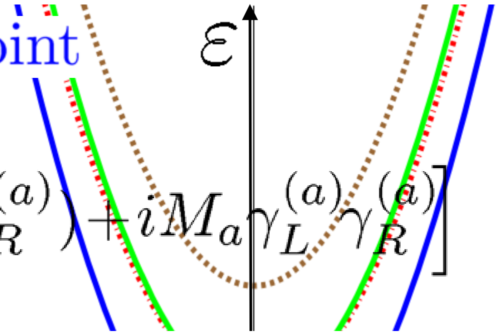
Multicritical point

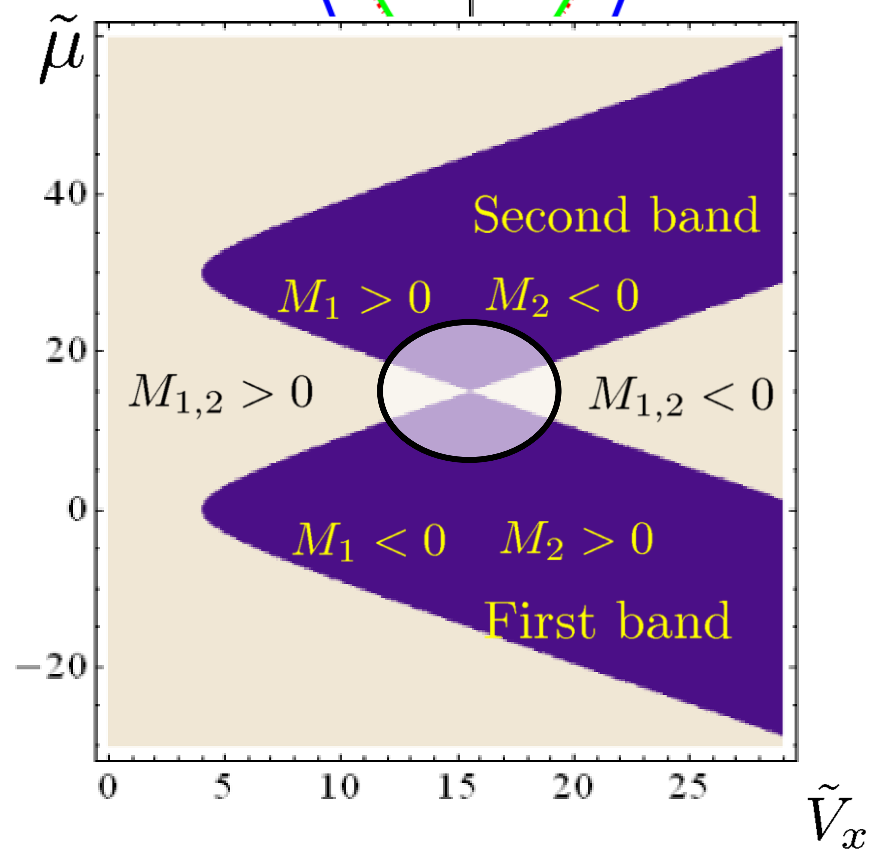
Derive an effective model at multicritical point



$$H_0 = \sum_{a=1,2}\int dx\left[iv\left(\gamma_L^{(a)}\partial_x\gamma_L^{(a)} - \gamma_R^{(a)}\partial_x\gamma_R^{(a)}\right) + iM_a\gamma_L^{(a)}\gamma_R^{(a)}\right]$$

magnetic field $V_x \uparrow$





$$M_1 = \delta\mu - \delta V_x$$

$$M_2 = \delta\mu + \delta V_x$$

$Z_2 \otimes Z_2$ symmetry

Lutchyn & M.P.A. Fisher, arXiv'11

Effect of band-mixing terms

What are allowed band-mixing terms ?

Hidden symmetry of total Hamiltonian $\psi_\lambda(x) \to \psi_{-\lambda}(-x)$

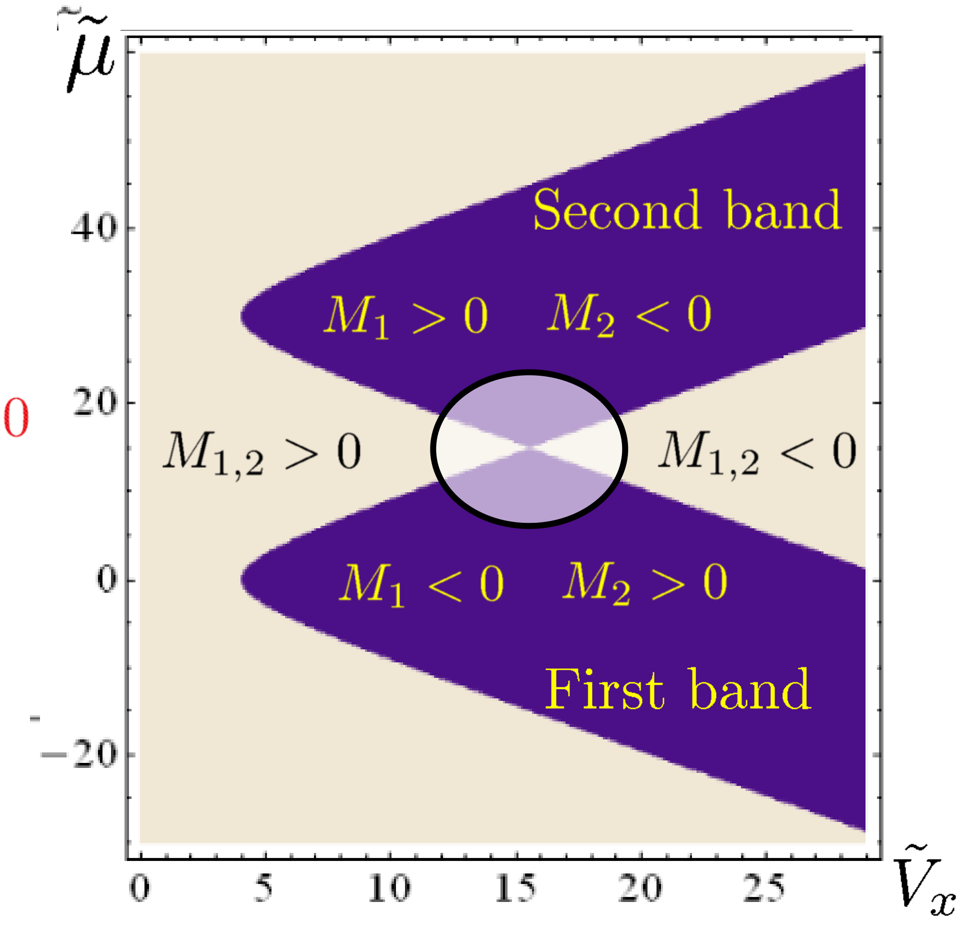
$$H_{12} = i\lambda_1 \int dx[\gamma_R^{(1)}\gamma_L^{(2)} - \gamma_L^{(1)}\gamma_R^{(2)}] + i\lambda_2 \int dx[\gamma_L^{(1)}\gamma_L^{(2)} + \gamma_R^{(1)}\gamma_R^{(2)}]$$

Spectrum for $\delta\mu = 0$; $\lambda_1 \neq 0$ and $\lambda_2 = 0$

$$E = \sqrt{v^2p^2 + (\delta V_x \pm \lambda_1)^2}$$

Phase transition occurs at

$$\delta V_x = \pm\sqrt{\lambda_1^2 + \delta\mu^2}$$



What is effect of interactions on topological superconducting phase ?

Conventional wisdom:

TP phase is protected as long as interactions are weak

Counterexample: Fidkowski & Kitaev'10

Noninteracting classification (Schnyder et al.'08; Kitaev'08)

$$Z \to Z_8$$

Add short-range interactions: $H = U \int dx :\rho(x)::\rho(x):$

Interaction commutes with fermion parity operator in each chain

⬇

Multicritical point is preserved

Effect of interactions on the phase boundary

Add weak interparticle interactions: → arrive at massive Thirring model

After bosonization, one arrives at

$$H = \frac{v}{2\pi}\int dx\left[K(\partial_x\theta)^2 + K^{-1}(\partial_x\phi)^2 - \delta\tilde{V}_x \sin 2\phi - \tilde{\Delta}_{12}\cos 2\theta\right]$$

RG equations

$$\frac{d\delta\tilde{V}_x}{d\ln b} = (2-K)\delta\tilde{V}_x$$

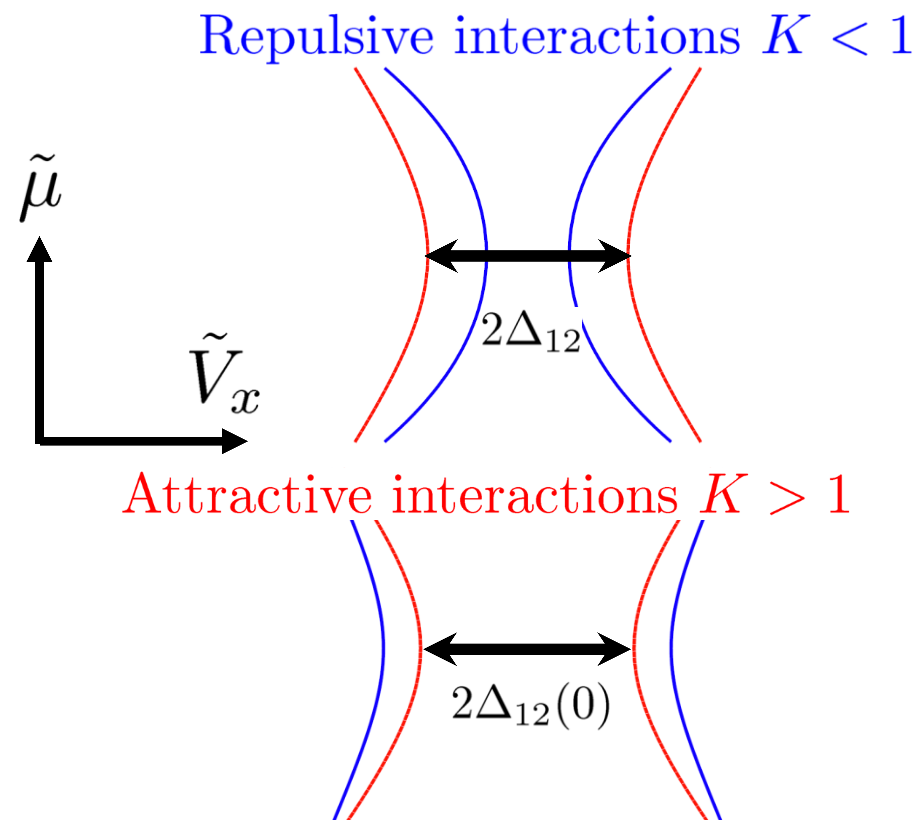
$$\frac{d\tilde{\Delta}_{12}}{d\ln b} = (2-K^{-1})\tilde{\Delta}_{12}$$

Scaling of free energy

$$f(V,\Delta) = b^{-2} f_s(b^{\lambda_V} V, b^{\lambda_\Delta}\Delta)$$

New phase boundary

$$\delta V_x = \left(\Delta_{12}^2 + \delta\mu^2\right)^{\frac{1}{2}\frac{2-K}{2-K^{-1}}}$$



R. M. Lutchyn & M. P. A. Fisher, arXiv'11

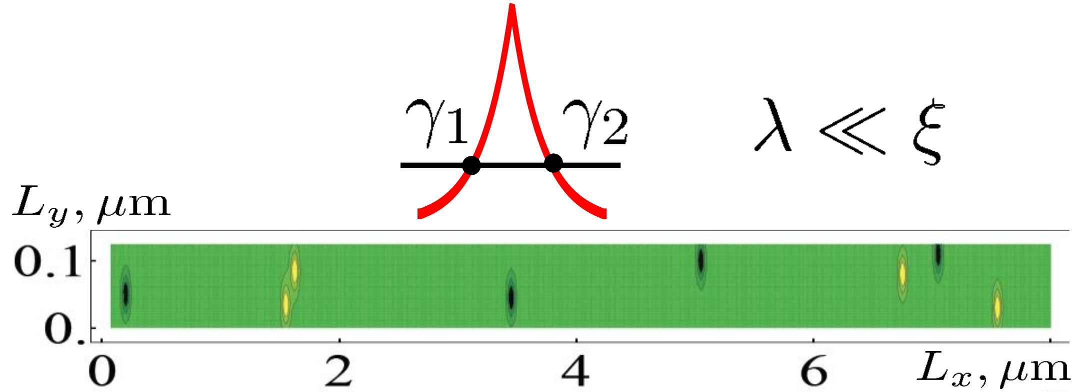
Effect of short-range disorder

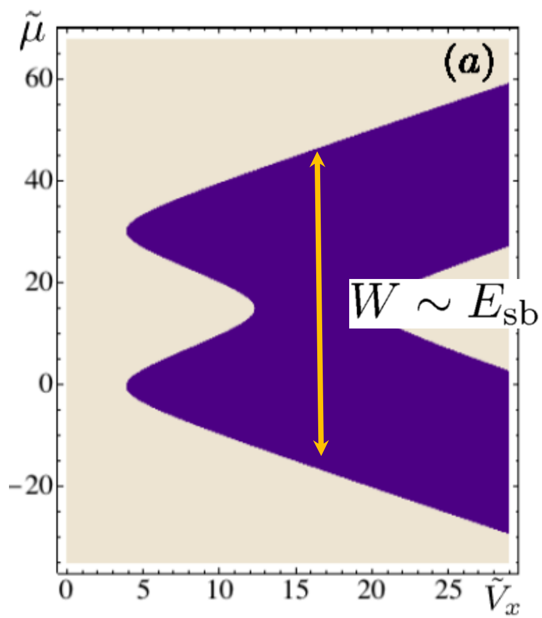
Impurity potential

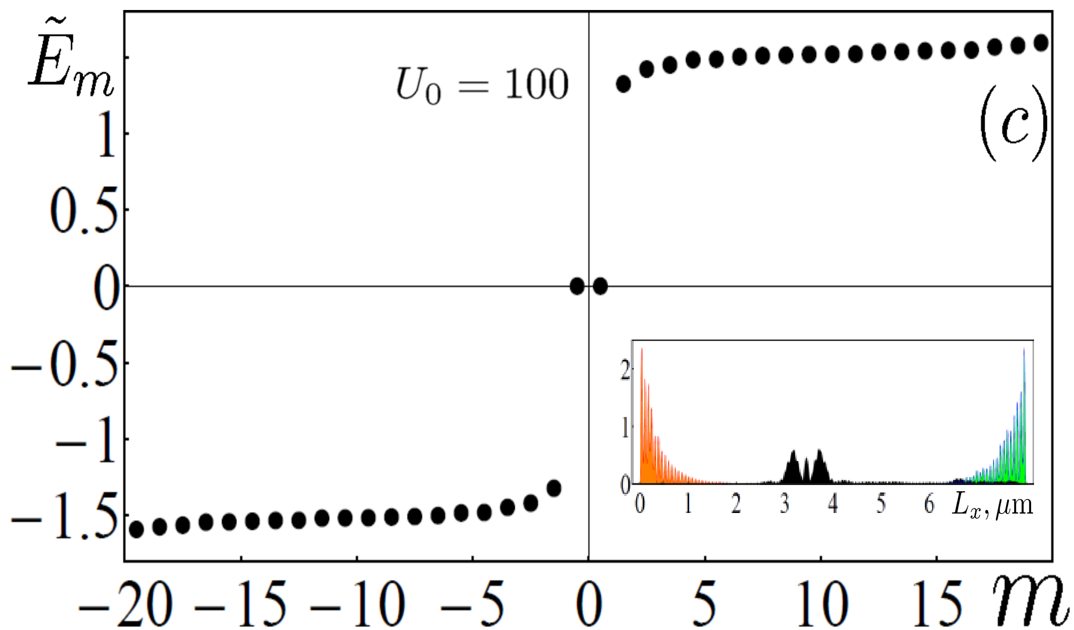
$$U_{\mathrm{imp}}(\boldsymbol{r}) = \sum_j U_{0j} \frac{\exp(-|\boldsymbol{r}-\boldsymbol{r}_j|/\lambda)}{1+|\boldsymbol{r}-\boldsymbol{r}_j|/d}$$

$U_{0j} = \pm U_0$ $\lambda = 16\mathrm{nm}$

$\lambda \ll \xi$

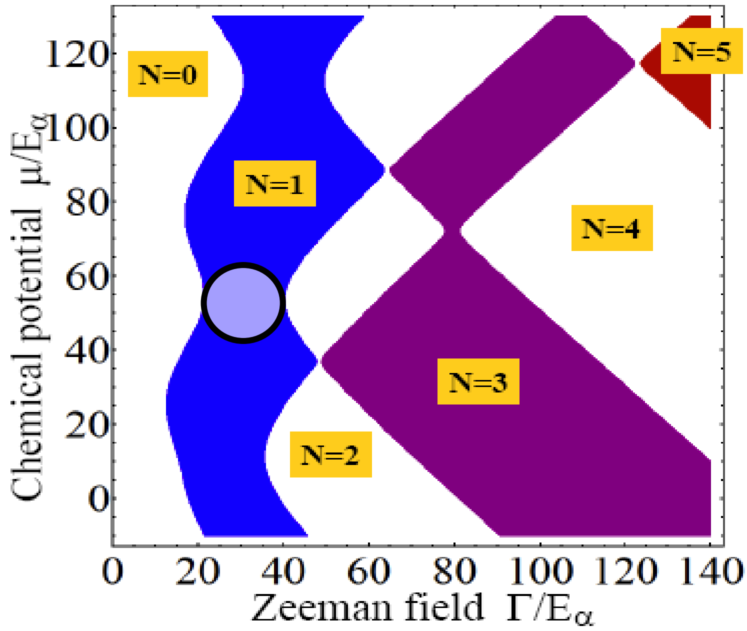


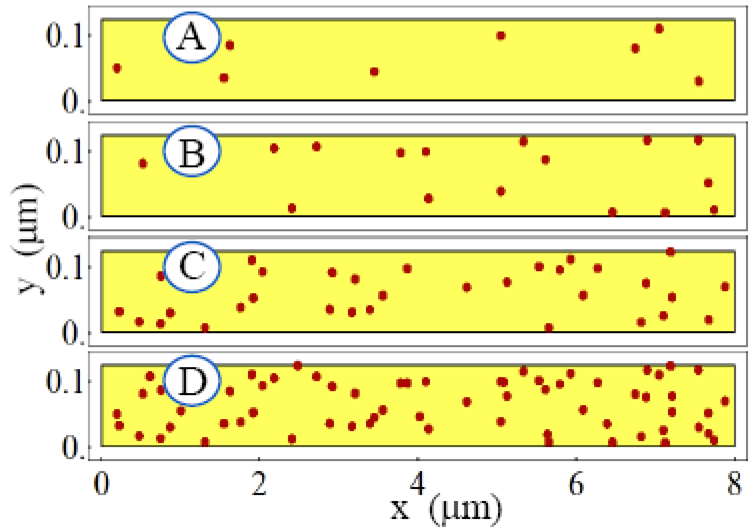


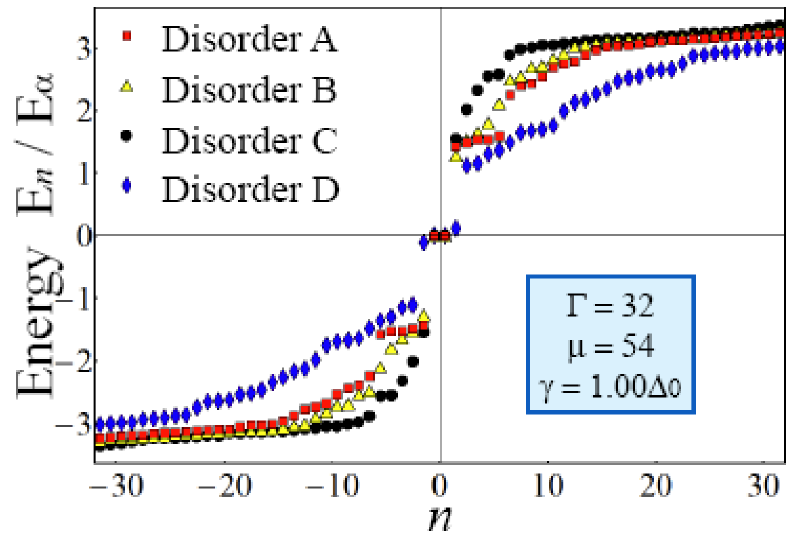


Lutchyn, Stanescu, Das Sarma, PRL'11

Effect of disorder in the multi-band nanowire







For single channel nanowire disorder drives topological phase transition $\tau_{\text{eff}}\Delta_{\text{eff}} \sim 1$

Motrunich, Damle & Huse, PRB (2001)

Gruzberg, Read & Vishveshwara, PRB (2005)

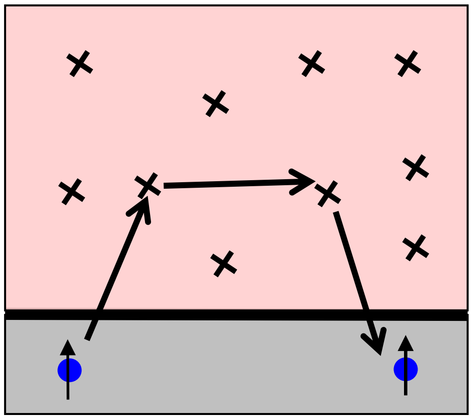
Brouwer et al., arXiv (2011)

Stanescu, Lutchyn, Das Sarma, PRB'11

Effect of disorder in the superconductor

s-wave superconductors are usually disordered $\tau\Delta_0 \ll 1$

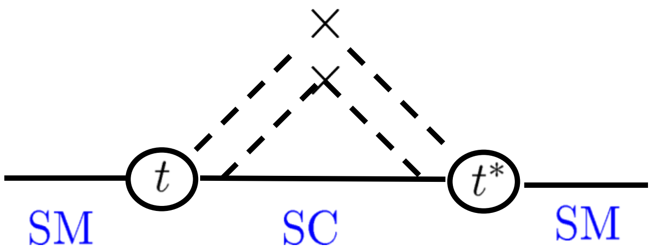
Potter & P.A. Lee, PRB'11



Green's function for disordered SCs (Abrikosov-Gorkov, JETP'61)

Proximity effect in the clean case

$$\hat{\Sigma}(\omega) = -\frac{\gamma}{\sqrt{\Delta_0^2-\omega^2}}(\omega\tau_0 + \Delta_0\tau_x)$$

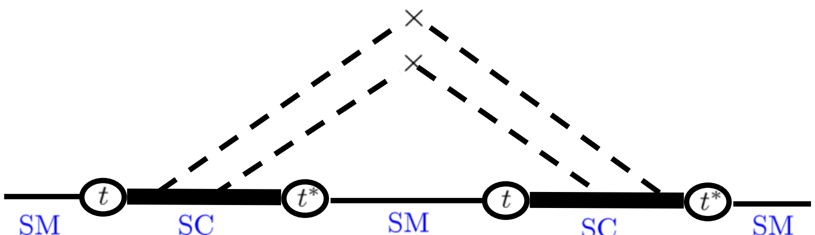


$$\omega \to \omega\eta_\omega \qquad \Delta_0 \to \Delta_0\eta_\omega$$

$$\eta_\omega = 1 + \frac{1}{2\tau\sqrt{\Delta_0^2-\omega^2}}$$

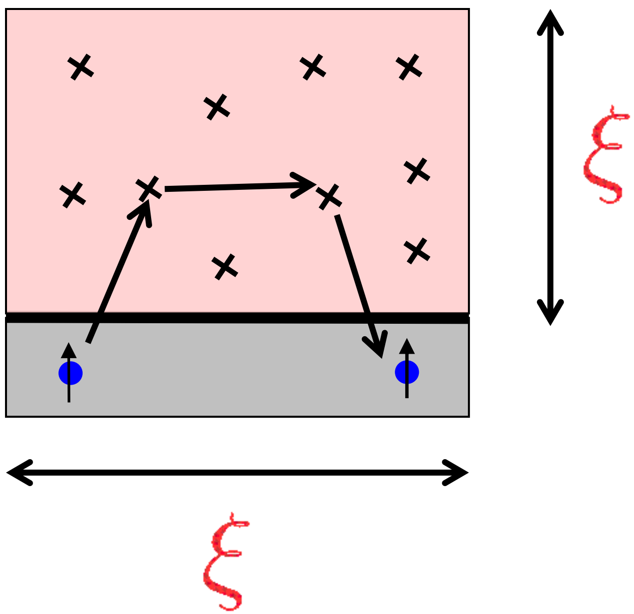
Reducible part

Irreducible part

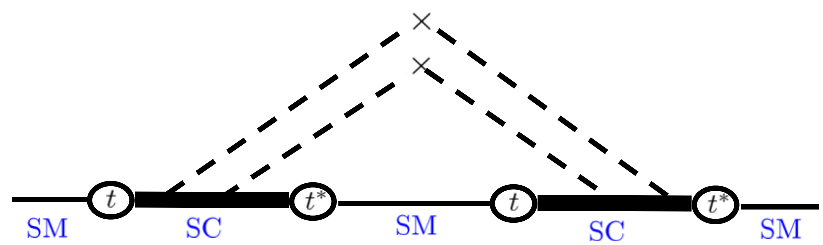


Irreducible part of the self-energy

Momentum relaxation rate $\Gamma = \mathrm{Im}[\Sigma_{\mathrm{irr}}(\omega)]$



Irreducible part



classical diffusion propagator

$$\Gamma \propto |t|^4 \nu_{3D} \nu_{2D} \int_0^\infty dt \int d^2 r P(t,r) e^{-2\Delta |t|}$$

Disorder in SC can be neglected !

$$\Gamma \propto t^2 \nu_{\mathrm{3D}} \frac{t^2 \nu_{\mathrm{2D}} \xi^2}{\xi^3} \frac{1}{\Delta} \propto \frac{\gamma^2}{\Delta} \frac{m^*}{m} \frac{1}{p_F \xi}$$

γ γ_{out}

Superconducting phase fluctuations

- Thus far, we have treated the superconducting order parameter as if it were classical.
- In reality, the SC order parameter might fluctuate due to finite-size and/or reduced dimensionality.
- How are the Majorana zero modes affected by SC order parameter fluctuations?

Fidkowski, Lutchyn, Nayak, Fisher, PRB11

Majorana zero modes via bosonization

- Consider a semiconductor quantum wire in contact with a bulk 3D superconductor
- Project to the lowest band and bosonize

$$H = \int_{-L/2}^{L/2} dx \left(\frac{v}{2\pi} \left[K(\partial_x \theta)^2 + K^{-1}(\partial_x \phi)^2 \right] - \frac{\Delta_P}{(2\pi a)} \cos 2\theta \right)$$

Δ_P is relevant for $K > 1/2$ and flows to strong coupling.

There are two degenerate ground states: $\theta = 0$ and $\theta = \pi$:

Spontaneously broken $\mathbb{Z}_2$ symmetry in bosonic variables corresponds to ground-state degeneracy in fermionic variables.

Fermion parity operator $(-1)^{N_F}$ transforms $\theta \to \theta + \pi$

$$|\text{even/odd}\rangle \equiv |\theta = 0\rangle \pm |\theta = \pi\rangle$$

Quasi-long-ranged superconducting order

- 1D SC wire, instead of 3D superconductor.
- LRO not possible; at best, power-law-decaying SC correlations.

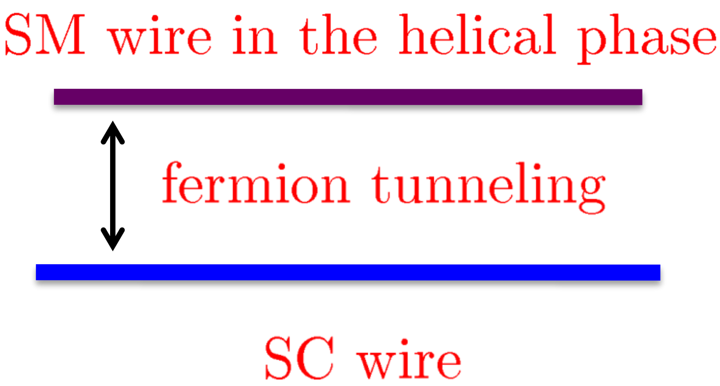
$$H_{\mathrm{SC}}^{(\rho)} = \frac{v_F}{2\pi} \int_{-L/2}^{L/2} dx \left[K_\rho (\partial_x \theta_\rho)^2 + K_\rho^{-1} (\partial_x \phi_\rho)^2 \right]$$

$$H_{\mathrm{SC}}^{(\sigma)} = \frac{v_F}{2\pi} \int_{-L/2}^{L/2} dx \left[K_\sigma (\partial_x \theta_\sigma)^2 + K_\sigma^{-1} (\partial_x \phi_\sigma)^2 \right] - \frac{2|U|}{(2\pi a)^2} \int_{-L/2}^{L/2} dx \cos(2\sqrt{2}\phi_\sigma)$$

- For attractive effective interaction U, pairs form so that a spin-gap opens, but long-ranged coherence does not.

$$\left\langle e^{i\sqrt{2}\theta_\rho(x)}\, e^{-i\sqrt{2}\theta_\rho(0)} \right\rangle \sim \frac{1}{x^{K_\rho}}$$

Single SM wire in proximity to a SC wire



Because of spin gap single fermion tunneling is blocked $\rightarrow$ pair hopping dominates

$$S_{\mathrm{PH}} \approx -\frac{\Delta_P}{(2\pi a)} \int d\tau \int_{-L/2}^{L/2} dx \sin\left(\sqrt{2}\theta_\rho - 2\theta\right)$$

at $v = v_F$, $2K_\rho = K$, the action decouples in $2\theta_\pm = \sqrt{2}\theta_\rho \pm 2\theta$

$$H = \frac{v}{2\pi} \int_{-L/2}^{L/2} dx \left[K_\rho (\partial_x \theta_+)^2 + K_\rho^{-1} (\partial_x \phi_+)^2\right]$$

$$+ \frac{v}{2\pi} \int_{-L/2}^{L/2} dx \left[K_\rho (\partial_x \theta_-)^2 + K_\rho^{-1} (\partial_x \phi_-)^2\right] - \frac{\Delta_P}{(2\pi a)} \int_{-L/2}^{L/2} dx \sin(2\theta_-)$$

Charging energy breaks the degeneracy between $|\mathrm{even}\rangle$ and $|\mathrm{odd}\rangle$ states

Two SM wires in proximity to a SC wire

Now we have two vacua
& topological degeneracy

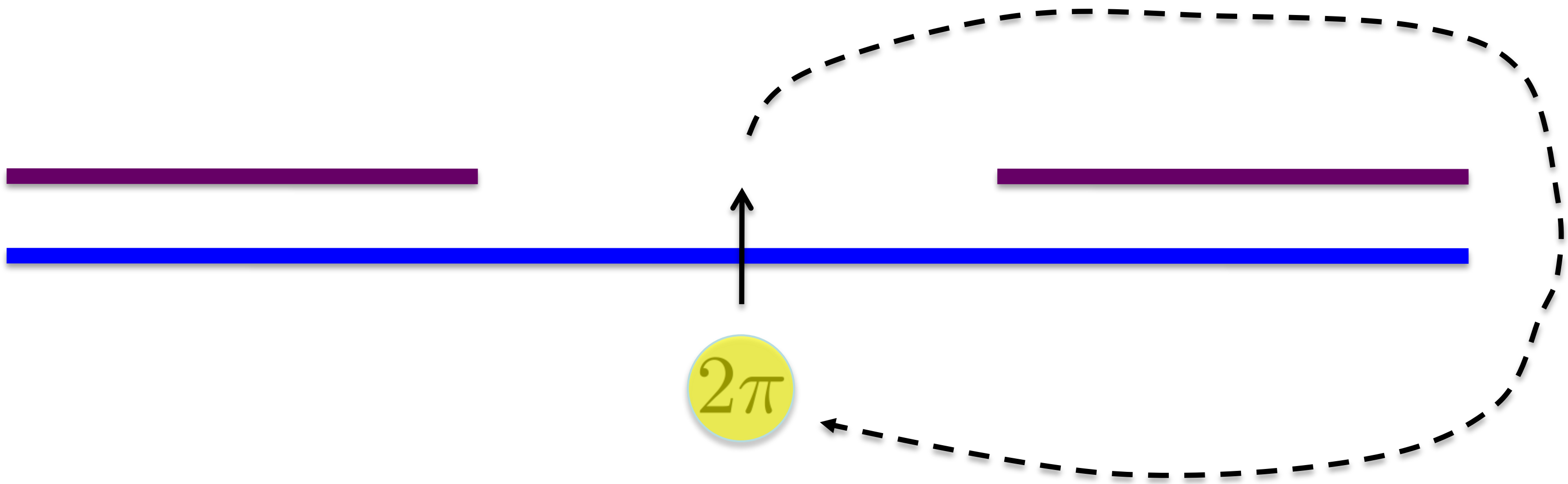
$|\mathrm{even},\mathrm{even}\rangle$ and $|\mathrm{odd},\mathrm{odd}\rangle$ $\quad (-1)^{N_F} = +1$

$|\mathrm{even},\mathrm{odd}\rangle$ and $|\mathrm{odd},\mathrm{even}\rangle$ $\quad (-1)^{N_F} = -1$

Exponentially small splitting by instanton analysis,
stable to perturbations about soluble point

Effects of impurity scattering

- Impurities can cause electron backscattering = 2π phase slips
- A phase slip allows a vortex to measure the qubit via Aharonov-Casher:



- Backscattering operator: $\cos(\sqrt{2}\phi_\rho)$

- Amplitude from one state of qubit to other: $\boxed{\Delta E \sim \frac{v}{L^{K_\rho/2}}}$

Summary

- Model for semiconductor nanowires
- Proximity-induced superconductivity
- Majorana zero modes detection schemes

- How important is one-dimensionality (single band)

Lutchyn, Stanescu, Das Sarma, PRL (2011); Lutchyn & Fisher, arXiv (2011)

- Disorder and chemical potential fluctuations

Stanescu, Lutchyn, Das Sarma, PRB (2011); Lutchyn, Stanescu, Das Sarma, arXiv (2011)

- Majorana fermions without long range SC order

Fidkowski, Lutchyn, Nayak, Fisher, PRB (2011)

Thank you !