

# Topological phases on honeycomb bilayer

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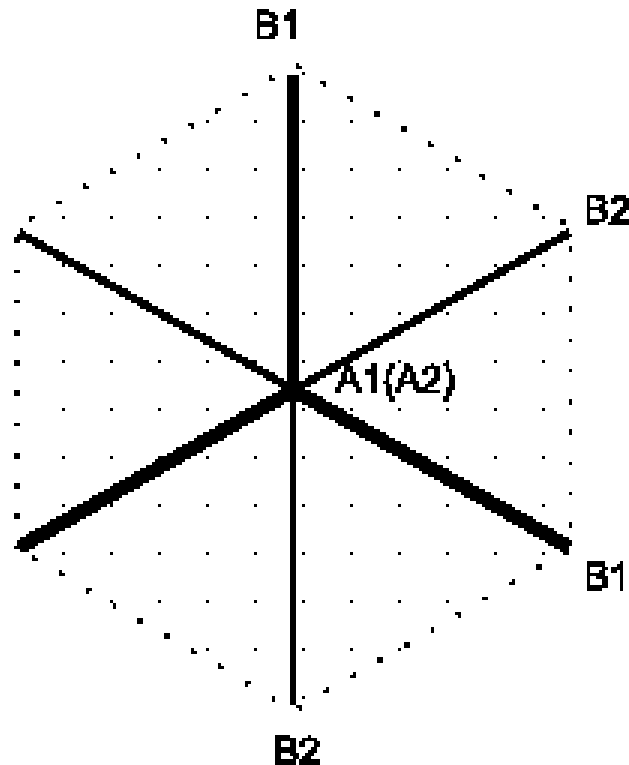
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# Acknowledgment

- J. Vučićević, D. Tanasković, M. Goerbig
- N. Regnault, A. Sterdyniak, I. Vidanović

# Definition: honeycomb bilayer



# graphene bilayer

Exps: insulating phase at charge neutrality point

J. Martin et al., PRL 105, 256806 (2010)

R. Weitz et al., Science 330, 812 (2010)

F. Freitag et al., arXiv: 1104.3816

J. Velasco Jr. et al., arXiv:1108.1609

theory: close competition of

AHE

LAF (opposite spins in opposite layers)

SHE

VHE (charge imbalance in opposite layers)

H. Min et al., PRB 77, 041407 (2008)

R. Nandkishore and L. Levitov, PRL 104, 156803 (2010);  
PRB 82, 115124 (2010)

F. Zhang et al., PRL 106, 156801 (2011)

J. Jung et al., PRB 83, 115408 (2011)

F. Zhang et al., PRB 81, 041402(R) (2010)

Y. Lemonik et al., PRB 82, 201408 (2010)

O. Vafek and K. Yang, PRB 81, 041401 (2010)

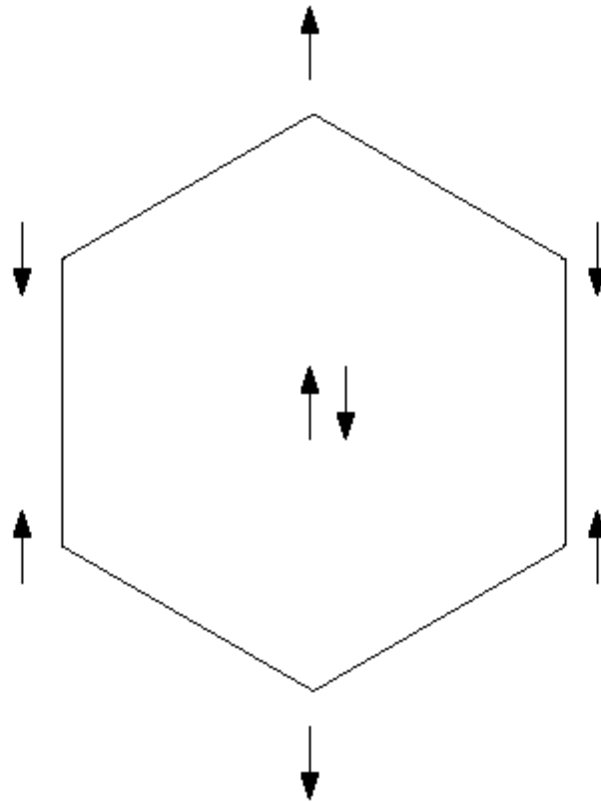
F. Zhang and A.H. MacDonald, arXiv:1107.4727

# Hubbard

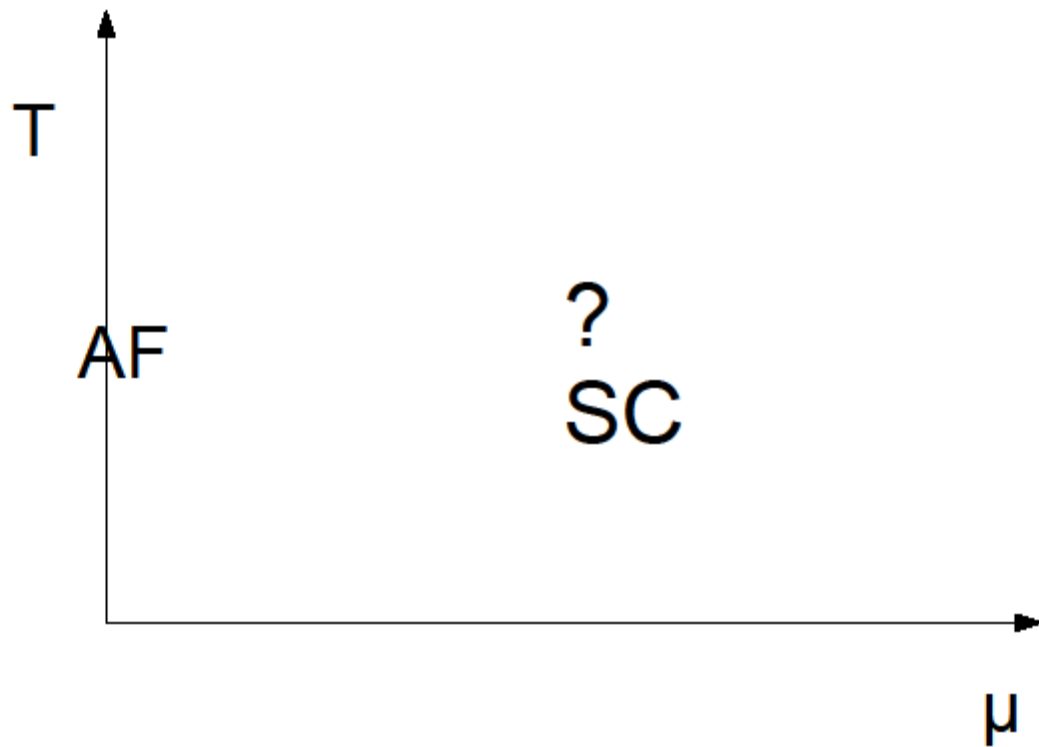
$$U \gg t \gg t_{\perp}$$

microscopically

locked AF order







AF even in weak coupling

$$t, t_{\perp} \gg U$$

O.Vafek, PRB 82, 205106 (2010))

easy to handle

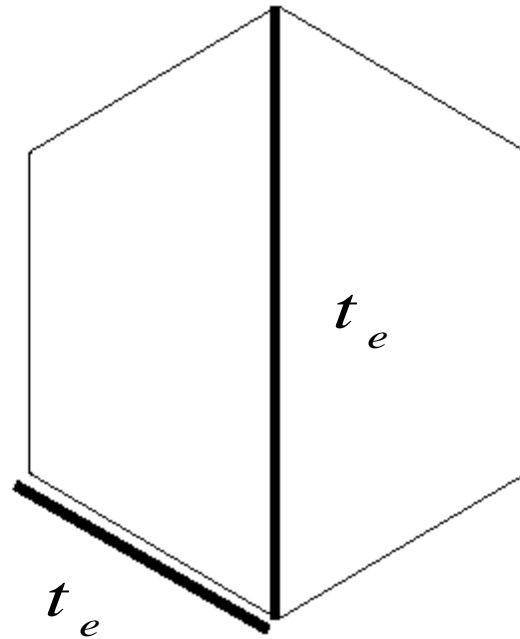
$$t_{\perp} \gg t \gg U$$

noninteracting electrons  $t_{\perp} \gg t$

$$\begin{bmatrix} -\mu & -t_e (S)^2 \\ -t_e (S^*)^2 & -\mu \end{bmatrix} \begin{bmatrix} \Psi_{B2} \\ \Psi_{B1} \end{bmatrix}$$

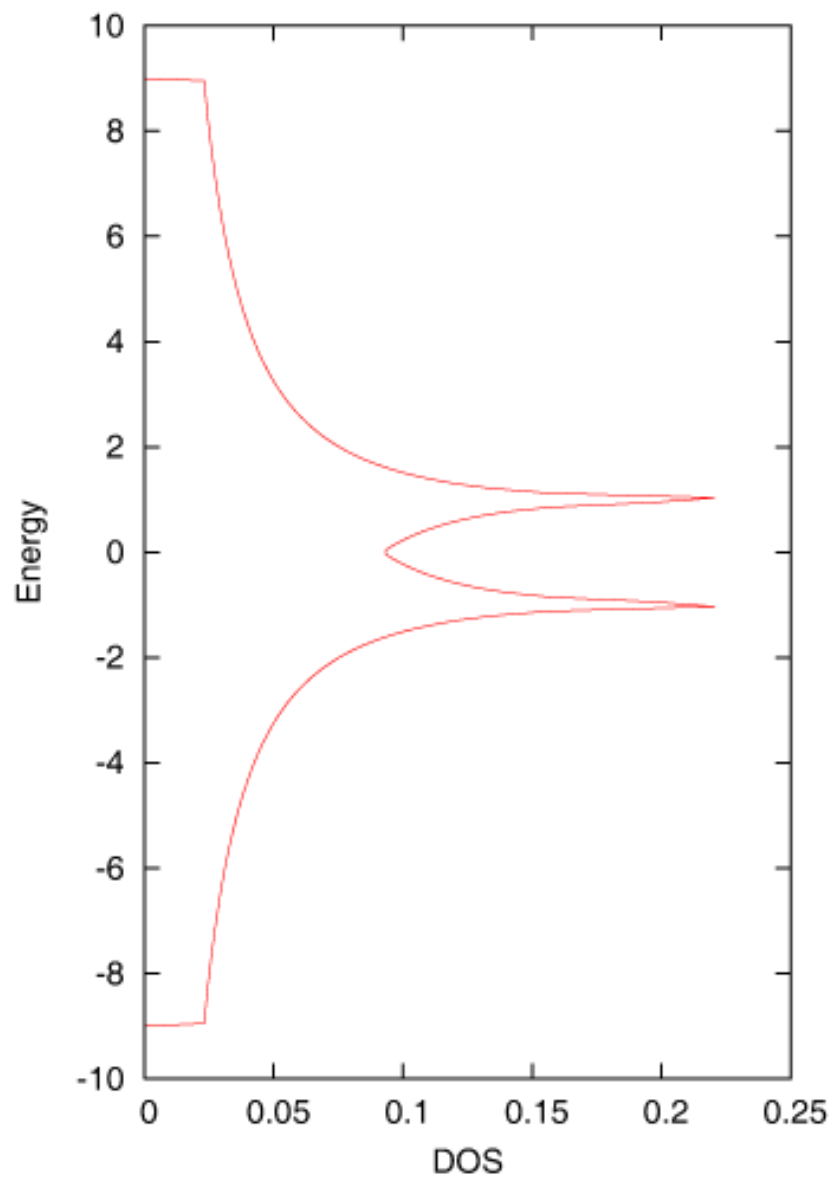
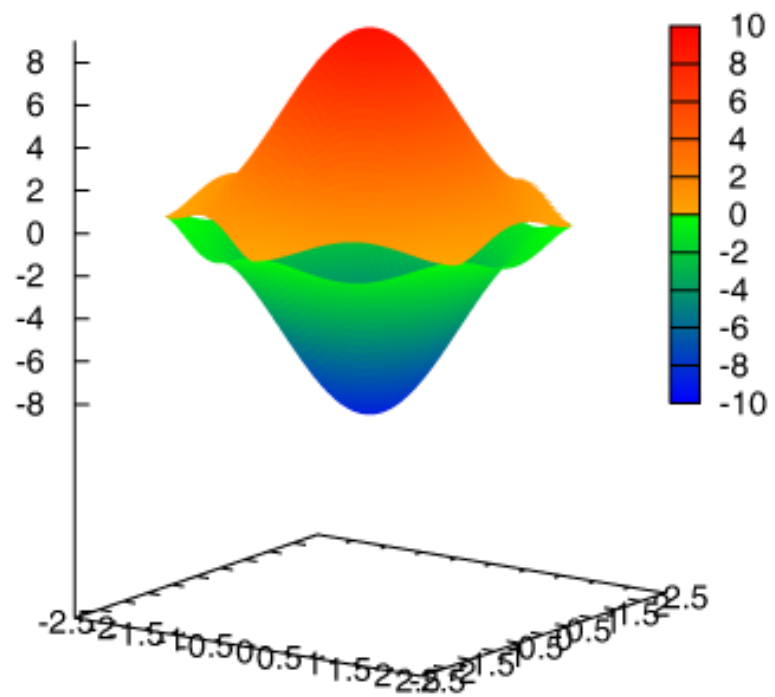
$$t_e = \frac{t^2}{t_{\perp}} \quad S(\vec{k}) = \sum_{\delta} \exp \{ i \vec{k} \cdot \vec{\delta} \}$$

in real space



NN and NNNN hopping of the same value

Dispersion



- introduce  $U$

- large  $t_{\perp}$   $\longrightarrow$  small  $t_e$

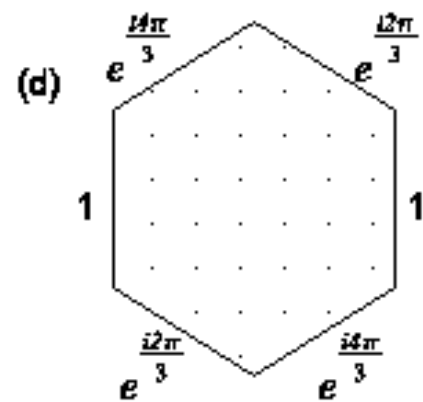
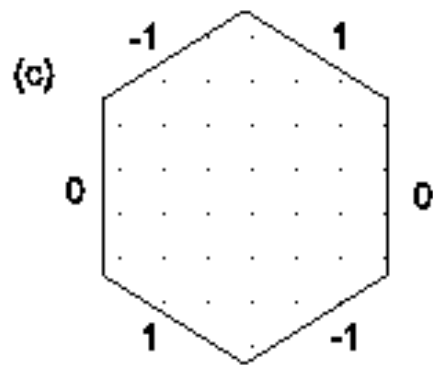
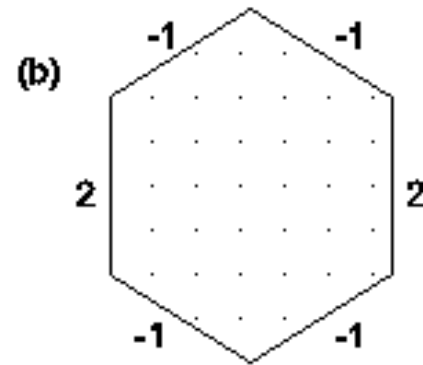
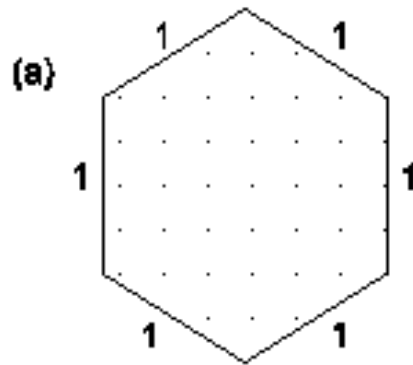
- superexchange and  $t$   $J$  model

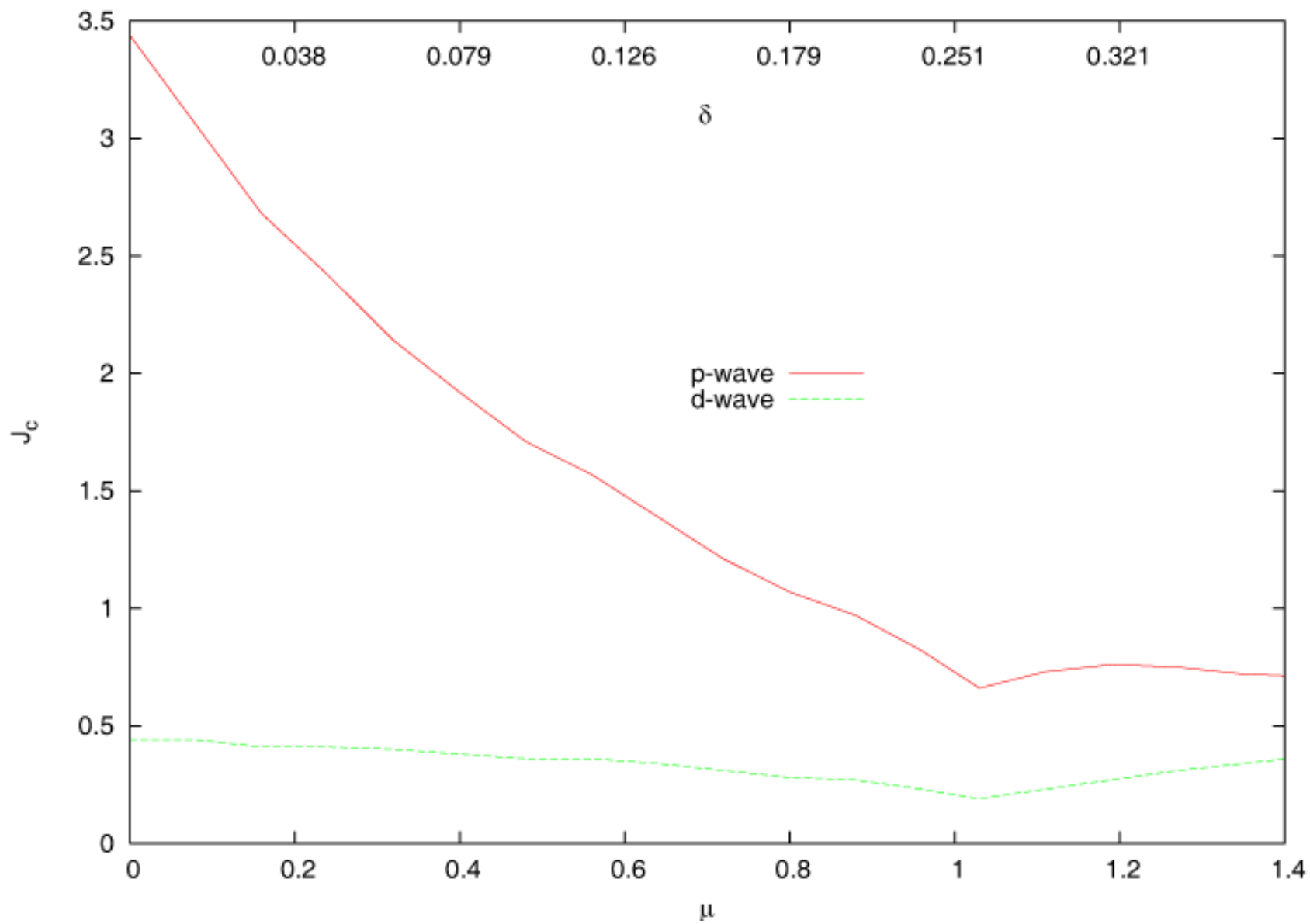
- consider  $t$   $J$  model on projected monolayer
- high doping, weak coupling limit
- mean field of a model with NN attractive ( $J$ ) interaction

A. Black-Schaffer and S. Doniach, PRB 75, 134512 (2007)



classify instabilities

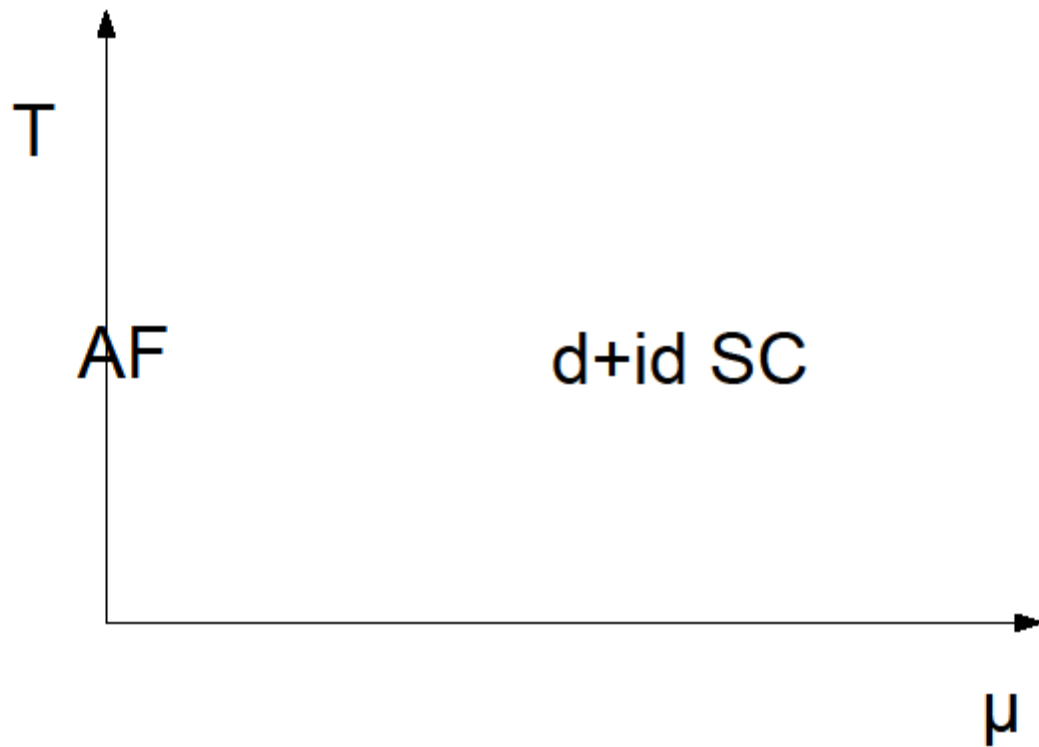


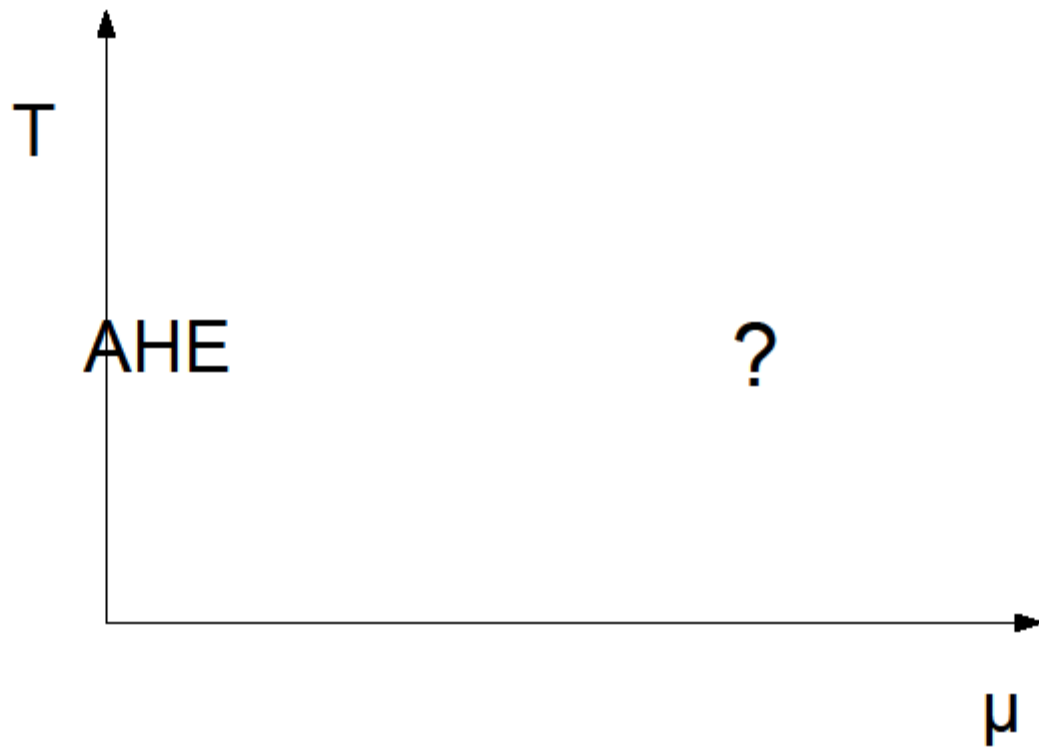


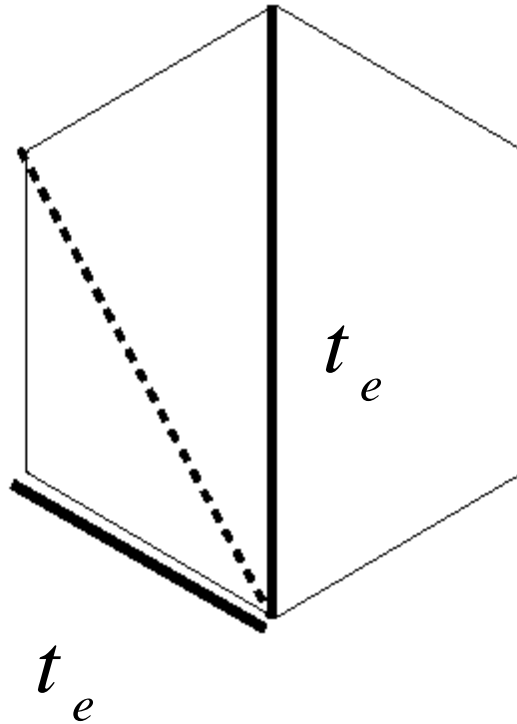
## BCS weak coupling analysis

**d + i d** - lowest lying instability

$$\Delta_{K_{\pm} + k} \sim \frac{(k_x + i k_y)^2}{|k|^2}$$







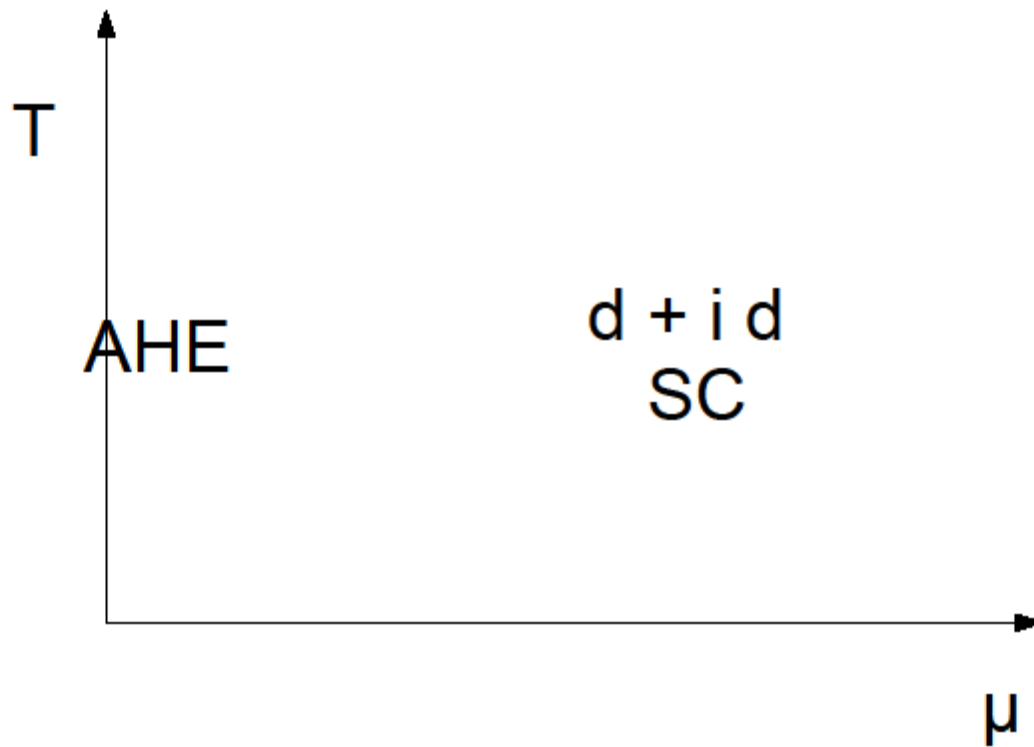
NNN complex hopping –

simple generalization of Haldane model

-C = 2 “flat” band

-no spin

-expect d + i d valley pairing for finite  $\mu$   
if attractive NN interaction exists





no surprise:

$C = 2$  QHE and  $d + i d$  topological  
superconductor are related

$$\mathcal{X}_2 = \begin{vmatrix} \mathbf{1} & \cdots & \mathbf{1} \\ \vdots & \ddots & \vdots \\ z_1^* & \cdots & z_N^* \\ \vdots & \ddots & \vdots \end{vmatrix}$$

$$\chi_2 = \det \left( \frac{z_{\uparrow}^* - z_{\downarrow}^*}{z_{\uparrow} - z_{\downarrow}} \right) \prod_{i,j} (z_{i\uparrow} - z_{j\downarrow})$$

$\uparrow, \downarrow$  division arbitrary

$\chi_2$  has premade pairs

need “spontaneous breakdown of permutation symmetry” and pairing agent to establish  $d + i d$

## Conclusions:

In the limit of  $t_{\perp} \gg t \gg g$  (*attractive*)  
d+id topological superconductor  
may be present on honeycomb bilayer

Graphene bilayer might support  
d + i d superconductivity at finite doping