KIT P, UCSB
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Berry curvature and Hall Effect of Magnons in Insulating Ferromagnets

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Ryuichi Shindou (Tokyo Inst. Tech.)

- R. Matsumoto, S. Murakami, Phys. Rev. Lett. 106, 197202 (2011).
- R. Matsumoto, S. Murakami, arXiv:1106.1987 (to appear in Phys. Rev. B)

outline

1. Introduction

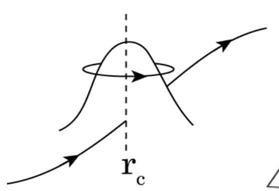
    spin Hall effect of electrons

    spin Hall effect of photons

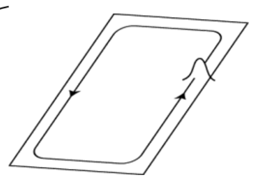
2. Semiclassical theory:

    rotational motion, Hall effect

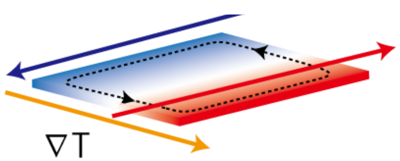
3. Linear response theory
4. Application ($Lu_2V_2O_7$, YIG)
5. Summary



self-rotation



edge current

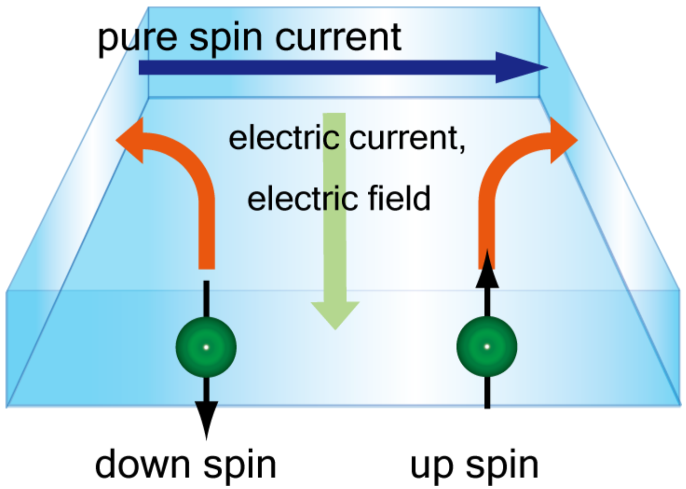


Thermal Hall effect

Spin Hall effect of electrons

Spin Hall effect

Electric field ➡ Transverse pure spin current



Magnetic field
Magnet

not needed

Electric manipulation of spins
(without magnetic field)

$$H_{so} = \lambda(\vec{\nabla} U \times \vec{k}) \cdot \vec{\sigma}$$

intrinsic spin Hall effect= spin-orbit coupling + Berry phase in k-space

- p-type semiconductors
  (SM, Nagaosa, Zhang, Science (2003))
- n-type semiconductors with Rashba coupling
  (Sinova, Culcer, Niu, Sinitsyn, Jungwirth, and MacDonald, Phys. Rev. Lett. (2004))

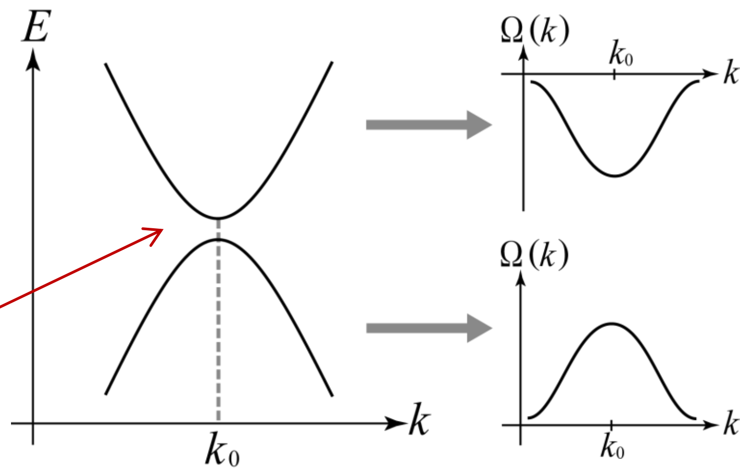
Semiclassical theory

Adams, Blount; Sundaram,Niu, …

$$\begin{cases} \dot{\vec{x}} = \boxed{\dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -e\vec{E} \end{cases}$$

Boltzmann
transport

- Determined by Bloch wf.
  → various Hall effects
  spin Hall effect
- Enhanced near band crossing



Energy band

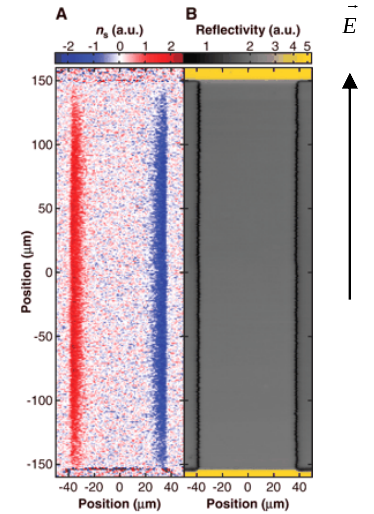
Berry curvature

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$ **: Berry curvature = "magnetic field in k-space"**

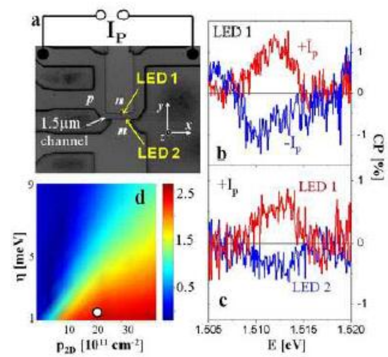
$u_{n\vec{k}}$: periodic part of the Bloch wf. $\quad \psi_{n\vec{k}}(\vec{x}) = u_{n\vec{k}}(\vec{x})e^{i\vec{k}\cdot\vec{x}}$

(n : band index)

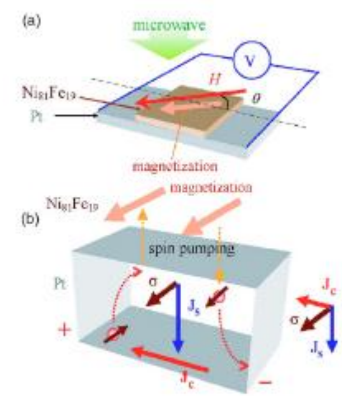
Experiments on spin Hall effect



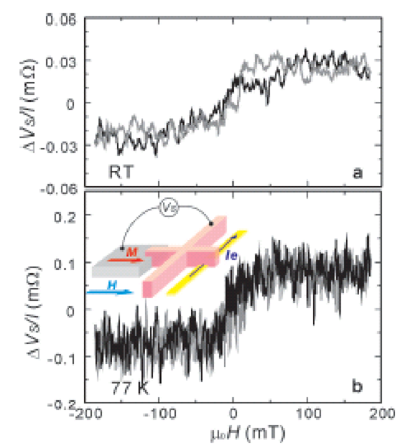
- **3D n-type semicond., Kerr rotation**
  - Kato, Myers, Gossard, Awschalom, Science (2004)
  - Sih et al. , Nature Phys. (2005) n-GaAs, n- (In,Ga)As
  - Sih et al., PRL (2006)
  - Stern et al., PRL(2006) n-ZnSe **RT**



- **2D p-type semicond., spin LED**
  - J. Wunderlich et al., PRL(2005)
    p-GaAs

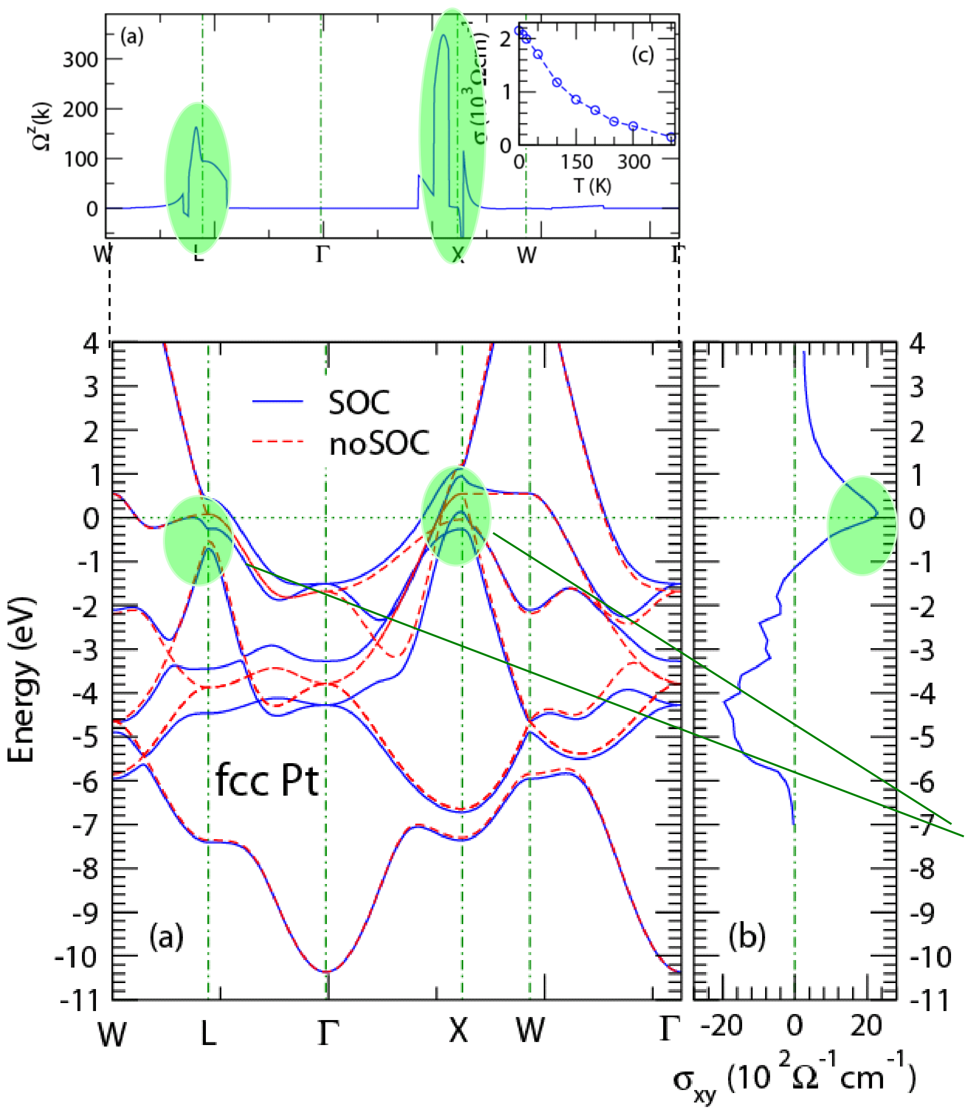


- **Metal (Pt, Al,Au…) -- Inverse SHE**
  - Saitoh, Ueda, Miyajima, Tatara, APL (2006) Pt **RT**
  - Valenzuela and Tinkham, Nature(2006) Al
  - Seki et al., Nature Materials(2008) Au **RT**



- **Metal (Pt) -- SHE & Inverse SHE**
  - Kimura, Otani, Sato,
    Takahashi, Maekawa, PRL (2007) Pt **RT**

Large spin Hall effect in Pt: first-principle calc.



$\sigma_s \approx 360 \ \Omega^{-1} cm^{-1}$

at RT

Cf. Experiment (Kimura et al.)

$\sigma_s \approx 240 \ \Omega^{-1} cm^{-1}$

Band crossing near Fermi energy (← d-orbitals)

G. Y. Guo, SM, T. W.Chen, N. Nagaosa, Phys. Rev. Lett. (2008)

Spin Hall effect of light

Spin Hall effect of light

-- Analogous to electrons --

Onoda, SM, Nagaosa, Phys. Rev. Lett. (2004)
Onoda, SM, Nagaosa, Phys. Rev. E (2006)
also in Dooghin et al. Phys. Rev. A (1992)
Libermann et al., Phys. Rev. A (1992)

Semiclassical eq. of motion

$$\begin{cases} \dot{\vec{r}} = v(\vec{r})\hat{k} + \dot{\vec{k}} \times \langle z | \vec{\Omega}(\vec{k}) | z \rangle \\ \dot{\vec{k}} = -k\nabla v(\vec{r}) \\ |\dot{z}\rangle = -i\dot{\vec{k}} \cdot \vec{\Lambda}(\vec{k}) | z \rangle \end{cases}$$

Shift of a trajectory of light beam
"Spin Hall effect of light"

Geometrical optics
"Fermat's principle"

Polarization change $\begin{cases} \text{Chiao,Wu('86) : theory} \\ \text{Tomita,Chiao('86) : experiment} \end{cases}$

$v(\vec{r}) = \dfrac{c}{n(\vec{r})}$: slowly varying

$|z\rangle$: polarization

$\vec{\Lambda}(\vec{k})$: gauge field

$\vec{\Omega}(\vec{k})$: Berry curvature

In the vacuum

$$\vec{\Omega}(\vec{k}) = \frac{\vec{k}}{k^3}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

→ Left circular pol.
→ right

← spin-orbit coupling of light
(transverse only)

Transverse shift at the interface

Onoda, SM, Nagaosa, Phys. Rev. Lett. (2004)
Onoda, SM, Nagaosa, Phys. Rev. E (2006)

Anomalous velocity due to Berry phase

$$\dot{\vec r} = v(\vec r)\frac{\vec k}{k} + \boxed{\dot{\vec k}\times (z|\vec\Omega_{\vec k}|z)} \qquad \dot{\vec k} = -k\nabla v(\vec r)$$

← spin-orbit coupling of light (transverse only)

Isotropic medium

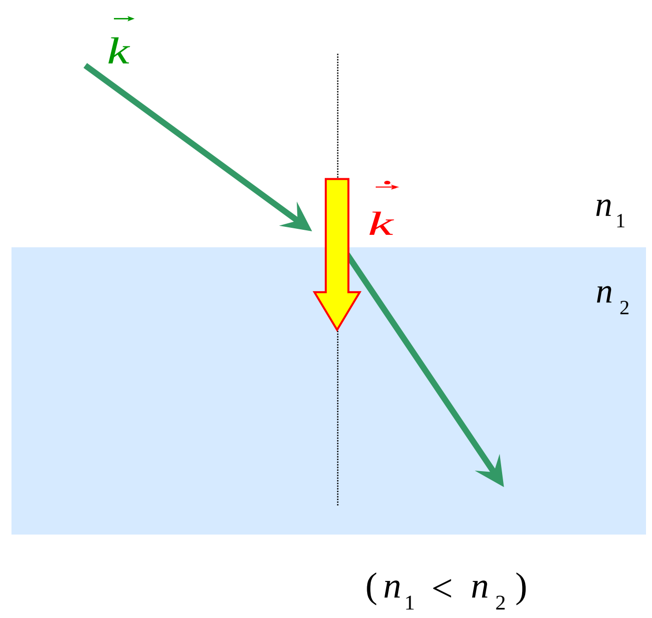
$$\vec\Omega(\vec k) = \frac{\vec k}{k^3}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

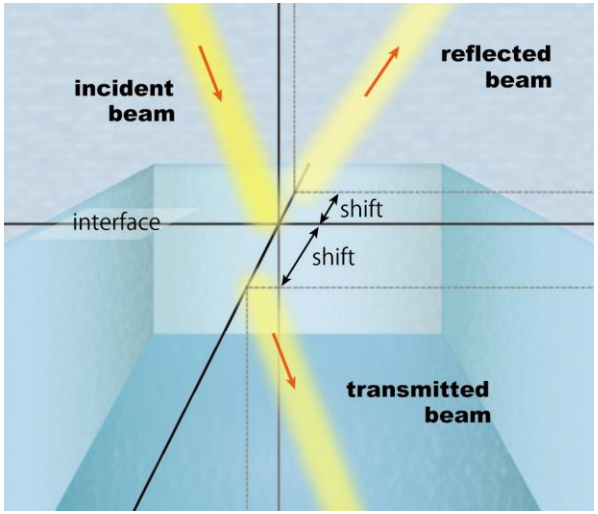
→ Left circular pol. → Anomalous velocity = ⊙

→ Right circular pol. → Anomalous velocity = ⊗

= transverse shift

Imbert shift
Theory: Fedorov (1955)
Experiment: Imbert(1972)



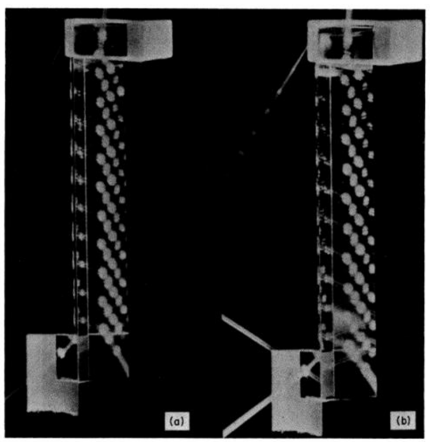


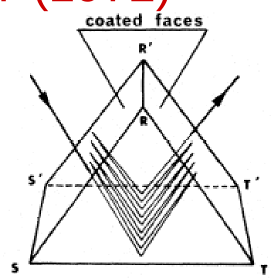
Experiments on imbert shift : Shift of light beam in reflection/refraction

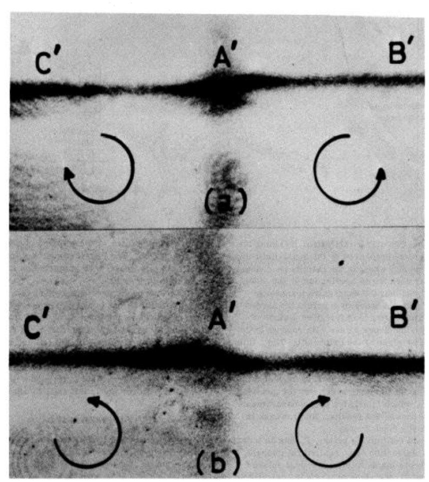
Magnitude of the shift $\approx \lambda$
Width of the beam is much larger → not easy to observe.

total reflection

Imbert, Phys. Rev. D5, 787 (1972)



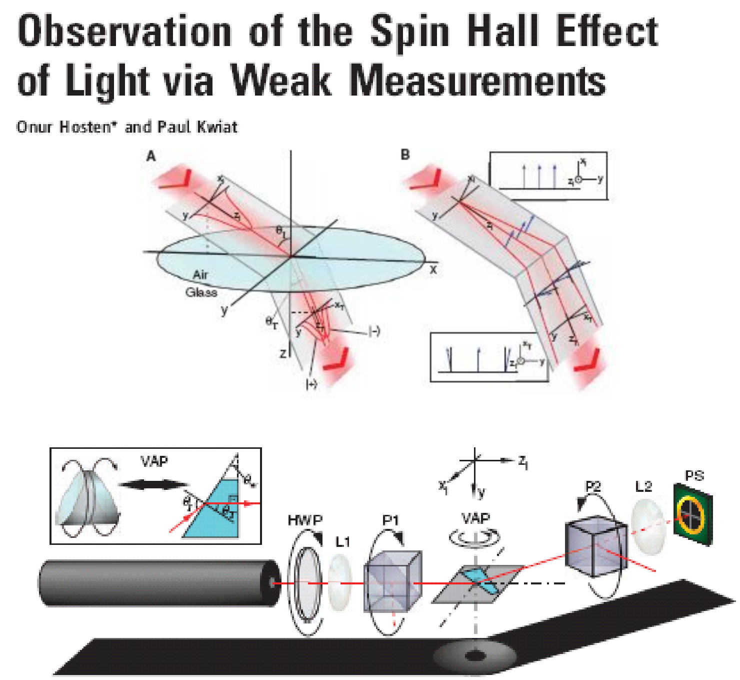




28 total reflections → shift is enhanced

refraction

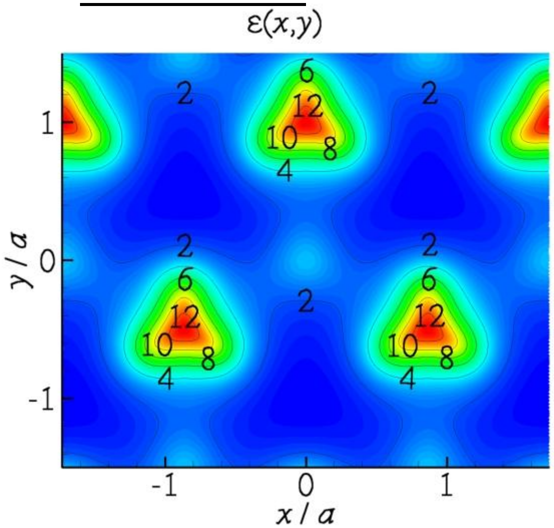
Hosten, Kwiat, Science 319,787 (2008)



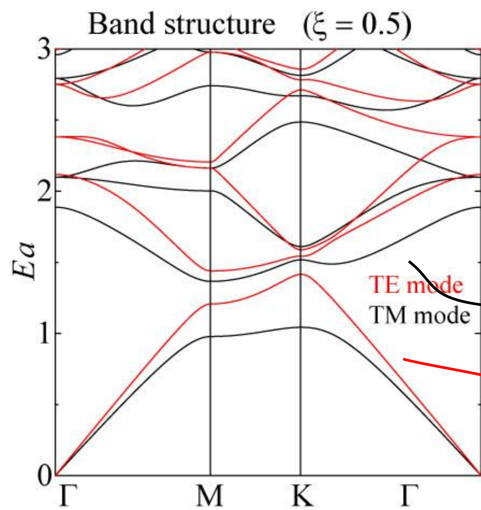
Spin Hall effect of light enhanced in photonic crystals

Onoda, SM, Nagaosa, Phys. Rev. Lett.93, 083901 (2004)

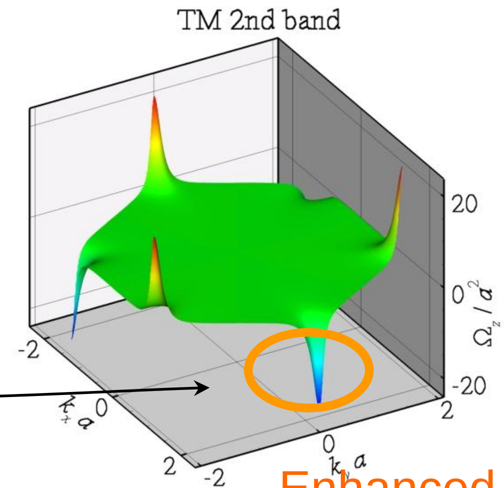
Simulation



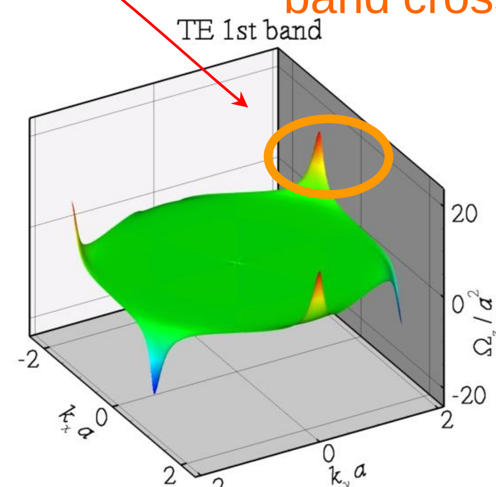
Dielectric constant

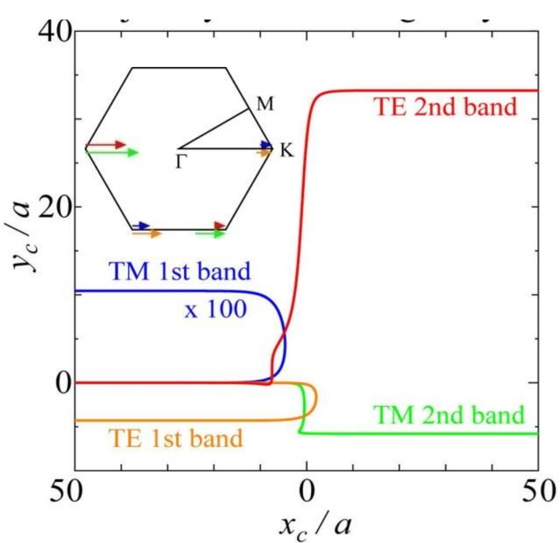


Photonic band structure



Enhanced Berry curvature near band crossing



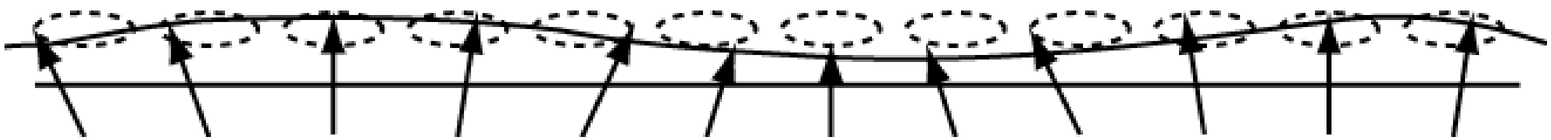


Large shift
← enhancement
(2 orders of magnitude)

Hall effect of magnons

Introduction : magnons

magnon (spin wave)



- low energy excitation in magnets

  (fluctuation from the ground state)

- Bose distribution function

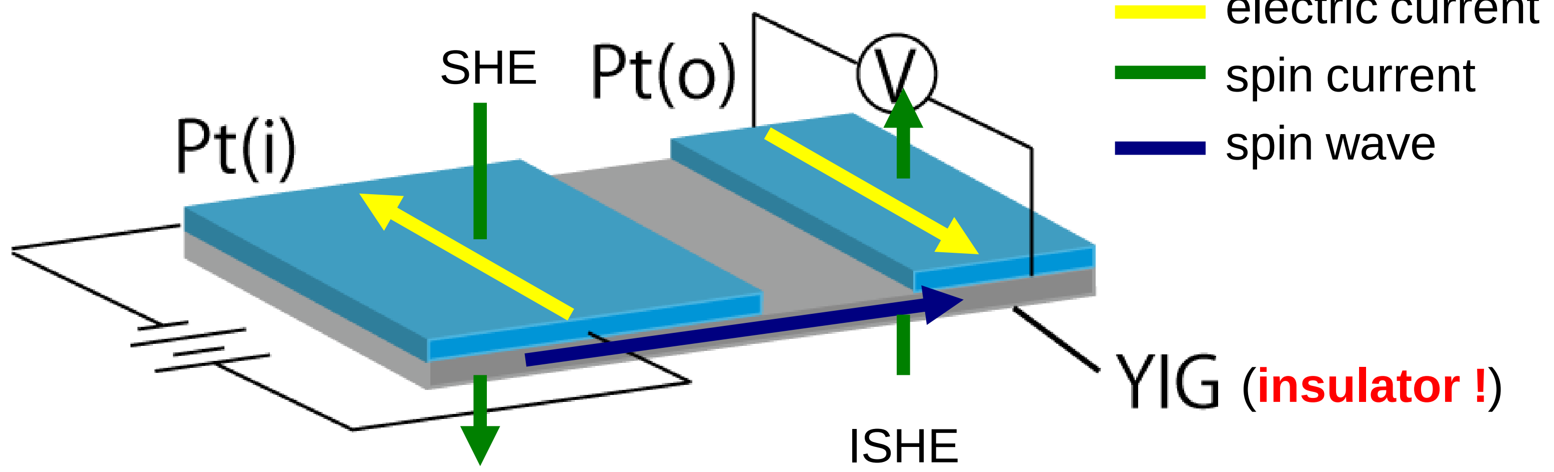
- spin current (magnon transport)

  e.g.) Kajiwara *et al., Nature* **464,** 262 (2010)

Introduction

- magnon transport in magnetic insulator

Kajiwara *et al.*, *Nature* **464,** 262 (2010)

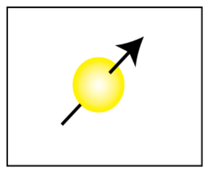


- long distance (~1[mm])
- no Joule heating by electrons
- room temperature

Motivation

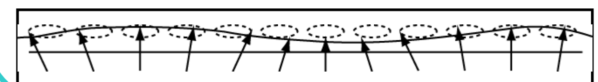
electron system

- charge $-e$
- fermion



magnon system

- *no* charge
- boson

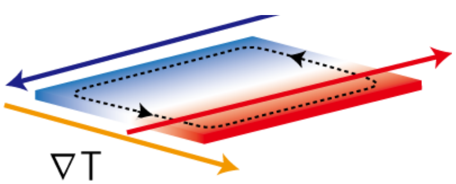


Berry phase in ***k***-space

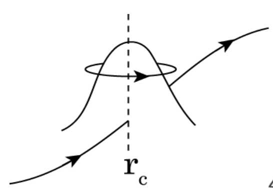
☑ wave nature

☑ band structure

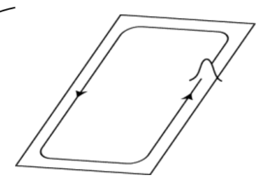
Berry curvature effect can be expected in magnon systems!



Thermal Hall effect



self-rotation



edge current

Magnon thermal Hall effect by Berry curvature
– previous works --

<u>Theory:</u>

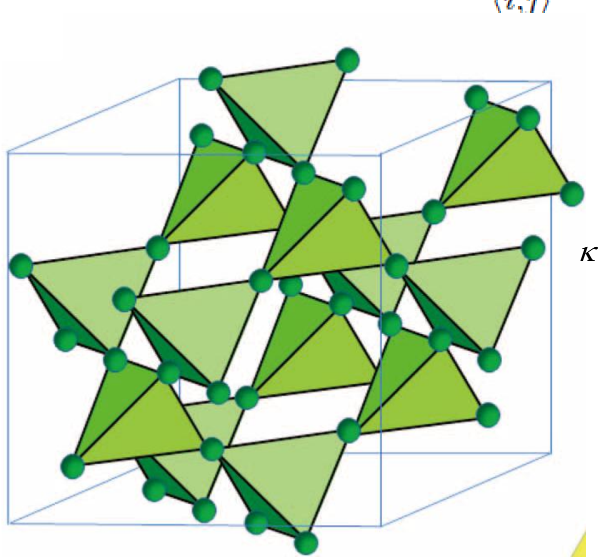
- S. Fujimoto, Phys. Rev. Lett. 103, 047203 (2009).
- H. Katsura, N. Nagaosa, and P. A. Lee, Phys. Rev. Lett.104, 066403 (2010).

<u>Experiment & theory:</u>

- Y. Onose, et al., Science 329, 297 (2010);

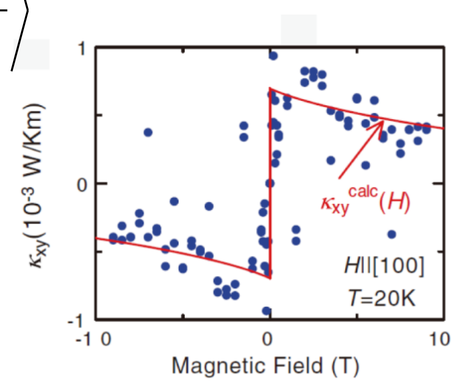
$Lu_2V_2O_7$: Ferromagnet

$$H_{\rm eff} = \sum_{\langle i,j\rangle} -J\boldsymbol{S}_i\cdot\boldsymbol{S}_j + \boldsymbol{D}_{ij}\cdot(\boldsymbol{S}_i\times\boldsymbol{S}_j) - g\mu_{\rm B}\boldsymbol{H}\cdot\sum_i \boldsymbol{S}_i.$$



Dyaloshinskii-Moriya interaction → Berry phase

$$\kappa^{xy} = \frac{2}{\hbar V}\sum_{n,k}\rho\left(\varepsilon_{n\vec{k}}\right)\mathrm{Im}\left\langle \frac{\partial u_{n\vec{k}}}{\partial k_x}\right|\left(\frac{H+\varepsilon_{n\vec{k}}}{2}\right)^2\left|\frac{\partial u_{n\vec{k}}}{\partial k_y}\right\rangle$$



Rotational motions of electrons in a magnetic field due to Berry curvature

1. self-rotation ($\approx$ cyclotron motion)

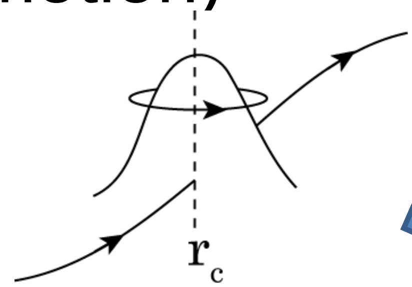
$$L = \langle W_0 | (\boldsymbol{r} - \boldsymbol{r}_c) \times \boldsymbol{p} | W_0 \rangle \neq 0$$

$|W_0\rangle$: wave packet

***L* is expressed by Berry curvature**

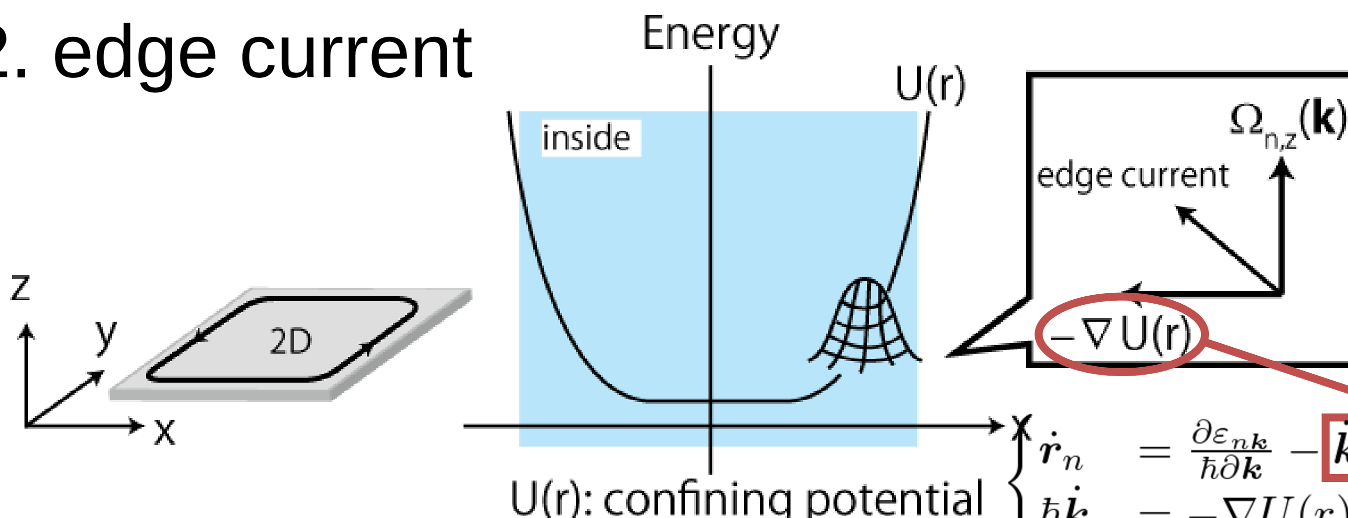
Yafet (1963)

Chang and Niu, PRB **53**, 7010 (1996)



Orbital magnetic moment

2. edge current



$$\begin{cases} \dot{\boldsymbol{r}}_n = \dfrac{\partial \varepsilon_{n\boldsymbol{k}}}{\hbar \partial \boldsymbol{k}} - \boxed{\dot{\boldsymbol{k}} \times \boldsymbol{\Omega}_{n\boldsymbol{k}}} \\ \hbar \dot{\boldsymbol{k}} = -\nabla U(r) \end{cases}$$

$\boldsymbol{\Omega}_{n\boldsymbol{k}}$: Berry curvature

Gradient of confinement potential → Hall current

From electrons to magnons (spin waves)

- Electrons: charge $-e$, fermion
- Magnons: no charge, boson

$\boxed{\text{Wave nature = described by Bloch wf.}}$

electrons

$$\begin{cases} \dot{\vec{x}} = \dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -e\left(\vec{E} + \dot{\vec{x}} \times \vec{B}\right) \end{cases}$$

magnons

$$\begin{cases} \dot{\vec{x}} = \dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}} - \dot{\vec{k}} \times \vec{\Omega}_n(\vec{k}) \\ \dot{\vec{k}} = -\nabla V \end{cases}$$

V(r): confinement potential

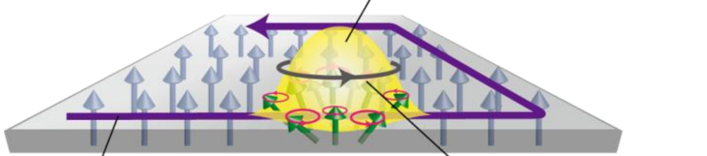
$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

: Berry curvature

- Exchange magnons (quantum-mechanical)
  e.g. $Lu_2V_2O_7$
- Magnetostatic spin waves (classical)
  e.g. YIG (yttrium iron garnet)

Orbital motions of magnon wavepacket

$\vec{l} = \langle \vec{r} \times \vec{v} \rangle$: (reduced) angular momentum



- self-rotation motion

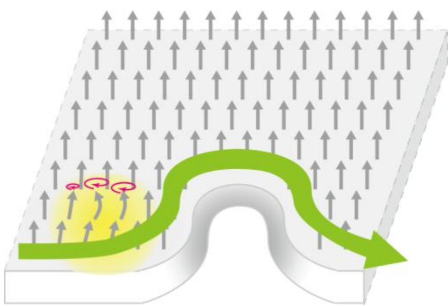
$$l_z^{\mathrm{self}} = -\frac{2}{\hbar V} \mathrm{Im} \sum_{n,\boldsymbol{k}} \rho_n \left\langle \frac{\partial u_n}{\partial k_x} \middle| (H - \varepsilon_{n\boldsymbol{k}}) \middle| \frac{\partial u_n}{\partial k_y} \right\rangle$$

- edge current of magnon

$$l_z^{\mathrm{edge}} = -\frac{2}{\hbar V} \sum_{n,\boldsymbol{k}} \int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} d\varepsilon \rho(\varepsilon) \Omega_{n,z}(\boldsymbol{k})$$

where

$$\boldsymbol{\Omega}_n(\boldsymbol{k}) = i \left\langle \frac{\partial u_n}{\partial \boldsymbol{k}} \right| \times \left| \frac{\partial u_n}{\partial \boldsymbol{k}} \right\rangle$$: Berry curvature

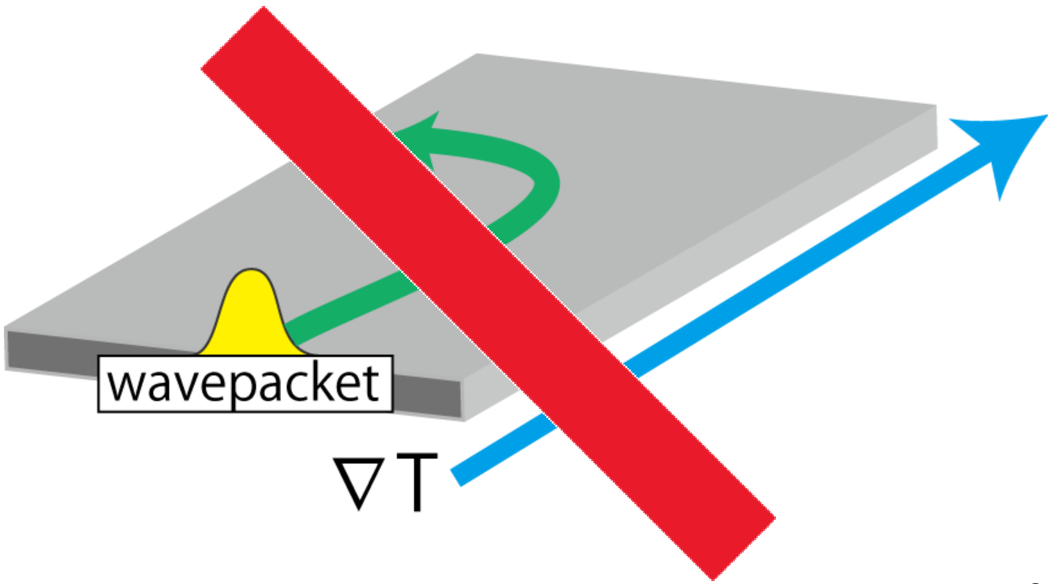


Along the edge

$$\begin{cases} \dot{\boldsymbol{r}}_n & = \frac{1}{\hbar} \frac{\partial \varepsilon_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} - \boxed{\dot{\boldsymbol{k}} \times \boldsymbol{\Omega}_n(\boldsymbol{k})} \\ \hbar \dot{\boldsymbol{k}} & = -\nabla U(\boldsymbol{r}) \end{cases}$$

: confining potential

Thermal Hall effect of magnon



- The gradient of temperature is not dynamical force but statistical force.
  →deflection does not occur

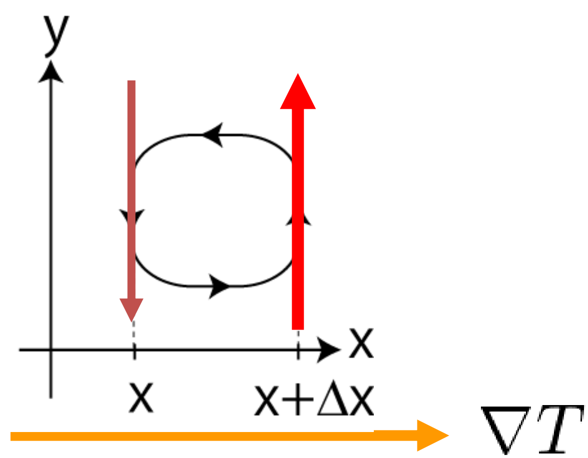
- How about in the semiclassical picture?

Thermal Hall effect of magnon

- semiclassical theory

Along the edge

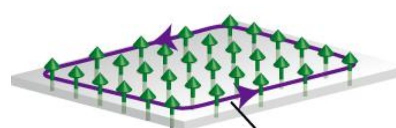
$$\begin{cases} \dot{\boldsymbol{r}}_n &= \frac{1}{\hbar}\frac{\partial \varepsilon_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} - \boxed{\dot{\boldsymbol{k}} \times \boldsymbol{\Omega}_n(\boldsymbol{k})} \\ \hbar\dot{\boldsymbol{k}} &= -\nabla U(\boldsymbol{r}) \end{cases}$$



Edge current

$$I_y = -\frac{1}{\hbar}\sum_{n,\boldsymbol{k}}\int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} d\varepsilon\rho(\varepsilon)\Omega^z_{n\boldsymbol{k}}$$

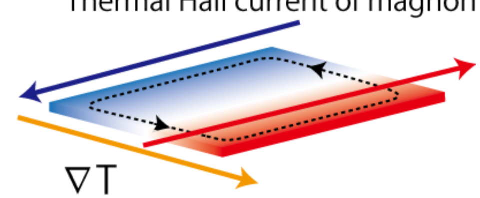
$$\rho(\varepsilon_{n\boldsymbol{k}}) = \frac{1}{(e^{\beta(\varepsilon_{n\boldsymbol{k}}-\mu)} - 1)}$$



Edge current of magnon

- If $\nabla T = 0$ there is no net current $\downarrow + \uparrow = 0$
- If $\nabla T \neq 0$ there appear net current $\downarrow + \uparrow \neq 0$

Thermal Hall current of magnon

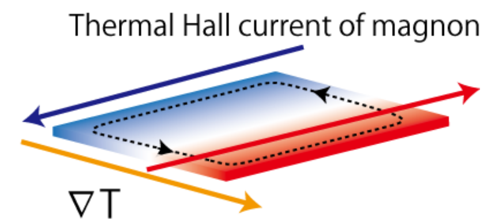


Thermal Hall effect of magnon

- semiclassical theory

• If $\nabla T \neq 0$ there appear net current

$\downarrow + \uparrow \neq 0$



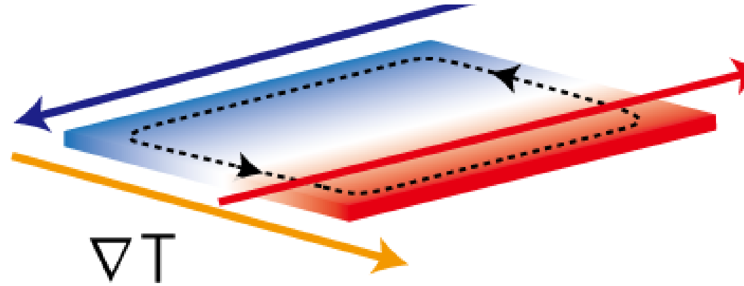
$$\boldsymbol{j} = \nabla \times \frac{1}{\hbar V} \sum_{n,\boldsymbol{k}} \int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} \rho(\varepsilon) \boldsymbol{\Omega}_n(\boldsymbol{k}) d\varepsilon.$$

$$\boldsymbol{j}_E = \nabla \times \frac{1}{\hbar V} \sum_{n,\boldsymbol{k}} \int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} \varepsilon \rho(\varepsilon) \boldsymbol{\Omega}_n(\boldsymbol{k}) d\varepsilon.$$

$$\left\{ \begin{aligned} (j)_x^{\nabla T} &= T \left[\partial_y \left(\frac{1}{T} \right) \right] \sum_{n,\boldsymbol{k}} \int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} \frac{\varepsilon - \mu}{\hbar V} \left(\frac{d\rho}{d\varepsilon} \right) \Omega_{n,z}(\boldsymbol{k}) d\varepsilon, \\ (j_E)_x^{\nabla T} &= T \left[\partial_y \left(\frac{1}{T} \right) \right] \sum_{n,\boldsymbol{k}} \int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} \frac{\varepsilon(\varepsilon - \mu)}{\hbar V} \left(\frac{d\rho}{d\varepsilon} \right) \Omega_{n,z}(\boldsymbol{k}) d\varepsilon. \end{aligned} \right.$$

$j_Q = j_E - \mu j$: heat current

Magnon Thermal Hall conductivity (Righi-Leduc effect)



$$(j_Q)_x = \kappa_{xy} (\nabla T)_y$$

Semiclassical theory

$$\kappa^{xy} = \frac{2k_{\mathrm{B}}^2 T}{\hbar V} \sum_{n,\boldsymbol{k}} \boxed{c_2(\rho(\varepsilon_{n\boldsymbol{k}}))} \mathrm{Im} \boxed{\left\langle \frac{\partial u_n}{\partial k_x} \middle| \frac{\partial u_n}{\partial k_y} \right\rangle}$$

where $c_2(\rho) = \int_0^\rho \left[\log\left(\frac{1+t}{t} \right) \right]^2 dt$

$$= (1+\rho)\left[\log\left(\frac{1+\rho}{\rho} \right) \right]^2 - (\log \rho)^2 - 2\mathrm{Li}_2(-\rho)$$

Note:

It is different from previous results by linear response theory

(Katsura *et al.* PRL (2010), Onose et al., Science (2010)).

$$\kappa^{xy} = -\frac{1}{2T} \mathrm{Im} \sum_{n,\boldsymbol{k}} \boxed{\rho(\varepsilon_{n\boldsymbol{k}})} \boxed{\left\langle \frac{\partial u_n}{\partial k_x} \middle| (H + \varepsilon_{n\boldsymbol{k}})^2 \middle| \frac{\partial u_n}{\partial k_y} \right\rangle}$$

Some terms missing !

→ modified linear response theory = identical result with semiclassical theory

Linear response theory

$$J = L_{11}\left[-\nabla U - T\nabla\left(\frac{\mu}{T}\right)\right] + L_{12}\left[T\nabla\left(\frac{1}{T}\right) - \nabla\psi\right]$$

$$J_E = L_{12}\left[-\nabla U - T\nabla\left(\frac{\mu}{T}\right)\right] + L_{22}\left[T\nabla\left(\frac{1}{T}\right) - \nabla\psi\right]$$

cf) for electrons

L. Smrčka and P. Středa, J. Phys. C, **10**, 2153 (1977)
H. Oji, P. Středa, PRB **31**, 7291 (1985)

- L_{11}: Hall effect
- $L_{12}=L_{21}$: Nernst effect
- L_{22}: thermal Hall effect

$\boldsymbol{J}$: magnon current

$\boldsymbol{J}_E$: energy current

ψ : gravitational field ← J. M. Luttinger, Phys. Rev. 135, A1505 (1964)

L_{ij}: transport coefficient

Linear response theory

- Linear response theory (to external field)

$$J = \mathrm{Tr}[\rho\, j(\vec{r})] = \mathrm{Tr}[\rho_0\, j_0(\vec{r})] + \mathrm{Tr}[\rho_1\, j_0(\vec{r})] + \mathrm{Tr}[\rho_0\, j_1(\vec{r})]$$

$$J_E = \mathrm{Tr}[\rho\, j_E(\vec{r})] = \mathrm{Tr}[\rho_0\, j_{0E}(\vec{r})] + \mathrm{Tr}[\rho_1\, j_{0E}(\vec{r})] + \mathrm{Tr}[\rho_0\, j_{1E}(\vec{r})]$$

$$\downarrow 0 \qquad \downarrow S_{ij} \qquad \downarrow M_{ij}$$

Density matrix

$$g(H) = \underline{f_0(H)} + \underline{f_1(H)}$$

equilibrium

deviation from equilibrium

Current

$$\boldsymbol{j}(\boldsymbol{r}) = \boldsymbol{j}^{(0)}(\boldsymbol{r}) + \boldsymbol{j}^{(1)}(\boldsymbol{r})$$

$$\boldsymbol{j}_E(\boldsymbol{r}) = \boldsymbol{j}_E^{(0)}(\boldsymbol{r}) + \boldsymbol{j}_E^{(1)}(\boldsymbol{r})$$

equilibrium deviation from equilibrium

- S_{ij}: calculated by Kubo formula

$$(S^B)_{ij}^{\alpha\beta} = i\hbar \int \rho(\eta) \mathrm{Tr}\left(j_i^\alpha \frac{dG^+}{d\eta} j_j^\beta \delta(\eta - H) - j_i^\alpha \delta(\eta - H) j_j^\beta \frac{dG^-}{d\eta} \right) d\eta$$

- Theory : H. Katsura, N. Nagaosa, and P. A. Lee, *PRL* **104,** 066403 (2010)
- Experiment : Y. Onose, et al., *Science* **329,** 297 (2010)

- M_{ij}: new correction term

Linear response theory

Equilibrium currents

$$j_0(\vec{r}) = \frac{1}{2}\sum_j \left\{ v_j, \delta\left(\vec{r} - \vec{r}_j\right)\right\}$$ Magnon current

$$j_{0E}(\vec{r}) = \frac{1}{2}\left\{ H, j_0(\vec{r})\right\}$$ Energy current

Currents in the presence of fields

$$H' = \sum_j U\left(\vec{r}_j\right) + \frac{1}{2}\left\{ H, \sum_j \psi\left(\vec{r}_j\right)\right\}$$

Correction terms of currents due to external fields

$$j(\vec{r}) = j_0(\vec{r}) + j_1(\vec{r}) = j_0(\vec{r}) + \boxed{\frac{1}{2}\sum_j \left\{ j_0(\vec{r}), \psi\left(\vec{r}_j\right)\right\}}$$

$$j_E(\vec{r}) = j_{0E}(\vec{r}) + j_{1E}(\vec{r})$$

$$= j_{0E}(\vec{r}) + \boxed{\frac{1}{2}\sum_j \left\{ j_0(\vec{r}), U\left(\vec{r}_j\right)\right\} + \frac{1}{4}\sum_j \left(\left\{ j_0(\vec{r}), \left\{\psi\left(\vec{r}_j\right), H\right\}\right\} + \left\{ H, \left\{\psi\left(\vec{r}_j\right), j_0(\vec{r})\right\}\right\}\right)}$$

$$J = \mathrm{Tr}\left[\rho\, j(\vec{r})\right] = \mathrm{Tr}\left[\rho_0\, j_0(\vec{r})\right] + \mathrm{Tr}\left[\rho_1\, j_0(\vec{r})\right] + \boxed{\mathrm{Tr}\left[\rho_0\, j_1(\vec{r})\right]}$$

$$J_E = \mathrm{Tr}\left[\rho\, j_E(\vec{r})\right] = \mathrm{Tr}\left[\rho_0\, j_{0E}(\vec{r})\right] + \mathrm{Tr}\left[\rho_1\, j_{0E}(\vec{r})\right] + \boxed{\mathrm{Tr}\left[\rho_0\, j_{1E}(\vec{r})\right]}$$

Linear response theory

Magnon current

$$J = L_{11}\left[-\nabla U - T\nabla\left(\frac{\mu}{T}\right)\right] + L_{12}\left[T\nabla\left(\frac{1}{T}\right) - \nabla\psi\right]$$

Energy current

$$J_E = L_{12}\left[-\nabla U - T\nabla\left(\frac{\mu}{T}\right)\right] + L_{22}\left[T\nabla\left(\frac{1}{T}\right) - \nabla\psi\right]$$

$$L_{ij} = S_{ij} + M_{ij}$$

$$S_{ij}^{\alpha\beta} = i\hbar\int\rho(\eta)\mathrm{Tr}\left(j_i^{\alpha}\frac{dG^{+}}{d\eta}j_j^{\beta}\delta(\eta - H) - j_i^{\alpha}\delta(\eta - H)j_j^{\beta}\frac{dG^{+}}{d\eta}\right)d\eta$$

$$M_{11}^{\alpha\beta} = 0$$

$$M_{12}^{\alpha\beta} = \frac{1}{2}\int\rho(\eta)\mathrm{Tr}\left(\delta(\eta - H)\left(r^{\alpha}v^{\beta} - r^{\beta}v^{\alpha}\right)\right)d\eta$$

$$M_{22}^{\alpha\beta} = \int\eta\rho(\eta)\mathrm{Tr}\left(\delta(\eta - H)\left(r^{\alpha}v^{\beta} - r^{\beta}v^{\alpha}\right)\right)d\eta + \frac{i\hbar}{4}\int\rho(\eta)\mathrm{Tr}\left(\delta(\eta - H)\left[v^{\alpha}, v^{\beta}\right]\right)d\eta$$

Correction terms of currents due to external fields

Linear response theory

Formula in terms of Berry curvature

$$L_{ij} = S_{ij} + M_{ij}$$

$$S_{ij}^{\alpha\beta} = \boxed{\frac{2}{\hbar V}\mathrm{Im}\sum_{n,\boldsymbol{k}}\rho_n\left\langle\frac{\partial u_n}{\partial k_\alpha}\right|\left(\frac{H+\varepsilon_{n\boldsymbol{k}}}{2}\right)^q\left|\frac{\partial u_n}{\partial k_\beta}\right\rangle}$$

Correction terms

$$\begin{cases} M_{11}^{\alpha\beta} = 0, \\ M_{12}^{\alpha\beta} = \boxed{\frac{2}{\hbar V}\mathrm{Im}\sum_{n,\boldsymbol{k}}\left[-\rho_n\left\langle\frac{\partial u_n}{\partial k_\alpha}\right|\left(\frac{H+\varepsilon_{n\boldsymbol{k}}}{2}\right)\left|\frac{\partial u_n}{\partial k_\beta}\right\rangle\right.} + (\mu\rho_n + 2k_{\mathrm{B}}Tc_1(\rho_n))\left.\left\langle\frac{\partial u_n}{\partial k_\alpha}\middle|\frac{\partial u_n}{\partial k_\beta}\right\rangle\right], \\ M_{22}^{\alpha\beta} = \boxed{\frac{2}{\hbar V}\mathrm{Im}\sum_{n,\boldsymbol{k}}\left[-\rho_n\left\langle\frac{\partial u_n}{\partial k_\alpha}\right|\left(\frac{H+\varepsilon_{n\boldsymbol{k}}}{2}\right)^2\left|\frac{\partial u_n}{\partial k_\beta}\right\rangle\right.} \\ \qquad + (\mu^2\rho_n + 2\mu k_{\mathrm{B}}Tc_1(\rho_n) + (k_{\mathrm{B}}T)^2 c_2(\rho_n))\left.\left\langle\frac{\partial u_n}{\partial k_\alpha}\middle|\frac{\partial u_n}{\partial k_\beta}\right\rangle\right]. \end{cases}$$

$$\left(\begin{array}{l} c_1(\rho) = (1+\rho)\log(1+\rho) - \rho\log\rho \\ c_2(\rho) = (1+\rho)\left[\log\left(\frac{1+\rho}{\rho}\right)\right]^2 - (\log\rho)^2 - 2\mathrm{Li}_2(-\rho) \end{array}\right)$$

Heat current $J_Q = J_E - \mu J$

$\longrightarrow$ Thermal Hall conductivity

$$\kappa^{xy} = (L_{22}^{xy} - 2\mu L_{12}^{xy} + \mu^2 L_{11}^{xy})/T$$

Linear response theory

- Thermal Hall conductivity

$$\kappa^{xy} = \frac{2k_{\mathrm{B}}^2 T}{\hbar V} \sum_{n,\boldsymbol{k}} c_2(\rho(\varepsilon_{n\boldsymbol{k}}))\mathrm{Im}\left\langle \frac{\partial u_n}{\partial k_x} \middle| \frac{\partial u_n}{\partial k_y} \right\rangle$$

$$c_2(\rho) = \int_0^\rho \left[\log\left(\frac{1+t}{t}\right)\right]^2 dt = (1+\rho)\left[\log\left(\frac{1+\rho}{\rho}\right)\right]^2 - (\log\rho)^2 - 2\mathrm{Li}_2(-\rho)$$

the same result as the semicalssical theory !

- New correction term

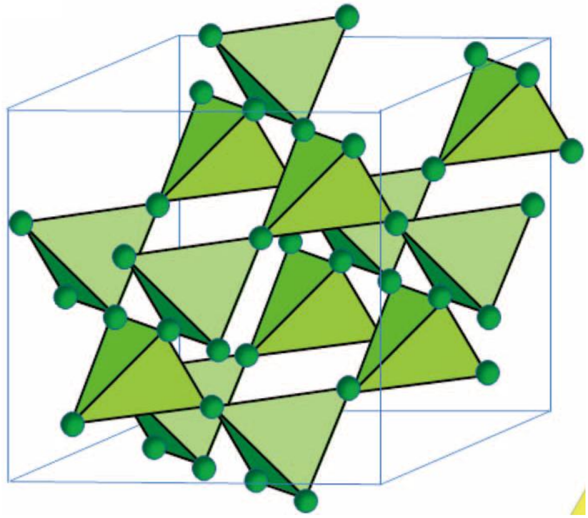
$$\left.\begin{aligned}
(M^B)_{11}^{\alpha\beta} &= 0 \\
(M^B)_{12}^{\alpha\beta} &= \frac{1}{2}\int \rho(\eta)\mathrm{Tr}[\delta(\eta - H)(r^\alpha v^\beta - r^\beta v^\alpha)]d\eta \\
(M^B)_{22}^{\alpha\beta} &= \int \eta\rho(\eta)\mathrm{Tr}\delta(\eta - H)(r^\alpha v^\beta - r^\beta v^\alpha)d\eta \\
&\quad + \frac{i\hbar}{4}\int \rho(\eta)\mathrm{Tr}\delta(\eta - H)[v^\alpha, v^\beta]d\eta
\end{aligned}\right\}$$

These arises from

the orbital motion of magnon.

ex) $2(M^B)_{12}^{\alpha\beta} = l_z^{\mathrm{self}} + l_z^{\mathrm{edge}}$

Example 1: Thermal Hall effect of magnons in $Lu_2V_2O_7$



H. Katsura et al., PRL 104, 066403 (2010)
Y. Onose, *et al.*, *Science* 329, 297 (2010);

$Lu_2V_2O_7$: Ferromagnet

$$H_{\text{eff}} = \sum_{\langle i,j\rangle} -J\boldsymbol{S}_i\cdot\boldsymbol{S}_j + \boldsymbol{D}_{ij}\cdot(\boldsymbol{S}_i\times\boldsymbol{S}_j) - g\mu_{\text{B}}\boldsymbol{H}\cdot\sum_i \boldsymbol{S}_i.$$

Dyaloshinskii-Moriya interaction → Berry phase

$$\kappa^{xy} = \frac{2}{\hbar V}\sum_{n,\boldsymbol{k}} \rho(\varepsilon_{n\boldsymbol{k}})\,\mathrm{Im}\left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial k_x}\middle|\left(\frac{H+\varepsilon_{n\boldsymbol{k}}}{2}\right)^2\middle|\frac{\partial u_{n\boldsymbol{k}}}{\partial k_y}\right\rangle$$

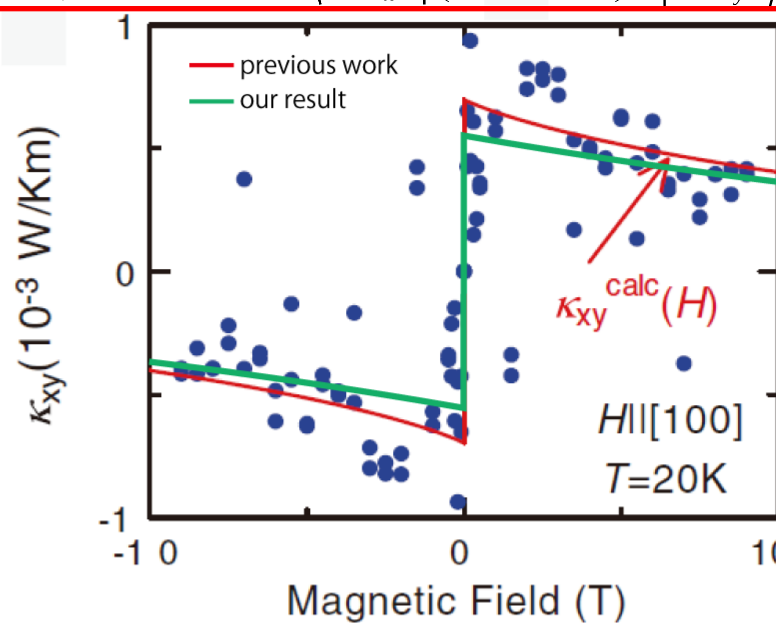
Our work:

$$\kappa^{xy} = \frac{2k_B^2T}{\hbar V}\sum_{n,\boldsymbol{k}} c_2(\rho(\varepsilon_{n\boldsymbol{k}}))\,\mathrm{Im}\left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial k_x}\middle|\frac{\partial u_{n\boldsymbol{k}}}{\partial k_y}\right\rangle$$

$$c_2(\rho) = \int_0^\rho \left(\log\frac{1+t}{t}\right)^2 dt$$

Matsumoto,Murakami, PRL106,197202 (2011)

arXiv: 1106.1987 (2011).



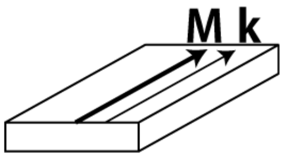
Magnons: classical vs. quantum

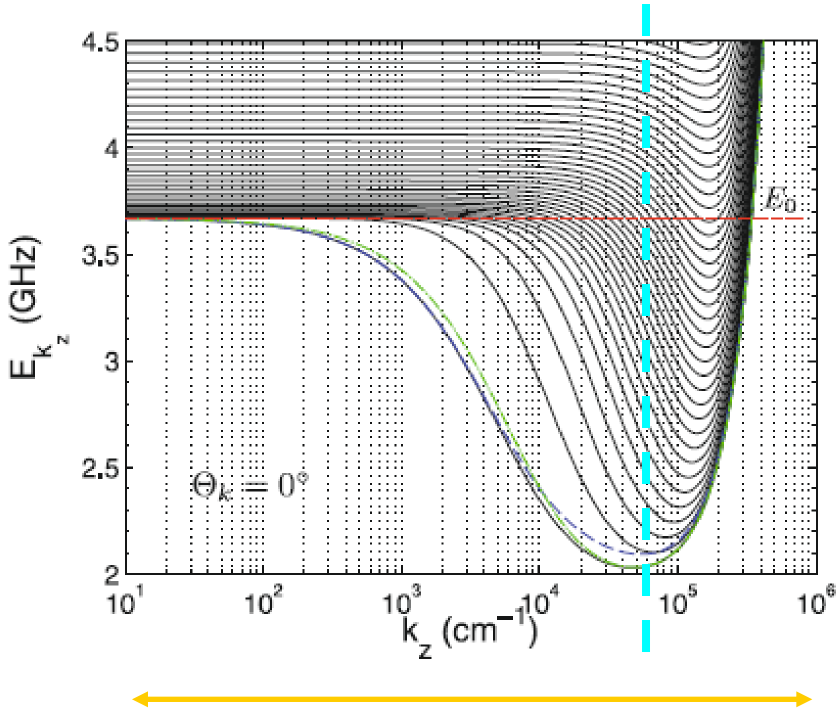
	magnon	magnetostatic spin wave
wave length	Short (nm)	Long (μm)
dominant interaction	exchange interaction	demagnetizing field (dipole interaction)
mechanism	quantum mechanical	classical electromagnetism
coherence length (typical value)	[nm]~[μm]	~[mm] in some magnets

We similarly apply this theory to the magnetostatic spin wave!

Magnetostatic mode in YIG film

E.g.: Magnetostatic backward volume mode (MSBVM)





Magnetostatic mode

(← demagnetizing field)

Magnon

(← exchange)

A. Kreisel, F. Sauli, L. Bartosch, and P. Kopietz, *Eur. Phys. J.* B **71,** 59 (2009)

Example 2: Magnetostatic modes in ferromagnetic films (YIG)

Exchange interaction = isotropic : $E = Jk^2$

Wavelength = $[\mu\mathrm{m}]$ - $[\mathrm{mm}]$

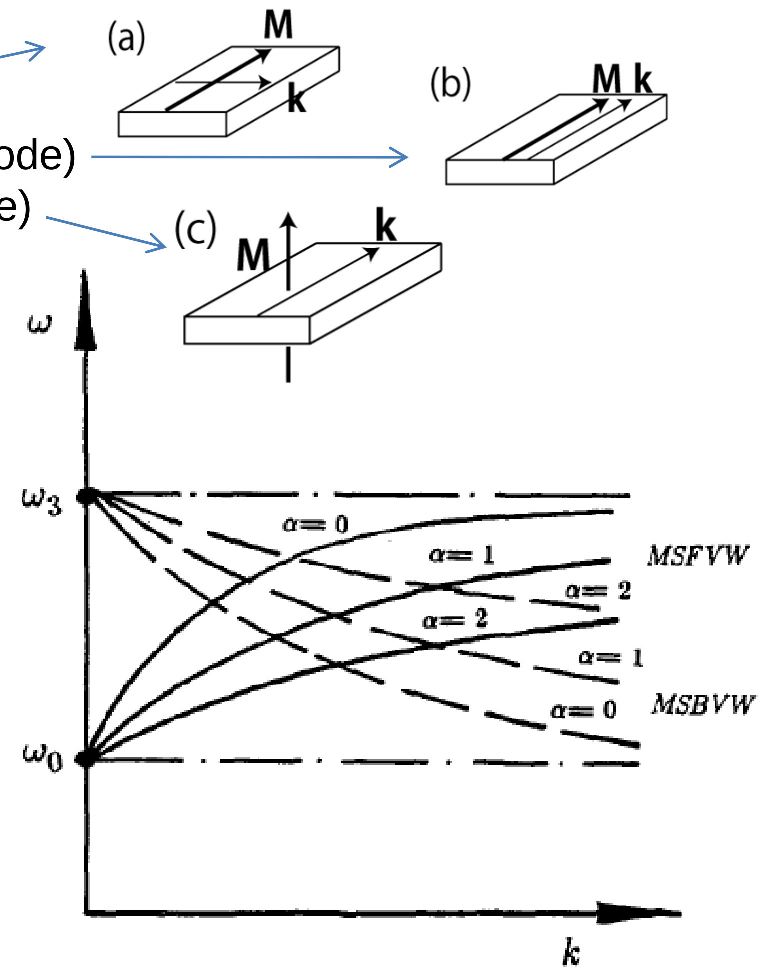
→ **exchange interaction negligible**

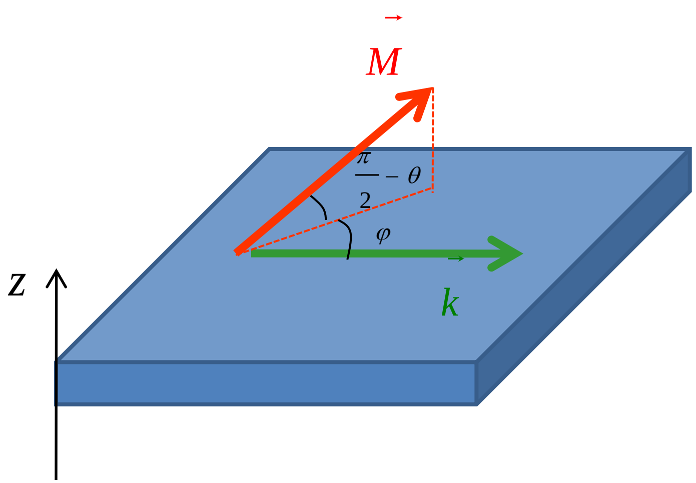
film → demagnetization field (i.e dipolar interacition) =dominant

(a) MSSW (magnetostatic surface mode)

(b) MSBVW (magnetostatic backward volume mode)

(c) MSFVW (magnetostatic forward volume mode)





- Multiband system

wavefunction : $\mathbf{m}(z) = \begin{pmatrix} m_x(z) \\ m_y(z) \end{pmatrix}$

Example 2: Magnetostatic modes in ferromagnetic films (YIG)

M: magnetization, γ: gyromagnetic ratio, ***H***: external magnetic field

- Landau-Lifshitz (LL) equation $\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M} \times \mathbf{H})$
- Maxwell equation $\nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \times \mathbf{H} = 0$ (magnetostatic limit)
- Boundary conditions $\mathbf{B}_{1\perp} = \mathbf{B}_{2\perp} \quad , \quad \mathbf{H}_{1//} = \mathbf{H}_{2//}$

Generalized eigenvalue eq.

B. A. Kalinikos and A. N. Slavin, *J. Phys. C* **19**, 7013 (1986)

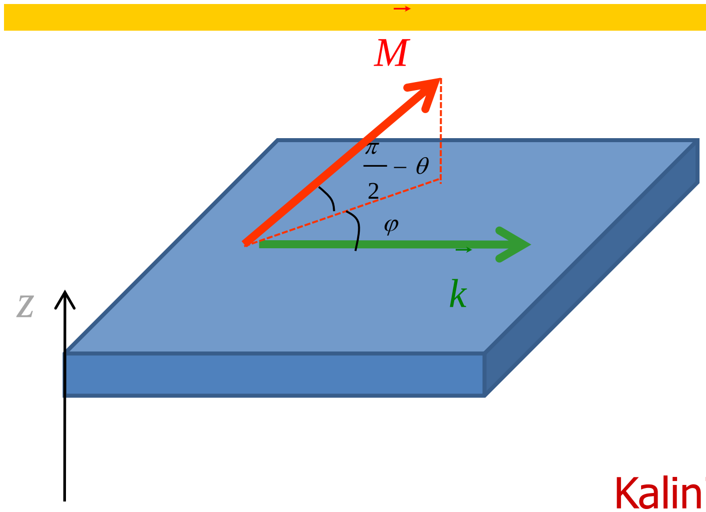
$$\boxed{\omega_H \mathbf{m}(z) - \omega_M \int_{-L/2}^{L/2} dz' \hat{G}(z, z')\mathbf{m}(z') = \omega \sigma_y \mathbf{m}(z)}$$

$\omega_H = \gamma H_0$, $\omega_M = \gamma M_0$, L : thickness of the film, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $z \perp$ film,

M_0 : saturation magnetization, H_0 : static magnetic field

$\hat{G}$: 2×2 matrix of the Green's function, ω : frequency of the spin wave

Example 2: Magnetostatic modes in ferromagnetic films (YIG)



Generalized eigenvalue eq.

$$\omega_H \mathbf{m}(z) - \omega_M \int_{-L/2}^{L/2} dz' \hat{G}(z,z')\mathbf{m}(z') = \omega \sigma_y \mathbf{m}(z)$$

$$\mathbf{m}(z) = \begin{pmatrix} m_x(z) \\ m_y(z) \end{pmatrix} \qquad \langle \mathbf{m}_n | \sigma_y | \mathbf{m}_n \rangle = 1$$

Kalinikos, Slavin, J. Phys. C: Solid State Phys. 19, 7013 (1986).

$$\hat{G}(z,z') = \begin{pmatrix} [G_P - \delta(z-z')]\sin^2\theta - iG_Q \sin 2\theta \cos\varphi - G_P \cos^2\theta \cos^2\varphi & -iG_Q \sin\theta \sin\varphi - \frac{1}{2}G_P \cos\theta \sin 2\varphi \\ -iG_Q \sin\theta \sin\varphi - \frac{1}{2}G_P \cos\theta \sin 2\varphi & -G_P \sin^2\varphi \end{pmatrix}$$

$$G_P(z,z') = \frac{k}{2}\exp\left(-k|z-z'|\right), \quad G_Q(z,z') = G_P(z,z')\,\mathrm{sgn}(z-z')$$

Berry curvature

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial \mathbf{m}_n}{\partial \vec{k}} \right| \times \sigma_y \left| \frac{\partial \mathbf{m}_n}{\partial \vec{k}} \right\rangle$$

Cf. Bogoliubov Hamiltonian

cf: for electrons: Schrödinger eq.

$$\vec{\Omega}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

Example 2: Magnetostatic modes in ferromagnetic films (YIG)

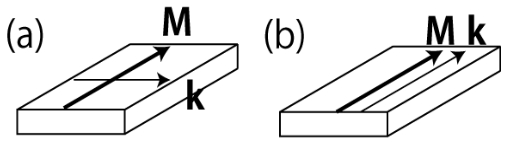
We introduce the Berry curvature for the magnetostatic spin wave:

$$\boxed{\Omega_n^{\gamma}(\mathbf{k}) = -\varepsilon_{\alpha\beta\gamma}\,\mathrm{Im}\left\langle \frac{\partial \mathbf{m}_{n,\mathbf{k}}}{\partial k_\alpha}\right|\sigma_y\left|\frac{\partial \mathbf{m}_{n,\mathbf{k}}}{\partial k_\beta}\right\rangle}$$

generalized eigenvalue problem.

(a) MagnetoStatic Surface Wave (MSSW)

(b) MagnetoStatic Backward Volume Wave (MSBVW)

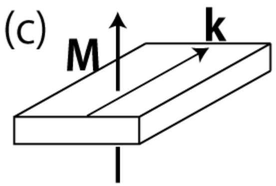


Zero Berry curvature

$\Omega_n^{z}(\mathbf{k}) = 0$ because of the symmetry

(2-fold in-plane rotation + time reversal)

(c) MagnetoStatic Forward Volume Wave (MSFVW)



We can expect the Berry curvature to be nonzero !

Magnetostatic forward volume waves (MSFVW) in ferromagnetic films (YIG)

Wavefunctions

$$\begin{pmatrix} m_x(z) \\ m_y(y) \end{pmatrix} = \frac{1}{\sqrt{N}} \begin{pmatrix} i\kappa k_x + \nu k_y \\ -\nu k_x + i\kappa k_y \end{pmatrix} \cos\left(\sqrt{p}kz + \tfrac{n\pi}{2}\right)$$

$$\sqrt{p}\tan\left(\sqrt{p}kz + \tfrac{n\pi}{2}\right) = 1,\ p = -1 - \kappa$$: Eigenmode eq. (n=0,1,2,…)

N : normalization constant

$$\kappa = \frac{\omega_M \omega_H}{\omega_H^2 - \omega^2}, \qquad \nu = \frac{\omega_M \omega}{\omega_H^2 - \omega^2}$$

R. W. Damon and H. van de Vaart, *J. Appl. Phys.* **36**, 3453 (1965)

→ Berry curvature for the MSFVW mode

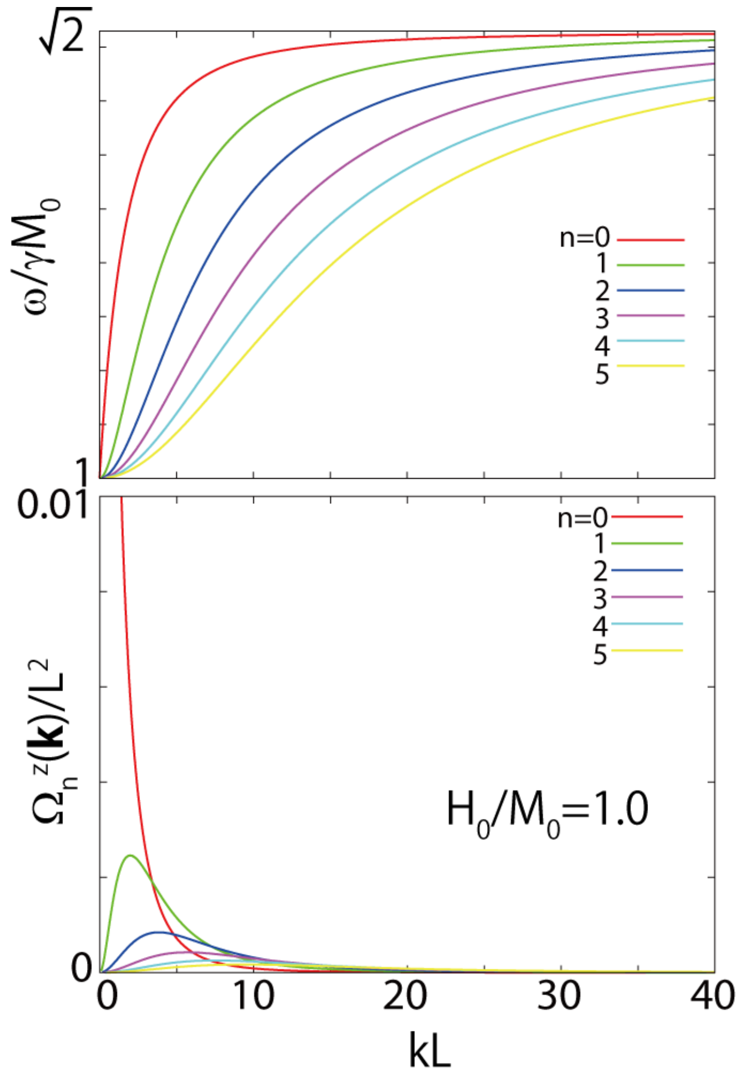
$$\Omega_n^z(\mathbf{k}) = \frac{1}{2\omega_H k} \frac{\partial \omega_n}{\partial k} \left(1 - \frac{\omega_H^2}{\omega_n^2}\right)$$

n : band index of the MSFVW mode

Numerical results:

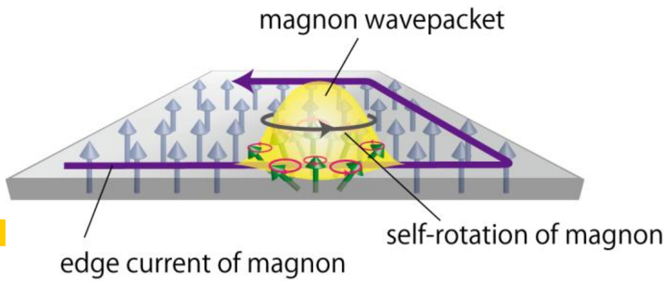
Dispersion rof the MSFVW mode (H_0/M_0=1.0), for n=0～5.

Berry curvature of MSFVW mode.



- R. Matsumoto, S. Murakami, Phys. Rev. Lett. 106,197202 (2011), arXiv: 1106.1987 (2011).

Summary



Wavepacket of spin wave (magnon)

- Berry curvature $\boldsymbol{\Omega}_n(\boldsymbol{k}) = i\left\langle \frac{\partial u_n}{\partial \boldsymbol{k}} \right| \times \left| \frac{\partial u_n}{\partial \boldsymbol{k}} \right\rangle$

- <u>2 types of rotational motions</u>

self rotation $$l_z^{\mathrm{self}} = -\frac{2}{\hbar V}\mathrm{Im}\sum_{n,\boldsymbol{k}} \rho_n \boxed{\left\langle \frac{\partial u_n}{\partial k_x} \right| (H - \varepsilon_{n\boldsymbol{k}}) \left| \frac{\partial u_n}{\partial k_y} \right\rangle}$$

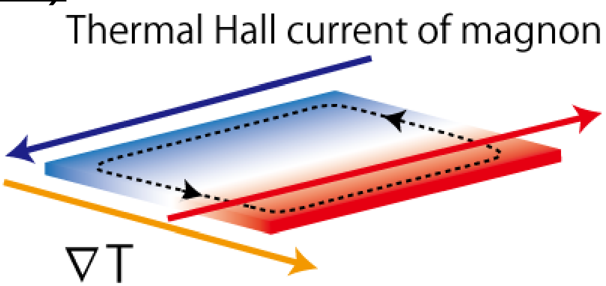
edge current $$l_z^{\mathrm{edge}} = -\frac{2}{\hbar V}\sum_{n,\boldsymbol{k}} \int_{\varepsilon_{n\boldsymbol{k}}}^{\infty} d\varepsilon \rho(\varepsilon) \boxed{\Omega_{n,z}(\boldsymbol{k})}$$

$$\vec{l} = \left\langle \vec{r} \times \vec{v} \right\rangle$$

- <u>Magnon thermal Hall effect (Righi-Leduc effect)</u>

$$\kappa^{xy} = \frac{2k_B^2 T}{\hbar V}\sum_{n,\boldsymbol{k}} c_2(\rho(\varepsilon_{n\boldsymbol{k}})) \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial k_x} \middle| \frac{\partial u_{n\boldsymbol{k}}}{\partial k_y} \right\rangle$$

$$c_2(\rho) = \int_0^{\rho} \left(\log \frac{1+t}{t}\right)^2 dt$$



- R. Matsumoto, S. Murakami, Phys. Rev. Lett. 106,197202 (2011)
- R. Matsumoto, S. Murakami, arXiv:1106.1987