Staggered current Mott insulator of kinetically frustrated bosons

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Strongly interacting bosons

Bose-Hubbard Hamiltonian: Simplest model of strong correlation effects

$$H = -\sum_{i,j} t_{i,j} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

M.P.A. Fisher et al, PRB 1989

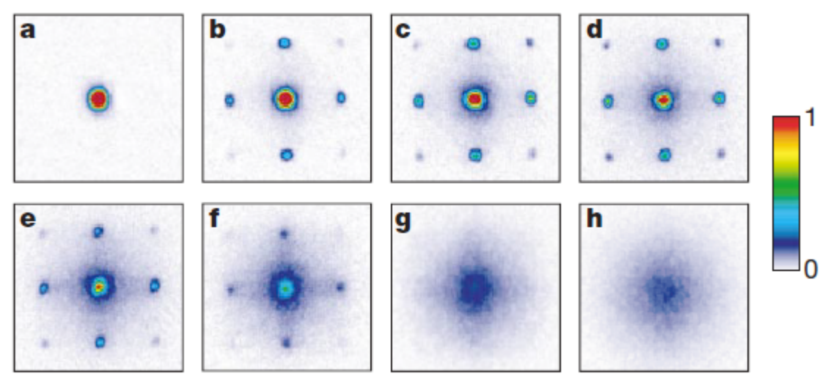
For large fillings, this is closely related to a Josephson junction array (JJA) model
U ~ charging energy, t ~ Josephson coupling

S. Chakravarty, et al, PRL 1986
Mooij group, PRL 1989
Clarke group, PRL 1997

Integer filling

(d+1) XY

Mott insulators of Rb-87 atoms



He-4, JJAs, Cold Atoms

Greiner et al, Nature 2002

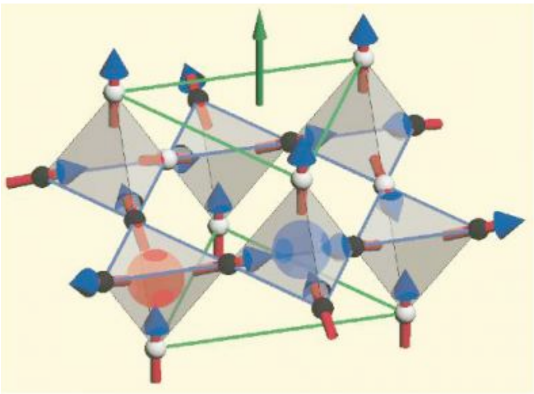
Frustration

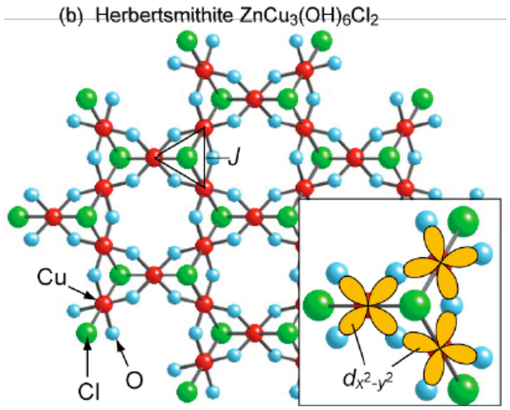
Why is frustration interesting?

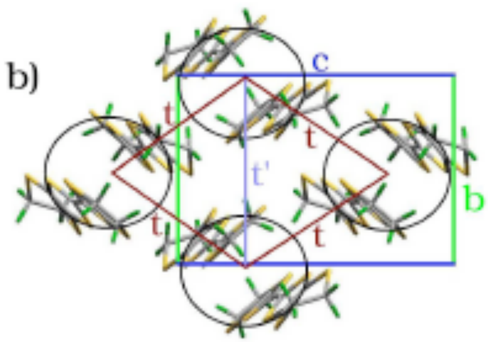
. Many many ground states at classical level (Entropy grows with size)
. Quantum effects have to select unique ground state (“zero” entropy)
. Source of interesting new states: FQHE, quantum spin liquids

Examples of frustrated spin liquid-like materials?

. Pyrochlore [spin ice] materials
. Kagome magnets [Herbertsmithite]
. Organic antiferromagnets [kappa-BEDT, Pd(MIT)2] - *near Mott transition*





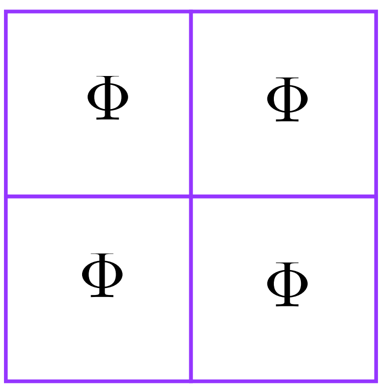


Bosons: Frustration + Strong Correlations

Fully frustrated Bose-Hubbard Hamiltonian

$$H = -\sum_{i,j} t_{i,j} b_i^\dagger b_j + \frac{U}{2}\sum_i n_i(n_i - 1)$$

Pick the kinetic term to include fluxes



For $\Phi = p/q$ *D. R. Hofstadter (PRB 1976)*

Get q bands with q-degenerate minima in the dispersion

Bosons are thus kinetically frustrated and have various ways in which to condense
This kinetic frustration must be resolved by interaction effects

What is the phase diagram as we vary U/t?

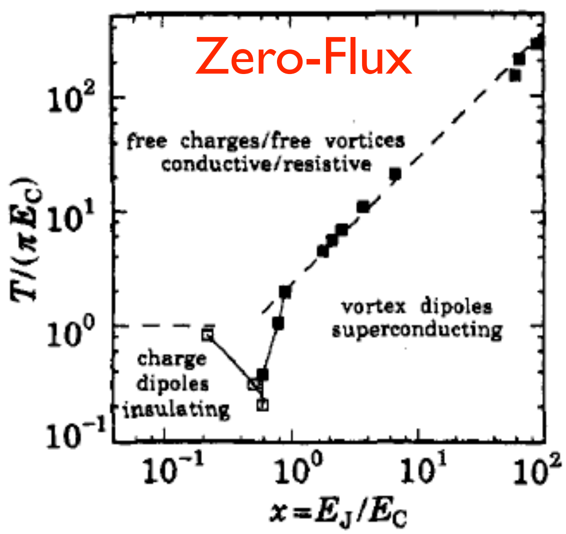
Why is this interesting or relevant?

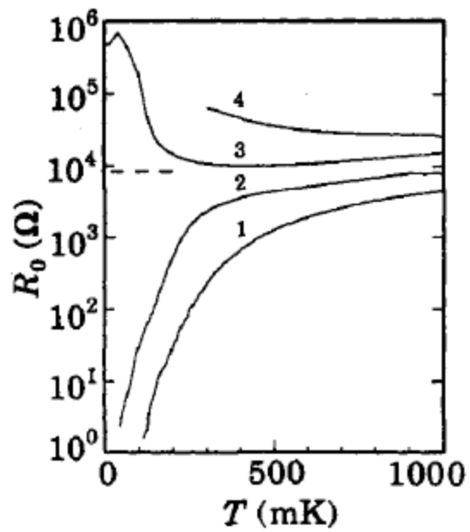
1. Classical variant ~ Fully frustrated Josephson junction array

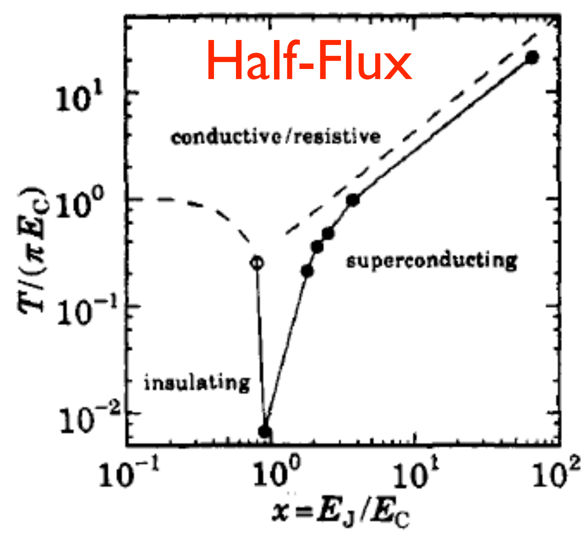
Teitel and Jayaprakash (PRB 1983); M.Y. Choi and S. Doniach (PRB 1985)
Olsson (PRL 1995) - indicates **two** thermal transitions in 2d

What happens with increasing charging energy?

Fully frustrated Josephson junction array - experiment







Mooij group (EPL 1992, PRB 1996)

Why is this interesting or relevant?

1. Fully frustrated Josephson junction array - some theory

For p/q flux, we have q boson "flavors"

Generalize to an n-component field

. $n = \infty$ decoupling: **O(n) transition**

. Assuming valid for n=2, expect (d+1) XY criticality

H. Kawamura -1988,1989
Granato, Kosterlitz - 1990

. Small system L=20 simulations indicate critical exponents ~ XY

. Universal conductivity at the transition for p/q seems to be roughly

$$\sigma_q^* = q \times \sigma_1^*$$

Granato, Kosterlitz - 1990; M.-C. Cha, Girvin - 1994

Epsilon expansion of O(n) model hints at a **chiral fixed point** for

$$n > 21.8 - 23.4\epsilon$$

(H. Kawamura -1988,1989)

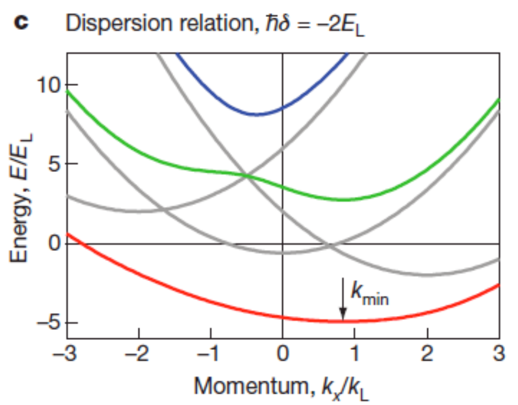
Large system [L>150] numerical studies on stacked triangular lattice indicates a very **weak first order** transition *(H. Diep, PRE 2008, 2010)*

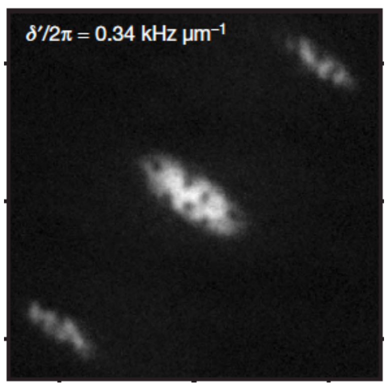
Why is this interesting or relevant?

2. Cold atoms with frustration are beginning to be explored in the weakly correlated regime

Synthetic magnetic fields

Spielman, et al (Nature 2009)
Observation of few vortices





Higher band bosons

Hemmerich group (Nat. Phys. 2010)
Observation of "T-breaking" superfluidity

Frustrated hoppings

Sengstock group (Science 2011)
Observation of "120-magnetic order" in triangular array of tubes

Natural extension is to explore Mott (correlated) regime

Why is this interesting or relevant?

3. Field theories of multiple bosons arise in various different contexts

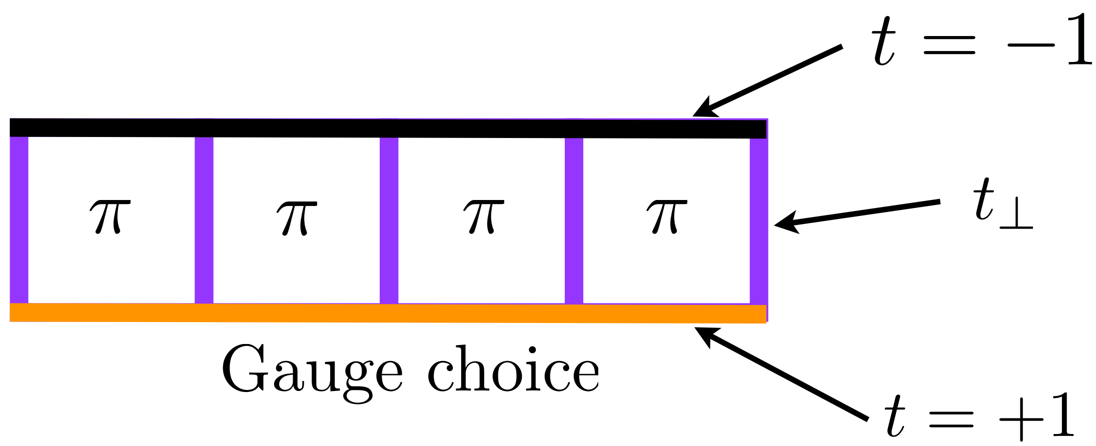
Eg: Deconfined critical point theories (XY variant) have multiple types of vortices (bosons) interacting with U(1) gauge fields

Senthil et al (Science 2004)

What happens in the absence of gauge fields?

4. Unusual connection to frustrated polyelectrolyte hexagonal bundles with JJA phase ~ displacement of counterions along rodlike DNA (rod direction ~ imaginary time) *Grason and Bruinsma (PRL 2006)*

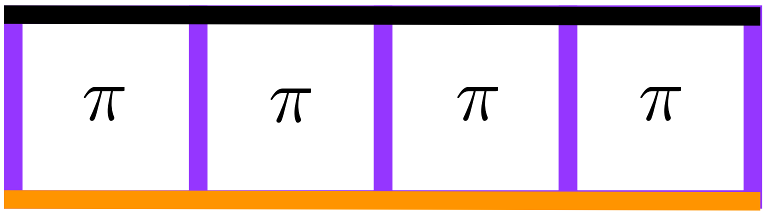
Fully Frustrated ladder



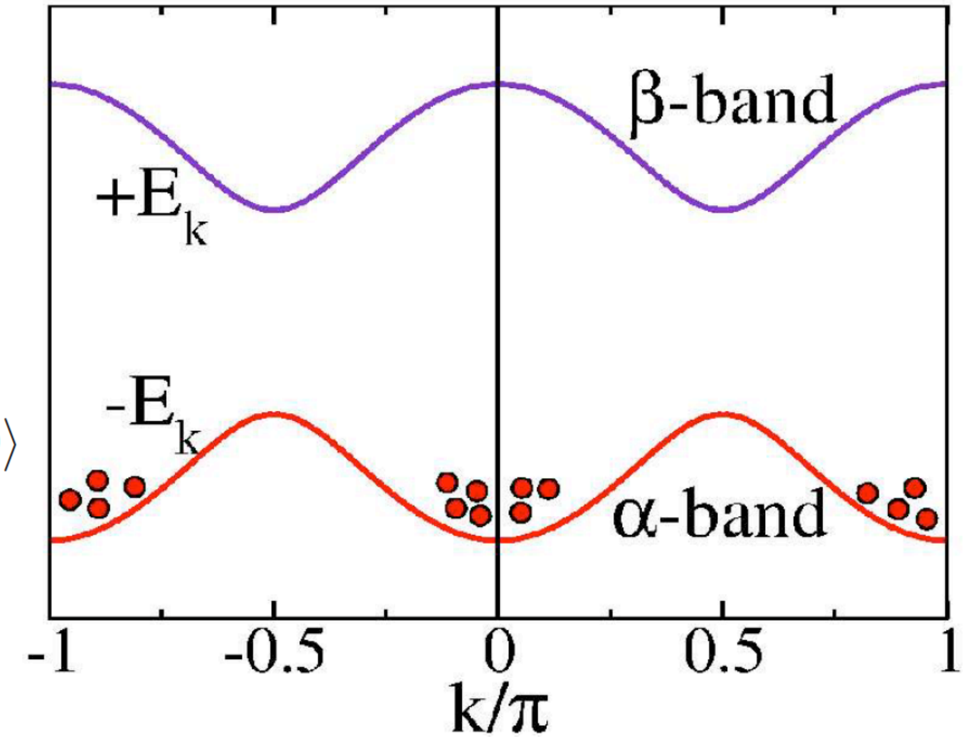
Study the phase diagram as a function of interactions and perpendicular hopping

. Bilayer classical model (JJA or quantum FFXY)
. DMRG study of bosons
. Physical picture

Fully Frustrated ladder: Boson dispersion



$$|\psi\rangle = \frac{1}{\sqrt{N!}}\left[e^{i\varphi}(\alpha_0^\dagger + e^{i\theta}\alpha_\pi^\dagger)\right]^N |0\rangle$$



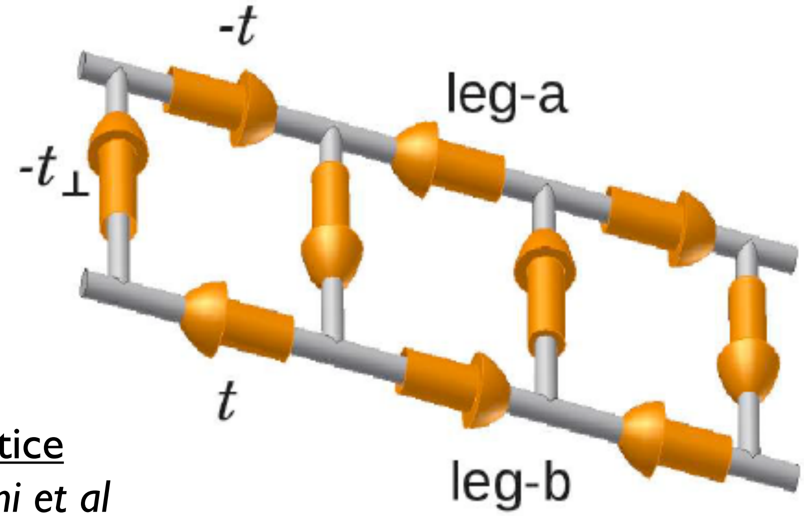
"Landau theory"

$$E_{\rm low}^{\rm mft} = (-E_0 - \mu)\sum_{i=0,\pi}|\varphi_i|^2 + U(u_0^4 + v_0^4)\sum_{i=0,\pi}|\varphi_i|^4 + 8U u_0^2 v_0^2 |\varphi_0|^2 |\varphi_\pi|^2 + 2U u_0^2 v_0^2 (\varphi_0^{*2}\varphi_\pi^2 + \varphi_\pi^{*2}\varphi_0^2)$$

Favors relative phase = $\pi/2$

Favors equal amplitude

Fully Frustrated ladder
Mean field state for weak interactions



Resulting mean field superfluid has uniform density but staggered current pattern:
“Chiral” Superfluid (CSF)

Known from various earlier works on square lattice

Eg: *L. K. Lim, C. M. Smith, A. Hemmerich (PRL 2008); M. Polini et al (PRL 2005); G. Moller and N. R. Cooper (PRA 2010); S. Sinha and K. Sengupta (EPL 2011); S. Powell, et al (PRL 2010)*

Reminiscent of electronic d-density wave metal proposed for cuprate pseudogap state

S. Chakravarty, R. Laughlin, D. Morr, C. Nayak (PRB 2001)
[Affleck, Marston staggered flux state (1988); Nori, Abrahams, Zimanyi (1990)]
Numerical studies of extended tJ models with many extra interactions
J. B. Marston, et al (PRL 2002); U. Schollwock, et al (PRL 2003); C. Weber et al (PRL 2009)

Fully Frustrated ladder
What happens for very strong interactions?

At integer filling and large U, the bosons must localize into a conventional Mott insulator

Mean field theory (2D: L. K. Lim et al, PRL 2008) suggests a direct continuous CSF-MI transition

MFT on ladder

$$\frac{1}{\sqrt{4t^2+t_\perp^2}} = \frac{n}{\mu - U(n-1)} + \frac{n+1}{Un-\mu}$$

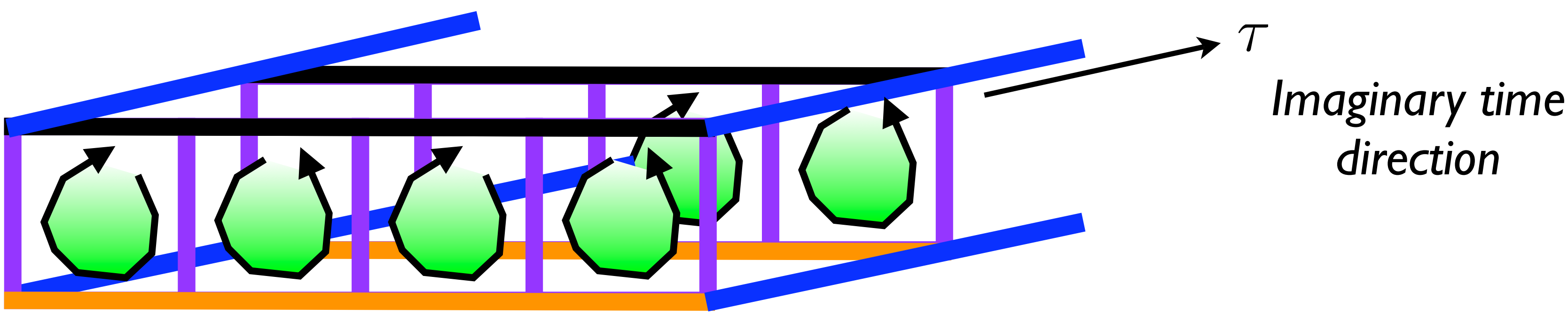
$$\frac{U_{c,ladder}^{\pi-\mathrm{flux}}}{U_{c,ladder}^{0-\mathrm{flux}}} = \frac{\sqrt{4t^2+t_\perp^2}}{2t+t_\perp} < 1$$

Frustration makes it easier to undergo the Mott transition

But the CSF breaks U(1) [phase] and Z(2) [Time reversal]
Do these get "unified"?

Fully Frustrated ladder
Classical Bilayer Model

$$H_{\mathrm{XY}} = -\sum_{i,\delta} J_\delta \cos\left(\varphi_i - \varphi_{i+\delta}\right)$$



Fully Frustrated ladder
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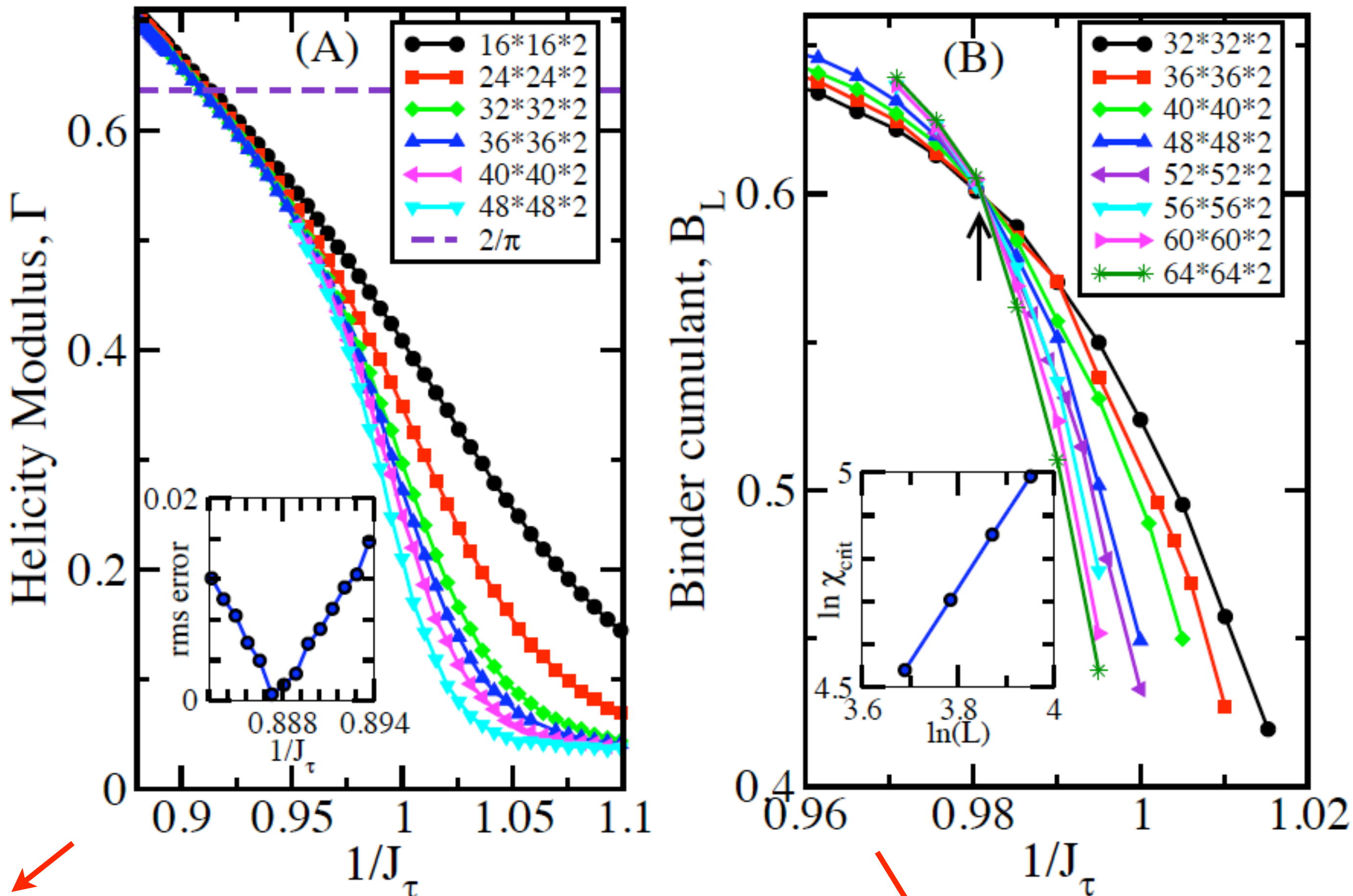
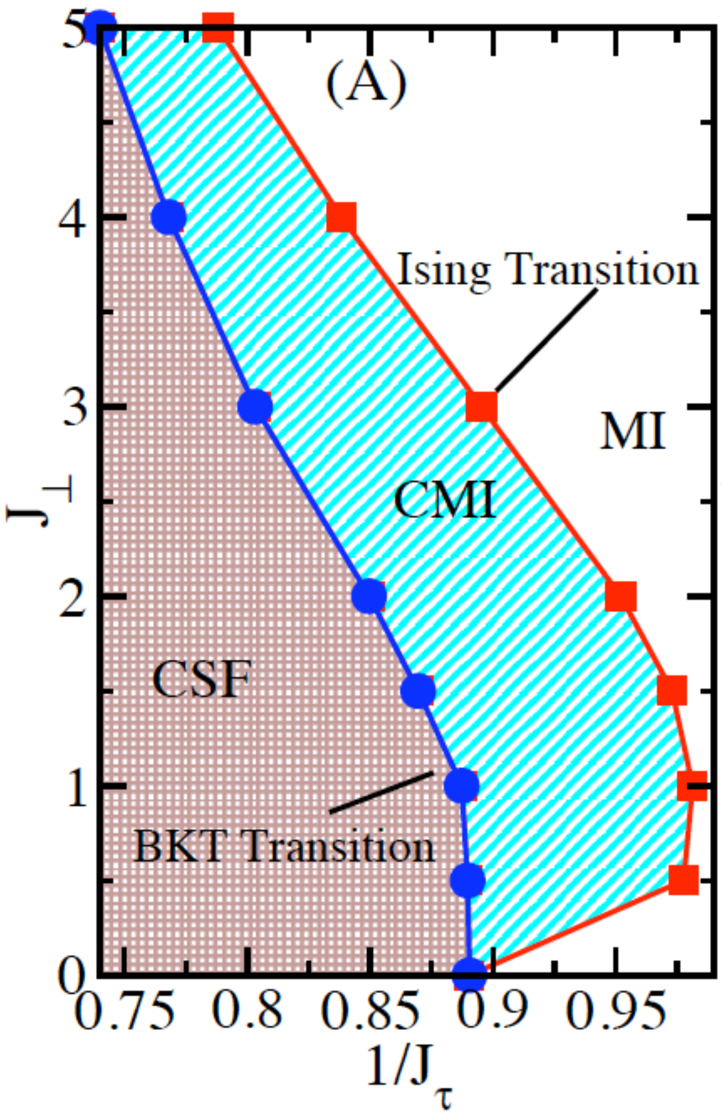


Helicity modulus: BKT transition
Finite size scaling form: $\Gamma(L) = A\left(1 + \frac{1}{2}\frac{1}{\log L + C}\right)$

Weber and Minnhagen (PRB 1988)

Binder cumulant for the staggered current order

Fully Frustrated ladder
Classical Bilayer Model

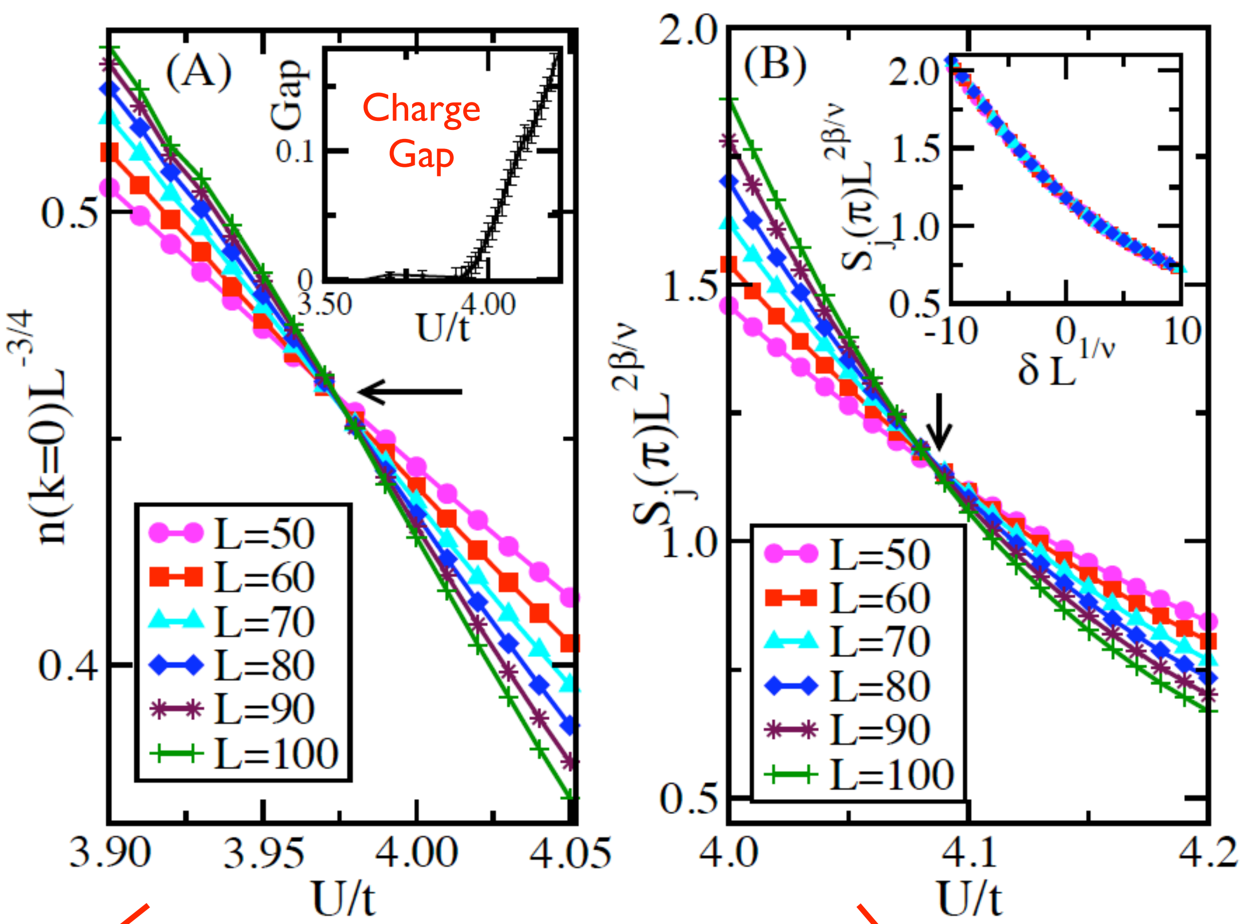


. CSF-MI transition splits into two transitions

. Intermediate CMI phase

. CMI=paramagnet with staggered currents
  on the ladder (interlayer)

Fully Frustrated ladder
DMRG study of the Hubbard model

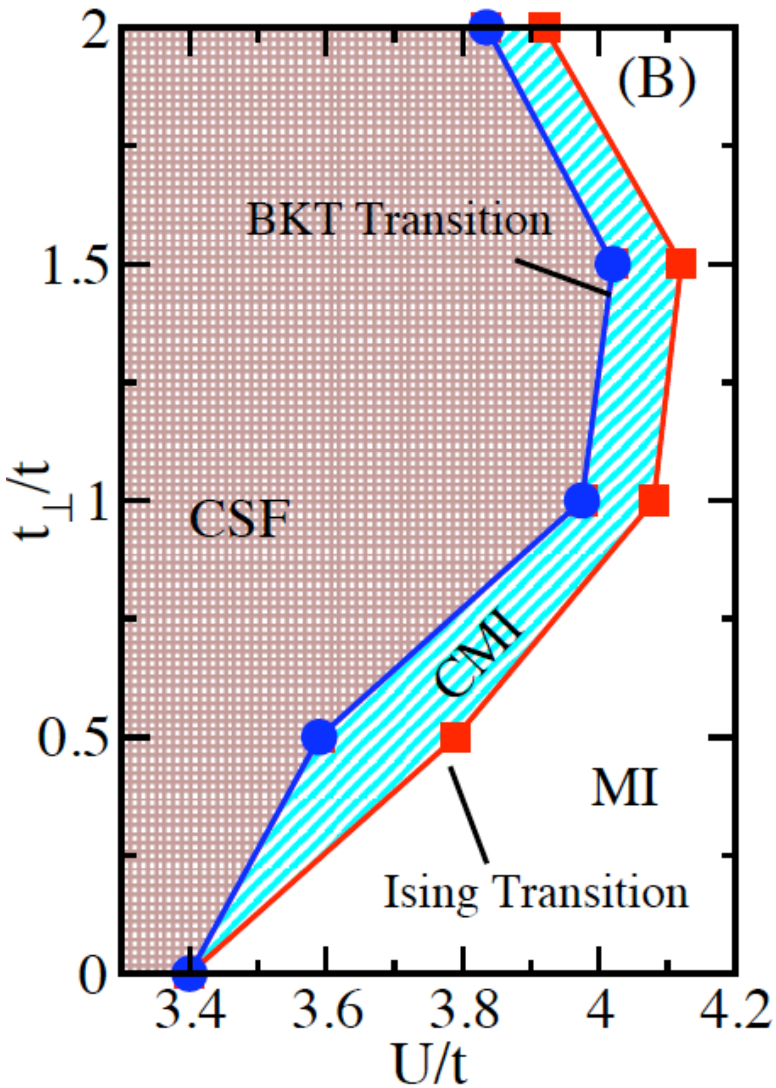


BKT transition implies universal scaling

$$n(k \to 0) \sim |k|^{-3/4}$$

Staggered Current
Order parameter - Ising scaling

Fully Frustrated ladder
DMRG study of the Hubbard model



. CSF-MI transition splits into two transitions

. Intermediate CMI phase

. CMI= insulator with staggered currents

Simple pictures for the “chiral” Mott insulator

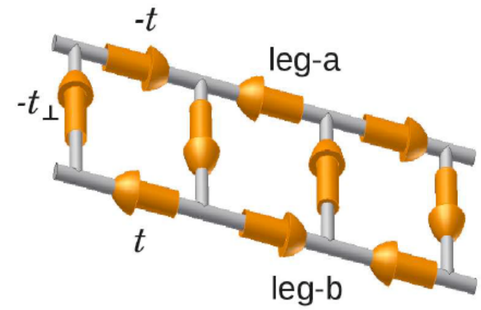
Chiral superfluid = Vortex Crystal

. Flux nucleates vortex or antivortex

.Vortex-vortex interaction is repulsive

. Equal number of V/AV

.“Antiferromagnetic” crystal



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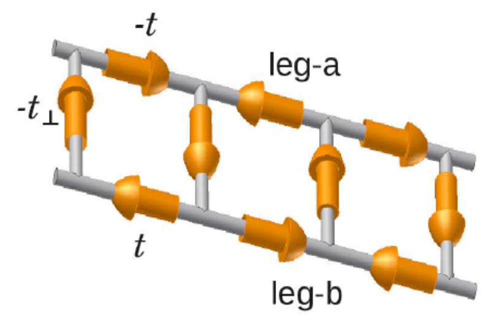

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Regular Mott insulator = Vortex Superfluid (*D. Haldane; Halperin/Dasgupta; Fisher/Lee*)

. Dual - proliferated quantum phase slips

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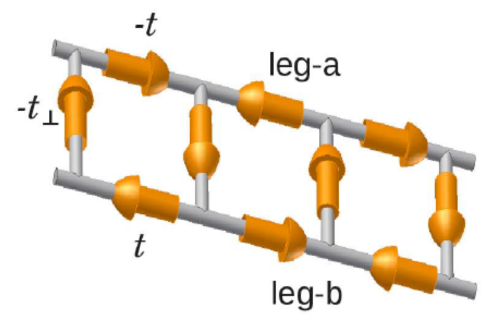

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. Dual - proliferated quantum phase slips

Chiral Mott insulator = Vortex Supersolid
. Defect in crystal: Extra vacancy/interstitial vortex/antivortex
. Proliferating and condensing dilute defects: Vortex superfluid
. Background current pattern preserved: Vortex crystal

Simple pictures for the “chiral” Mott insulator

Excitations of a Conventional Mott insulator

. Gapped Particles: “double occupancy”

. Gapped Holes: “vacancy”

. Dispersing particles/holes: Like a “semiconductor”

Simple pictures for the "chiral" Mott insulator

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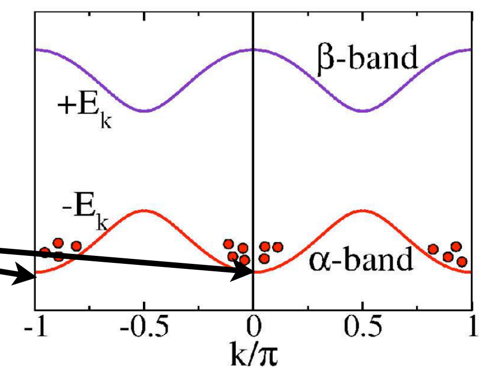
Excitations of a Conventional Mott insulator **with flux**

. Gapped Particles: "double occupancy"
. Gapped Holes: "vacancy"
. Dispersing particles/holes **with multiple minima**
Like a "semiconductor" with multiple valleys



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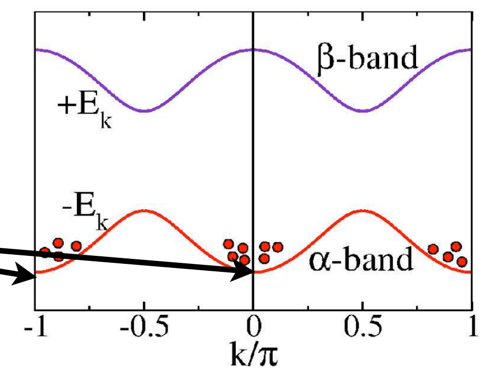


Chiral Mott insulator: Indirect Exciton condensate!

Wavefunction for the "chiral" Mott insulator

$$\Psi(r_1, r_2, \ldots, r_N) = e^{-\sum_{i,j} \tilde{v}(r_i - r_j)} \Psi_{MF}(r_1, r_2, \ldots, r_N)$$

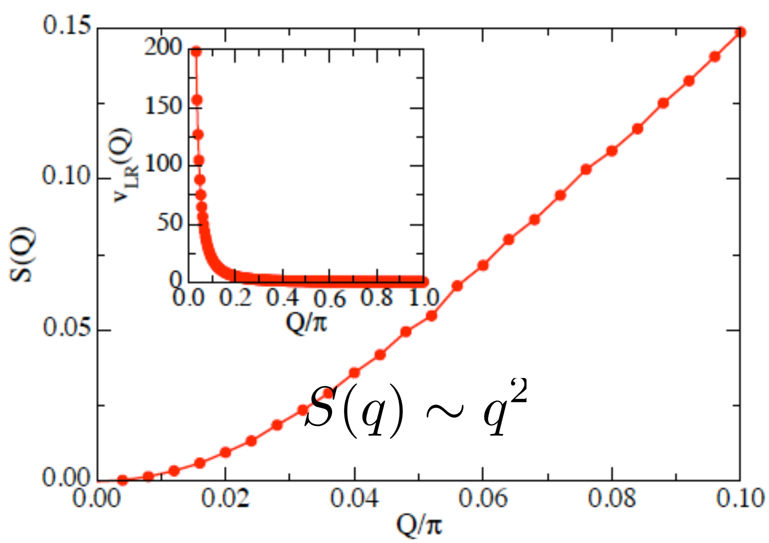
Pick mean field state to be the chiral Bose condensed superfluid

Choose a singular long range Jastrow: $v(q \to 0) \sim 1/q^2$

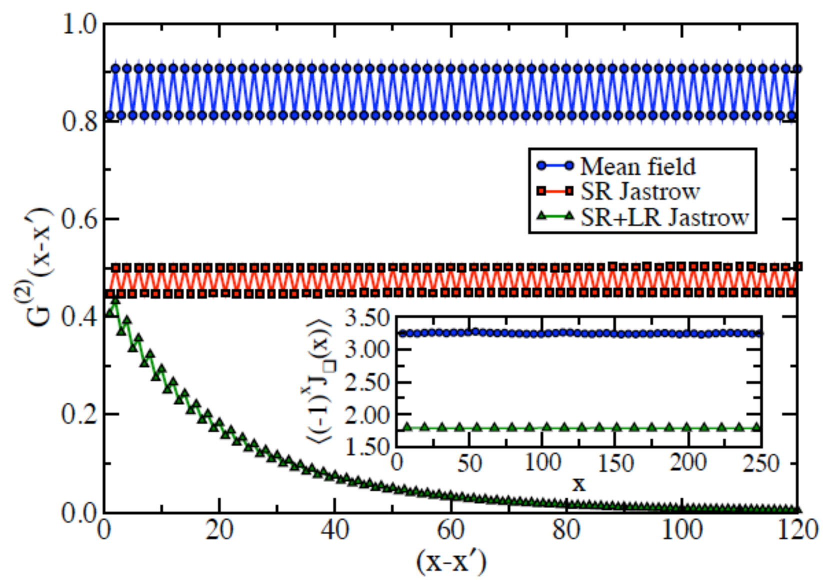
$$|\psi\rangle = \frac{1}{\sqrt{N!}} \left[e^{i\varphi} (\alpha_0^\dagger + e^{i\theta} \alpha_\pi^\dagger) \right]^N |0\rangle$$

Disorder Preserve

Study using variational MC



Density structure factor
suggestive of gap (Feynman single mode)



Boson density matrix
becomes short-ranged

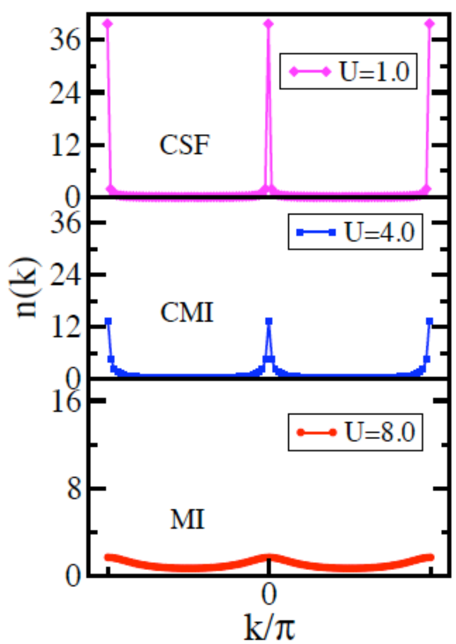
Experimental signatures

1. Josephson junction array realization:

. Insulator in transport
. Spontaneous staggered fields (SQUID microscopy)
$J \sim 1K$ coupling; $1\mu m \times 1\mu m$ cell; $1nT$ fields

2. Cold atom realization:

. No sharp peaks in n(k)
. Look for residual interference between the two peaks



Summary

1. Fully frustrated Bose Hubbard model supports a **staggered current Mott insulator** on a ladder

2. Simple pictures for the “chiral” Mott insulator

3. Field theory understanding:
   Coupled sine-Gordon model - In progress

4. Higher dimensional generalizations: In progress

5. Other models of frustration and their Mott limits?