Chiral p-wave superconductivity in mesoscopic systems

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Outline

- Introduction and motivation
- S-wave superconductivity in a small disk:
  vortexless and vortex state
- Chiral p-wave superconductivity in a small disk:
  predicted re-entrant superconductivity
- Equal spin pairing state in p-wave superconductor
  half quantum flux

Topology: winding number

Issue and basic results

Study superconductivity in mesoscopic system with size comparable to the coherence length.

The small size makes vortex state in s-wave unfavorable, but favors a vortex state in chiral p-wave.

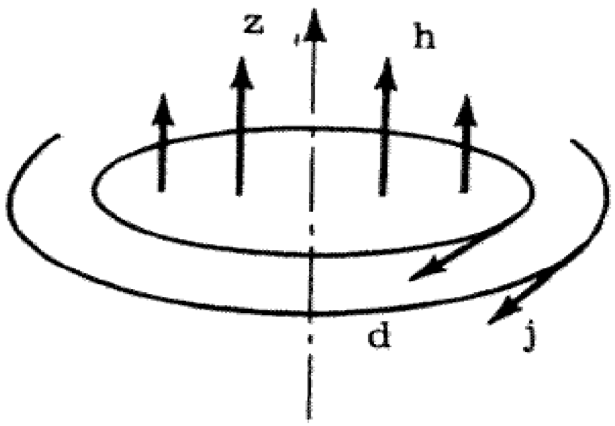
The total winding number W=0 stable agianst small size.

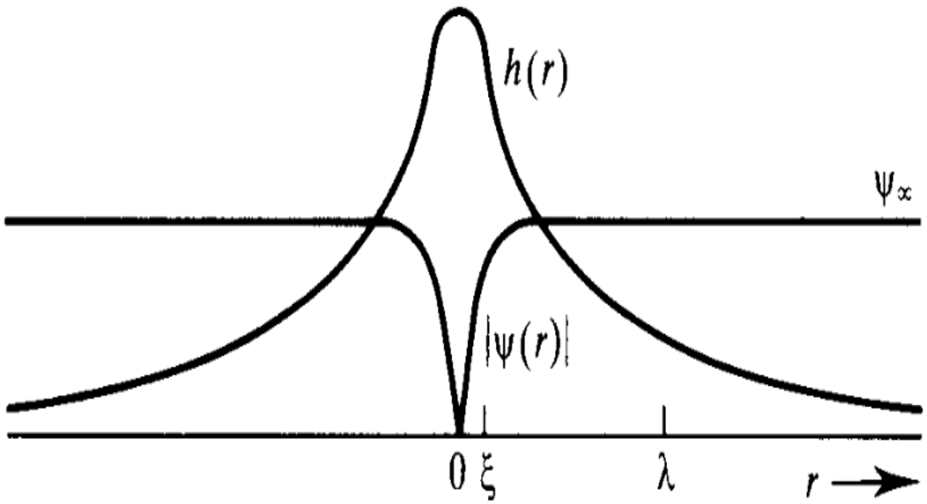
The method: Bogolibov de-Gennes equations solutions of microscopic models

Introduction and motivation

Vortex

- A topological defect.

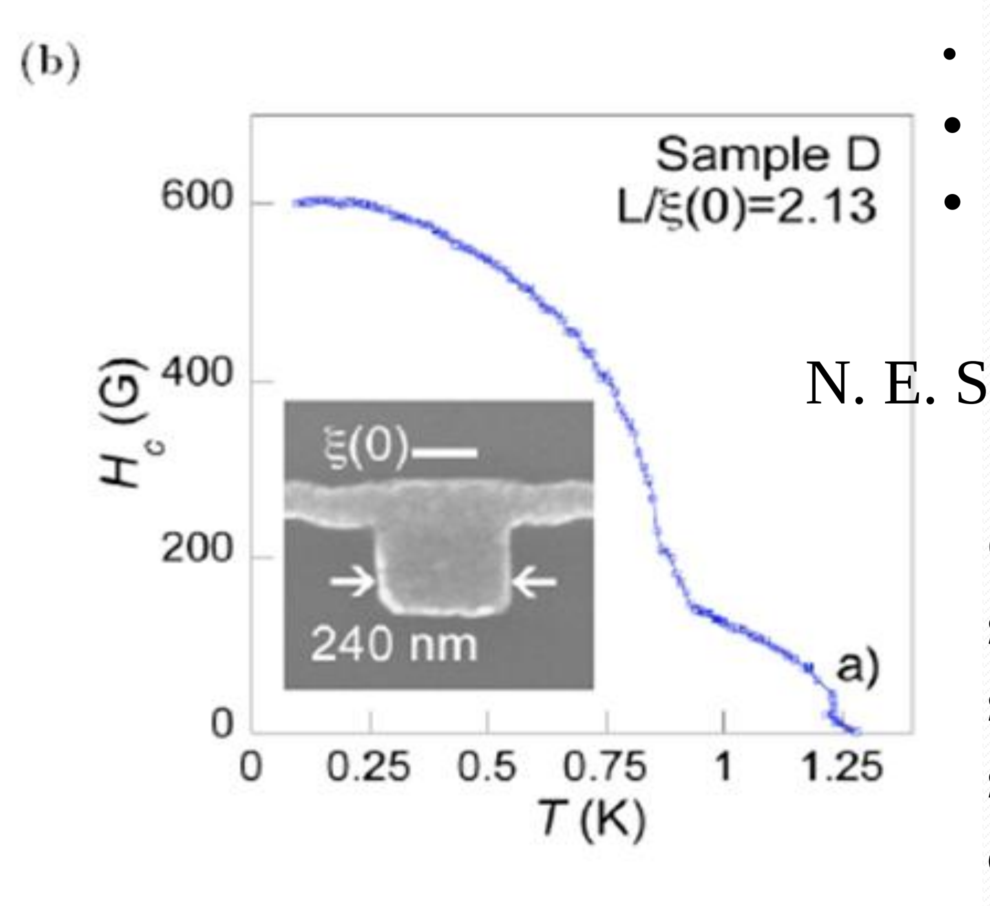




- Vortex core: size ≈ coherence length, SC order parameter is suppressed.
- Screening current distributed in a length scale of penetration depth.
- The finite size effect: sample size $\sim \xi(0)$

Motivation

recent nano-fabrication has made ultra small sample size at hundreds of nm



- **Al film**
- **L = 240 nm ≈ 2.13 ξ(0)**
- **kink in critical field Hc(T)**

N. E. Staley et al. arXiv1005.3843

Ginzburg Landau theory shows no vortices exist in s-wave supercond as sample size smaller than a critical value .
Schweigert- Peeters (98)

motivation

Renewed interest in study of p-wave supercond, which may support exotic object such as half quantum vortices and Majorana quasiparticles

Mesoscopic effect of p-wave may be of interest

A vortex in s-wave supercond

Gygi and Schlueter (1991)

- BdG equations:

$$\begin{bmatrix} h_0(\boldsymbol{r}) & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_0^*(\boldsymbol{r}) \end{bmatrix} \begin{bmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{bmatrix} = E_i \begin{bmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{bmatrix}.$$

$$h_0(\boldsymbol{r}) = \frac{1}{2m}\left[-i\hbar\nabla - \frac{e}{c}\boldsymbol{A}(\boldsymbol{r})\right]^2 - \mu,$$

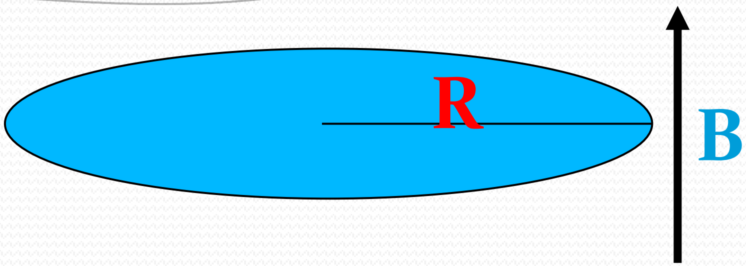
A – vector potential μ – chemical potential

- Self consistent equation:

$$\Delta(\boldsymbol{r}) = g\sum_{E_i<\Lambda} u_i(\boldsymbol{r})v_i^*(\boldsymbol{r})[1-2f(E_i)], \qquad \nabla\times\nabla\times\boldsymbol{A} = \frac{4\pi}{c}\boldsymbol{j}$$

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{e\hbar}{2mi}\sum_i \left\{ f(E_i)u_i^*(\boldsymbol{r})\left[\nabla - \frac{ie}{\hbar c}\boldsymbol{A}(\boldsymbol{r})\right]u_i(\boldsymbol{r}) + [1-f(E_i)]v_i(\boldsymbol{r})\left[\nabla - \frac{ie}{\hbar c}\boldsymbol{A}(\boldsymbol{r})\right]v_i^*(\boldsymbol{r}) - \text{H.c.}\right\}$$

- Disk geometry with radius R (describing a thin film or a quasi-2D cylinder-like sample)



- Boundary condition: $u_i(r=R,\theta) = v_i(r=R,\theta) = 0.$
- Eigenfunction of h_0:

$$\phi_{j,l}(r,\theta) = \frac{\sqrt{2}}{R J_{l+1}(\alpha_{jl})} J_l\left(\alpha_{jl}\frac{r}{R}\right) e^{il\theta}$$

l – angular momentum, $J_l(x)$ – l^{th} order Bessel function of first kind
α_{jl} – the j^{th} zero of $J_l(x)$.

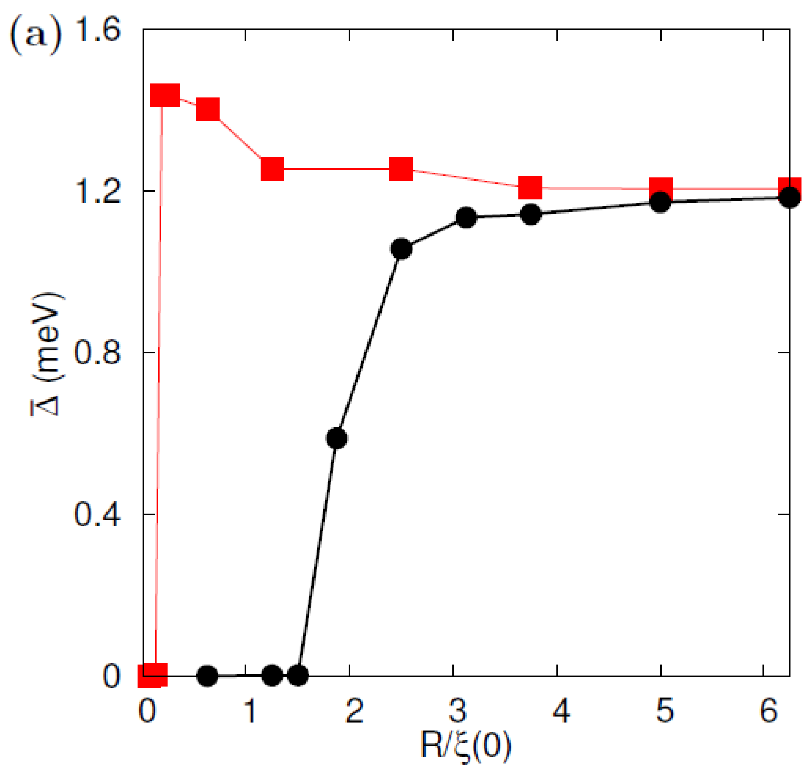
- Rotational invariance:

$$u_i(\mathbf{r}) = u_i(r)e^{il\theta}$$
$$v_i(\mathbf{r}) = v_i(r)e^{i(l-n)\theta},$$
$$\Rightarrow \Delta(\mathbf{r}) = \Delta(r)e^{in\theta}$$

n – vorticity. n= 0: vortex-free state, n = 1: single vortex state.

Result at T = 0

g=0.256, Λ = 30meV, m = m_e ➡ $\xi(T=0) \approx$ 40nm, $\Delta \approx$ 1.2meV



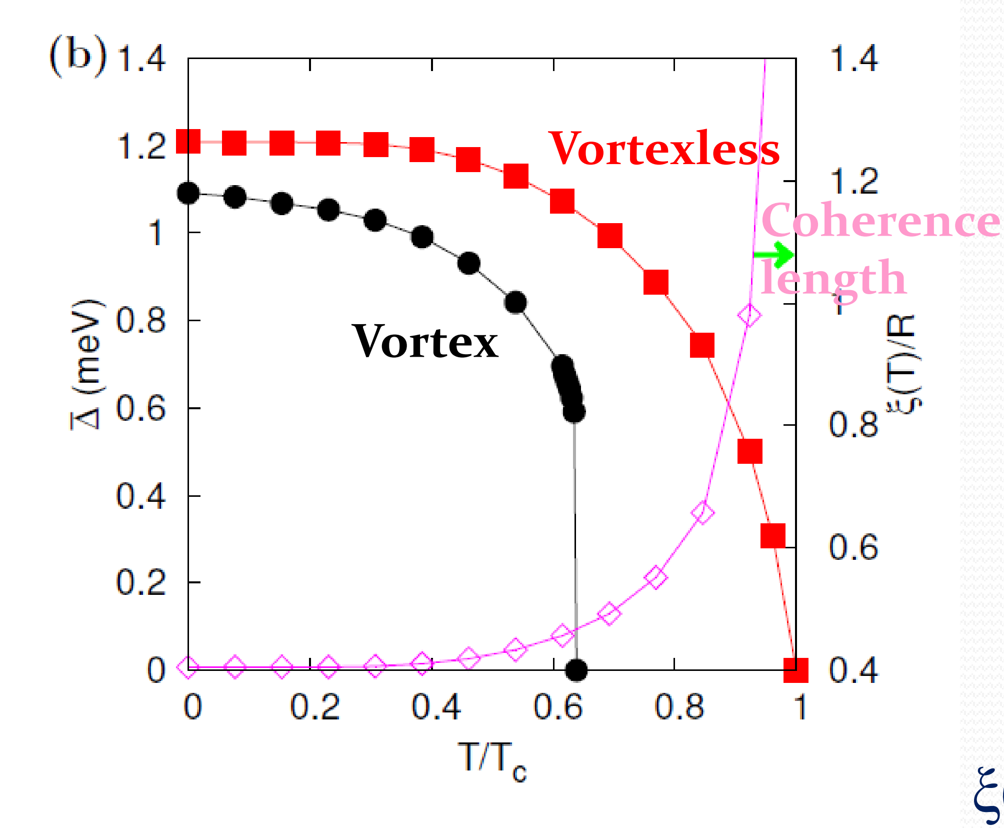
Spatially averaged order parameter $\bar{\Delta}$

Red: vortex-free state

Black: vortex state

- **Vortex-free state robust**
- **No vortex state at R < 1.5 ξ(0)**
- **A crossover from vortex to vortexless supercond**
- **Consistent with G-L theory.**

Temperature dependence

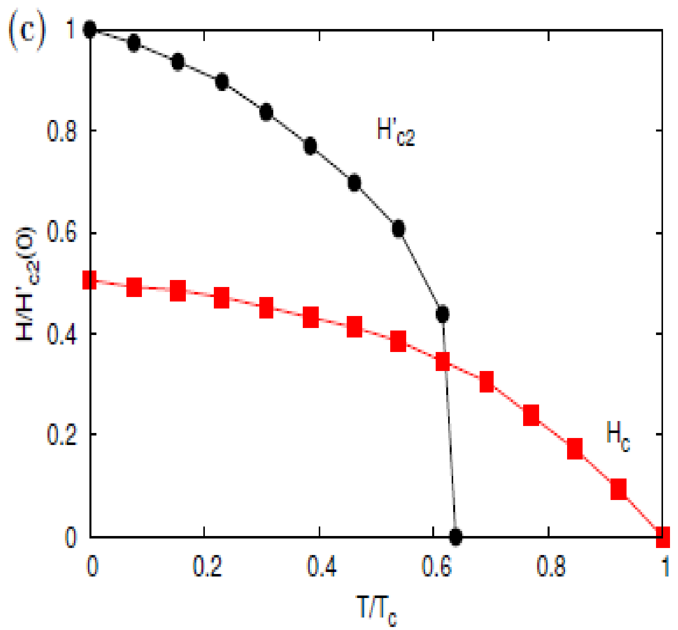


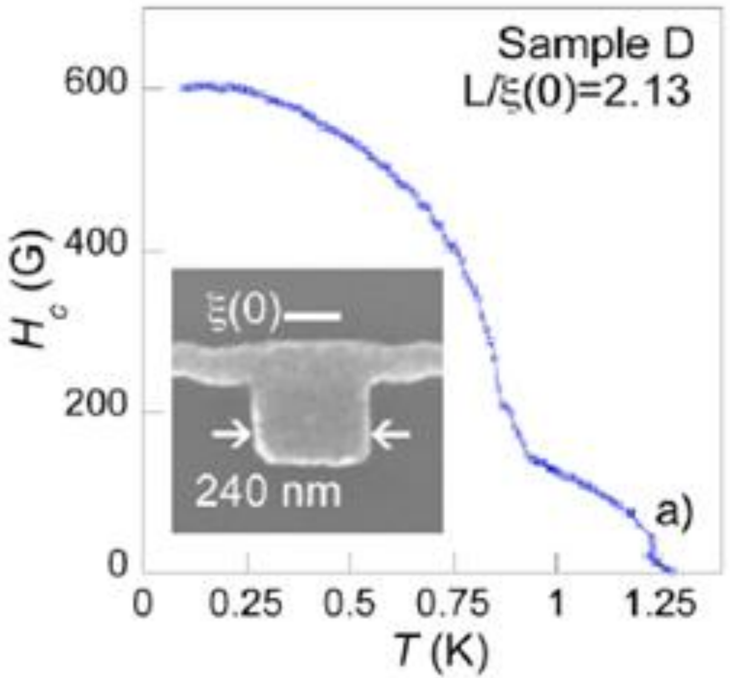
R = 100nm ≈ 2.5 ξ(0)

Vortex state has much lower Tc than the vortex-free state at $R \gtrsim R_c$, because of the increasing coherence length.

ξ(T)=(\hbar /\pi) v_F/gap

T-dependence of critical field





The sharp drop indicate the suppression of vortex state in small sample.

Ying Liu's group, arXiv:1005.3843

P-wave superconductivity: spin triplet, odd parity, described by a d-vector. A chiral state:

$$\Psi = e^{i\varphi}[d_x(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) + i d_y(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) + d_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)](k_x + ik_y).$$

Chiral p-wave case($p_x \pm ip_y$)

- Topological nontrivial
- Sr2RuO4: odd parity spin triplet, likely chiral p-wave pairing symmetry, but no edge current seen in expt.
- $p_x \pm ip_y$ are degenerate in thermodynamic limit, but they are mixed in a finite size system.
- We focus on the state with $p_x + ip_y$ dominant

Chiral p-wave case

- BdG equation for chiral p-wave superconductor:

$$\begin{bmatrix} h_0(\boldsymbol{r}) & \Pi(\boldsymbol{r}) \\ -\Pi^*(\boldsymbol{r}) & -h_0^*(\boldsymbol{r}) \end{bmatrix} \begin{bmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{bmatrix} = E_i \begin{bmatrix} u_i(\boldsymbol{r}) \\ v_i(\boldsymbol{r}) \end{bmatrix}$$

with $\Pi(\boldsymbol{r}) = -\frac{i}{k_\mathrm{F}} \sum_\pm \left[\Delta_\pm \Box_\pm + \frac{1}{2}(\Box_\pm \Delta_\pm) \right]$ and $\Box_\pm = e^{\pm i\theta}(\partial_r \pm \frac{i}{r}\partial_\theta)$

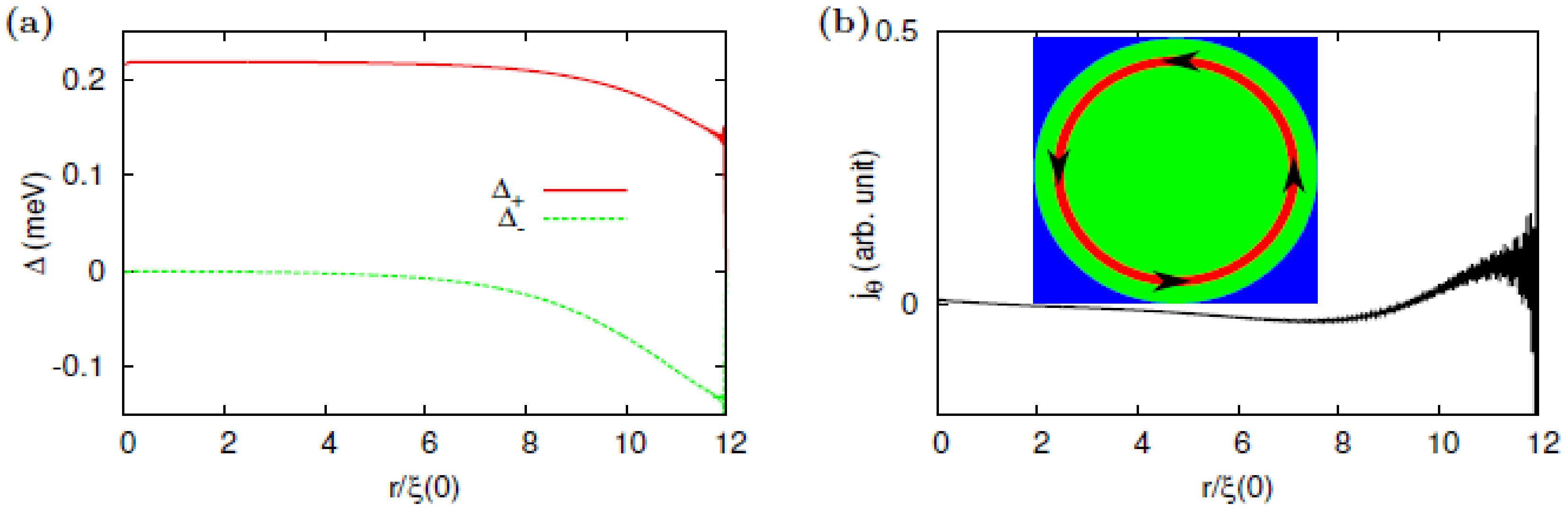
- Ansatz:

$$\Delta_+(\boldsymbol{r}) = \Delta_+(r) e^{in\theta} \qquad \Delta_-(\boldsymbol{r}) = \Delta_-(r) e^{i(n+2)\theta}$$

- Self consistent equation:

$$\Delta_\pm(\boldsymbol{r}) = -i\frac{g}{2k_\mathrm{F}} \sum_{E_i<\Lambda} \left[v_i^*(\boldsymbol{r}) \Box_\mp u_i(\boldsymbol{r}) - u_i(\boldsymbol{r}) \Box_\mp v_i^*(\boldsymbol{r}) \right] \left[1 - 2f(E_i) \right]$$

Vortex-free state at R≈840nm

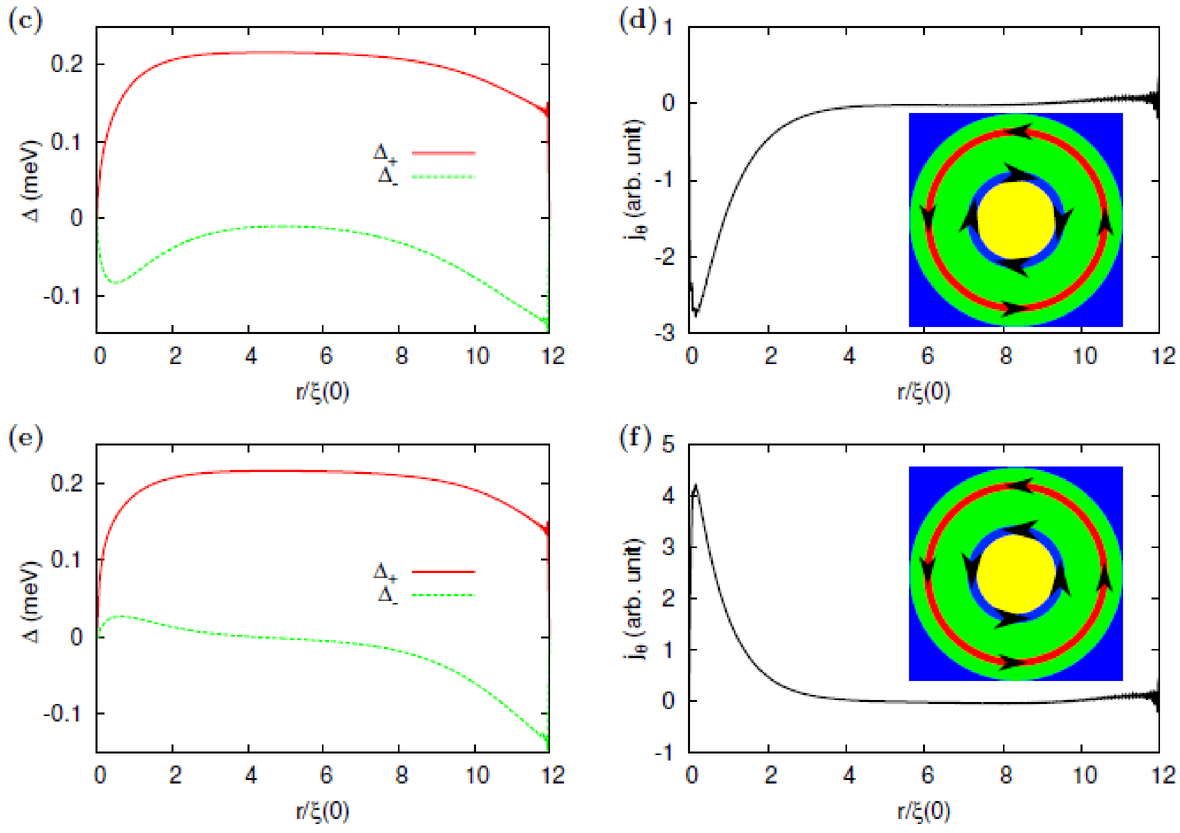


Para: g = 0.2, μ=Λ=16.32meV ➡ Δ=0.2meV, ξ(0) ~70 nm

- **p+ip dominates. At edge p-ip component substantial.**
- **Counter clock-wise edge current within scale ξ(0) and clock-wise screening current within penetration depth (Meissner)**
- **Edge current oscillate with a wave vector $2k_F$.**

  (both $\mathbf{\Delta}_+$ and $\mathbf{\Delta}_-$ vanish at the edge, but change sharply to finite value – note added after the talk)

Vortex states at R=840nm



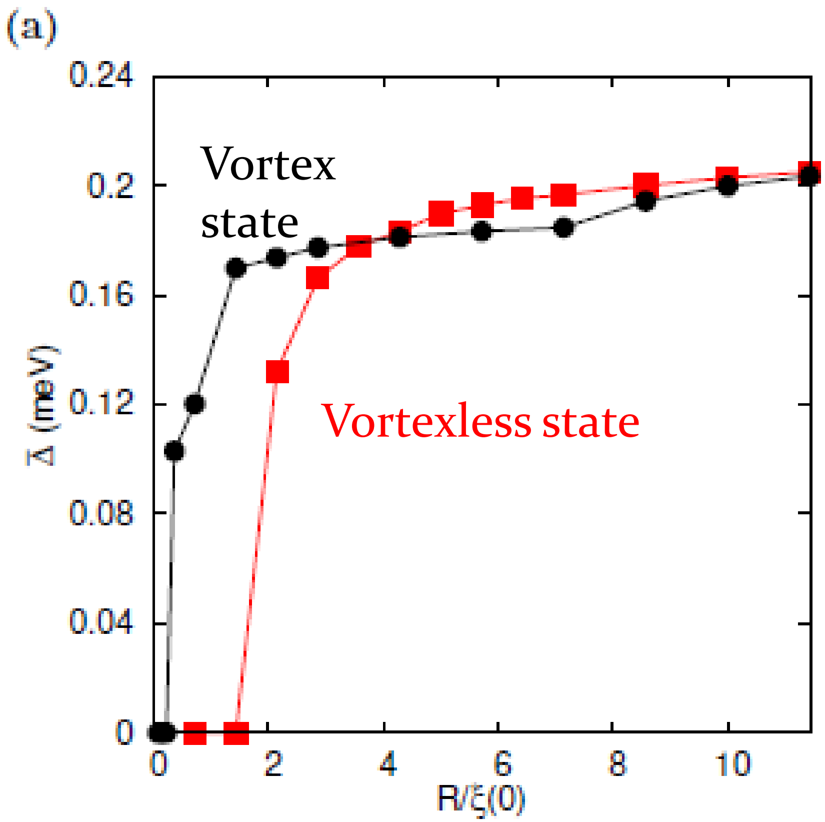
Negative vortex: n = -1

Current in vortex core flows in opposite direction of edge current.

Positive vortex: n = 1

Core current flows in same direction of edge current

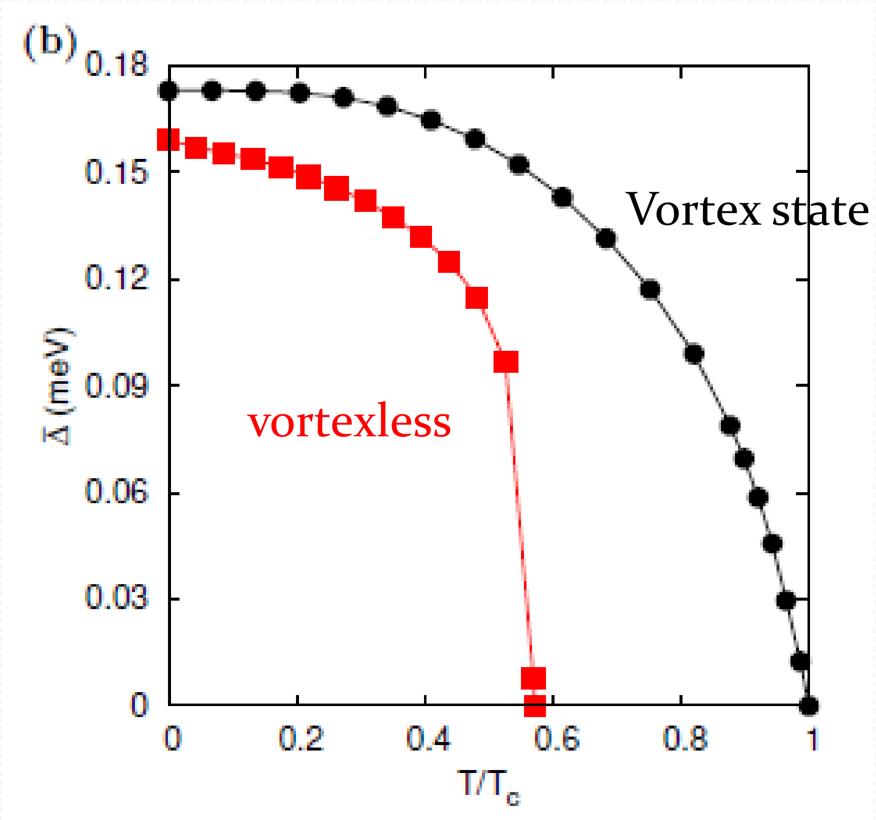
Size effect in chiral p-wave at T=0



Contrary to S-wave case:

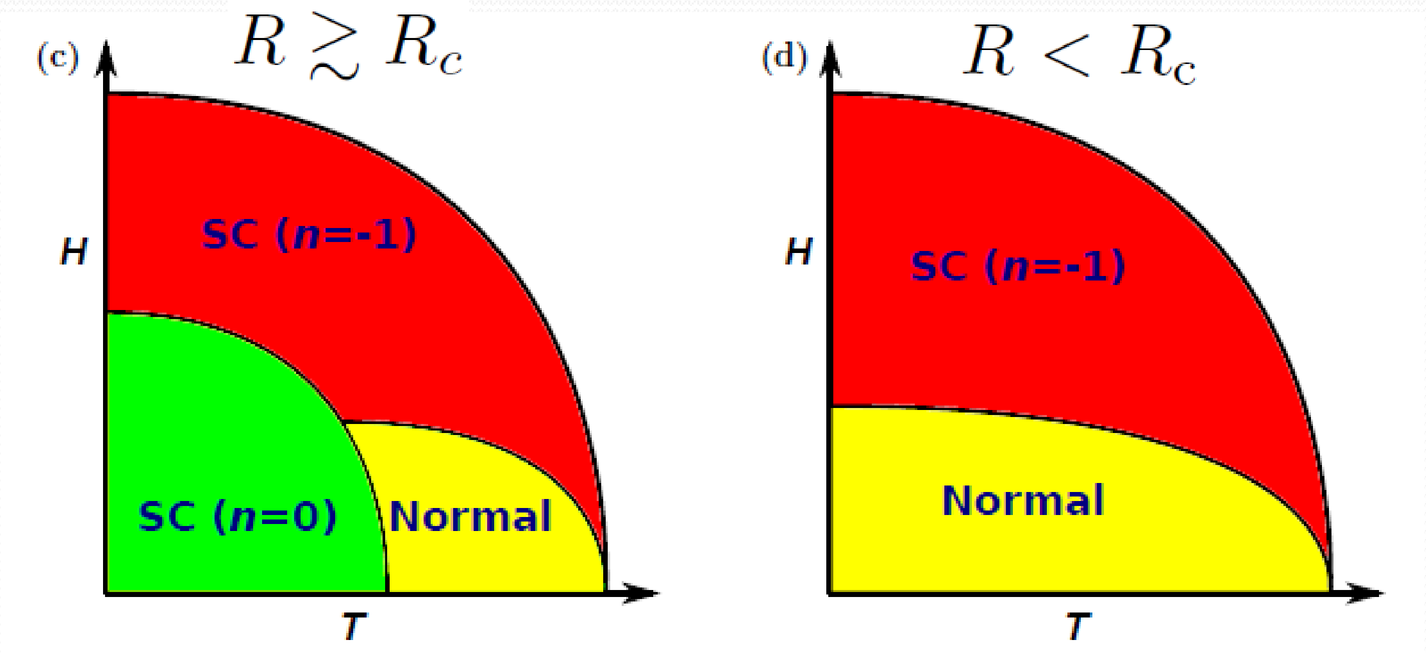
- No vortex-free state at $R < R_c \approx 1.5\xi(0)$.
- Boundary region is similar to the vortex core in s-wave case.
- Negative vortex state robust against small size.
- Edge current and core current are partially cancelled.

T-dependence



- R=165nm ≈ 2 ξ(0)
- In the absence of magnetic field, the superconductivity disappears above 0.6Tc, because ξ increases with T.
- Reentrant SC phase produced by magnetic field.

Schematic phase diagram



- For R < Rc, Field-cooled samples will exhibit superconductivity whereas zero-field cooled samples do not.
- Experimental observation of such phenomena can provide a very strong evidence of chiral p-wave pairing symmetry.

Topological point of view

- Winding number associated with SC order parameter

$$\mathcal{W} = \frac{1}{2\pi i} \oint \frac{\mathrm{d}\Delta}{\Delta},$$

- S-wave case: w = vorticity.
- p+ip case: w= vorticity + 1.
- State with w = 0 (s-wave vortex-free state and p+ip negative vortex state) is robust against small size
- State with w ≠ 0 (s-wave vortex state and p+ip vortex-free state) vanishes below a critical size comparable to ξ.

Ginzburg-Landau Analysis

- Assumption:

  GL theory predicts correct asymptotic behaviors in small size

- Free energy density

  The dominant gradient term $|\mathcal{D}\Delta|^2$

$$\mathcal{D} \equiv -i\nabla - 2\boldsymbol{A} = -i\partial_r \hat{\boldsymbol{e}}_r - (i\frac{1}{r}\partial_\theta + 2A_\theta)\hat{\boldsymbol{e}}_\theta$$

$\frac{1}{r}\partial_\theta \Delta$ dominates and makes SC disfavor in small size

Therefore $\partial_\theta \Delta$ has to vanish, *i.e.*, $\mathcal{W} = 0$

Equal spin pairing case

- Two weakly interacting condensates with $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ pairing.
- Both condensates see the same vector potential.
- Pairing potential:

$$\Psi(\boldsymbol{r}) \propto (\Delta_{\uparrow+}(r)e^{\pm i\theta}|\uparrow\uparrow\rangle + \Delta_{\downarrow+}(r)|\downarrow\downarrow\rangle) * (p_x + ip_y) \\ + e^{2i\theta}(\Delta_{\uparrow-}(r)e^{\pm i\theta}|\uparrow\uparrow\rangle + \Delta_{\downarrow-}(r)|\downarrow\downarrow\rangle) * (p_x - ip_y),$$

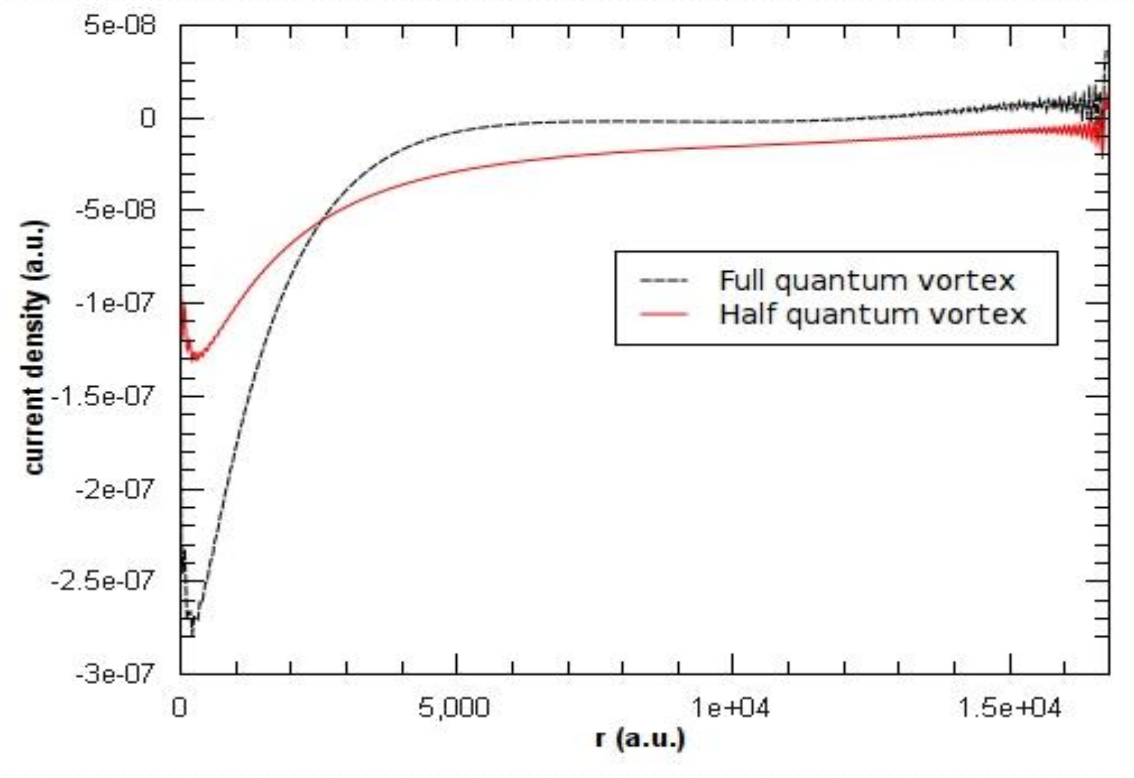
* – symmetrized product.

- Vortex state in spin up pairing, vortex-free state in spin down pairing.
- At R < Rc, pairing in $|\downarrow\downarrow\rangle$ breaks down, only pairing in $|\uparrow\uparrow\rangle$ with n=-1 is robust.

Spinless chiral p-wave superconductor at R<Rc.

Half quantum vortex

Small size system allows only spin up pairing state hence only half quantum vortex



Summary

- We studied the superconductivity in small 2D superconducting disk with and without vortex.
- For s-wave pairing, vortex-free state is still stabilized when sample size approaches to coherence length, while vortex state can not exist below a critical radius.
- There will be magnetic induced superconductivity in ultrasmall chiral p-wave superconductor. (also exists on a square lattice in the tight-binding limit)
- The winding number plays a determining role in understanding the geometry constrained effect in mesoscopic superconductors.

Thank you !