Topological phases protected by point group symmetry

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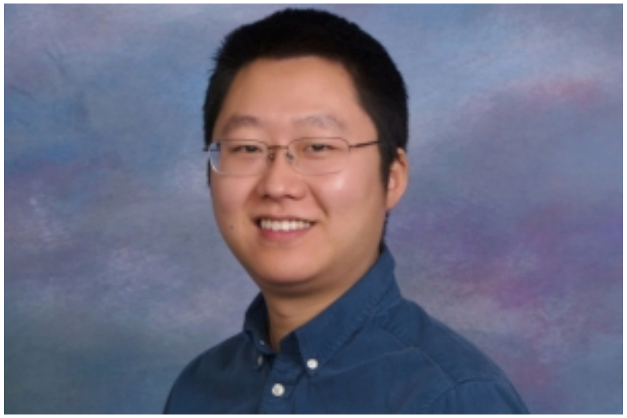
Thanks to...



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Symmetry protected topological (SPT) phases

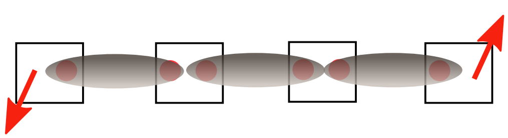
<u>*T*=0 phases of matter characterized by:</u>

1. Energy gap
2. Symmetry *G* not spontaneously broken
3. Ground state becomes trivial if *G* explicitly broken

→ *No intrinsic topological order* in the bulk, *i.e.* no non-trivial braiding statistics or ground state degeneracy on torus

<u>Classic examples:</u>

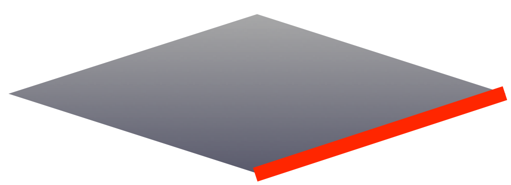
d=1



Haldane *S=1* chain

Symmetry: time reversal, or SO(3) spin rotation, or reflection

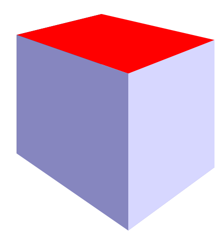
d=2



Quantum spin-hall insulator

Symmetry: charge conservation + time reversal

d=3



Topological band insulator

Symmetry: charge conservation + time reversal

What SPT phases to study?

Some guidance (paraphrase):
"*The life without reflection is not worth living*." - Socrates

- Most theory of SPT phases focuses on internal symmetries, or on non-interacting fermions
- Discrete symmetries of crystal lattices, including reflection, are pervasive in solids, we should not ignore them when studying SPT phases
- For crystalline SPT phases with strong interactions, some examples and case studies, but no general theory

This talk: point group SPT phases

- Focus on SPT phases protected by point group symmetry (= pgSPT phases)
- A surprise: “pgSPT phases are *easier* than SPT phases protected by internal symmetry in the same spatial dimension.”
- There is a mapping between pgSPT states in spatial dimension d and certain lower-dimensional topological states with internal symmetry

Overview of results

- Classification and characterization of pgSPT phases in terms of lower-dimensional topological states with internal symmetry
- pgSPT phase $\simeq$ stack/array of lower-dimensional topological phases with internal symmetry
- For simplicity, this talk will mostly focus on reflection symmetry, but the approach applies to any point group.
- Remark: reflection pgSPT's are related to time-reversal SPT's if one assumes a Lorentz-invariant field theory description.
  I will not make this assumption (more comments at the end).

Prior work on interacting point group SPT phases

- d=1 (inversion symmetry):
  Z.-C. Gu & X.-G. Wen
  Pollmann, Turner, Berg, Oshikawa
  X. Chen, Z.-C. Gu, X.-G. Wen
  Schuch, Perez-Garcia, Cirac
  Fuji, Pollmann, Oshikawa

- Higher dimensions:
  Y. Qi & L. Fu; Isobe & L. Fu
  G.-Y. Cho, C.-T. Hsieh, R. Leigh, T. Morimoto S. Ryu, O. Sule
  A. Furusaki, T. Morimoto, C. Mudry, T. Yoshida
  Ware, Kimchi, Parameswaran, Bauer
  Lapa, Teo & Hughes
  Y.-Z. You & C. Xu
  Kapustin, Thorngren, Turzillo & Zitao Wang
  MH & X. Chen

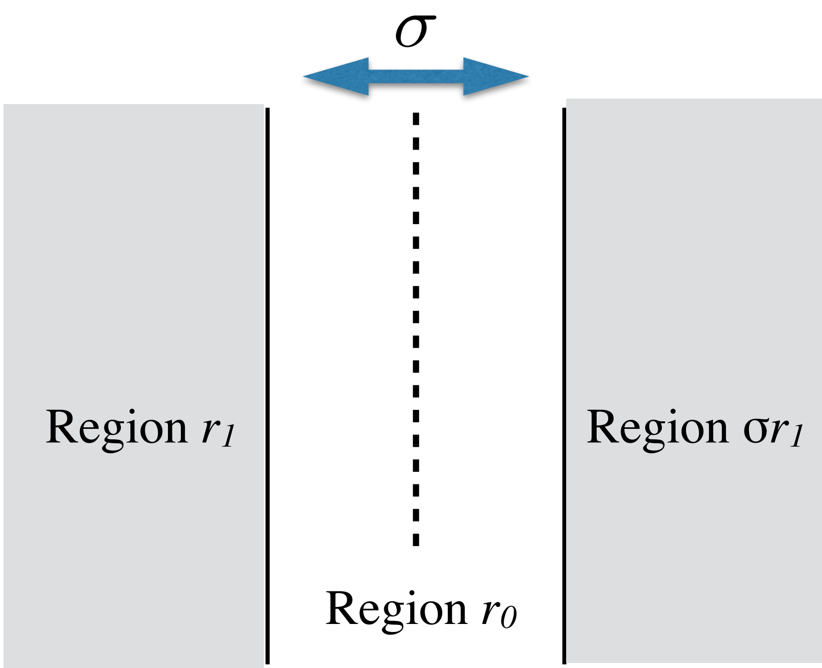
Outline

1. Bosonic mirror SPT phases in d=3
2. Electronic topological crystalline insulators with interactions
3. Point groups beyond reflection
4. Odds and ends, outlook

"Simplest interesting example"

Consider bosonic system in *d=3* with *only* mirror (reflection) symmetry $\sigma : (x,y,z) \rightarrow (-x,y,z)$. (Ignore any other symmetry present.)

Cross section through *bulk*



Adiabatic continuity (preserving symmetry):

SPT ground state

$$|\psi\rangle \rightarrow |T\rangle_{r_1} \otimes |\tilde{\psi}\rangle_{r_0} \otimes |T\rangle_{\sigma r_1}$$

Product states in regions r_1, σr_1

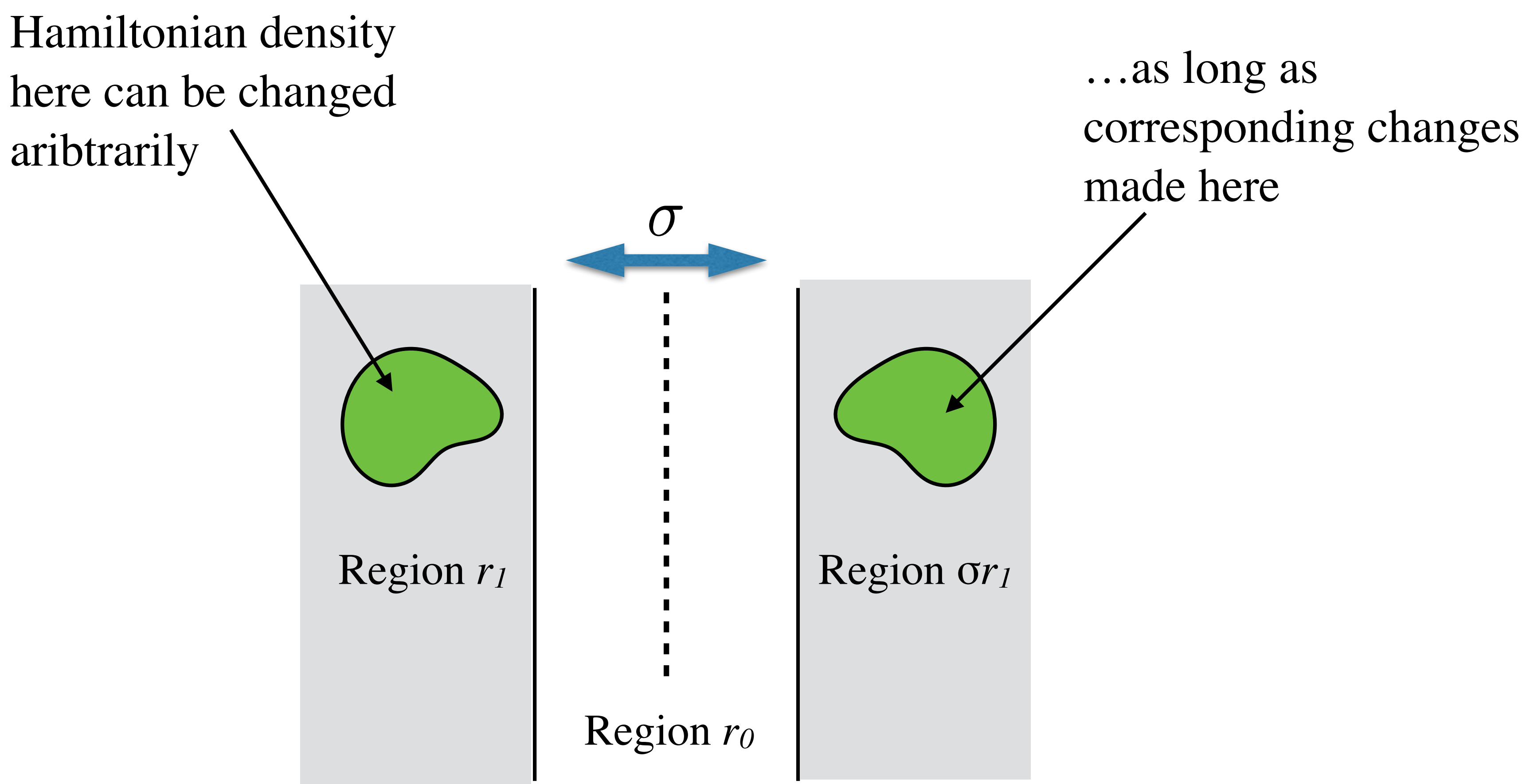
Non-trivial state in *d=2* region r_0

- The mirror symmetry becomes an internal Z_2 symmetry.
- *The d=3 point group SPT state is equivalent to a d=2 state on the mirror plane, with Z_2 internal symmetry.*

Why can we dimensionally reduce?

$$|\psi\rangle \to |T\rangle_{r_1} \otimes |\tilde{\psi}\rangle_{r_0} \otimes |T\rangle_{\sigma r_1}$$

- Quick argument: can locally trivialize any patch away from the mirror plane



Why can we dimensionally reduce?

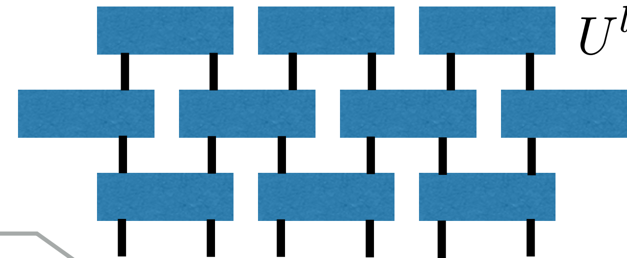
$$|\psi\rangle \rightarrow |T\rangle_{r_1} \otimes |\tilde{\psi}\rangle_{r_0} \otimes |T\rangle_{\sigma r_1}$$

- More detailed argument based on "cutting" a finite-depth quantum circuit

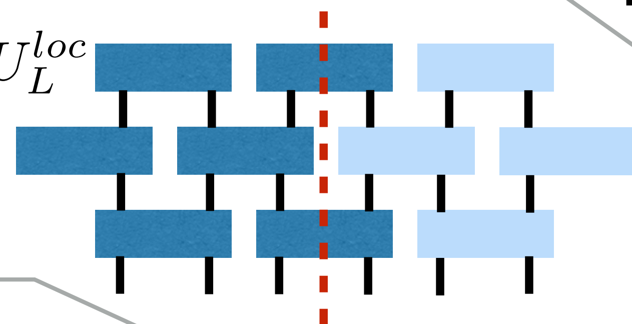
$U^{loc}|\psi\rangle = |\text{product}\rangle$

U^{loc} breaks symmetry

U^{loc} can be represented as a finite-depth quantum circuit:

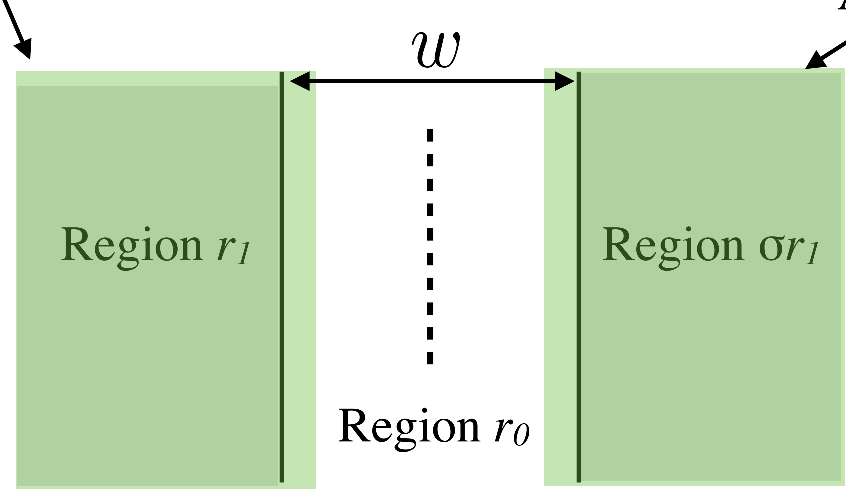


We can "cut" U^{loc}, to get a new quantum circuit U_L^{loc} acting only in region r_1



Act here with U_L^{loc}

Act here with $U_R^{loc} = \sigma U_L^{loc} \sigma^{-1}$



$$U_L^{loc} U_R^{loc} |\psi\rangle = |T\rangle_{r_1} \otimes |\tilde{\psi}\rangle_{r_0} \otimes |T\rangle_{\sigma r_1}$$

Width $w \gg \xi$, correlation length

Dimensional Reduction → Classification

Proceed in two steps:

First: What *d=2* quantum phases can occur on mirror plane?

Second: How to group d=2 states on mirror plane into equivalence classes of d=3 quantum phases?

Dimensional Reduction → Classification

First: What *d=2* quantum phases can occur on mirror plane?

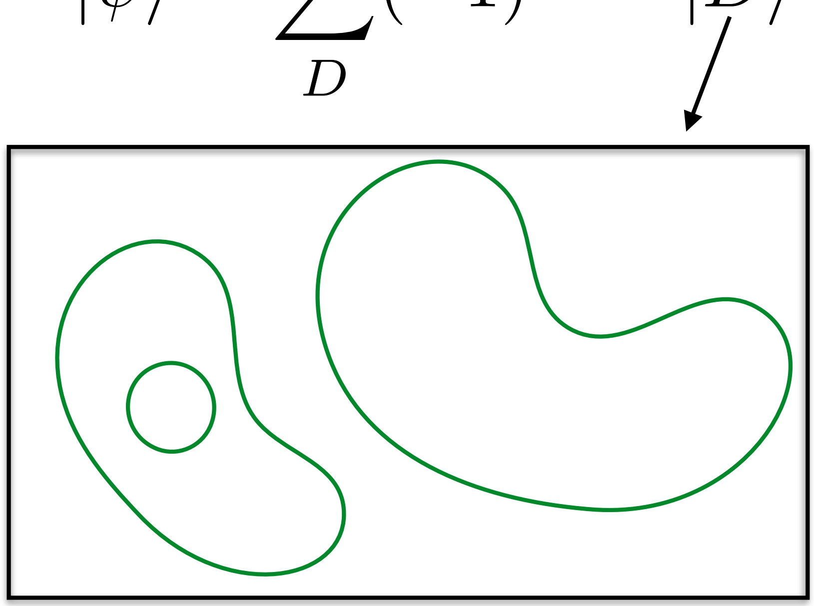
"Integer" topological phases (gap, no anyons), preserving Z_2 symmetry

Two possibilities…

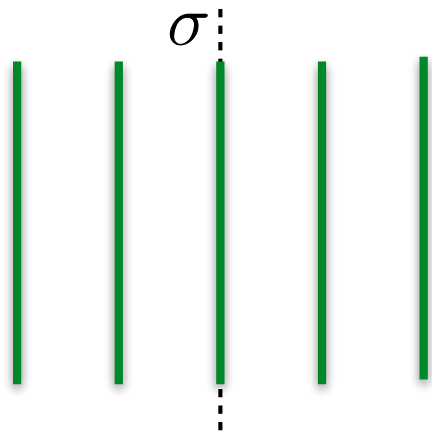
A. Non-trivial *d=2* SPT phase with Z_2 symmetry
(Levin & Gu; X. Chen, Z.-C. Gu, Z.-X. Liu, X.-G. Wen)

Domain wall picture

$$|\psi\rangle = \sum_D (-1)^{N(D)} |D\rangle$$



- Gapless edge modes protected by Z_2 symmetry
- Stack to get non-trivial pgSPT phase



Dimensional Reduction → Classification

First: What *d=2* quantum phases can occur on mirror plane?

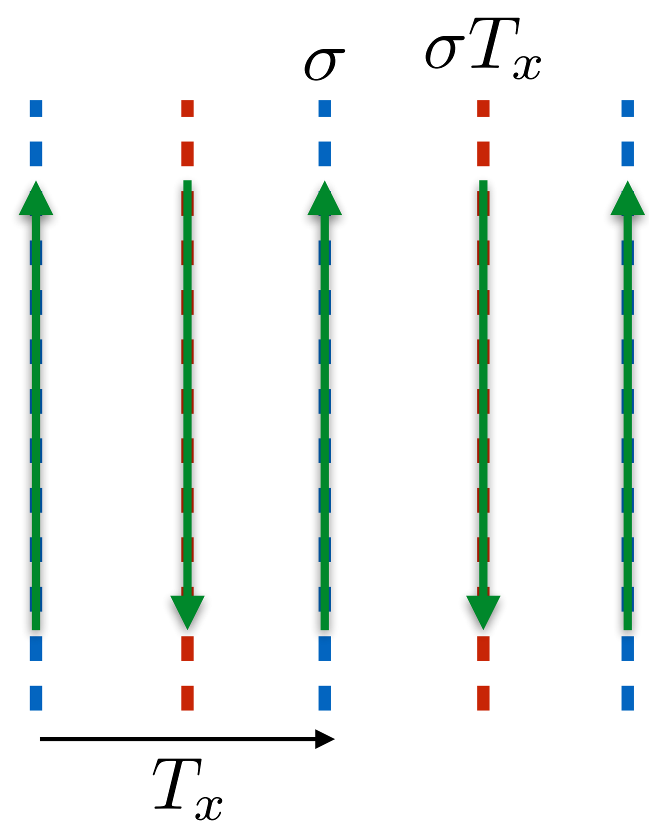
B. E_8 state (Kitaev)

- This state has 8 co-propagating edge modes → quantized thermal Hall conductance

$$K_H = 8\frac{\pi^2}{3}\frac{k_B^2}{h}T$$

- No fractional excitations (anyons) in bulk
- Like IQH state, but in bosonic system, *not* a SPT phase

To get a d=3 pgSPT state, make alternating-chirality stack of E_8 states →



Dimensional Reduction → Classification

Second: How to group *d*=2 states on mirror plane into equivalence classes of *d*=3 quantum phases?

- Naively, classification of d=2 phases directly gives a classification of d=3 pgSPT phases.

- In this case, classification of d=2 phases is $Z_2 \times Z$

  From Z_2 SPT

  From E_8 states

- But this is *not* the correct classification of d=3 phases, instead it collapses to a coarser classification

Dimensional Reduction → Classification

Second: How to group d=2 states on mirror plane into equivalence classes of d=3 quantum phases?

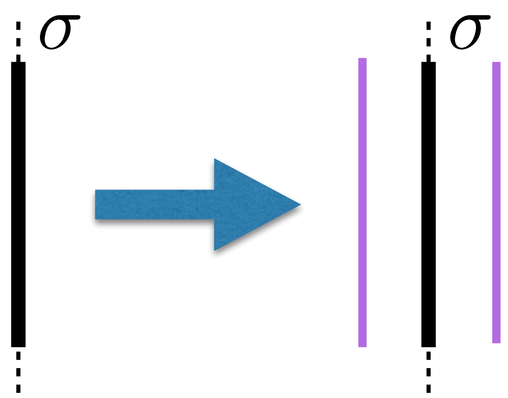
Three equivalence operations

A. Adiabatic continuity (preserving symmetry)

B. Stable equivalence (adding trivial degrees of freedom)

Operations for d=2 phases. Gives $Z_2 \times Z$

C. "Adjoining layers"



$$|\psi\rangle_{r_0} \to |L\rangle \otimes |\psi\rangle_{r_0} \otimes |R\rangle$$

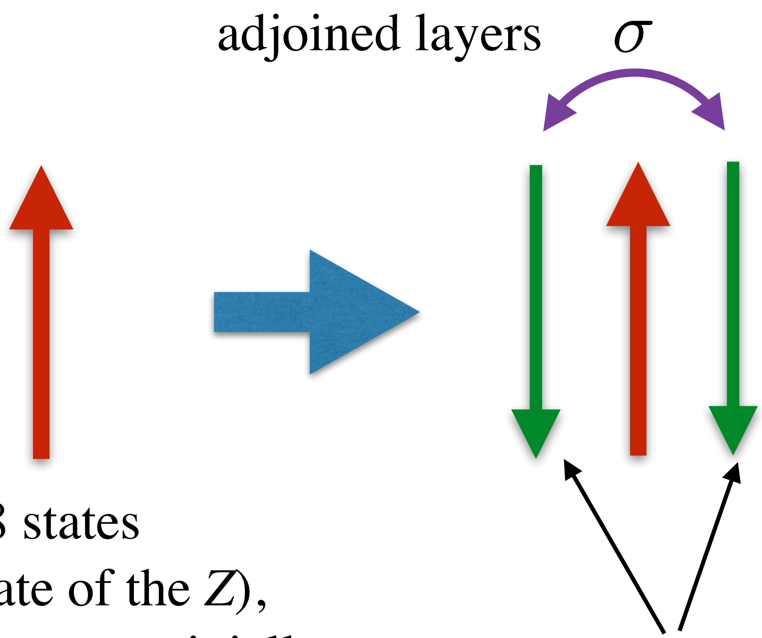
- Corresponds to making region surrounding the mirror plane wider
- Adjoined layers can be E_8 states

Collapse of *d*=2 classification

<u>Second</u>: How to group *d*=2 states on mirror plane into equivalence classes of *d*=3 quantum phases?

- *d*=2 classification is $Z_2 \times Z$
- Study effect of adjoining layers on two E8 states:

Reflection exchanges adjoined layers



Two E8 states (*n*=2 state of the *Z*), reflection acts trivially

Adjoined E8 layers of opposite chirality.

Resulting state is non-chiral, two possibilities:

1. Trivial → $Z_2 \times Z_2$
2. d=2 Z2 SPT state → Z_4

Can show it's trivial by analyzing edge theory

Classification collapses to $Z_2 \times Z_2$

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2. Electronic topological crystalline insulators with interactions
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Topological crystalline insulators (mirror reflection)

Theory: Teo, Fu & Kane; T. Hsieh, H. Lin, J. Liu, W. Duan, A. Bansil, L. Fu; …
Experiment: Tanaka, … , Y. Ando; P. Dziawa, …, T. Story;
S.-Y. Xu, …, M. Z. Hasan; …

- Consider electrons with U(1) charge conservation and mirror reflection $\sigma : (x,y,z) \to (-x,y,z)$
- SPT phases = topological crystalline insulators (TCIs)
- Non-interacting electrons: Z classification, "mirror Chern number"
- TCI predicted and observed in $Pb_{1-x}Sn_xTe$

Interacting electrons:

Z is reduced to Z_8. Isobe & Fu showed this by:

(1) Adding spatially varying Dirac mass terms to "dimensionally reduce" the surface theory to 1d lines
(2) Using bosonization to show n=8 surface can be gapped

Interacting TCIs

Q: Is the Z_8 classification complete?

A: Full classification is Z_8 x Z_2

Intrinsically strongly-interacting electron TCI

Electron TCI: non-interacting limit

First, reproduce non-interacting Z classification using dimensional reduction

1. What can go on mirror plane?

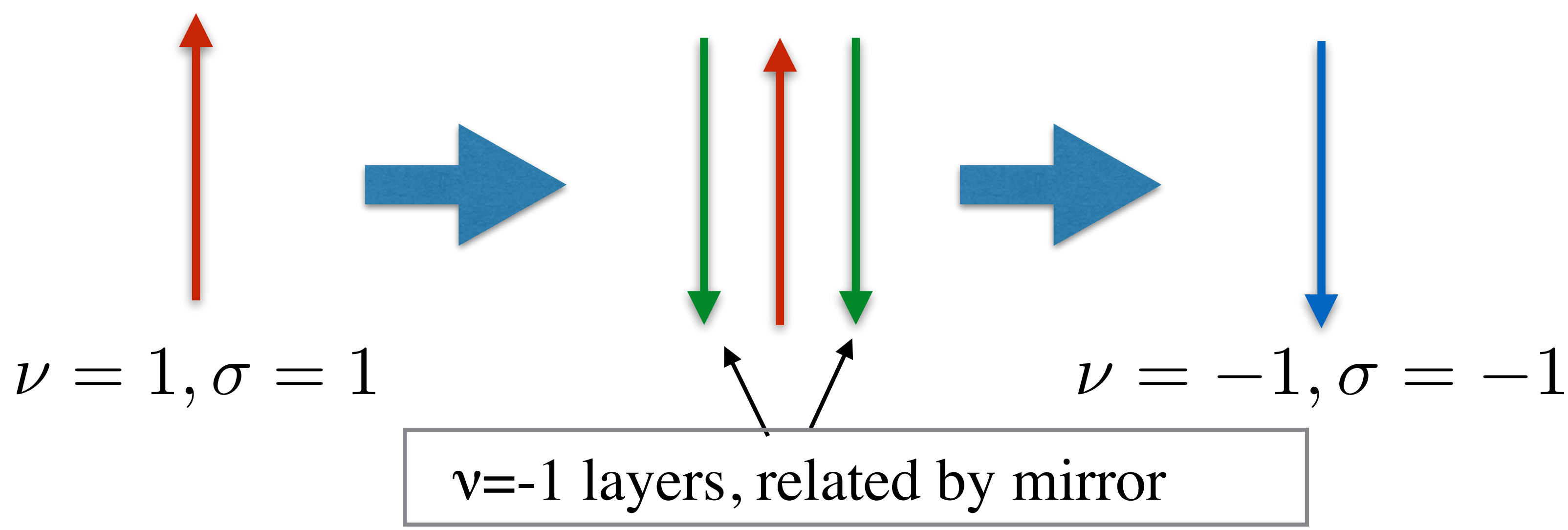
   A. Integer quantum Hall state (ν=1) with σ=+1

   B. Integer quantum Hall state (ν=1) with σ=-1

σ is the reflection eigenvalue of the fermion field

Naively gives $Z \times Z$ classification, which is too big.

2. Effect of adjoining layers



$Z \times Z$ collapses to correct Z classification

Electron TCI: interacting case

1. What can go on mirror plane?

IQH states with σ=+1

E_8 paramagnets:
Spin sector in E_8 state,
trivial action of reflection

d=2 classification is: $Z^{IQH} \times Z_4^{SPT} \times Z^{E_8}$

2d SPT phases
protected by U(1) × Z_2

Bilayer of opposite-chirality IQH states:

$\nu = n,$
$\sigma = +1$

$\nu = -n,$
$\sigma = -1$

n=4 state is trivial:
can be gapped out at edge (Isobe & Fu)

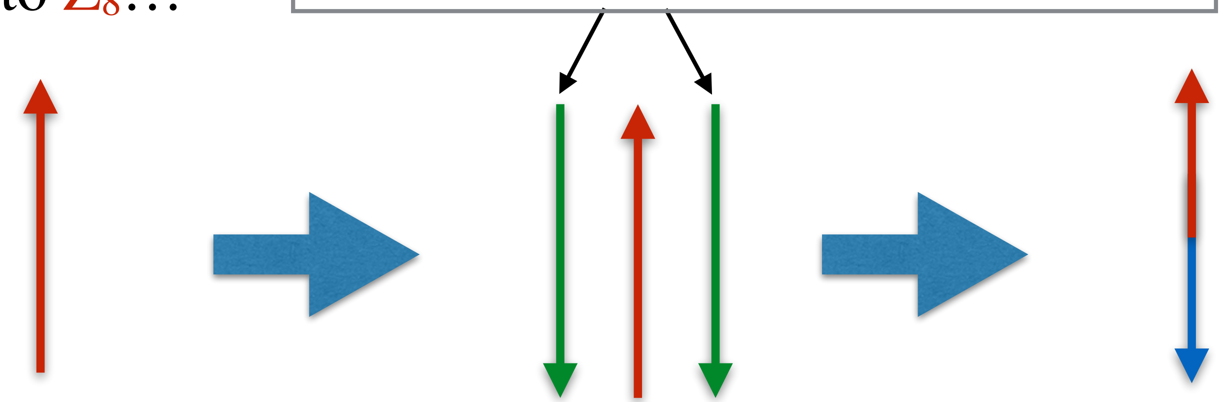
Electron TCI: interacting case

2. Collapse of d=2 classification (under adjoining layers operation)

d=2 classification is: $Z^{IQH} \times Z_4^{SPT} \times Z^{E_8}$ ⟶ Collapses to Z_2, as for bosonic mirror SPTs

Collapses to Z_8…

ν=-1 layers, related by mirror



Start with *n*=2 state of the Z^{IQH}

Get root state of the Z_4^{SPT}

Obtain $Z_8 \times Z_2$ classification

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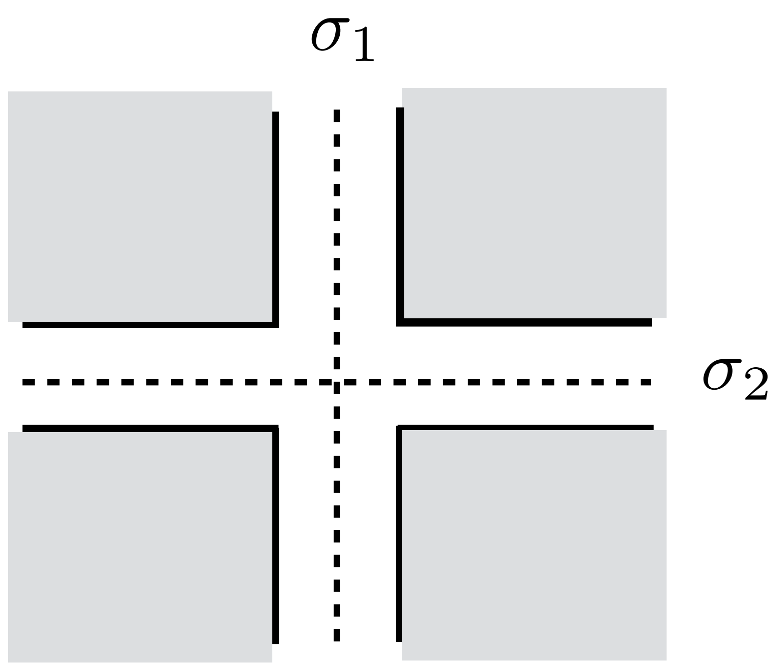
Point groups beyond mirror reflection

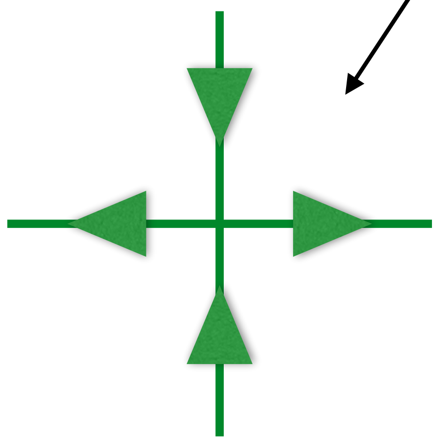
Example: bosonic system with C_{2v} symmetry in $d=3$

C_{2v} is generated by two perpendicular mirror planes



Root states → $(Z_2)^4$ classification

- $d=2$ Z_2 SPT phase on either mirror plane
- $d=1$ Haldane phase on $d=1$ axis
- $d=2$ E_8 states on mirror planes, with chiralities as shown



Reduce onto "cross-shaped region:"

Two planes with Z_2 internal symmetry

$d=1$ axis with $Z_2 \times Z_2$ symmetry

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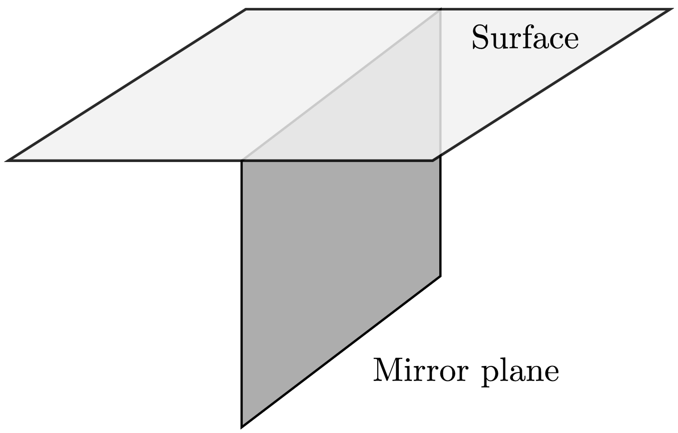
Surface properties

All the d=3 bosonic mirror SPT phases admit gapped, topologically ordered surfaces with anomalous implementations of the symmetry.



- Dimensional reduction shows surfaces can be studied in "T-junction" geometry.
- Anomaly of the 2+1 dimensional surface can be canceled by anomaly of a 2+1 dimensional bulk

see also recent work by Ethan Lake, arXiv:1608.02736

- Z_2 SPT root state: surface with toric code topological order, mirror squares to (-1) on both bosonic particles "ePmP"
- E_8 root state: surface with 3 fermion topological order, preserving reflection (impossible in strict d=2)

Reflection and time reversal

- Classifications for reflection and time-reversal are related:

d=3 bosonic system, reflection	d=3 bosonic system, time-reversal	$Z_2 \times Z_2$
d=3 fermions, $\sigma^2 = 1$	d=3 fermions, $T^2 = (-1)^F$	Z_{16}
d=3 fermions, $\sigma^2 = (-1)^F$	d=3 fermions, $T^2 = 1$	Trivial
d=3 fermions, U(1) x Reflection*	d=3 fermions, U(1) x Time reversal*	$Z_8 \times Z_2$

- Follows from assuming a Lorentz-invariant field theory description (see e.g. Witten arXiv:1508.0471)

* All fermions carry odd U(1) charge, bosons carry even U(1) charge

Summary & Outlook

Summary

- Point group SPT phases can be classified and studied by a dimensional reduction to lower-dimensional topological phases with internal symmetry
- All point group SPT phases can be constructed as stacks/arrays

Outlook

- Physical realizations, connections to other approaches, etc.
- Formal classification: what is the mathematical structure?
- Space group symmetry
- Dimensional reduction for point group symmetry enriched topological (SET) phases