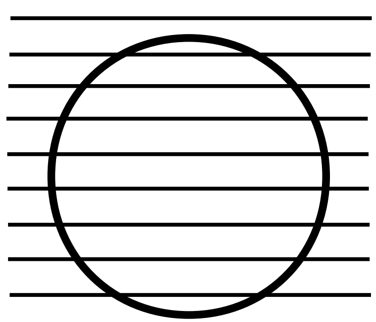
Numerical studies of Composite Fermion Liquid

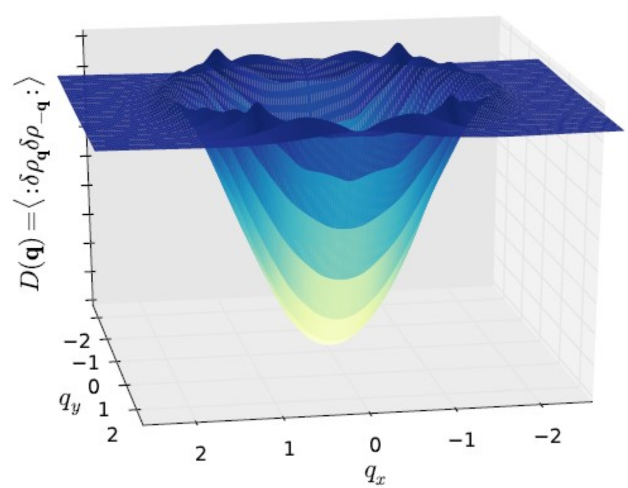
* Scott Geraedts, Michael Zaletel, Roger Mong, Max Metlitski, Ashvin Vishwanath, and OIM [Science 352, p.197 (2016)]





* Ryan Mishmash and OIM [PRB 94, 081110 (2016)]





Outline

* Review of Halperin-Lee-Read (HLR) theory (talks by Senthil, Haldane, Young, Metlitski, Wang) and relation to other gapless fractionalized phases

* Infinite-cylinder DMRG study of the half-filled lowest Landau level with Coulomb interactions – evidence for the Composite Fermion Liquid (CFL) state

* Particle-hole symmetry in the LLL and Son's proposal for PH-symmetric CFL with “Dirac composite fermions” (talks by Senthil, Seiberg, Haldane, Metlitski, Wang)

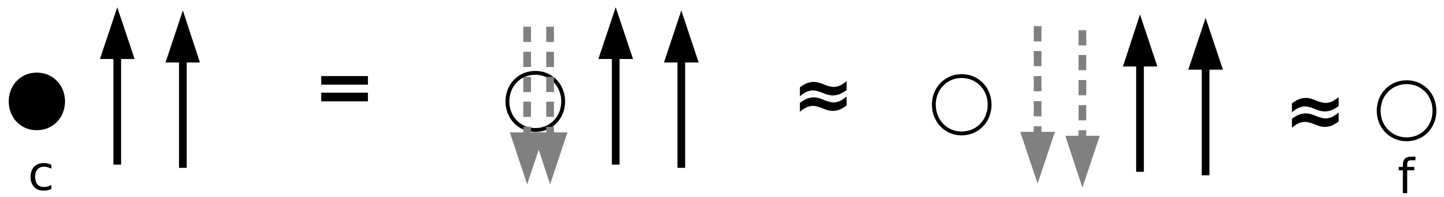
* Evidence for the Son's theory in the DMRG study

* VMC study of entanglement entropy in trial CFL wavefunctions

* Future directions

Halperin-Lee-Read (HLR) composite fermion liquid

2d electron gas in strong magnetic field at filling fraction $\nu=1/2$, i.e., two magnetic flux quanta per electron:



"Flux attachment" (Chern-Simons) transformation and "flux-smearing" mean field -> "composite fermions" see zero average field and form "Composite Fermion Liquid" (CFL). Schematic wavefunction:

$$\Psi_{\mathrm{el}}(z_1, \ldots, z_N) = \left[\prod_{i<j} (z_i - z_j)^2 \right] \Psi_{\mathrm{CF}}(z_1, \ldots, z_N)$$

filled Fermi sea of f fermions

Alternative parton description:

$c = d_1 \, d_2 \, f, \qquad \Psi_c = \Psi_{d1} \, \Psi_{d2} \, \Psi_f$

dividing electron charge:

$e = e/2 \; + e/2 \; + 0$

d_1 and d_2 see the external magnetic field, each at an effective filling fraction 1, while f's see no field and form a Fermi sea state

Beyond meanfield – Chern-Simons field theory

$$\mathcal{L} = \Psi^\dagger_{CF}\left(\partial_\tau - \mu - ia_0 - iA_0^{\text{ext}} - \frac{(\boldsymbol{\nabla} - i\boldsymbol{a} - i\boldsymbol{A}^{\text{ext}})^2}{2m}\right)\Psi_{CF}$$
$$+\frac{i}{4\pi}a_0(\boldsymbol{\nabla}\wedge\boldsymbol{a}) + V_{\text{Coul}}$$

HLR theory: RPA-like treatment & predictions for experiments

CFL is a non-Fermi-liquid (non-FL)!

* Obvious non-FL aspect: Electrons are gapped, but the state is still metallic

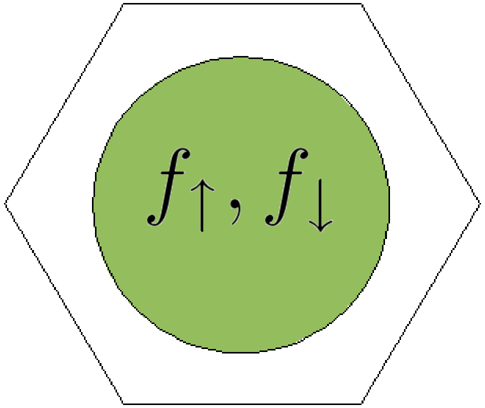
* More subtle non-FL aspect: Beyond mean field, there is also an emergent fluctuating gauge field, and the CFL does not have a quasiparticle description (unlike FL)

Similarity to other "non-FL states" ~ gapless fractionalized states

Many microscopic non-FL theories obtained via parton construction with fermionic partons forming some Fermi sea

Spinon Fermi sea spin liquid: $\vec{S} = f^\dagger \frac{\vec{\sigma}}{2} f, \quad f_\uparrow^\dagger f_\uparrow + f_\downarrow^\dagger f_\downarrow = 1$

$\Psi_{\rm spin} =$ PG (



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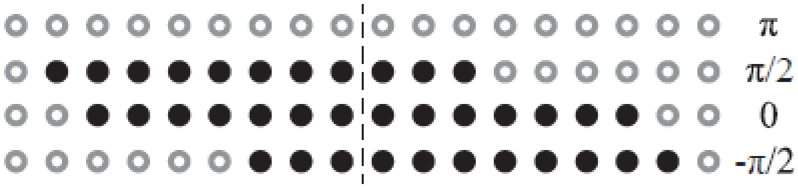
Candidate model: Heisenberg plus ring exchanges on a triangular lattice - relevant for organic spin liquids near metal-Mott insulator transition

The largest unbiased numerical study to date (Block, Sheng, OIM, & Fisher): DMRG on a 4-leg ladder, finding 3+3 slices through the spinon Fermi seas

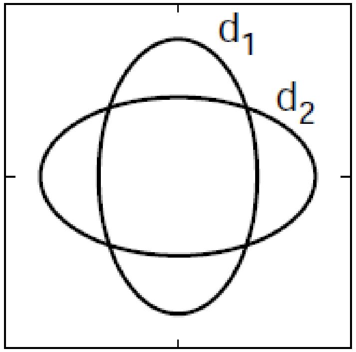
$f_\uparrow, f_\downarrow$:



More non-FLs and “slicing through the FSs” DMRG

Bose metal: $b^\dagger = d_1^\dagger d_2^\dagger, \qquad d_1^\dagger d_1 = d_2^\dagger d_2 = b^\dagger b$

$$\Psi_{\text{boson}} = \Psi_{d1}\,\Psi_{d2} = \det{}_1\,\det{}_2 = P_G\,($$



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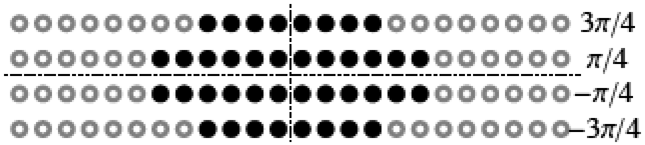
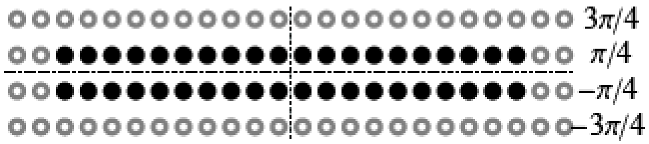
Candidate model: Bosons with hopping plus frustrating ring exchanges on a square lattice

The largest unbiased study to date (Mishmash, Block, Sheng, OIM, Fisher): DMRG on a 4-leg ladder; 4+2 slices through the parton Fermi seas:

d_1 :



d_2 :



Electron “d-wave metal”: $c_\sigma^\dagger = d_1^\dagger d_2^\dagger f_\sigma^\dagger$

$$\Psi_{\text{electron}} = \det{}_1\,\det{}_2\,\det{}_{\uparrow,\downarrow}$$

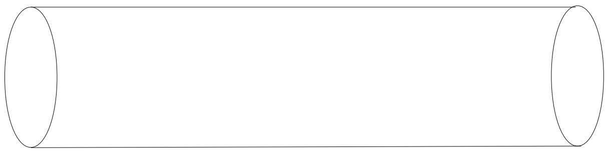
Candidate model: Electron t-J model plus electron ring exchanges on a square lattice

The largest unbiased study (Jiang, Block, Mishmash, Sheng, OIM, Fisher): DMRG on a 2-leg ladder; 2+1+1+1 slices through the parton Fermi seas

Numerical study of the half-filled Landau level

S. Geraedts, M. Zaletel, R. Mong, M. Metlitski, A. Vishwanath, & OIM

Electrons in continuum with Coulomb interactions, on a cylinder of infinite length and finite circumference L_y

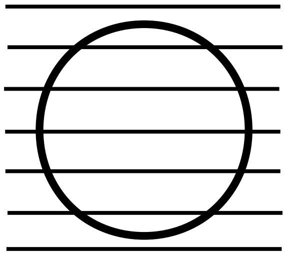


Solve the Coulomb interactions projected into the lowest Landau level (LLL) using infinite-cylinder DMRG (developed for FQH by M.Zaletel, R.Mong, F.Pollman)

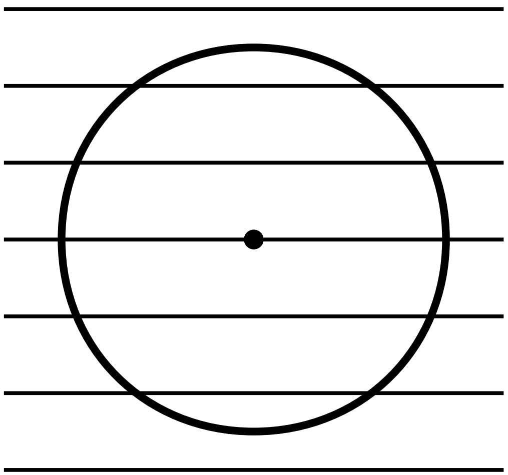
CFL state in 2d LLL: [magnetic length $\ell_B = \sqrt{\hbar c/(eB)}$]

$$\frac{\pi k_F^2}{(2\pi)^2} = \nu \frac{1}{2\pi \ell_B^2}; \qquad \nu = \frac{1}{2} \implies k_F = \frac{1}{\ell_B}$$

CFL state on a cylinder with finite L_y: slice through the CF Fermi sea with discrete k_y in steps of $2\pi/L_y$



Slicing through the CF Fermi sea



$k_F = 1$ in units where magnetic length is $l_B = 1$

$\mathbf{k_y=2\pi n_y/L_y}$ (periodic boundary conditions for f)

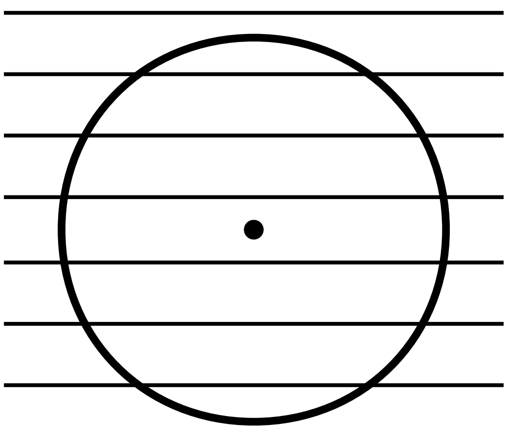
$0 < L_y < 2\pi$: 1 slice

$2\pi < L_y < 4\pi$: 3 slices

$4\pi < L_y < 6\pi$: 5 slices

$6\pi < L_y < 8\pi$: 7 slices

...



$\mathbf{k_y=2\pi(n_y+1/2)/L_y}$ (anti-periodic b.c. for f)

$\pi < L_y < 3\pi$: 2 slices

$3\pi < L_y < 5\pi$: 4 slices

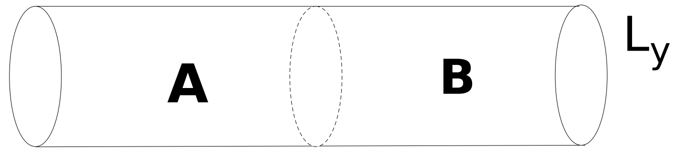
$5\pi < L_y < 7\pi$: 6 slices

$7\pi < L_y < 9\pi$: 8 slices

...

Electrons have periodic b.c. in all cases. The p.b.c./a.b.c. for f's can be accommodated by b.c. for one of the d-partons. Infinite-DMRG can access different such sectors!

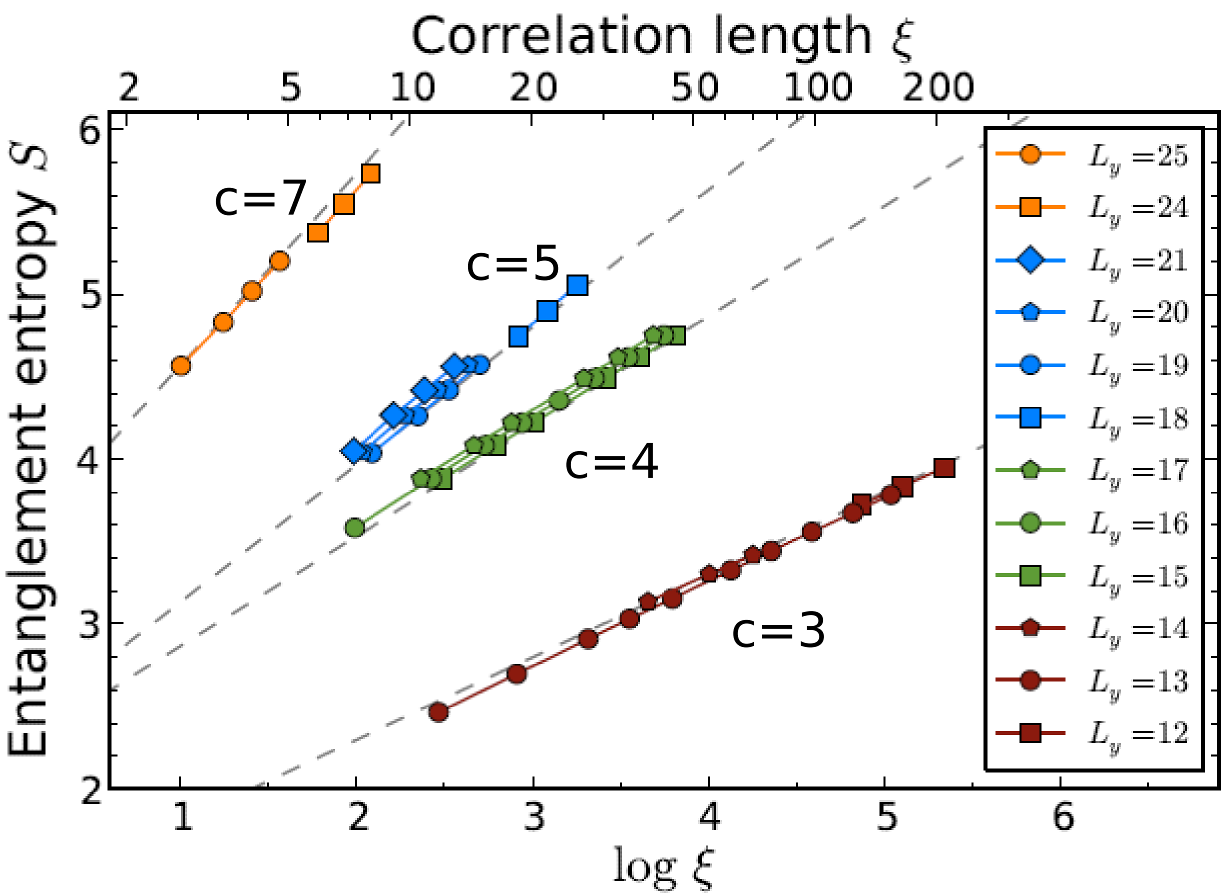
Slicing through the CF Fermi sea – entanglement entropy (EE) study



Infinite-cylinder DMRG: measure EE of half-cylinder with the other half.
"Finite-bond-dimension scaling" (developed by F.Pollman and J.Moore):

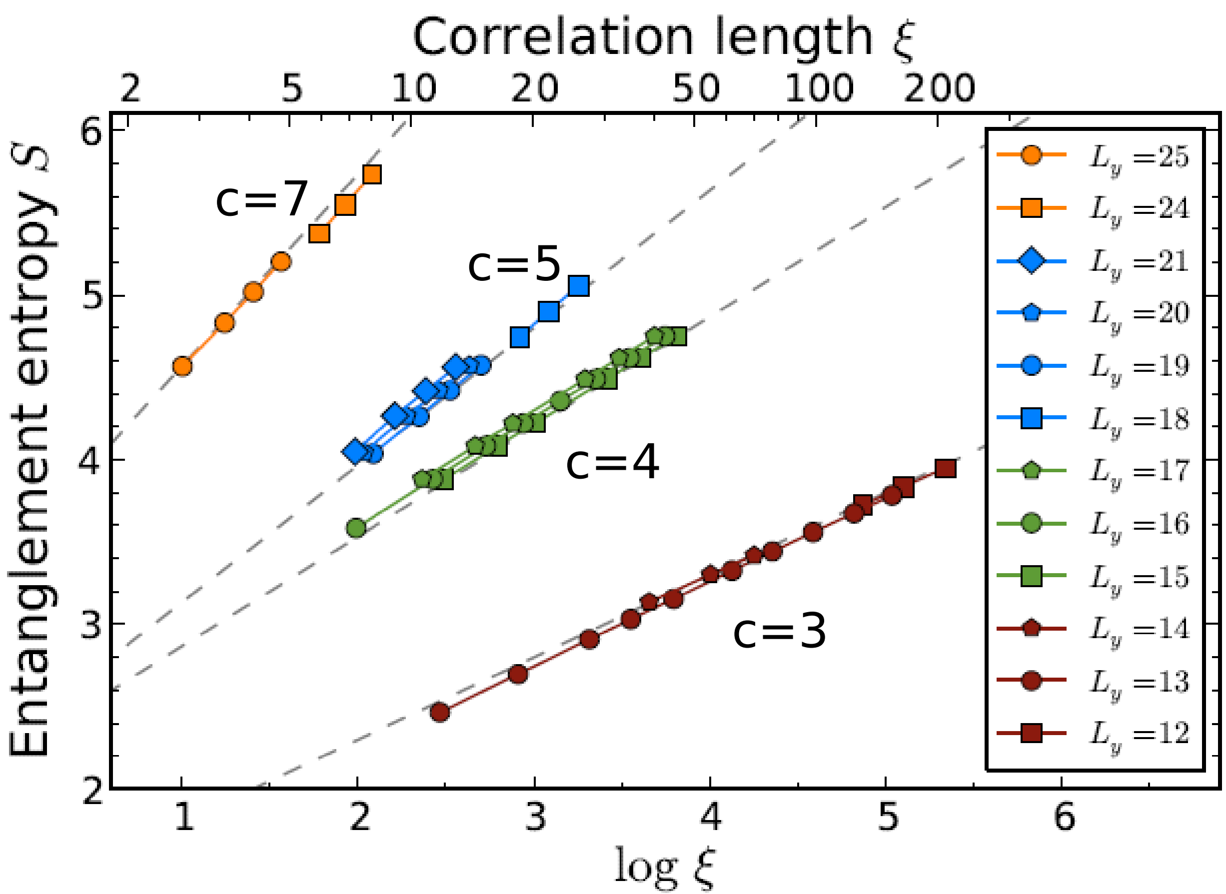
$$S = \frac{c}{6}\log(\xi)$$

--> can extract central charge c



Upon increasing L_y, see c = 3, 4, 5, 7

Slicing through the CF Fermi sea – entanglement entropy (EE) study



$\mathbf{k_y = 2\pi n_y / L_y}$

$0 < L_y < 2\pi$: 1 slice

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…

$\mathbf{k_y = 2\pi (n_y + 1/2) / L_y}$

$\pi < L_y < 3\pi$: 2 slices

$3\pi < L_y < 5\pi$: 4 slices

$5\pi < L_y < 7\pi$: 6 slices

$7\pi < L_y < 9\pi$: 8 slices

…

Quasi-1d gauge theory for the CFL: $c = N_{slices} - 1$

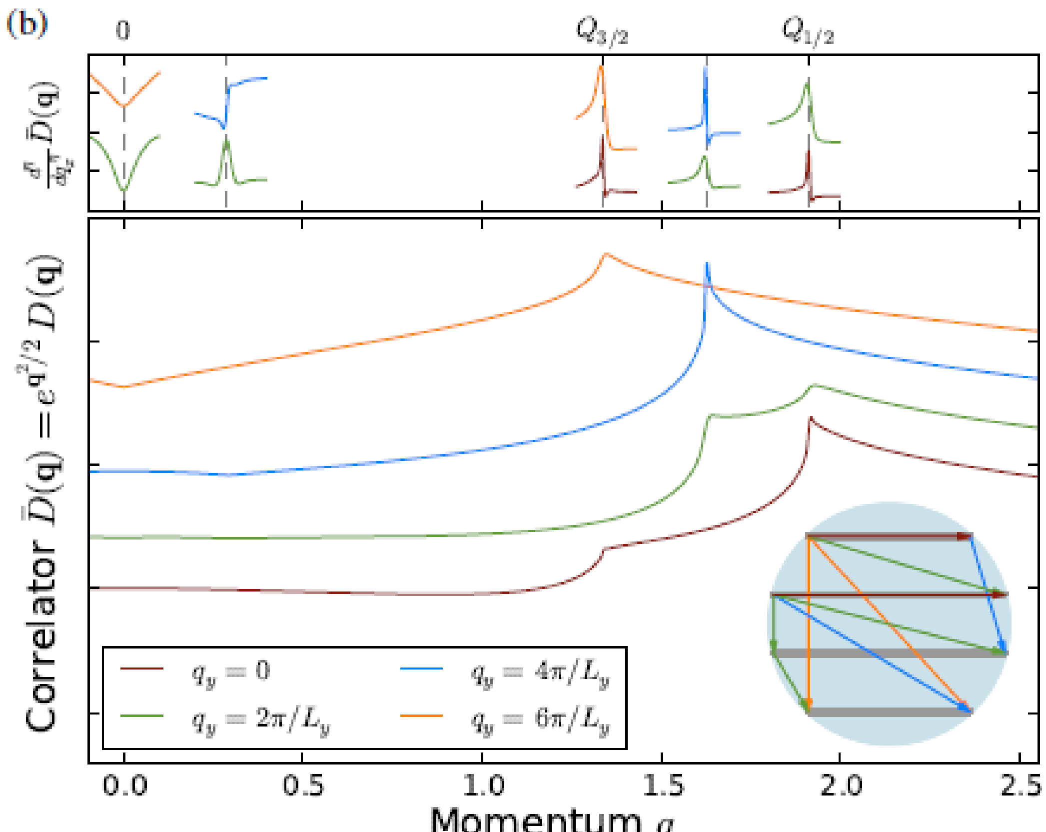
Upon increasing L_y, see N_{slices} = 4, 5, 6, 8!

Detailed characterization using electron density-density structure factor

On general symmetry grounds, electron density operator obtains contributions from CF bilinear (gauge-neutral) combinations:

$$\rho_{\rm el}(\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_j) \sim \psi^\dagger_{\rm CF}(\boldsymbol{k}_i)\psi_{\rm CF}(\boldsymbol{k}_j)$$

$L_y = 13\ l_B$: find 4 slices through the CF Fermi sea



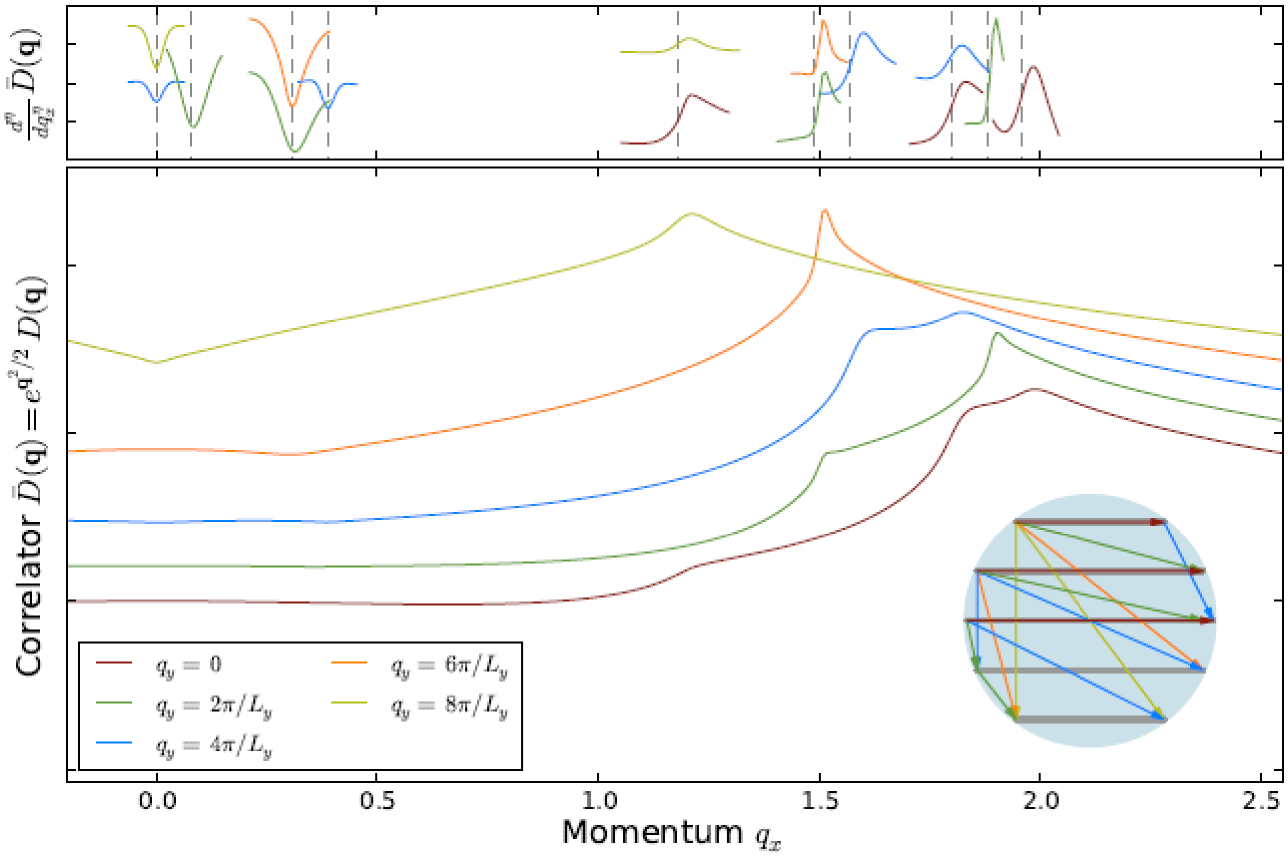
See all wavevectors expected from the bilinears!

Using derivatives of the structure factor we can detect also many higher-order "processes" including six-fermion terms!

Detailed characterization using electron density-density structure factor

$$\rho_{\rm el}(\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_j) \sim \psi^\dagger_{\rm CF}(\boldsymbol{k}_i)\psi_{\rm CF}(\boldsymbol{k}_j)$$

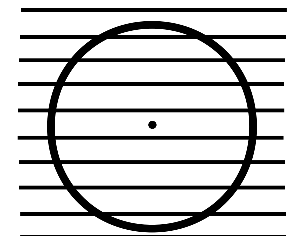
$L_y = 16\ l_B$: find 5 slices through the CF Fermi sea



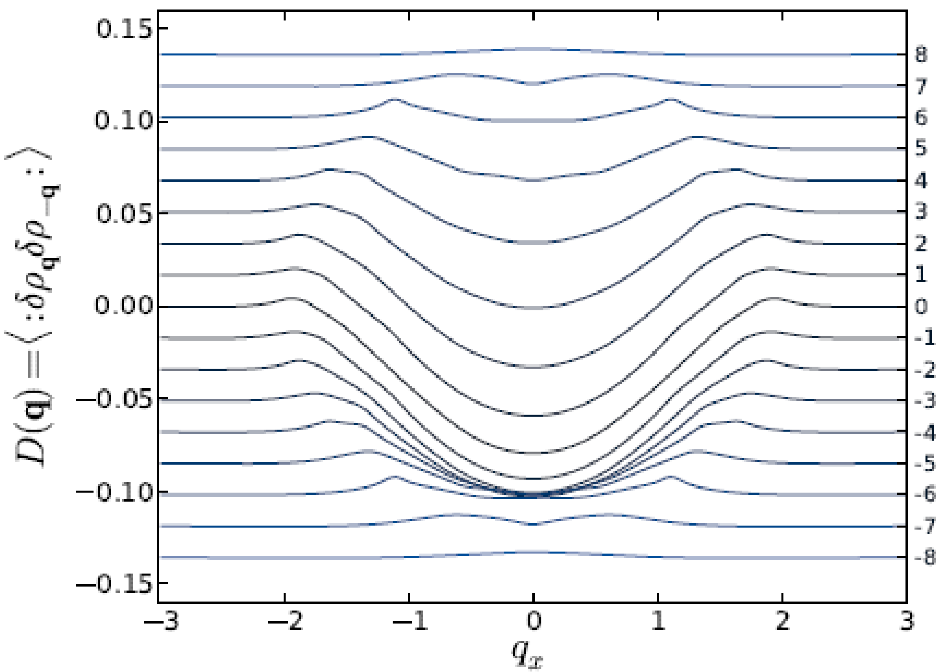
See all wavevectors expected from the bilinears!

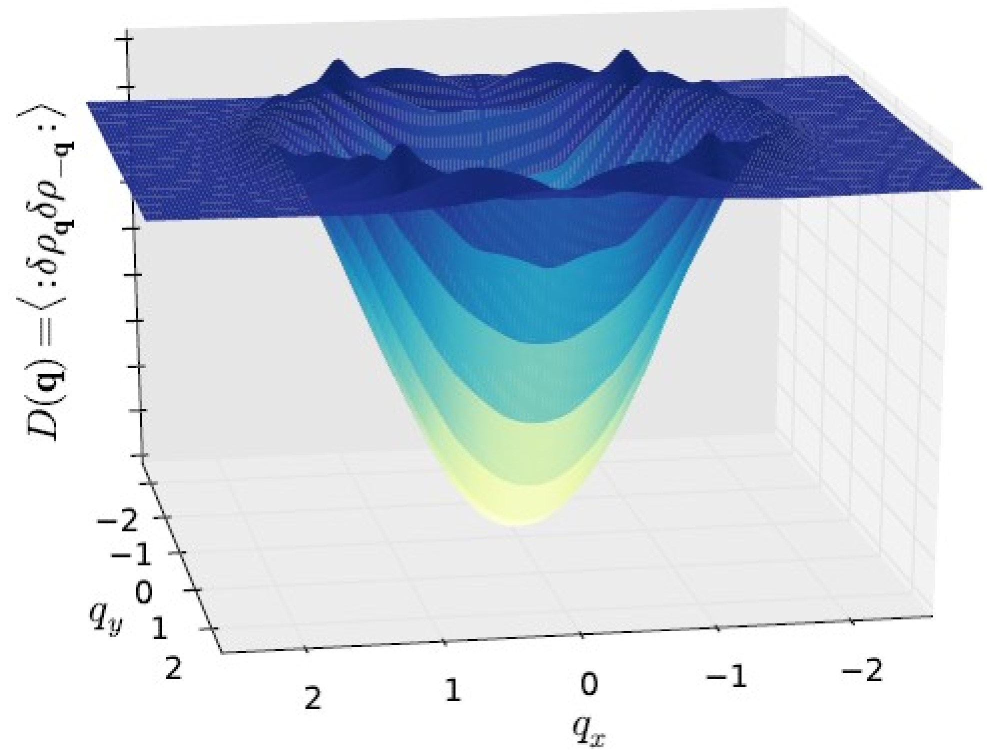
Wide cylinders approaching 2d

$L_y = 24\, l_B$: find 8 slices through the CF Fermi sea



Electron density-density structure factor:





Dominant feature --- "$2k_F$ circle" --- accumulation surface for low-energy CF particle-hole excitations

$D(q_x, q_y)$ already looks 2d-like (the closest approach to 2d for any non-FL state to date)

Particle-hole symmetry in the LLL

* LLL spanned by orbitals $\phi_j(r)$; electron $c(\boldsymbol{r}) = \sum_j \phi_j(\boldsymbol{r}) c_j$

Anti-unitary operation ("particle-hole transformation"):

$$c(\boldsymbol{r}) \to c^\dagger(\boldsymbol{r}), \quad i \to -i; \qquad \Longrightarrow \qquad c_j \to c_j^\dagger \text{ (in any orbital basis)}$$

--- can be symmetry at $\nu=1/2$ and is symmetry for any two-body interactions projected to LLL (most of ED studies of FQH!)

* HLR construction operates in the full electron Hilbert space and not just in the LLL and has no way to incorporate PH

* Trial wave functions motivated by the HLR theory are not PH-symmetric. For a long time, the small PH-breaking was not considered a serious issue with the HLR

* ED for small numbers of electrons in the putative HLR phase found PH-symmetric ground state (Rezayi and Haldane)

* Recent proposal that perhaps CFL breaks PH spontaneously, similarly to Moore-Read Pfaffian (Barkeshli, Mulligan, & Fisher)

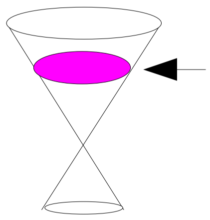
Son's proposal of PH-symmetric "Dirac CFL"

* Fermi surface of "composite fermions" which are not the same as the HLR CFs but have an underlying gapless Dirac character

* New CFs are coupled to a dynamical gauge field (similar to the HLR), but with no Chern-Simons term (different from the HLR)

* CFs do not carry electric charge; instead, the electric charge currents are encoded as fluxes of the gauge field:

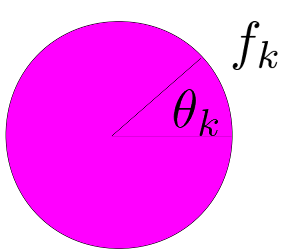
$$j_{\rm el} = \frac{\nabla \times a}{4\pi}$$



* PH acts as familiar time reversal on the Dirac CFs

$$\Psi_{\rm CF} \to i\sigma^y \Psi_{\rm CF}$$

(e.g., familiar from action of physical time reversal on a single Dirac fermion on the surface of a 3d topological insulator)



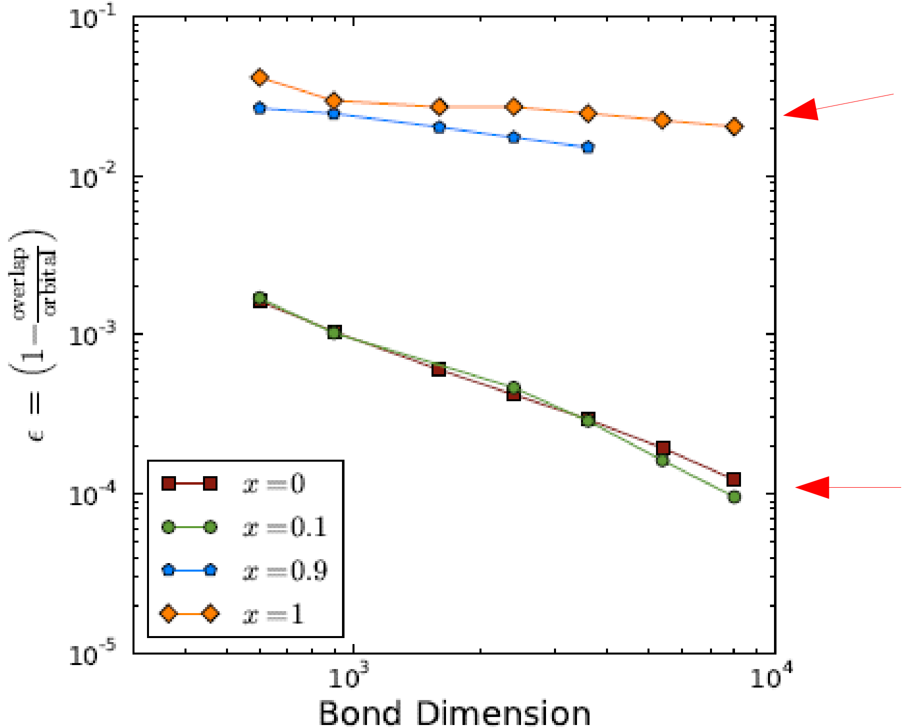
$$\text{PH}: \quad f_k \to e^{i\theta_k} f_{-k}$$

$A f_k^\dagger f_{-k} + \text{H.c.}$ – odd under PH!

DMRG study of PH in the half-filled LLL

* Checked absence of PH-breaking by studying appropriate “order parameters”

* Checked absence of PH-breaking by calculating overlap between the ground state and its PH-conjugate:
Considered a potential interpolating between the Coulomb interactions projected into the 0^{th} and 1^{st} Landau levels: $V = (1-x)\, V_0 + x\, V_1$



Moore-Read phase in the 1^{st} LL spontaneously breaks PH

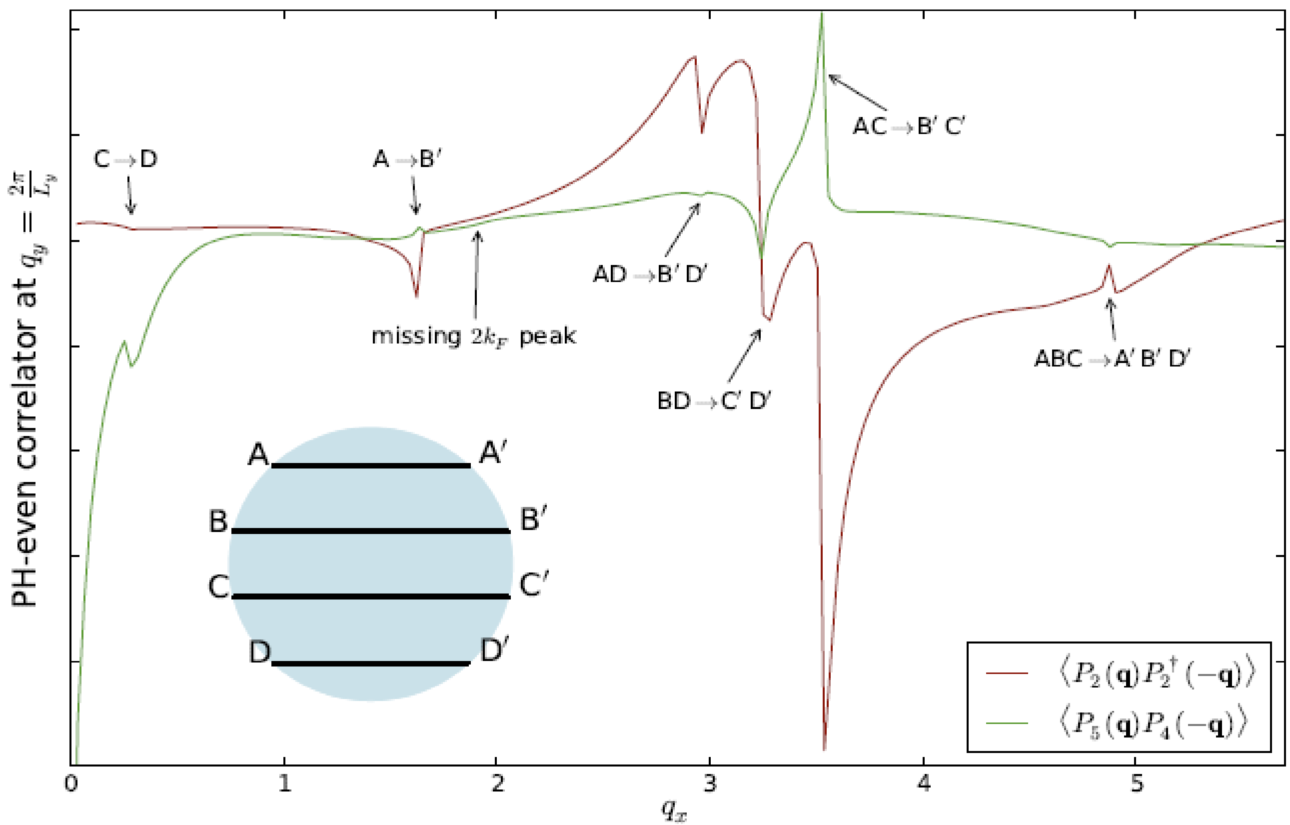
CFL phase in the 0^{th} LL does not break PH

DMRG test of the Son's theory: absence of backscattering from PH-symmetric impurities

Familiar property of the single Dirac cone on the surface of 3d TI: absence of back-scattering from non-magnetic (i.e. T-preserving) impurities

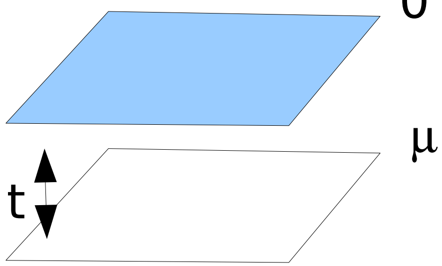
Analogous property in Son's theory: absence of backscattering from PH-preserving impurities.
Static (equal-time) analog: Correlation functions of PH-symmetric operators do not have $2k_F$ signatures corresponding to precise back-scattering:

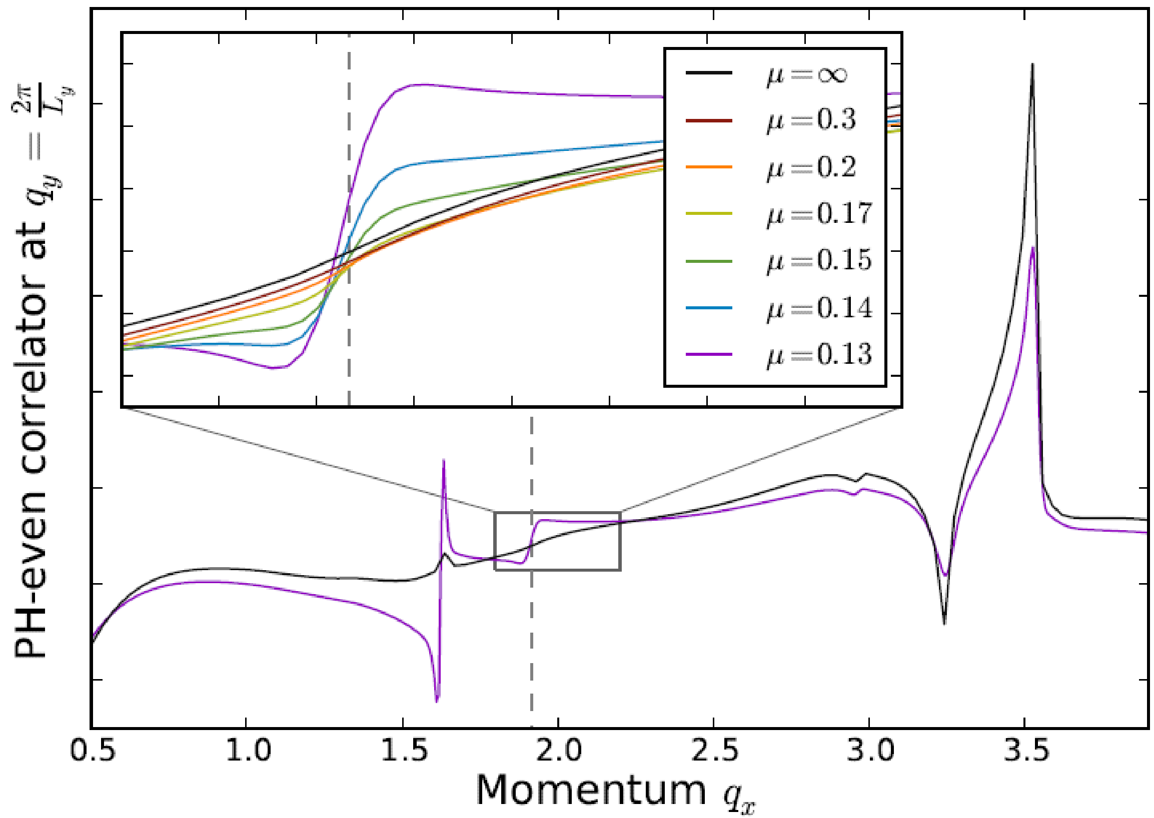
$L_y=13$



DMRG test of the Son's theory: recovery of backscattering upon removing PH-symmetry

Remove PH-symmetry by coupling to another 2DEG layer:





$2k_F$ signal is recovered!

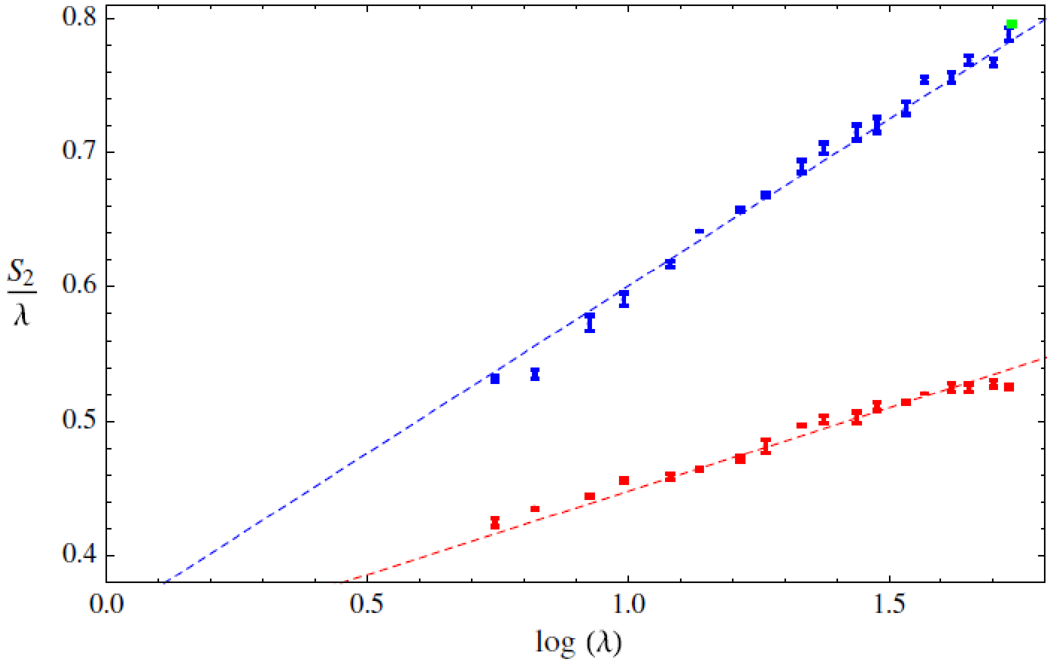
Entanglement Entropy in the CFL

DMRG study used c = N_{slices} – 1, i.e., essentially counting number of slices through the Fermi sea. Such counting is a key step behind Widom's formula for the multiplicative-log violation in EE for free fermions; in 2d:

$$S = \frac{L \log(L)}{24\pi} \int_{\text{boundary}} \int_{\text{Fermi surface}} dS_x dS_k |\hat{n}_x \cdot \hat{n}_k|$$

Senthil & Swingle proposed that EE for non-FLs is given by the same formula. However, recent numerical study by Shao, Kim, Haldane, & Rezayi found significantly larger EE in a trial wave function for the CFL:

N_{el} = 37

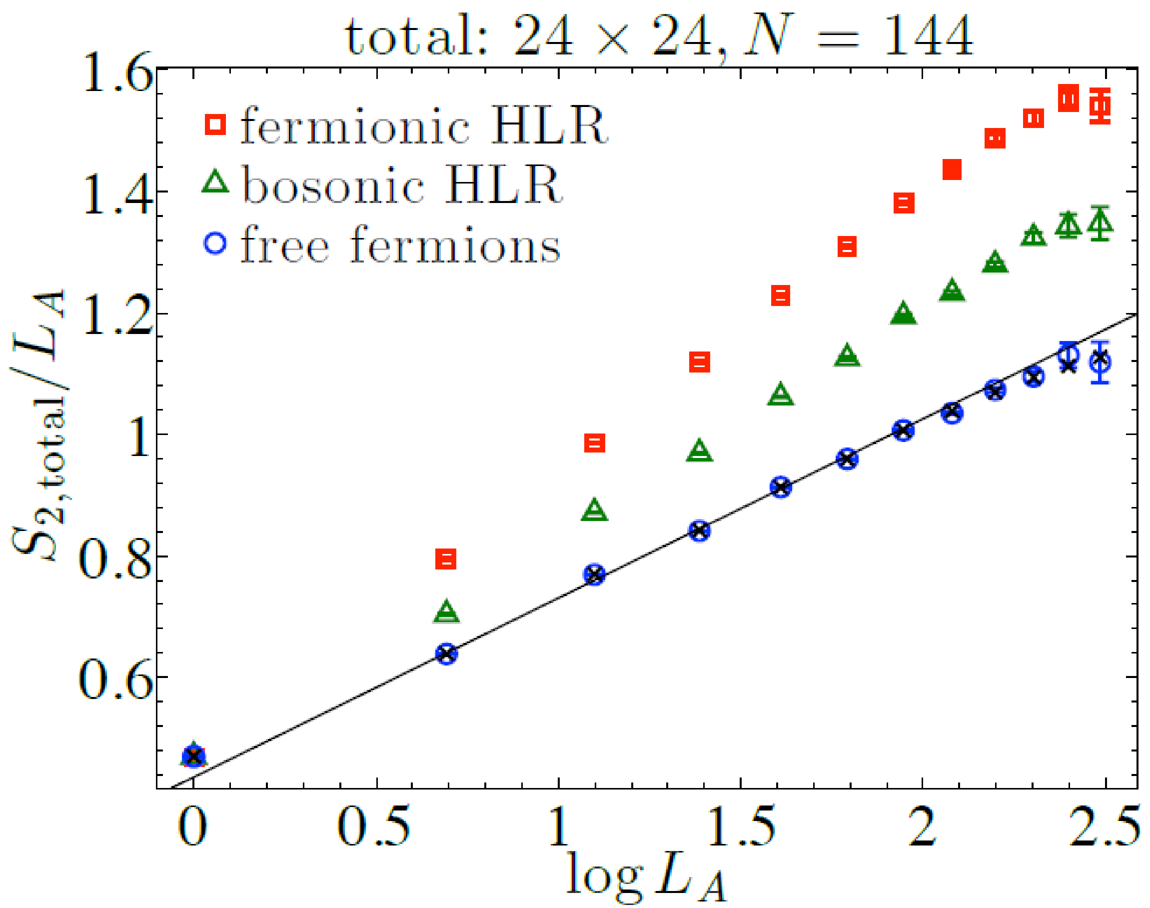


$\Psi_{\nu=1/2}$ electron CFL

free fermions

EE in the CFL – N_{el}=144 study on 24x24

VMC study (Ryan Mishmash & OIM, PRB 2016):
Lattice version of the CFL wavefunction (electrons on a triangular lattice at density 1/4, in a magnetic field corresponding to ν=1/2)



Naively, we see roughly similar increase in the prefactor for the electron HLR compared to free-ferms (and a smaller increase in EE for the boson HLR).

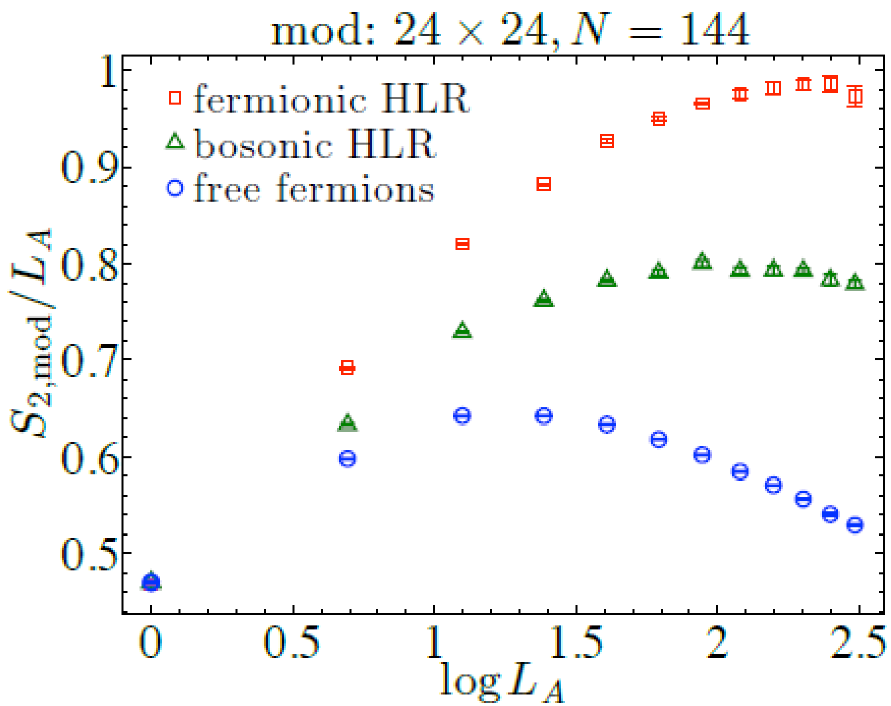
However, examination of contributing pieces suggests strong crossover at these length scales and that ultimately there is no such increase.

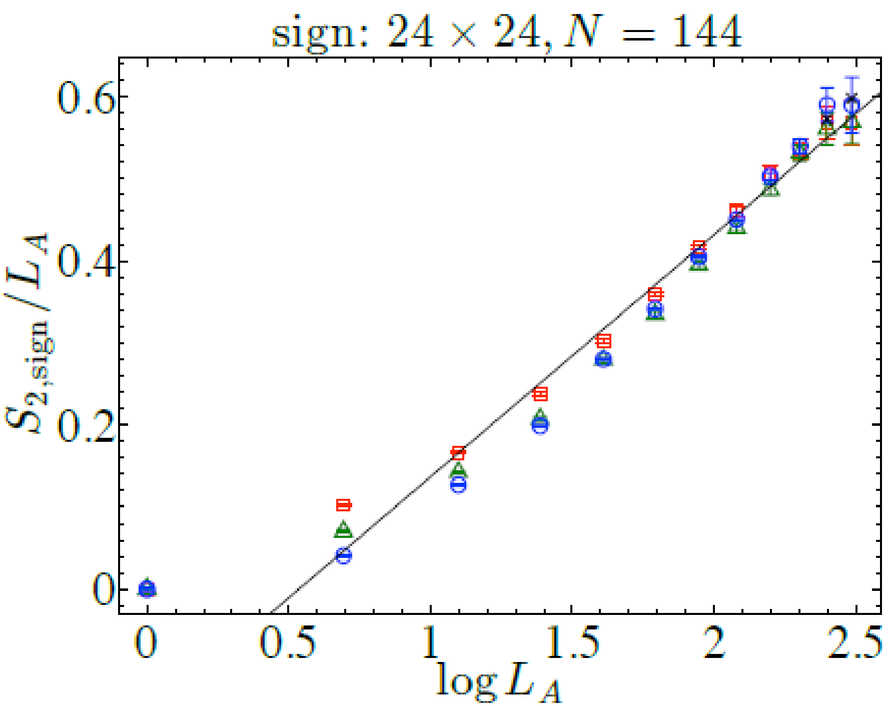
Pieces of the EE: “mod” and “sign”

Natural decomposition in VMC for the Renyi entropy (Zhang, Grover, Vishwanath):

$$S_{2,\mathrm{total}} = S_{2,\mathrm{mod}} + S_{2,\mathrm{sign}}$$

Renyi entropy for $|\Psi|$





We believe the curves saturate
-no log-violation of the area law for $|\Psi|$
(rigourours bound on EE for “Jastrow” $|\Psi|$)

$S_{2,\mathrm{sign}}$ behaves very similarly for the CFL states and free fermions; all log-violation comes from here!

Future directions

* Experimental signatures of Dirac CFs? Recent proposal by Potter, Serbyn, and Vishwanath to use Nernst measurements. Other probes?

* Construction of PH-symmetric trial CFL wavefunctions?

* Search for bosonic HLR at $\nu=1$ in bosonic FQH problems – not found so far

* Bosonic HLR with PH-symmetry – not natural in FQH contexts (no PH even when projected into the LLL), but can be realized/interesting as a surface state of 3d bosonic TIs (Senthil & Wang; Xu & You; Mross, Alicea & OIM)

* Detailed study of non-FL properties of the CFLs ($2k_F$ singularity in 2d)

* Application of infinite-cylinder DMRG to other non-Fermi-liquid problems (gapless spin liquids, Bose-metals, non-FL electronic metals)

* Application of fermionic dualities to other non-FLs

THANK YOU!

Relation among non-FL states;
"Slicing through the Fermi surface" DMRG studies

Beyond mean field – all these examples (including also CFL) lead to parton-gauge-type theories with Fermi surfaces of partons coupled to a dynamical gauge field. Partons are very strongly scattered by the gauge field fluctuations and are not true "quasiparticles" - non-FL aspect (the CS term in the CFL case is not so important for the non-FL aspects)
Status of such field theories in 2d is still not fully resolved – do they give stable phases? (S.S.Lee, M.Metlitski, S.Sachdev, D.Mross, T.Senthil)

Unbiased numerical studies (D.N.Sheng, M.P.A.Fisher, M.Block, R.Mishmash, R.Kaul, OIM): Idea of using DMRG to study N-leg ladders slicing through the gapless surfaces. Successful with the Spinon Fermi sea and Bose-metal states for up to 4-leg ladders (but really pushing it/close to being inconclusive), which are still very far from 2d, with many "quasi-1d" details still in play.

Thanks to recent developments in DMRG for Fractional Quantum Hall (FQH) problems (M.Zaletel, R.Mong, F.Pollman, S.Geraedts) -> Composite Fermion Liquid can be reliably studied on effectively much wider systems (so far up to 8 slices through the Fermi sea), which is much closer to 2d --- ideal setting for exploring such non-FL phases!