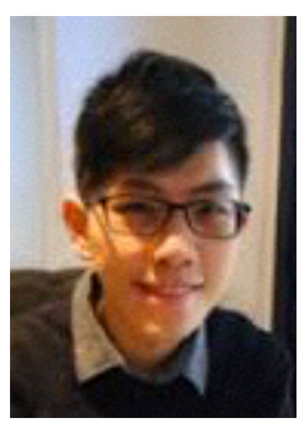
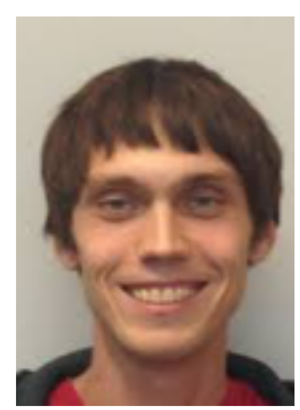
Dynamic topological orders in periodically driven systems

Andrew C. Potter





Adrian Po (Berkeley)
Lukasz Fidkowski (Stony Brook)
Takahiro Morimoto (Berkeley)
Ashvin Vishwanath (Harvard)

TEXAS
The University of Texas at Austin

Plan

New non-equilibrium dynamical phases from periodic driving

$$H(t+T) = H(t)$$

Floquet symmetry protected topological phases (SPTs)

- aka driven + interacting topological insulators

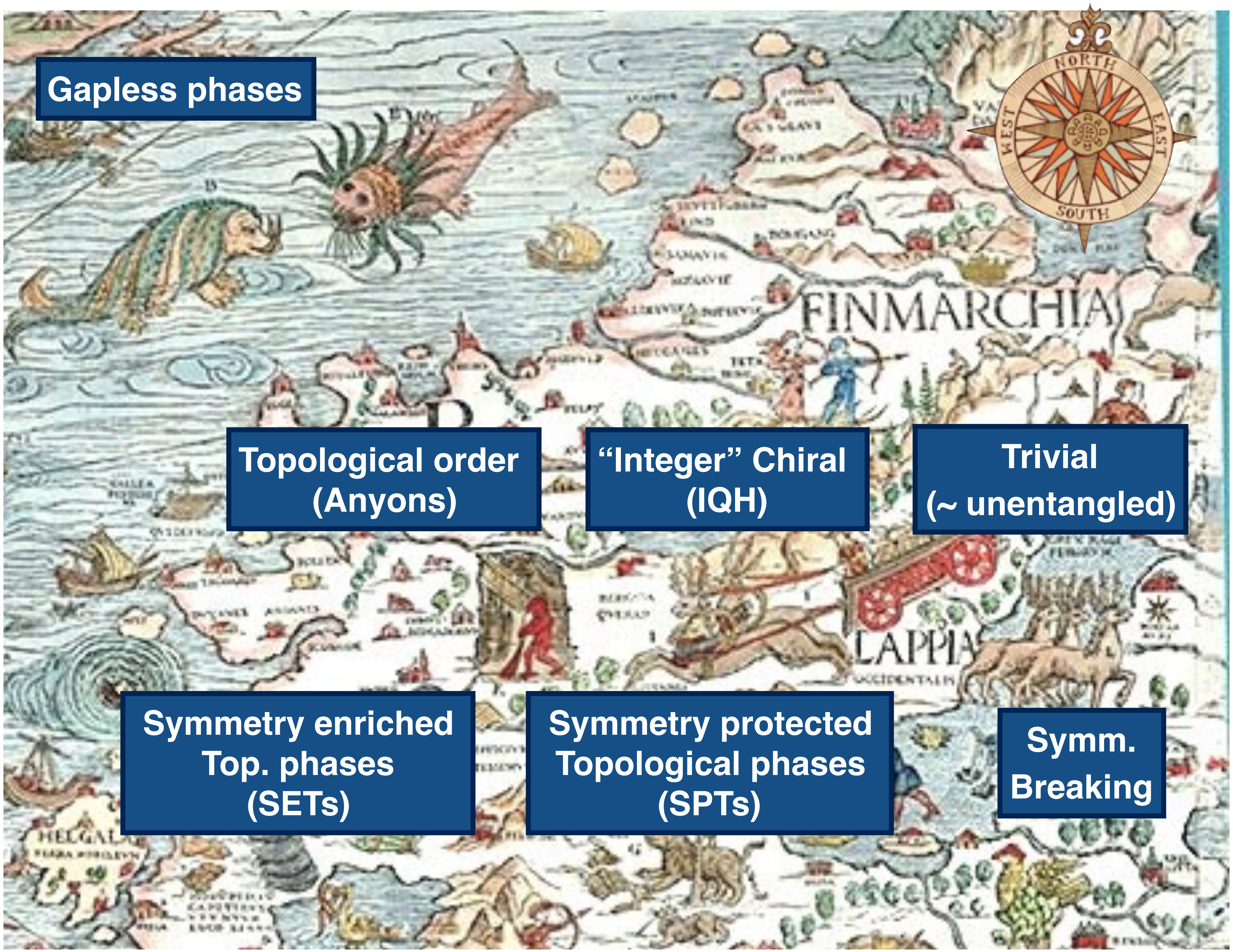
1. *ACP, T. Morimoto, A Vishwanath Phys. Rev. X* ***6****, 041001 (2016)*
2. *ACP. T Morimoto, arXiv:1610.03485*

Chiral Floquet phases

- Edge: one-way pumping of quantum information

3. Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

“World Map” of quantum matter (Equilibrium, T=0)



Symmetry required

(Ground-state) Entanglement

Isolated ("closed") quantum many-body systems

Ultracold atoms

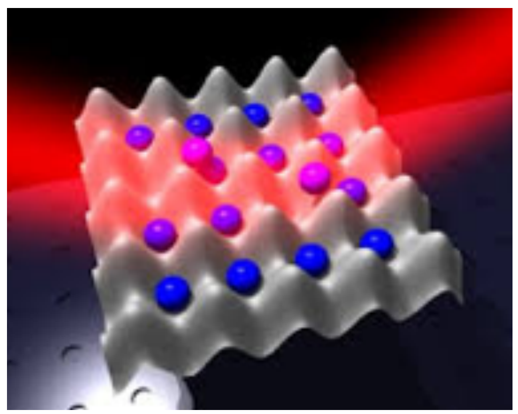


image: http://www.lens.unifi.it

Trapped Ions

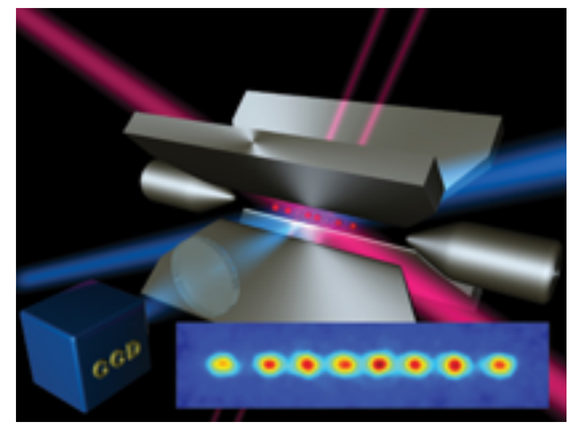


image: www.laserfocusworld.com

NV Centers

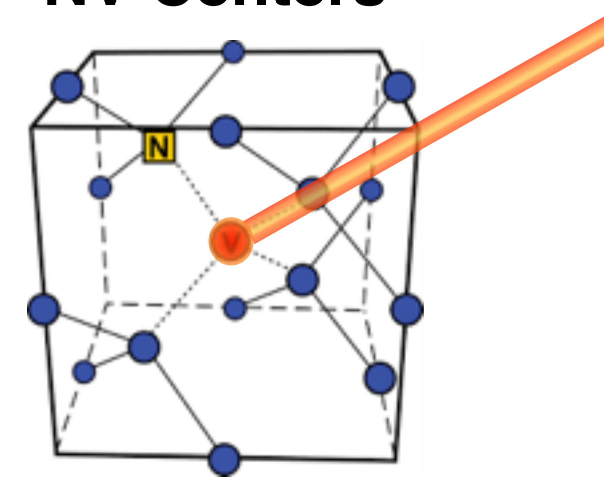
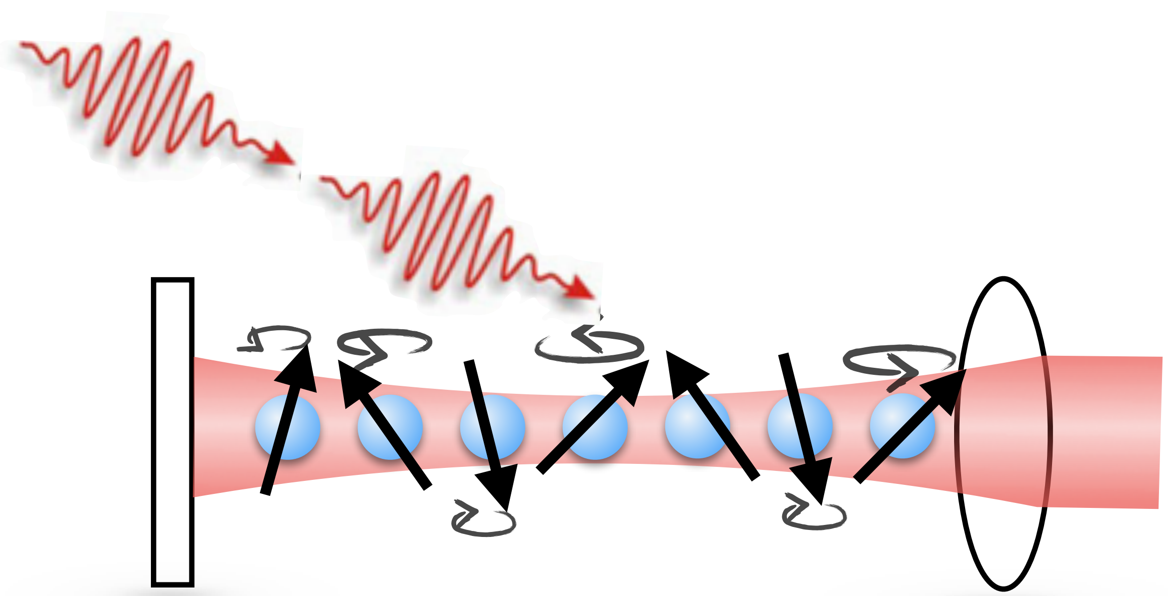


image: www.labnews.co.uk



$$|\Psi(0)\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_L\rangle = \sum_n c_n |n\rangle$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

$$H(t+T) = H(t)$$

New universal quantum phenomena in dynamics?

Periodic Driving 1: Engineer new interactions

$$H(t+T) = H(t) \quad \Rightarrow \quad H_{\text{eff}} \approx H_0 + \frac{iT}{2}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H(t_2)] + \ldots$$

$$U(T) = e^{-iH_{\text{eff}}T}$$

Lindner, Refael, Galitski Nat. Phys. '11
Many others…

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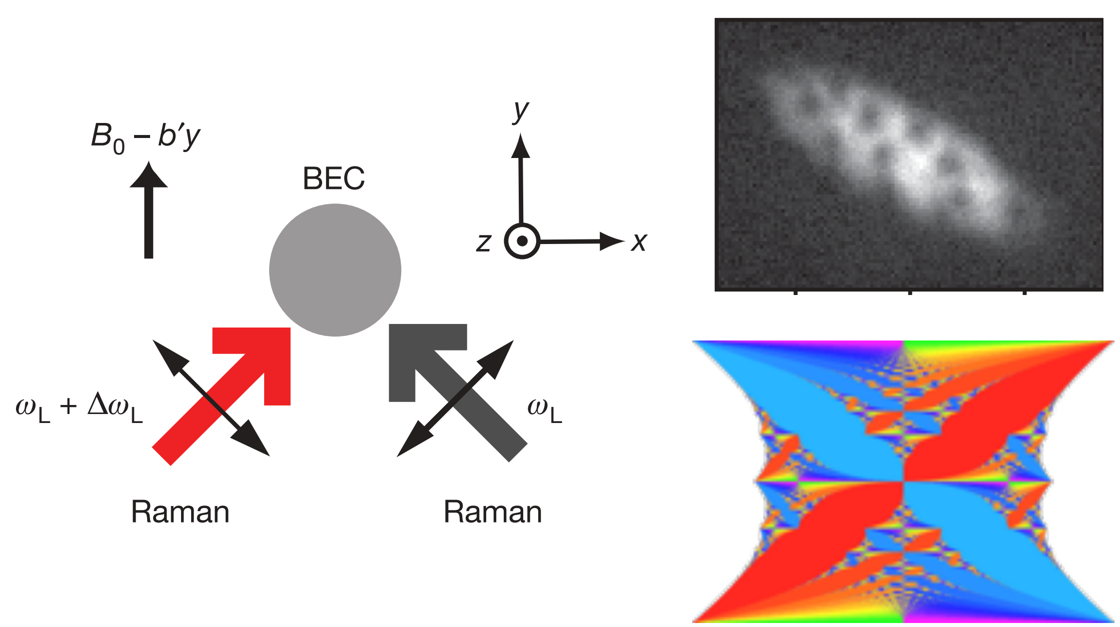
Cold atoms



Synthetic Gauge Fields

Lin et al. (Speilman group) Nature '09;
Aidelsburger et al. (Bloch group) PRL '11

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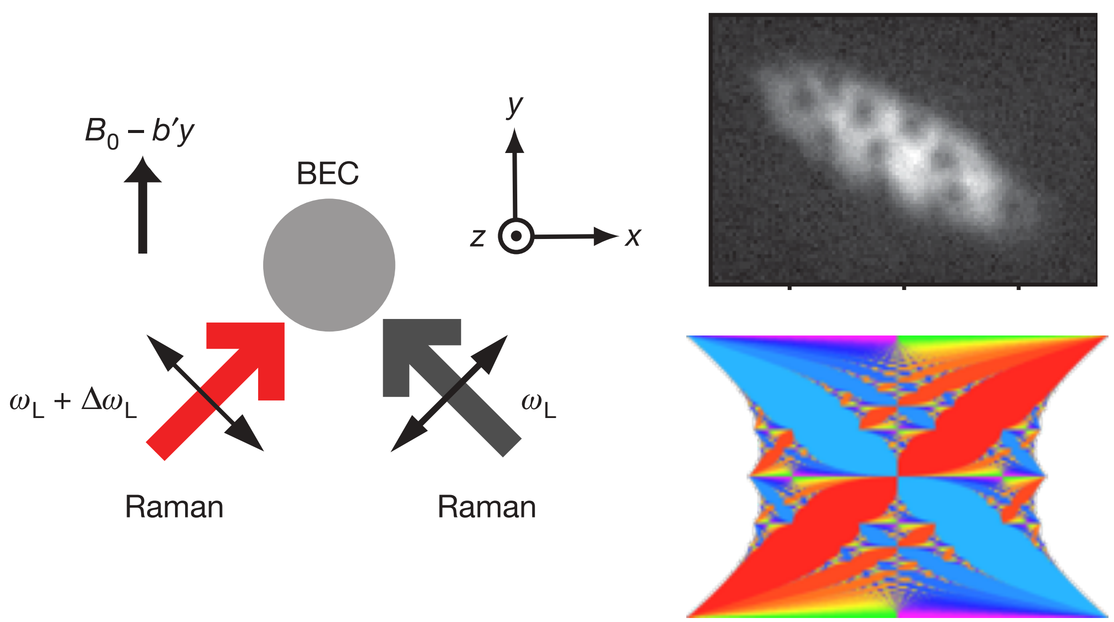

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Solids

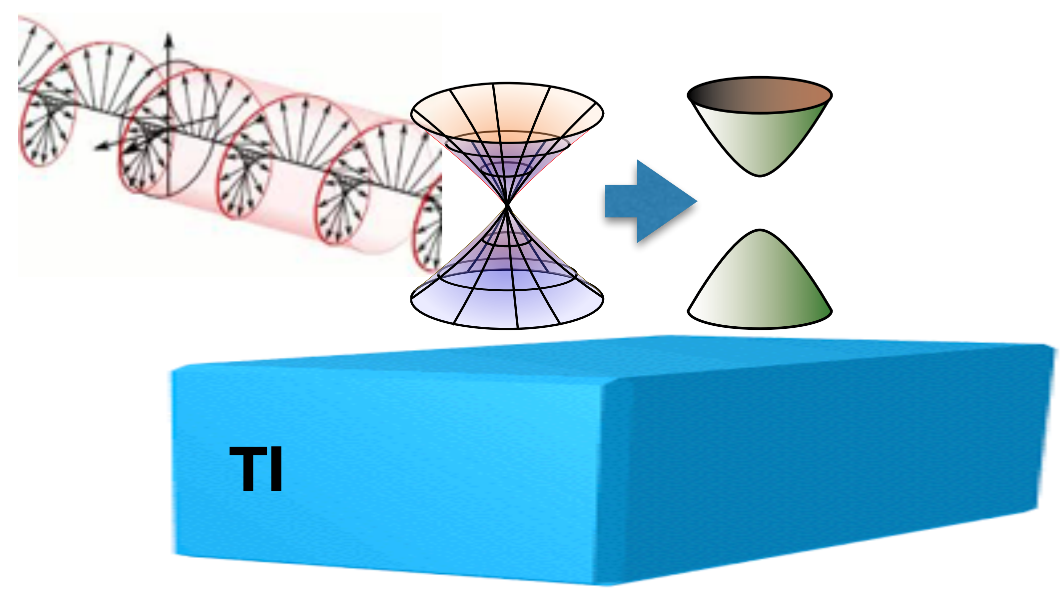
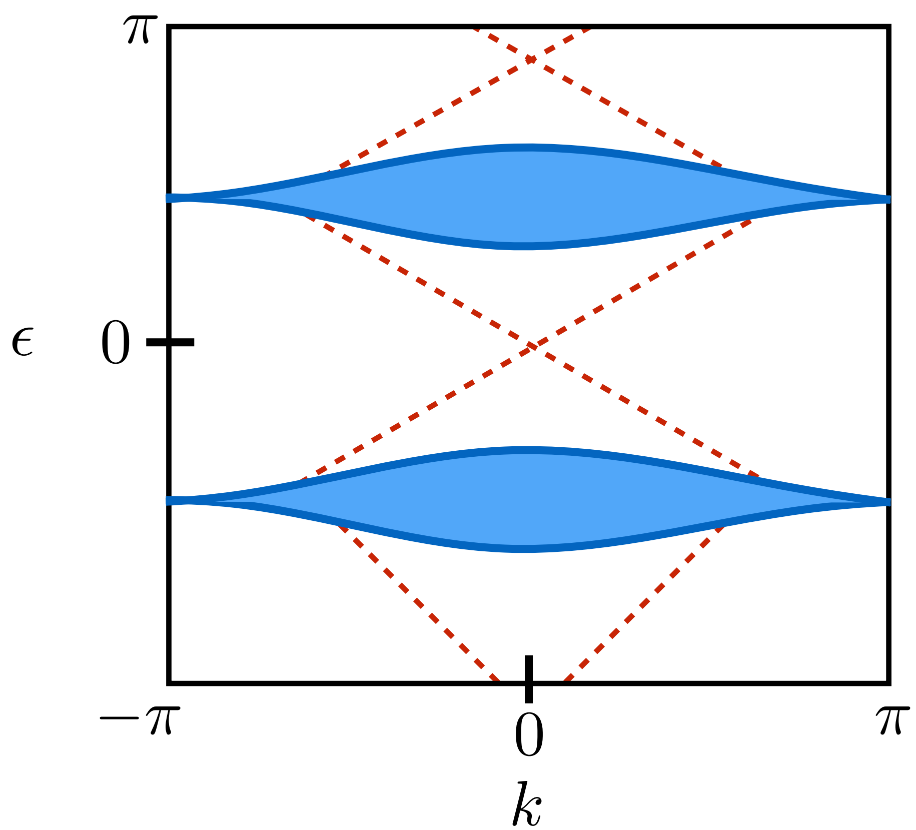


Photo-induced Anomalous Quantum Hall
state in TI surface

Wang et al. (Gedik Group) Science '13

Periodic Driving 2: Fundamentally new dynamical phases



New types of band topology due to periodicity of quasi-energy

$$U(T) = \mathcal{T}\{e^{-\int_0^T dt H(t)}\} \qquad \epsilon_n \simeq \epsilon_n + \frac{2\pi}{T}\mathbb{Z}$$

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Kitagawa, Berg, Rudner, Demler PRB '10; Jiang et al. PRL '11; many others…

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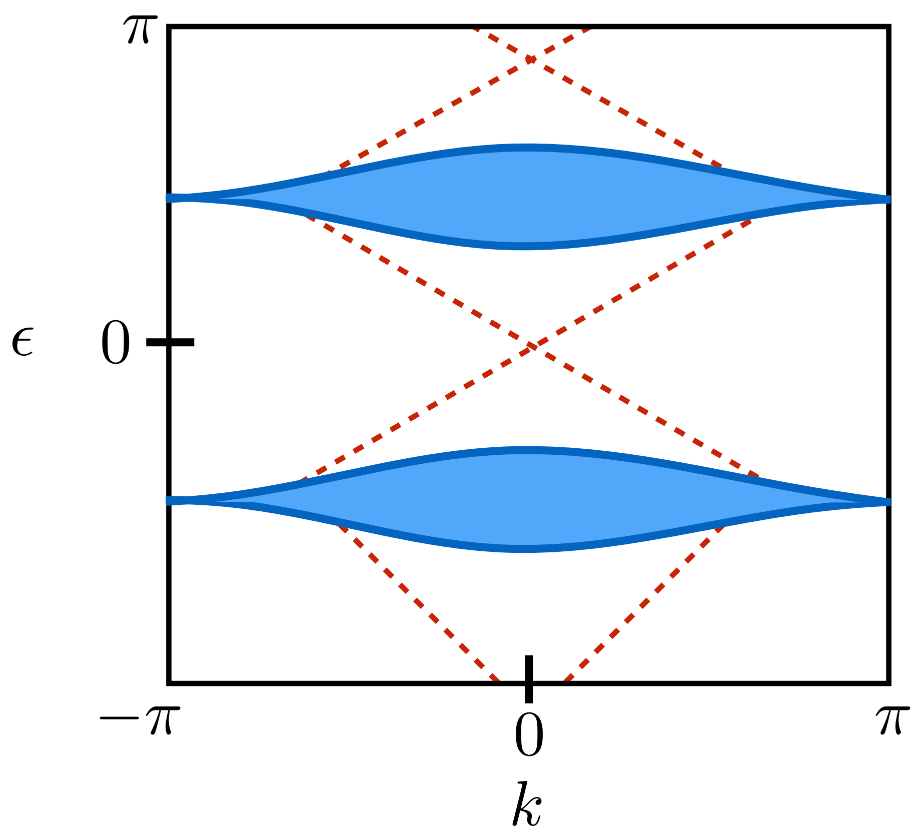

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(all systems are interacting, not perfectly clean, etc…)

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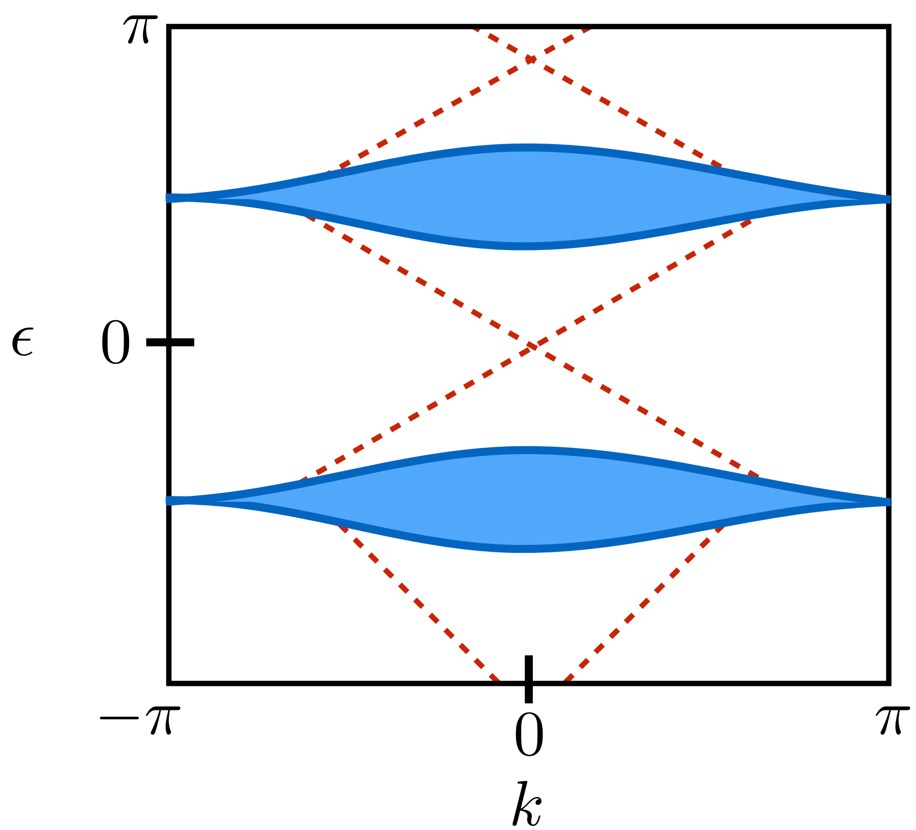

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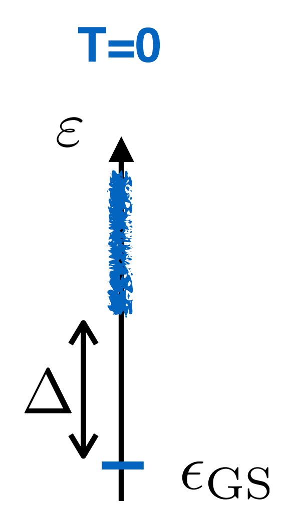
Usual (equilibrium, T -> 0):

- ground-state + gap => physics dominated by filled bands
- weak interactions/disorder don't close the gap (band topology inherited by many-body system)



Many-body spectrum

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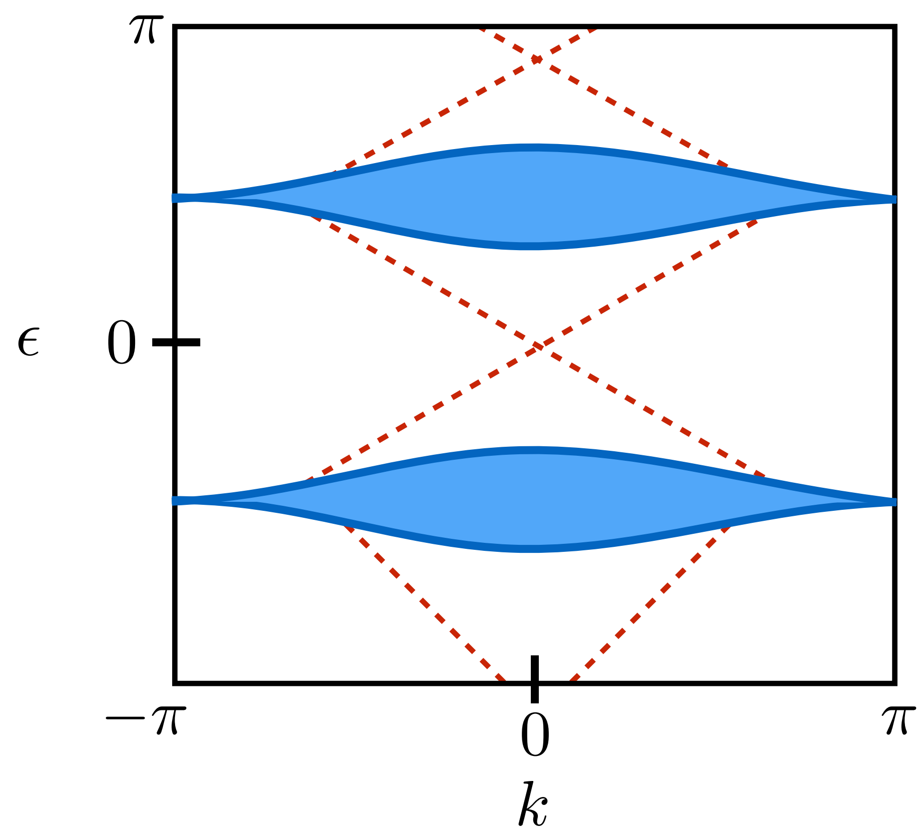

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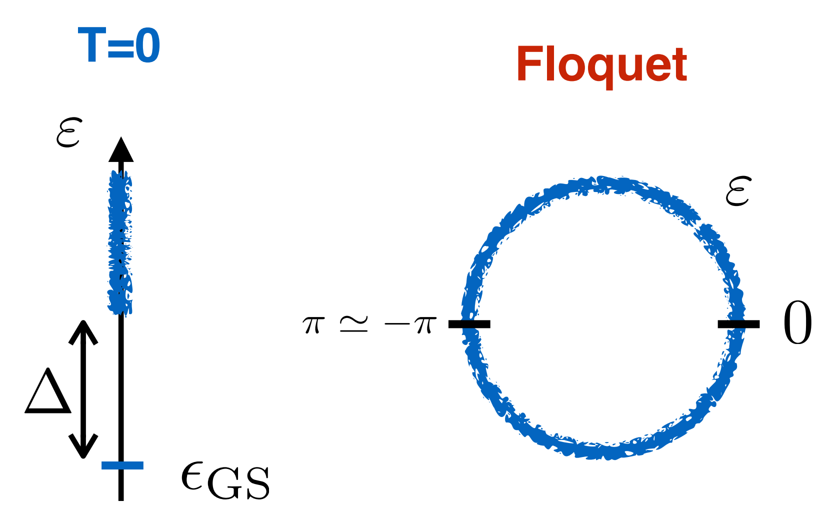
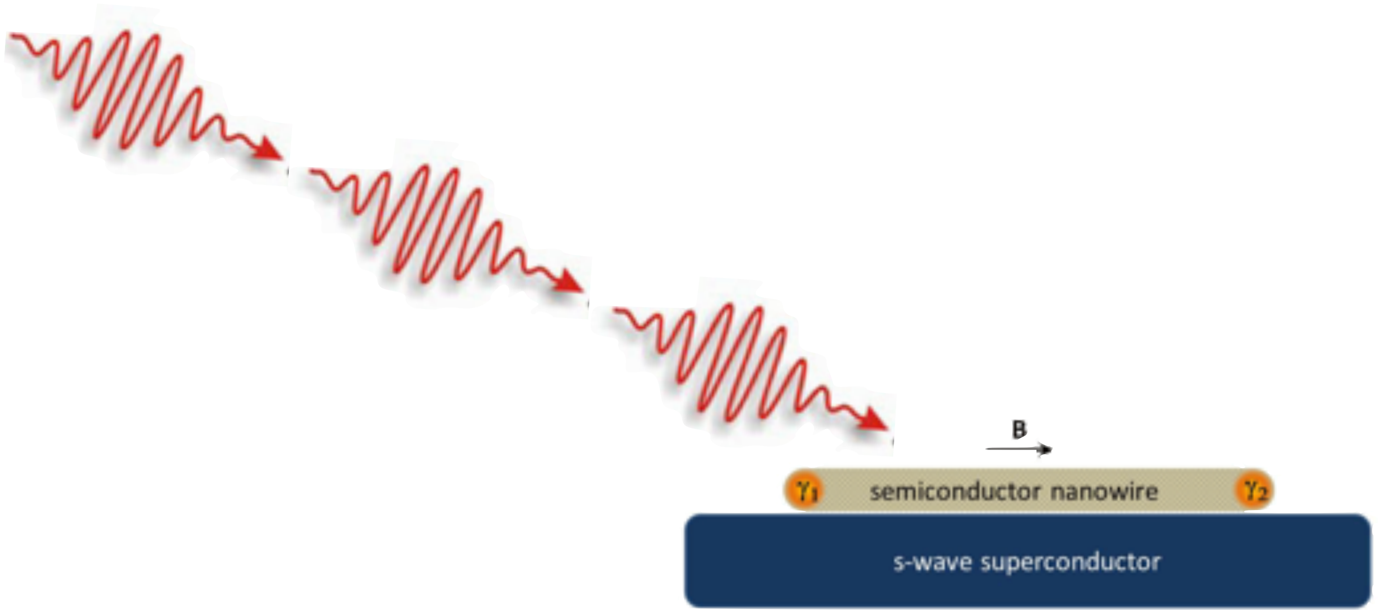
Floquet:

- no "lowest" energy state,
- What single-particle levels are "filled"? (is there a sense of equilibrium?)

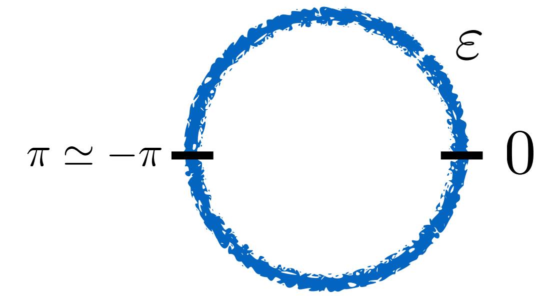


Many-body spectrum

Fate of generic driven/interacting system?



Floquet
Many-body spectrum



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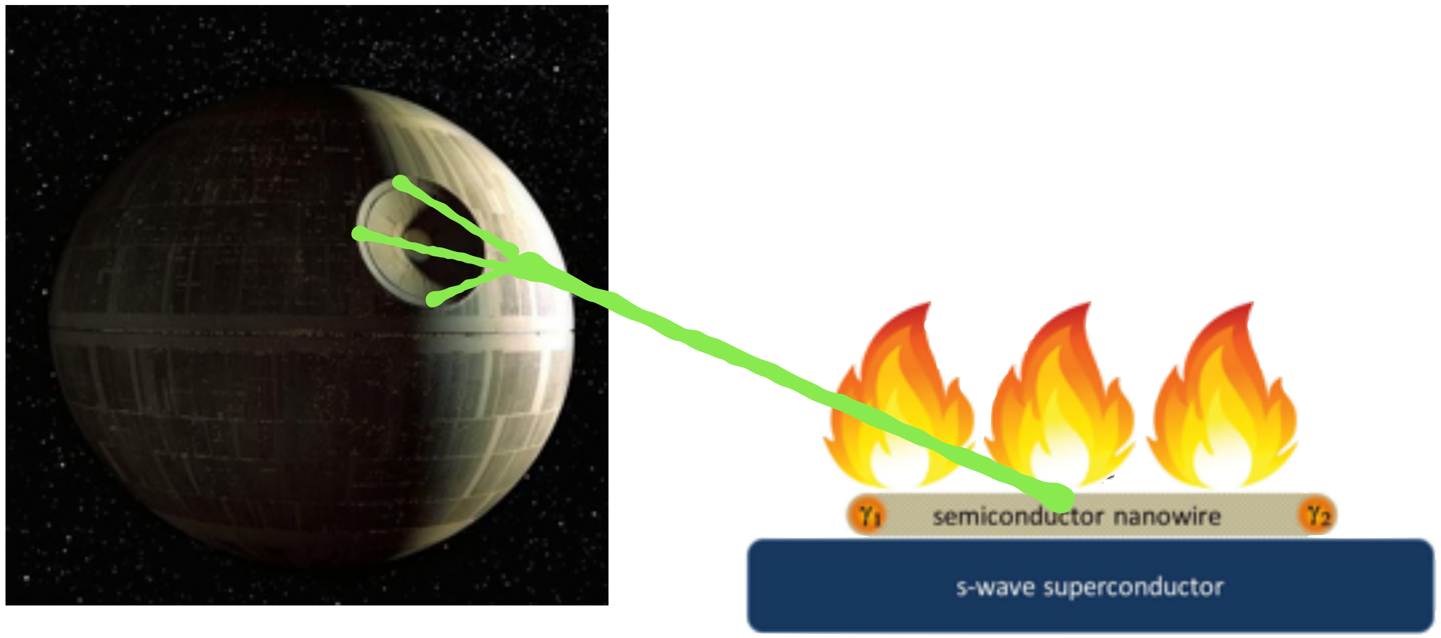

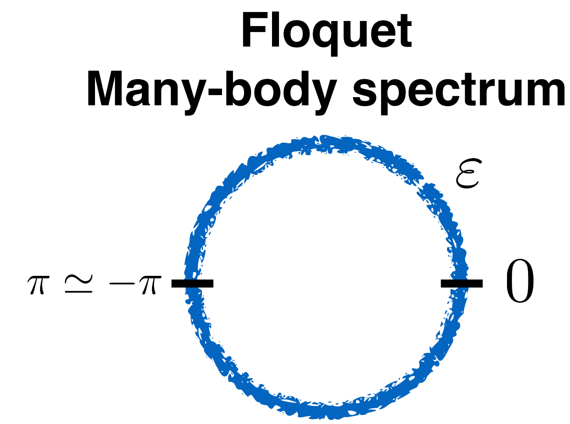

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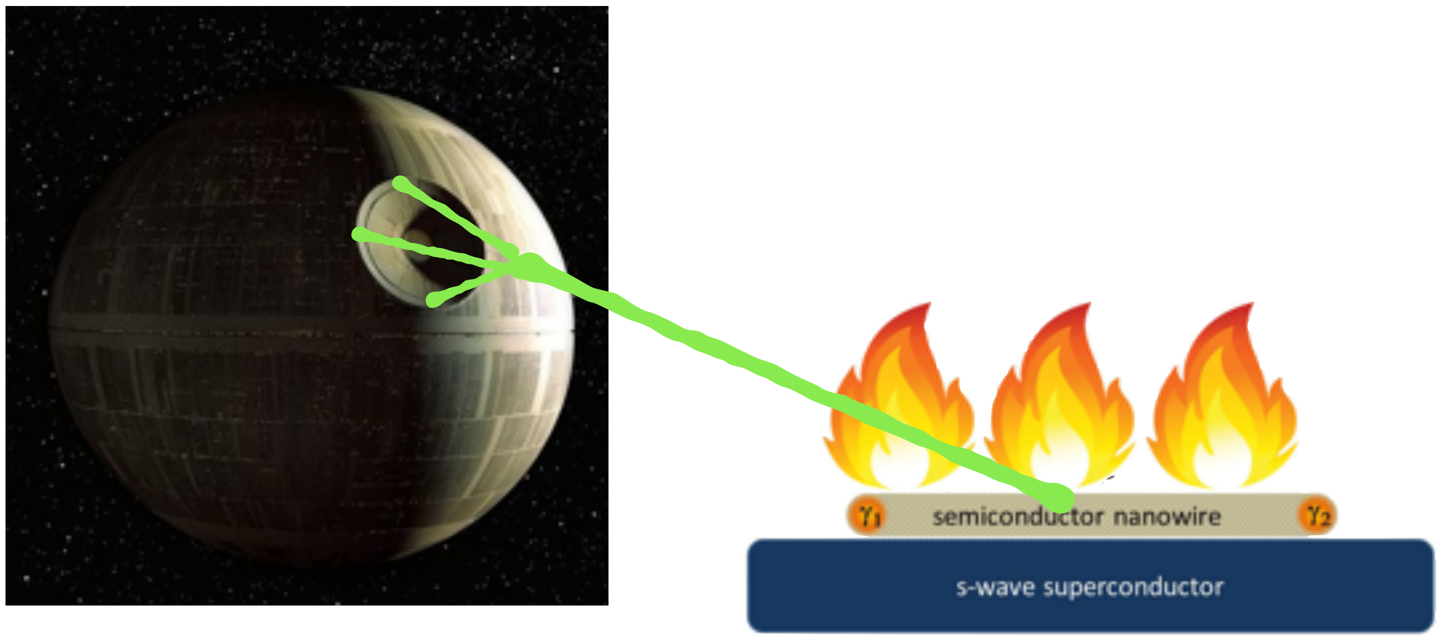
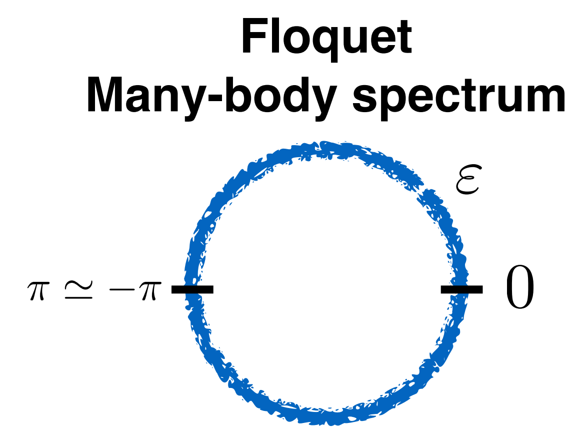




- runaway heating to effective infinite temperature steady state
- no sense of distinct dynamical "phases"
  (all infinite temperature states are equivalent without a phase transition)
- bulk band topology does not govern physics system
  (even for arbitrarily weak interactions)

Solution: Many body localization (MBL)

Strong disorder + Isolated system => No thermal equilibrium

Basko, Aleiner, Altshuler; Pal, Huse; many others

All eigenstates have local (area-law) entanglement
(like gapped ground-states)

Bauer, Nayak

“T=0” Quantum order possible in all eigenstates

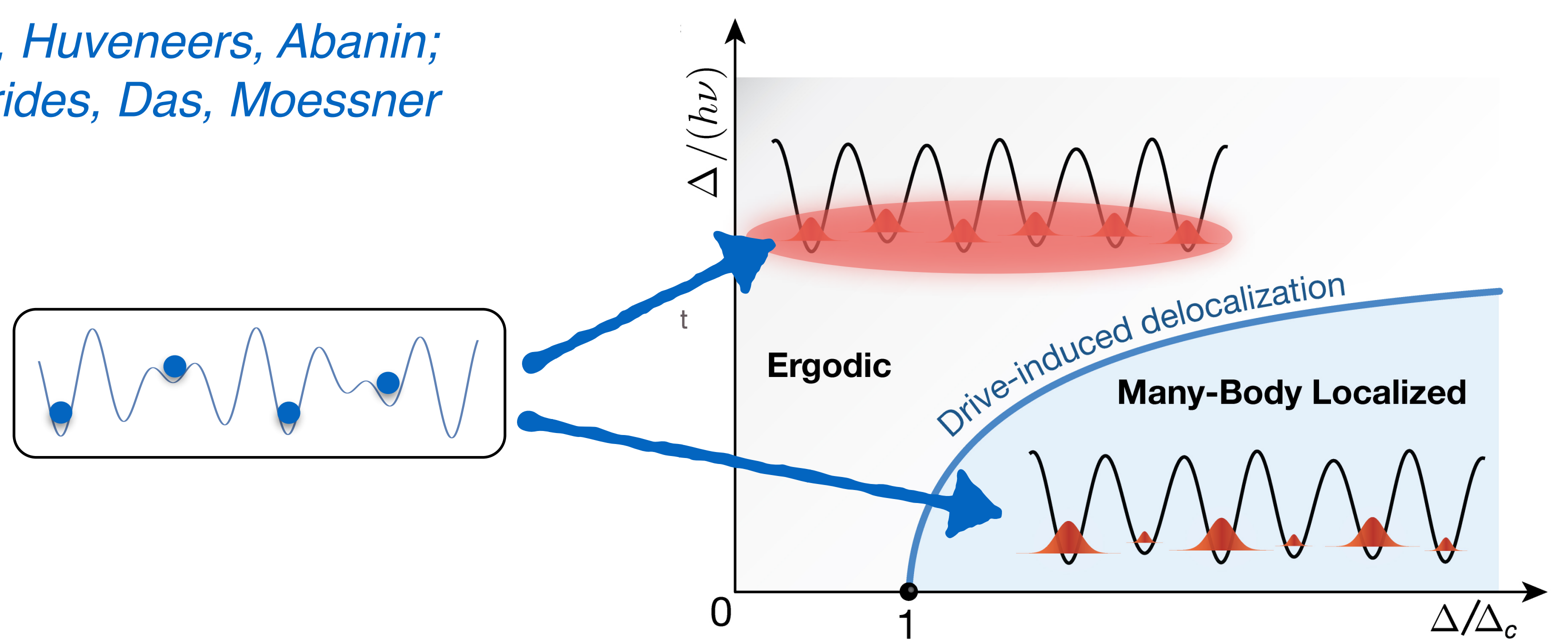
Huse et al.; Bauer, Nayak

Experimental realizations: cold atoms, trapped ions,
(likely many more to come)

Bloch & Monroe Groups

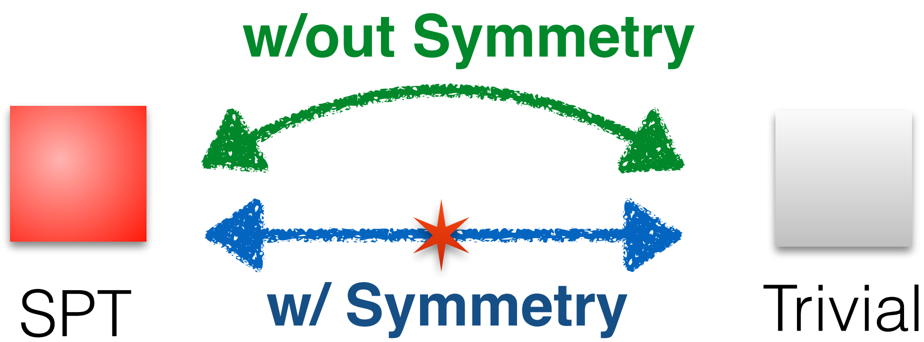
Can (periodically) drive without heating

Ponte, Papic, Huveneers, Abanin;
Lazarides, Das, Moessner



Bordia et al. (Bloch group) - arXiv:1607.07868

Driven SPT phases



New SPT phases from driving?

Haldane Spin-Chain, Topological Insulators,…

1D: ACP, T. Morimoto, A Vishwanath PRX ’16
[see also: Keyserlingk, Sondhi; Else, Nayak; Roy, Harper]

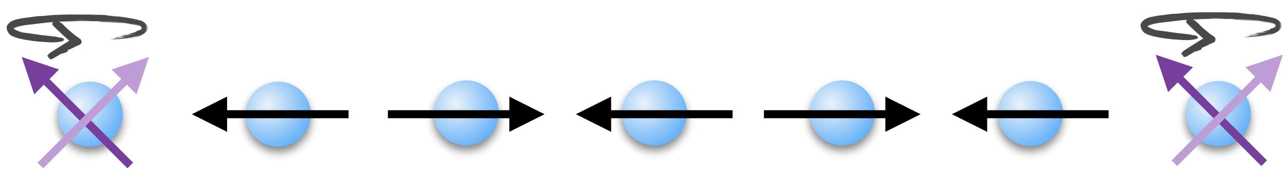
2D: ACP, T Morimoto arXiv ’16

Takahiro Morimoto
(Berkeley)

Ashvin Vishwanath
(Harvard)

Ex: Periodically driven Ising chain

(dual of pi-SG model in Khemani, Lazarides, Moessner, Sondhi PRL '16)



$$g = \prod_i \sigma_i^x$$

Z$_2$ Symmetry

No Eq. SPT

$$H(t) = \begin{cases} \sum_i h_i \sigma_i^x & 0 \leq t < T_1 \\ \sum_i J \sigma_i^z \sigma_{i+1}^z & T_1 \leq t < T \end{cases}$$

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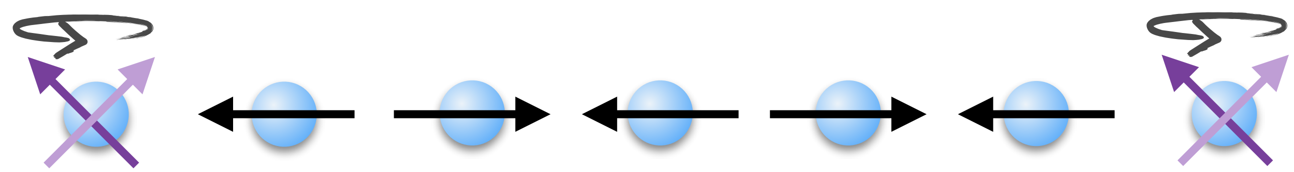

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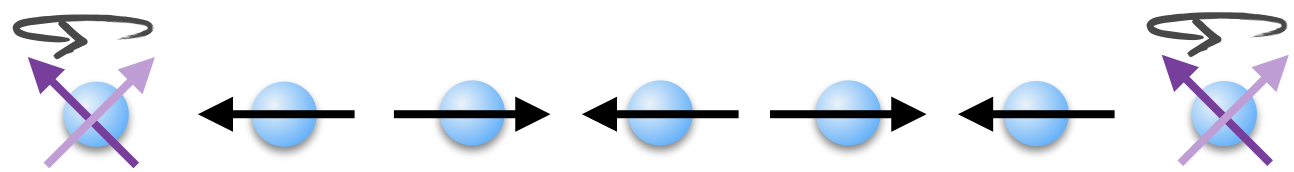

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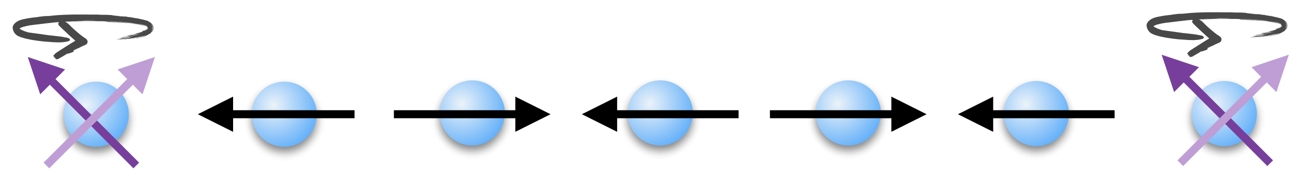

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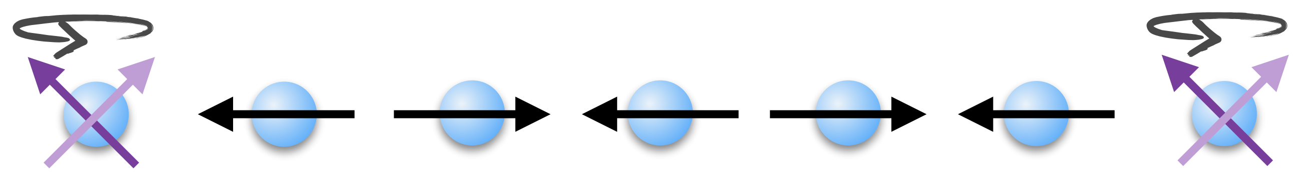

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- Edge spins flip around z each period,
- "Spin-echo" away any symmetry preserving edge field
- Spin-echo usually requires fine-tuning (pi-pulse), **but this is stable to errors!**

ACP, T. Morimoto, A Vishwanath PRX '16
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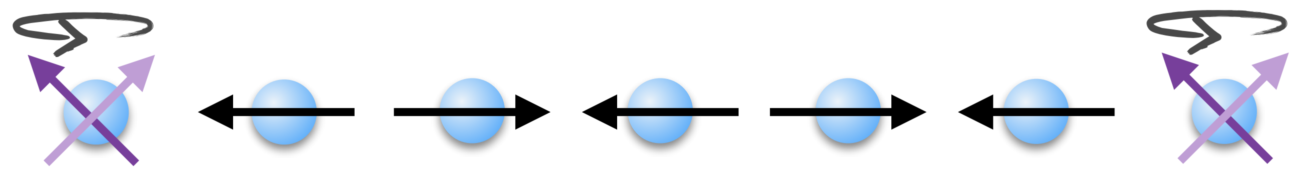
What protects the edges?



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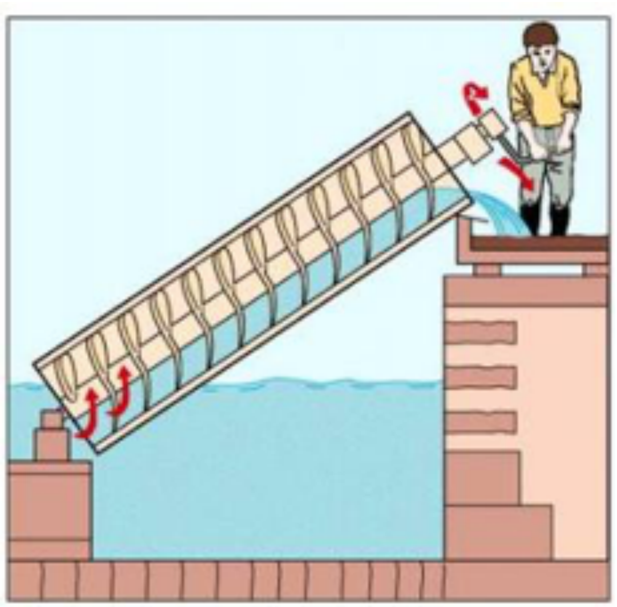
$$\sigma^x_{\text{edge}} = \pm 1 \to \mp 1$$



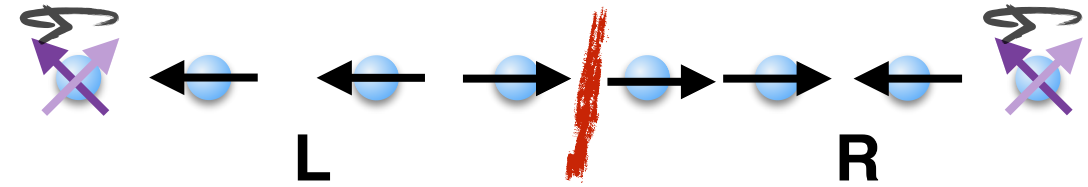
"Quantum archimedes screw"

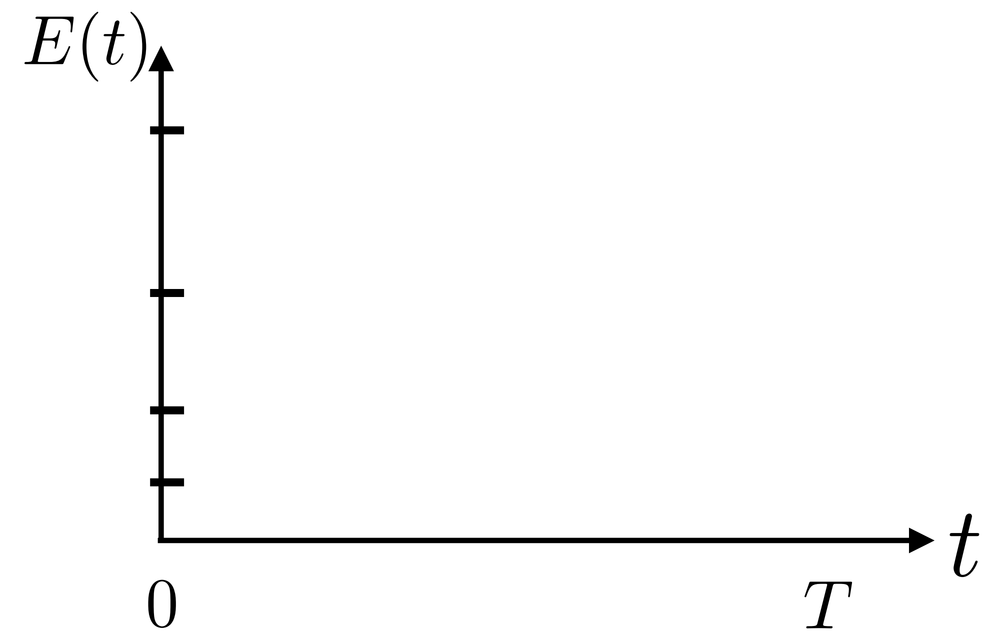
- Symmetry "charge" (irreducible representation) pumped onto edge during each period
- Possible charges are discrete, "quantized"
  can't be continuously altered by small perturbations
- Pumping different symmetry charges
  (irreducible representations) <=> different FSPT phases

ACP, T. Morimoto, A Vishwanath PRX '16
(see also von Keyserlingk & Sondhi, Else & Nayak, Harper & Roy)

Formal classification

Entanglement spectrum (micromotion)





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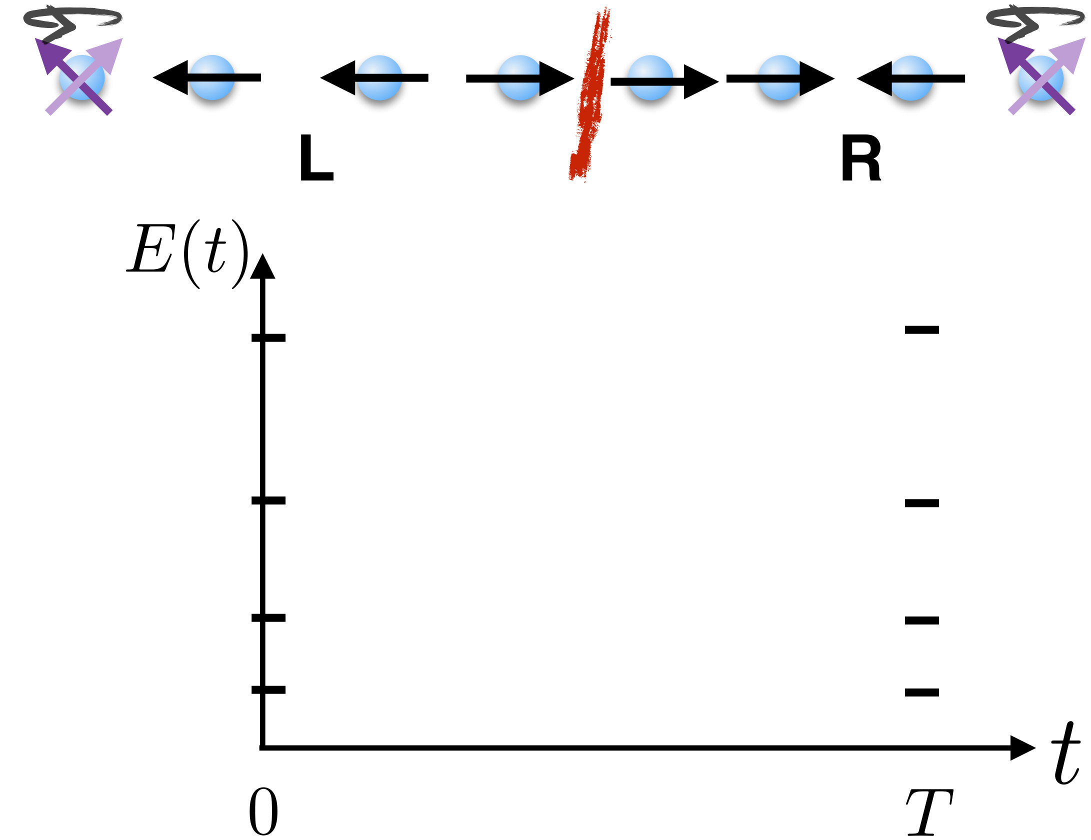

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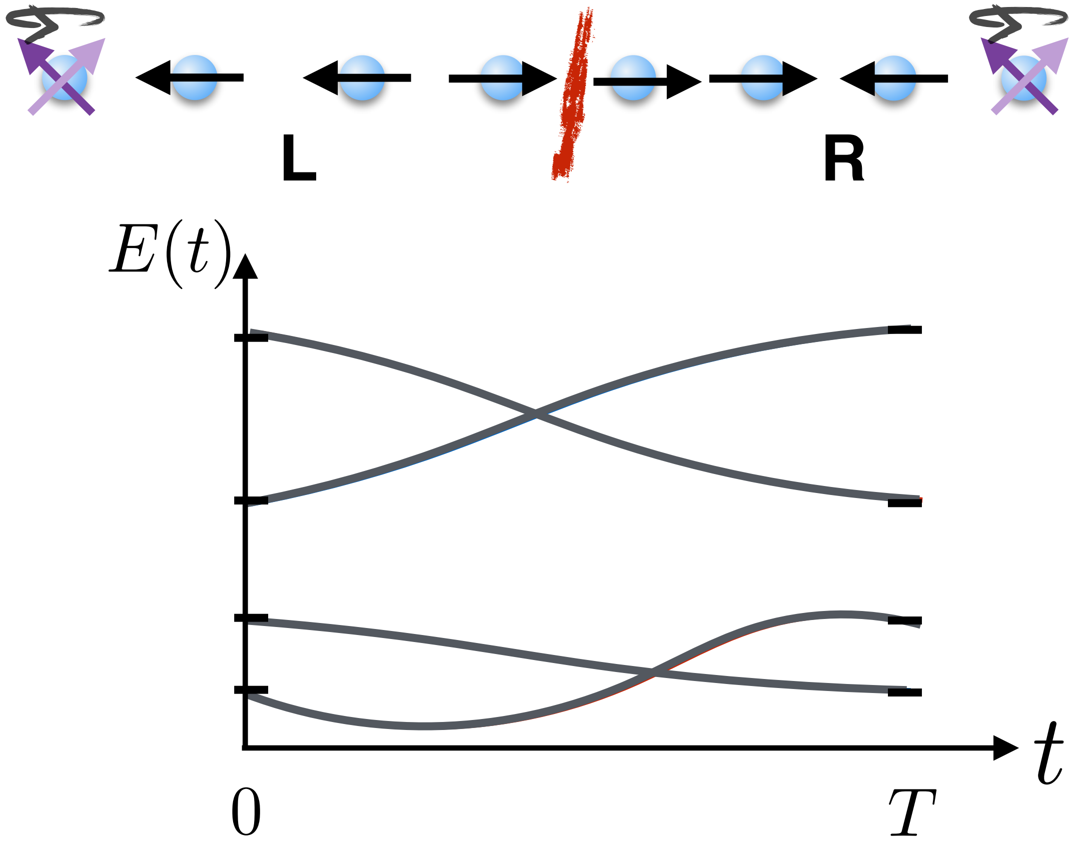

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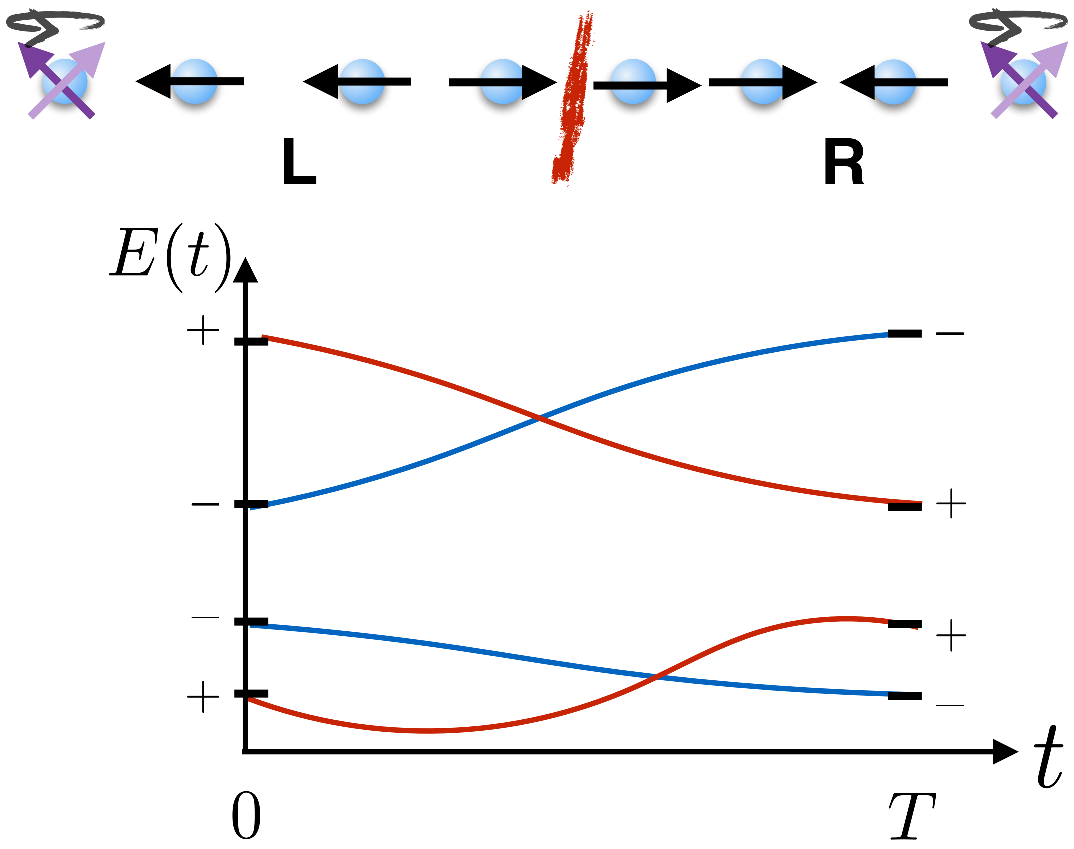

ACP, T. Morimoto, A Vishwanath PRX '16 (see also Else & Nayak)

Formal classification

Entanglement spectrum (micromotion)

$$|\Psi(t)\rangle = \underbrace{e^{-i\epsilon t/T}}_{\text{phase}} \; \underbrace{|u(t)\rangle}_{\text{periodic "micro-motion"}}$$

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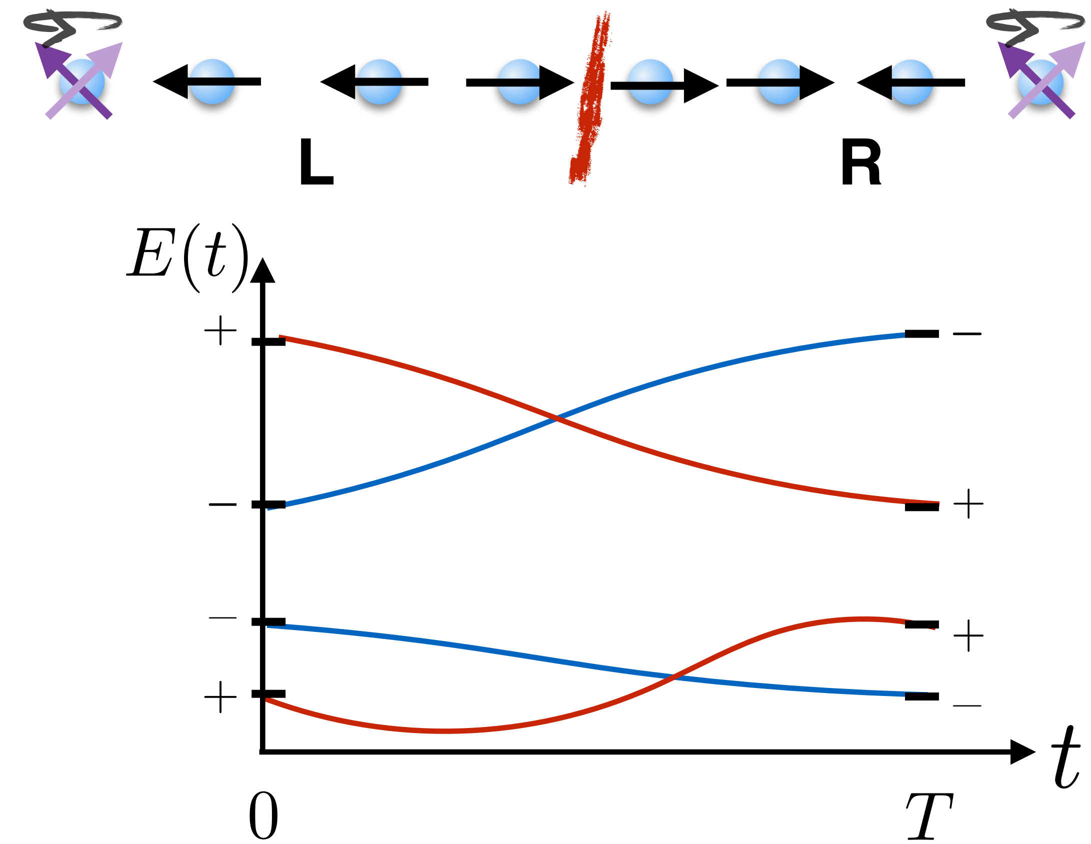


Cohomology: Extra time-translation symmetry $\quad G \longrightarrow G \times \mathbb{Z}$

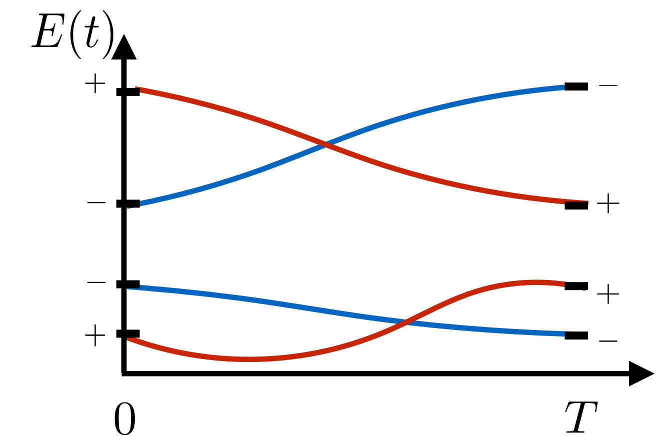
Kunneth Formula: $\quad \mathcal{H}^2(G \times \mathbb{Z}, U(1)) = \underbrace{\mathcal{H}^2(G, U(1))}_{\textbf{1D Equilibrium}} \times \underbrace{\mathcal{H}^1(G, U(1))}_{\textbf{New Floquet Phases}}$

(boson phases)

Possible charges = $H^1(G,U(1))$

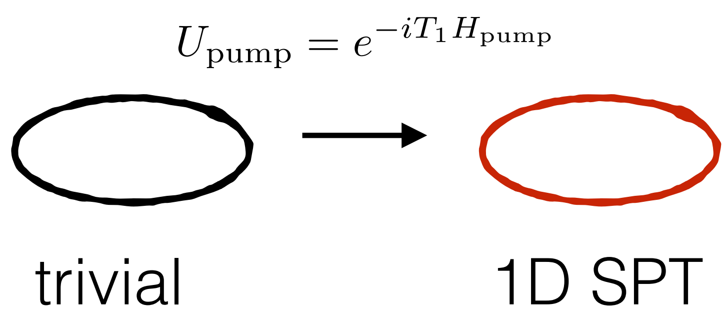
ACP, T. Morimoto, A Vishwanath PRX '16 (see also Else & Nayak)

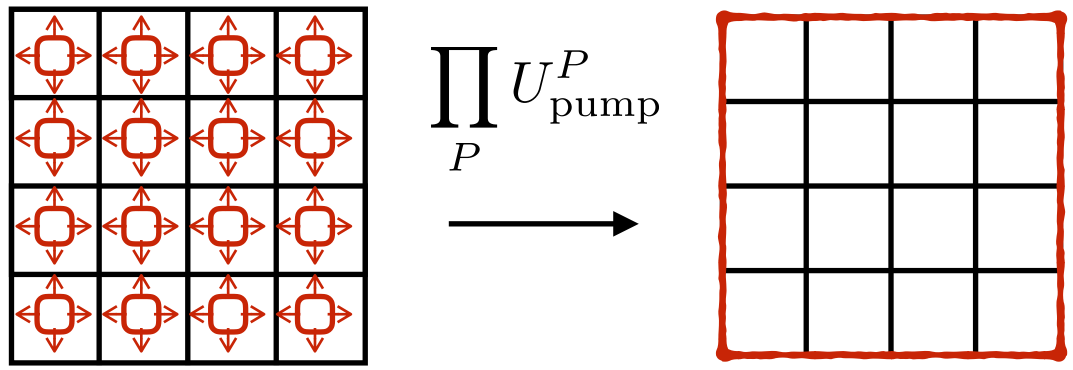
Generalizations to 2D



2D Floquet SPTs:

- 0D charge = 0D SPT => 1D SPT





ACP, T. Morimoto ACP, T. Morimoto arXiv 1610.03485
(see also Else & Nayak PRB '16)

Floquet enriched topological phases

- Example: gauged Floquet SPT
- More general: pumping 1D topological chains of emergent anyons
- Anyons get permuted each pumping cycle

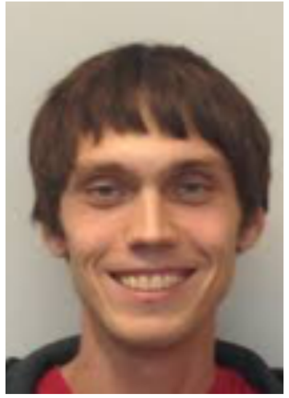
ACP, T. Morimoto arXiv 1610.03485

Chiral Floquet phases

So far: discrete time-translation = extra symmetry of dynamics

Is this all?

Adrian Po
(Berkeley)



Lukasz Fidkowski
(Stony Brook)

Takahiro Morimoto
(Berkeley)

Ashvin Vishwanath
(Harvard)

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

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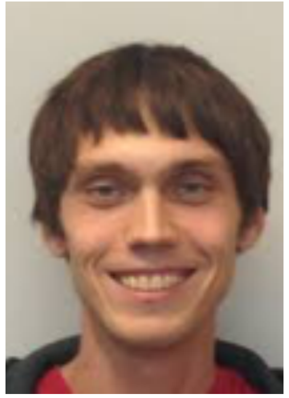

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Intrinsically topological dynamics (no symmetry)

Adrian Po
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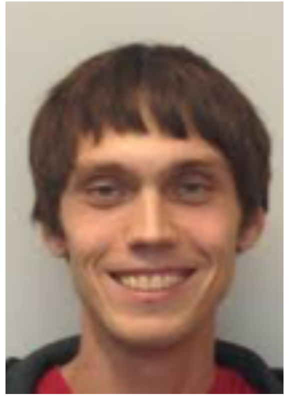

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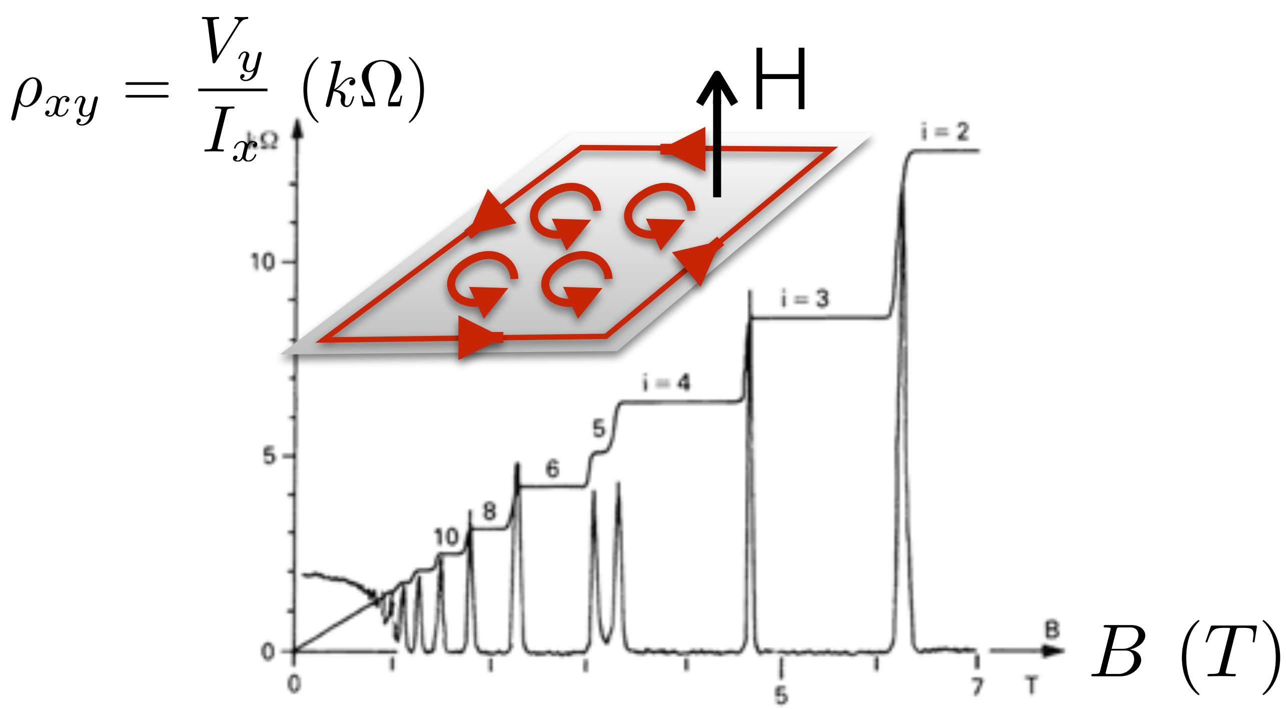
Non-Equilibrium chiral matter?



No chiral edges in energy conserving MBL systems

Halperin, ''82
Nandkishore ACP, PRB '14

- Chern number = obstacle to localization
- Can prove even in the presence of arbitrary interactions
  - any MBL system will not have "gravitational anomaly" (no thermal quantum Hall effect)

Kitaev '06
ACP, Vishwanath arXiv '15

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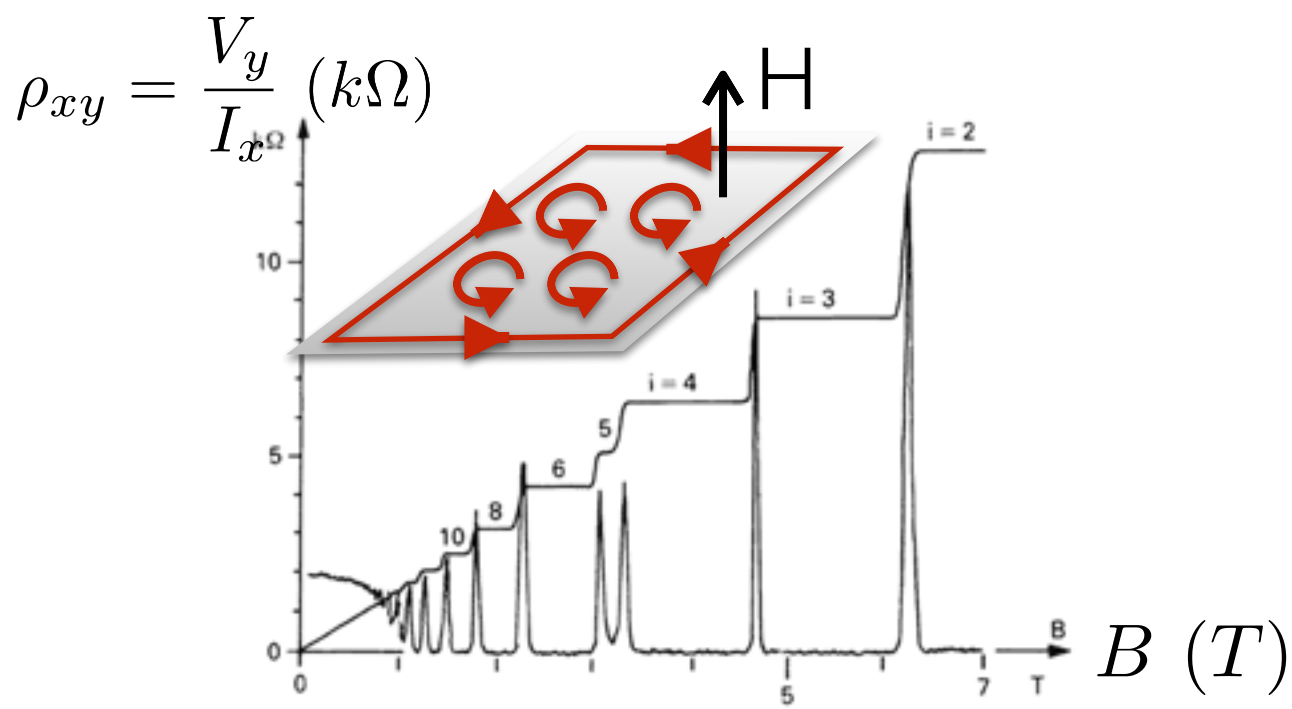

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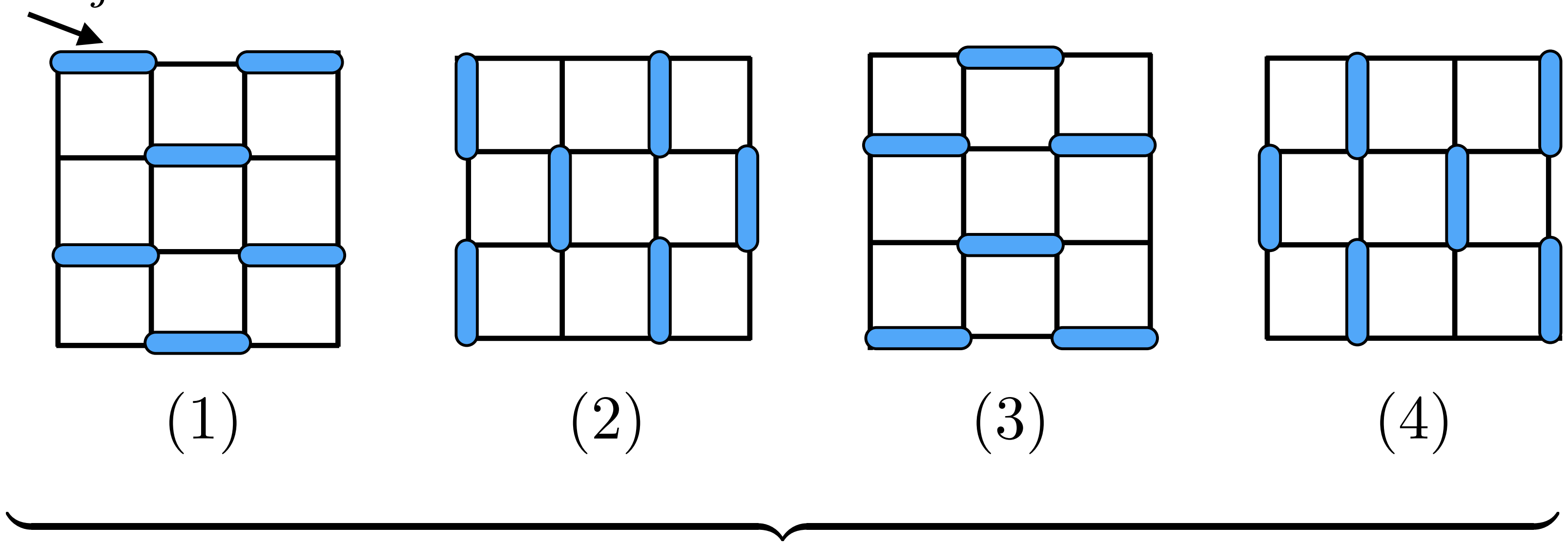
What if energy is not conserved?

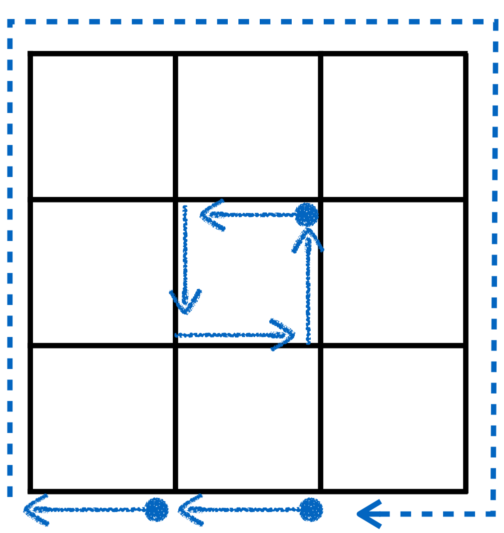
SWAP model

(Direct bosonic analog of free-fermion version by Rudner, Berg, Levin, Titum, Lindner, Refael PRX '13, '16)

$$U(T) = e^{-iH_5} e^{-iH_4} e^{-iH_3} e^{-iH_2} e^{-iH_1}$$

$$\mathrm{SWAP}_{ij} = e^{i\pi \vec{S}_i \cdot \vec{S}_j}$$





$$U_{1\ldots4}(T) = \begin{cases} 1 & \text{bulk} \\ \hat{T}_1 & \text{edge} \end{cases}$$

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

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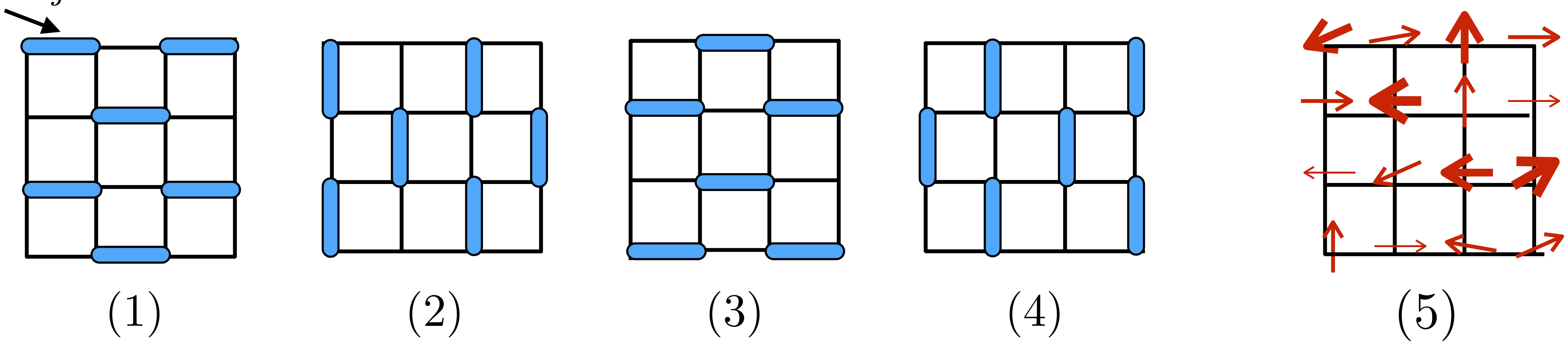
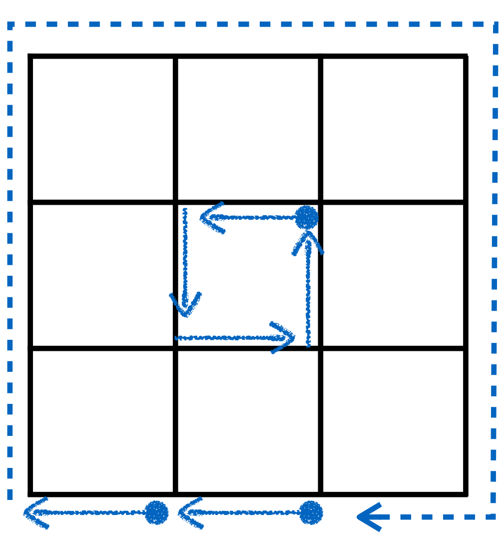
Disorder step



$$H_5 = \sum_i \vec{h}_i \cdot \vec{S}_i$$



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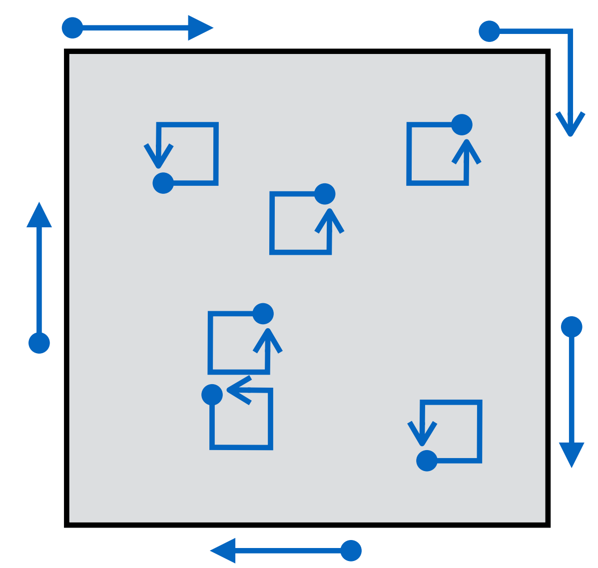
Localized bulk:
edge dynamics occurs separate from bulk

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

What is topological about this state?

Rudner, Berg, Levin, PRX '2013



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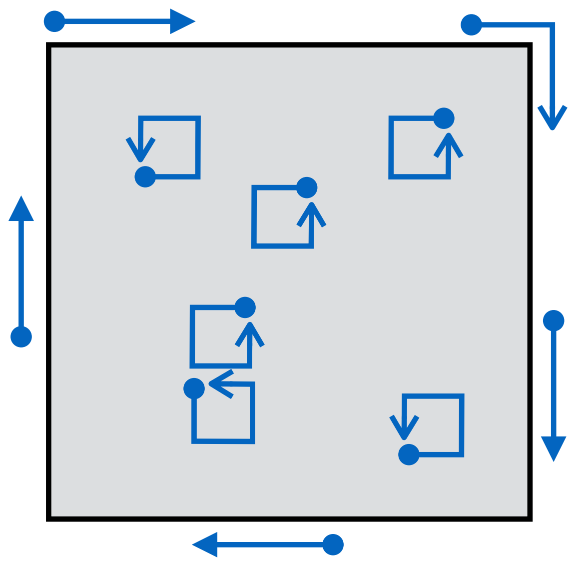
Single particle winding invariant (fermion version) *Rudner, Berg, Levin, PRX '2013*



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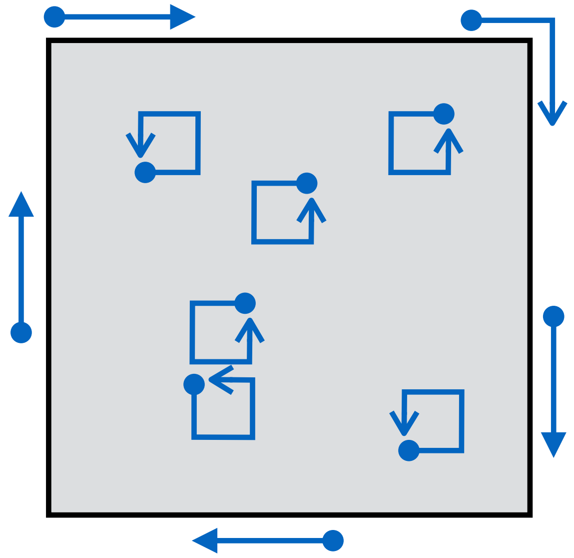

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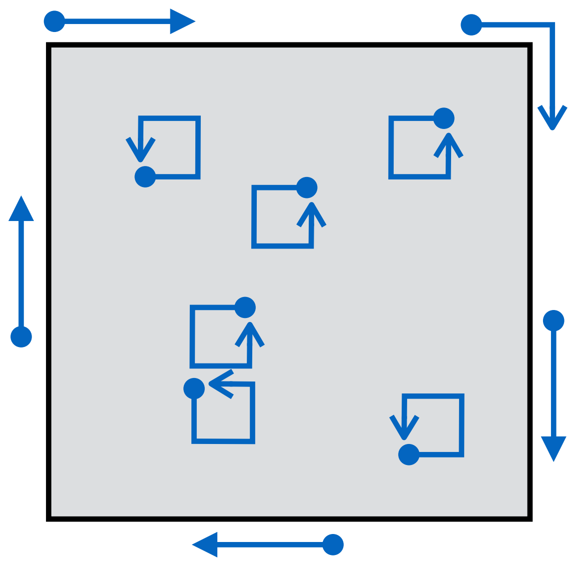

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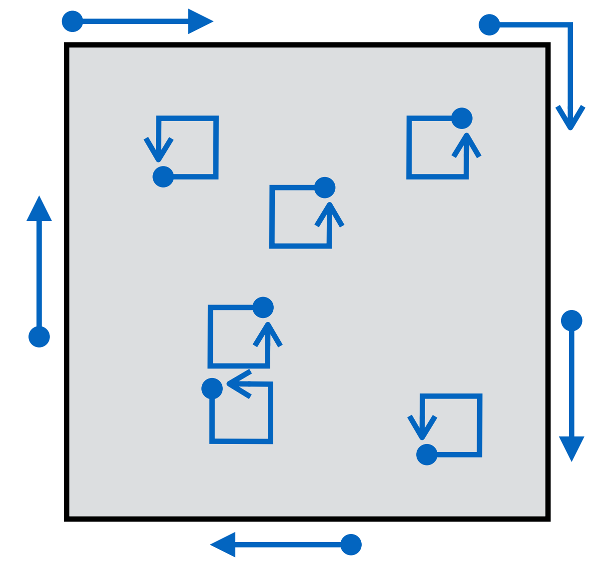
  (related: magnetization density)
  Nathan et al. arXiv:1610.03590



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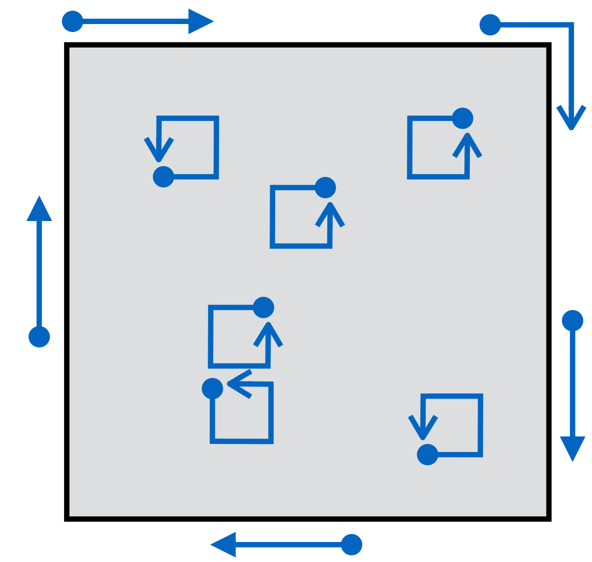

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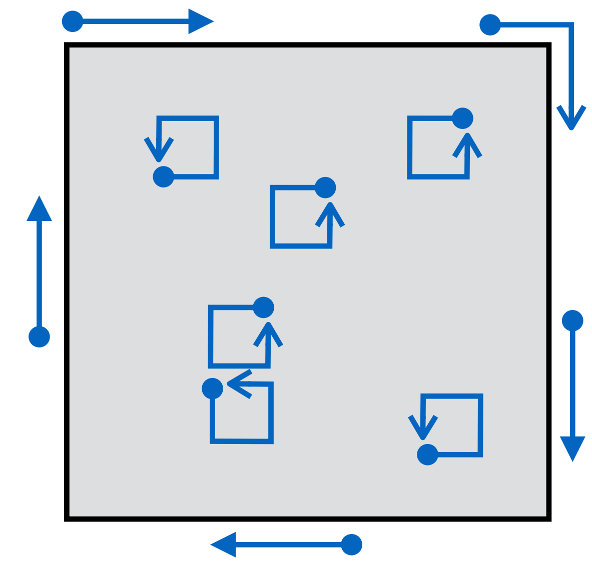

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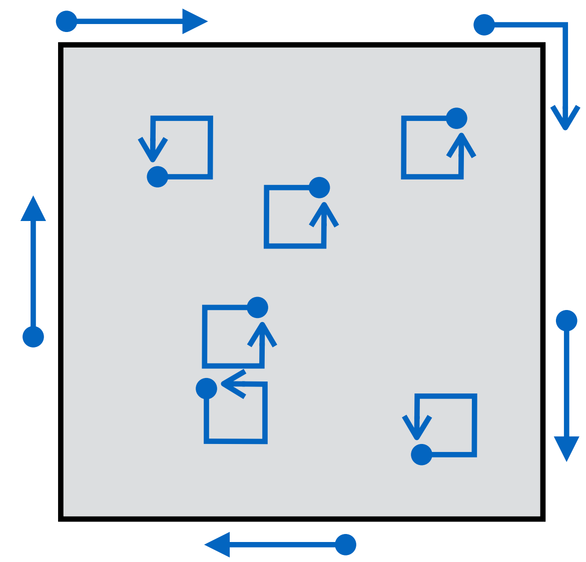

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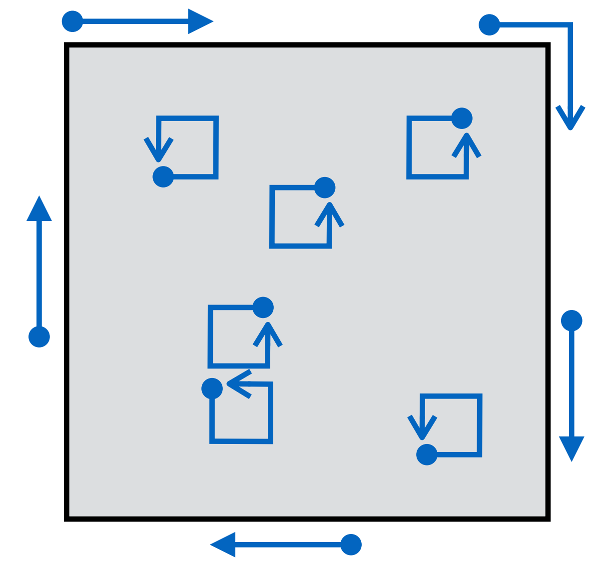

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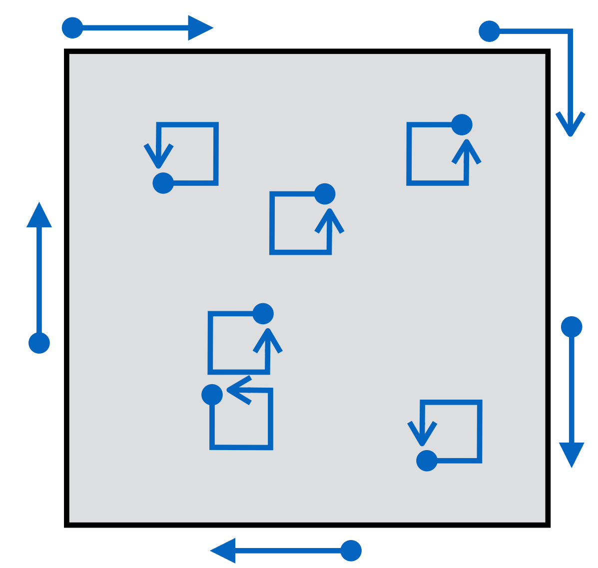

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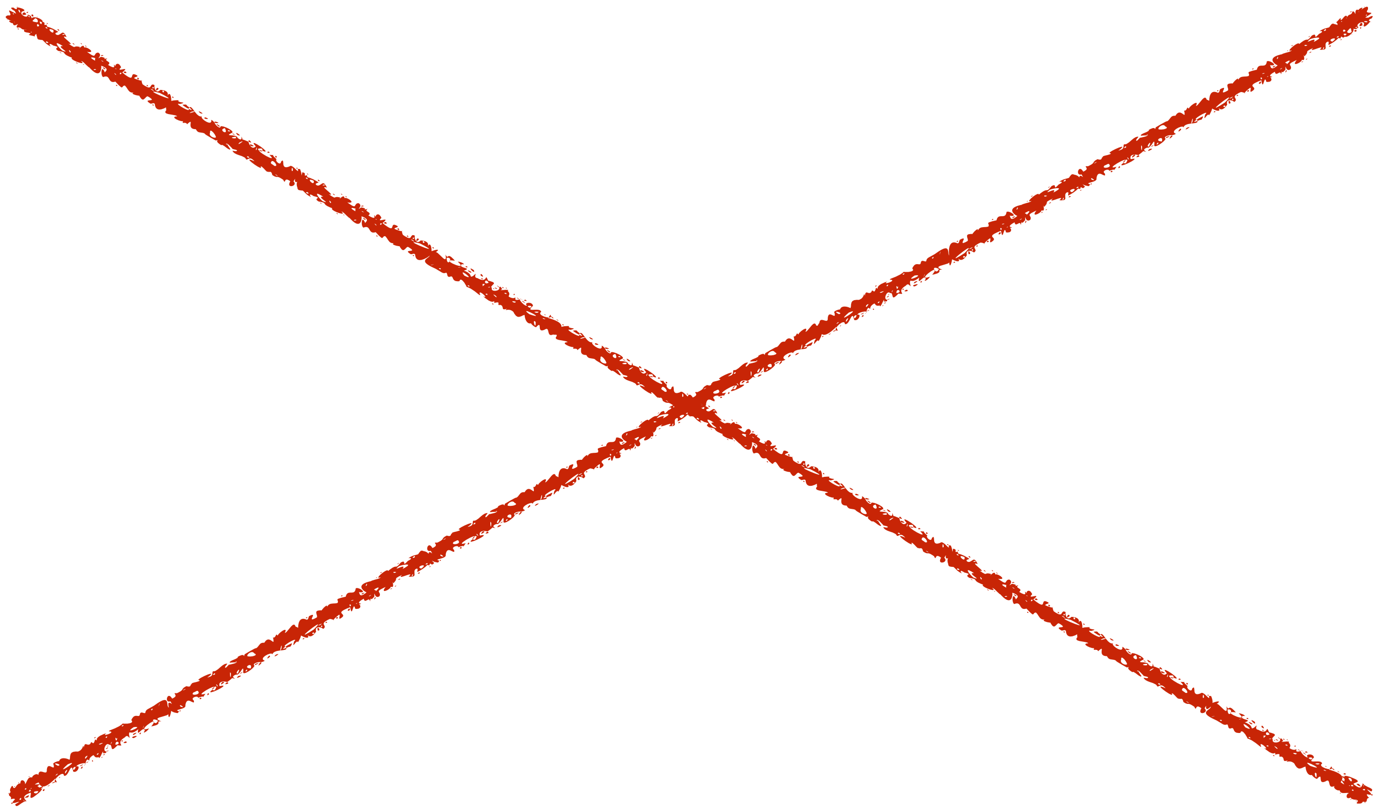

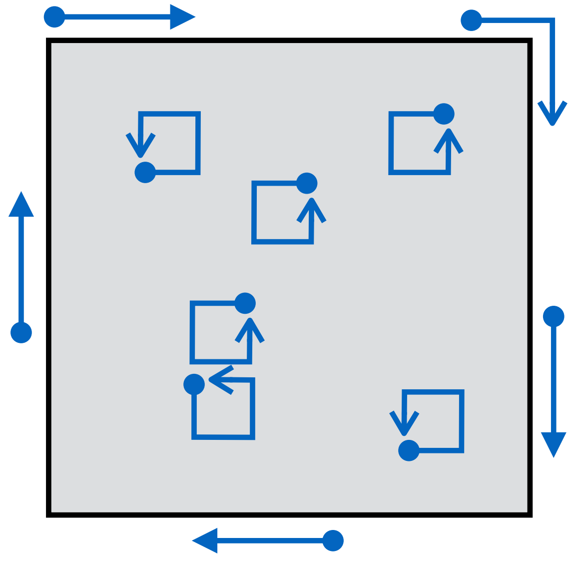

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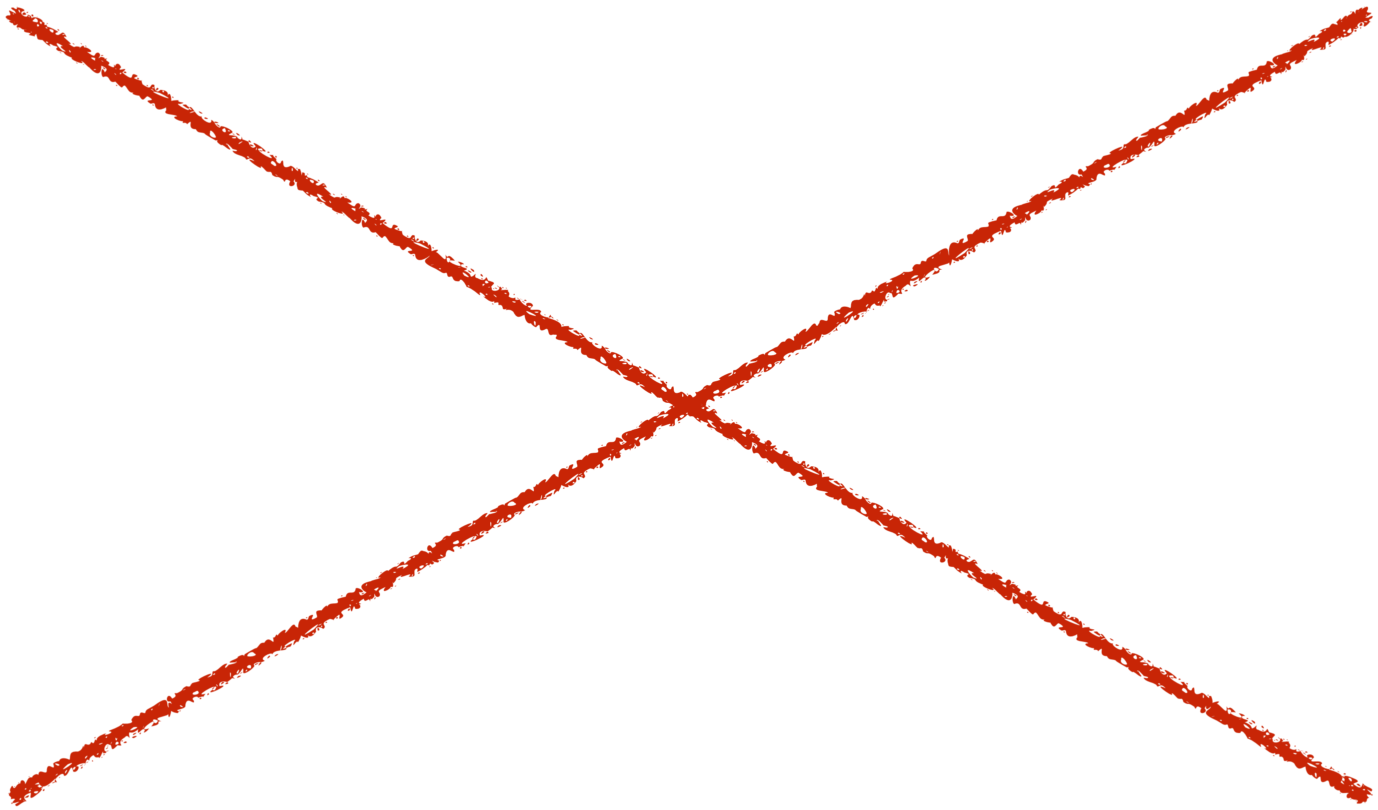

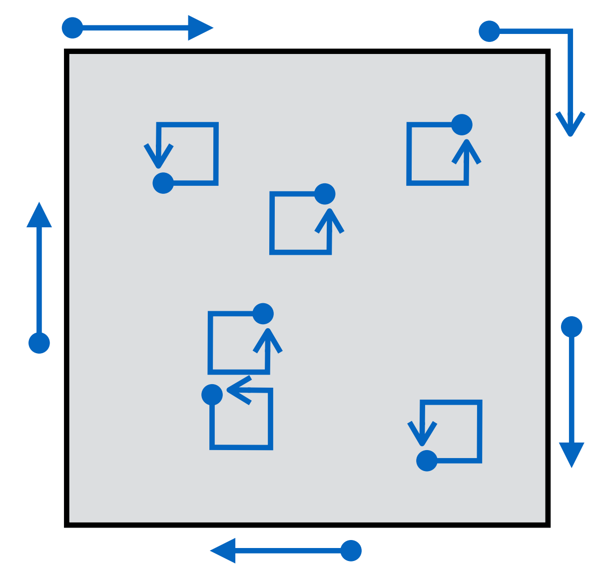

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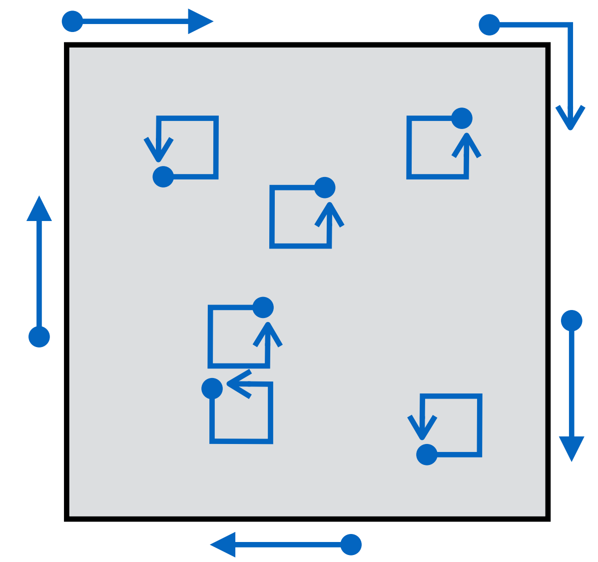

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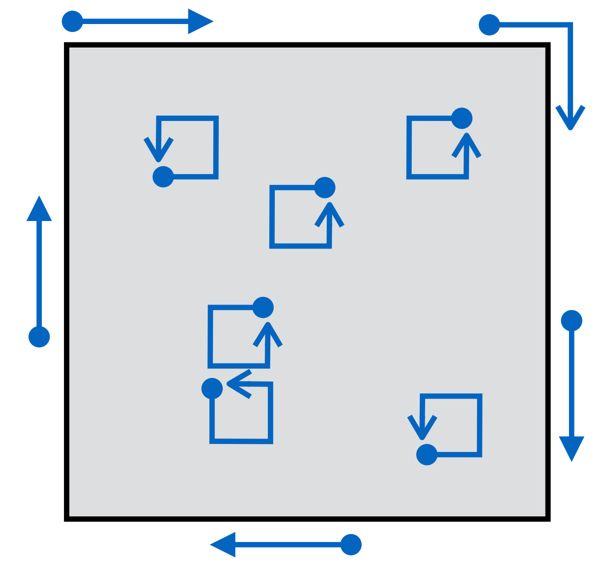

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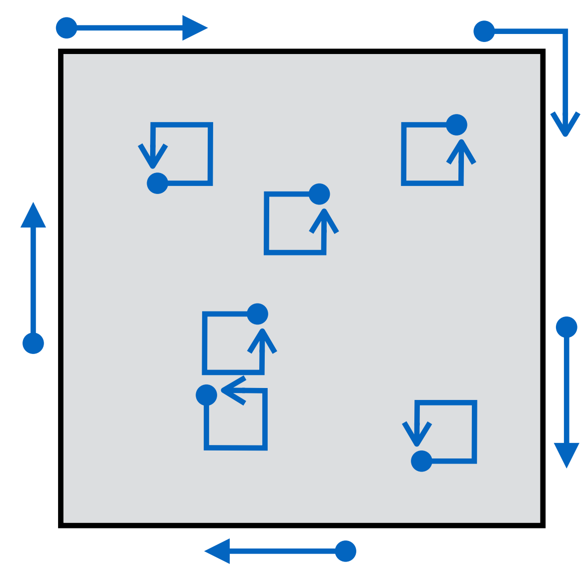

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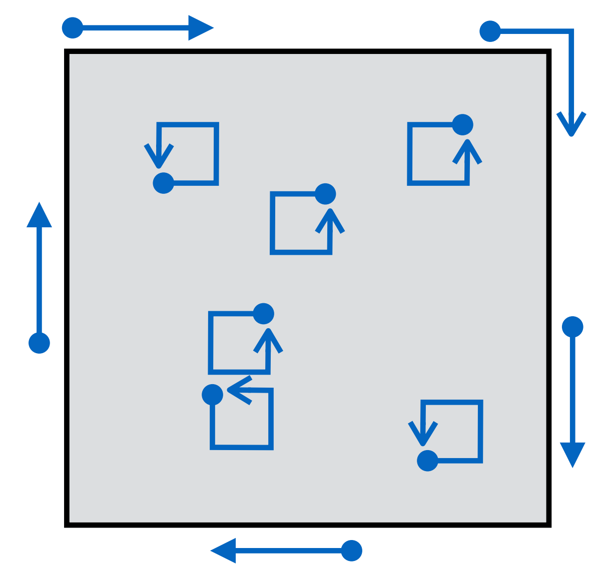

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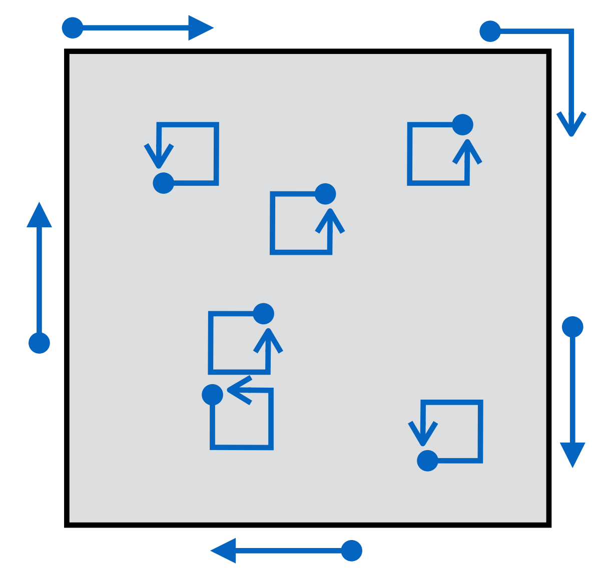

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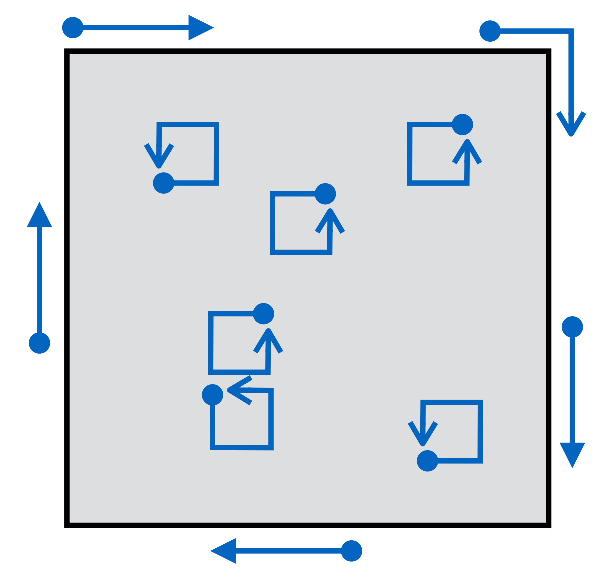

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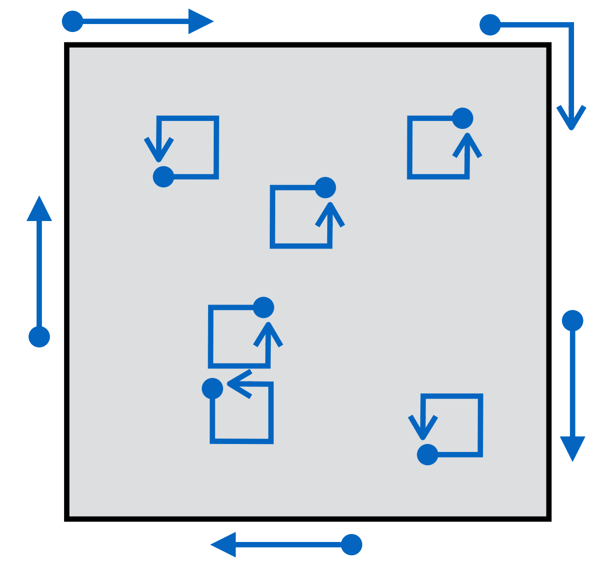

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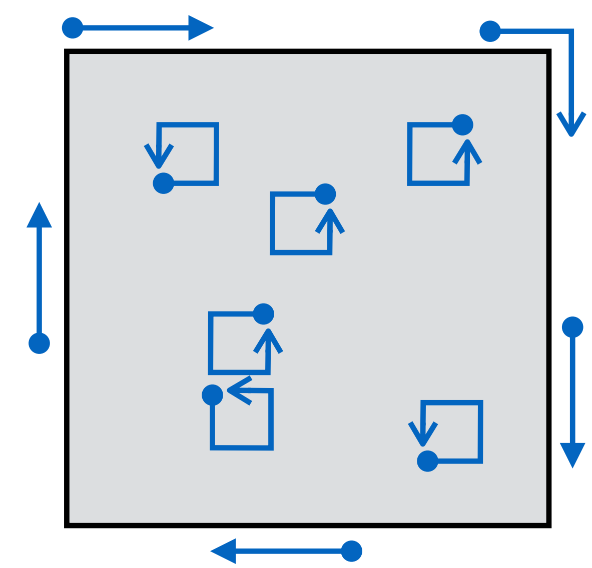

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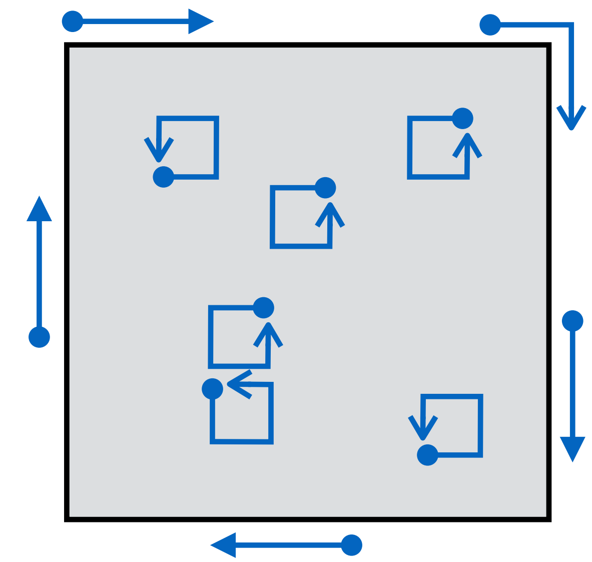

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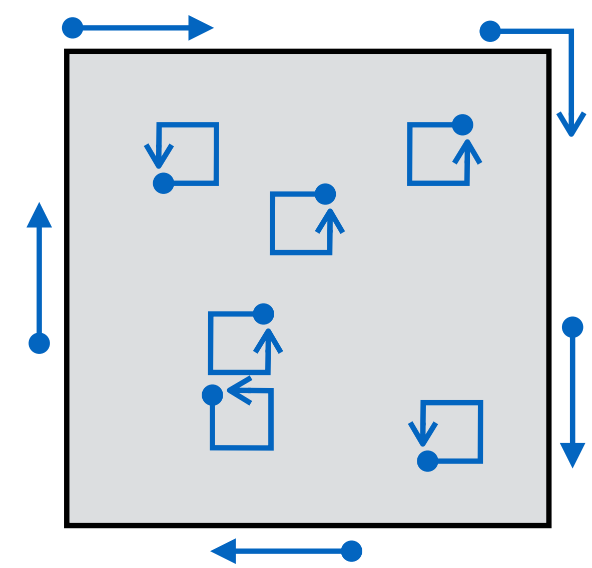
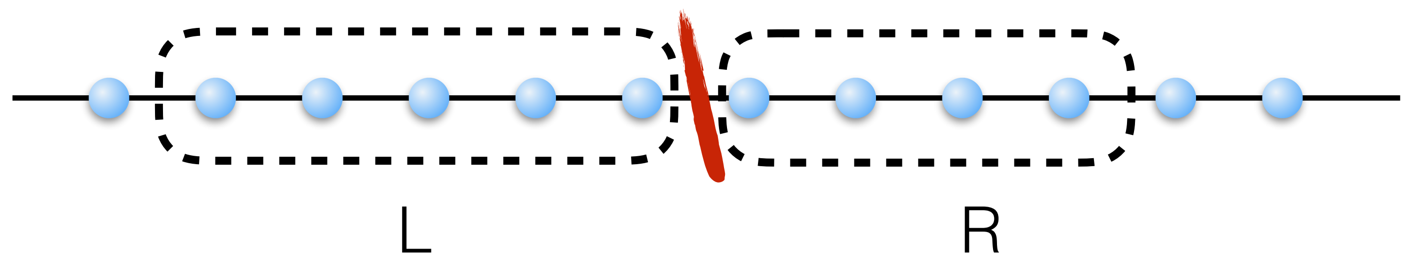


Quantized pumping of <u>quantum information</u> around edge

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

A quantum information flow gauge



*take L/R >> v*T*

Algebra of observables: $\mathcal{A} = \{ \sum_{i,j=1}^{\mathcal{D}_\mathcal{A}} a_{ij} |i\rangle\langle j| \;;\;\; a_{ij} \in \mathbb{C} \}$ $\qquad e_{ij} = |i\rangle\langle j|$

Overlap of algebras:

$$\langle \mathcal{A}, \mathcal{B} \rangle = \frac{\sqrt{\mathcal{D}_\mathcal{A} \mathcal{D}_\mathcal{B}}}{\mathcal{D}_{\text{tot}}} \sqrt{\sum_{i,j=1}^{\mathcal{D}_\mathcal{A}} \sum_{l,m=1}^{\mathcal{D}_\mathcal{B}} \left| \text{tr}\left(e_{ij}^{a\dagger} e_{lm}^{b} \right) \right|^2} \qquad \langle \mathcal{A}, \mathcal{B} \rangle = \begin{cases} 1 & [\mathcal{A}, \mathcal{B}] = 0 \\ \mathcal{D}_\mathcal{A} & \mathcal{A} = \mathcal{B} \end{cases}$$

log <A,B> = "how much information about A is contained in B"

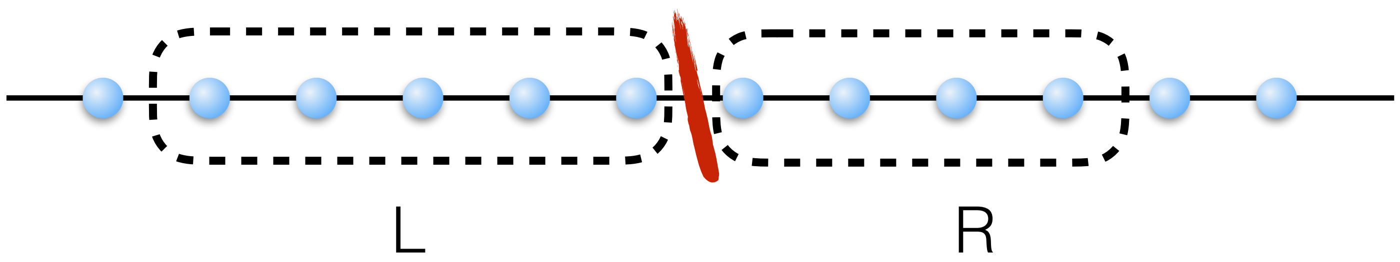
Topological index (GNVW):

$$\nu = \log \frac{\langle U(\mathcal{A}_L), \mathcal{A}_R \rangle}{\langle \mathcal{A}_L, U(\mathcal{A}_R) \rangle} \in \log \mathbb{Q}_+$$

imbalance of information flow (-> left) - (<- right)

A quantum information flow gauge

D. Gross, V. Nesme, H. Vogts, R.F. Werner arXiv:0910.3675



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Algebra of observables: $\mathcal{A} = \{ \sum_{i,j=1}^{\mathcal{D}_{\mathcal{A}}} a_{ij} |i\rangle\langle j| \; ; \quad a_{ij} \in \mathbb{C} \}$ $\qquad e_{ij} = |i\rangle\langle j|$

Overlap of algebras:

$$\langle \mathcal{A}, \mathcal{B} \rangle = \frac{\sqrt{\mathcal{D}_{\mathcal{A}} \mathcal{D}_{\mathcal{B}}}}{\mathcal{D}_{\text{tot}}} \sqrt{\sum_{i,j=1}^{\mathcal{D}_{\mathcal{A}}} \sum_{l,m=1}^{\mathcal{D}_{\mathcal{B}}} \left| \text{tr} \left(e_{ij}^{a\dagger} e_{lm}^{b} \right) \right|^2} \qquad \langle \mathcal{A}, \mathcal{B} \rangle = \begin{cases} 1 & [\mathcal{A}, \mathcal{B}] = 0 \\ \mathcal{D}_{\mathcal{A}} & \mathcal{A} = \mathcal{B} \end{cases}$$

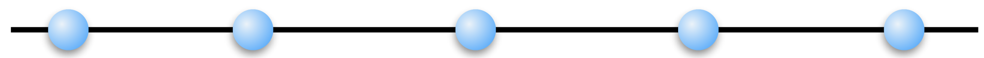
log <A,B> = "how much information about A is contained in B"

Topological index (GNVW):

$$\nu = \log \frac{\langle U(\mathcal{A}_L), \mathcal{A}_R \rangle}{\langle \mathcal{A}_L, U(\mathcal{A}_R) \rangle} \in \log \mathbb{Q}_+$$

imbalance of information flow (-> left) - (<- right)

Yes! (Dynamical edge anomaly)



$$\cdots \otimes |\psi_{j-1}\rangle \otimes |\psi_j\rangle \otimes |\psi_{j+1}\rangle \otimes \dots$$

$\hat{T}_1$:

$$\cdots \otimes |\psi_{j-2}\rangle \otimes |\psi_{j-1}\rangle \otimes |\psi_j\rangle \otimes \dots$$

time = T

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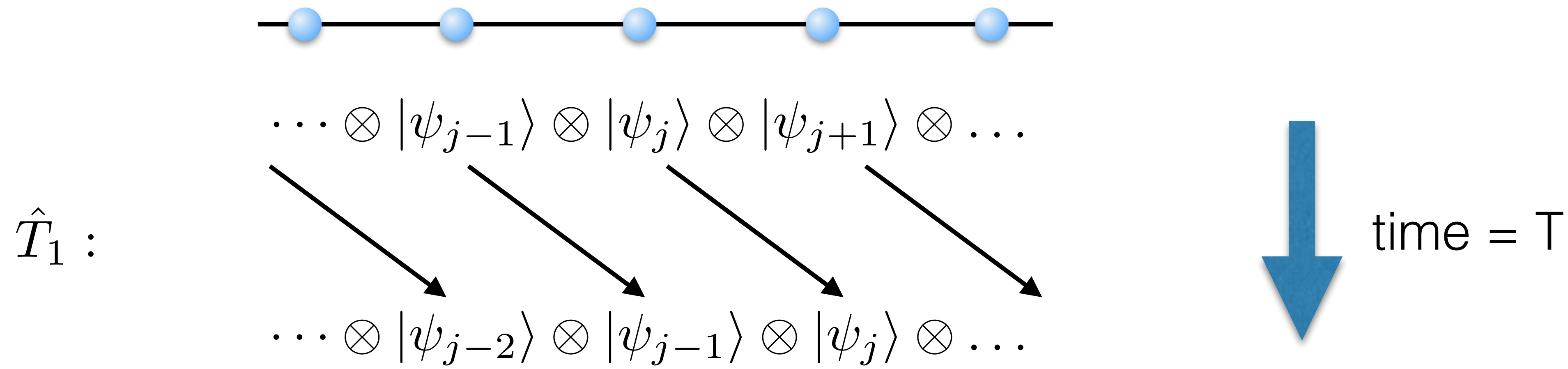

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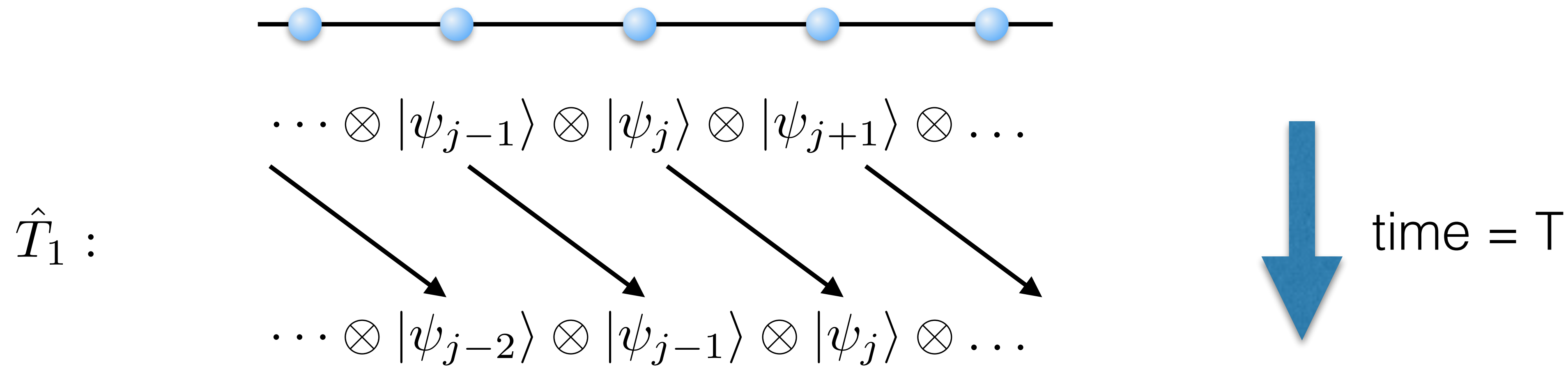

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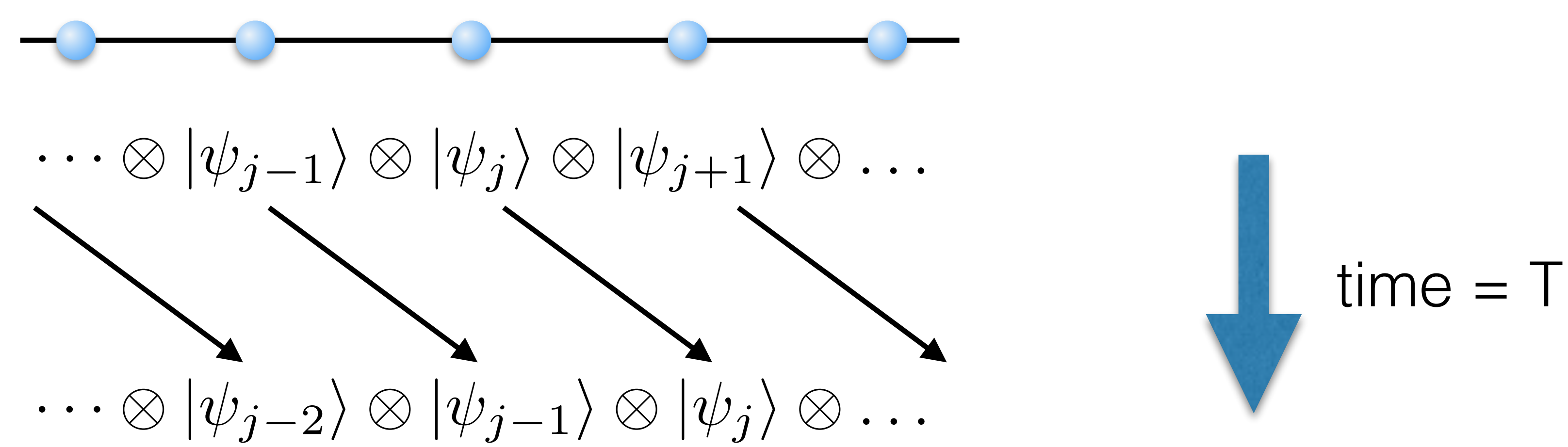

$$\hat{T}_1 : \quad \cdots \otimes |\psi_{j-1}\rangle \otimes |\psi_j\rangle \otimes |\psi_{j+1}\rangle \otimes \dots \;\longrightarrow\; \cdots \otimes |\psi_{j-2}\rangle \otimes |\psi_{j-1}\rangle \otimes |\psi_j\rangle \otimes \dots$$

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But….

Cannot be realized with time evolution by a local Hamiltonian

- Intuitive argument:
  - local Hamiltonian can always have edge
  - behavior at edge is sick! ``pile-up” of states (non-unitary)

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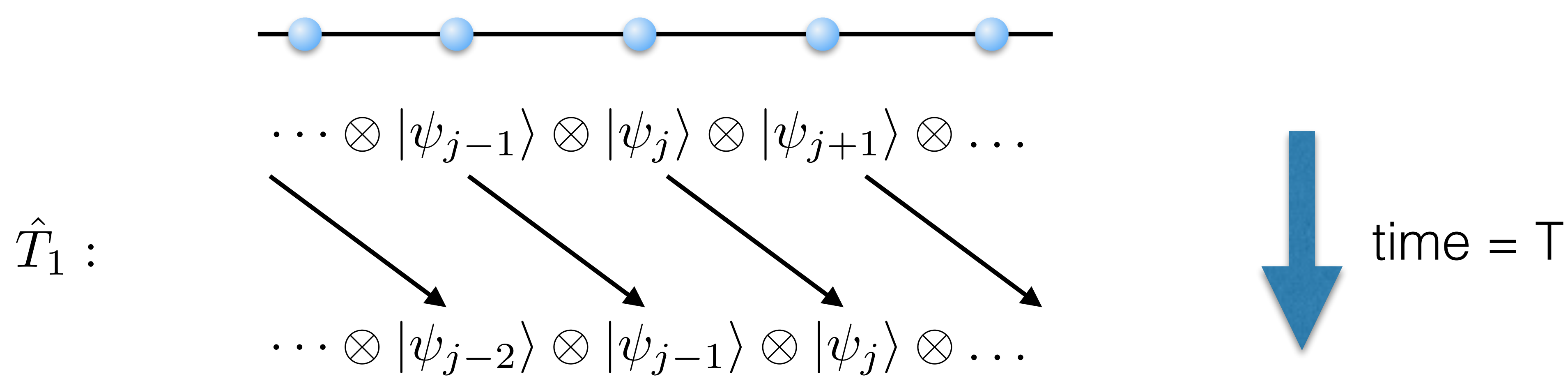

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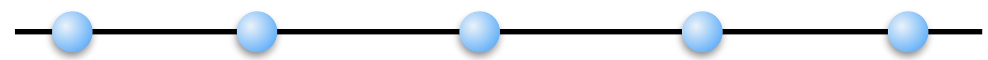

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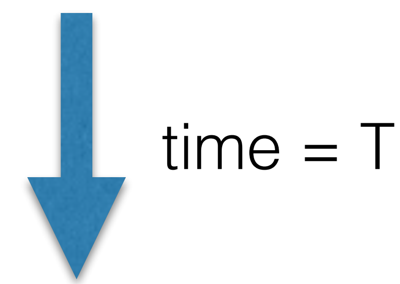

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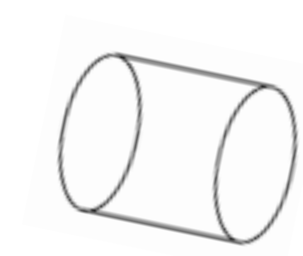
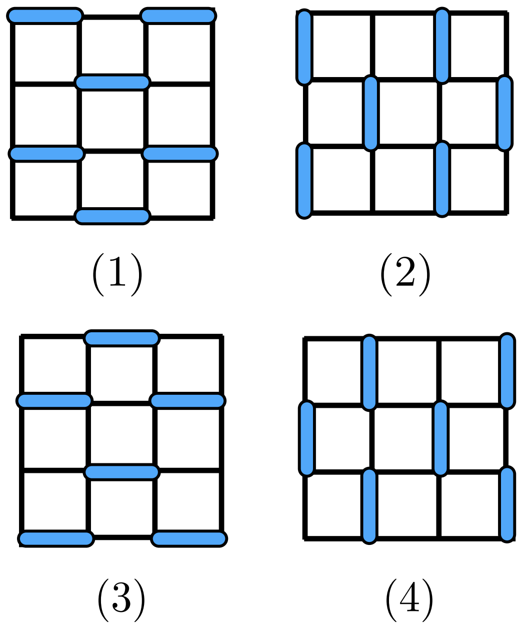
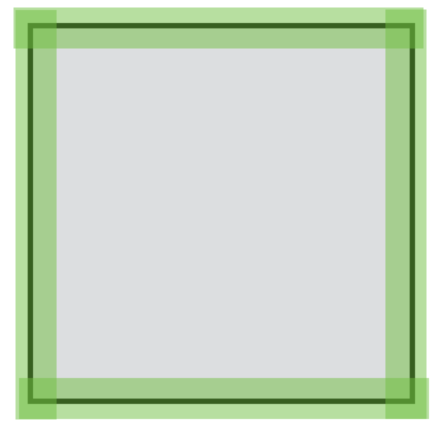
but, OK for boundary of local 2D system (no edge)

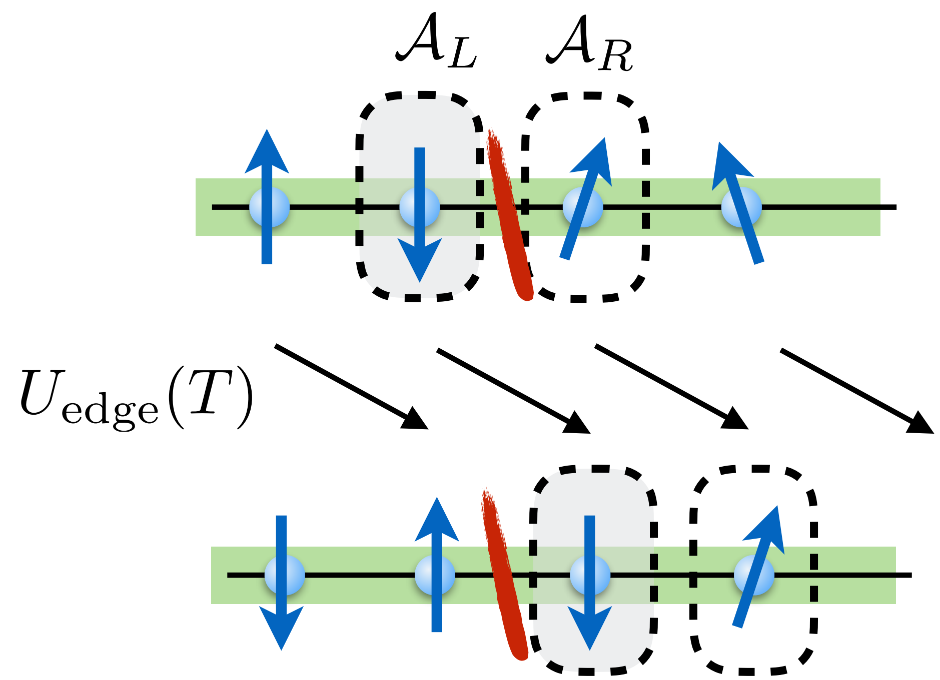


Dynamical chiral index of SWAP model





$$U \underset{\text{(MBL)}}{=} U_{\text{edge}} U_{\text{bulk}}$$



$$U(\mathcal{A}_L) = \mathcal{A}_R$$

$$\nu_{\text{SWAP}} = \log \frac{\langle U(\mathcal{A}_L), \mathcal{A}_R \rangle}{\langle \mathcal{A}_L, U(\mathcal{A}_R) \rangle} = \log(2)$$

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

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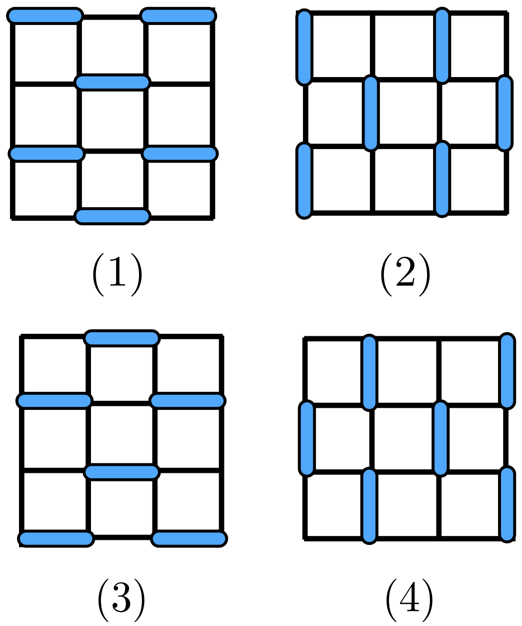

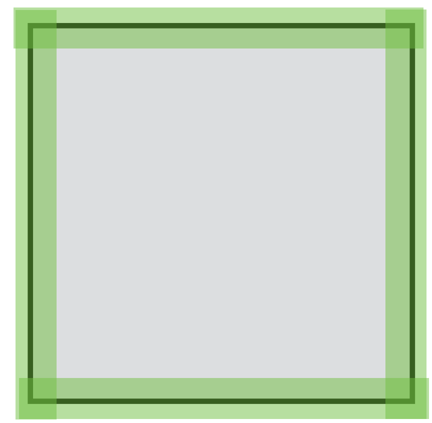

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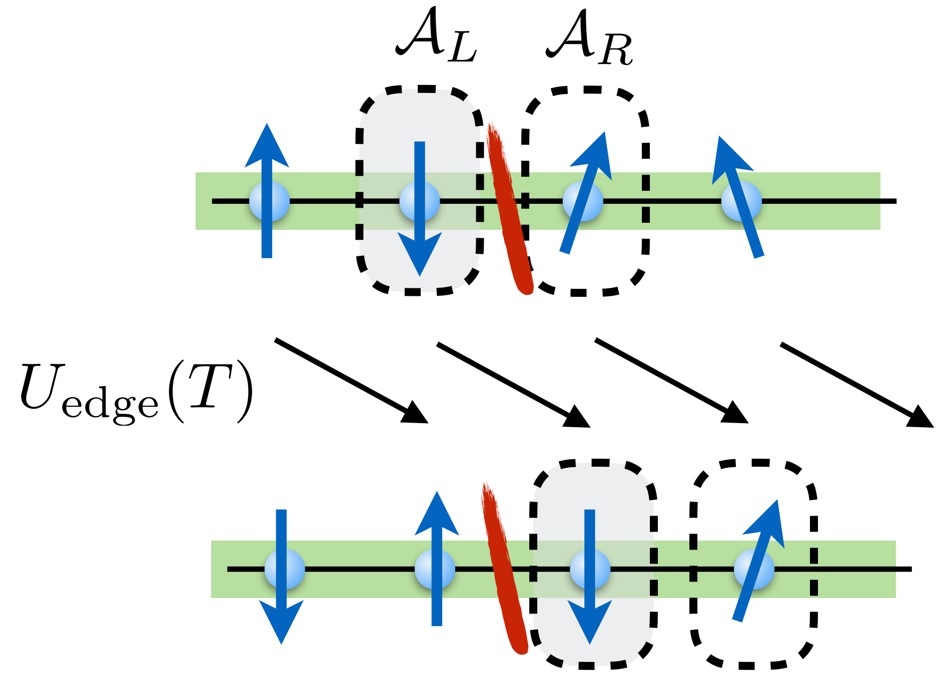

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Contrast to quantum Hall edge states

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

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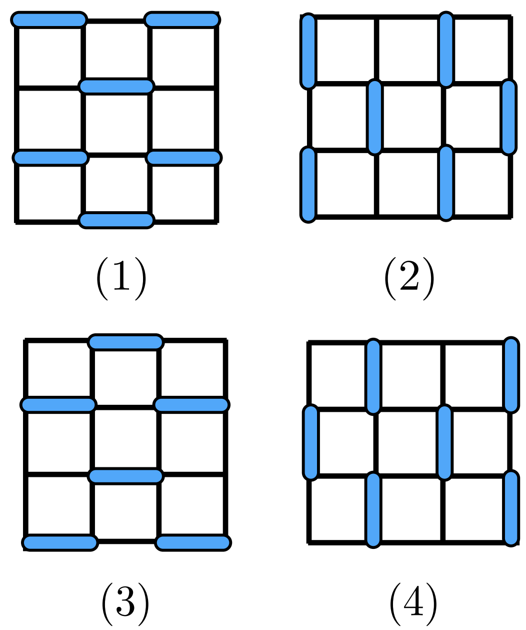

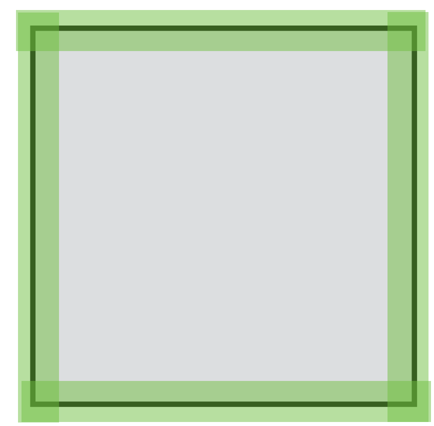

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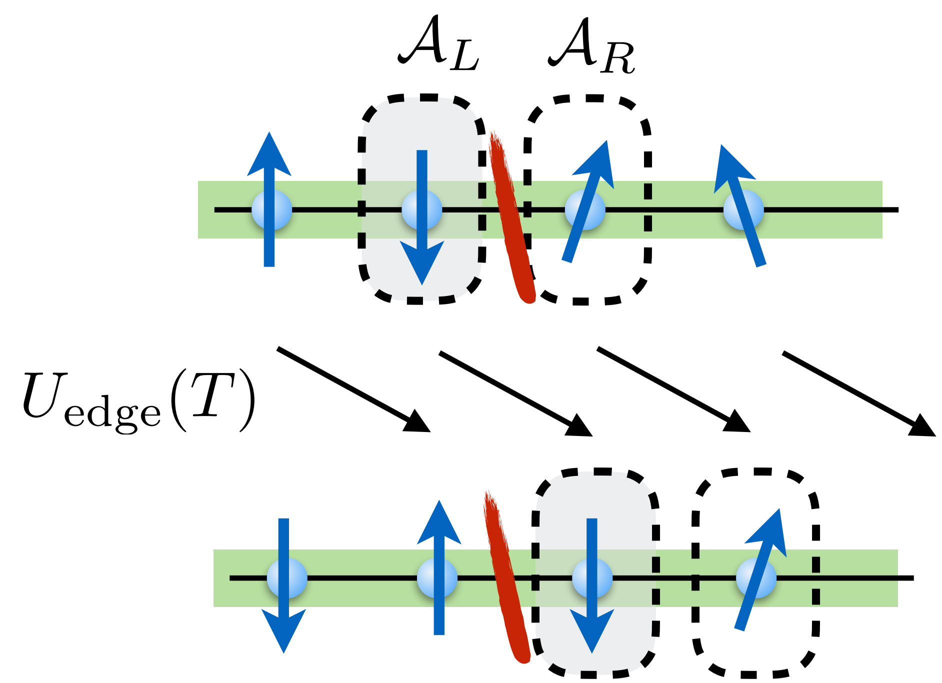

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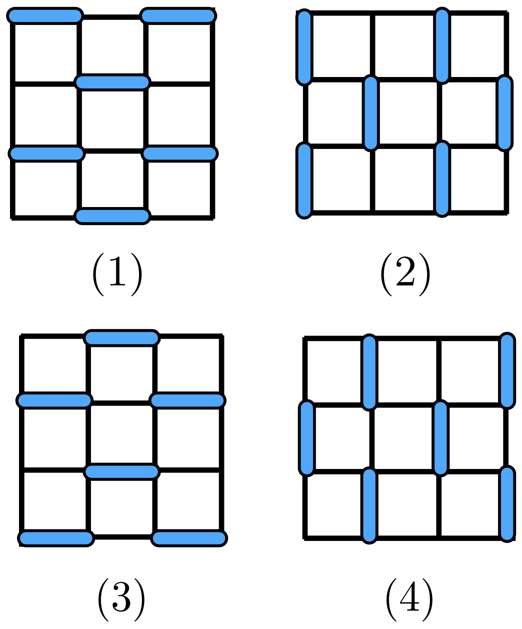

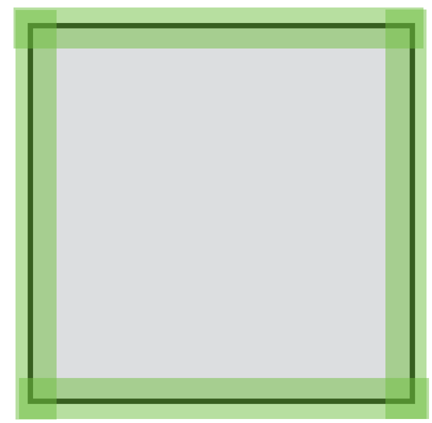

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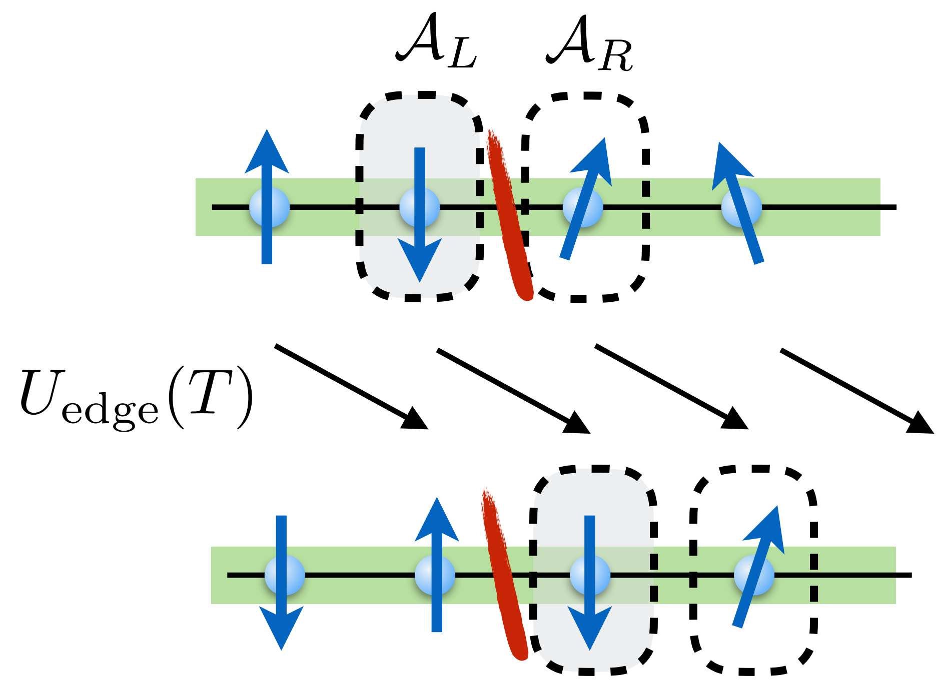

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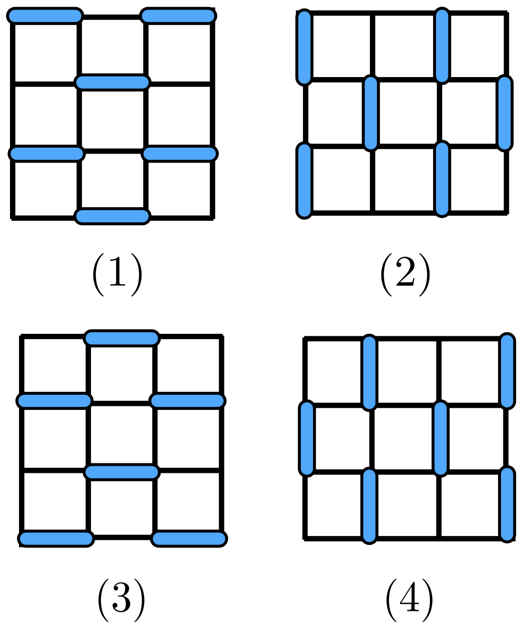

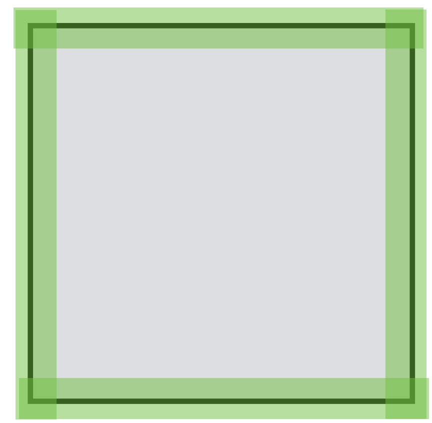

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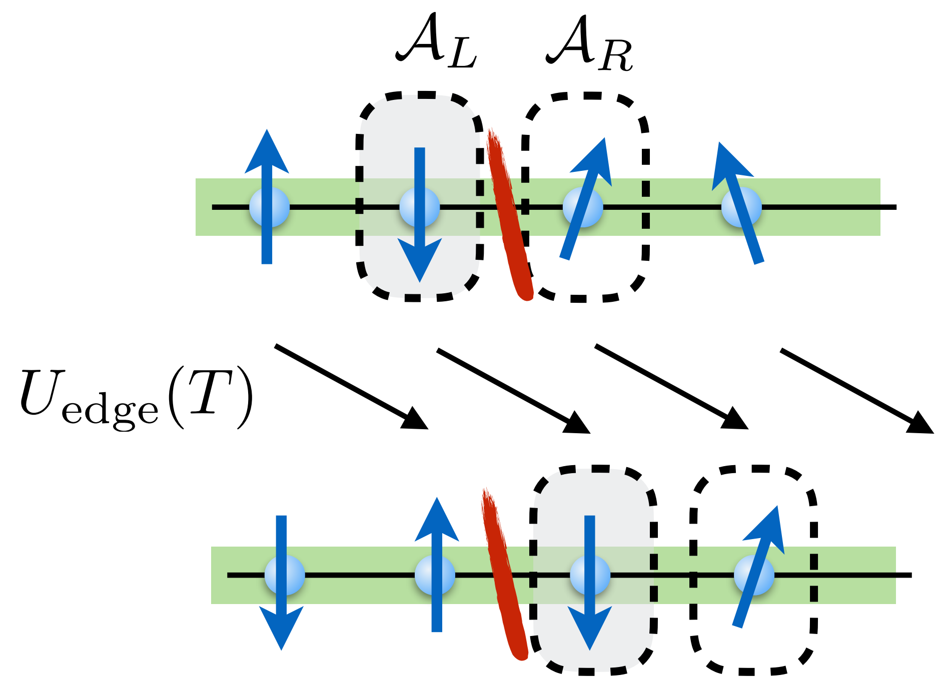

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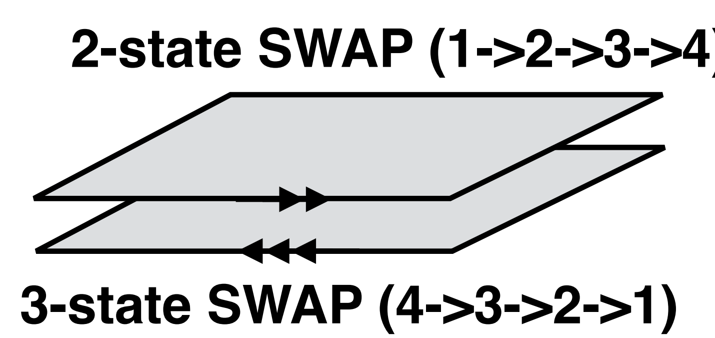
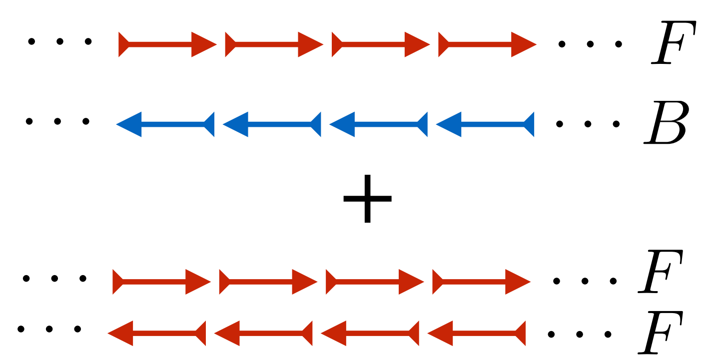
- QH edges don't sharply exist out of equilibrium
- Multiplicative rather than additive structure of index



$$\nu = \log \frac{2}{3}$$

Po, Fidkowski, Morimoto, ACP, Vishwanath arXiv:1609.00006

Fermion chiral Floquet phases



= trival

Fermion state equivalent to 2-state boson w/ nu=log(2)

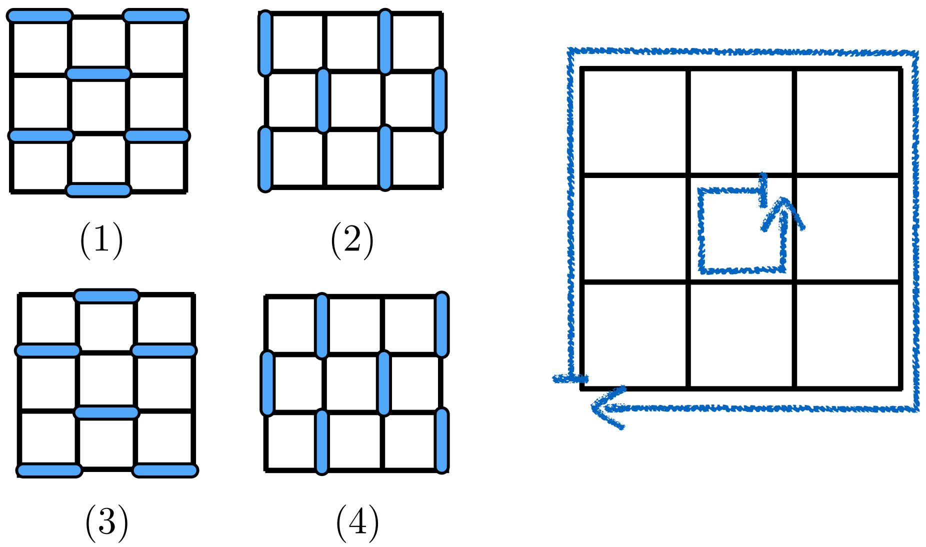
- Can show: counter-propagating fermion and boson edges can be deformed to a trivial state
  (requires edge-reconstruction by non-chiral modes — stable topological equivalence)
- Contrast to equilibrium:
  8x minimal fermion chiral state = minimal Boson c.s.
- Remaining challenge: generalize formal index to deal with fermionic operator algebras

$$\nu_F = \log 2$$

Fractional chiral Floquet phases?

Po, Fidkowski, Morimoto, ACP, Vishwanath, arXiv ‘16

Fractional chiral Floquet phases



Po, Fidkowski, Morimoto, Vishwanath, ACP to appear

Fractional chiral Floquet phases

Non-fractional:

- U(T) ~ 1 (bulk),
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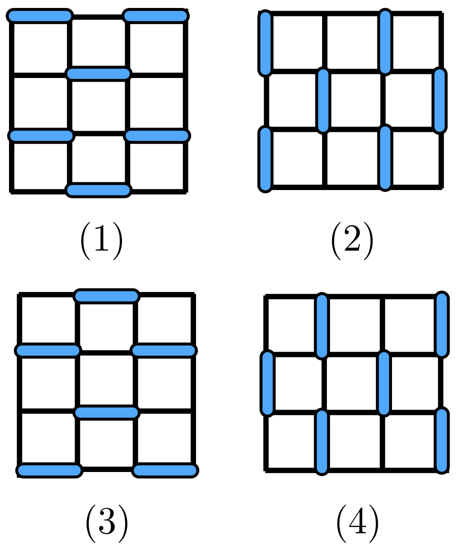
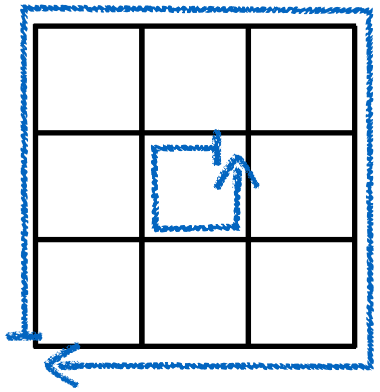
- Edge: pumps qudits with integer d





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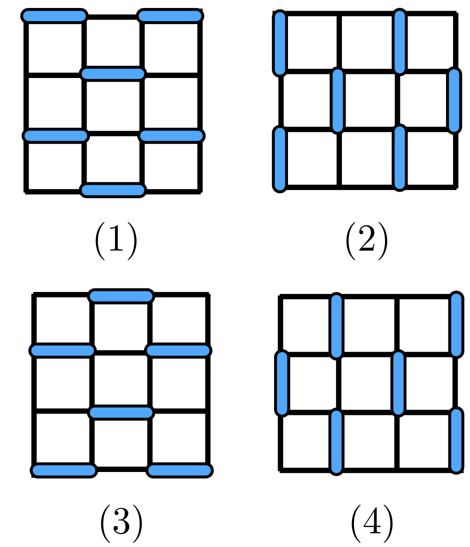

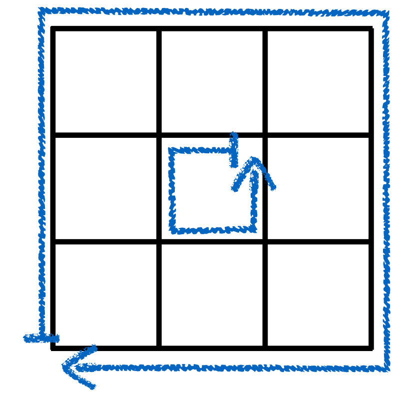

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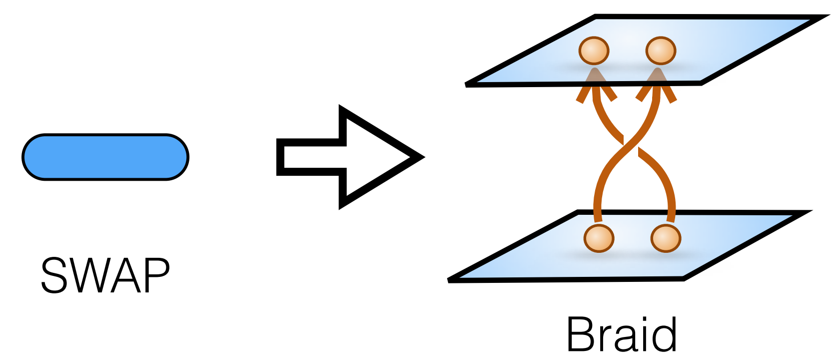

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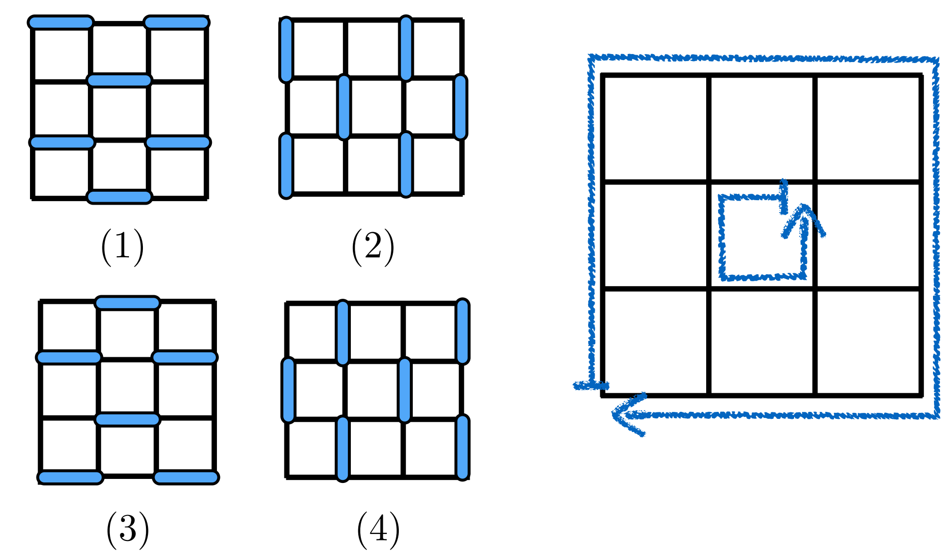

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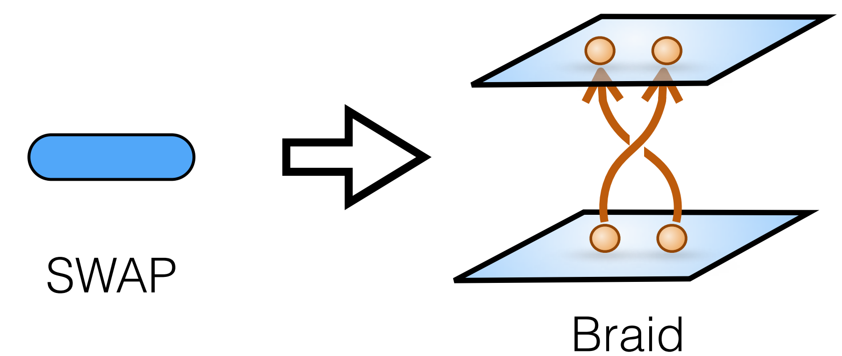

Localization: requires Abelian topological order

Potter, Vasseur arXiv '16

- Looks like d=1 only?
- No!: Can pump "sqrt" of an Abelian anyon

Po, Fidkowski, Morimoto, Vishwanath, ACP to appear

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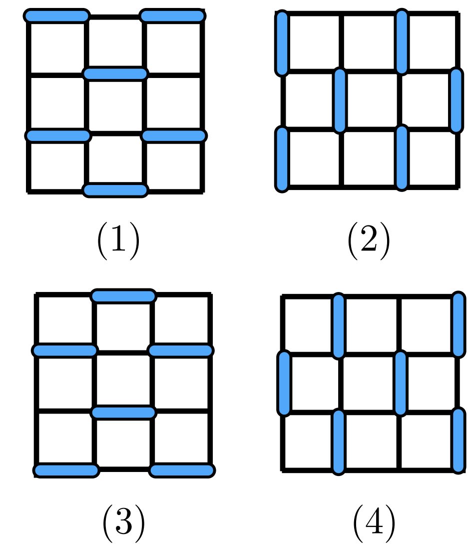

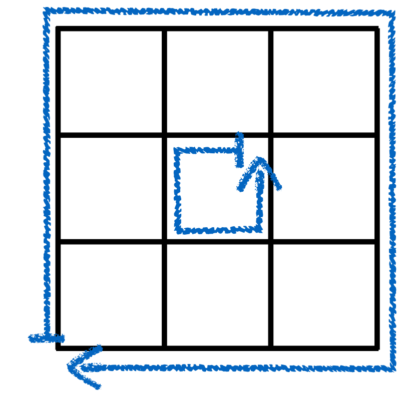

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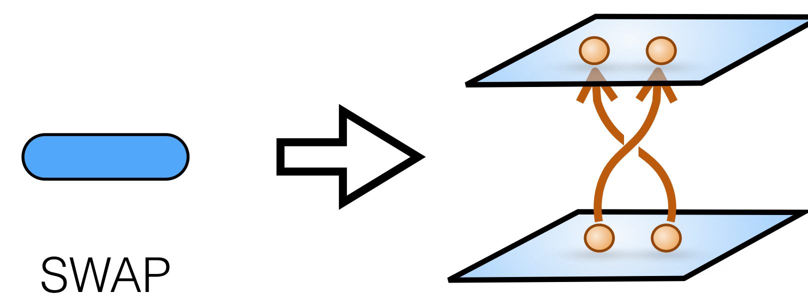

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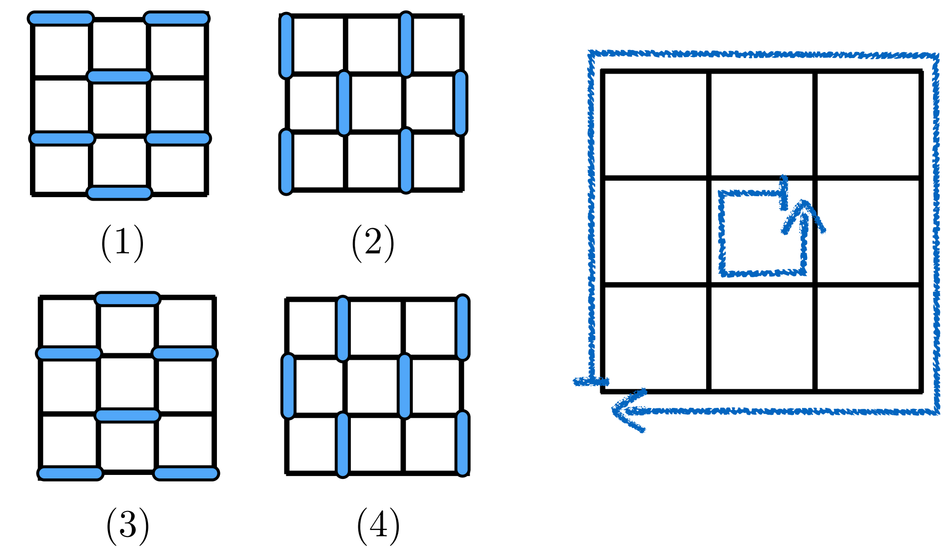

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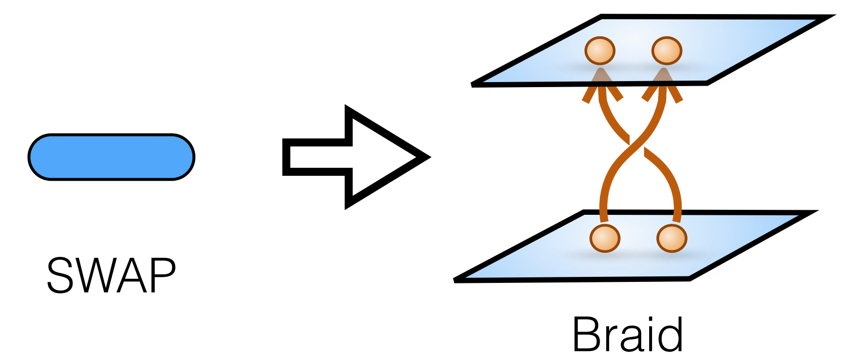

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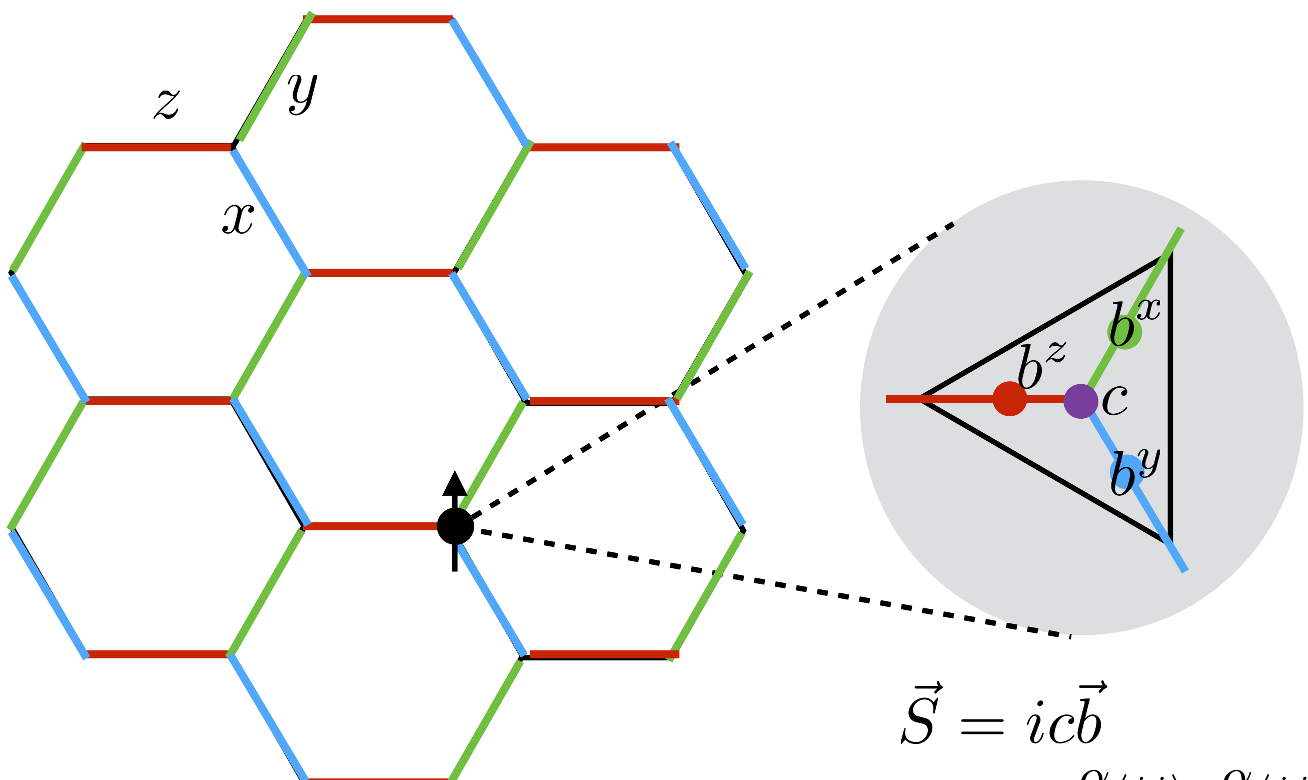
$\mathbf{Z}_2$ Example - Driven Kitaev Honeycomb model



Kitaev '05

$$\vec{S} = ic\vec{b}$$

$$\sigma_{ij} = ib_i^{\alpha\langle ij\rangle} b_j^{\alpha\langle ij\rangle}$$

$$S_i^\alpha S_j^\alpha = ic_i \sigma_{ij} c_j$$

- $H_1 = \frac{\pi}{2} \sum_{<ij>\in \mathrm{X}} S_i^x S_j^x$
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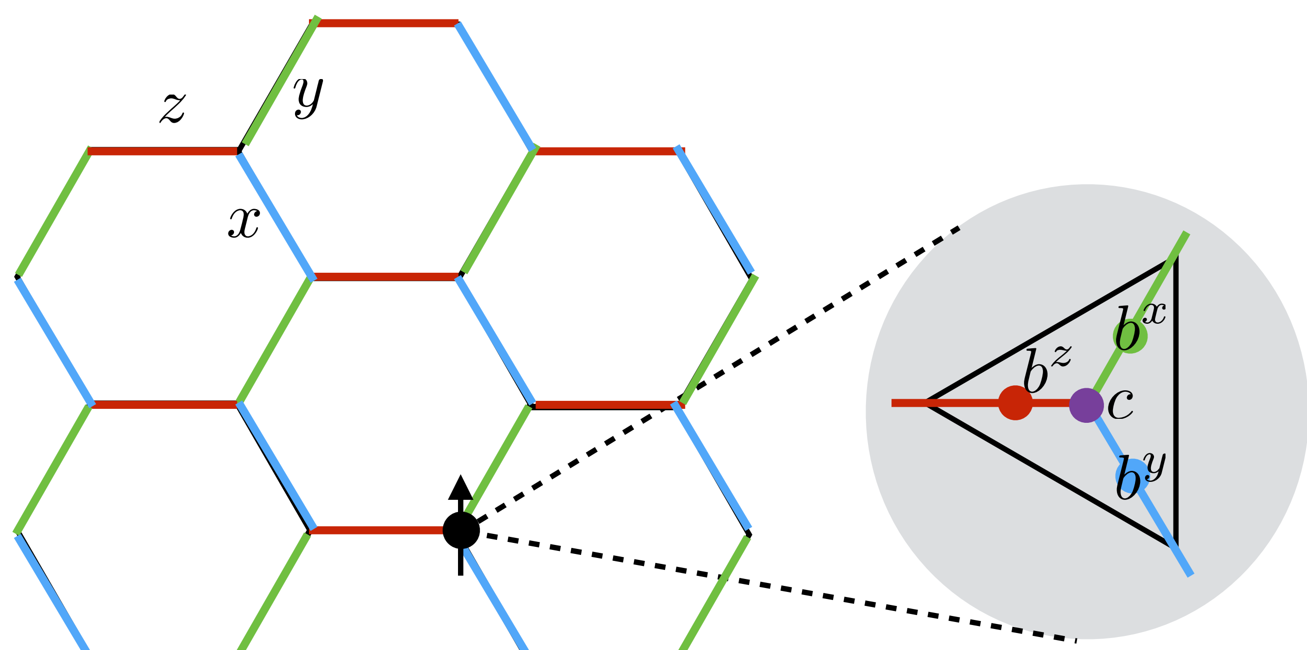

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Bulk:

- Bulk evolution = evolution w/r.t. static Hamiltonian w/ Z2 TO

Edge:

- c's pumped around edge chirally
- 2x Majorana edge = 1x fermion edge = spin-1/2 boson
- Fractional chiral index: $\nu_M = \frac{1}{2}\log 2$

Po, Fidkowski, Morimoto, Vishwanath, ACP to appear

“World Map” of (Driven) quantum matter