Identifying topological phases from microscopic models by DMRG

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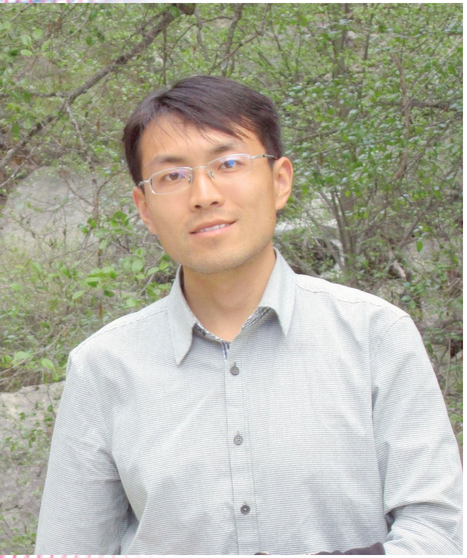
Collaborators



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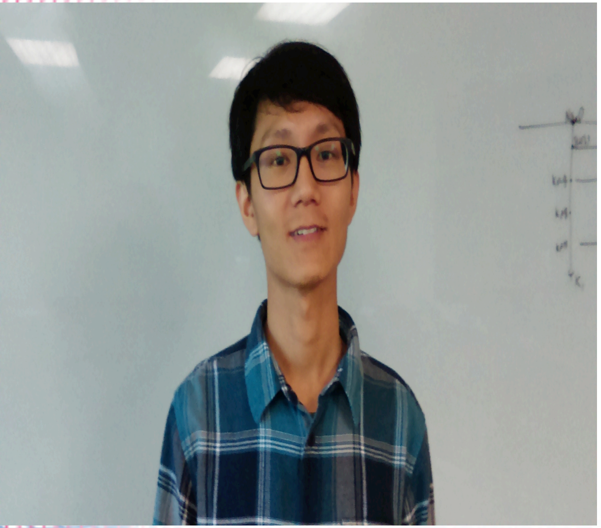

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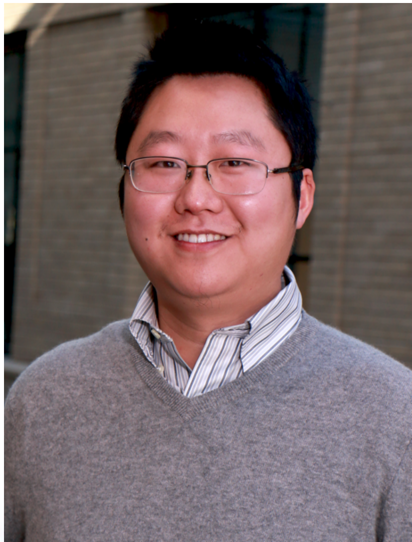

Outline

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Introduction

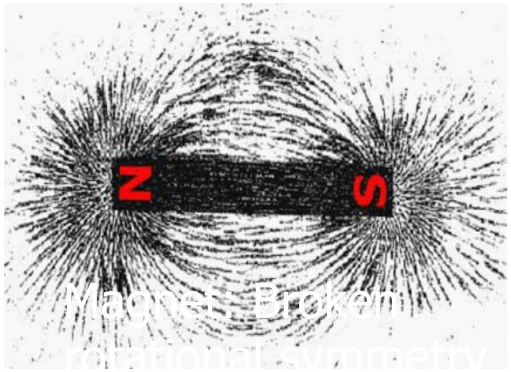
Interaction driven spontaneous QHE in time-reversal symmetric systems

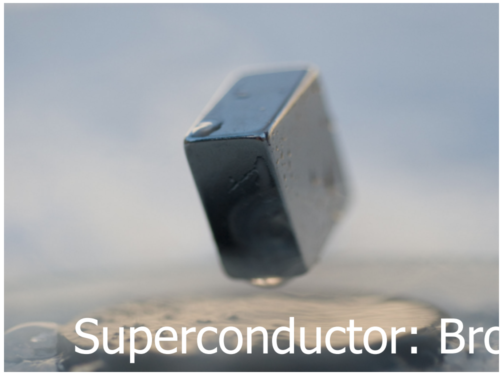
Identifying non-Abelian FQHEs

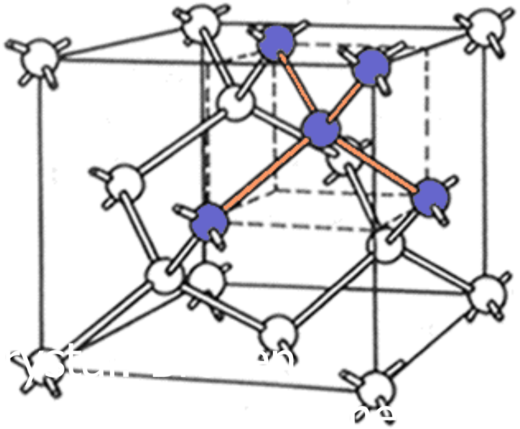
12/5 as Read-Rezayi state

1/2 FQHE driven by tunneling (331 Halperin to Moore-Read Pfafffian transition)

Different States of Matter





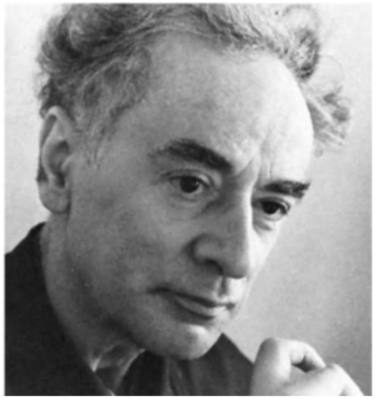


To understand the different states of matter,
and to classify them is a task of condensed matter physics!

Landau's Symmetry breaking theory

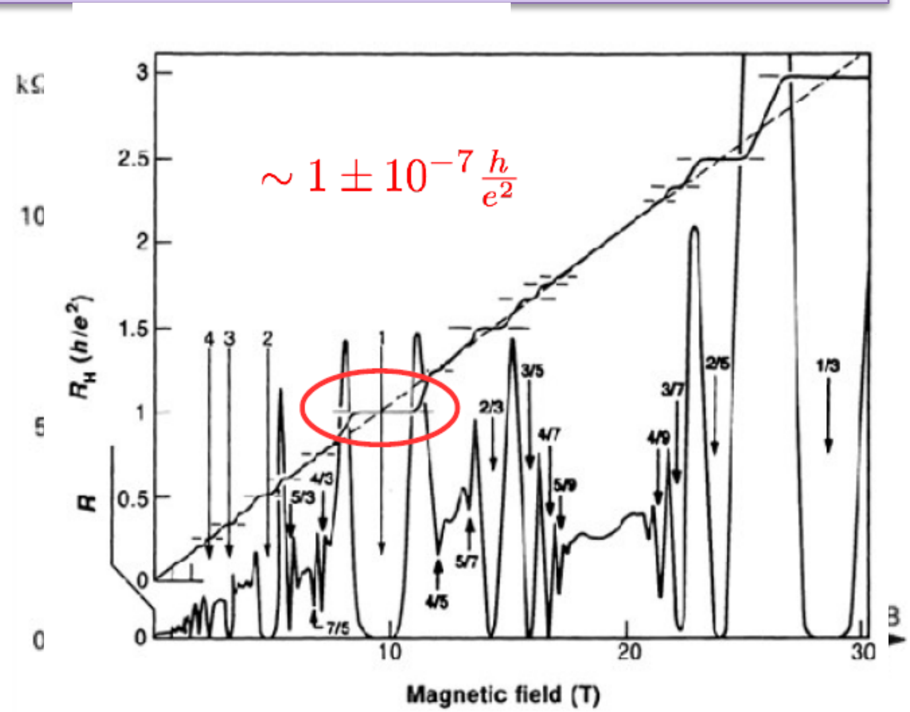
1. Different states of matter can be described by the different local order parameters.

2. Different orders come from different symmetry breaking.



Beyond Landau Theory

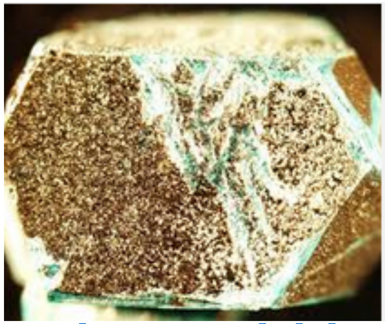
Integer quantum Hall state
Fractional quantum Hall state



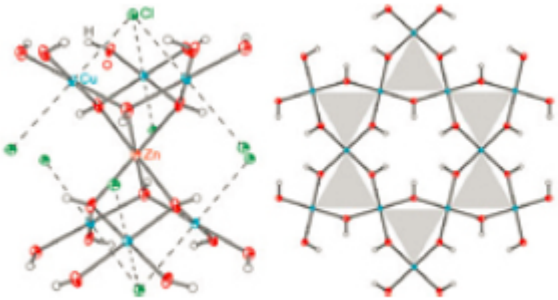
No local order parameter

No conventional symmetry breaking

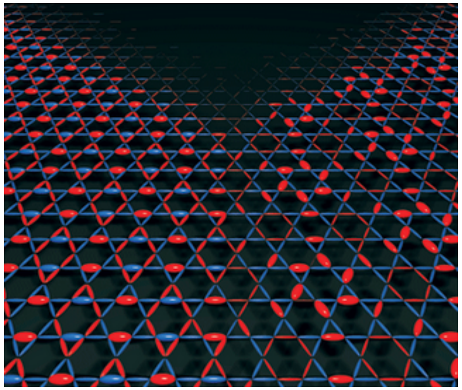
Gapped quantum spin liquids in magnetic materials



Herbertsmithite



$ZnCu_3(OH)_6Cl_2$



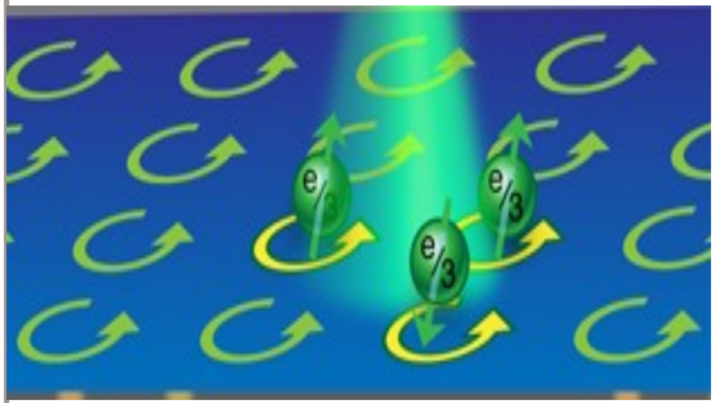
Topological States of Matter

Phase of matter, with topological order and long range entanglement. Wen et al (1989-1990) Kitaev (2003, 2006)

Bulk topological invariant and protected edge excitations.

Topological Order

- Fractional Quantum Hall
- Top degeneracy, and fractionalized quasiparticles



Topological band insulator (kane-Mele)

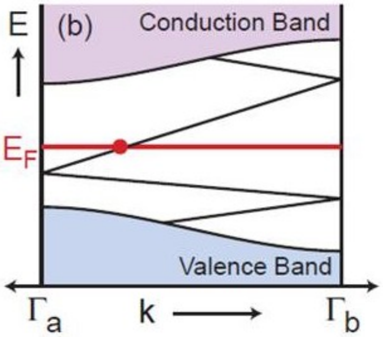
Integer Quantum Hall

Quantum Spin Hall

Quantum anomalous Hall (Haldane)

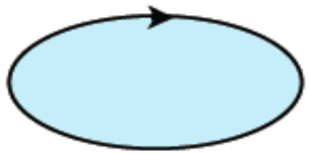
Energy gap in band structure

Non-zero topological number



Quantized Hall Conductance and Topological Chern Number Thouless et al (1982), Niu et al (1985)

- Bloch states $\psi_n(r) = e^{ik\cdot r} u_{n,k}(r)$
- Berry gauge field $\vec{\mathcal{A}}_n = i\langle u_n|\vec{\nabla}_k|u_n\rangle$



$$\vec{\mathcal{B}}_n = \vec{\nabla}_k \times \vec{\mathcal{A}}_n$$

- Net Berry flux gives Chern number

$$q_n = \frac{1}{2\pi}\int d^2k\, \mathcal{B}_n^z \;\in \mathbb{Z}$$

$$\sigma_{xy} = \frac{e^2}{h}\sum_{\epsilon_n<\epsilon_F} q_n$$

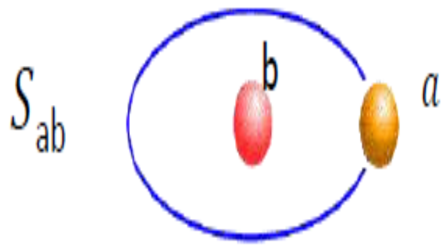
FQHE--- m degenerating groundstates will share a total integer Chern number—Sheng, Wan et al (2003)

Identifying different topological order (for FQHE states with the same Hall conductance)

•Topological order is beyond the Laudau symmetry-breaking paradigm, which cannot be detected by any local observable

•Numerical methods

- Groundstate degeneracy
- Wavefunction overlap with model wavefuctions
- Topological entanglement entropy (*Kitaev & Preskill, Levin & Wen*)
- Entanglement spectrum (*Li & Haldane*)
- Modular matrix (*Wen 1990, Zhang & Vishwanath*,201

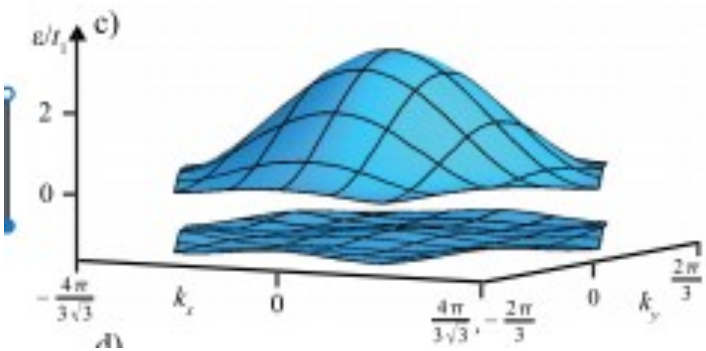


•Modular matrix

- S matrix: the element S_{ij} *determines* the mutual statistics of i'th quasiparticle with respect to the j'th quasiparticle
- U matrix: the element Uii determines the self-statistics (topological spin) of the i'th quasiparticle.

Application : Flat-band model

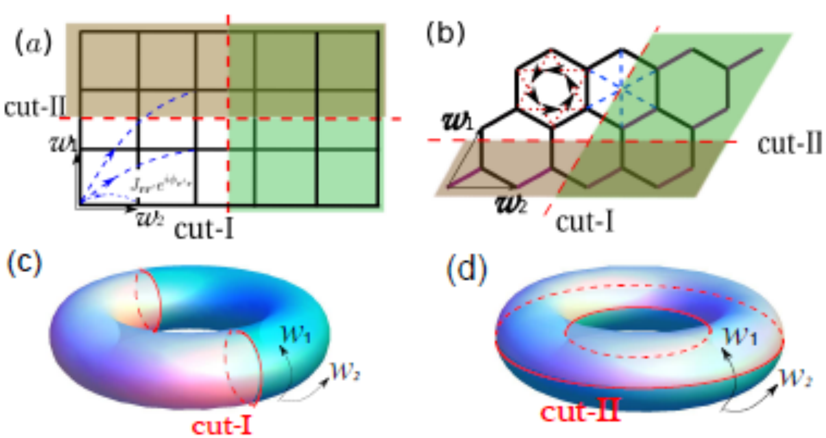
$$H = \sum_{\mathbf{r}\mathbf{r}'} \left[J_{\mathbf{r}\mathbf{r}'} e^{i\phi_{\mathbf{r}'\mathbf{r}}} b^\dagger_{\mathbf{r}'} b_{\mathbf{r}} + \mathrm{H.c.} \right] + \sum_n \frac{U_n}{n!} \sum_{\mathbf{r}} (b^\dagger_{\mathbf{r}})^n (b_{\mathbf{r}})^n$$

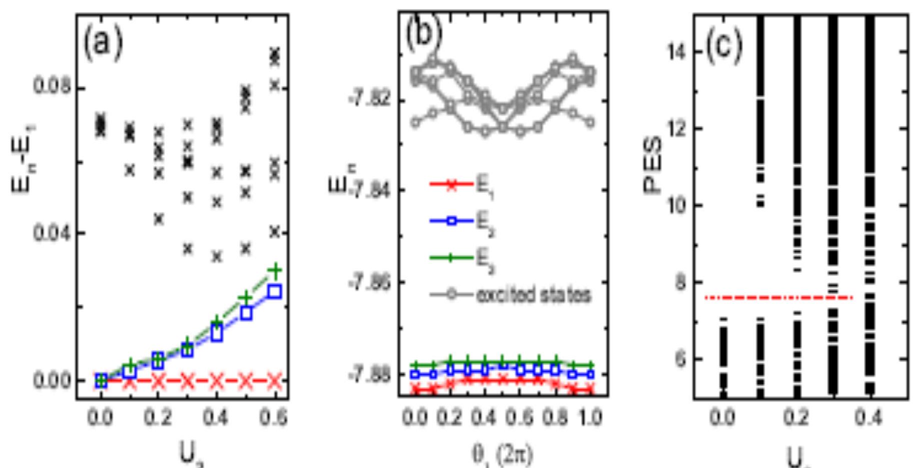


F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).
E. Kapit, et. al, Phys. Rev. Lett. Lett. 105, 215303 (2010).
E. Tang, et. al, Phys. Rev. Lett. **106**, 236802 (2011).
T. Neupert, et al, Phys. Rev. Lett. **106**, 236804 (2011).
K. Sun, et. Al, Phys. Rev. Lett. **106**, 236803 (2011).

Modular matrices: Zhu et al (Phys. Rev. 2013-2015)
Cicino & Vidal PRL(2013)

Moore-Read (SQ lattice)





Moore-Read State

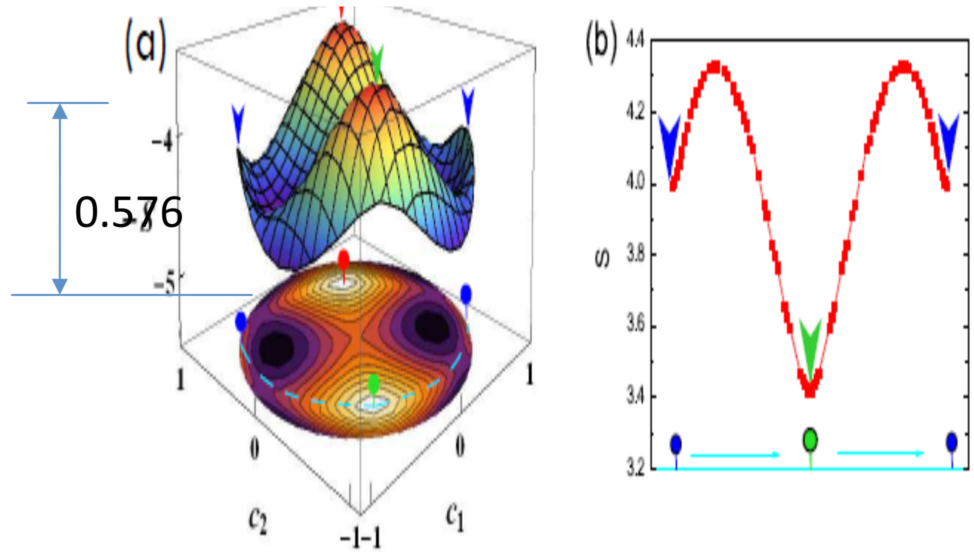
Zhu et al (2013)

Find MESs

$$|\Psi_{(c_1,c_2,\phi_2,\phi_3)}> = c_1|\xi_1> + c_2e^{i\phi_2}|\xi_2> + c_3e^{i\phi_3}|\xi_3>$$

$$<\Xi^{II}|\Xi^{I}> = \mathcal{U}^n\mathcal{S}^l\mathcal{U}^m$$

$$\mathcal{S} \approx \frac{1}{1.961}\begin{pmatrix} 1.000 & 1.025 & 1.373 \\ 0.929 & 0.920 & -1.431 \\ 1.392 & -1.376 & 0.037 \end{pmatrix}$$



Fusion rule

$$a \times b = \sum_c N_{ab}^c c \text{ where } N_{ab}^c = \sum_m \mathcal{S}_{am}\mathcal{S}_{bm}\mathcal{S}^*_{mc}/\mathcal{S}_{1m}$$

$$\sigma \times \sigma \approx 1.0051 + 1.056\psi + 0.096\sigma$$

CFT prediction

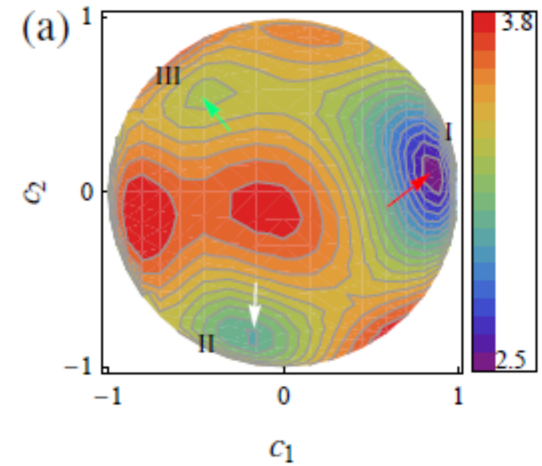
$$\mathcal{S} = \frac{1}{2}\begin{pmatrix} 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 0 \end{pmatrix}$$

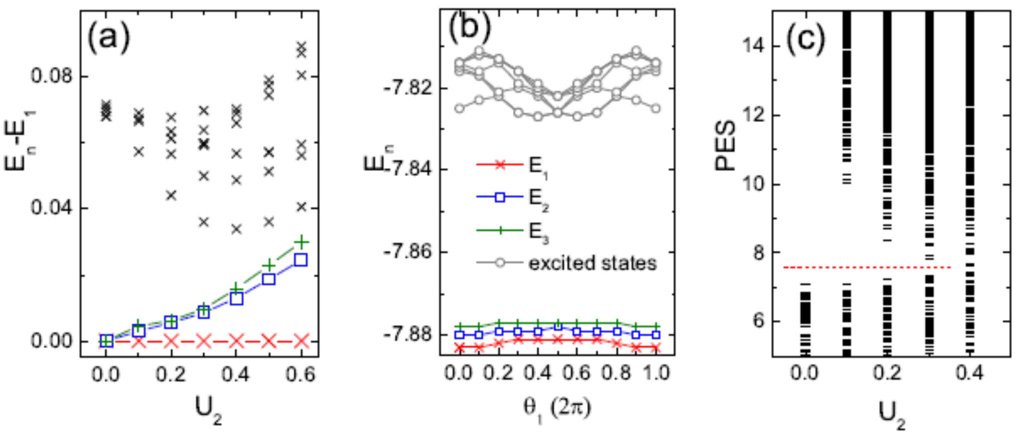
$$\sigma \times \sigma = \mathbb{1} + \psi$$

Quantum Phase Transition

Modular matrix can be used as ‘order parameter’ for topological phase transition

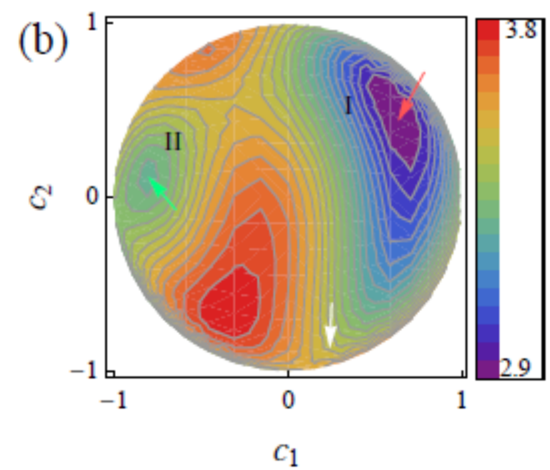
$$\mathcal{S} = \frac{1}{1.965}\begin{pmatrix} 1.000 & 1.041 & 1.316 \\ 1.006 & 0.888 & -1.448 \\ 1.334 & -1.440 & 0.028 \end{pmatrix}$$

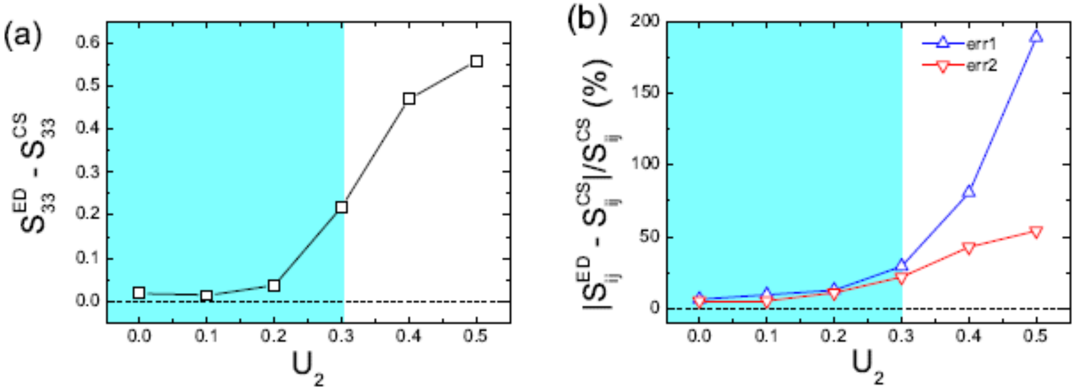




Quantum phase transition occurs at 0.3<U2<0.4

$$\mathcal{S} = \frac{1}{3.731}\begin{pmatrix} 1.000 & 2.873 & 2.037 \\ 2.899 & 0.750 & 2.354 \\ 2.015 & 2.354 & 2.082 \end{pmatrix}$$

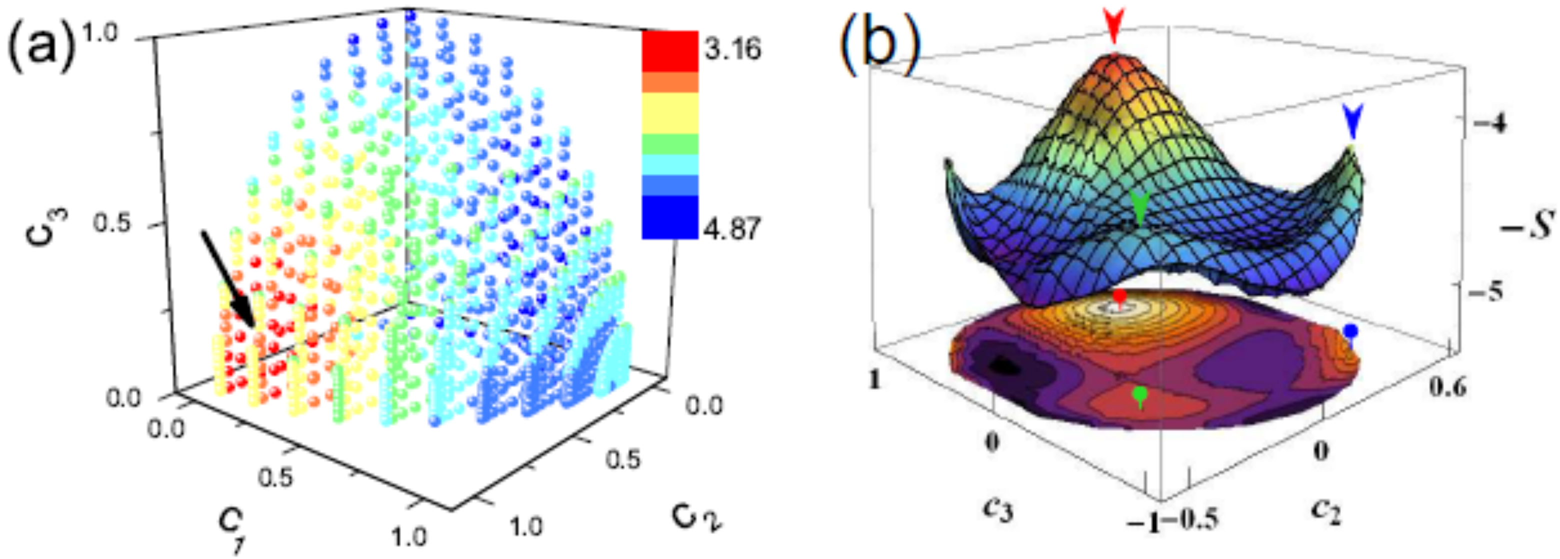




S faithfully represents the quasiparticle information for *U2 <=0.3 with the accuracy error* less than 30%

Read-Rezayi State

$$|\Psi>= c_1|\xi_1> + c_2 e^{i\phi_2}|\xi_2> + c_3 e^{i\phi_3}|\xi_3> + c_4 e^{i\phi_4}|\xi_4>$$



$$\mathcal{S} \approx \mathcal{S}_{pf} \otimes \mathcal{S}_{U(1)} + 10^{-2} \times \begin{pmatrix} 4.3 & -3.4 & -0.4 & 1.1 \\ -2.7 & -3.1 & -0.9 & -0.1 \\ 2.1 & -0.8 & 2.4 & -0.7 \\ 0.0 & -1.0 & -0.1 & -1.9 \end{pmatrix}$$

Fibonacci quasiparticle

$$\mathcal{S} = \mathcal{S}_{pf} \otimes \mathcal{S}_{U(1)} = \frac{1}{\sqrt{2+\phi}} \begin{pmatrix} 1 & \phi \\ \phi & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\phi = \frac{1+\sqrt{5}}{2}$$

golden ratio number

Entanglement spectrum and edge excitations

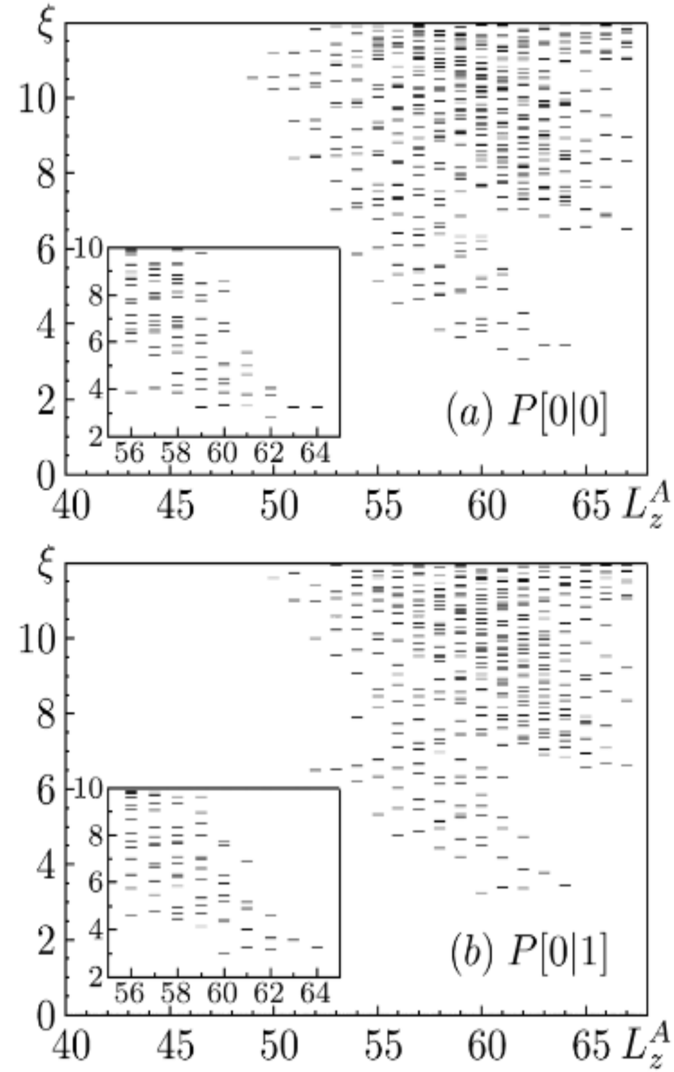
Li and Haldane (2008)--friendly for DMRG

the ground state can be written, according to Schmidt decomposition, as

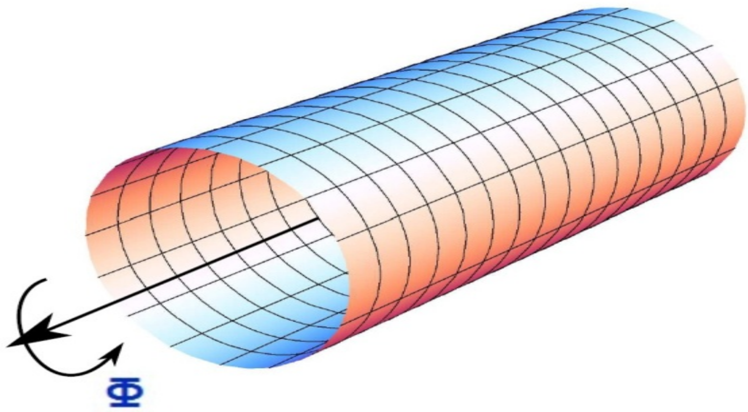
$$|\psi\rangle = \sum_i e^{-\xi_i/2} |\psi_A^i\rangle \otimes |\psi_B^i\rangle, \quad (2)$$

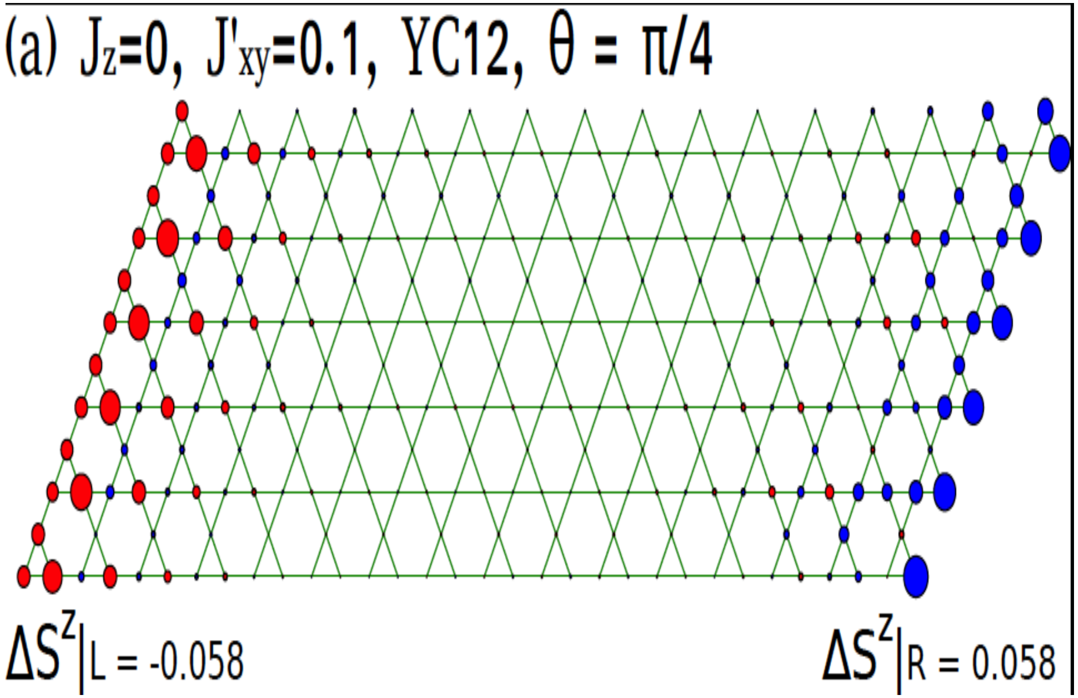
TABLE I. In this table, we count the root configurations of the MR Pfaffian edge excitations the $\Delta L_z = 0, 1, 2, 3, \cdots$ sector.

$\Delta L_z = 0$	$\Delta L_z = 1$	$\Delta L_z = 2$
1100110011|0000	1100110010|1000	1100110010|0100
		1100110001|1000
		1100101010|1000



Adiabatically adding flux into cylinder to detect topological phase

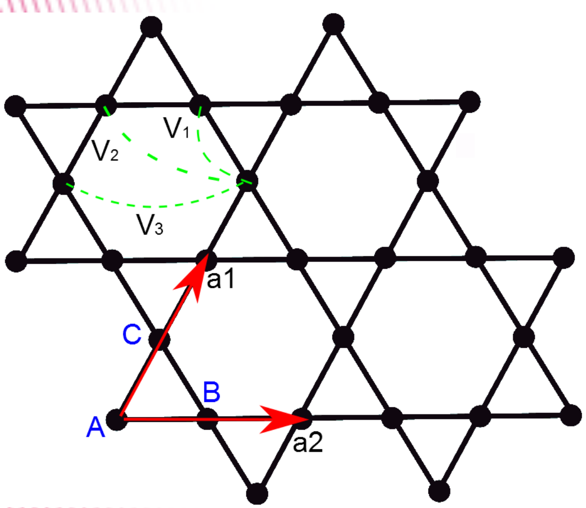


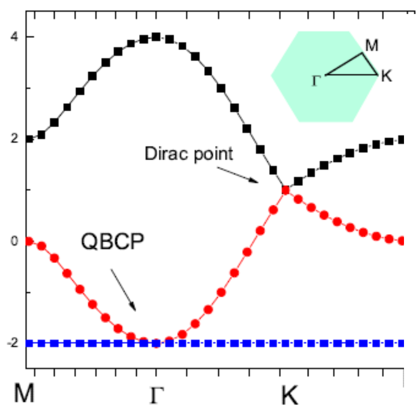


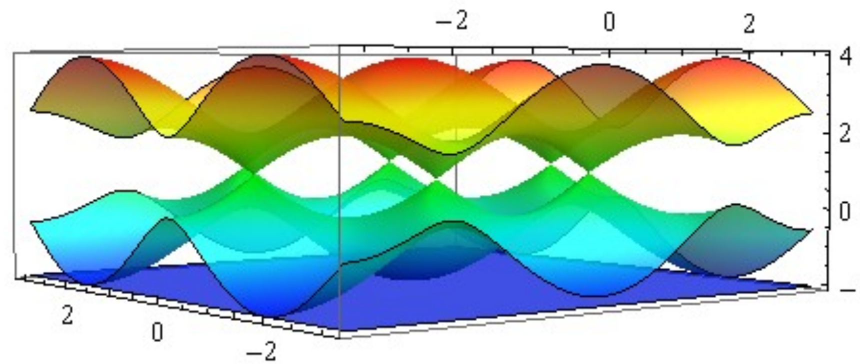
Gong, Zhu, Sheng   Scientific Report 2014
He, Sheng, Chen, PRL 2014
Zeletal et al.   2013, 2014.

Interaction-driven QAH ---- Fermion-Hubbard on kagome at 1/3

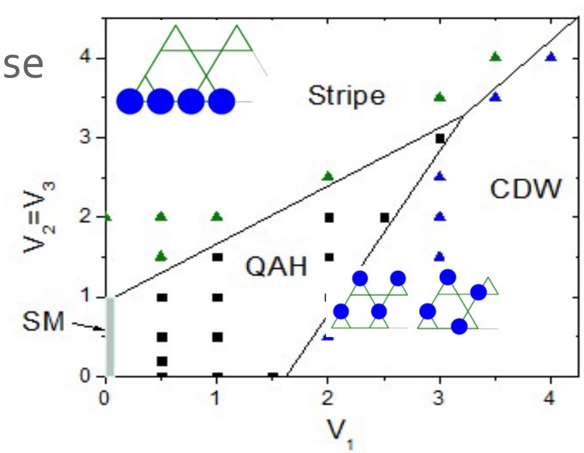
$$H = t\sum_{\langle\mathbf{r}\mathbf{r}'\rangle}\left[c^{\dagger}_{\mathbf{r}'}c_{\mathbf{r}} + \mathrm{H.c.}\right] + V_1\sum_{\langle\mathbf{r}\mathbf{r}'\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_2\sum_{\langle\langle\mathbf{r}\mathbf{r}'\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_3\sum_{\langle\langle\langle\mathbf{r}\mathbf{r}'\rangle\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'}$$







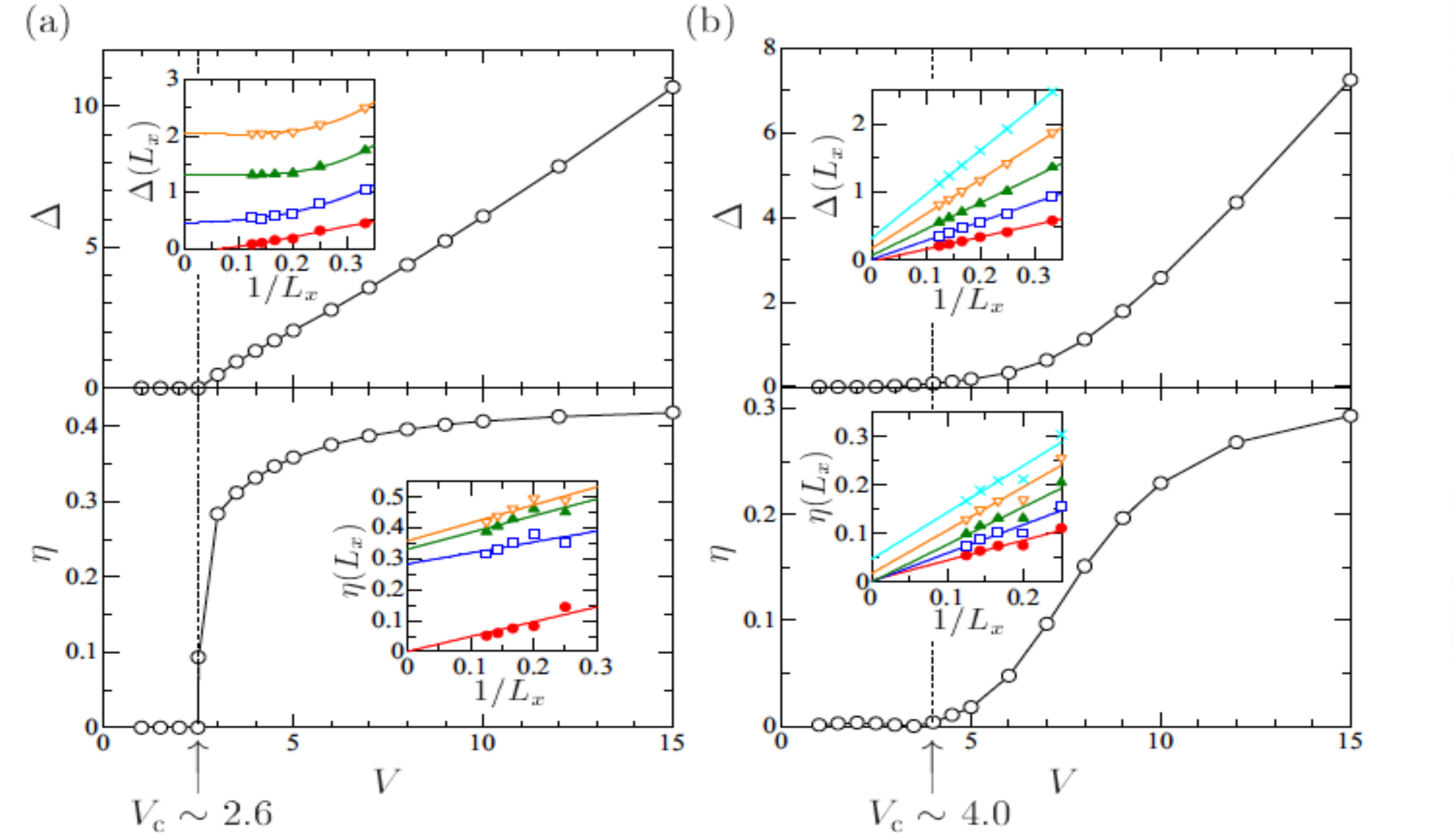
Mean-Field Phase Diagram



QAH

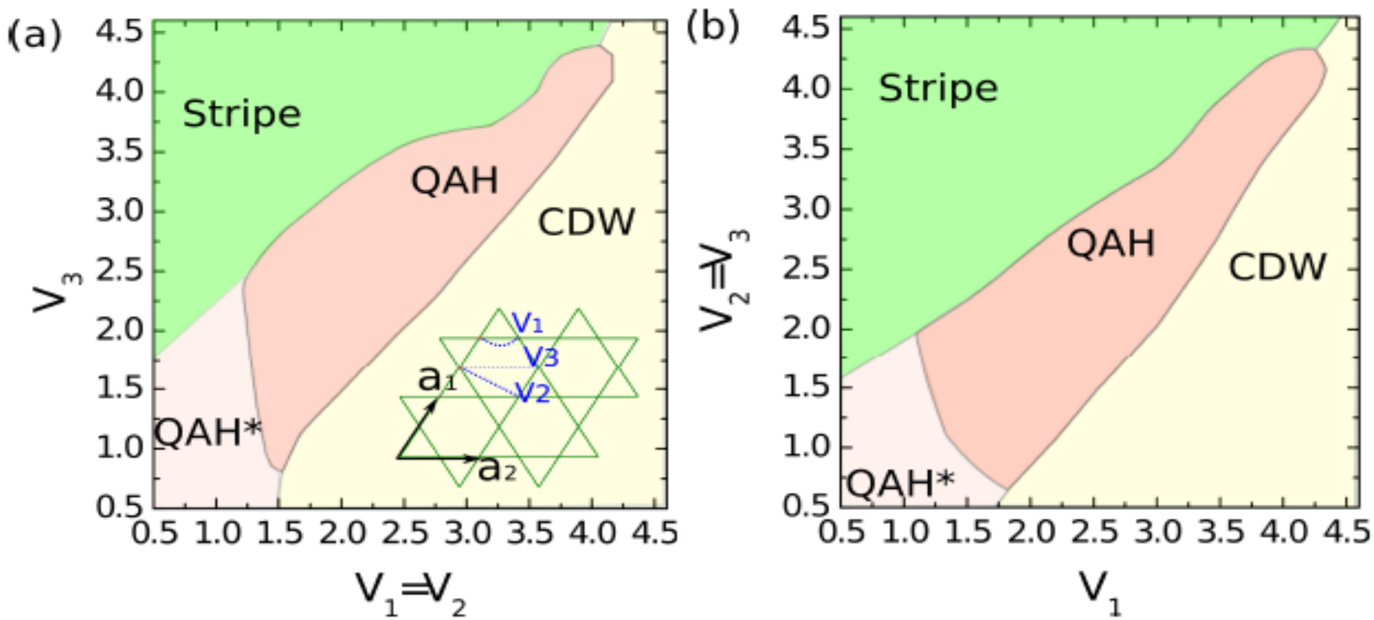
C. Wu et al. (2007),Bermann et al (2008)
D. Sun et al (2009)
Zhu, Gong, Zeng, Fu, Sheng (2016)

DMRG finds gapless and charge ordered phases for V1 (NN) interaction model (Nishimoto et al 2010)



Interaction-driven QAH ---- Fermion-Hubbard on kagome at 1/3

$$H = t \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left[c^{\dagger}_{\mathbf{r}'} c_{\mathbf{r}} + \mathrm{H.c.} \right] + V_1 \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_2 \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} n_{\mathbf{r}} n_{\mathbf{r}'} + V_3 \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} n_{\mathbf{r}} n_{\mathbf{r}'}$$



How to identify QAH? Using ED and DMRG

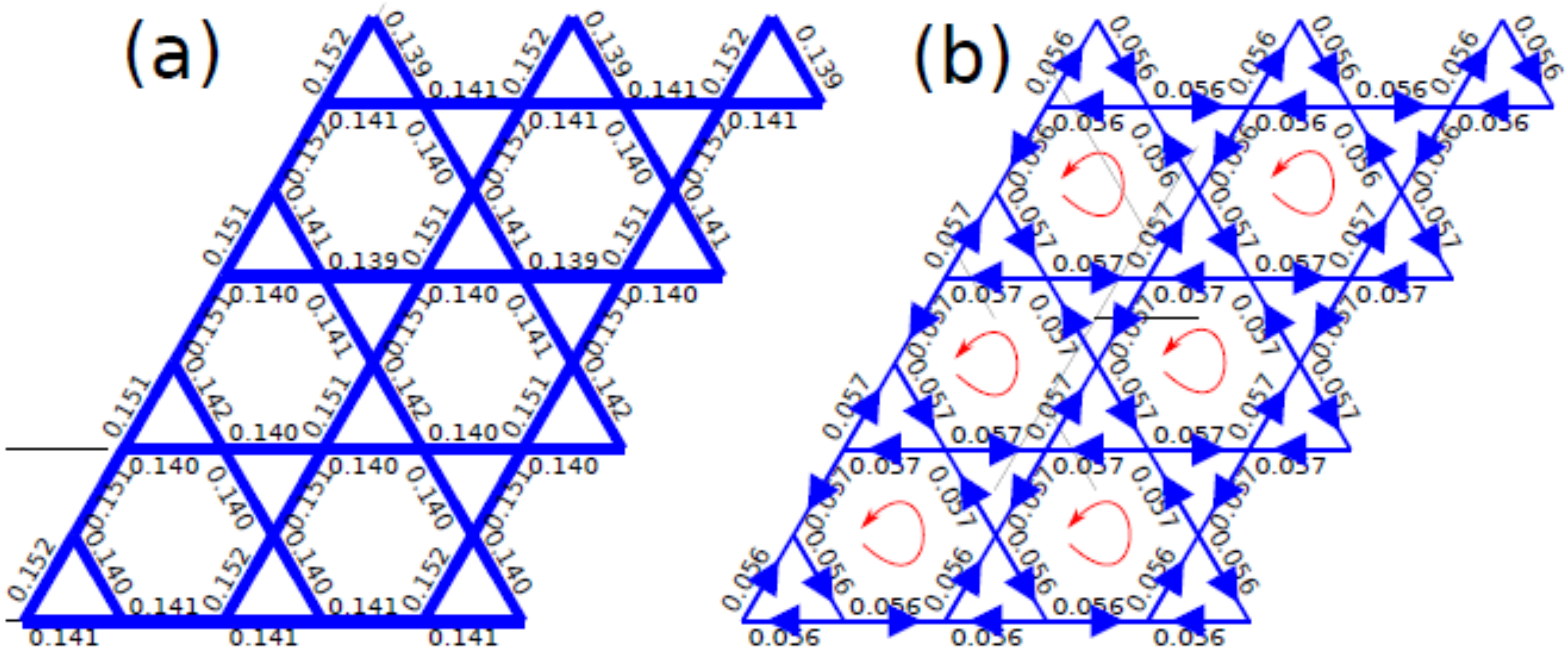
- Time-Reversal Symmetry Spontaneously Breaking
  - Doublet Ground state Degeneracy
  - Emergent Staggered Magnetic Flux
- Topological Chern Number (Hall Conductance)

Interaction-driven QAH ---- Emergent Magnetic Flux

$$H = t\sum_{\langle \mathbf{r}\mathbf{r}'\rangle}\left[c^{\dagger}_{\mathbf{r}'}c_{\mathbf{r}} + \mathrm{H.c.}\right] + V_1\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_2\sum_{\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_3\sum_{\langle\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'}$$

$$\mathcal{J}_{ij} = \langle \Psi^L | c^{\dagger}_i c_j | \Psi^L \rangle$$

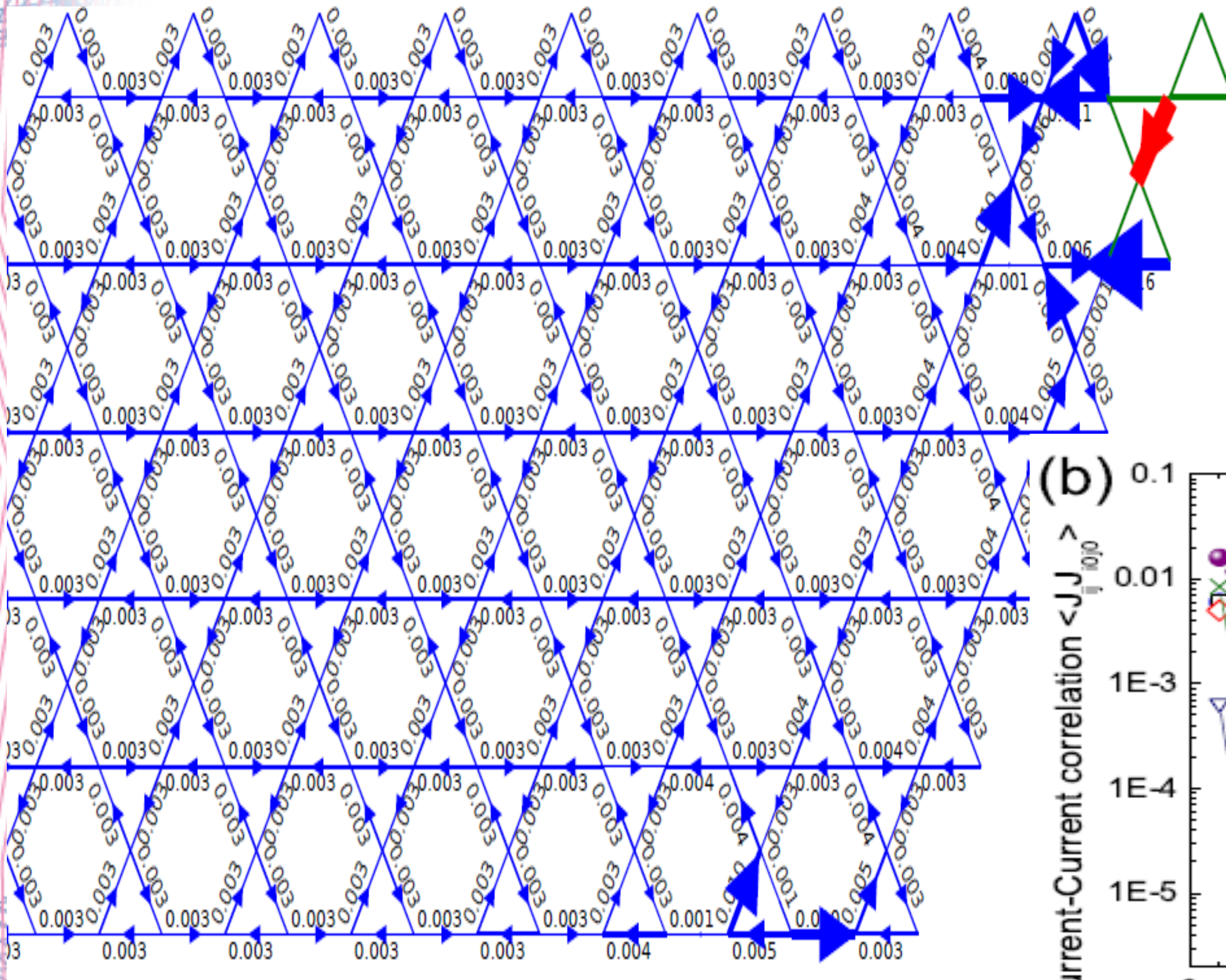
$Re\mathcal{J}_{ij}$ $Im\mathcal{J}_{ij}$

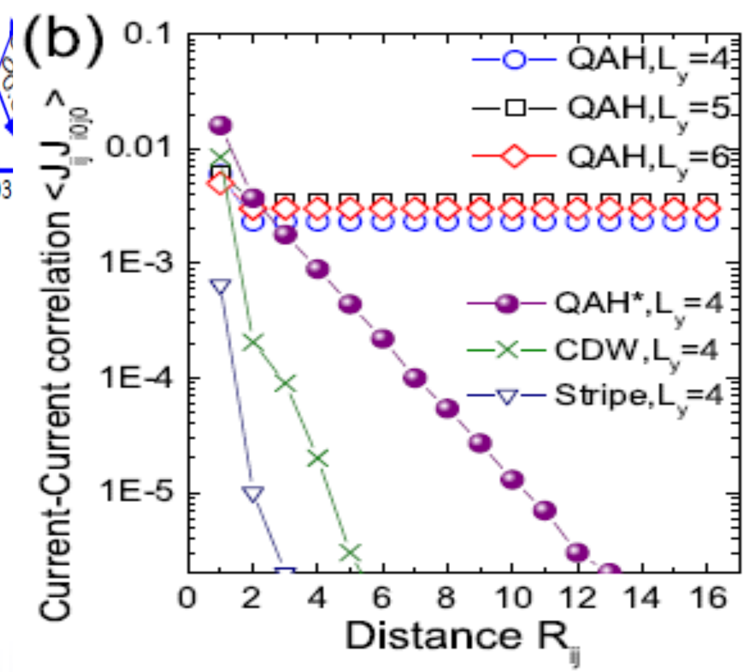


Staggered Magnetic Flux!!

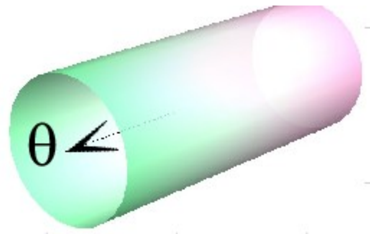
Current-Current correlation

$$\mathcal{J}_{ij} = i\langle\Psi^{L(R)}|c_i^\dagger c_j - c_j^\dagger c_i|\Psi^{L(R)}\rangle$$

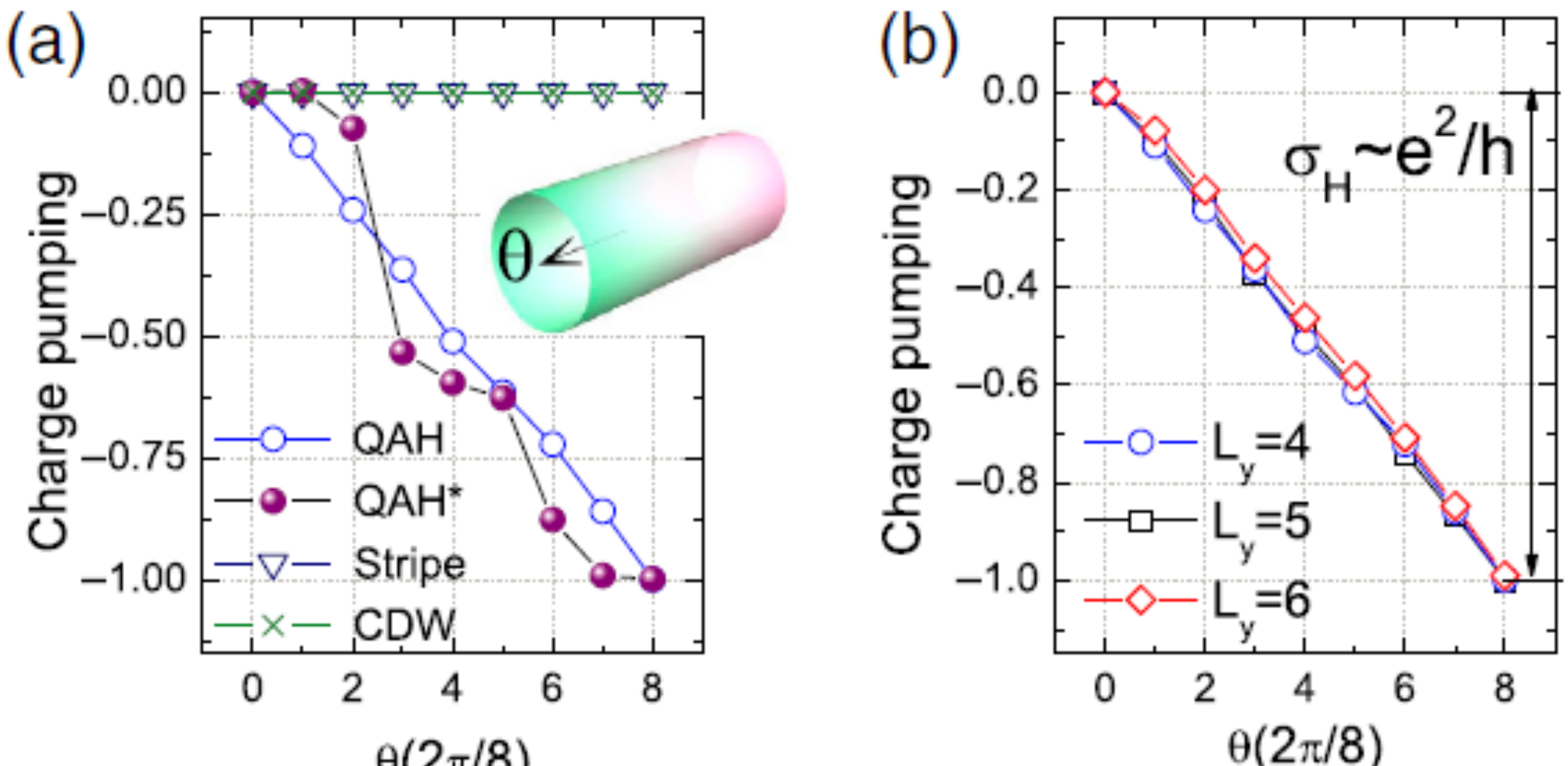




Interaction-driven QAH ---- Chern Number

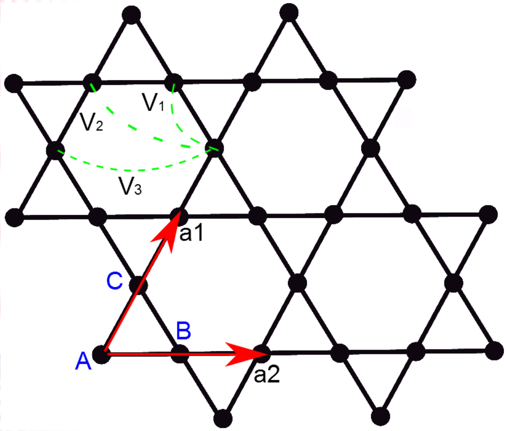


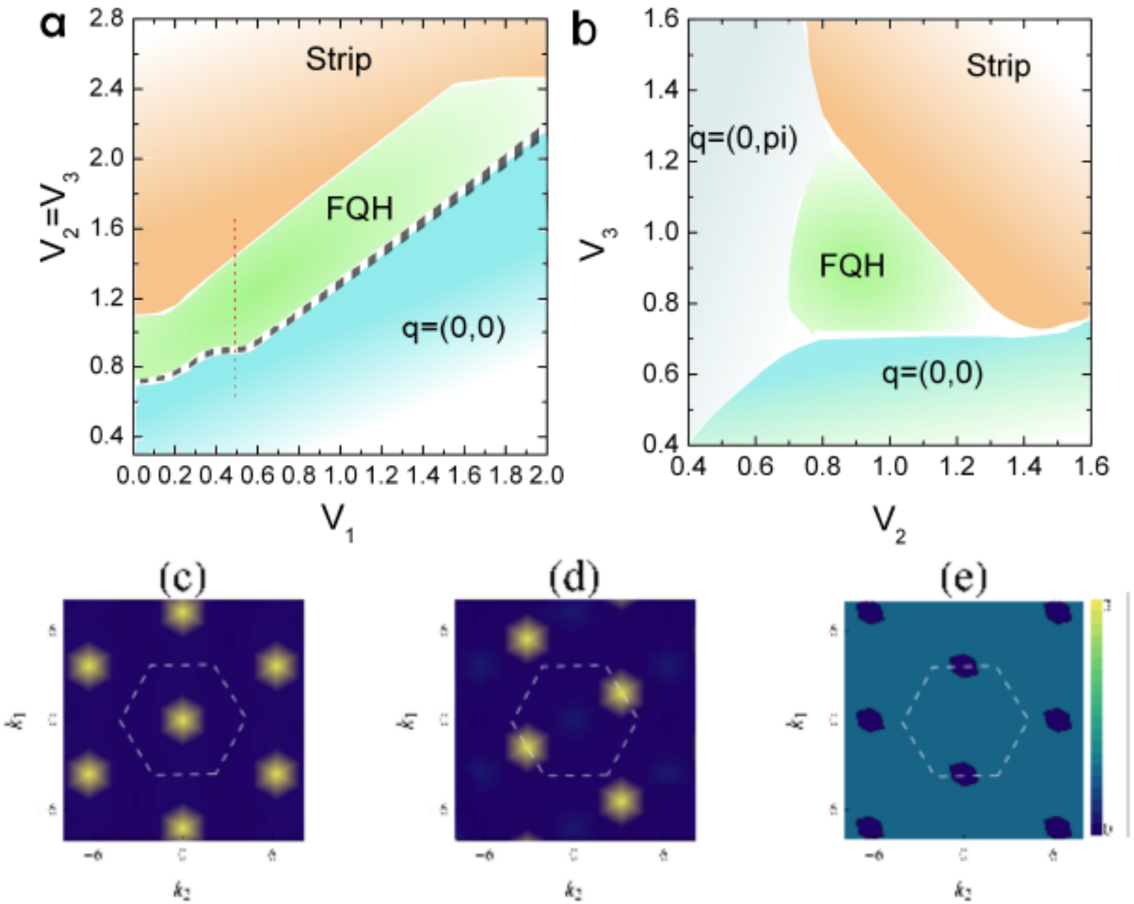
Chern Number (Hall conductance):
C = 1 (Left chirality)
C= -1 (Right chirality)



Boson at filling 1/3 ---- CSL (FQH)

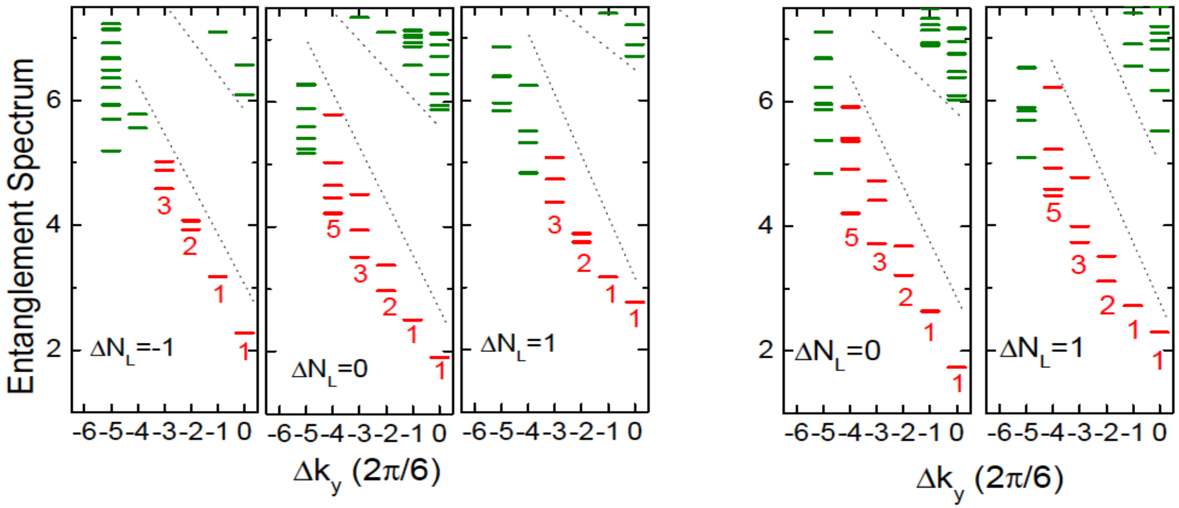
$$H = t\sum_{\langle \mathbf{r}\mathbf{r}'\rangle}\left[b^\dagger_{\mathbf{r}'}b_{\mathbf{r}} + \mathrm{H.c.}\right] + V_1\sum_{\langle \mathbf{r}\mathbf{r}'\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_2\sum_{\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'} + V_3\sum_{\langle\langle\langle \mathbf{r}\mathbf{r}'\rangle\rangle\rangle} n_{\mathbf{r}}n_{\mathbf{r}'}$$





Boson at filling 1/3: W Zhu, et. al, PRB 2016

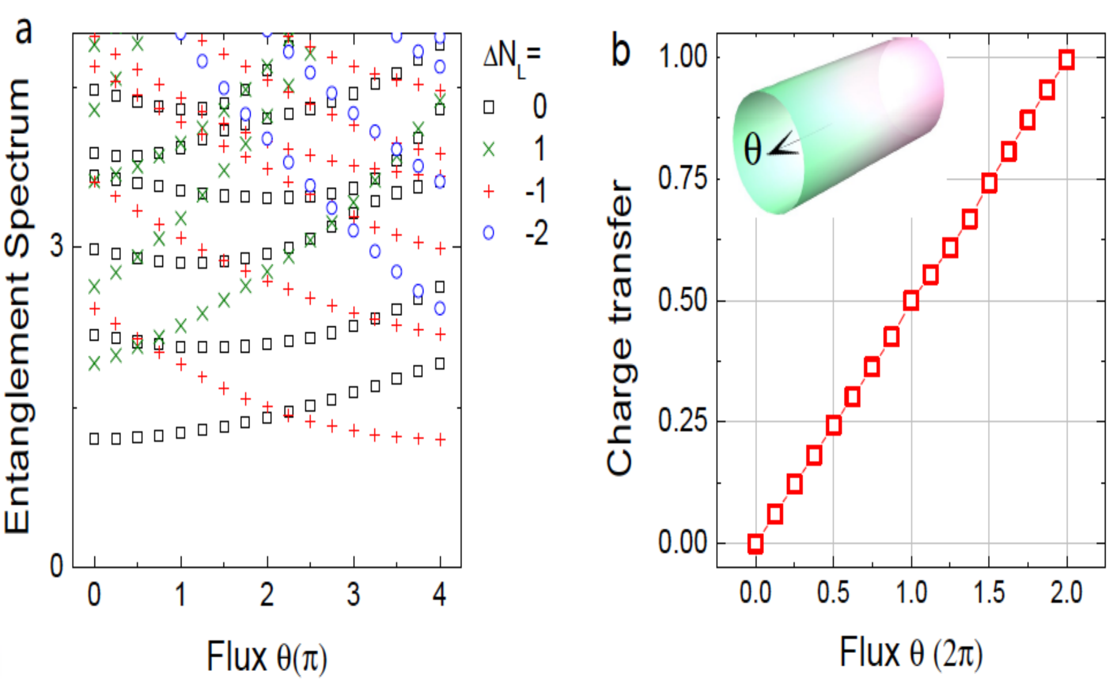
Entanglement spectra of CSL: emerging Laughlin ν=1/2 FQHE at 1/3 filling (Nb=1/3 N_site)



Inserting flux turns the ground state from vacuum to spinon sector

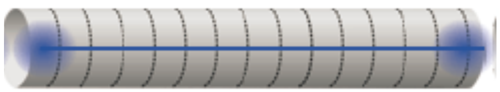
Chern number is Quantized at 1/2.

Fermion system has Chern number quantized at 1



Identifying nature of FQHE (in magnetic field)

Using "momentum" states for cylinder /spheric geometry as "local orbits" to construct DMRG
--- Mapping FQHE problem to 1D



$$\psi_{N,j}(x,y) = \left(\frac{1}{2^N N! \pi^{1/2} L_y l}\right)^{1/2} exp[i\frac{X_j}{l^2}y - \frac{(X_j - x)^2}{2l^2}] H_N(\frac{X_j - x}{l})$$

where $X_j = \frac{2\pi l^2}{L_y} j, j = 1, 2, \ldots, N_s$ is the center in x axis and l is the magnetic length. $H_N(x)$ is the Hermite polynomial.
Introducing the destruction (creation) operator $a_{N,j}(a^\dagger_{N,j})$ for $\psi_{N,j}$, the Coulomb interaction can be written as

$$H_C = \sum_{N_1,\ldots,N_4} \sum_{j_1,\ldots,j_4} V_{N_1,j_1,\ldots,N_4,j_4} a^\dagger_{N_1,j_1} a^\dagger_{N_2,j_2} a_{N_3,j_3} a_{N_4,j_4}$$

Feiguin et al (2008)
Zhao, Sheng, Halden (2011), Zhu et al. (2015-2016)
Hu et al (2012)) Zalletal et al , (2013)

Entanglement Spectrum: 12/5 FQHE as Read-Rezayi state Wei Zhu et al

τ

Detected by Entanglement Spectrum:
Root: 1110011100….. spectrum 1,1,3,6,12…
Root: 1110011…. spectrum 1,2,5,9,….

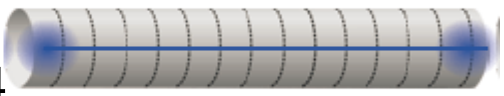
2x5 degenerating states
5 from center of mass
(Fibonacci qs)

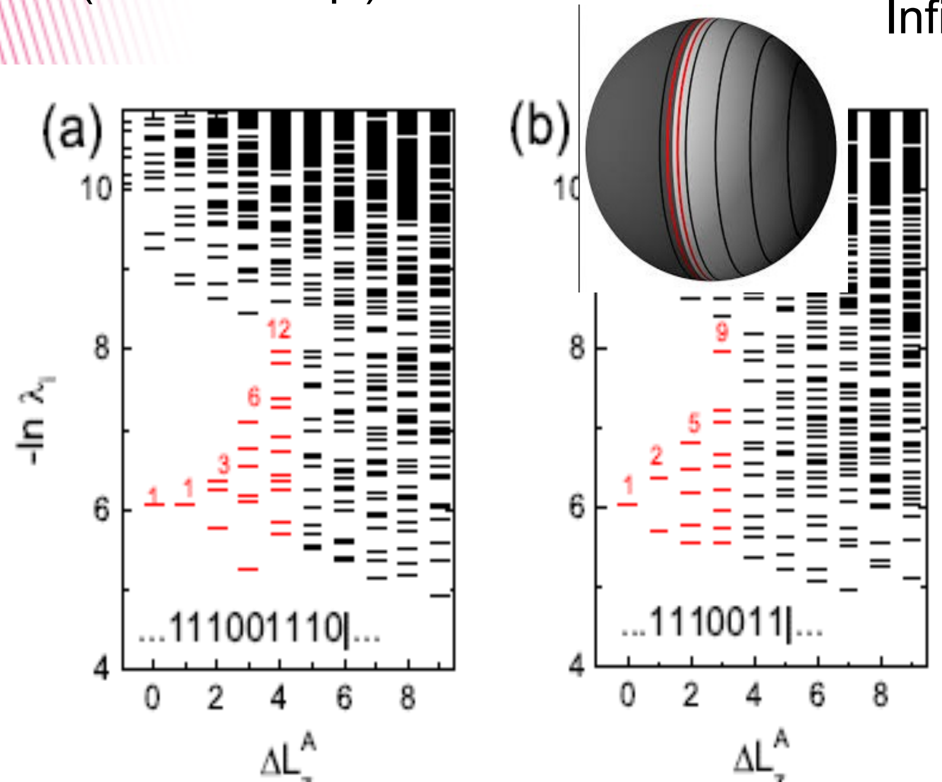
On cylinder, we also find another ground state: 10101 10101 10101

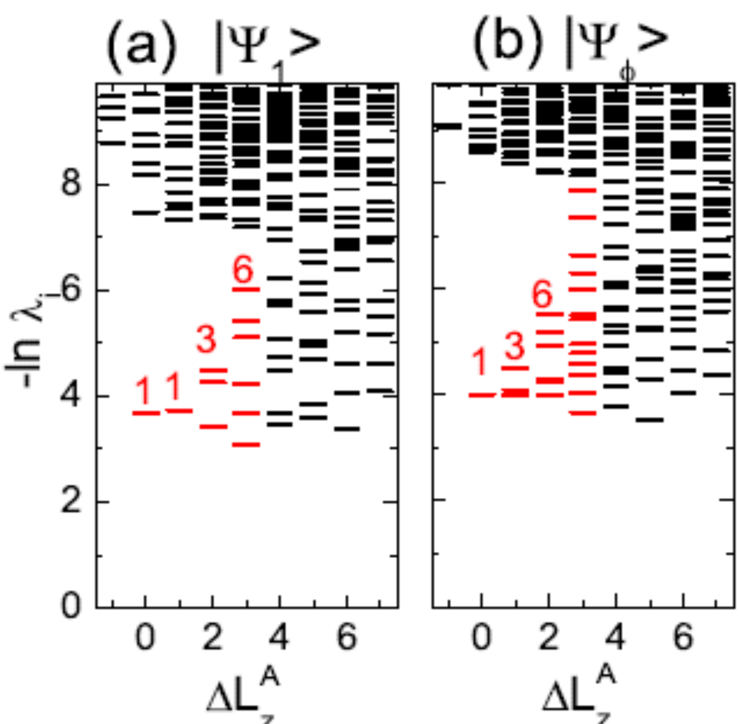
Entanglement spectrum:
1,1,3,6 ... and 1,3,6, (13)

τ

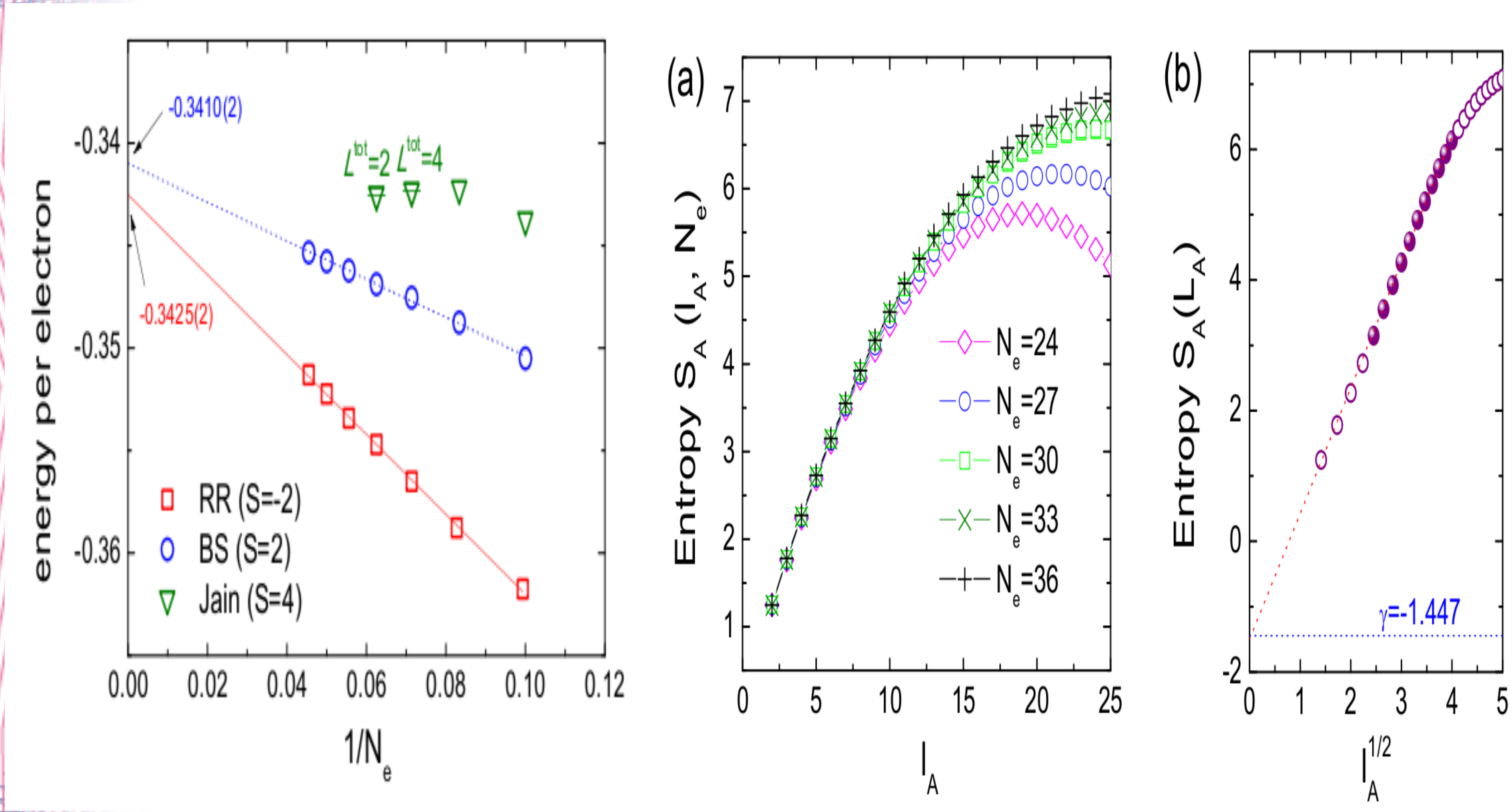
Infinite Cylinder Ly=24







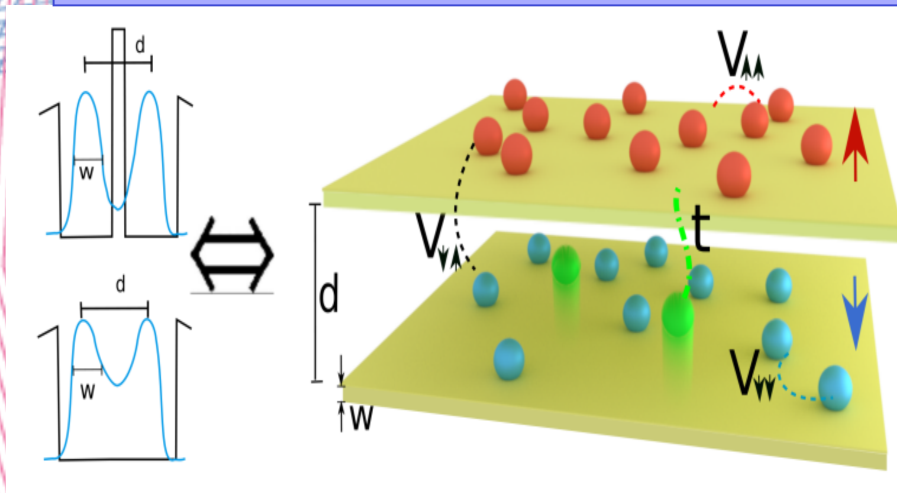
12/5 FQHE as Read-Rezayi state



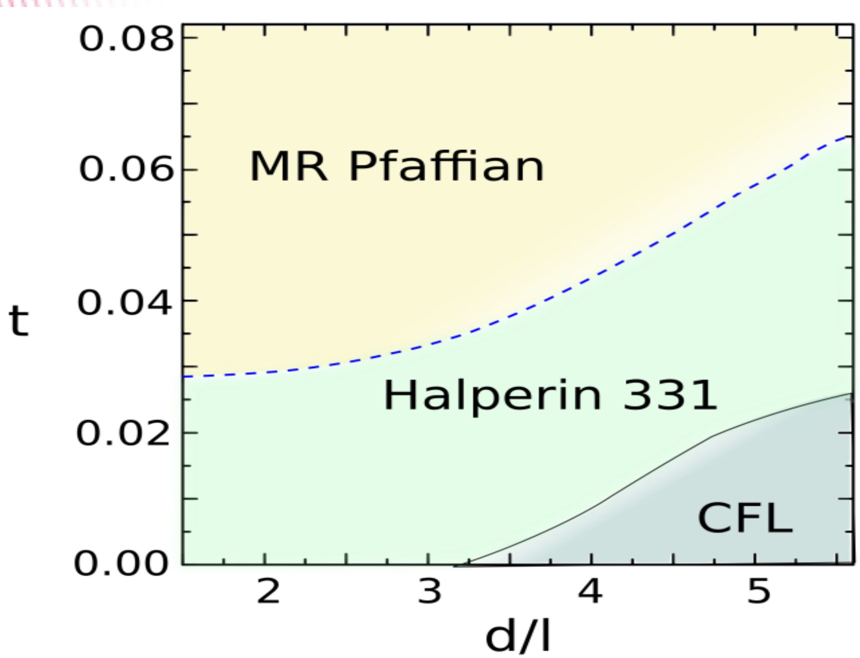
Read-Rezayi vs. Bonderson and Slingerland

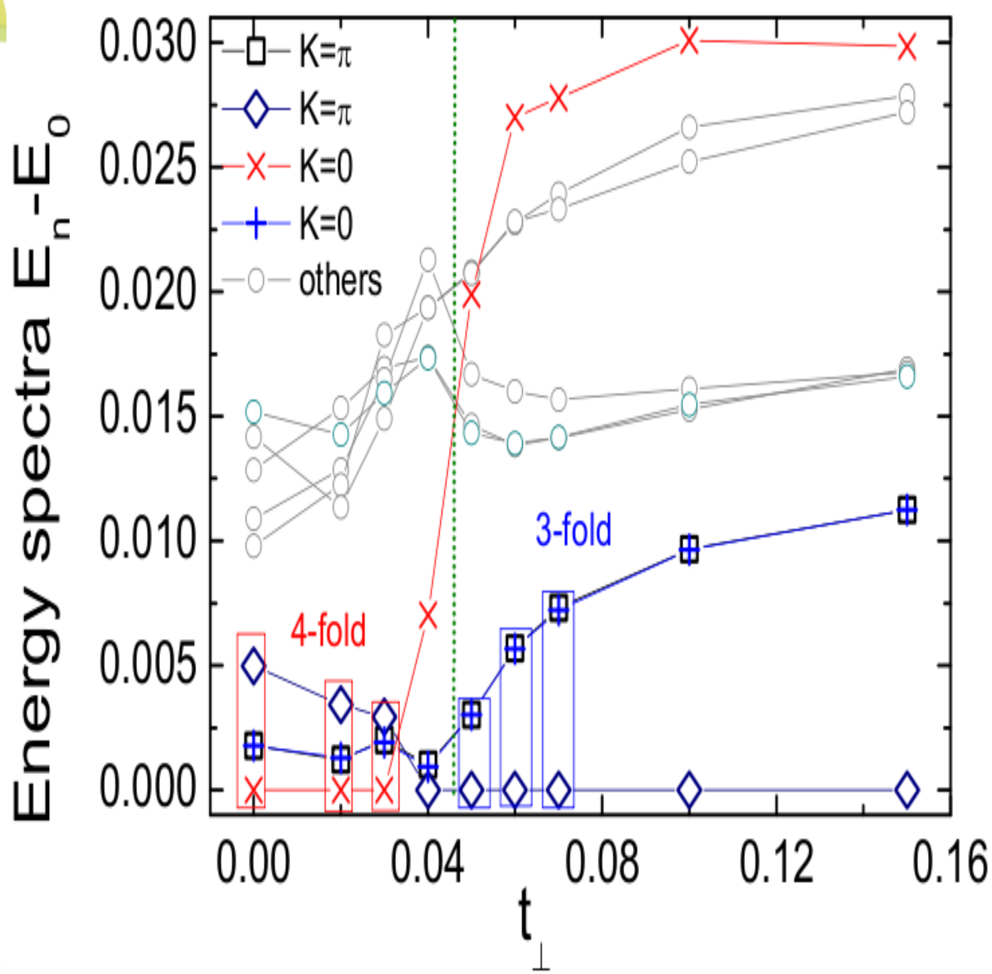
See also Roger Mong arXiv 2015,

Fractional Quantum Hall Bilayers at Half-Filling: Tunneling-driven Non-Abelian Phase Wei Zhu et al.



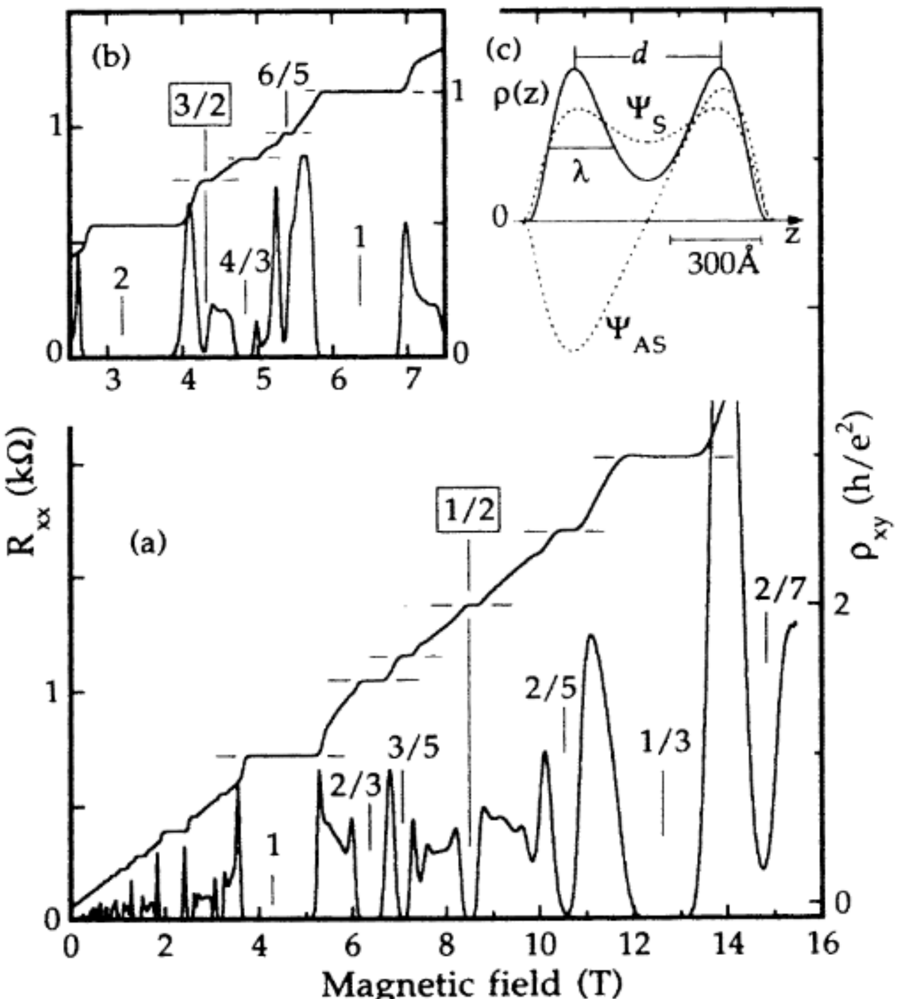
Predicted by Xiao-Gang Wen & other works

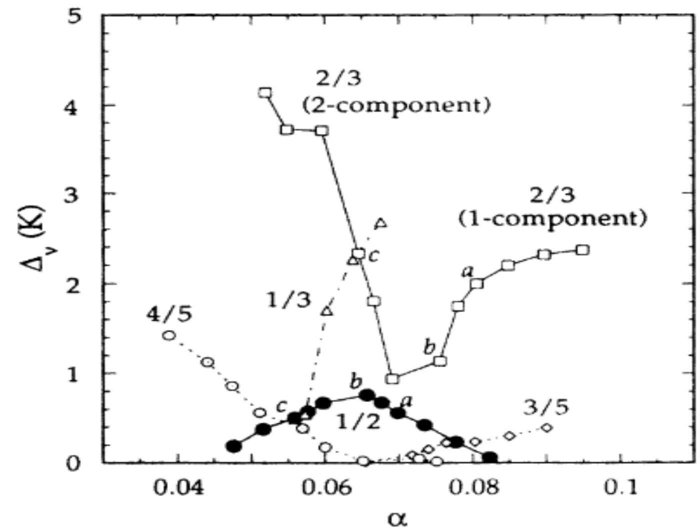




Experimental observation and theory

Suen et al. , PRL 1994, Eisenstein et al.,





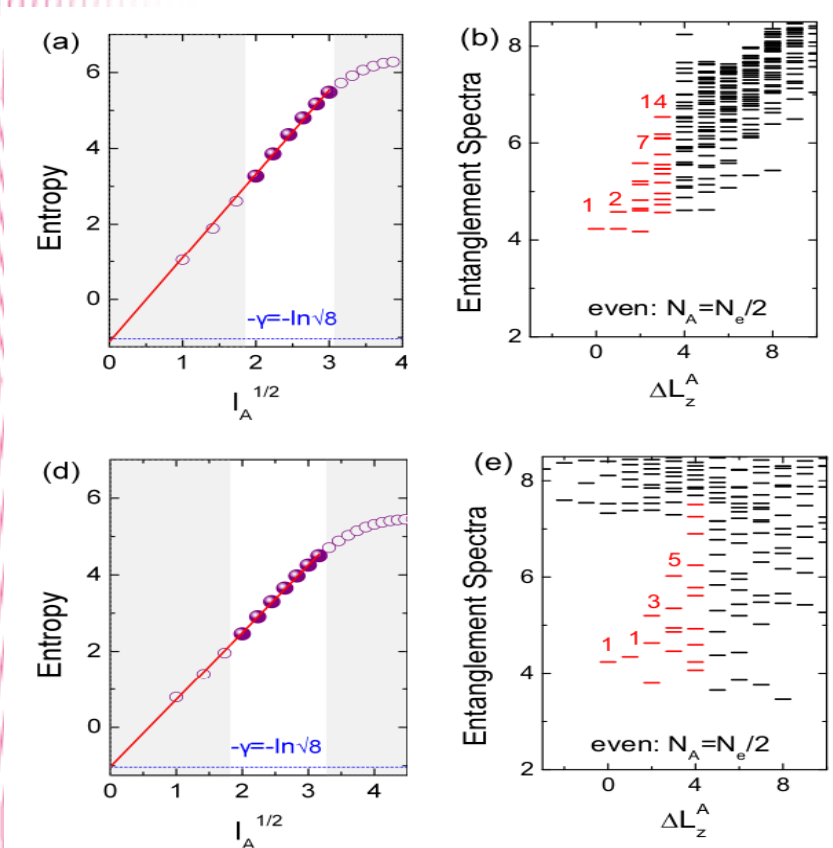
. 2. The quasiparticle excitation gaps Δ_ν

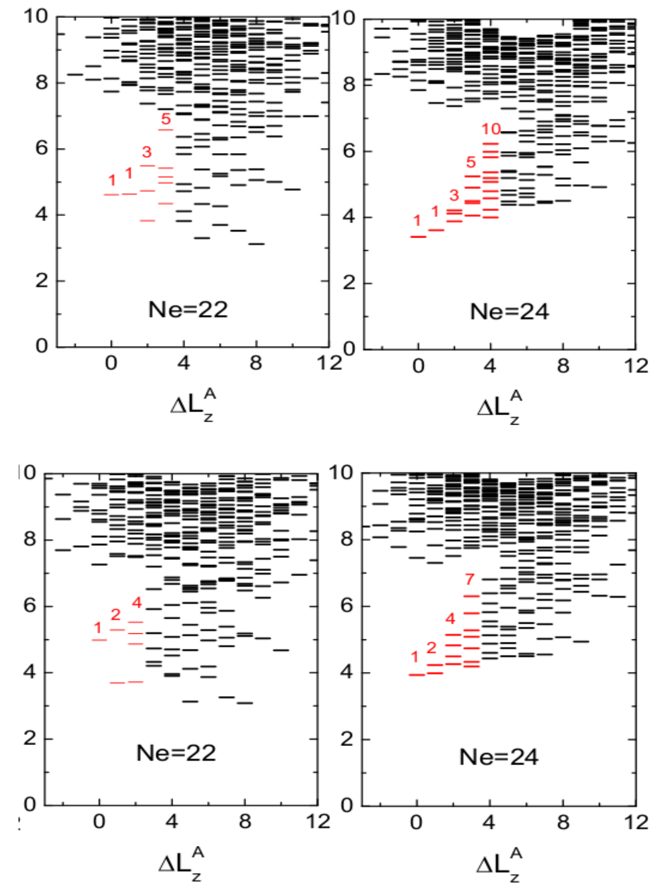
Ho 1995, Greiter et al (1992)
Halperin state 331
Wen 2000
Read & Green 2000
Barkeshli & Wen 2011
Peterson et al (2010)
Papic et al (2009-2010)

Entanglement spectrum and entropy, Moore-Read vs. Halperin 331 (sphere geometry)

Halperin 331 state t=0.02

Moore-Read for different cuts

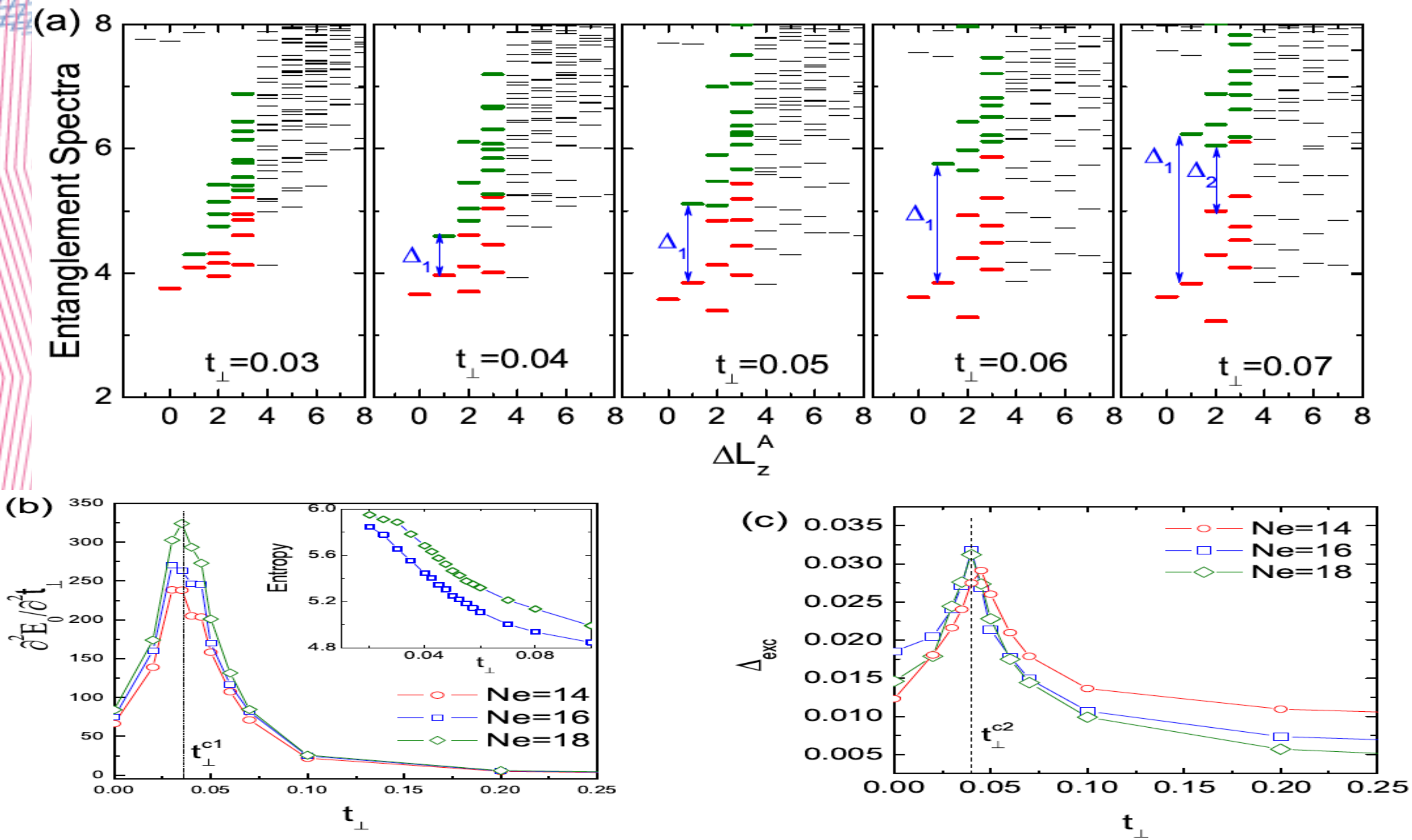




Ne=22 Moore-Read state t=0.1

Better spectrum for largest system (Ne=24)

Quantum phase transition through gapping out low energy state



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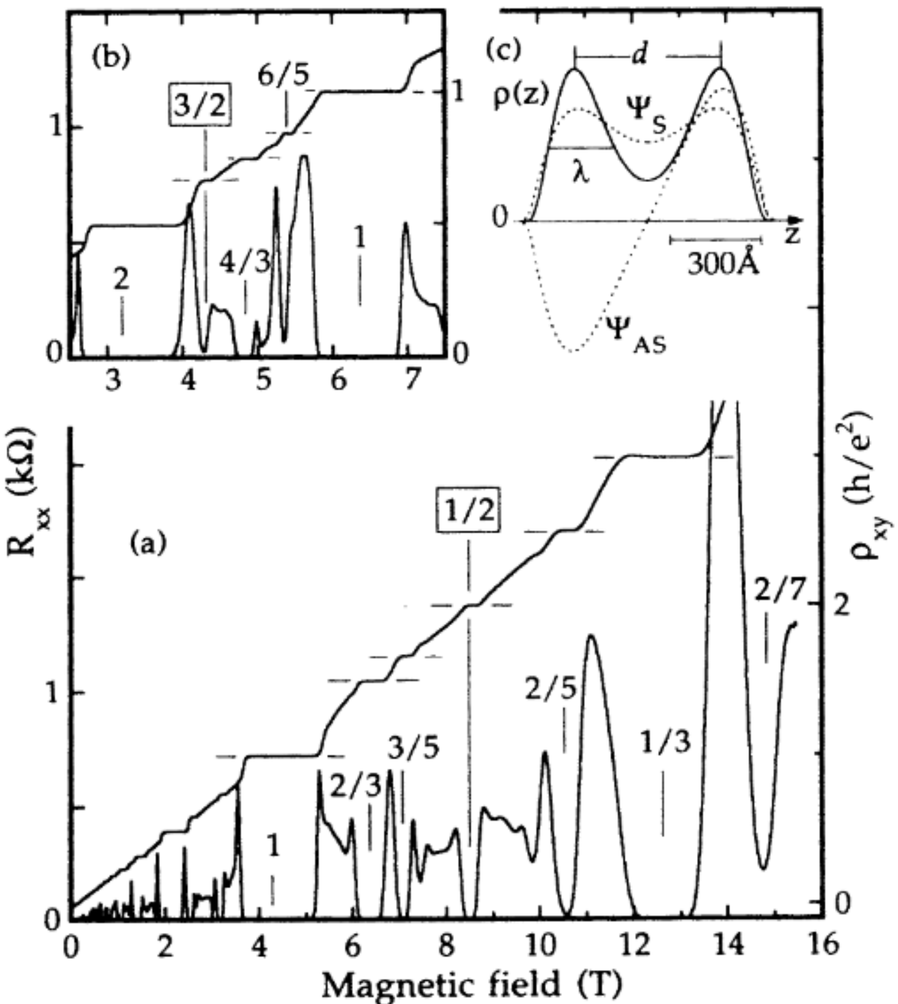

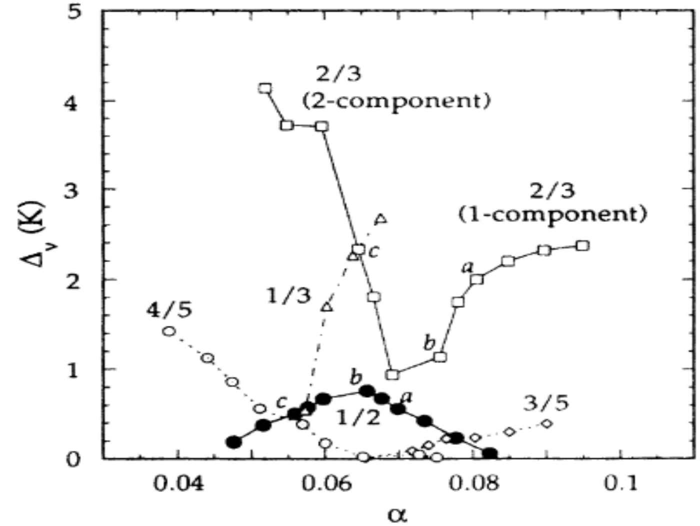

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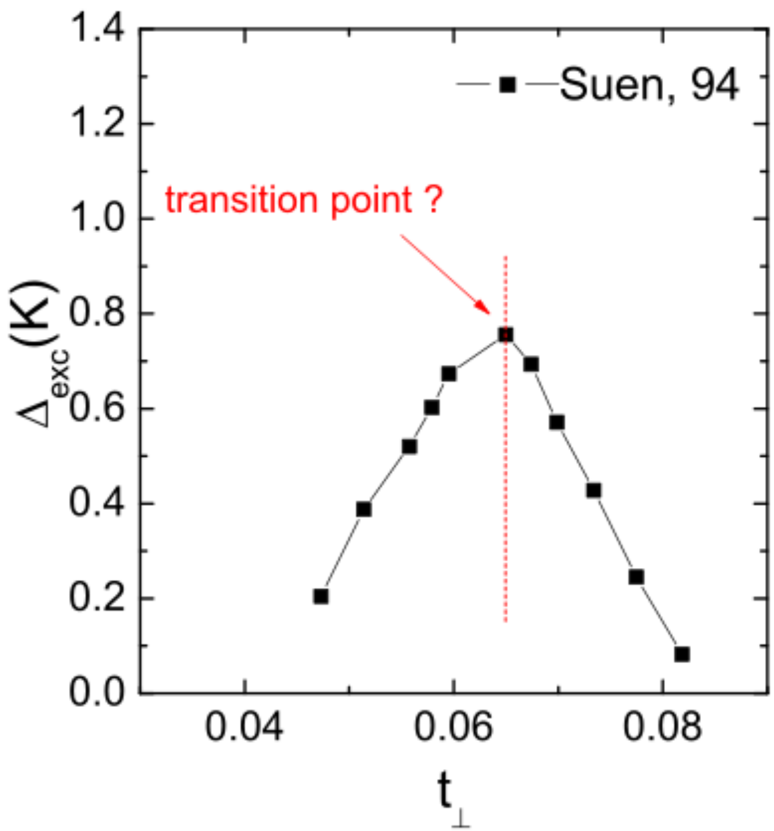
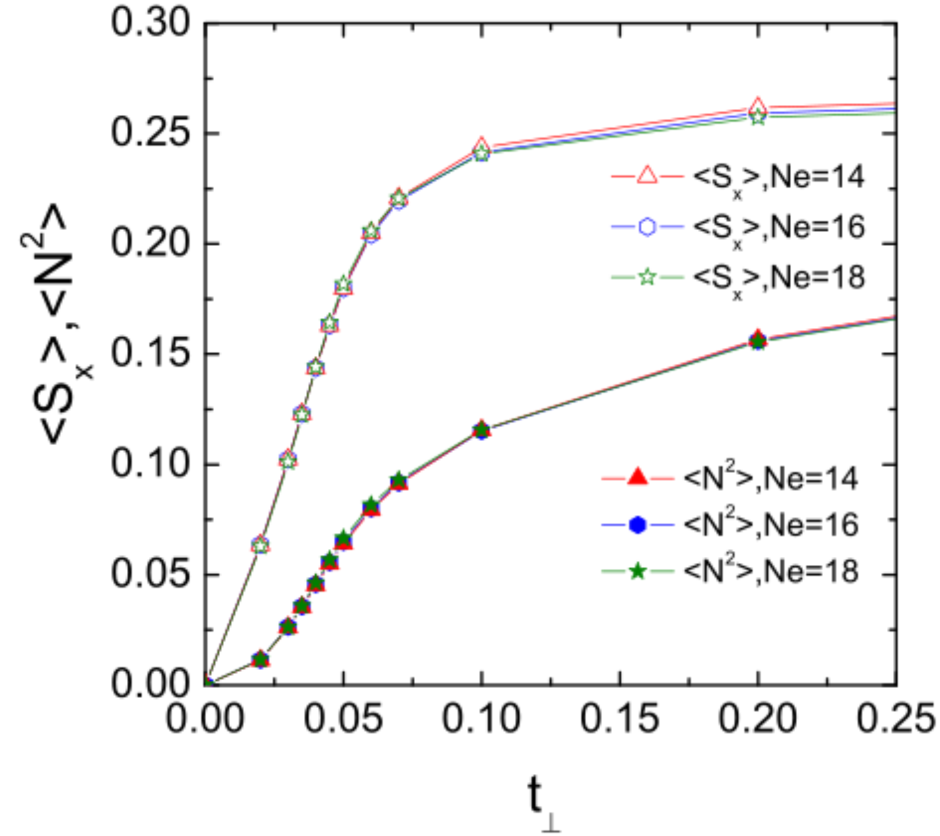
Comparing with experiments

Suen et al. , PRL 1994





Summary

Interaction driven spontaneous QHE in time-reversal symmetric systems

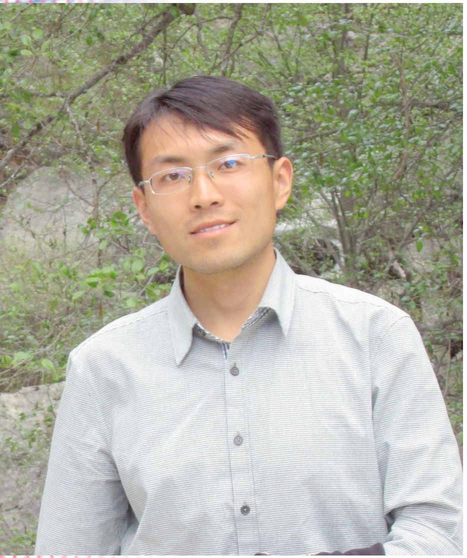
--- can be fully identified based on inserting flux in DMRG

Identifying non-Abelian FQHEs
(12/5 as parafermion Read-Rezayi)

Bilayer 1/2 FQHEs (331 Halperin to Pfafffian transition)

Supported by DOE and NSF

Collaborators



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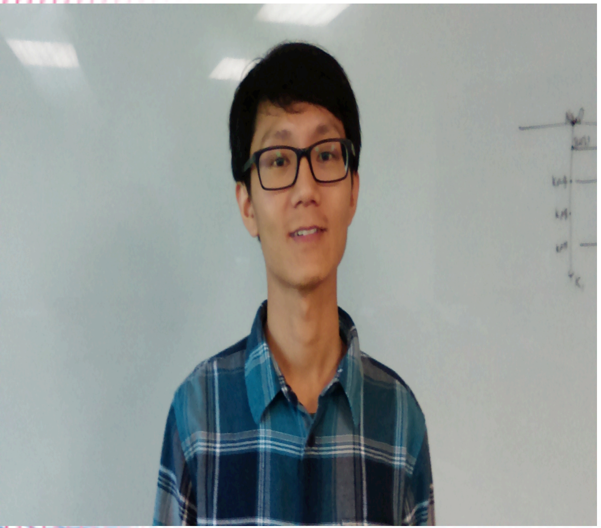
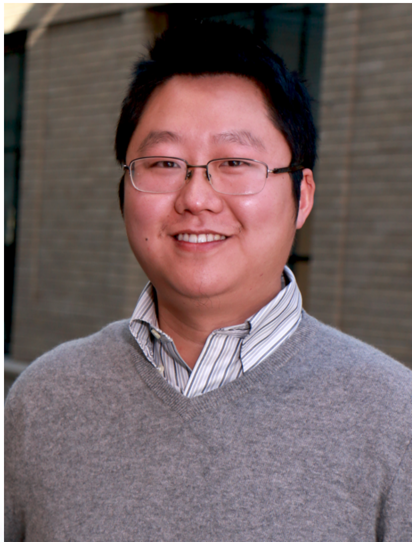
Duncan Haldane (Princeton)





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