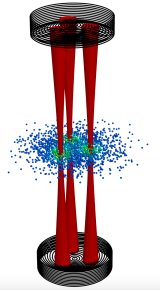


Topological Photonics

Landau Levels in Curved Space



Jon Simon

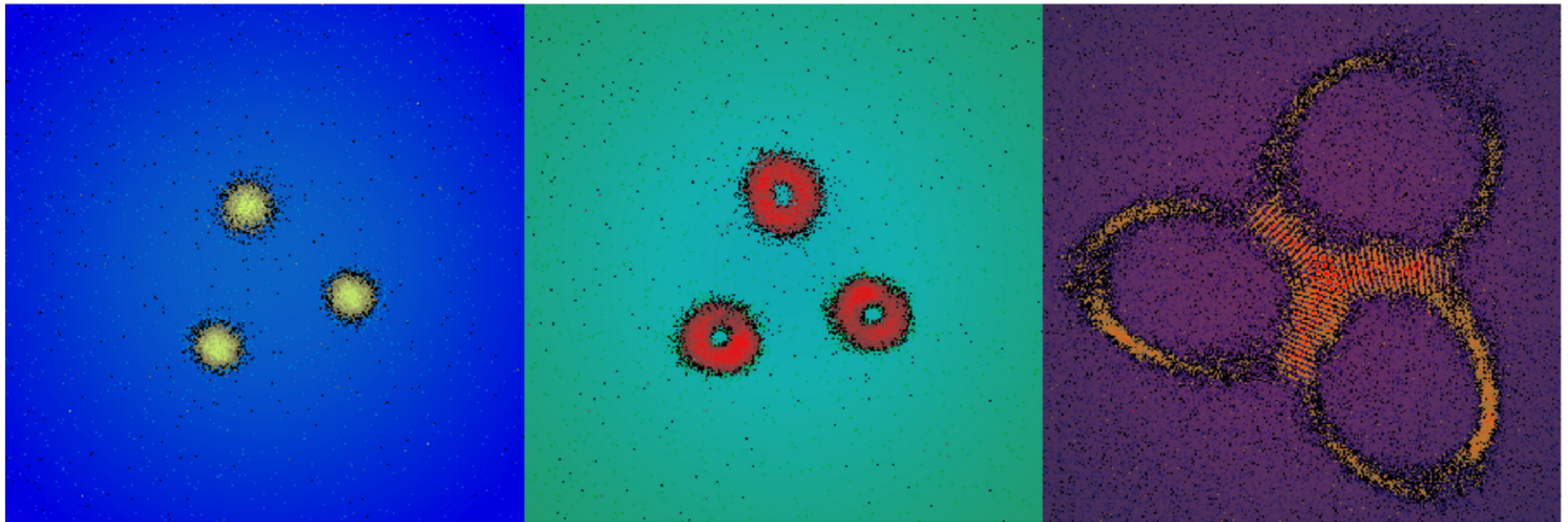
University of Chicago

Chicago, Illinois

Topological Quantum Matter

KITP @ UCSB

October 18th, 2016



quantum.uchicago.edu & simonlab.uchicago.edu

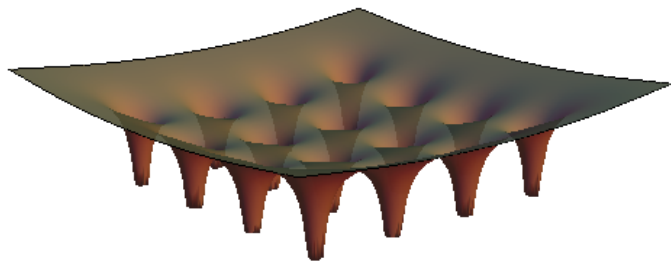
Synthetic Material Systems

Solid State

Interacting Electrons

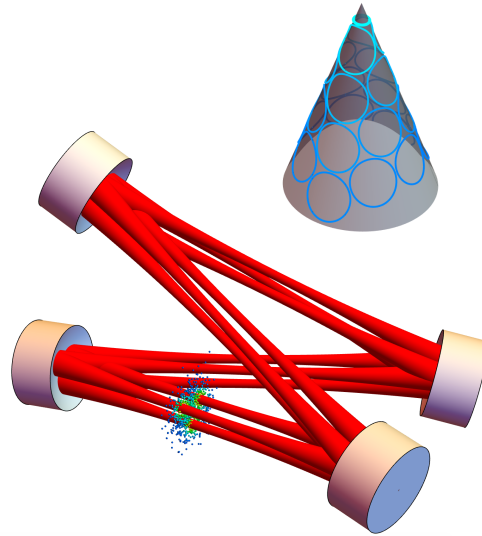
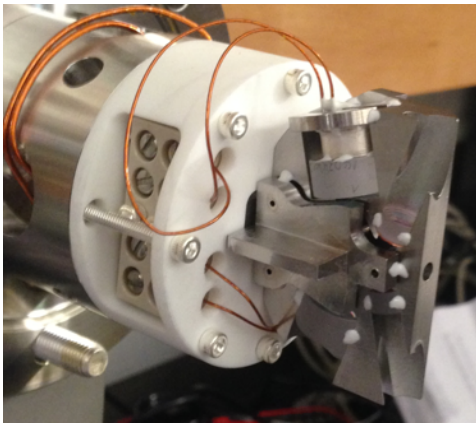


Ionic Lattice



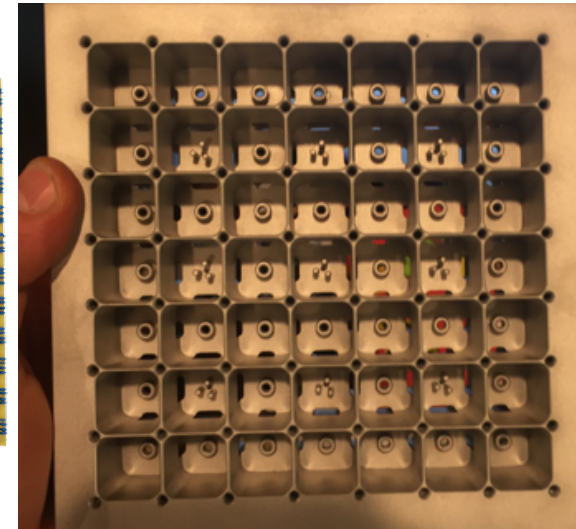
Topological Cavity QED

Twisted Optical Resonators



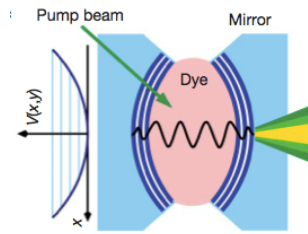
- **Bulk Landau Levels**
- **Atoms Break T**
- **Rydberg EIT** for Interactions
- **Flux threading** to build manybody states

μ -wave Lattices



- **Lattice Z_2 & Chern Bands**
- **Ferrites Break T**
- **Transmons** for Interactions
- **Chem pot'l** to build manybody states

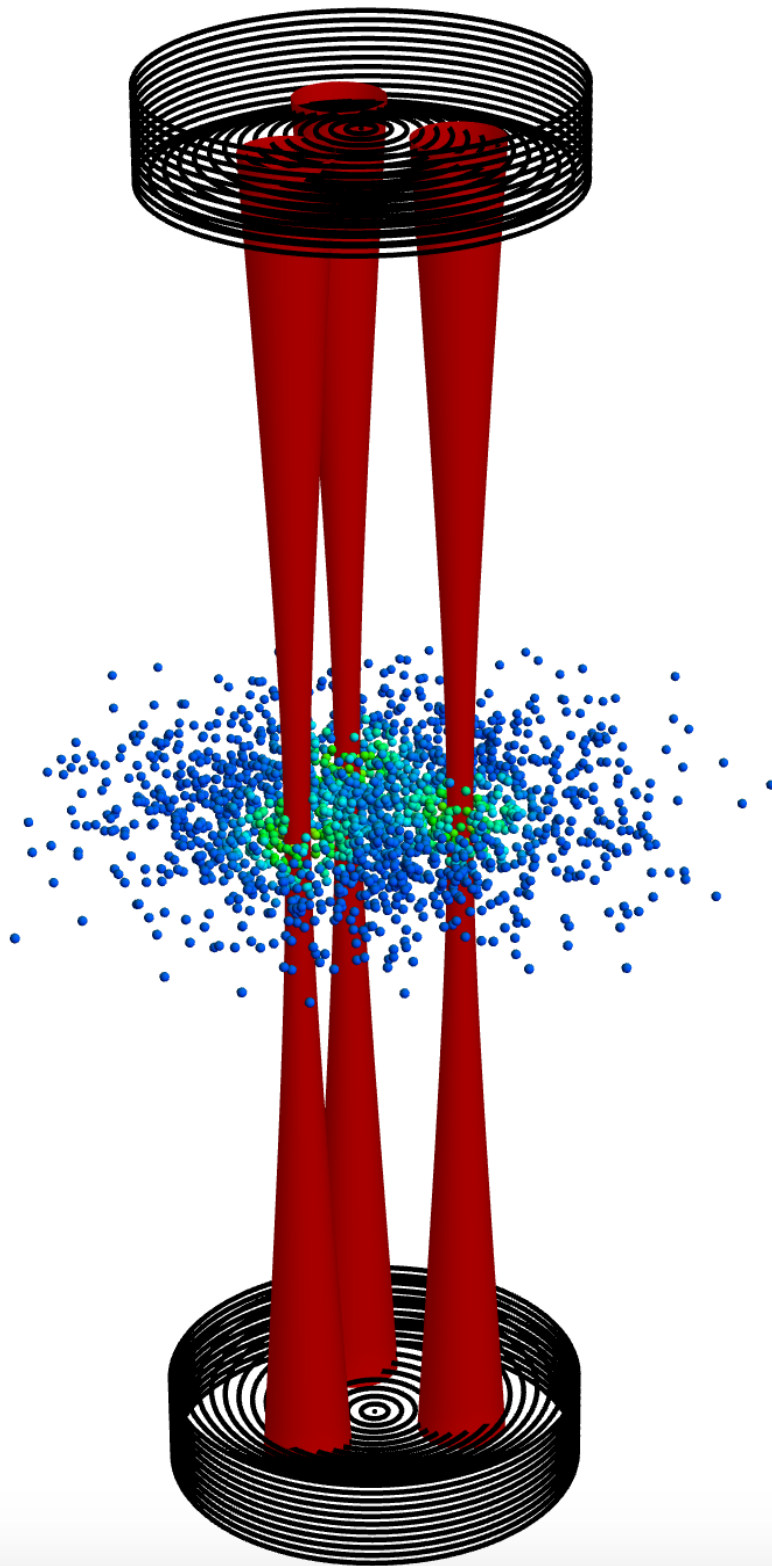
Synthe



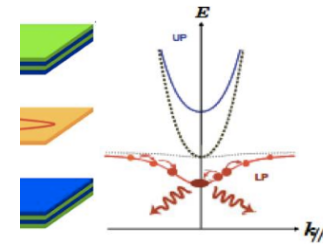
Photon Bl

1D

- [1] Klaers *et al.*, Nature
- [3] Peyronel *et al.*, Nat



aterials



Polaritons [2]

LS [3]

ence **298**, 199 (2002)

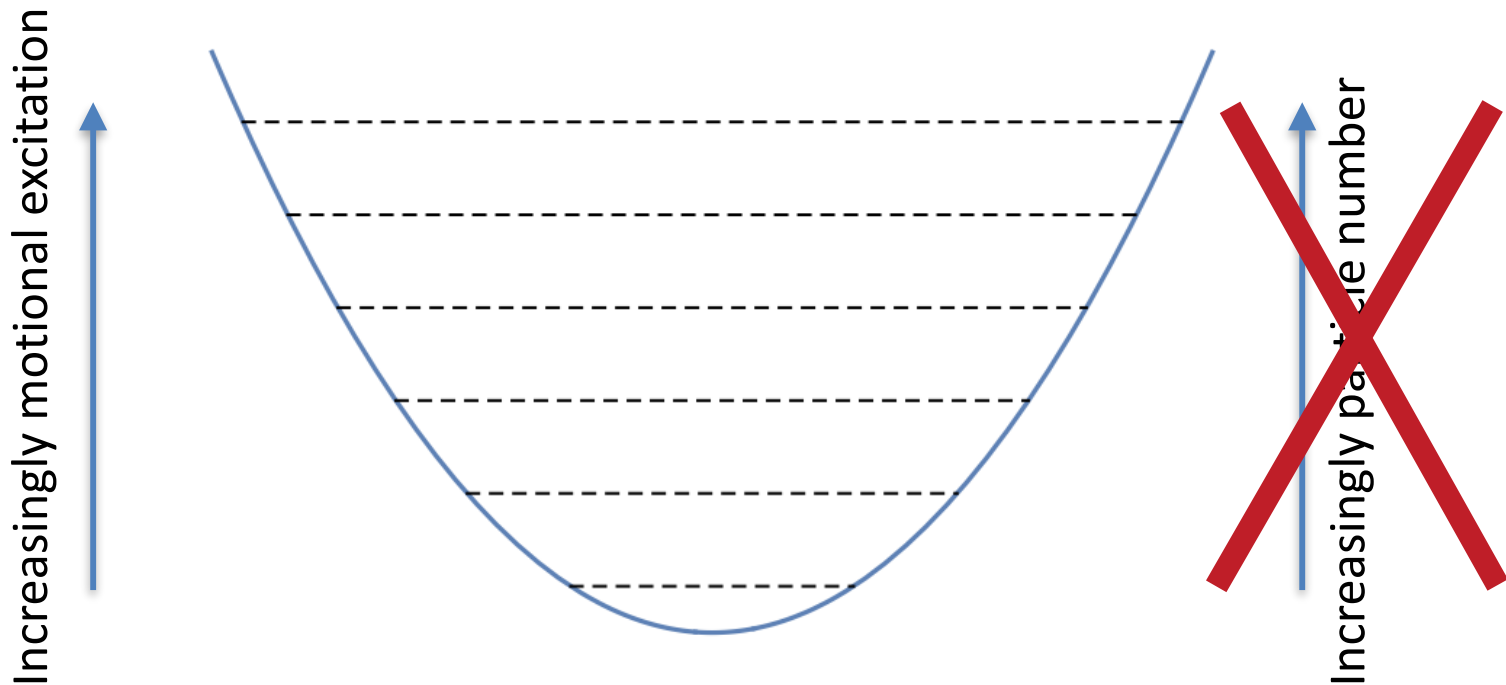
Central Premise

Photons in Multimode Resonator



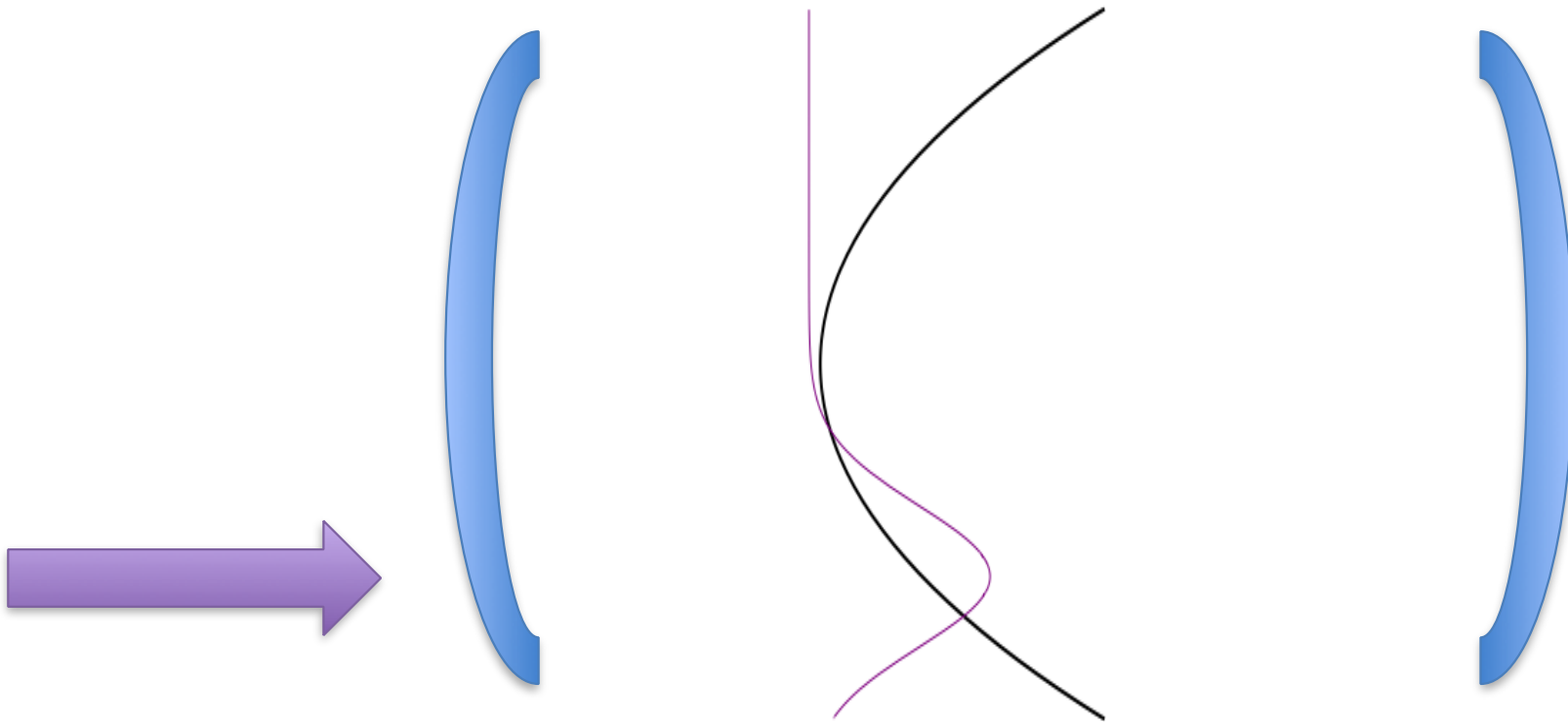
Massive Particles in Harmonic Trap

- This is a **real space-** not **Fock space-** analogy

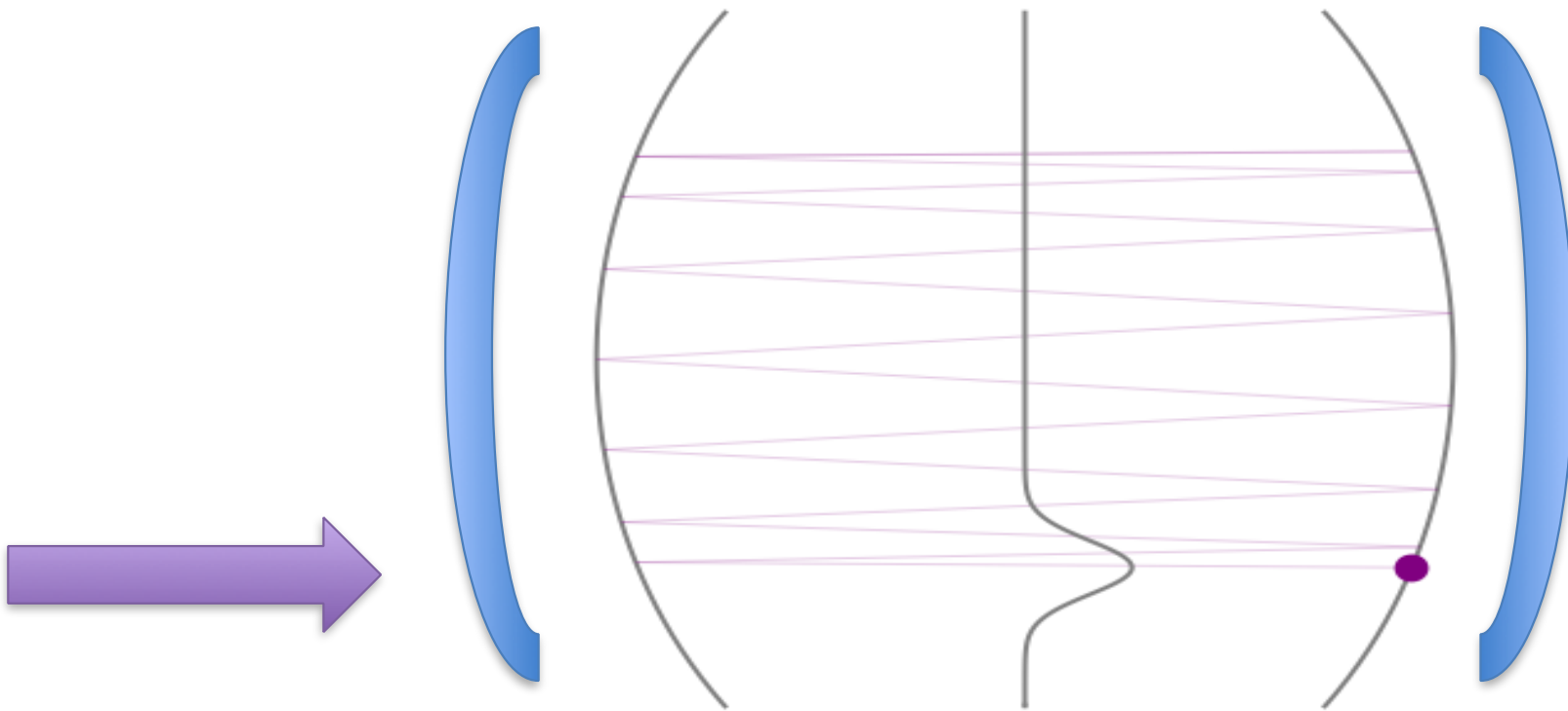


Understanding photons in optical resonators

A photon in a curved-mirror optical resonator behaves like a massive particle in a harmonic trap.

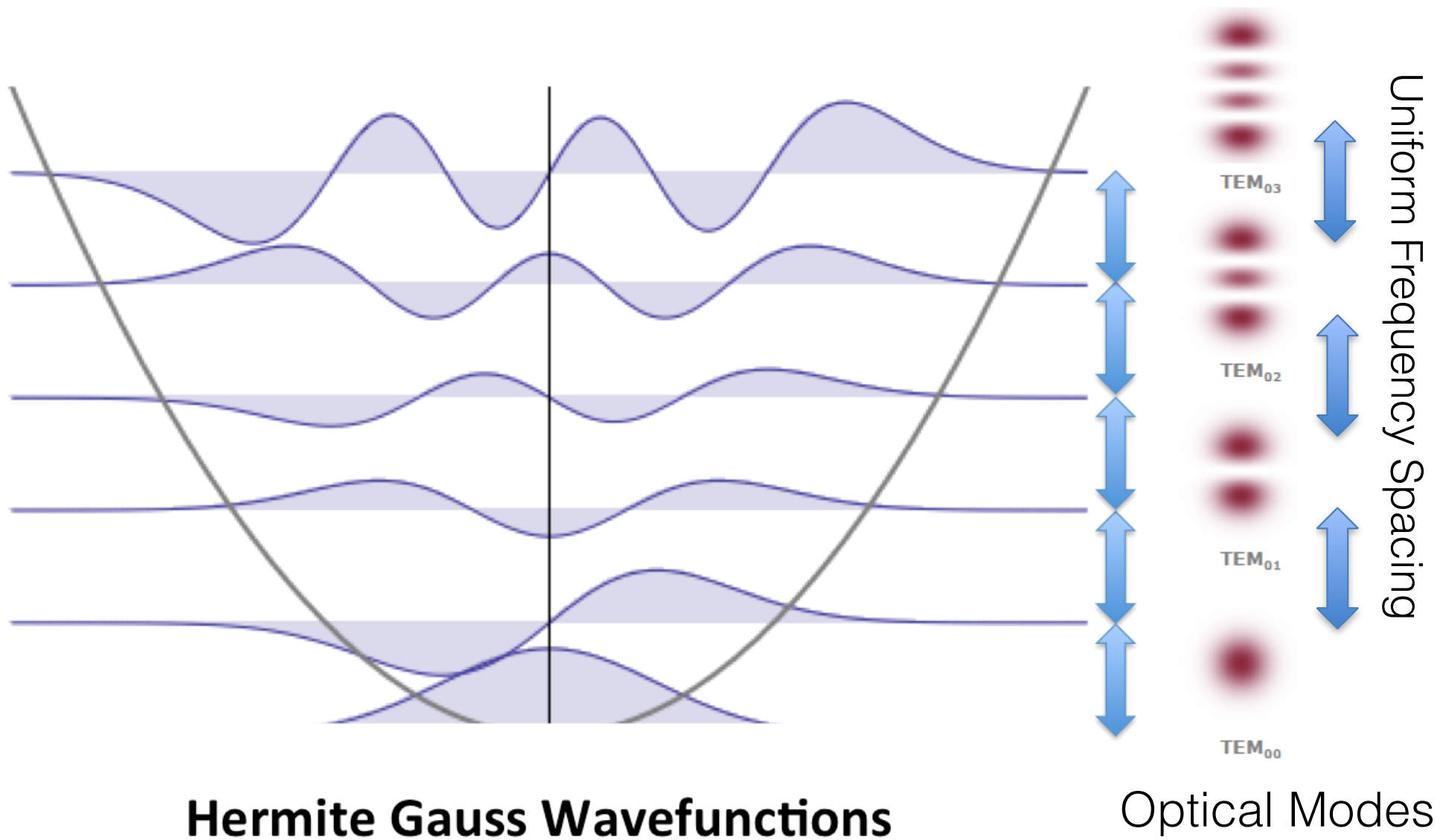


Understanding photons in optical resonators

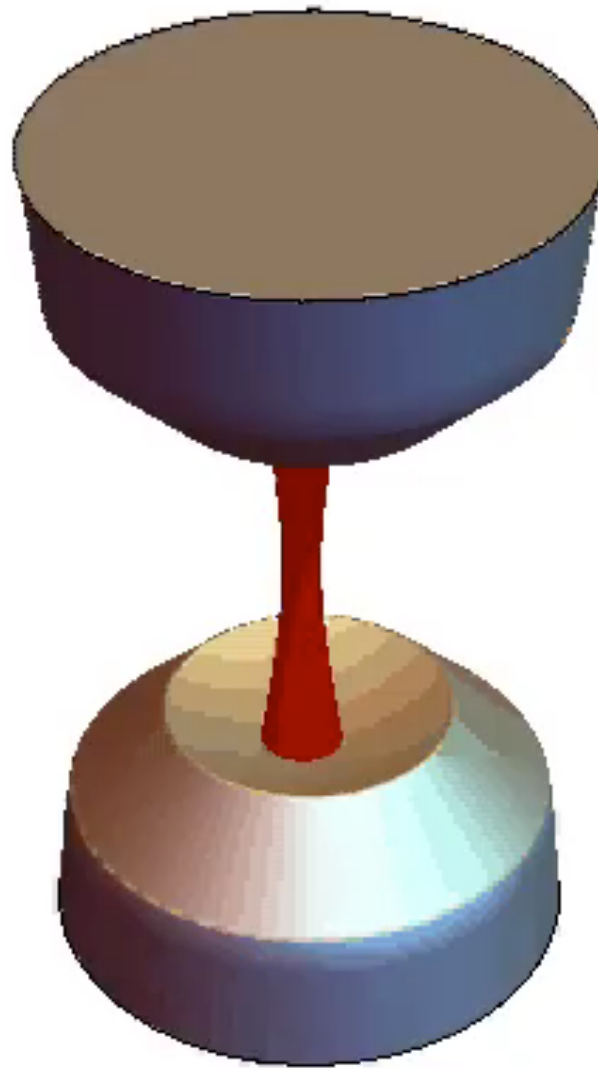
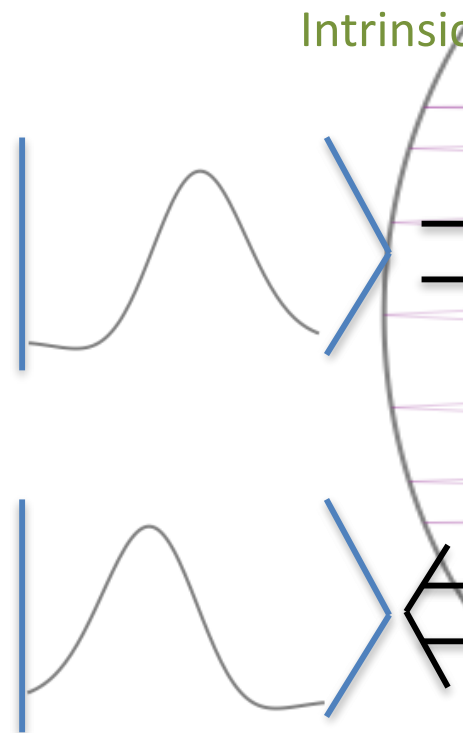
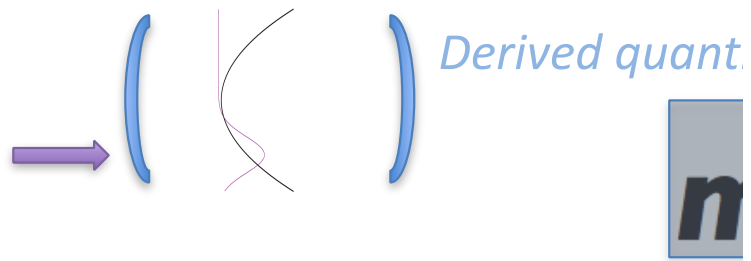


[1] Sommer et. al., **NJP** 18, 035008 (2016)

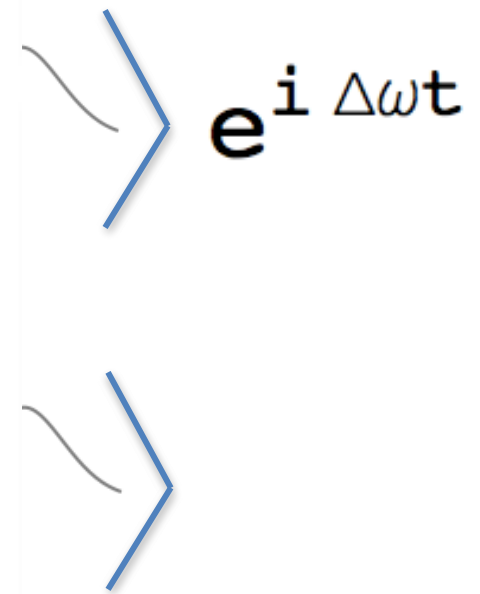
Comparing Harmonic Oscillators & Optical Resonators



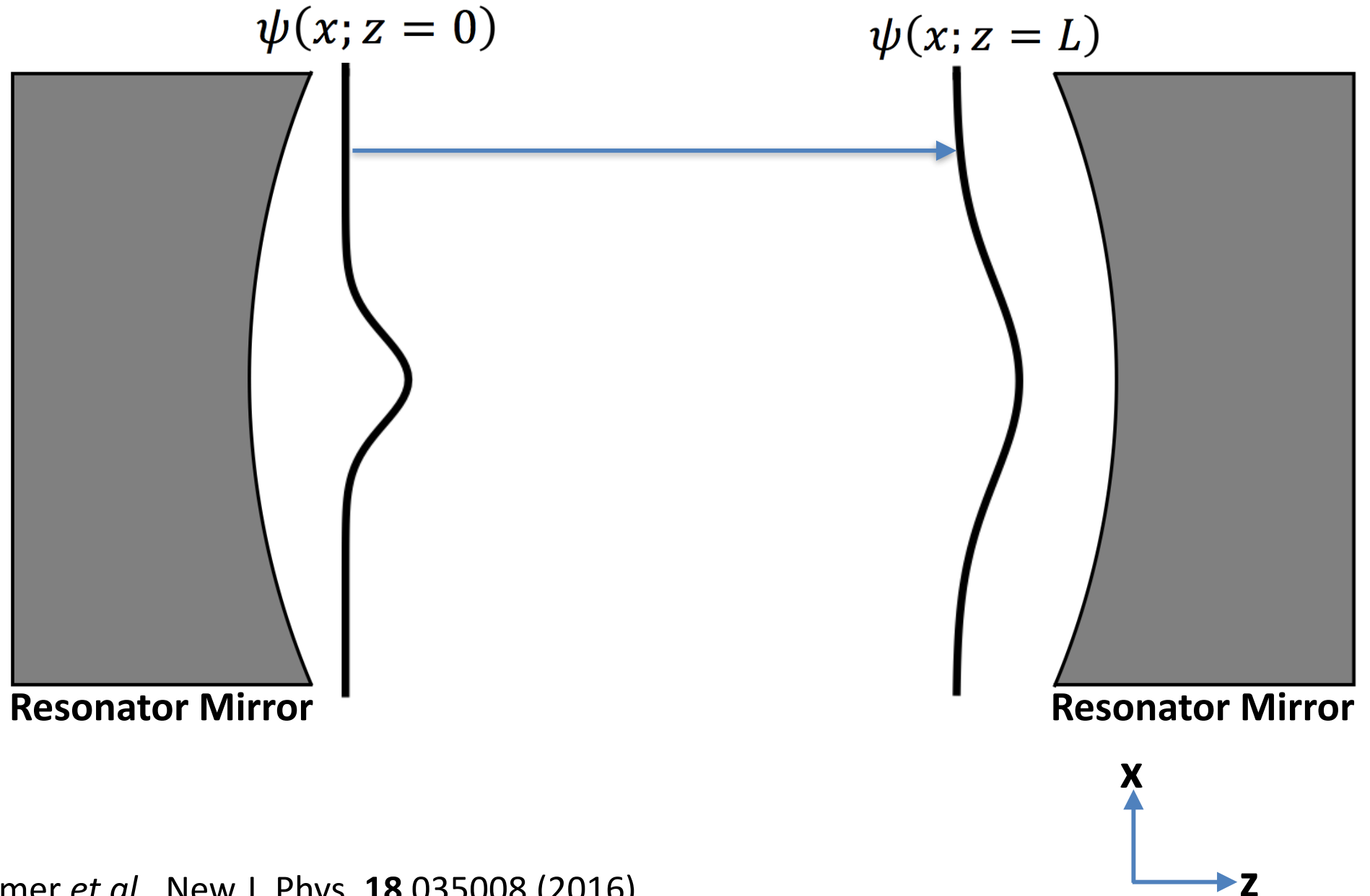
Understanding photons in optical resonators



electron

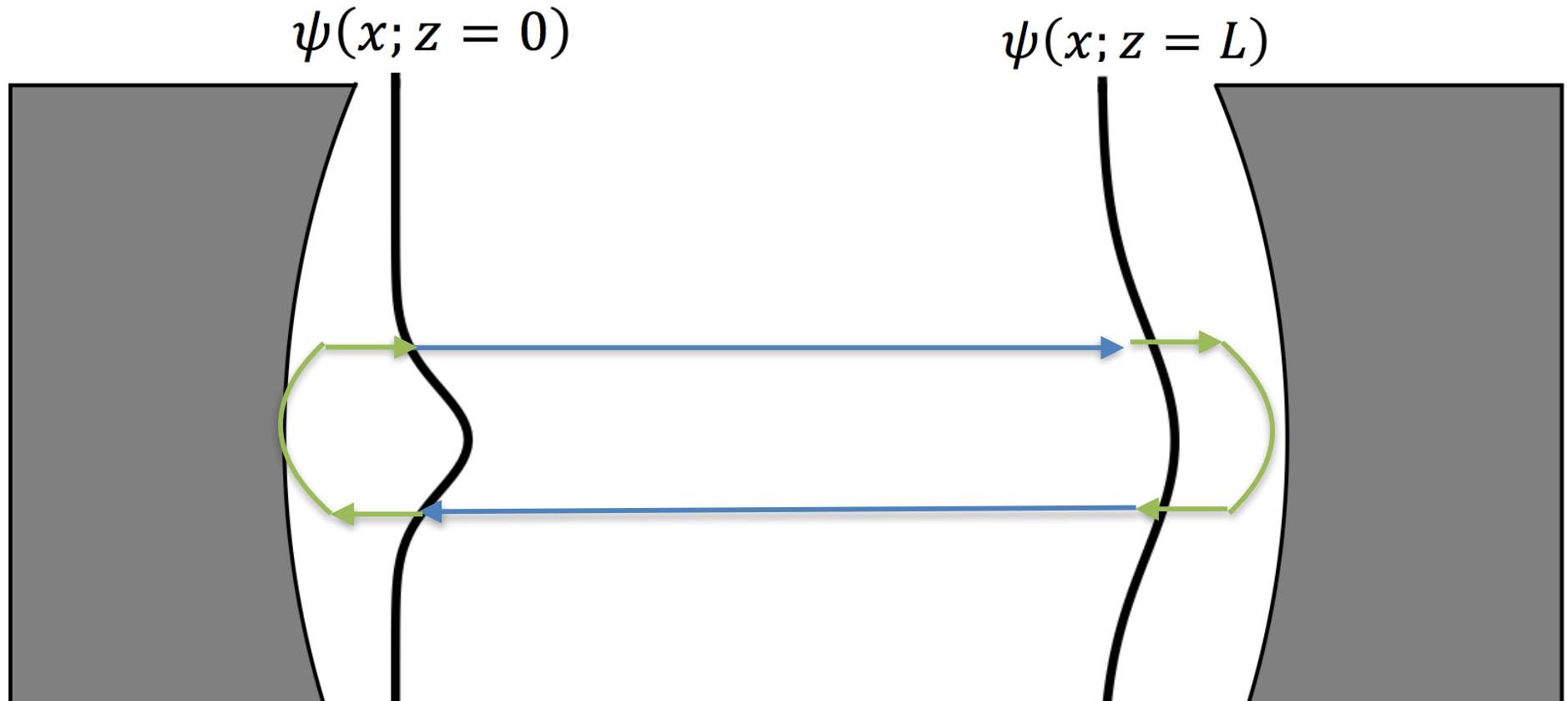


Formal Picture: Floquet Theory



Formal Picture: Floquet Theory

Formal Picture: Floquet Theory



$$H_{\text{floquet}} = \frac{1}{i T_{RT}} \text{Log} \left[\begin{array}{|c|c|c|c|} \hline e^{-i\frac{kx^2}{2R}} & e^{-i\frac{L}{2k}\left(\frac{\partial}{\partial x}\right)^2} & e^{-i\frac{kx^2}{2R}} & e^{-i\frac{L}{2k}\left(\frac{\partial}{\partial x}\right)^2} \\ \hline \end{array} \right]$$

Schine *et al.*, *Nature* **534**, 671–675 (2016)

Sommer *et al.*, *New J. Phys.* **18** 035008 (2016)

Rechtsman *et al.*, *Nature* **496**, 196–200 (2013)

Synthetic Magnetic Fields for Photons



Free Space Propagation → Mass
Mirror Curvature → Trapping
Resonator Twist → B Field

$$H_{\text{floquet}} = \frac{1}{iT_{RT}} \text{Log} \left[e^{i\theta L_z} e^{-i\frac{kx^2}{2R}} e^{i\frac{L}{2\hbar^2 k} p^2} e^{-i\frac{kx^2}{2R}} e^{i\frac{L}{2\hbar^2 k} p^2} \right]$$

*is there a (simple) way to add
a synthetic magnetic field??*

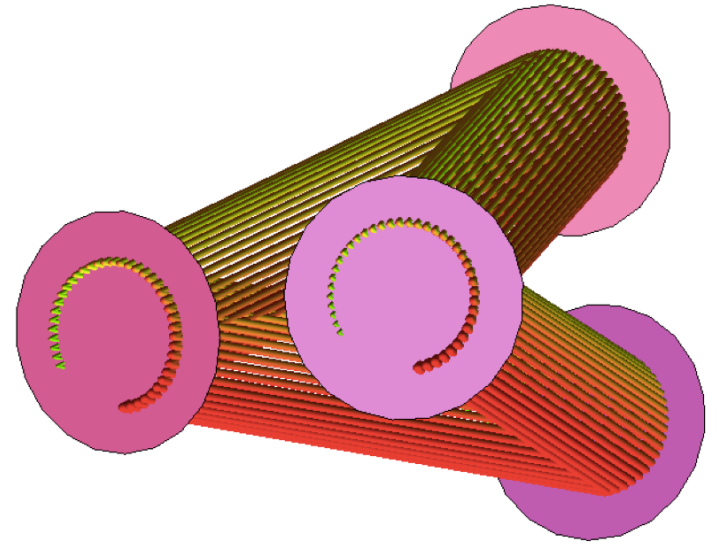
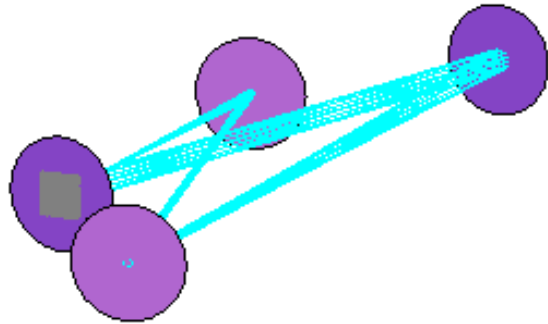
[1] Cooper, Phys. Rev. Lett. **106**, 175301 (2011)

[2] Otterbach, Phys. Rev. Lett. **104**, 033903 (2010)

[3] Maghrebi et al, arXiv:1411.6624 (2014)

[4] Karzig et al, arXiv: 1406.4156 (2014)

Synthetic Gauge Fields for Resonator Photons



Twisting the resonator out of the plane makes the lab frame a rotating frame!

$$H_{\text{floquet}} = \frac{1}{iT_{RT}} \text{Log} \left[e^{i\theta L_z} e^{-i\frac{kx^2}{2R}} e^{i\frac{L}{2\hbar^2 k} p^2} e^{-i\frac{kx^2}{2R}} e^{i\frac{L}{2\hbar^2 k} p^2} \right]$$

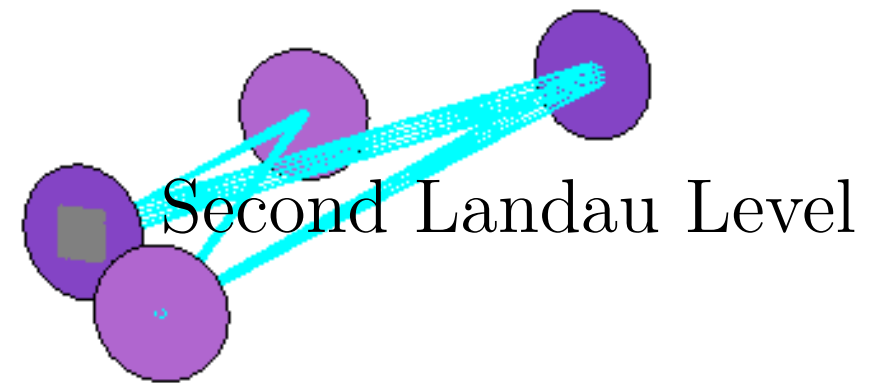
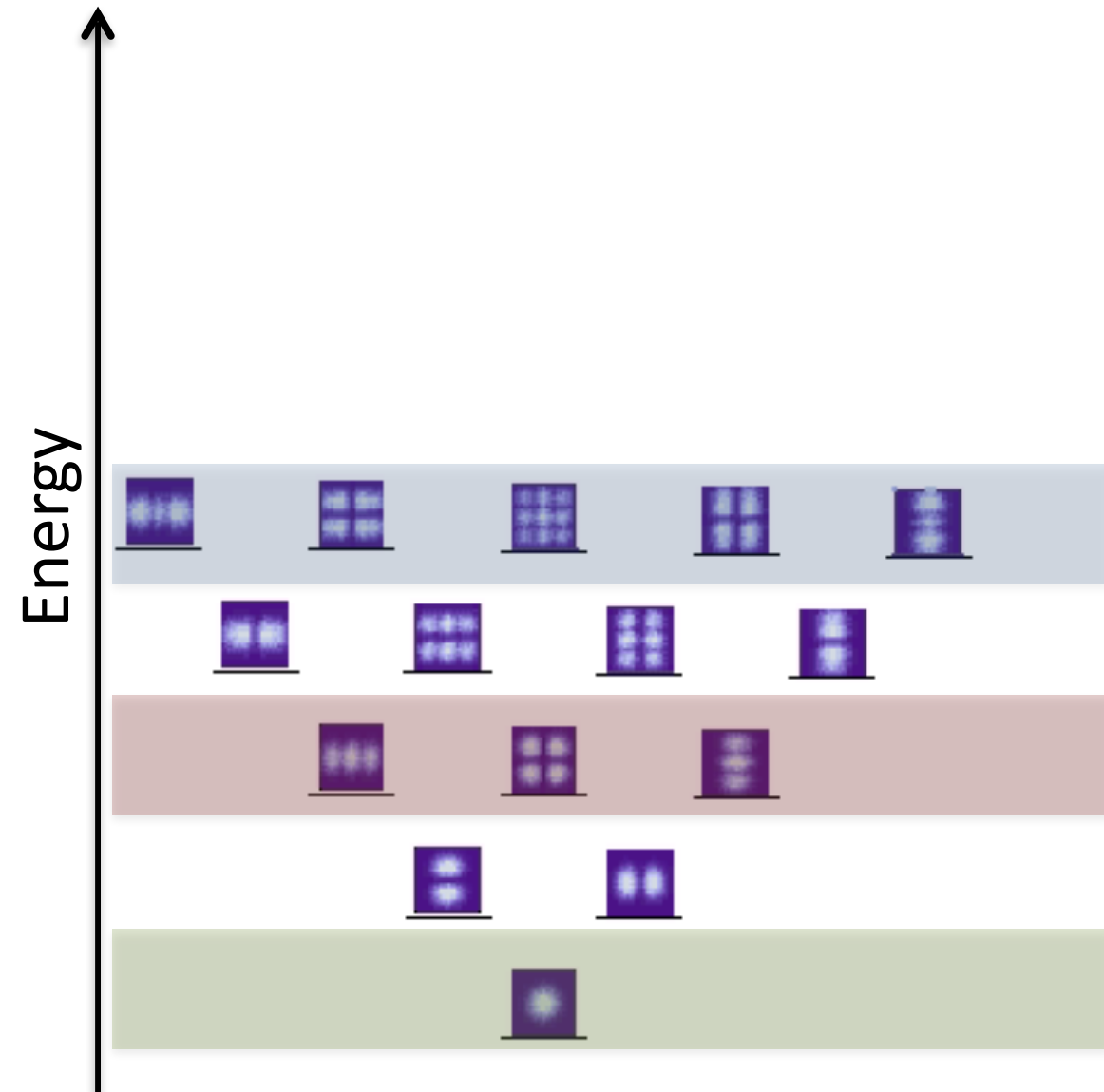
[1] Cooper, Phys. Rev. Lett. **106**, 175301 (2011)

[2] Otterbach, Phys. Rev. Lett. **104**, 033903 (2010)

[3] Maghrebi et al, arXiv:1411.6624 (2014)

[4] Karzig et al, arXiv: 1406.4156 (2014)

Flattening Photonic Landau-Levels



Second Landau Level

First Landau Level

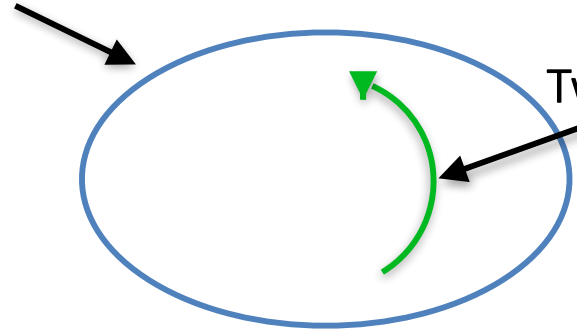
Lowest Landau Level

$$\Psi_L(z \equiv x + iy) = \frac{z^L}{\sqrt{\pi L!}} e^{-z z}$$

Photonic Centrifugal Barrier

- Twisted Cavity \rightarrow Lab Frame Coriolis Force + Anti-Trapping
- Slightly Astigmatic Harmonic Confinement (**drives $\Delta l = \pm 2$ transitions**)

Mirror-Induced Trapping



Twist-Induced Coriolis+Anti-Trapping

[1] Fetter, Rev. Mod. Phys. **81**, 647 (2009)

[2] Cooper, Adv. in Phys., **57**, 539 (2008)

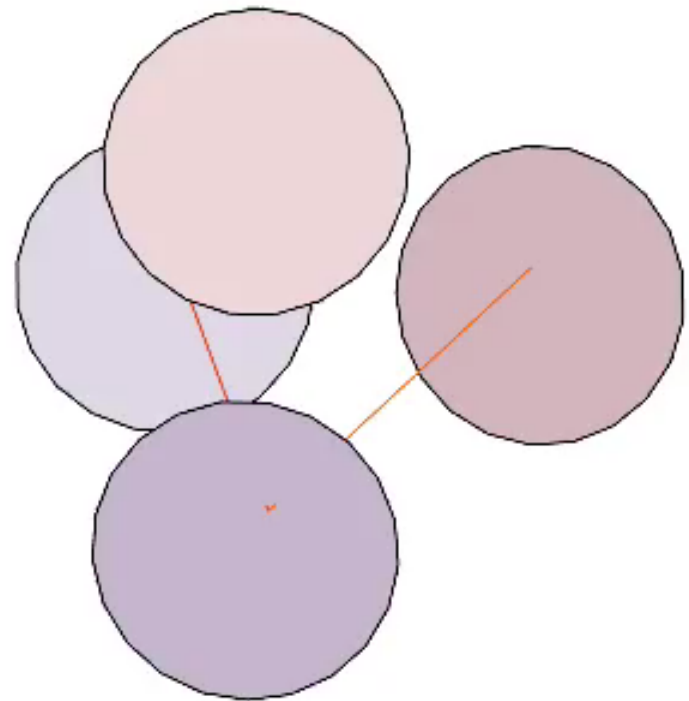
Stabilizing the System

- Floquet instability at degeneracy due to $\Delta l = \pm 2$ transitions
- Modify LLL to include only $l=0,3,6,9\dots$ removes instability

$$\Psi_M(z \equiv x + iy) = \frac{z^{3M}}{\sqrt{3\pi M!}} e^{-z\bar{z}}$$

- How? Keep twisting resonator!

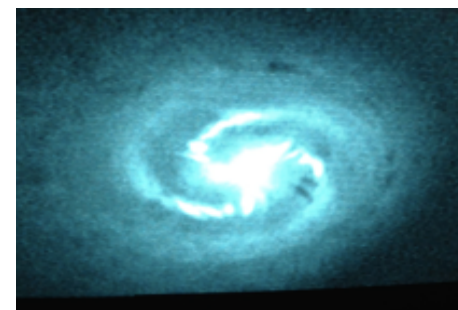
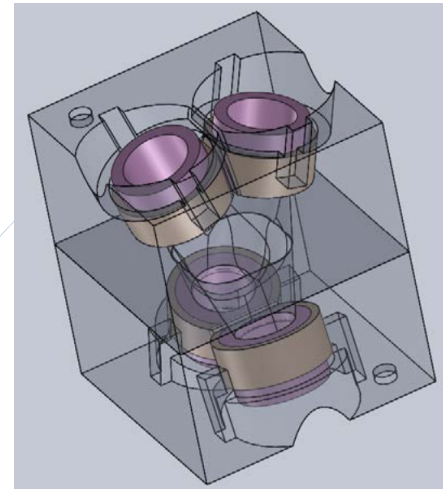
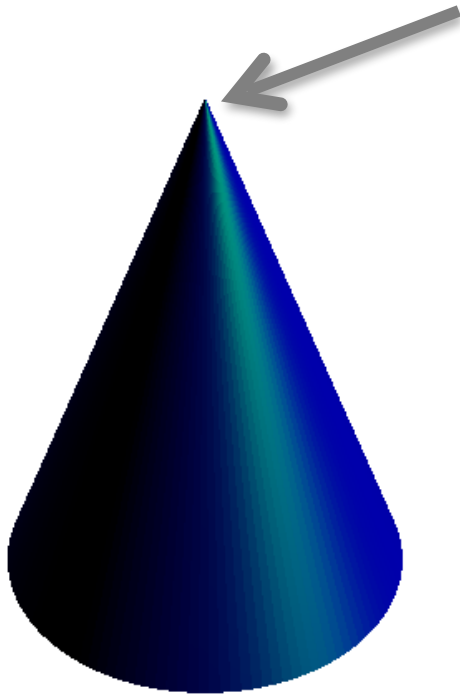
LLL for different
Floquet manifolds
(longitudinal modes)



What is this new, stable “LLL,” **physically**?

Landau Levels on a Cone

All Gaussian Curvature
Concentrated Here



Spectroscopy of Weakly Trapped Landau Levels on a Cone

No Centrifugal Barrier!

(24,0)



THREE-FOLD
SYMMETRY
DOES NOT COUPLE
TO ASTIGMATISM

(12,0)



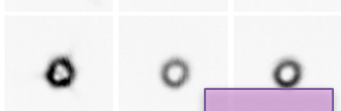
(9,0)



(6,0)



(3,0)

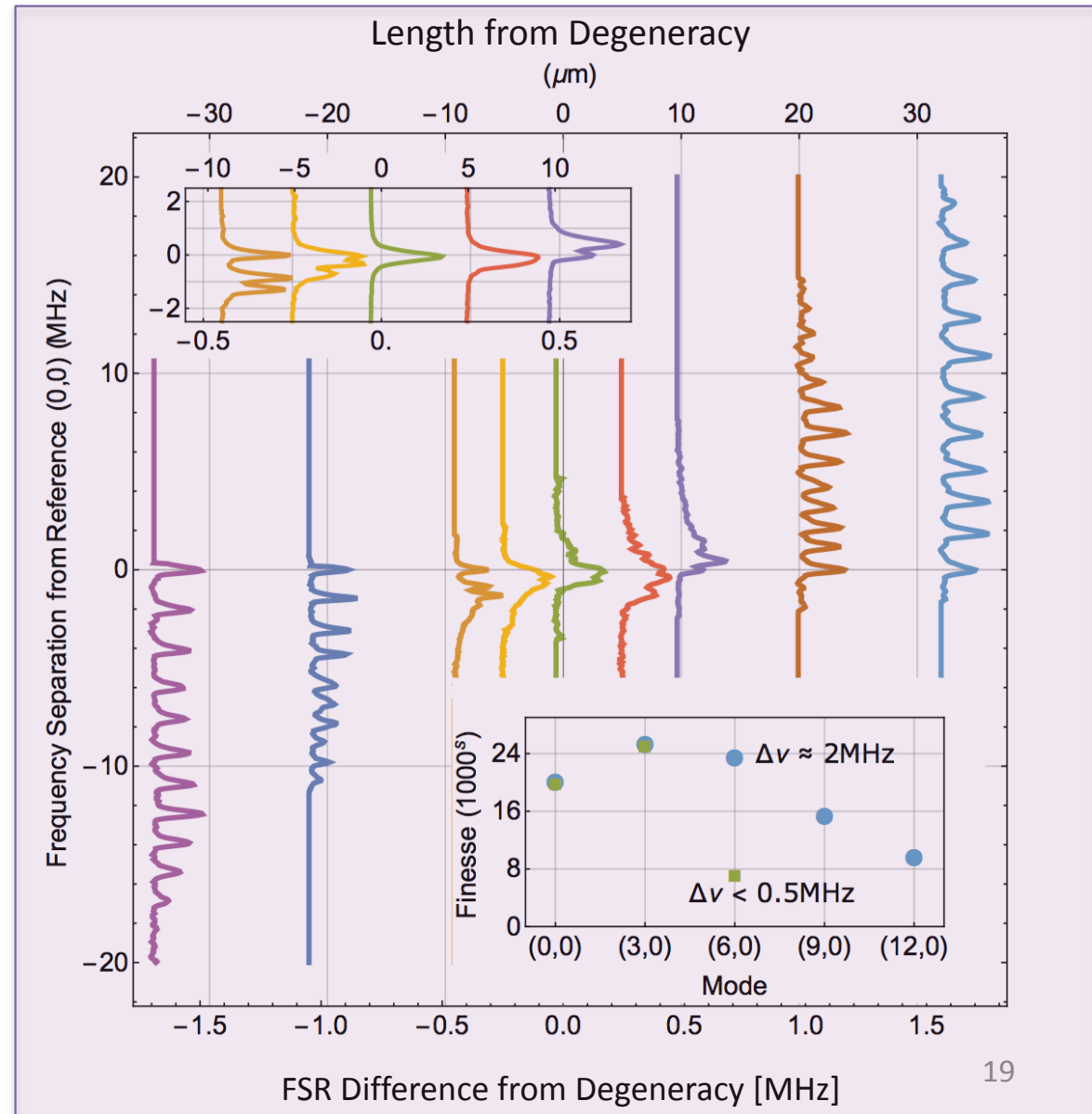


(0,0)



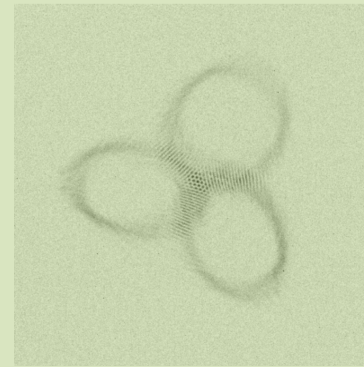
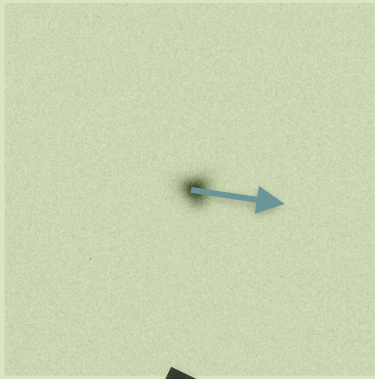
Mode -0.15 1.55 6.05

LLL



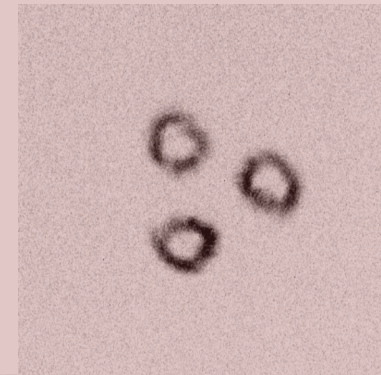
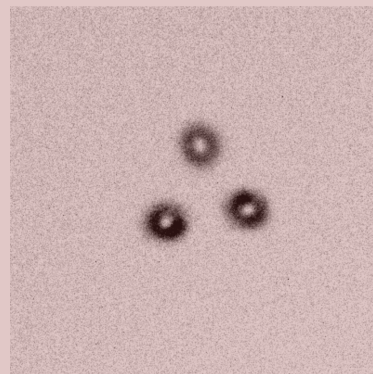
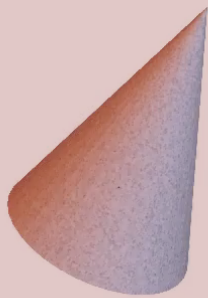
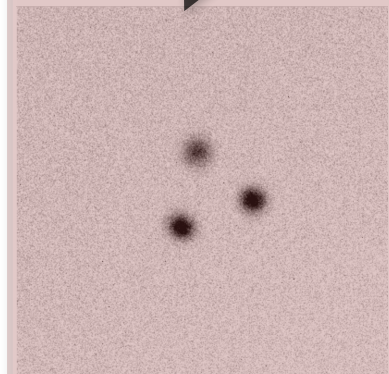
Exploring the Lowest Landau Level (on a Cone)

Sensitive to Conical Geometry



Translate

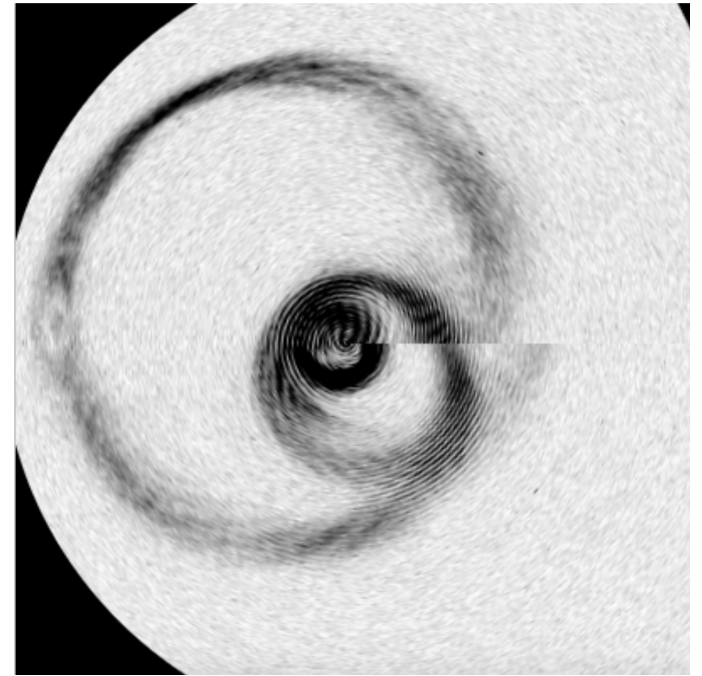
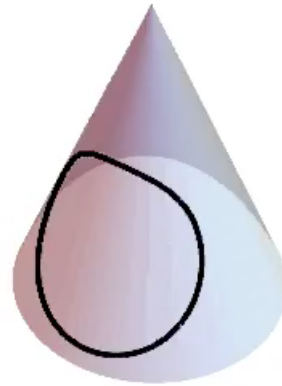
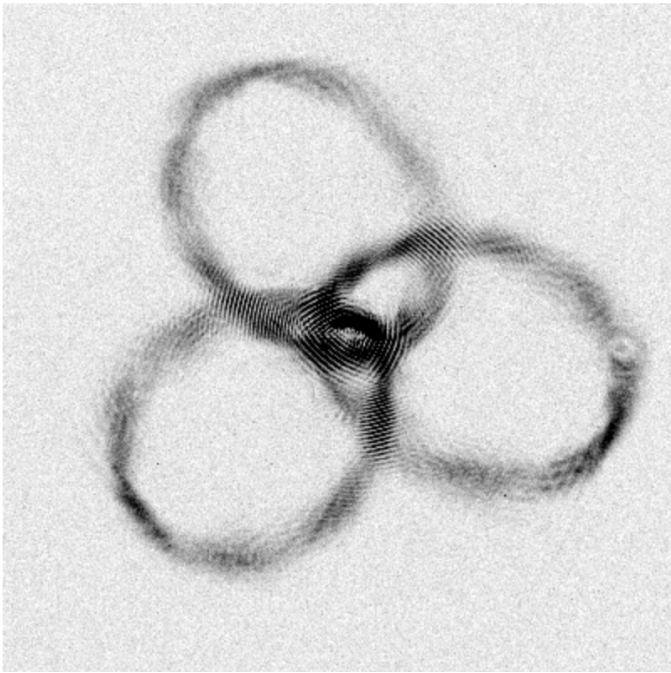
Insensitive to Conical Geometry



20

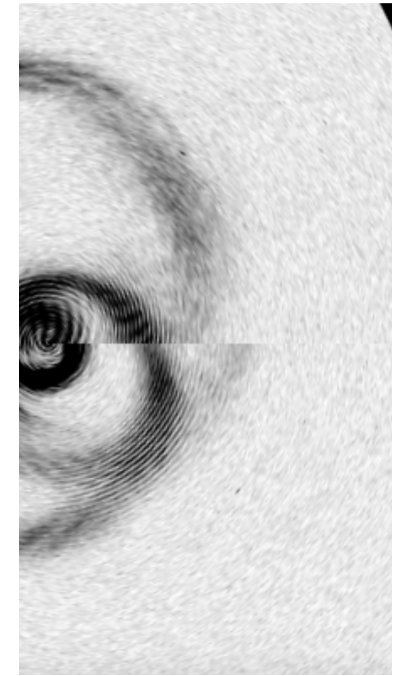
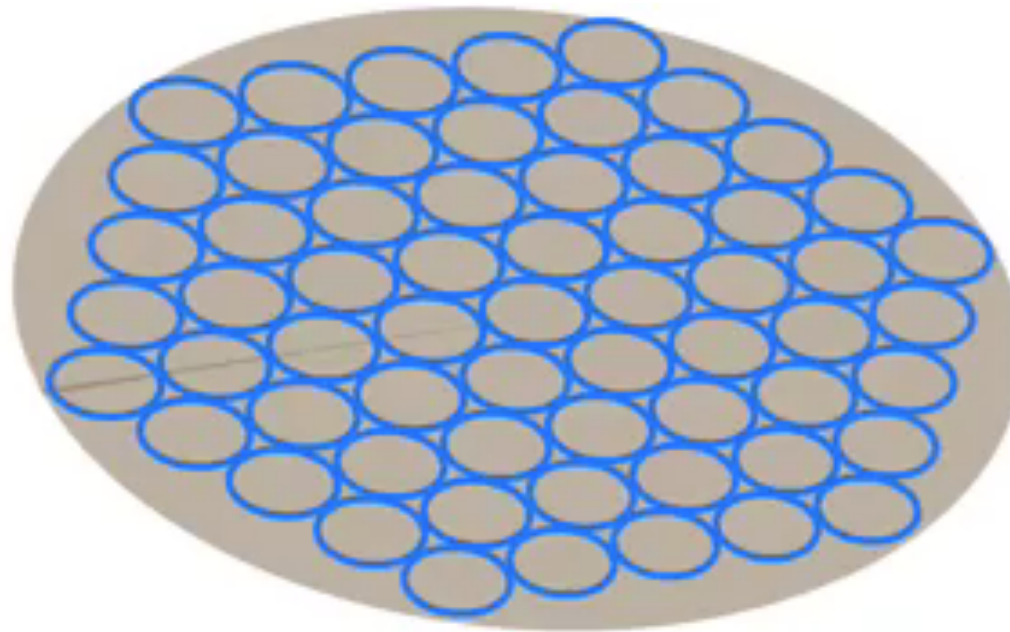
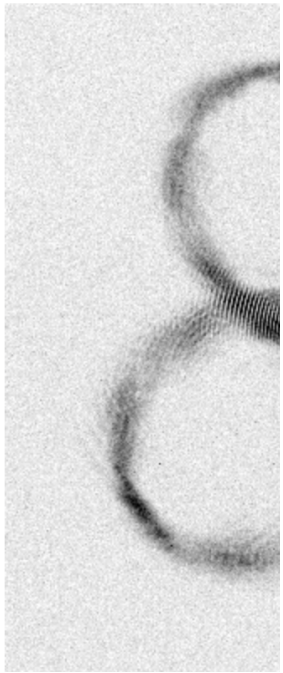
Increasing Angular Momentum

Orbiting the Cone Tip



Biswas *et al.*, *arXiv 1412.3809* (2014)

Orbiting the Cone Tip

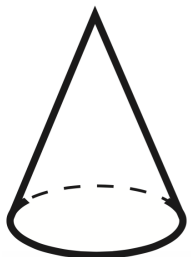
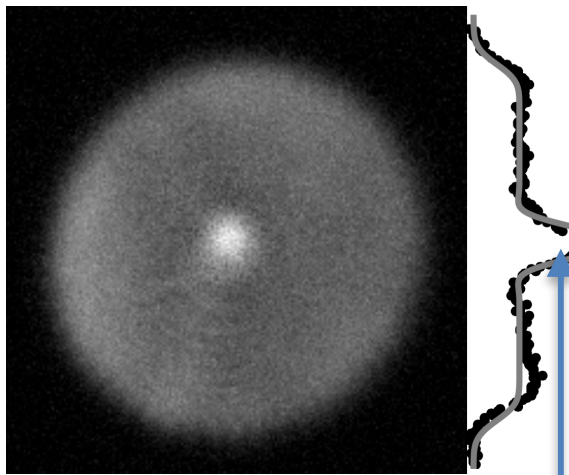


Biswas *et al*

Interplay of Topology and Manifold Curvature

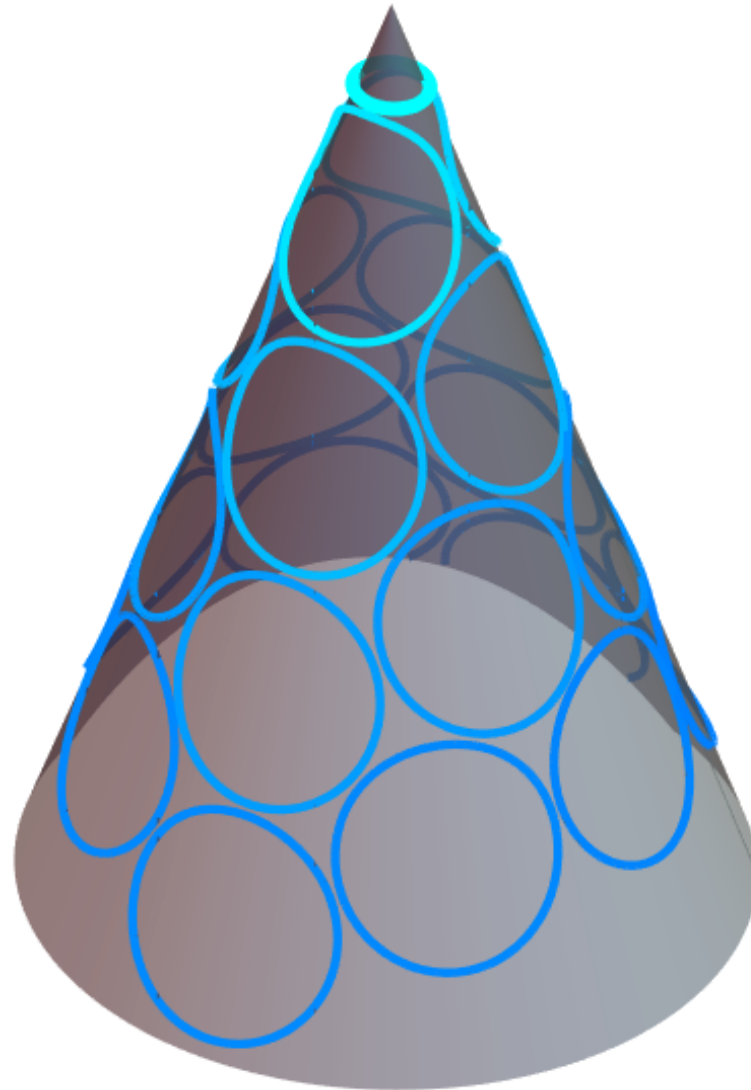
Prediction (Wen-Zee)

$l=0,3,6,9\dots$



0.31[2]

Can *et al.*, Phys. Rev. Lett



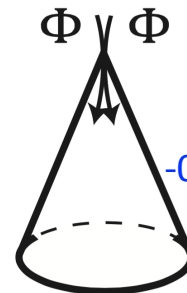
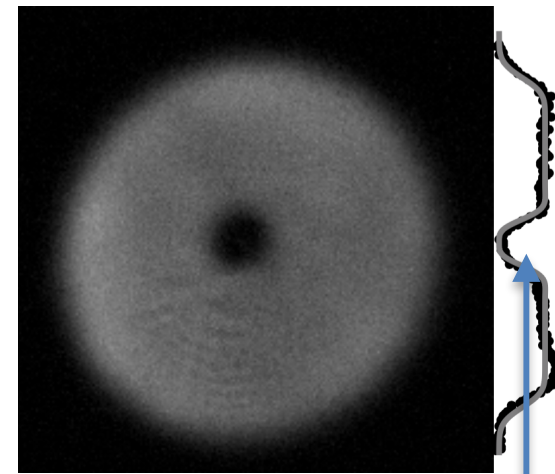
Mean orbital spin \bar{s}

$$+ \frac{1}{2} \frac{R(x)}{4\pi}$$

Cone Tip

$$\frac{1}{2} \frac{R(x)}{4\pi} + \frac{e \delta B(x)}{h}$$

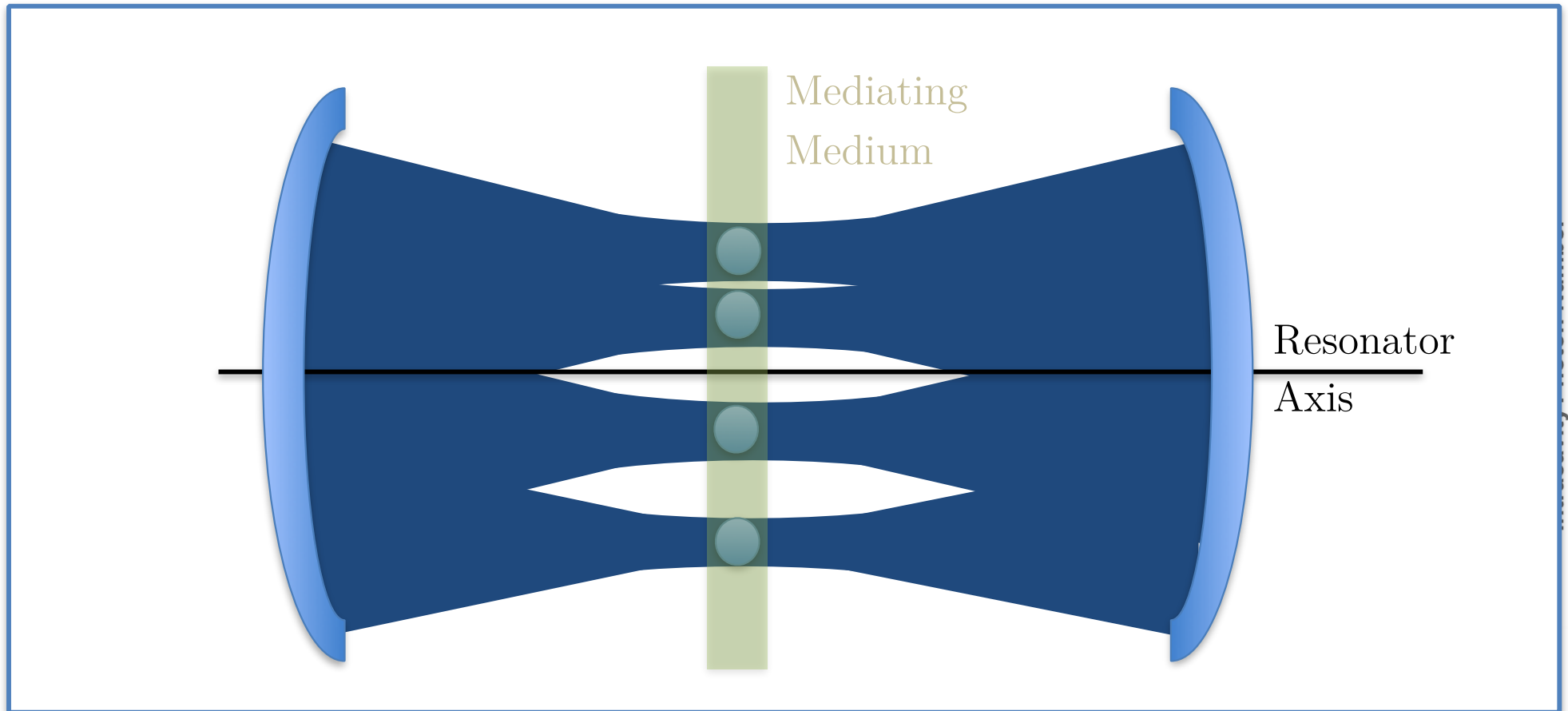
$l=2,5,8,11\dots$



-0.35[2] states

23
ev. B **90**, 014435 (2014)

Adding Photon-Photon Interactions

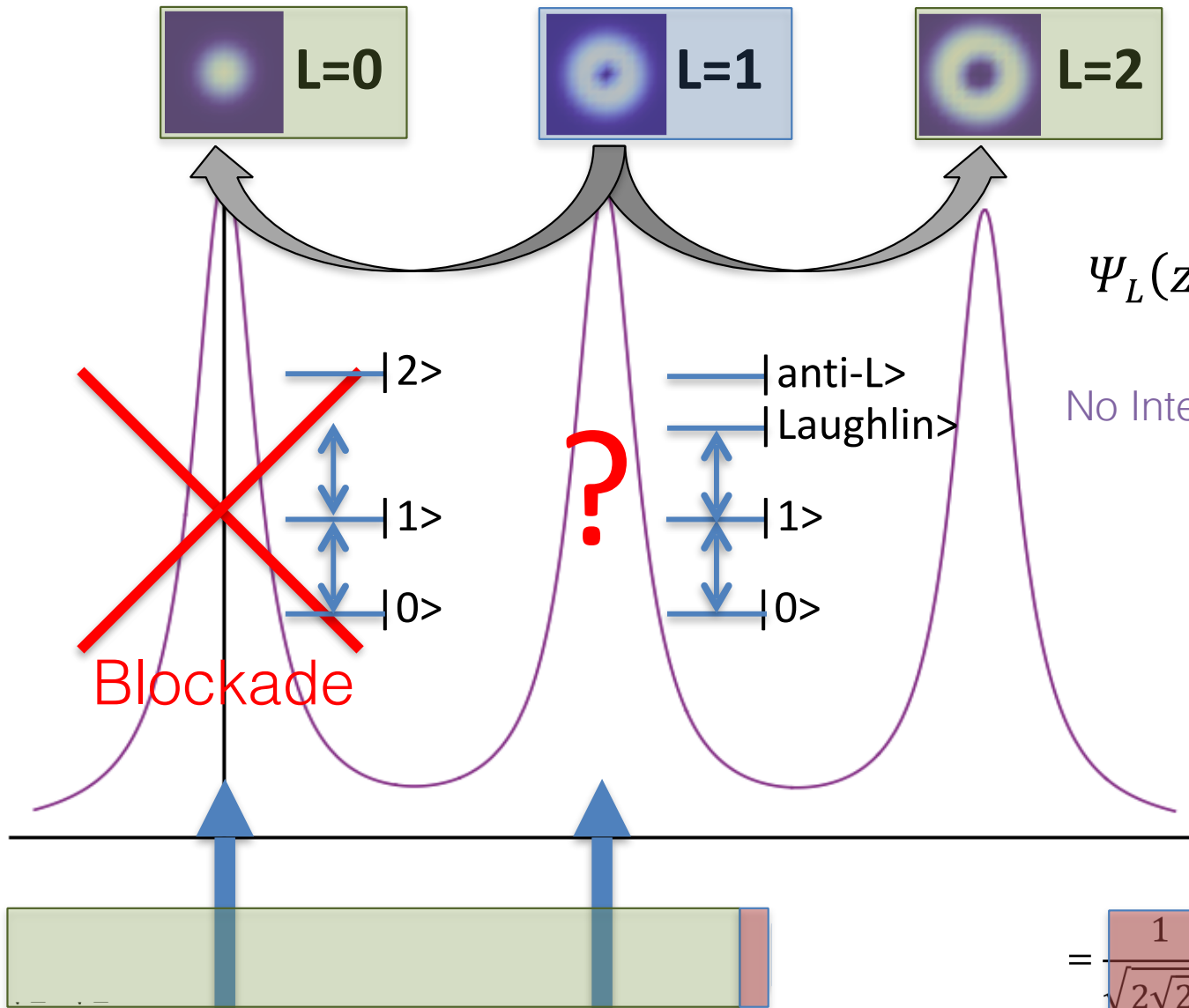


- Rydberg atoms interact \rightarrow photons interact...
- *Now what? How do we make manybody ground states?*

Ningyuan *et al.*, PRA **93**, 041802 (2016), Parigi *et al.*, PRL **109**, 233602 (2016)

Peyronel *et al.*, Nature **488**, 57-60 (2012)

Blockade-Assembly of Small Laughlin States *the “Quantum Optics” way*



$$\Psi_L(z \equiv x + iy) = \frac{z^L}{\sqrt{\pi L!}} e^{-z \bar{z}}$$

No Interactions → 2 photon transmission
LAUGHLIN FILTER

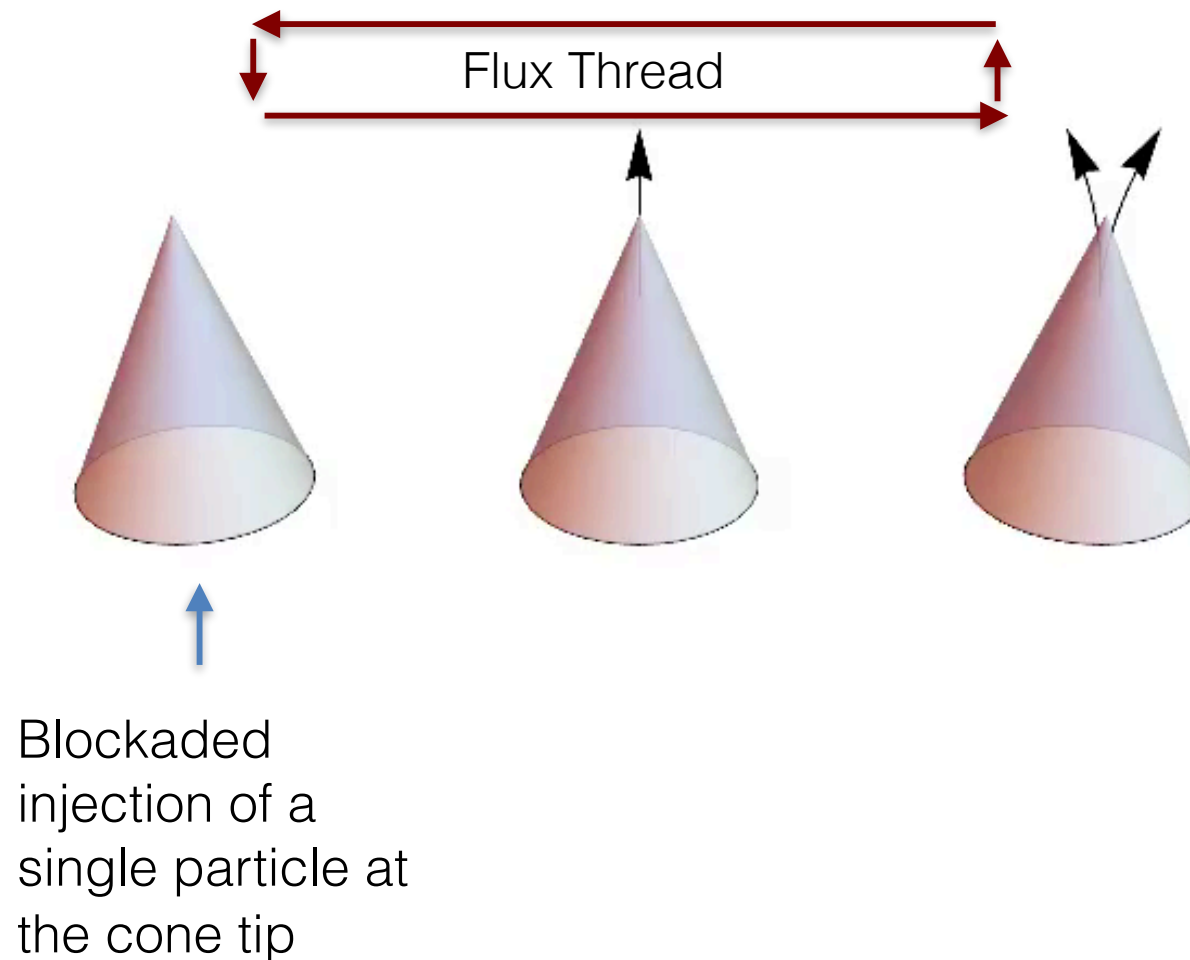
$$= \frac{1}{\sqrt{2\sqrt{2\pi}}} (z_1 - z_2)^2 e^{-(z_1 \bar{z}_1 + z_2 \bar{z}_2)}$$

[1] Gemelke *et al.*, arXiv: 1007.2677 (2010), [2] Umucalilar *et al.*, PRL **108**, 206809 (2012)

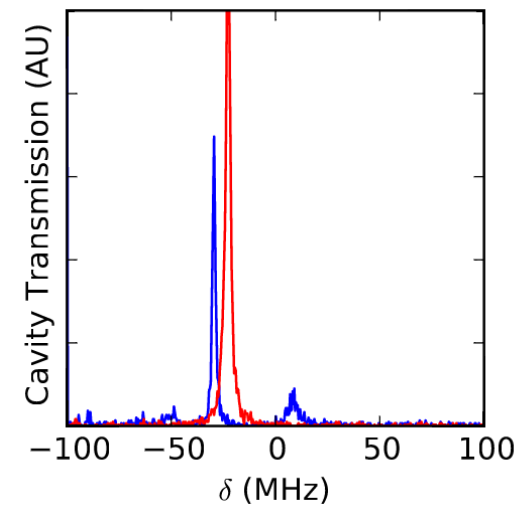
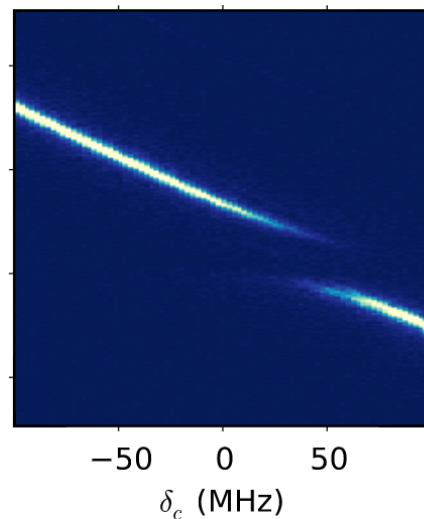
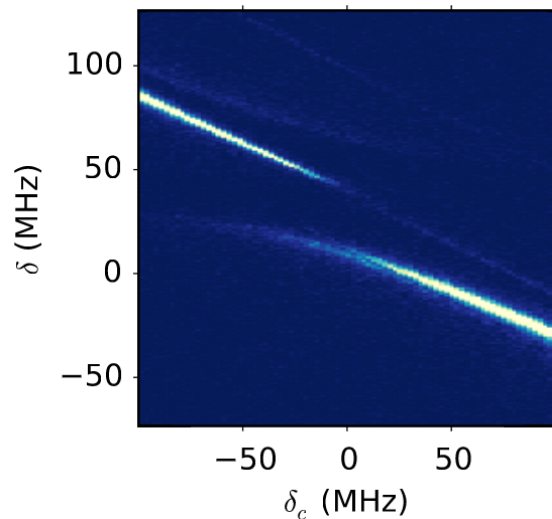
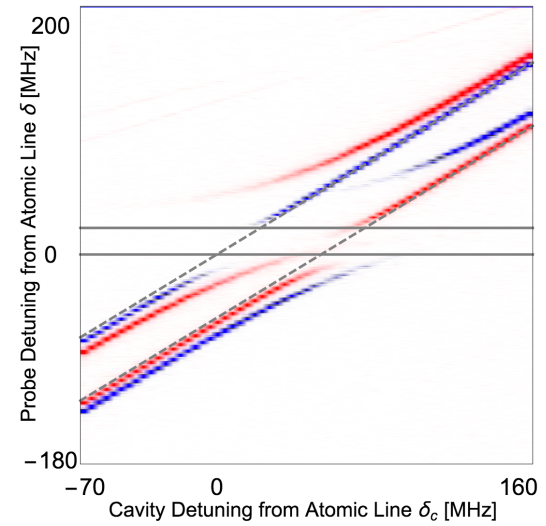
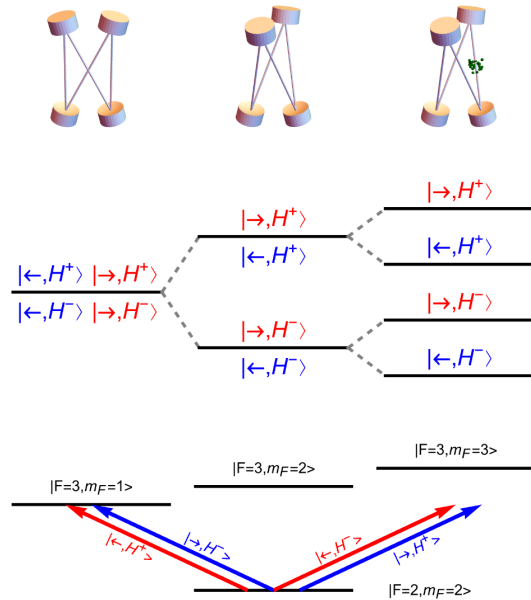
[3] Baur *et al.*, PRA **78**, 061608 (2008), [4] Hafezi *et al.*, NJP **15**, 063001 (2013).

[5] Schauß *et al.*, Nature **491**, 87-91 (2012)

Thouless-Pumping to Laughlin States *the “Quantum Optics” way*

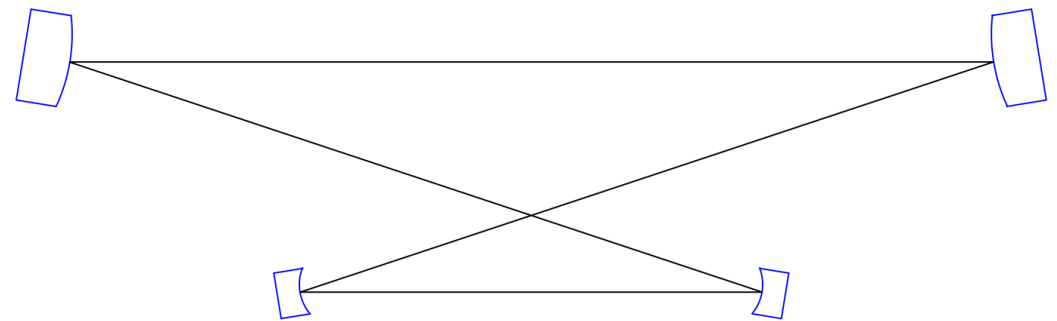


T-Breaking in Twisted Optical Cavities



Next-Generation Resonator

Ostracizing the Rydbergs



Convex Mirrors



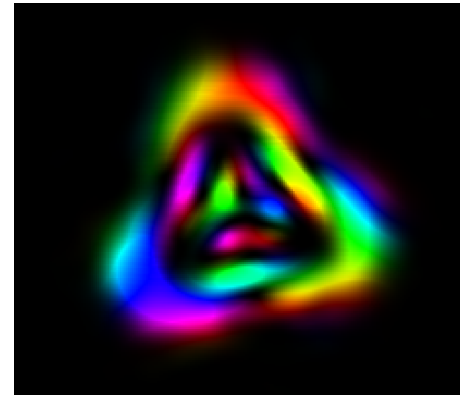
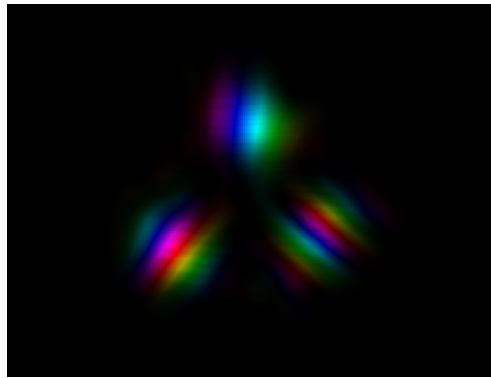
Smaller Waist (12 microns)

@ 1cm to atoms
stable (confocal)

Outlook

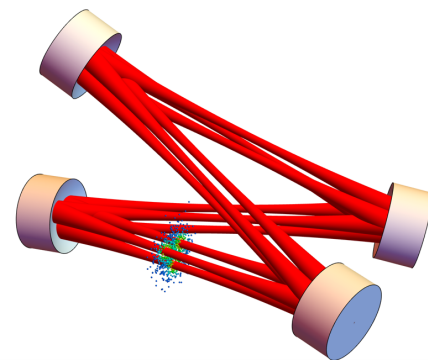
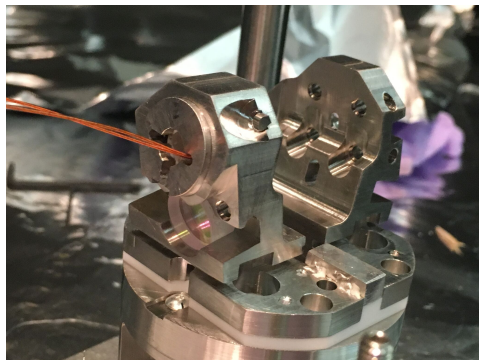
- Chern Number (Kitaev's Real-Space Formalism):

Holographic Reconstruction of Band Projectors

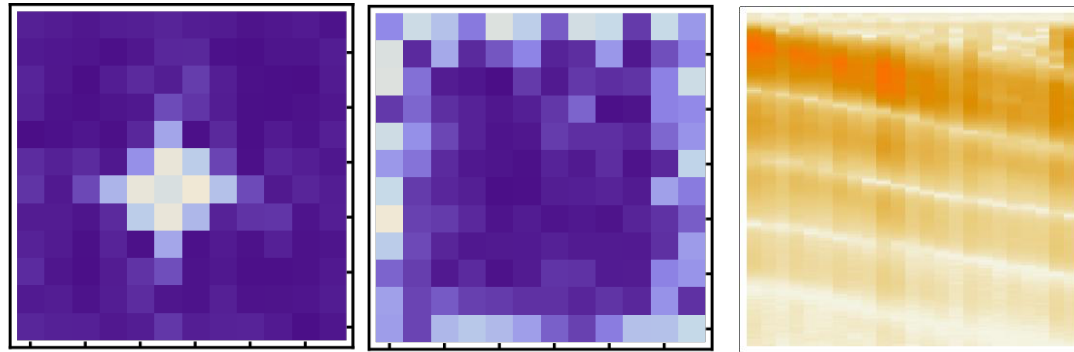


- Strong Interactions:

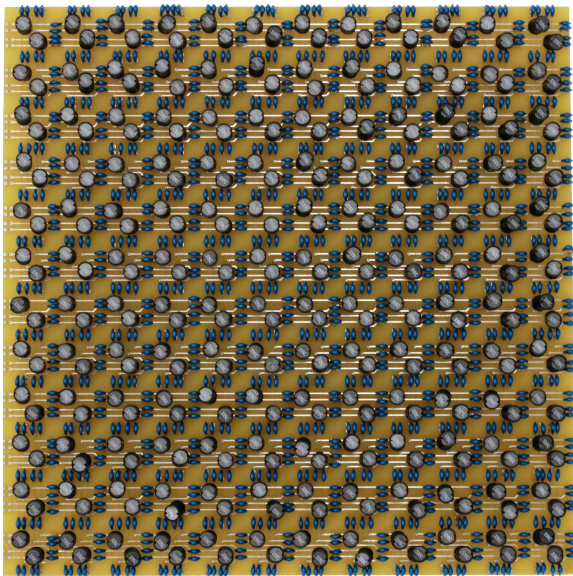
(0-d) Polariton Quantum Dots to (2-d) Laughlin Puddles



Engineering μ Wave Quantum Materials



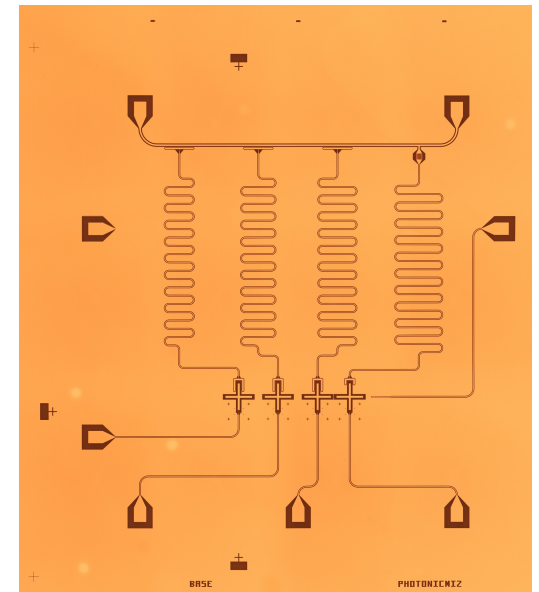
Z_2 TI (QSH)



Lattice QH



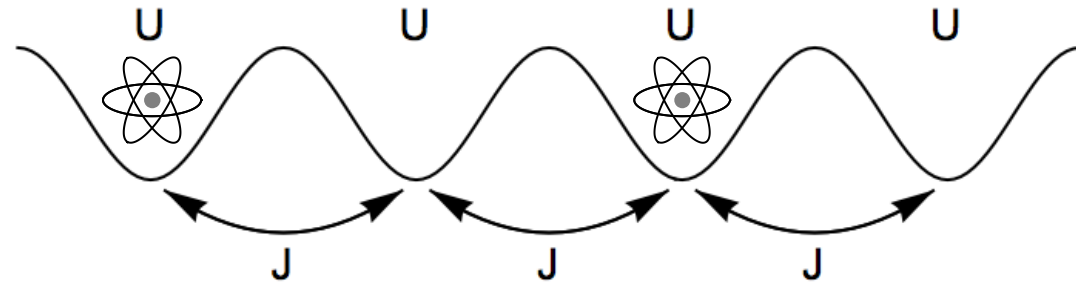
Chemical Potential



The Analogy

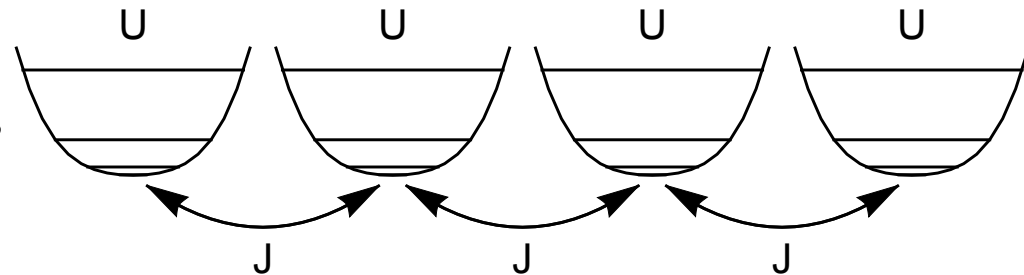
Cold Atoms

- Hopping in an optical lattice
- S-wave contact *interactions*



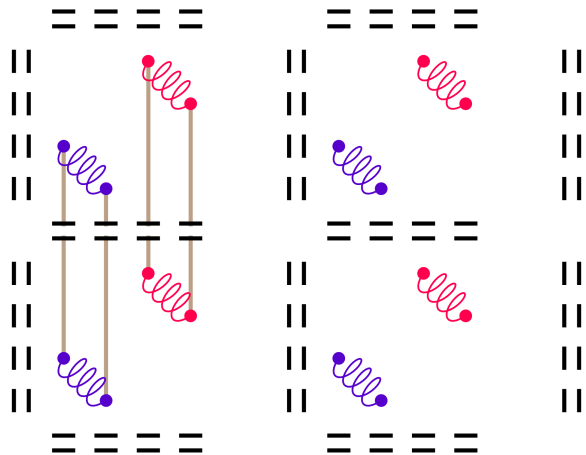
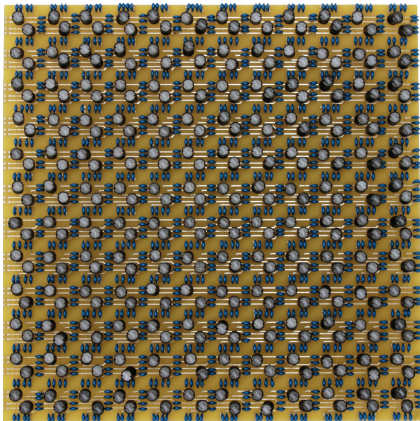
Microwave photons

- Hopping btwn cap.-coupled SC resonators
- Qubit-mediated *interactions*



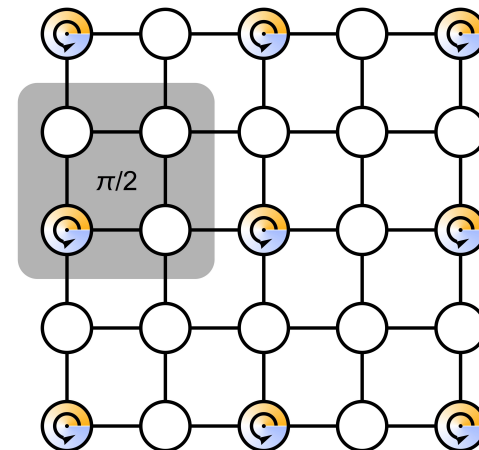
Two Approaches to Synthetic Magnetic Flux in Circuits

Weave Tunnel Couplers to Create Peierls Phase



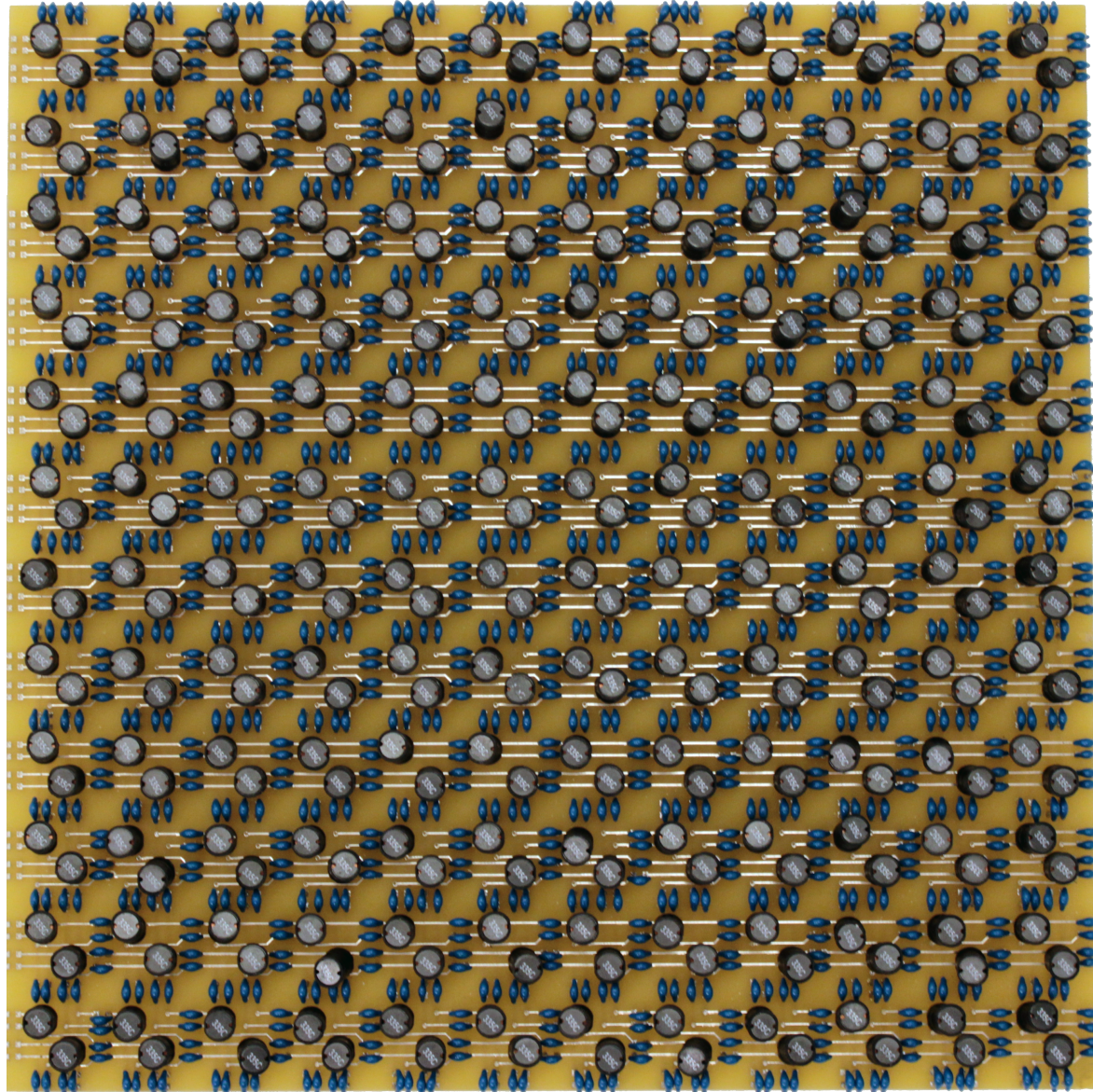
Ningyuan *et al.*, **PRX** 5 (2), 021031

Employ Chiral Orbitals to Create Peierls Phase

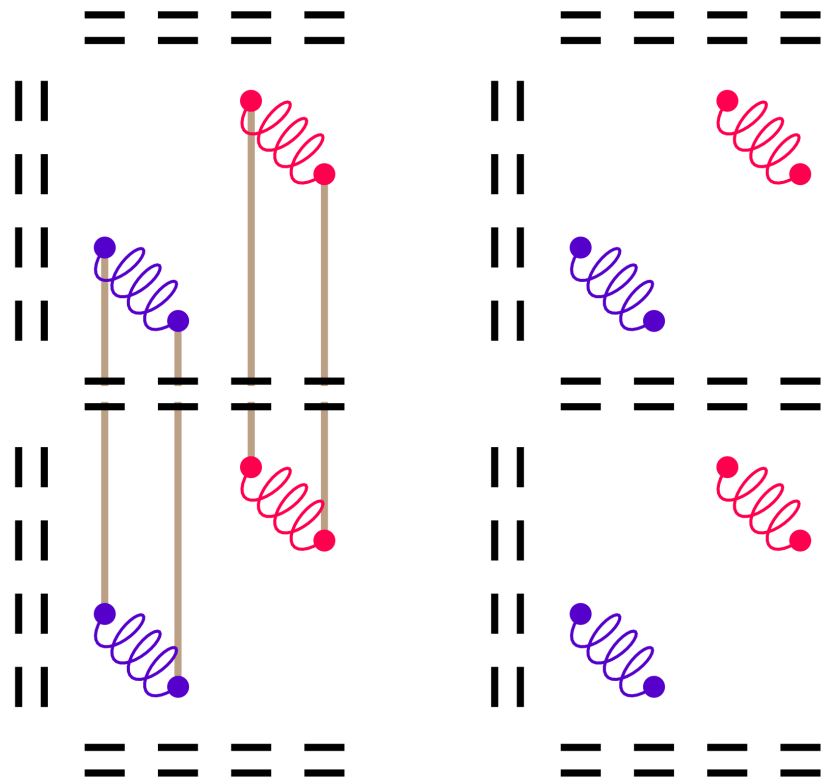


Anderson *et al.*, **arXiv**: 1605.03177, in press **PRX**

(Braided) Topological Circuits



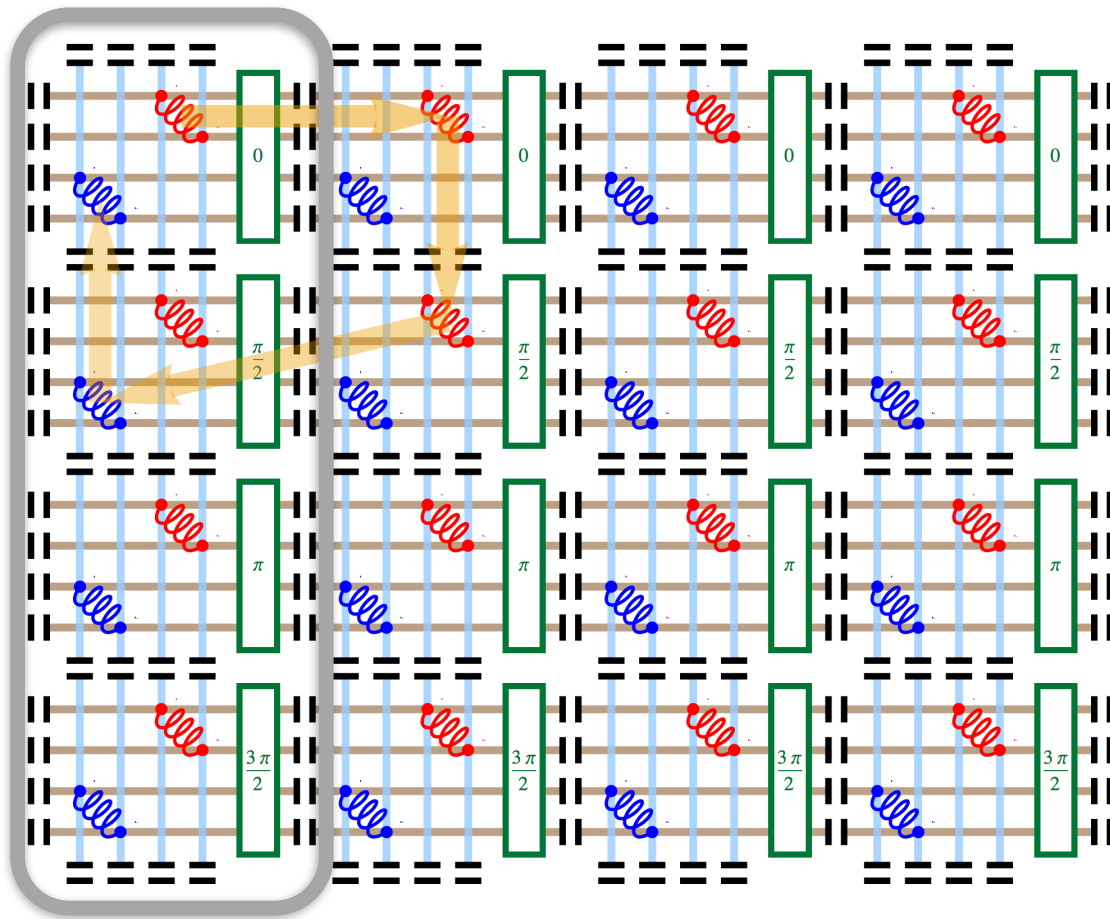
“Braiding” the Planes @ $\alpha=\pi/2$



The diagram shows four equations illustrating the transformation of vector pairs (V_1, V_2) under a 90-degree rotation. Each equation is preceded by a vertical double-line symbol.

$$\begin{aligned} & (V_1, V_2) \longrightarrow (V_1, V_2) \\ & (V_1, V_2) \longrightarrow (V_1, V_2) \\ & (V_1, V_2) \longrightarrow (-V_2, V_1) \\ & (-V_2, V_1) \longrightarrow (-V_2, V_1) \end{aligned}$$

“Braiding” the Planes



Unit Cell

$$|\uparrow\rangle = \mathbf{A} + i\mathbf{B}$$

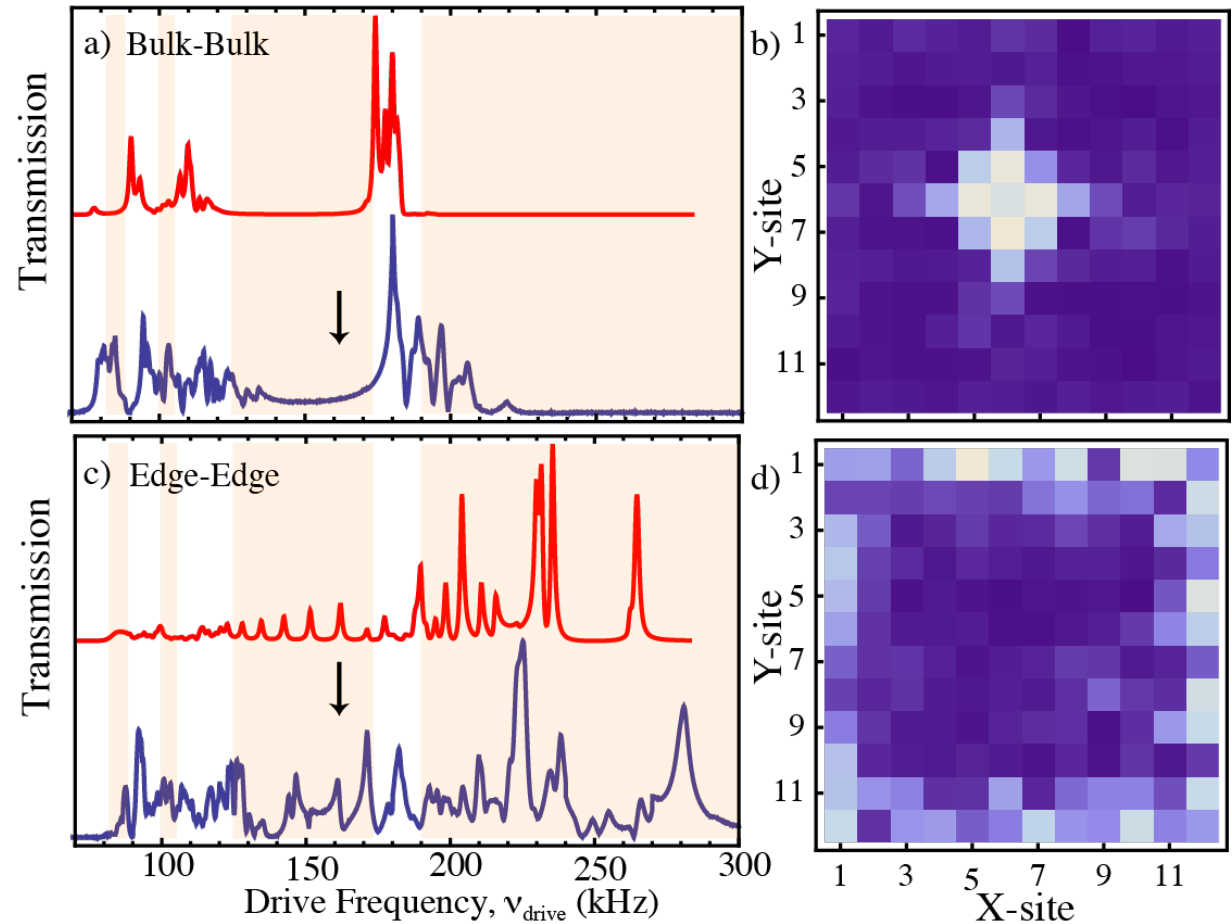
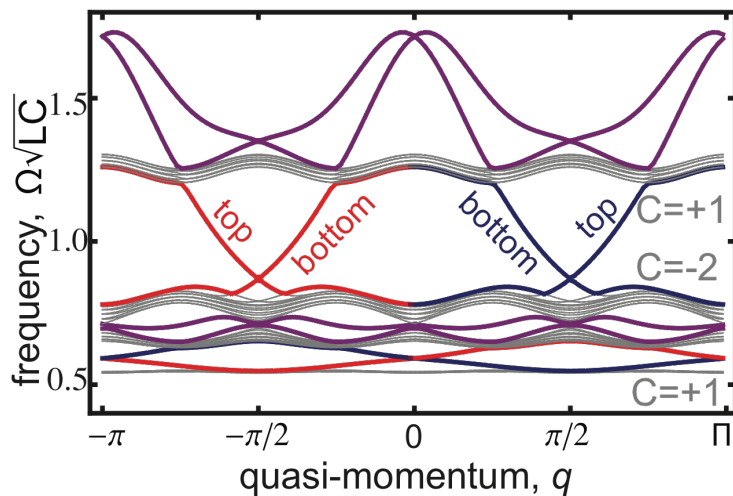
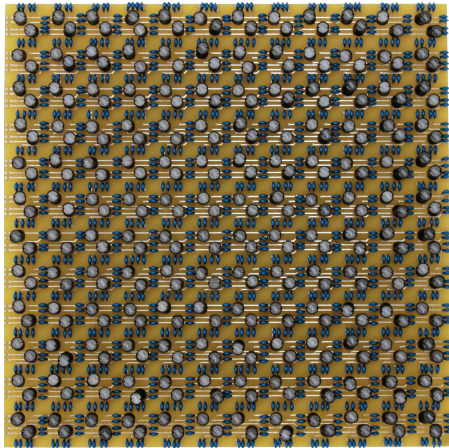
$$|\downarrow\rangle = \mathbf{A} - i\mathbf{B}$$

$$|\uparrow\rangle \rightarrow -i|\uparrow\rangle = e^{i\pi/2}|\uparrow\rangle$$

$$|\downarrow\rangle \rightarrow -i|\downarrow\rangle = e^{-i\pi/2}|\downarrow\rangle$$

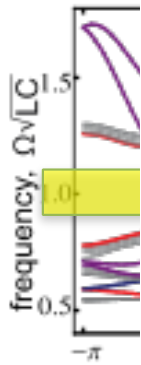
So $\alpha = \pm\pi/2$: Spin-Hall

A Topologically Insulating Circuit

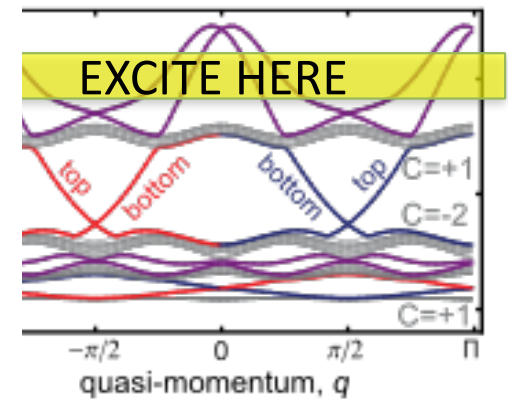


Spin- and Time- Resolved Dynamics

Topolo

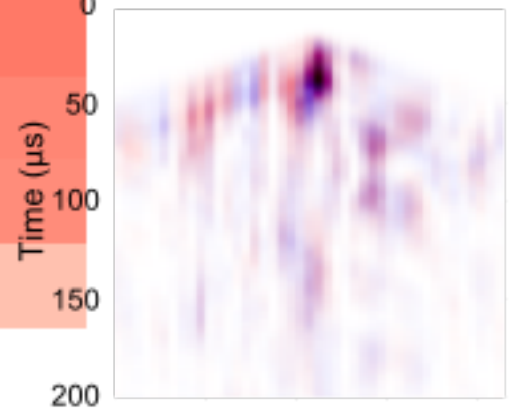
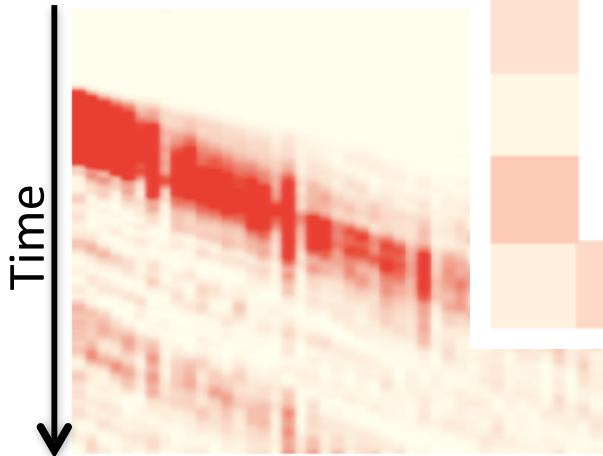


Unprotected Gap

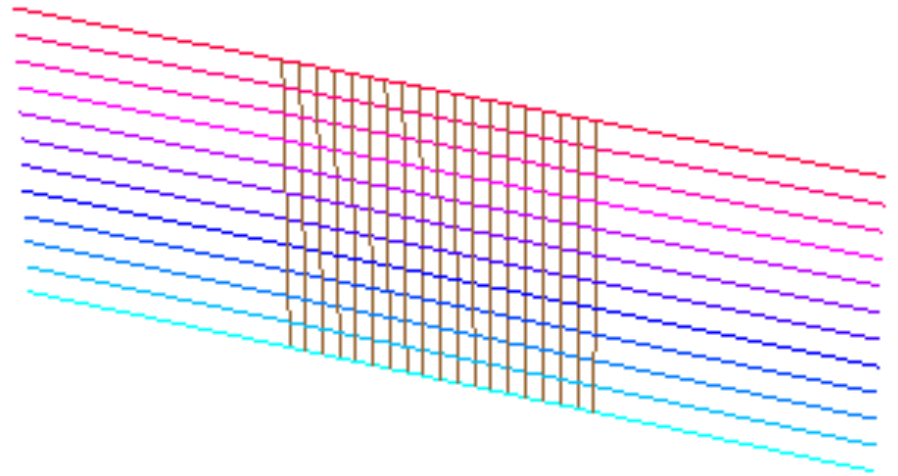
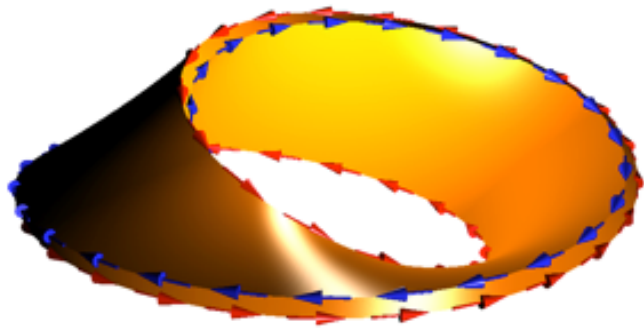


Excite One Spin State

Excite Both States



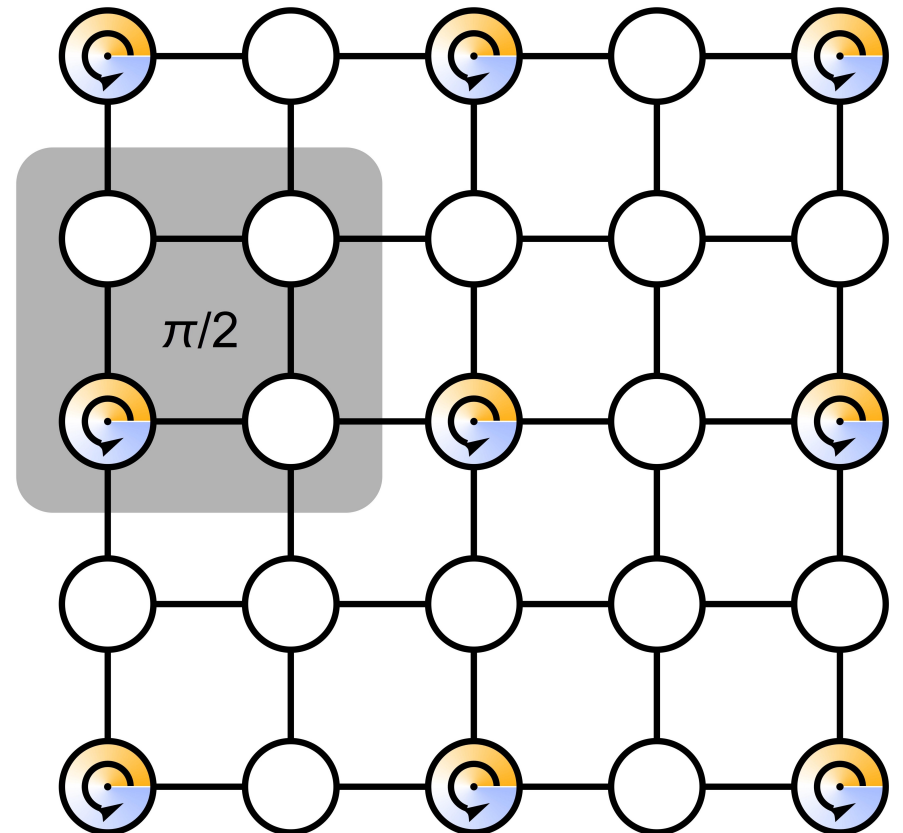
Exotic Global Topologies: Möbius TI



- Large wavelength compared to material size
- Useful for making tori, cylinders, etc.. for FQH physics

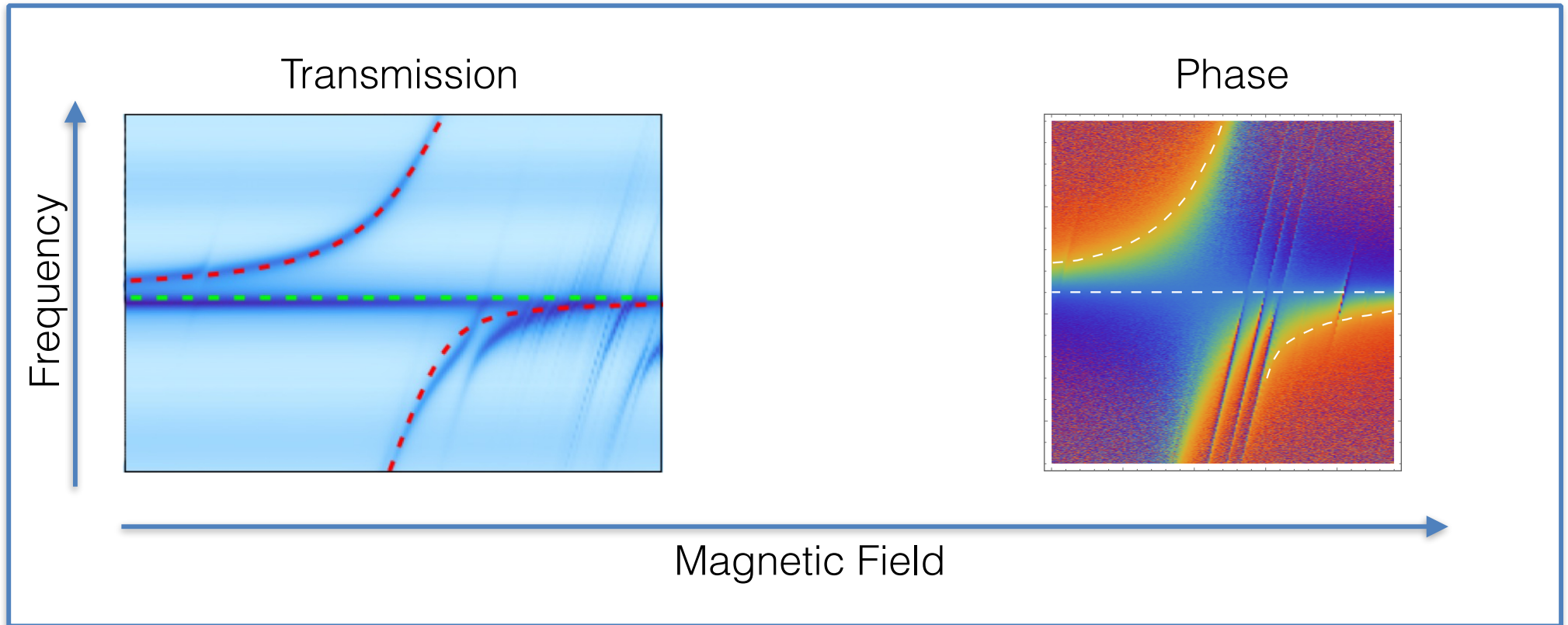
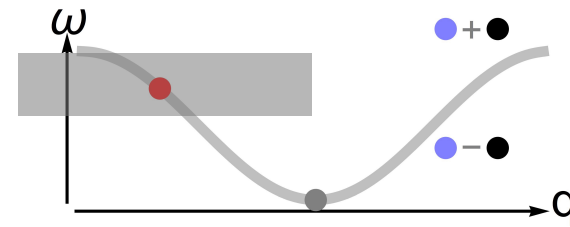
Breaking T Symmetry

Microwave Chern Insulators

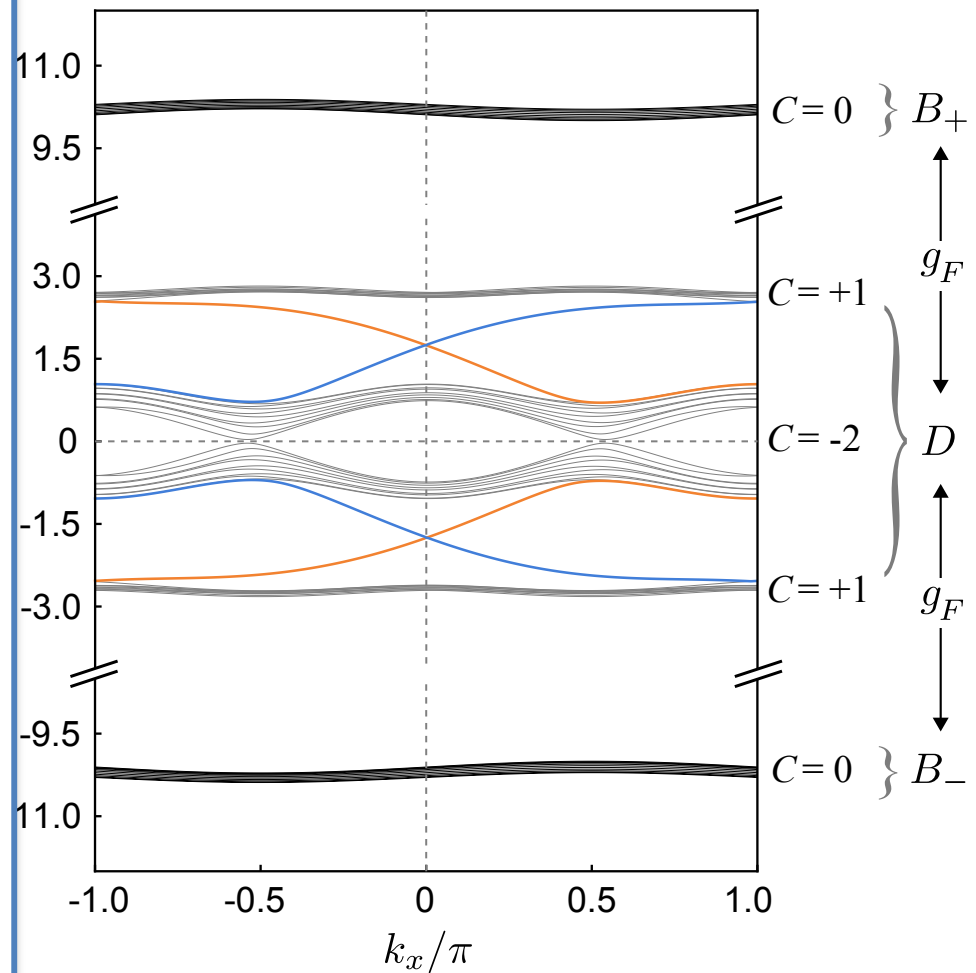
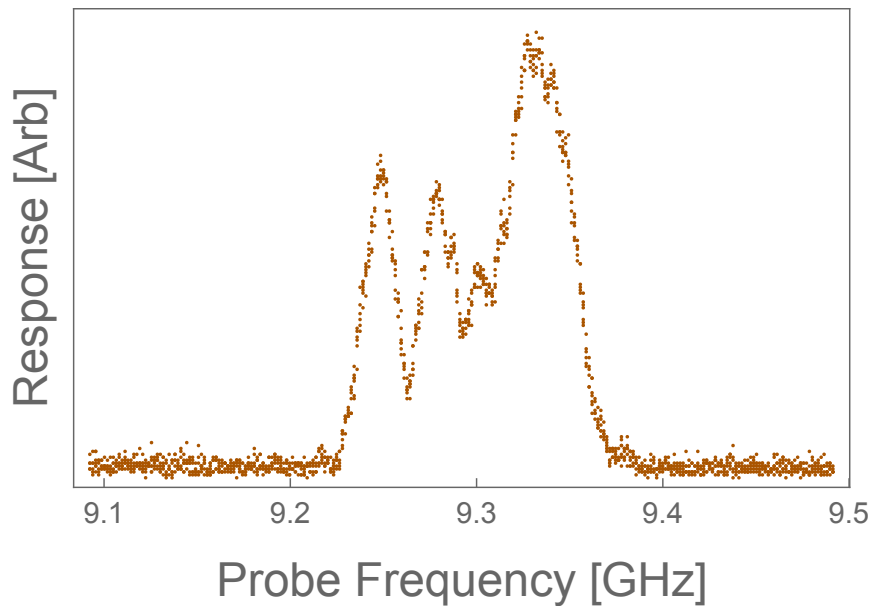
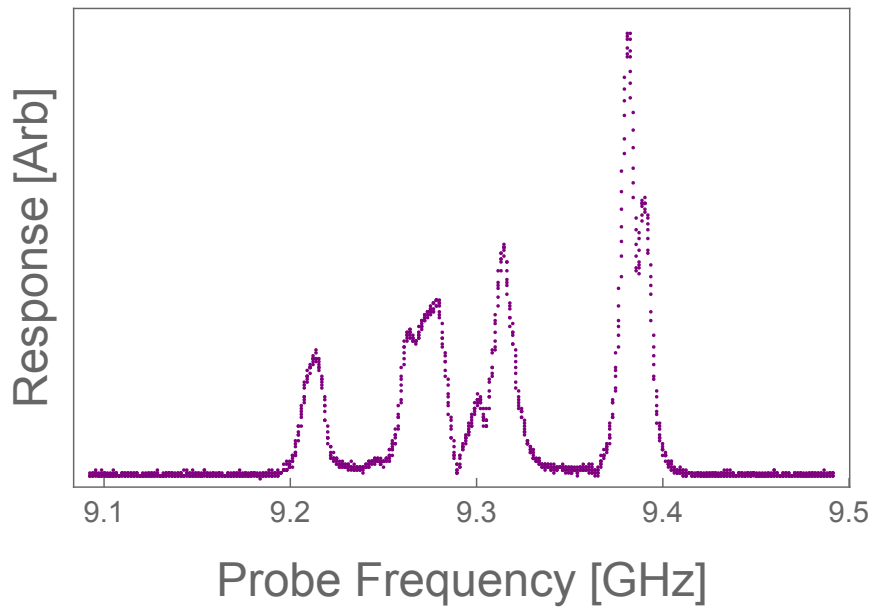


Building a Superconducting Photonic Lattice: A Chiral (p_x+ip_y) Site

Isolated Chiral Mode
(unspoiled by YIG)

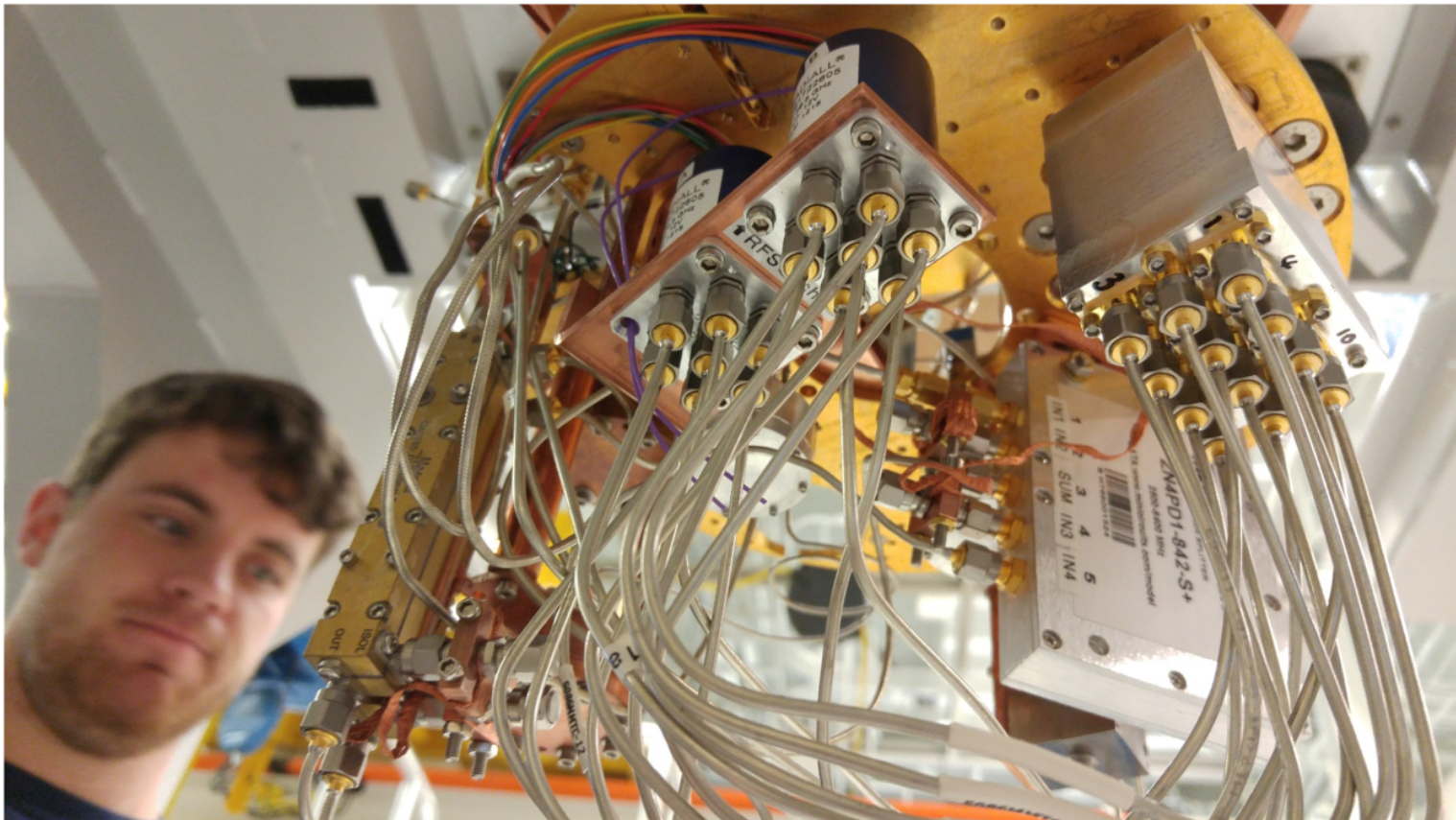


A 7×7 $\alpha = 1/4$ Harper-Hofstadter Lattice



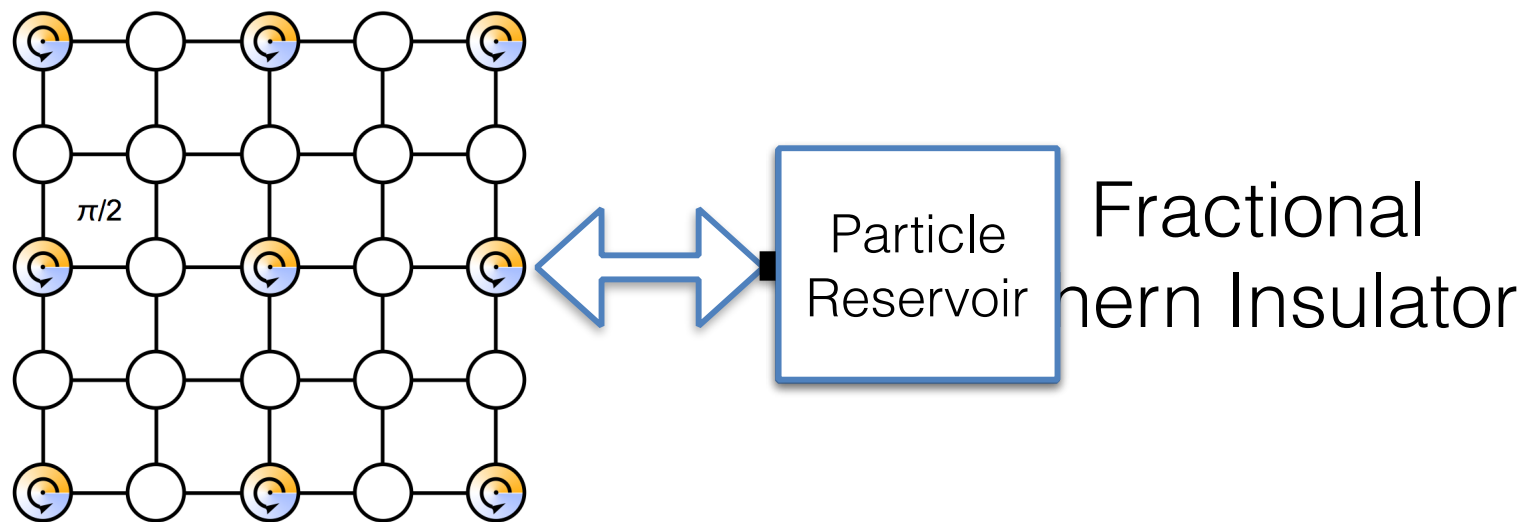
Temporal Dynamics in an $\alpha=1/4$ Hofstadter Lattice

Time



How do we build manybody states?

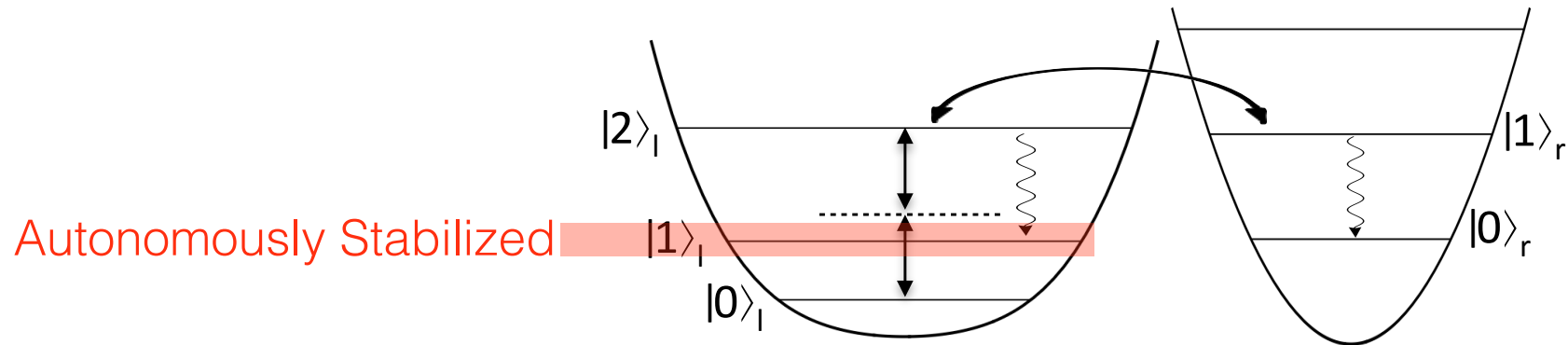
- Add interactions via J-J qubits on each site
... but how do we create correlated, interacting phases?
- Connect the lattice to a $T=0$ chemical potential—populates the system with particles!



Hafezi *et al.*, PRB **92**, 174305 (2015)

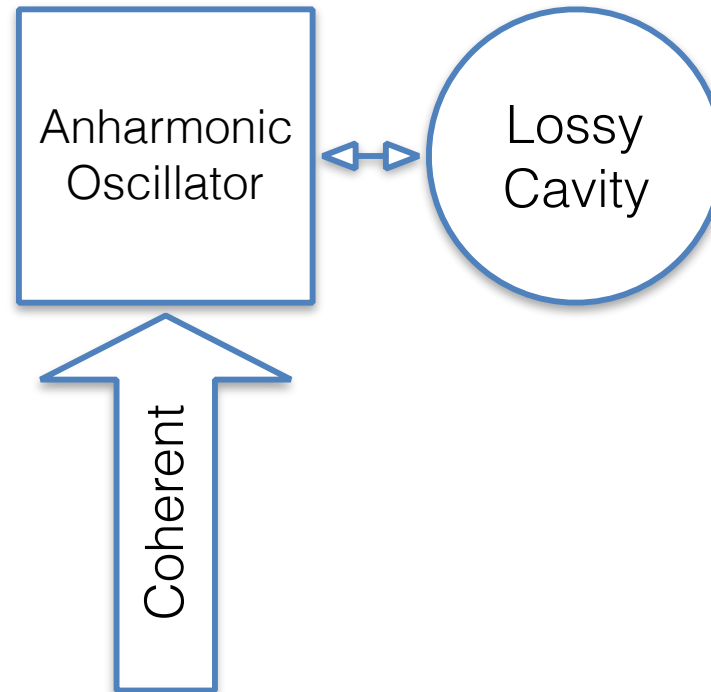
Kapit *et al.*, PRX **4**, 031039 (2016)

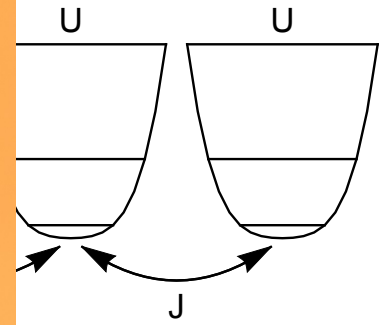
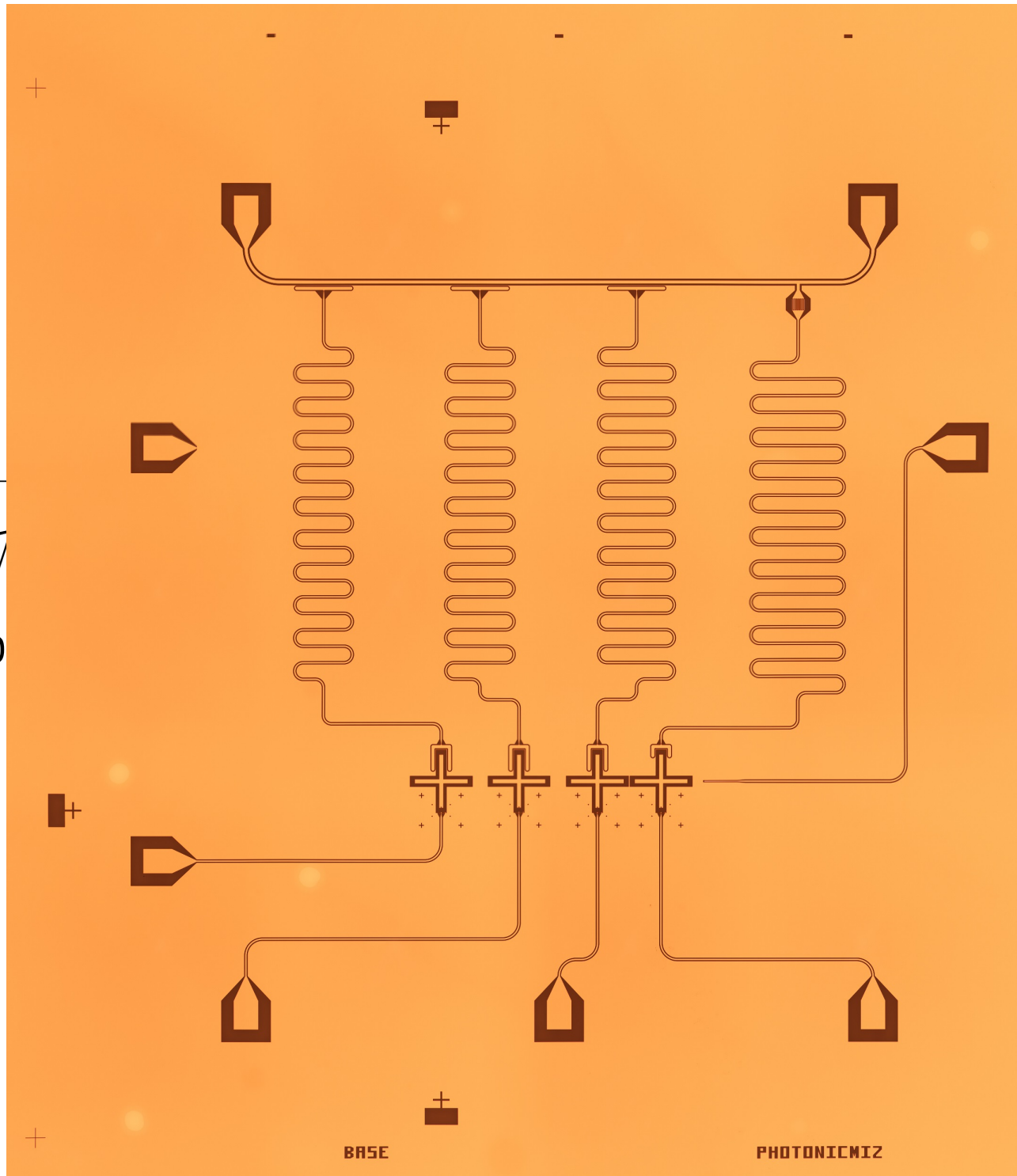
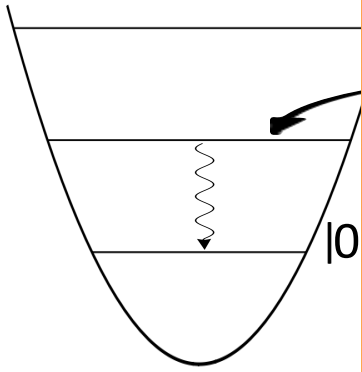
Building a Particle Reservoir



Stabilized
 $n=1$

=





Towards Photonic Materials

Experiment

- Schine *et al.*, “Synthetic Landau levels for photons,” **Nature** 534, 671–675 (2016).
- Ningyuan *et al.*, “Observation and characterization of cavity Rydberg polaritons,” **PRA** 93 (4), 041802 (2016).
- Ningyuan *et al.*, “Time-and site-resolved dynamics in a topological circuit,” **PRX** 5 (2), 021031 (2015).
- Ryou *et al.*, “Active Cancellation of Acoustical Resonances with an FPGA FIR Filter,” **arXiv**: 1604.04668 (2016).
- Sommer *et al.*, “Time-Reversal Symmetry Breaking in a High-Finesse Non-Planar Cavity,” *In Preparation*.

Theory

- Sommer *et al.*, “Engineering Photonic Floquet Hamiltonians through Fabry–Pérot resonators,” **NJP** 18 (3), 035008 (2016).
- Anderson *et al.*, “Engineering topological materials in microwave cavity arrays,” accepted to **PRX**.
- Ma *et al.*, “Hamiltonian Tomography of Photonic Lattices,” **arXiv**:1607.05180 (2016).
- Sommer *et al.*, “Quantum Crystals and Laughlin Droplets of Cavity Rydberg Polaritons,” **arXiv**: 1506.00341 (2015).
- Ma *et al.*, “A Simple Chemical Potential for Photons,” *In Preparation*.

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Andrey Gromov
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Thanks!



Bulk Polaritonic Materials

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Michelle Chalupnik (UG)

Lattice Microwave Materials

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Brendan Saxberg (PhD)
Aman LaChapelle (UG)

Optical/mmWave Hybrid

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Mark Stone (PhD)
Scott Eustice (UG)



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