

Multipolar frustration instead of geometric frustration

Gang Chen
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Multipolar frustration instead of geometric frustration

from realty to fantasy

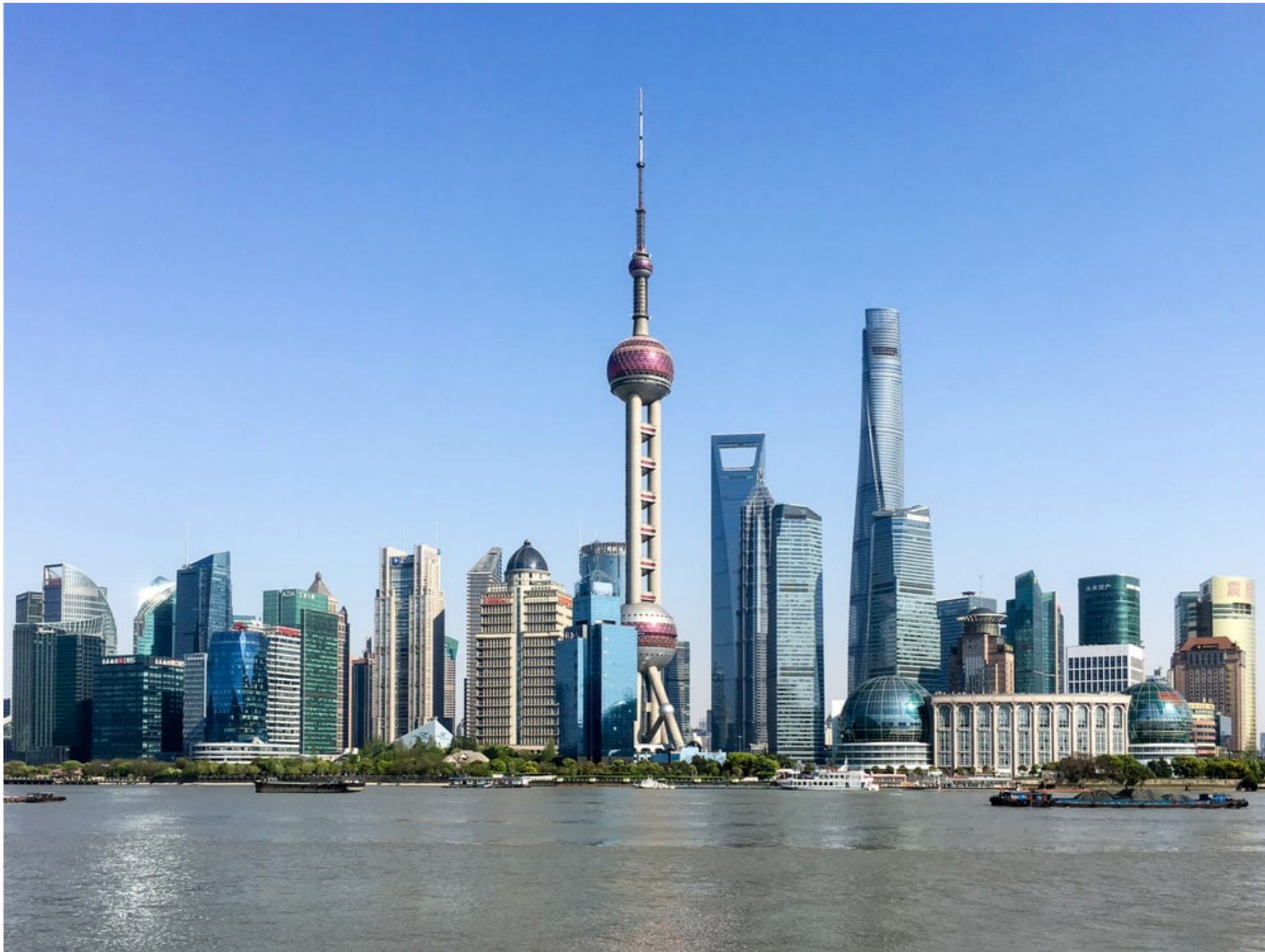
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Abstract Submission: 10/15/2019 - 12/15/2019

Early Registration: 1/1/2020 - 2/14/2020

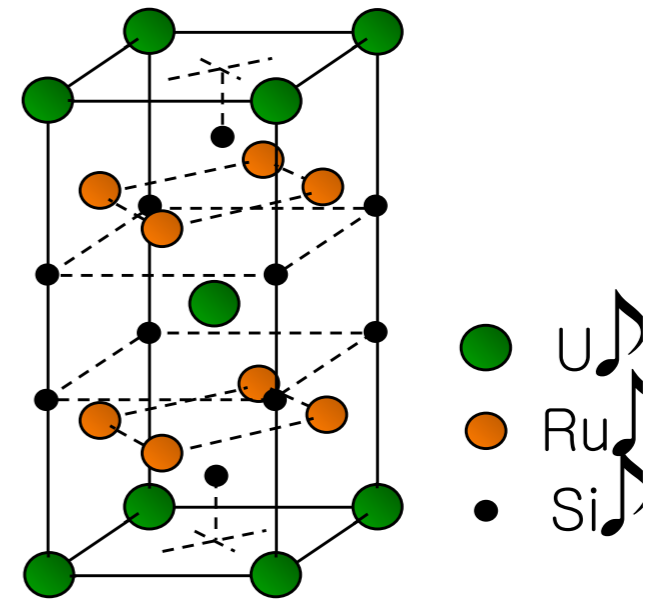
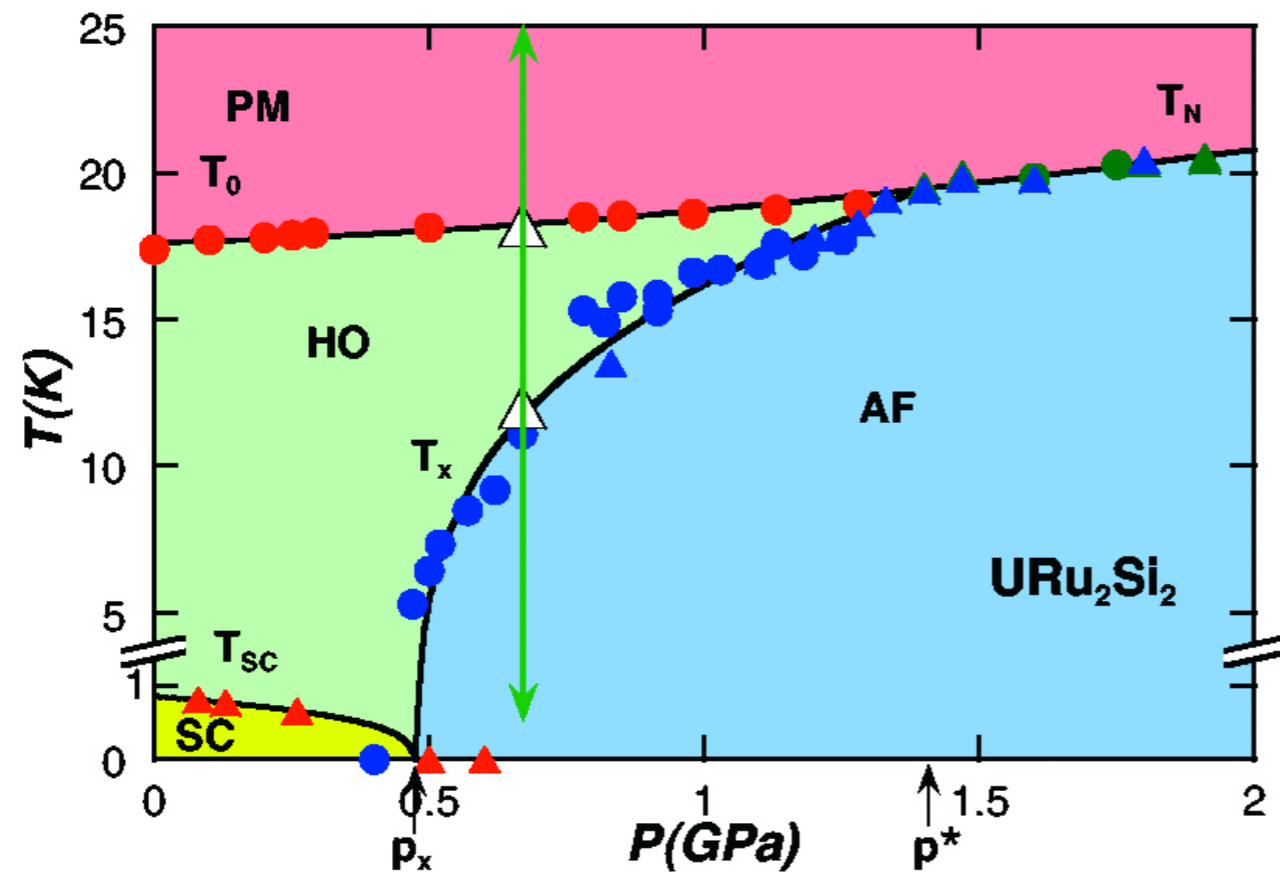


Organizing committee: Leon Balents, [Gang Chen](#), Michel Gingras, Sungbin Lee, Jie Ma, Rajiv Singh, Yuan Wan, Xiaoqun Wang

Outline

1. Identifying hidden multipolar order in triangular lattice magnet
2. Symmetry enriched $U(1)$ spin liquids, field-driven Anderson-Higgs transition,

Hidden order in condensed matter



- Hidden order: “dark matter” in CMT
- URu_2Si_2
 - Second order transition at $\sim 17\text{K}$, $\Delta S \sim 0.42 R \ln 2$
 - Order parameters unknown after decades

Hidden Order Behaviour in URu₂Si₂

(A Critical Review of the Status of Hidden Order in 2014)

J. A. Mydosh^{a*} and P. M. Oppeneer^b

Table 1. Summary of ongoing contemporary experiments to characterise the heavy fermion precursor, the HO transition and the HO and superconducting states of URu₂Si₂.

Angular resolved photoemission (ARPES) [7–11]
Quantum oscillations (QO) [12–14]
Elastic and inelastic neutron scattering [15–19]
Nuclear magnetic and quadrupolar resonance (NMR, NQR) [20–22]
Scanning tunneling microscopy (STM) and spectroscopy (STS) [23, 24]
Ultrafast time-resolved ARPES and reflection spectroscopy [25, 26]
Phononic Raman [27] and electronic Raman spectroscopy [28]
Optical spectroscopy [29–31]
Polar Kerr effect [32]
Magnetic torque measurements [5, 33]
Cyclotron resonance [34]
X-ray diffraction [35, 36]
X-ray resonant scattering (XRS) [37, 38]
Point contact spectroscopy (PCS) [39–41]
Resonance ultrasonics [42]
Core-level spectroscopy (XAS, EELS) [43]
Elasto-resistivity [44]

Every quantum material is a universe, and our telescopes are the experimental probes.
More telescopes can be applied if there are more kinds of degrees of freedom.

Table 2. Summary of analytic theories and models proposed to explain the HO, with an emphasise on the recent contributions. For proposals of specific multipolar magnetic order on the U ions, see Table 3.

Barzykin & Gorkov (1995)	three-spin correlations [45]
Kasuya (1997)	uranium dimerisation [46]
Ikeda & Ohashi (1998)	d-spin density wave [47]
Okuno & Miyake (1998)	CEF & quantum fluctuations [48]
Chandra et al. (2002)	orbital currents [49]
Viroszek et al. (2002)	unconv. spin density wave [50]
Mineev & Zhitomirsky (2005)	staggered spin density wave [51]
Varma & Zhu (2006)	helicity (Pomeranchuk) order [52]
Elgazzar et al. (2009)	dynamical symmetry breaking [53]
Kotetes et al. (2010)	chiral d-density wave [54]
Dubi & Balatsky (2011)	hybridization wave [55]
Pepin et al. (2011)	modulated spin liquid [56]
Fujimoto (2011)	spin nematic order [57]
Riseborough et al. (2012)	unconv. spin-orbital density wave [58]
Das (2012)	spin-orbital density wave [59]
Chandra et al. (2013)	hastatic order [60]
Hsu & Chakravarty (2013)	singlet-triplet d-density wave [61]

Various theoretical proposals,
still unresolved

Table 3. Summary of proposals for a specific multipolar magnetic ordering on the uranium ion to explain the HO, with an emphasise on the recent contributions. Note that different symmetries are possible for high-rank multipoles, therefore some kind of multipoles appear more than once.

Nieuwenhuys (1987)	dipole (2^1) order [62]
Santini & Amoretti (1994)	quadrupolar (2^2) order [63]
Kiss & Fazekas (2005)	octupolar (2^3) order [64]
Hanzawa & Watanabe (2005)	octupolar order [65]
Hanzawa (2007)	incommensurate octupole [66]
Haule and Kotliar (2009)	hexadecapolar (2^4) order [67]
Cricchio et al. (2009)	dotriacontapolar (2^5) order [68]
Harima et al. (2010)	antiferro quadrupolar order [69]
Thalmeier & Takimoto (2011)	$E(1,1)$ -type quadrupole [70]
Kusunose & Harima (2011)	antiferro hexadecapole[71]
Ikeda et al. (2012)	E^- -type dotriacontapole [72]
Rau & Kee (2012)	E -type dotriacontapole [73]
Ressouche et al. (2012)	dotriacontapolar order [16]

How to identify the nature of the “hidden orders”?

Yaodong Li, Xiaoqun Wang, GC, PRB (R) 94, 201114 (2016): **hidden octupolar order**
Changle Liu, Yaodong Li, GC, PRB 98, 045119 (2018): **hidden quadrupolar order**

How to identify the nature of the “hidden orders”?

Our simple proposal : Orthogonal operator approach

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How to identify the nature of the “hidden orders”?

Our simple proposal : Orthogonal operator approach

Find physical observables whose operators do not commute with the “proposed” hidden order operators, and these observables are easier to detect experimentally. The dynamic correlations or spectra reveals the structure and the nature of the hidden orders.

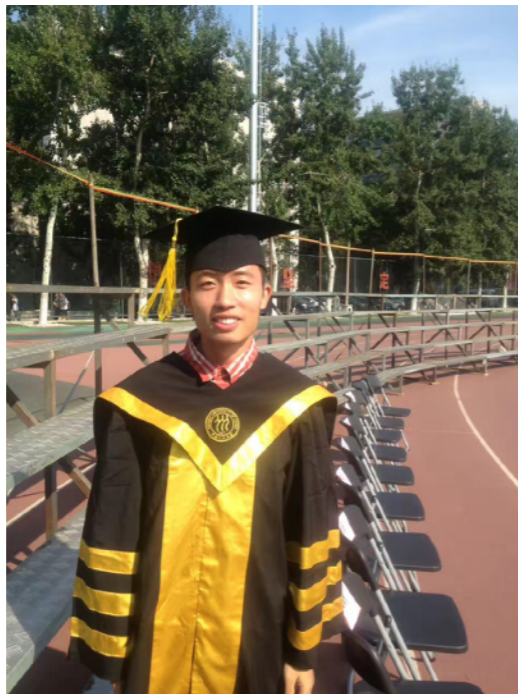
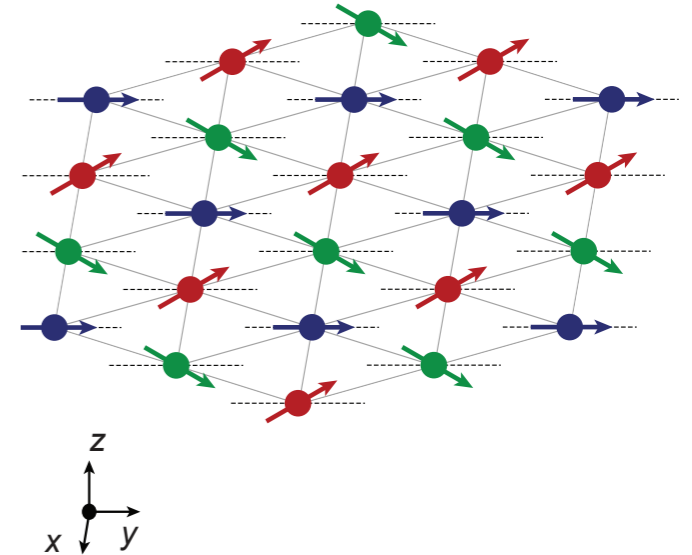
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Intertwined multipolar structure in TmMgGaO₄

arXiv 1810.05054

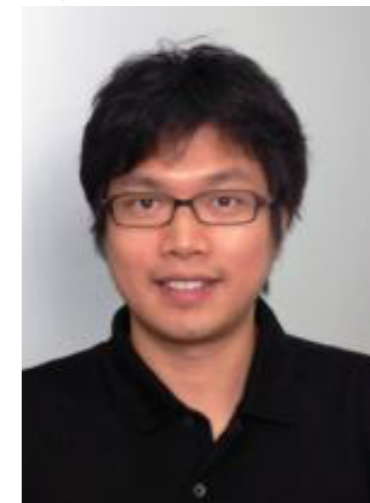
[To appear in Nature Communications]



Changle Liu
(Fudan)



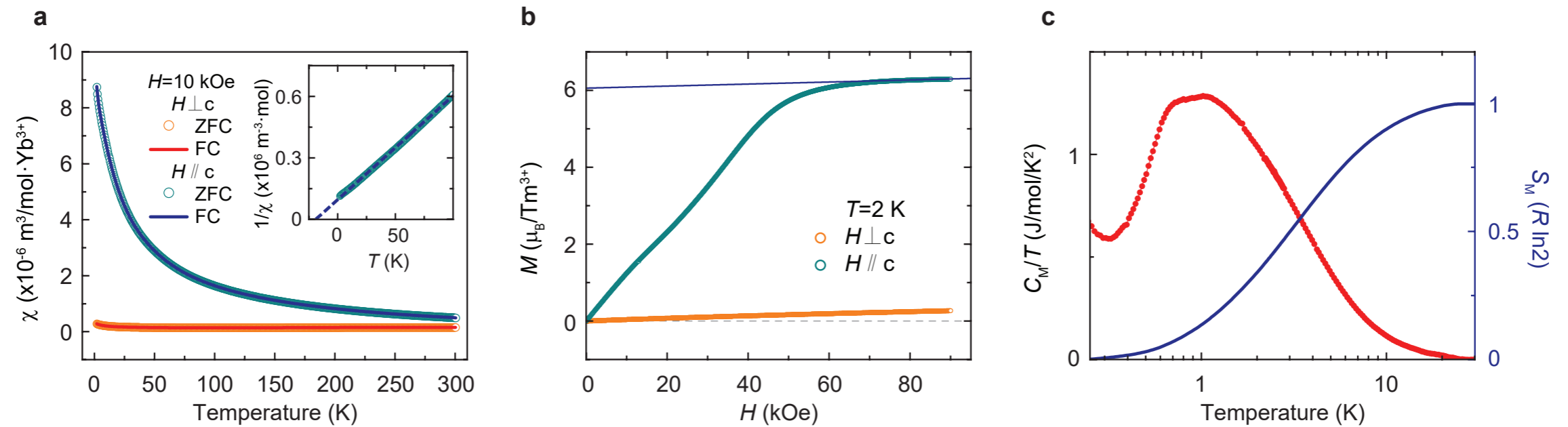
Yao Shen
(Fudan)



Jun Zhao
(Fudan)

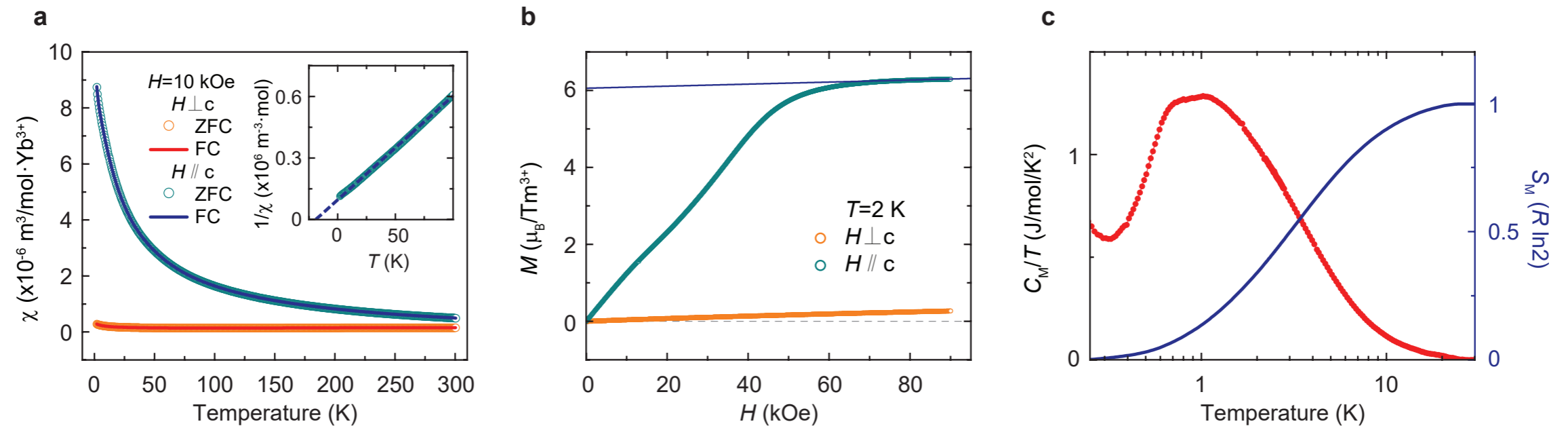
This material is not our motivation, but our application.

Tm in TmMgGaO₄ looks like non-Kramers doublets



Y Shen, Changle Liu, ..., GC, Jun Zhao, arXiv 1810.05054
[To appear in Nature Communications]

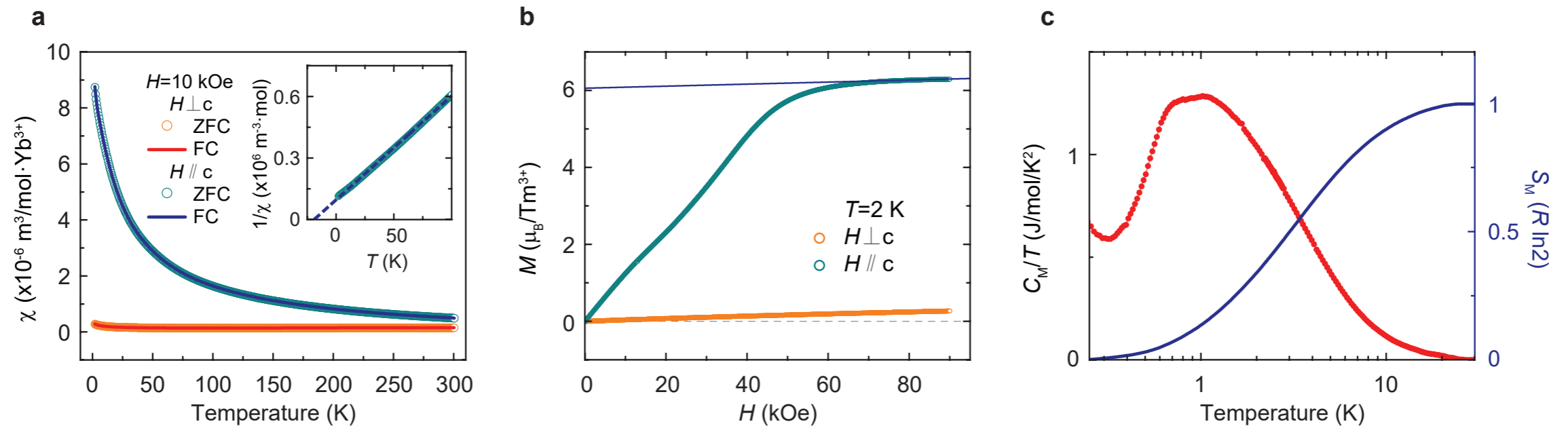
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Transverse components are hidden, only the z component is visible
in magnetic fields.

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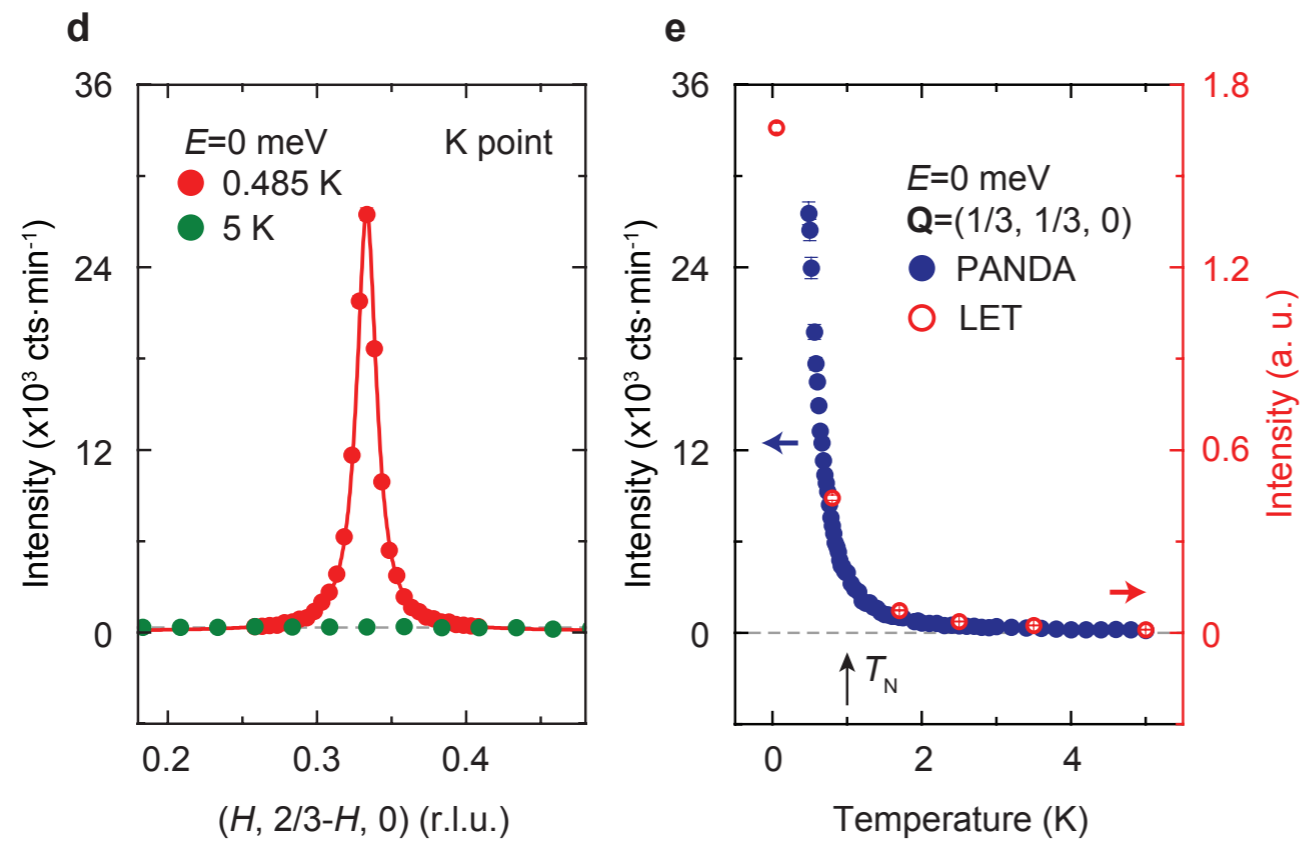
Transverse components are hidden, only the z component is visible
in magnetic fields.

Actually, it is thought to be Ising.

Li, Y., Bachus, S., Tokiwa, Y., Tsirlin, A. A. & Gegenwart, P. Absence of zero-point entropy in a triangular Ising antiferromagnet.

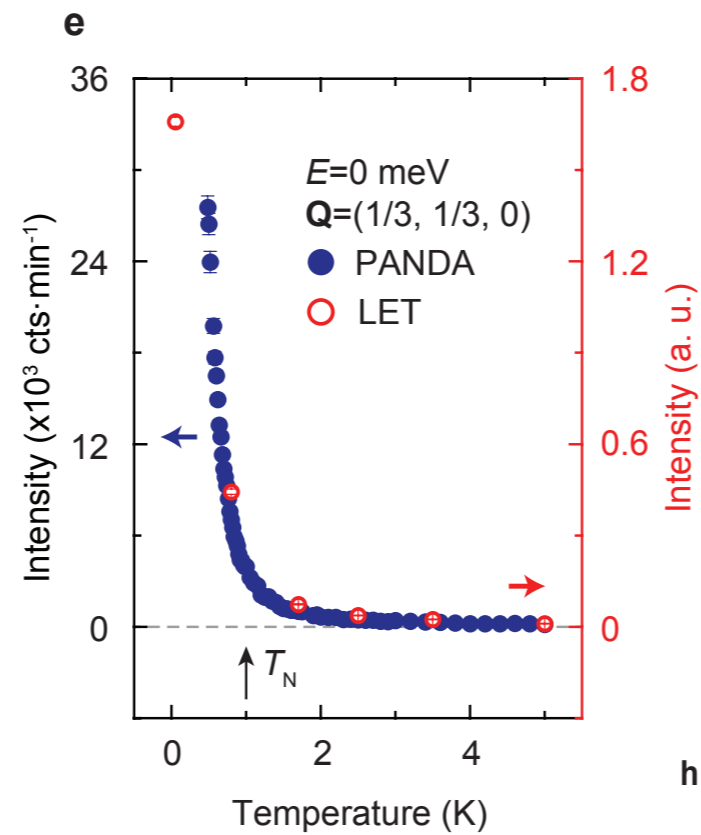
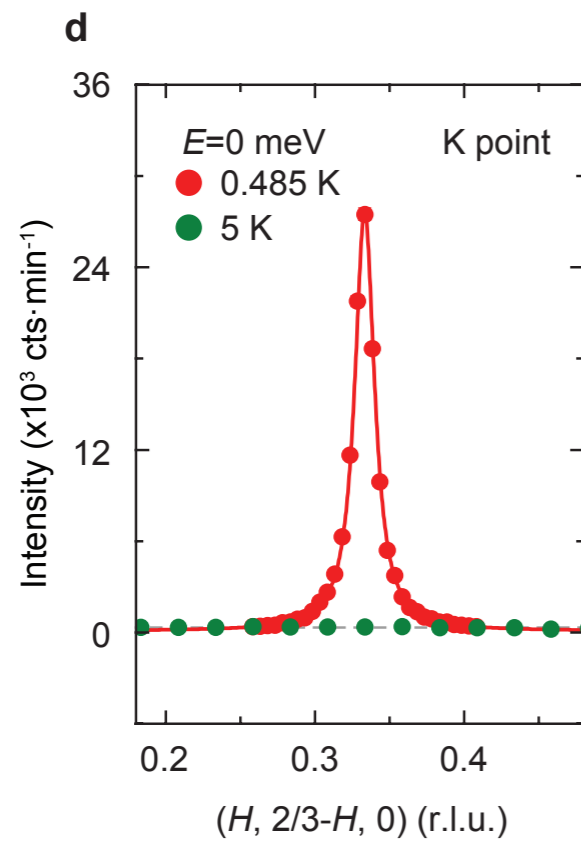
at <https://arxiv.org/abs/1804.00696> (2018).

The system orders antiferromagnetically

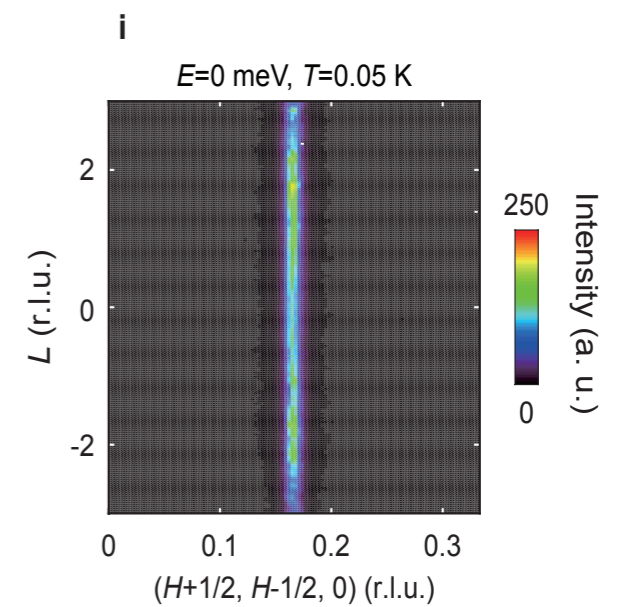
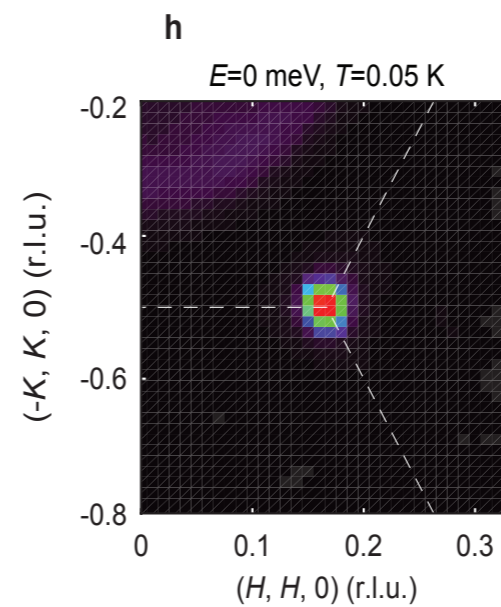


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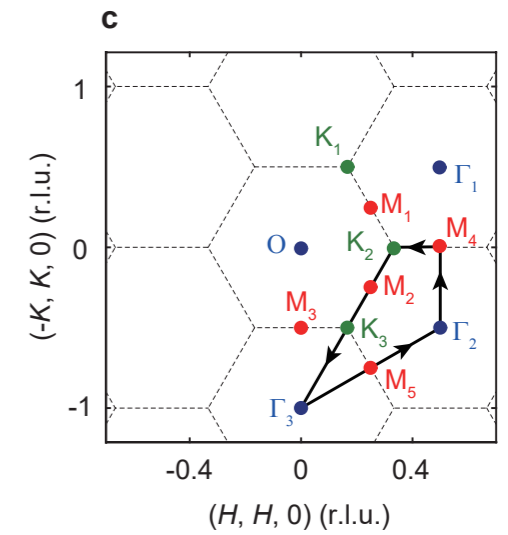
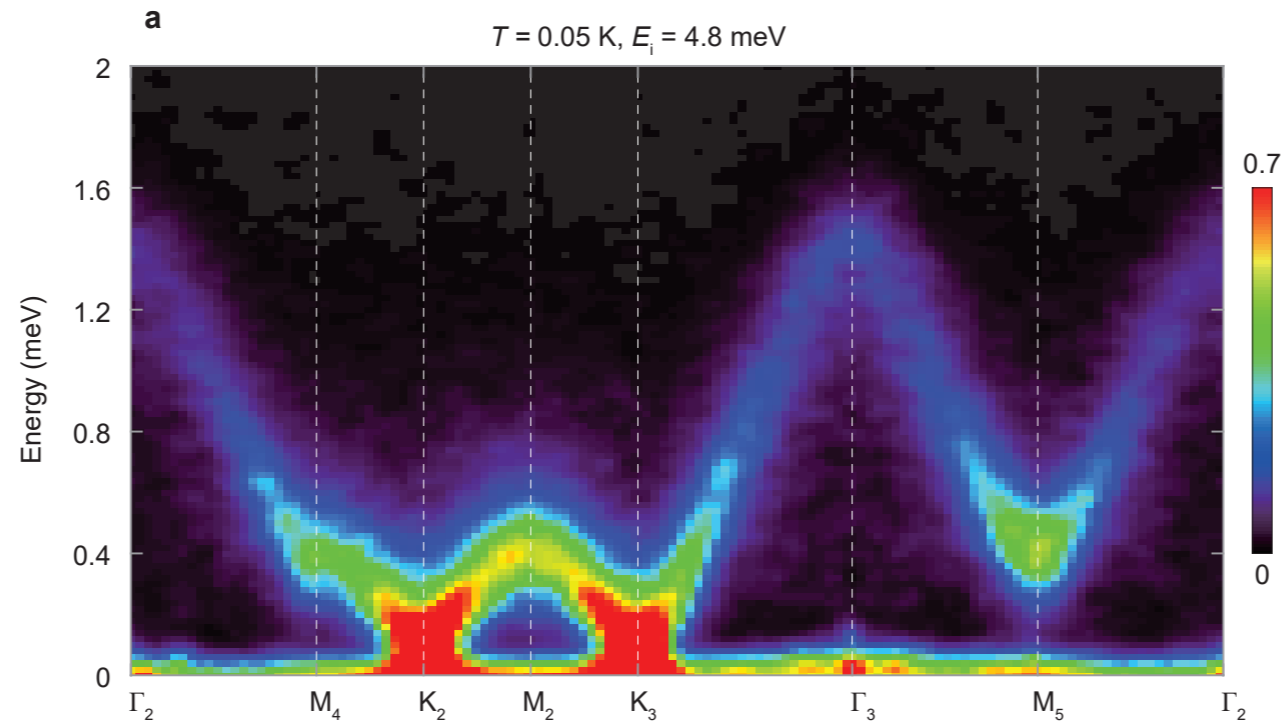
along c direction,
 truly 2D

“Spin-wave”-like dispersions: ω - k relation

Exp

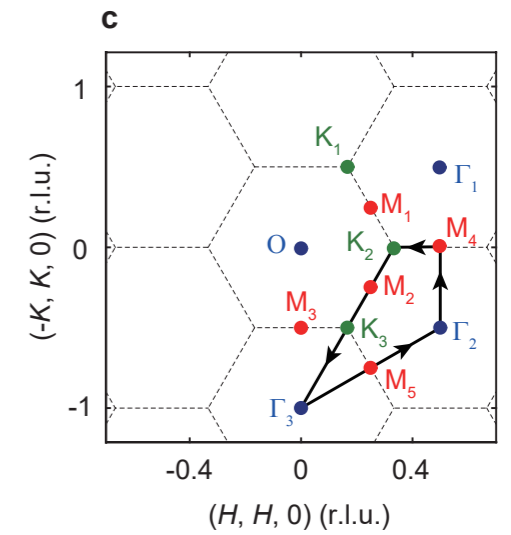
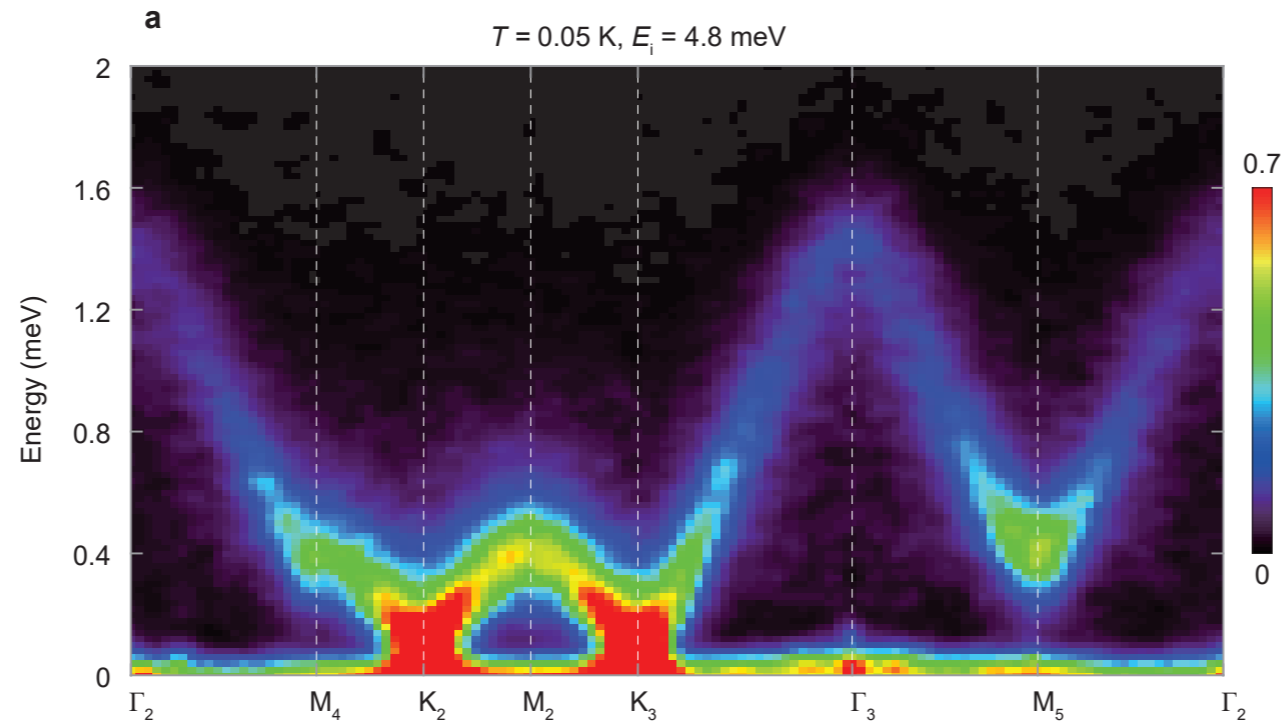
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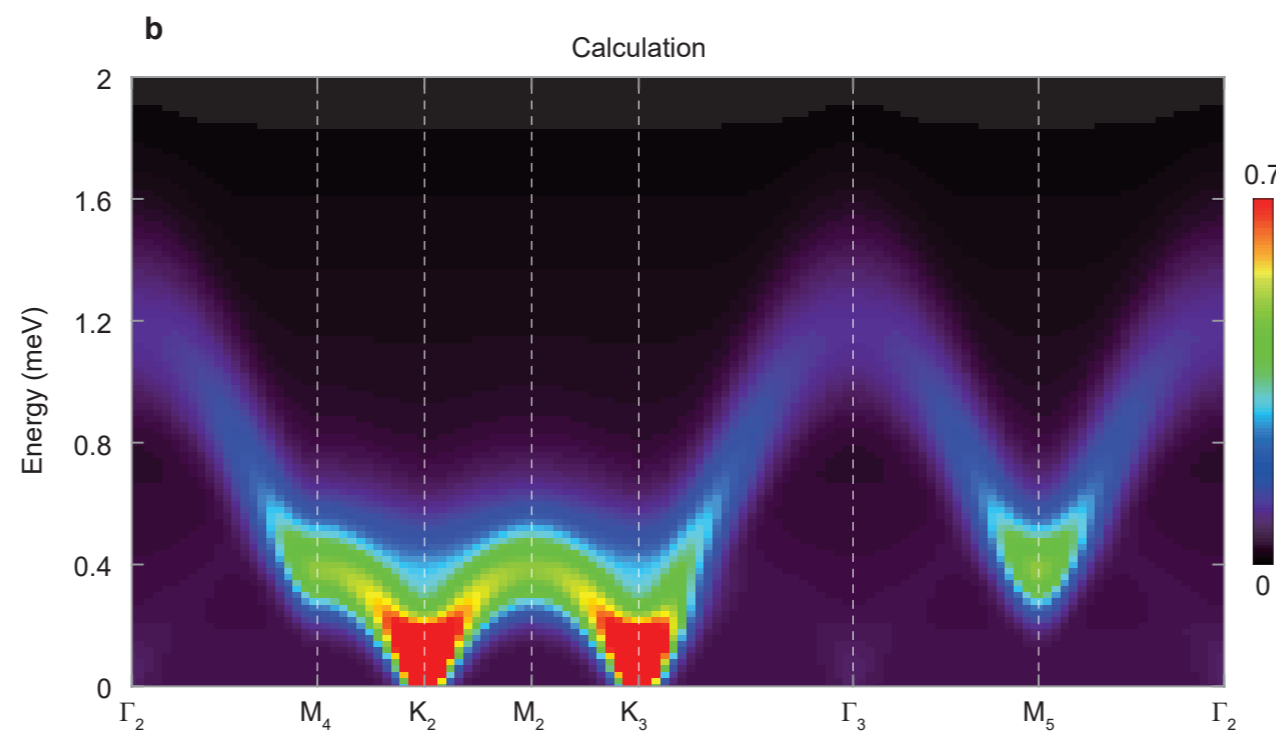


“Spin-wave”-like dispersions: ω - k relation

Exp



Theory



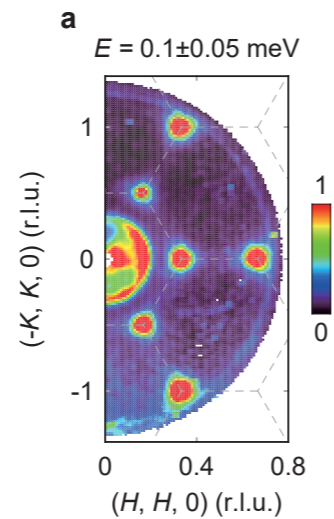
Well-defined spin wave

exp

The presence of well-defined spin wave indicates
the presence of the “hidden order” ?

Well-defined spin wave

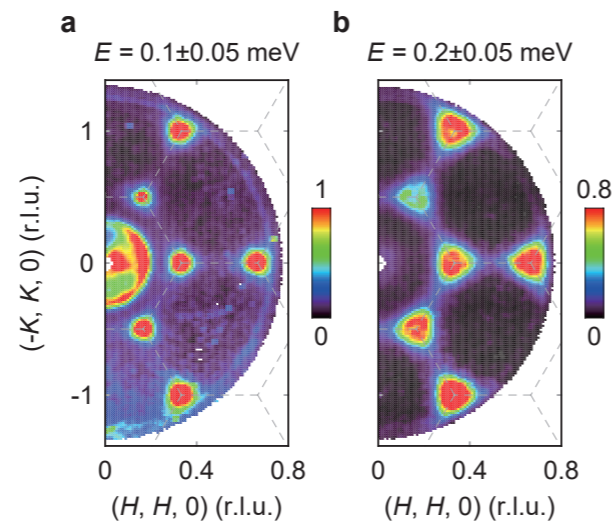
exp



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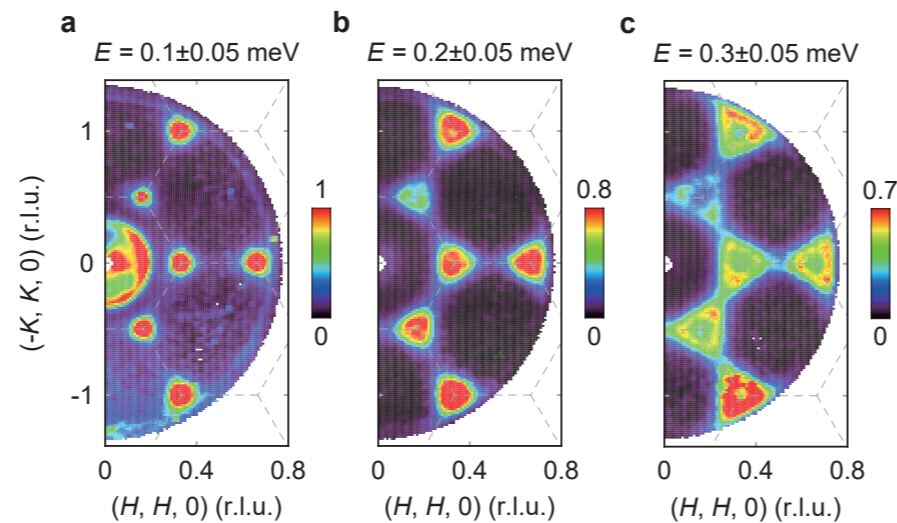
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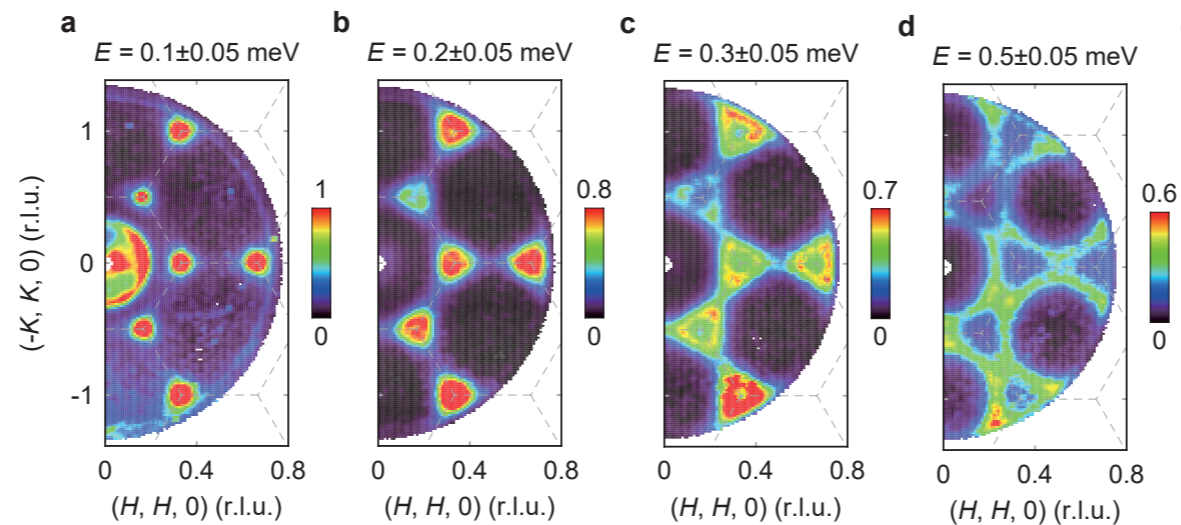
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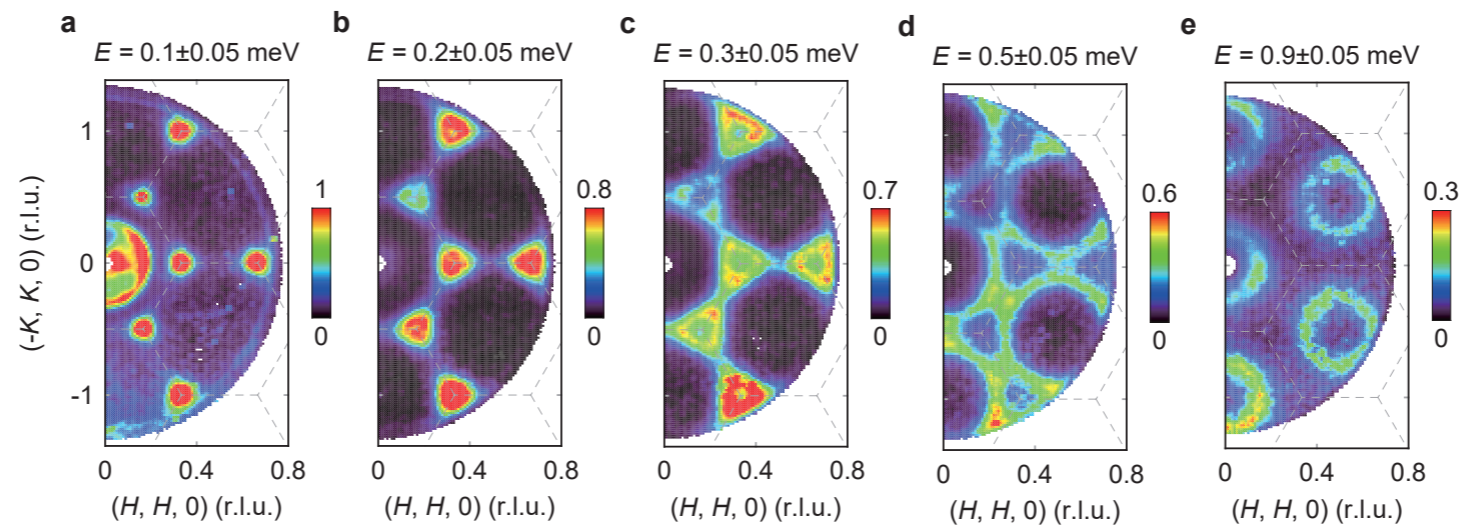
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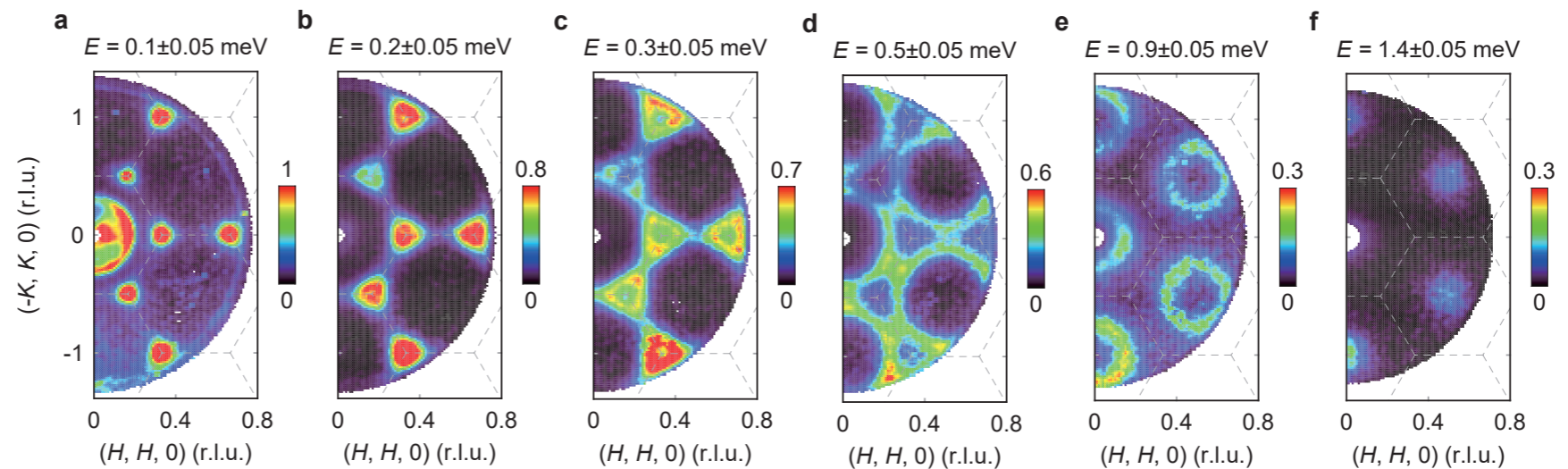
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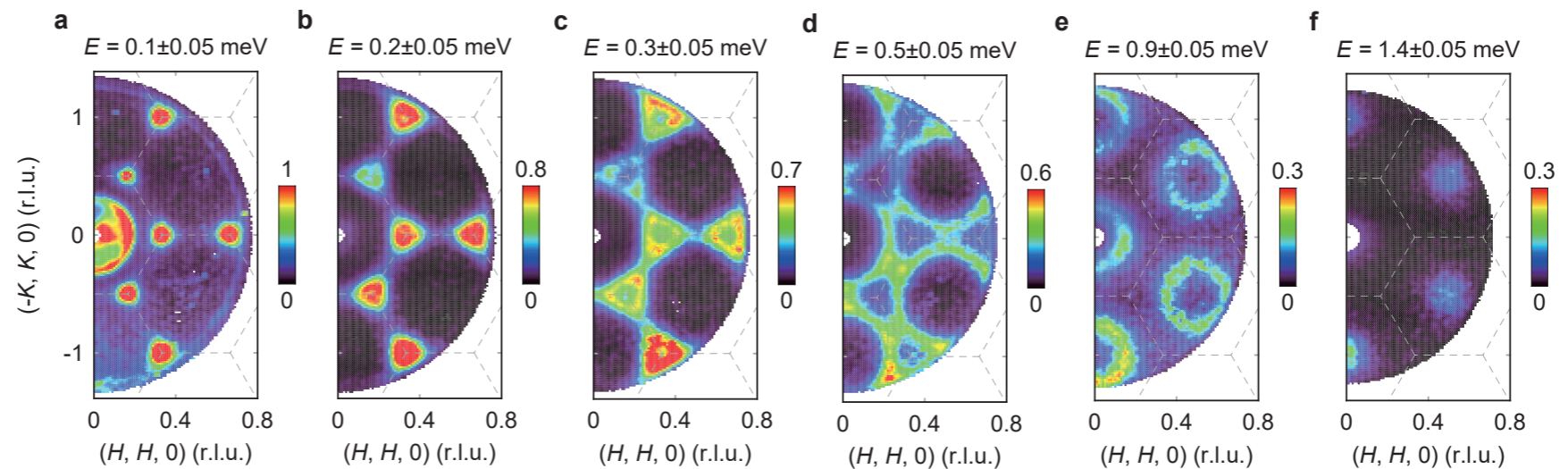
exp



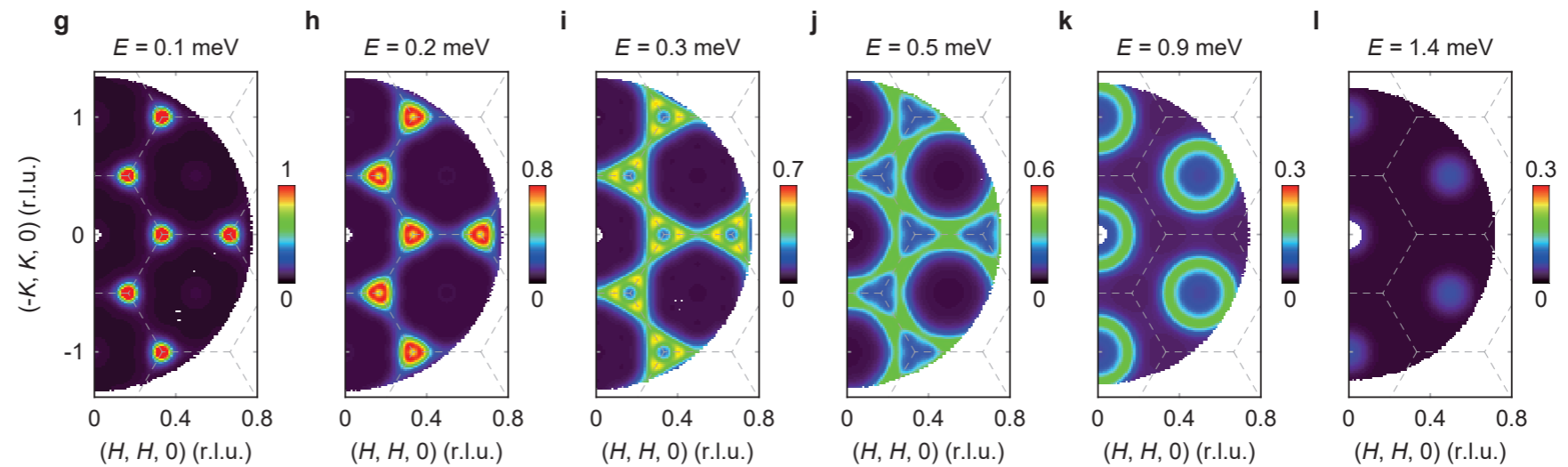
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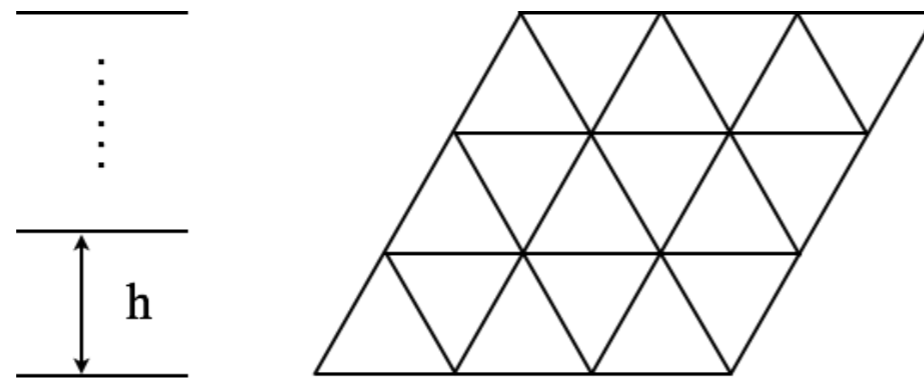
theory



The presence of well-defined spin wave indicates the presence of the “hidden order” ?

Actually, they are not non-Kramers doublets

lattice¹. In TmMgGaO_4 , the Tm^{3+} ion possess an electron configuration $4f^{12}$, in which the orbital and spin angular momentum ($L = 5$, $S = 1$) are entangled into the total angular momentum $J = 6$ due to



Tm^{3+} in triangular lattice magnet TmMgGaO_4

$$|\Psi^+\rangle \sim |J^z = 6\rangle + |J^z = -6\rangle + \dots,$$

$$|\Psi^-\rangle \sim |J^z = 6\rangle - |J^z = -6\rangle + \dots$$

FIG. 1. The Tm^{3+} magnetic ions in TmMgGaO_4 form a triangular lattice¹⁸⁻²⁰. The lowest two crystal field singlets can be modelled as an effective spin-1/2 degree of freedom,

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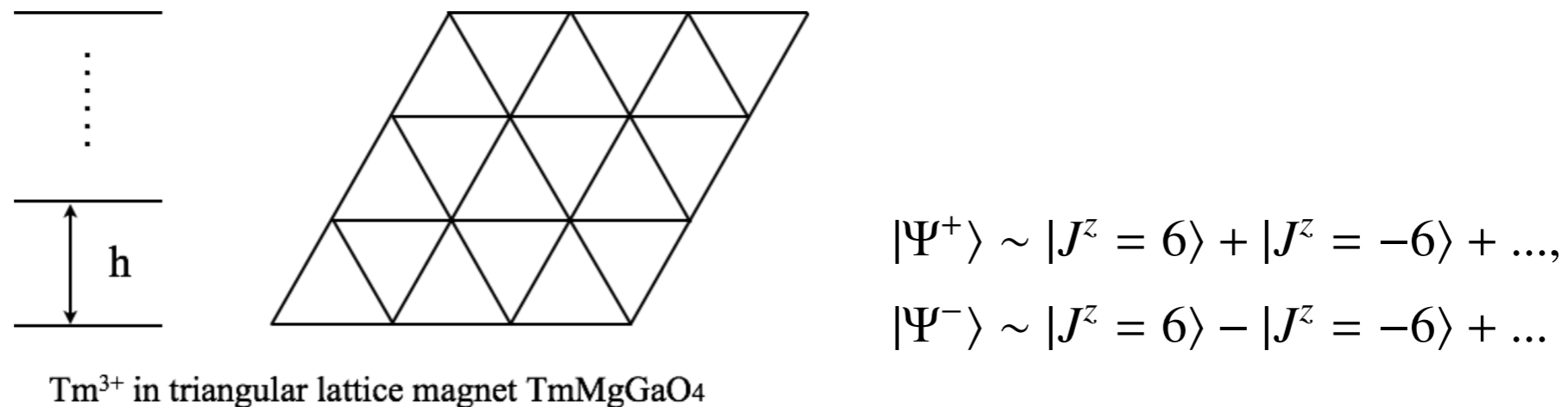


FIG. 1. The Tm^{3+} magnetic ions in TmMgGaO_4 form a triangular lattice¹⁸⁻²⁰. The lowest two crystal field singlets can be modelled as an effective spin-1/2 degree of freedom,

$$S_i^x = \frac{i}{2}(|\Psi_i^-\rangle\langle\Psi_i^+| - |\Psi_i^+\rangle\langle\Psi_i^-|),$$

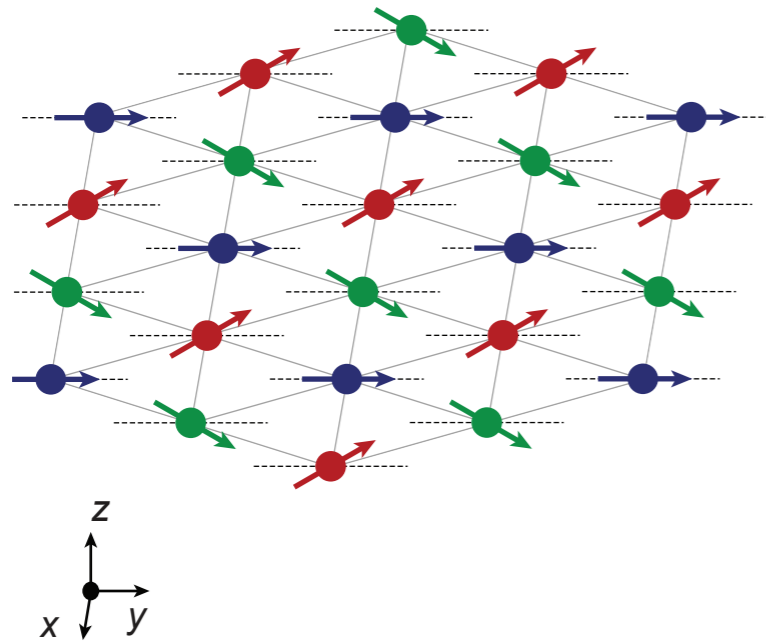
$$S_i^y = \frac{1}{2}(|\Psi_i^+\rangle\langle\Psi_i^+| - |\Psi_i^-\rangle\langle\Psi_i^-|),$$

$$S_i^z = \frac{1}{2}(|\Psi_i^+\rangle\langle\Psi_i^-| + |\Psi_i^-\rangle\langle\Psi_i^+|)$$

Here, these two singlets, $|\Psi_i^+\rangle$ and $|\Psi_i^-\rangle$, carry A_{1g} and A_{2g} representation of the D_{3d} group, respectively.

Intrinsic quantum Ising model

“Intrinsic” means the transverse field has an intrinsic origin.

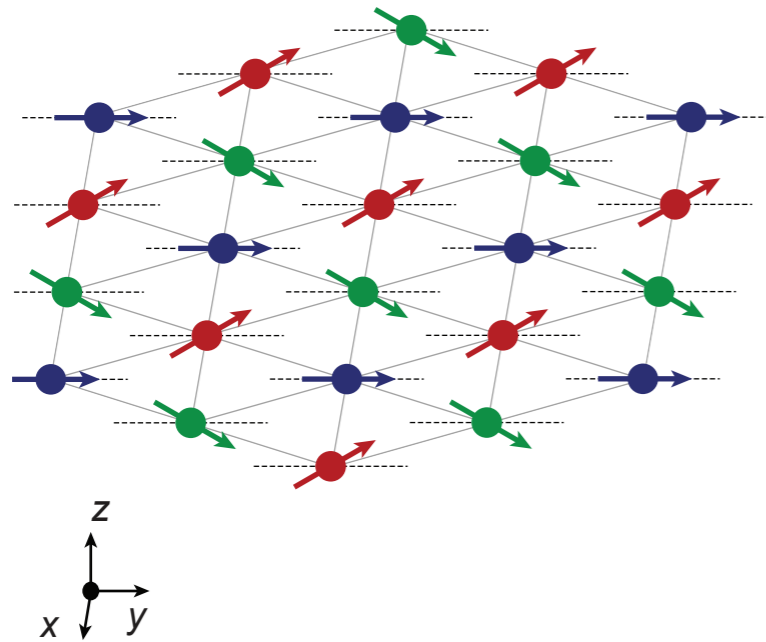


$$\mathcal{H} = \sum_{\langle ij \rangle} J_1^{zz} S_i^z S_j^z + \sum_{\langle\langle ij \rangle\rangle} J_2^{zz} S_i^z S_j^z - h \sum_i S_i^y.$$

$$J_1^{zz}=0.54(2) \text{ meV}, J_2^{zz}=0.026(6) \text{ meV}, h=0.62(2) \text{ meV}.$$

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For the case of quasi-1d magnets CoNb_2O_6 , $\text{BaCo}_2\text{V}_2\text{O}_8$ and $\text{SrCo}_2\text{V}_2\text{O}_8$, because of the local Co^{2+} environment and the special lattice geometry, the system realizes the Ising interactions between the local moments. The transverse field is then introduced externally by applying a magnetic field normal to the Ising spin direction. This

Extrinsic quantum Ising model

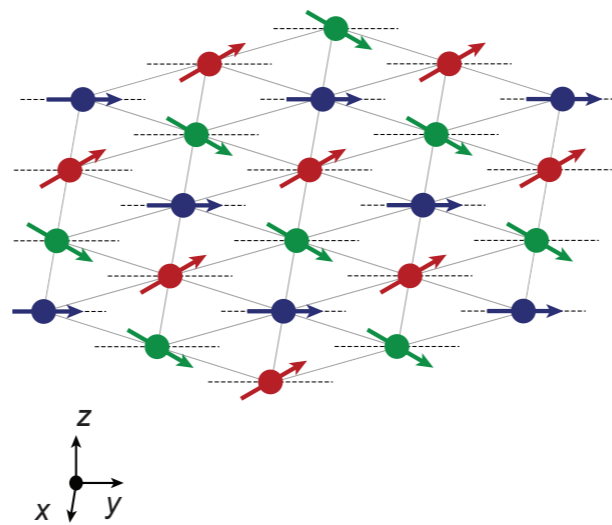
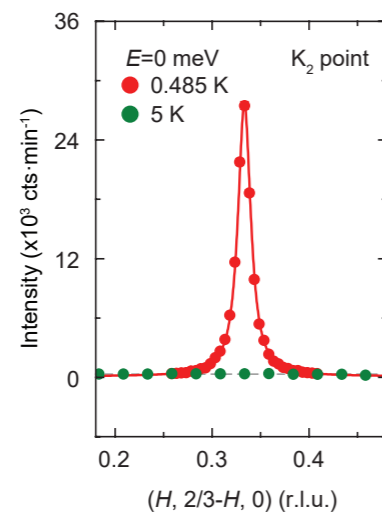
R Coldea, Sungbin Lee, Balents,...
Bella Lake, Congjun Wu, Alois Loidl
Jianda Wu.....

GC, unpublished 2019

Orthogonal operator: S_z

Transverse components are hidden.

The 3-sublattice S_z order [at K] is a quantum effect, arising from the geometrical frustration and quantum order by disorder. [known from weak field limit, Sondhi, Moessner]

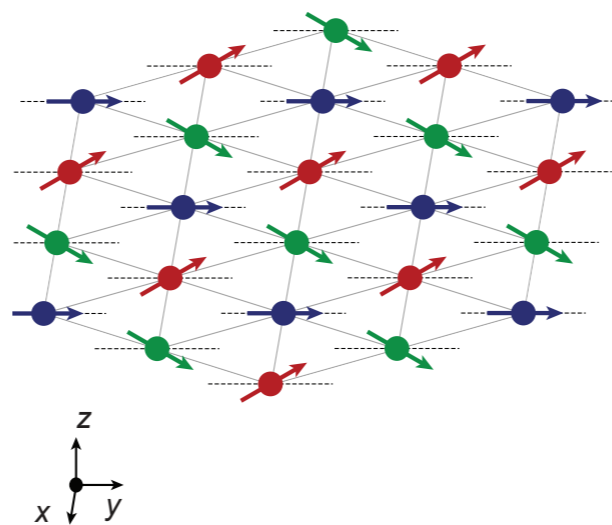
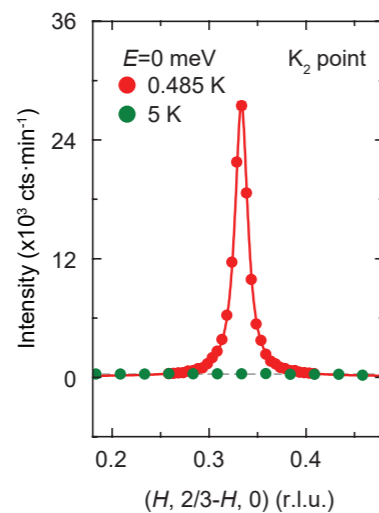


Y Shen, Changle Liu, ..., GC, Jun Zhao,
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Dynamic measurement: only Sz is visible by neutron spin.

$$\begin{aligned} \mathcal{S}^{zz}(\mathbf{q}, \omega > 0) \\ = \frac{1}{2\pi N} \sum_{ij} \int_{-\infty}^{+\infty} dt e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j) - i\omega t} \langle S_i^z(0) S_j^z(t) \rangle. \end{aligned}$$

as if it is polarized neutron scattering.

Changle Liu, Yaodong Li, GC,
PRB 98, 045119 (2018)

Summary-1

1. The interplay between geometrical frustration and multipolar local moments leads to rich phases and excitations.
2. The manifestation of the hidden multipolar orders is rather non-trivial, both in the static and dynamic measurements.
3. The **orthogonal operator approach** can be used to reveal the dynamics of hidden orders. **This is general** and can be adapted to many other hidden order systems. Think about URu₂Si₂.

2. Symmetry enriched $U(1)$ spin liquid,
field-driven Anderson-Higgs transition,

Candidate Quantum Spin Liquid in the Ce^{3+} Pyrochlore Stannate $\text{Ce}_2\text{Sn}_2\text{O}_7$

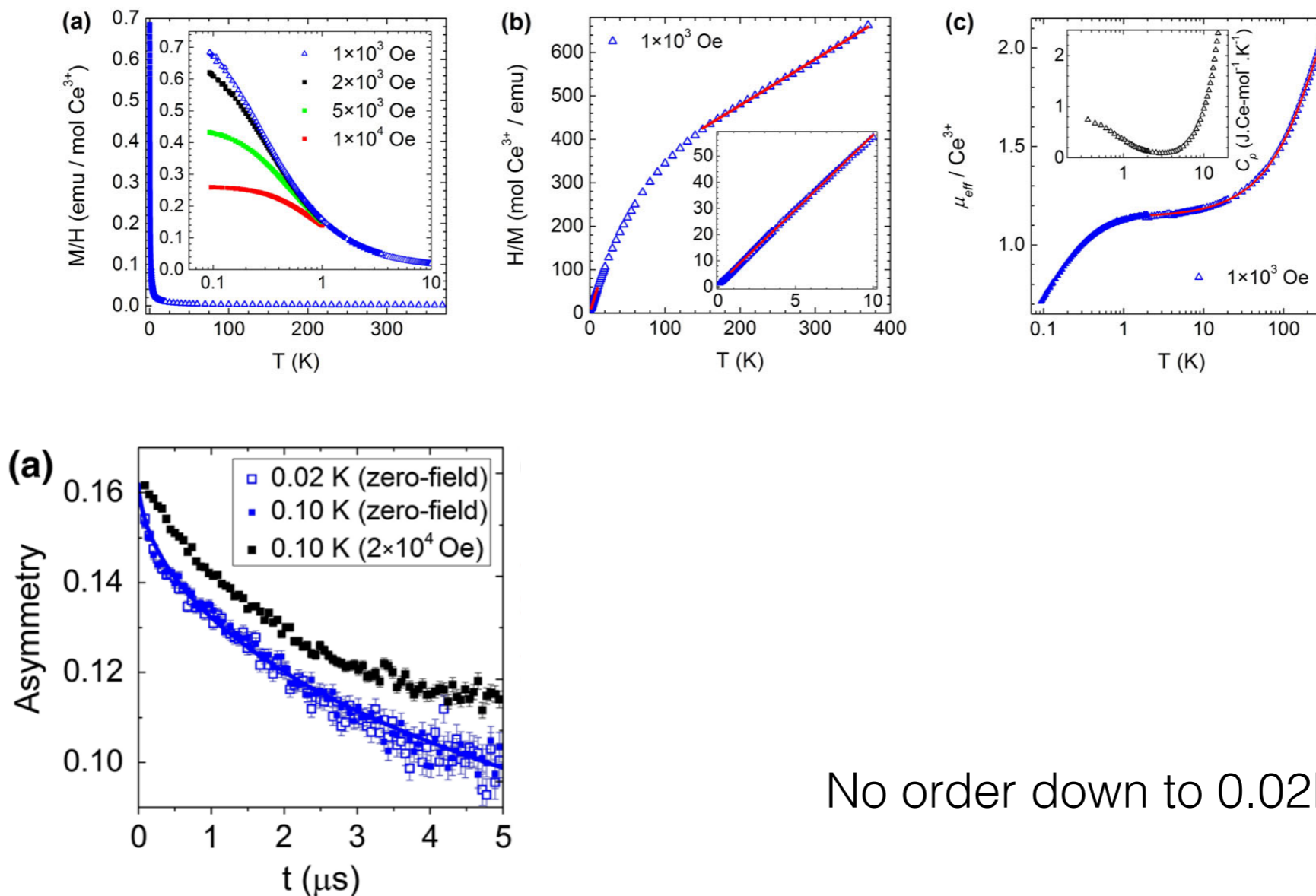
Romain Sibille,^{1,*} Elsa Lhotel,² Vladimir Pomjakushin,³ Chris Baines,⁴ Tom Fennell,^{3,†} and Michel Kenzelmann¹

¹Laboratory for Scientific Developments and Novel Materials, Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

²Institut Néel, CNRS, and Université Joseph Fourier, BP 166, 38042 Grenoble Cedex 9, France

³Laboratory for Neutron Scattering and Imaging, Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

⁴Laboratory for Muon Spin Spectroscopy, Paul Scherrer Institut, 5232 Villigen PSI, Switzerland



No order down to 0.02K !

Candidate Quantum Spin Liquid in the Ce^{3+} Pyrochlore Stannate $\text{Ce}_2\text{Sn}_2\text{O}_7$ Romain Sibille,^{1,*} Elsa Lhotel,² Vladimir Pomjakushin,³ Chris Baines,⁴ Tom Fennell,^{3,†} and Michel Kenzelmann¹

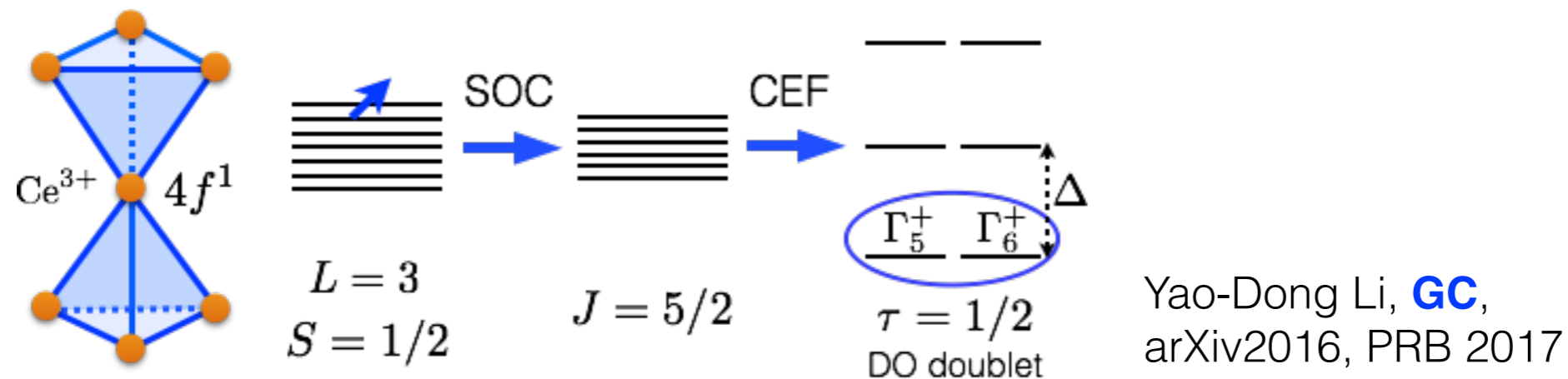
$4f^1$ ion in D_{3d} local symmetry to the susceptibility was realized between $T = 1.8$ and 370 K, and the resulting calculation of the single ion magnetic moment is shown in Fig. 2(c). The wave functions of the ground state Kramers doublet correspond to a linear combination of $m_J = \pm 3/2$ states. The fitted coefficients result in energy levels at $50 \pm$

Huang, [GC](#), Hermele, PRL, 112, 167203 (2014), arXiv Nov 2013
Yao-Dong Li, [GC](#), PRB Rapid Comm 2017
Yao-Dong Li, XQ Wang, [GC](#), PRB Rapid Comm 2016

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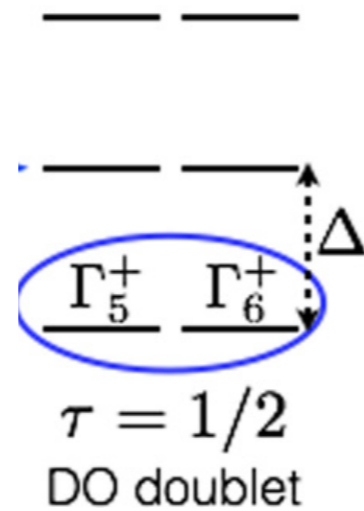
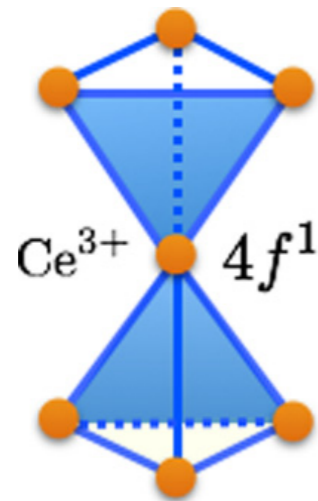
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This doublet is **dipole-octupole doublet**

Huang, [GC](#), Hermele, PRL, 112, 167203 (2014), arXiv Nov 2013
 Yao-Dong Li, [GC](#), PRB Rapid Comm 2017
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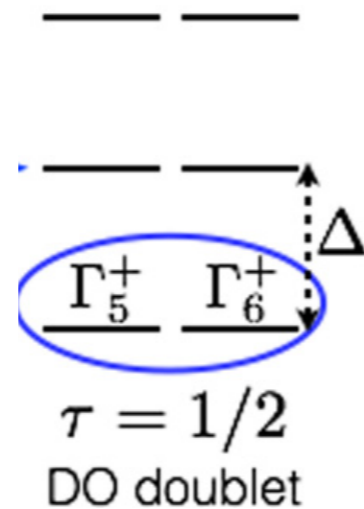
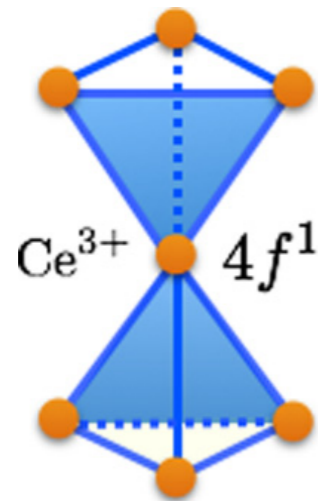
How does it work? why so special?



$$|\Psi_+\rangle = a_1 |J^z = \frac{3}{2}\rangle + a_2 |J^z = -\frac{3}{2}\rangle$$

$$|\Psi_-\rangle = a_1^* |J^z = -\frac{3}{2}\rangle + a_2^* |J^z = \frac{3}{2}\rangle$$

How does it work? why so special?



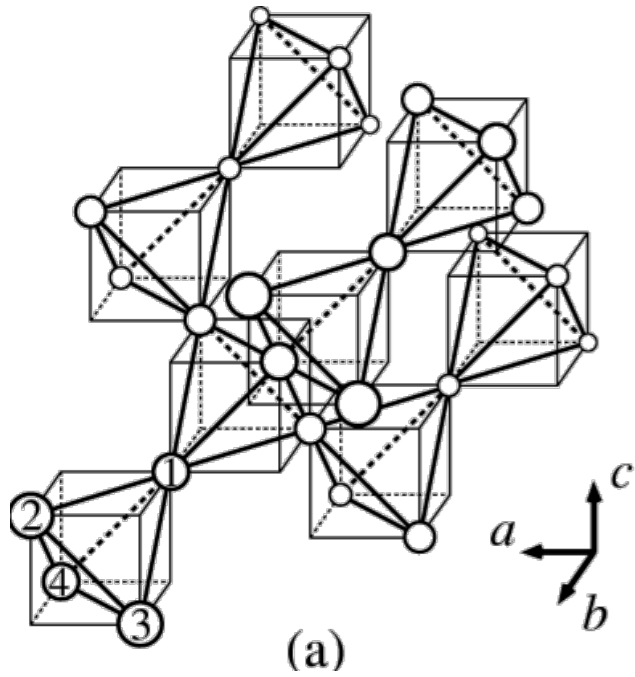
$$|\Psi_+\rangle = a_1 |J^z = \frac{3}{2}\rangle + a_2 |J^z = -\frac{3}{2}\rangle$$

$$|\Psi_-\rangle = a_1^* |J^z = -\frac{3}{2}\rangle + a_2^* |J^z = \frac{3}{2}\rangle$$

$$e^{-i\frac{2\pi}{3}J^z} |\Psi_{\pm}\rangle = e^{-i\frac{2\pi}{3}\frac{3}{2}} |\Psi_{\pm}\rangle = -|\Psi_{\pm}\rangle$$

One can then build up an effective spin-1/2 degree of freedom.

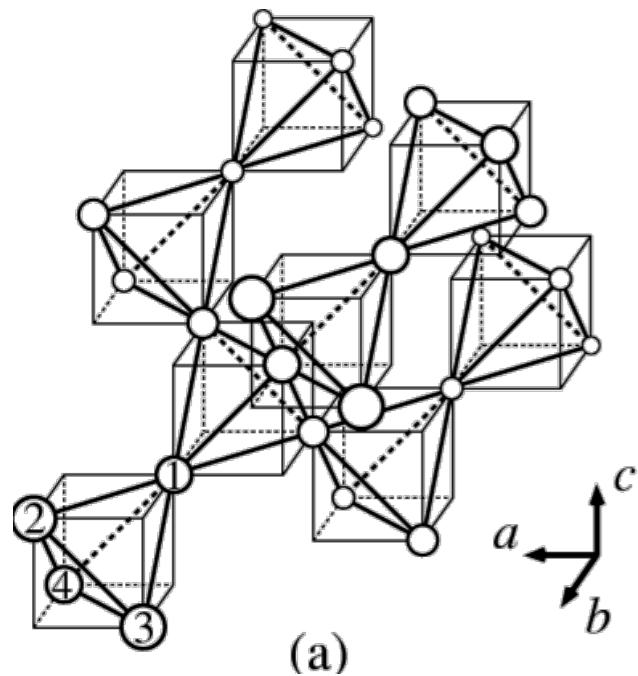
Generic model: XYZ model



$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$

$T_d \times \mathcal{I} \times translations$

Generic model: XYZ model

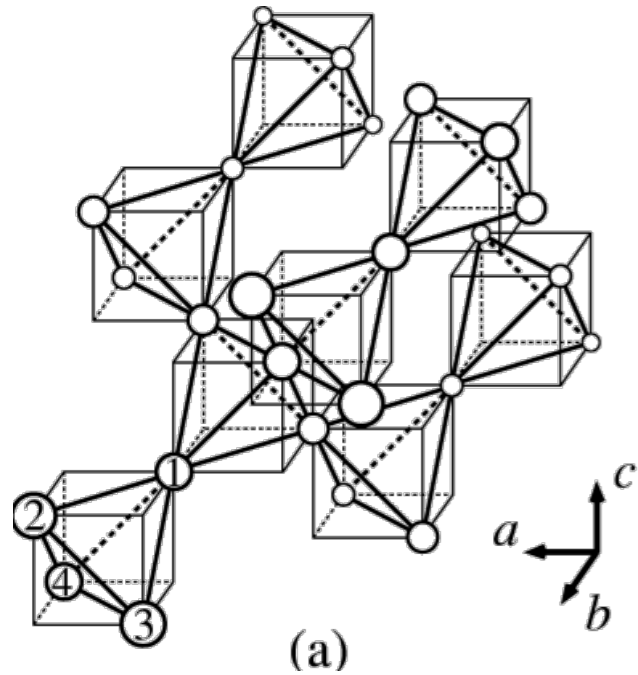


$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$

$T_d \times \mathcal{I} \times \text{translations}$

Important: \mathbf{S}^x and \mathbf{S}^z transform identically (as a dipole),
while \mathbf{S}^y transforms as an **octupole** moment under *mirror*.

Generic model: XYZ model



$T_d \times \mathcal{I} \times translations$

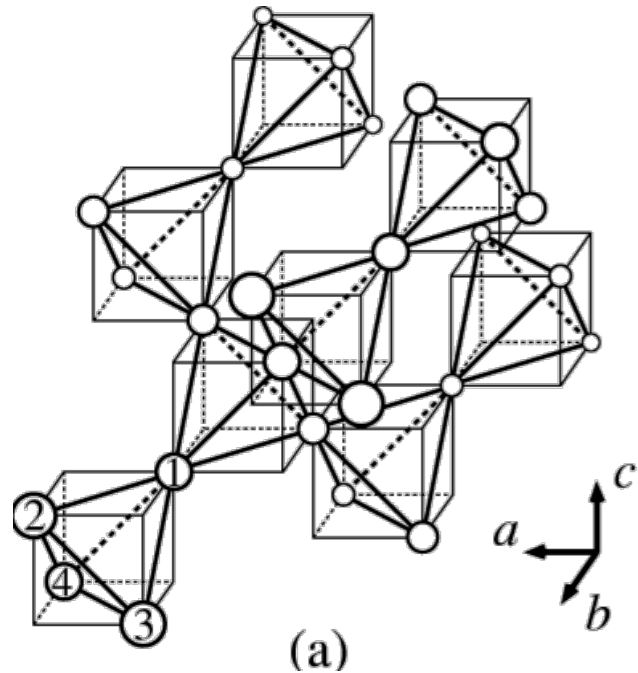
$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$



Rotation around **the y axis**
in the effective spin space

Important: \mathbf{S}^x and \mathbf{S}^z transform identically (as a dipole),
while \mathbf{S}^y transforms as an **octupole** moment under *mirror*.

Generic model: XYZ model



$T_d \times \mathcal{I} \times translations$

$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$



Rotation around **the y axis**
in the effective spin space

$$H_{\text{XYZ}} = \sum_{\langle ij \rangle} \tilde{J}_z \tilde{S}_i^z \tilde{S}_j^z + \tilde{J}_x \tilde{S}_i^x \tilde{S}_j^x + \tilde{J}_y \tilde{S}_i^y \tilde{S}_j^y \quad \text{XYZ model}$$

Important: \mathbf{S}^x and \mathbf{S}^z transform identically (as a dipole),
while \mathbf{S}^y transforms as an **octupole** moment under *mirror*.

XXZ model as the starting point for understanding

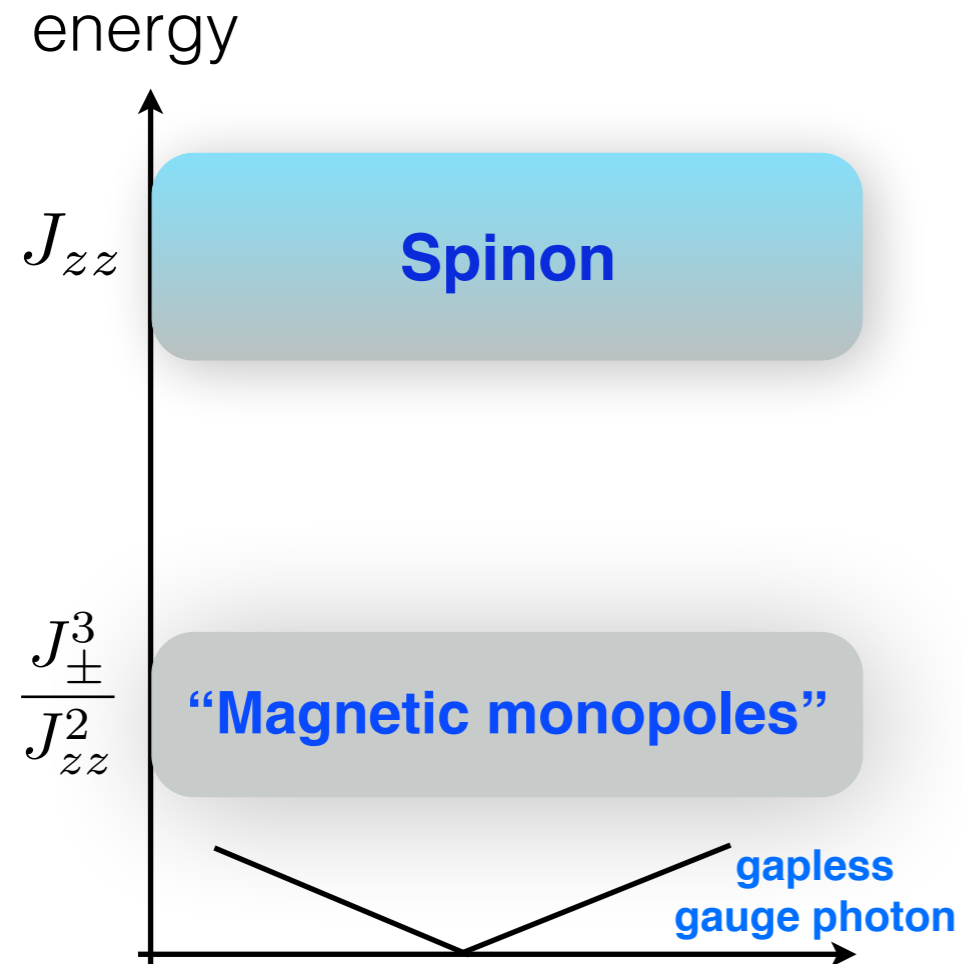


illustration for XXZ model

If I am asked later, I can explain
how to detect them.

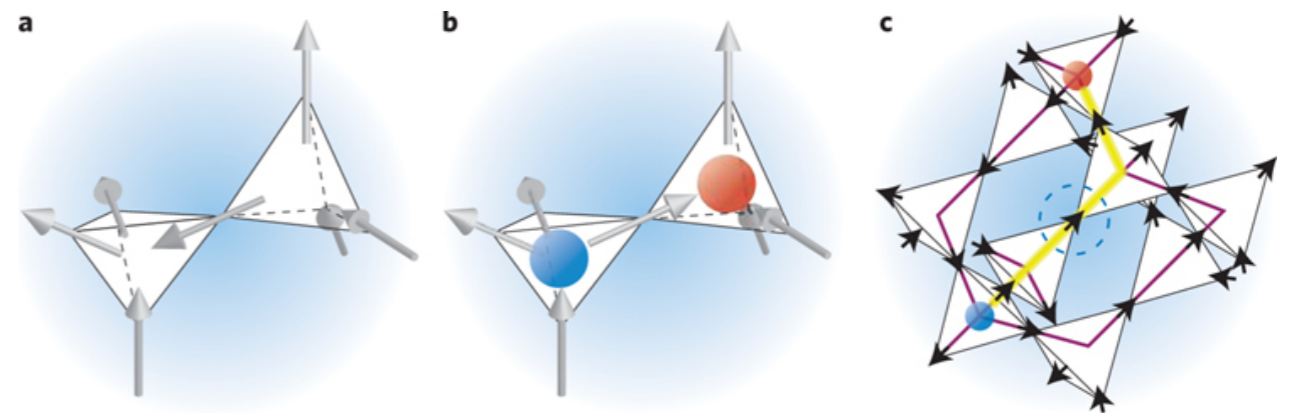


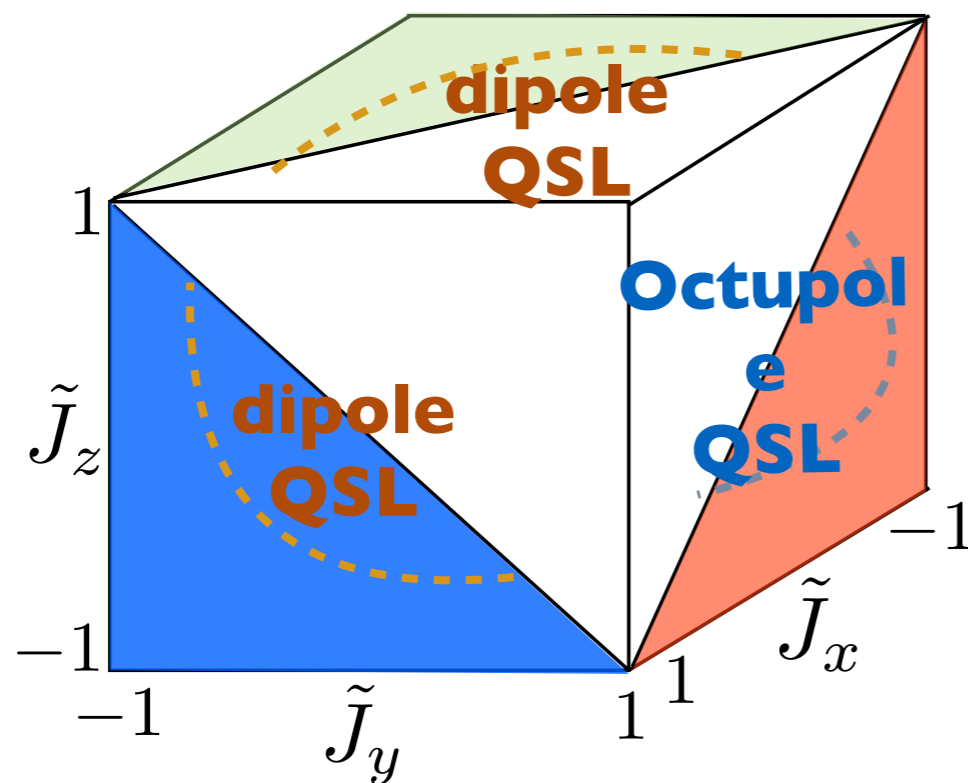
Figure credit: Moessner&Schiffer, 2009

Spinon deconfinement

Hermele, Fisher, Balent 2004
Nic Shannon, etc 2012
Lucile Savary, Balents, 2012

XYZ model is the generic model that describes the interaction between DO doublets.

$$H_{\text{XYZ}} = \sum_{\langle ij \rangle} \tilde{J}_z \tilde{S}_i^z \tilde{S}_j^z + \tilde{J}_x \tilde{S}_i^x \tilde{S}_j^x + \tilde{J}_y \tilde{S}_i^y \tilde{S}_j^y$$



3D phase diagram

Each component (not just S_z) can be emergent electric field, depending on the parameters !

The shady part does not have sign problem for quantum Monte Carlo

Study phase on a cube: $-1 \leq \tilde{J}_{x,y,z} \leq 1$.

Infinite anisotropic g-factor

$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x) - h \sum_{\tilde{i}} (\hat{n} \cdot \hat{z}_{\tilde{i}}) S_{\tilde{i}}^z$$

Different U(1) QSLs	Heat capacity	Inelastic neutron scattering measurement
Octupolar U(1) QSL for DO doublets	$C_v \sim T^3$	Gapped spinon continuum
Dipolar U(1) QSL for DO doublets	$C_v \sim T^3$	Both gapless gauge photon and gapped spinon continuum
Dipolar U(1) QSL for non-Kramers' doublets [23]	$C_v \sim T^3$	Gapless gauge photon
Dipolar U(1) QSL for usual Kramers' doublets [22]	$C_v \sim T^3$	Both gapless gauge photon and gapped spinon continuum

TABLE I. List of the physical properties of different U(1) QSLs on the pyrochlore lattice. “Usual Kramers doublet” refers to the Kramers doublet that is not a DO doublet. They transform as a two-dimensional irreducible representation under the D_{3d} point group. Although the dipolar U(1) QSL for DO doublets behaves the same as the one for usual Kramers' doublets, their physical origins are rather different [31].

Our understanding has been improved since 2016.

Yao-Dong Li, **Gang Chen**, PRB Rapid 2017
 Gang Chen, PRB 2017
 Gang Chen, PRB 2017

Emergent Anderson-Higgs transition

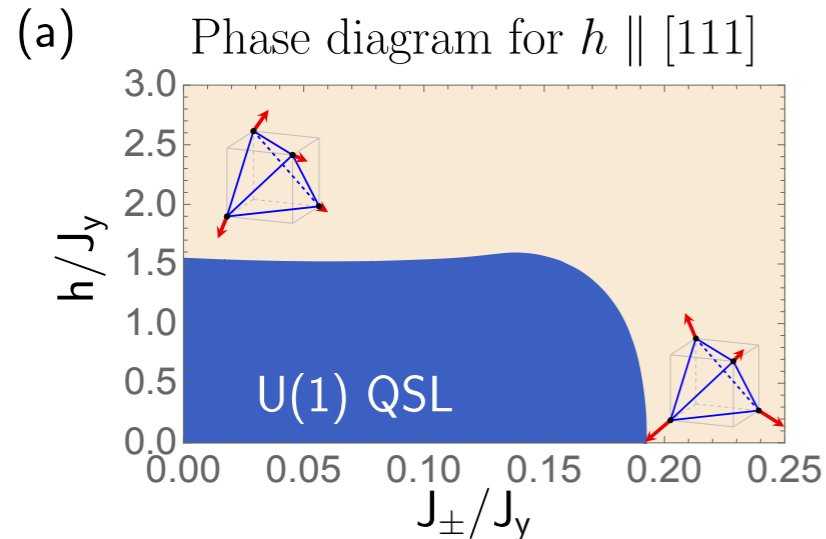
How to tell if **Ce₂Sn₂O₇** is an octupolar U(1) QSL or not ?

Here we apply external magnetic field, and expect a field-driven Higgs transition to magnetic ordering as the **field only couples to the matter field** (spinons).

$$H = \sum_{\langle ij \rangle} J_y S_i^y S_j^y - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) - h \sum_i (\hat{n} \cdot \hat{z}_i) S_i^z$$
$$S_i^{\pm} \equiv S_i^z \pm i S_i^x$$

Yaodong Li, Gang Chen, PRB 95,041106 (2017)

Emergent Anderson-Higgs transition



How to tell if **Ce₂Sn₂O₇** is an octupolar U(1) QSL or not ?

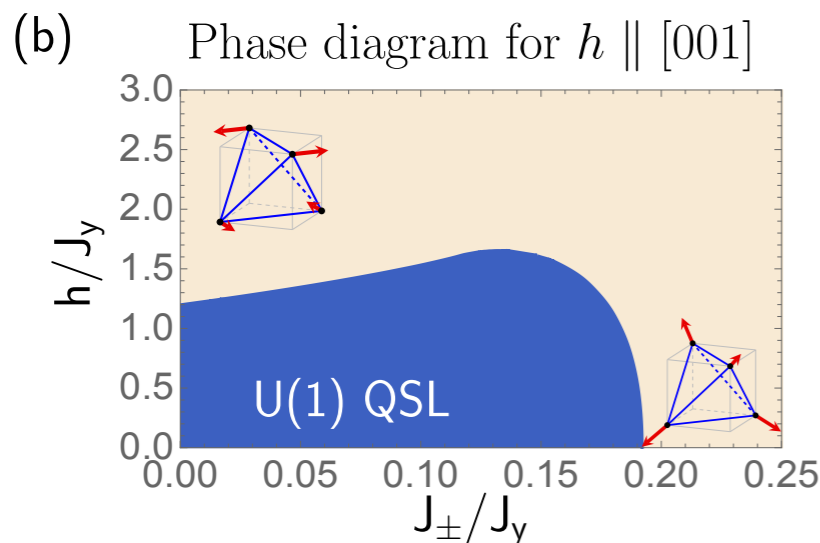
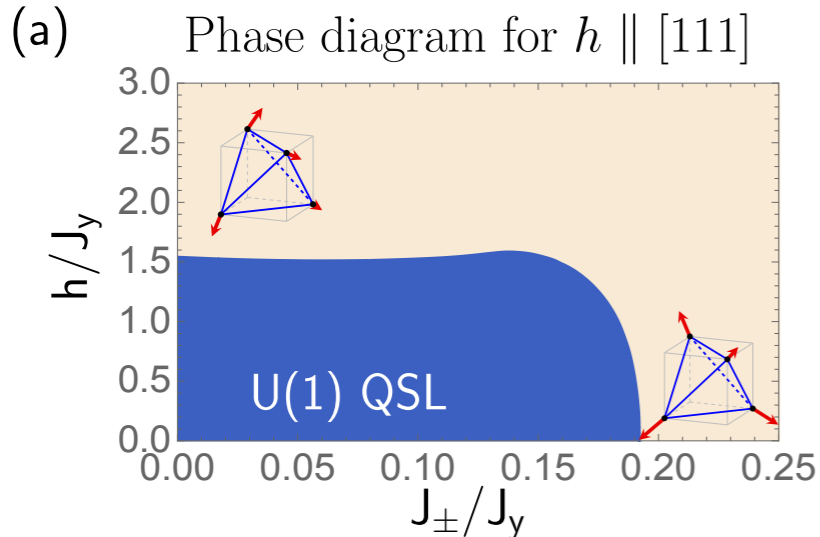
Here we apply external magnetic field, and expect a field-driven Higgs transition to magnetic ordering as the **field only couples to the matter field** (spinons).

$$H = \sum_{\langle ij \rangle} J_y S_i^y S_j^y - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) - h \sum_i (\hat{n} \cdot \hat{z}_i) S_i^z$$

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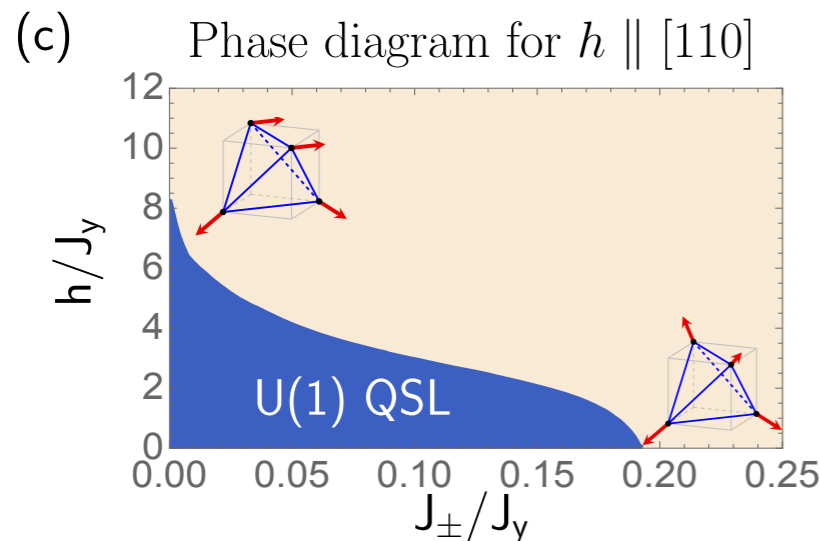
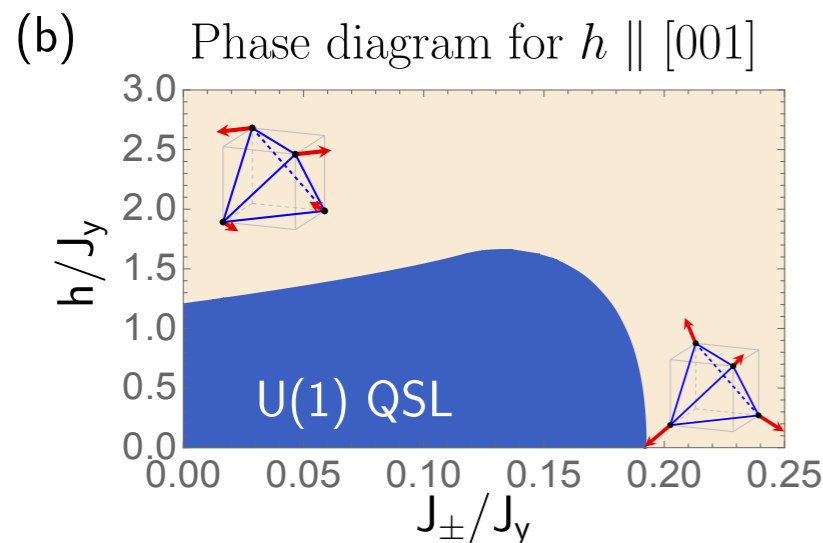
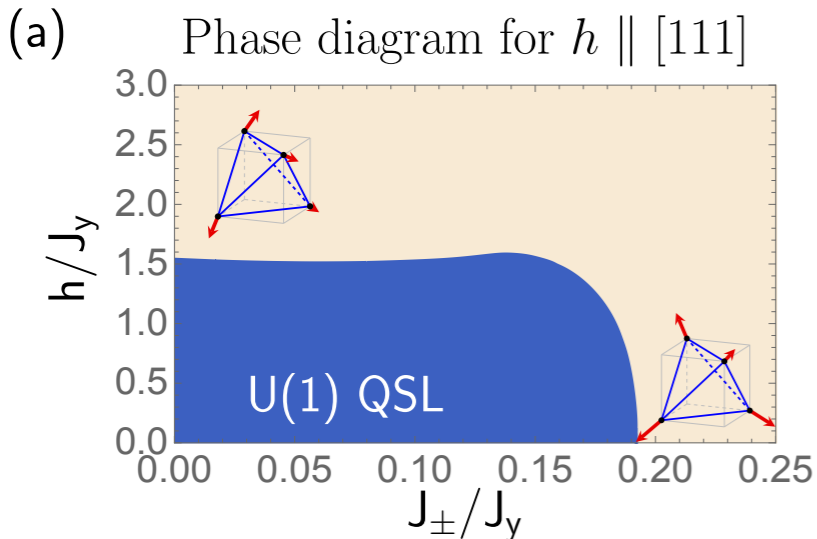
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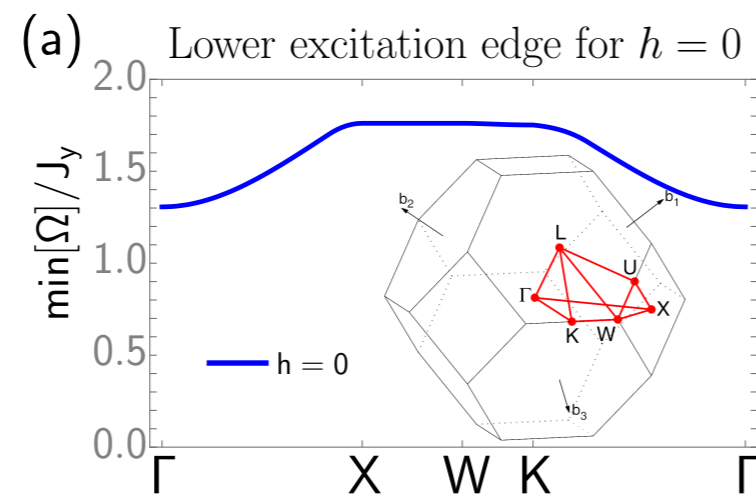
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$$S_i^{\pm} \equiv S_i^z \pm i S_i^x$$

Yaodong Li, Gang Chen, PRB 95,041106 (2017)

Lower excitation edge of 2-spinon continuum

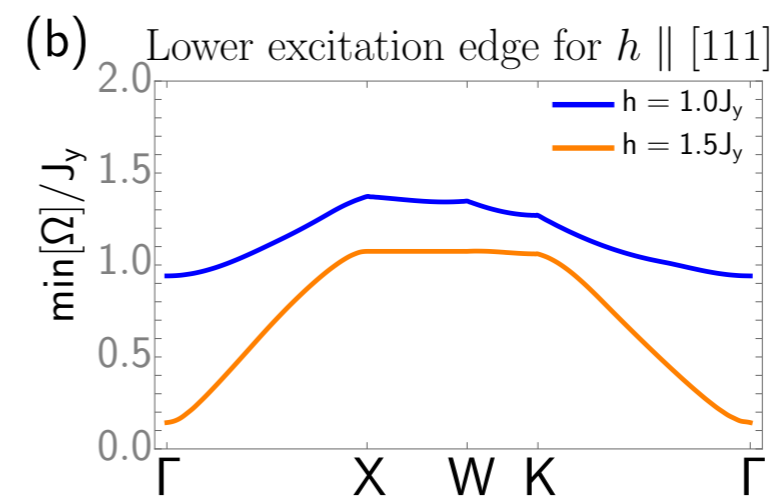
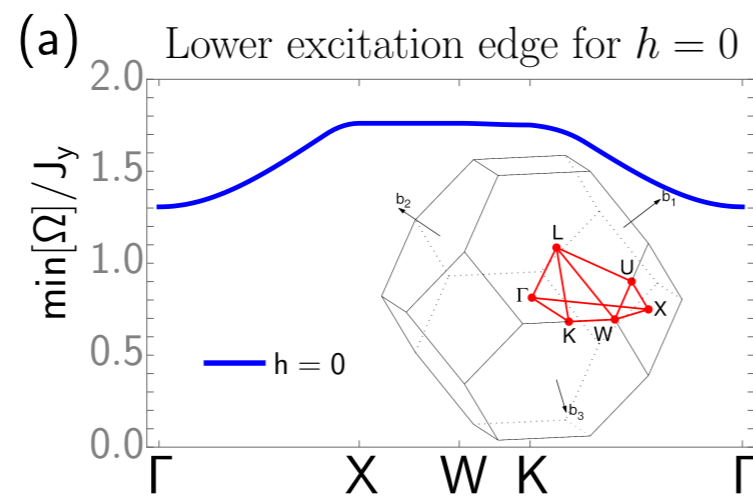
$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2,$$
$$\Omega(\mathbf{q}) = \omega_i(\mathbf{k}_1) + \omega_j(\mathbf{k}_2),$$



Lower excitation edge of 2-spinon continuum

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2,$$

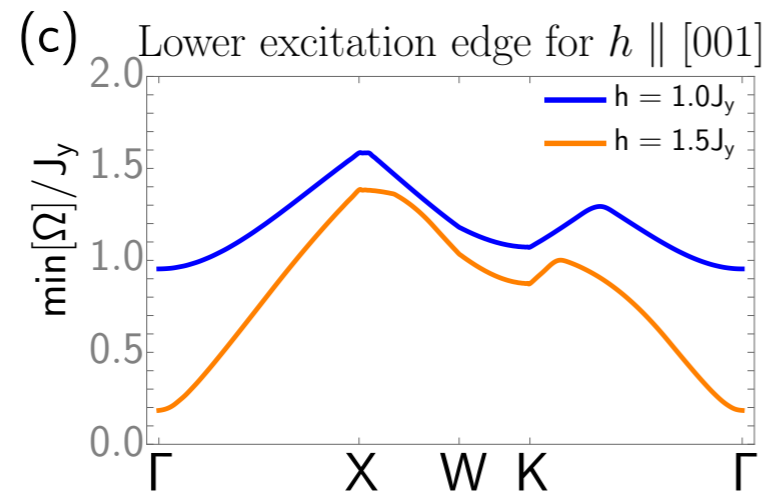
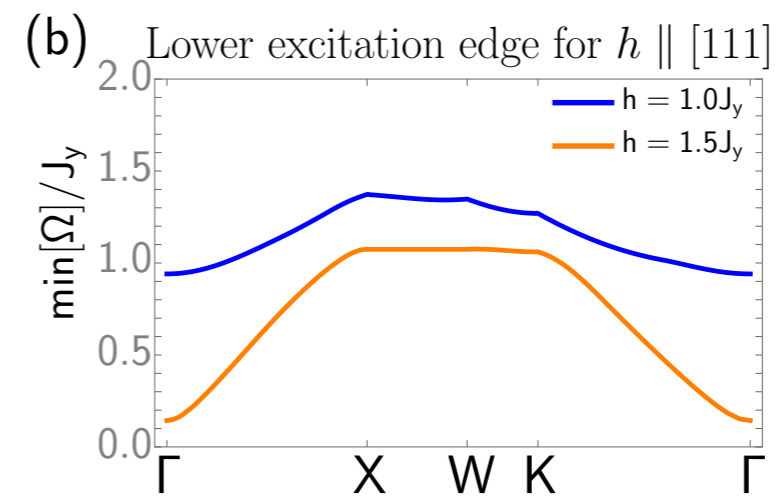
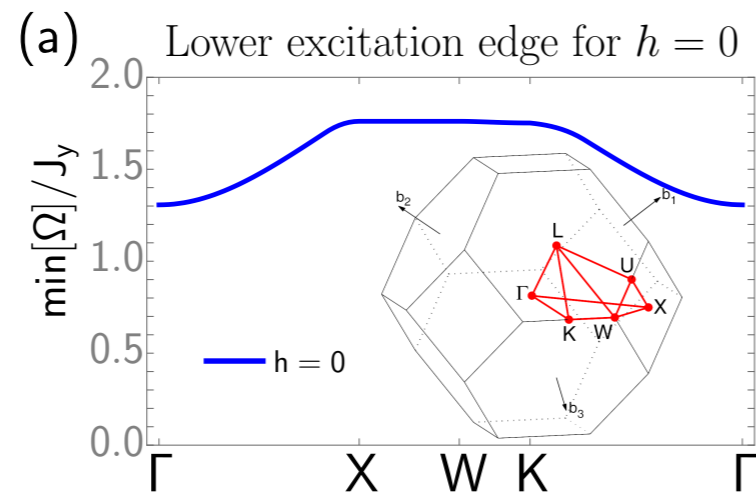
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Lower excitation edge of 2-spinon continuum

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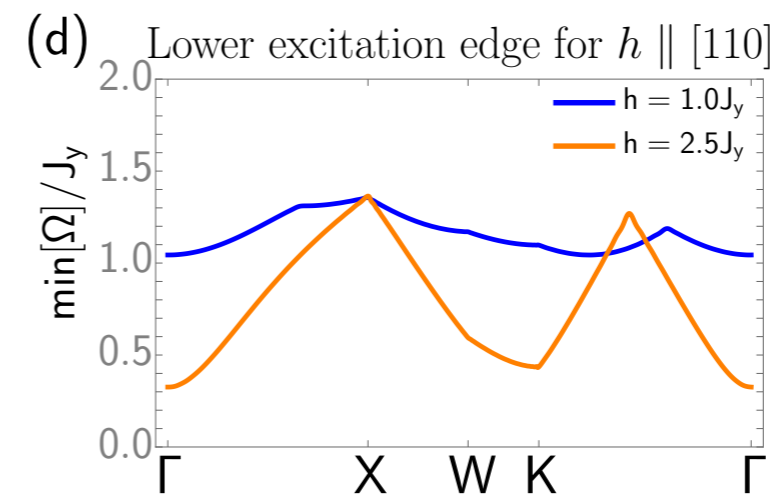
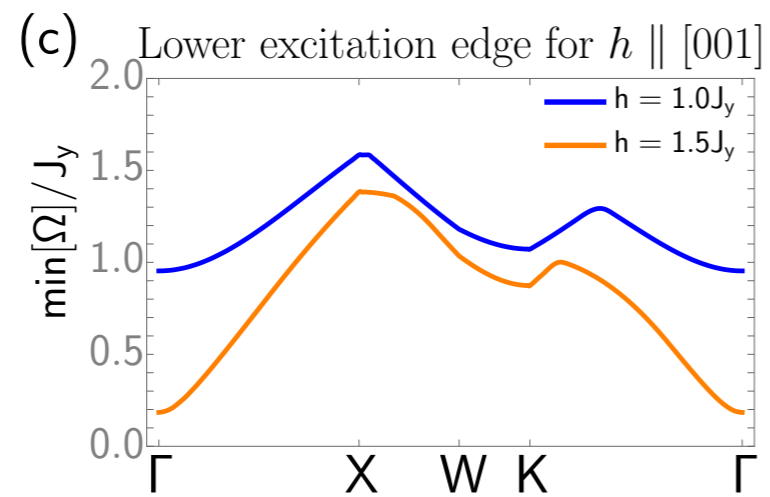
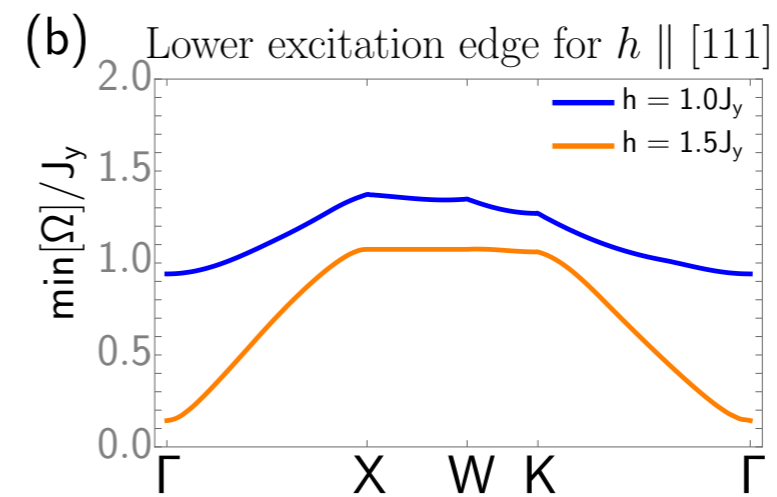
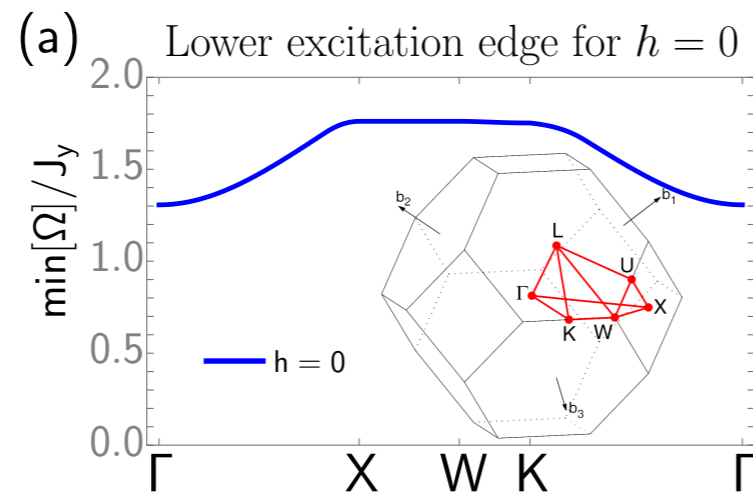
$$\Omega(\mathbf{q}) = \omega_i(\mathbf{k}_1) + \omega_j(\mathbf{k}_2),$$



Lower excitation edge of 2-spinon continuum

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2,$$

$$\Omega(\mathbf{q}) = \omega_i(\mathbf{k}_1) + \omega_j(\mathbf{k}_2),$$





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A re-evaluation of the heat capacity of cerium zirconate ($\text{Ce}_2\text{Zr}_2\text{O}_7$)

K. Popa¹, R.J.M. Konings*, F. Wastin, E. Colineau, N. Magnani, P.E. Raison

European Commission, Joint Research Centre, Institute for Transuranium Elements, P.O. Box 2340, 76125 Karlsruhe, Germany

Received 18 April 2007; received in revised form 18 July 2007; accepted 27 July 2007

Abstract

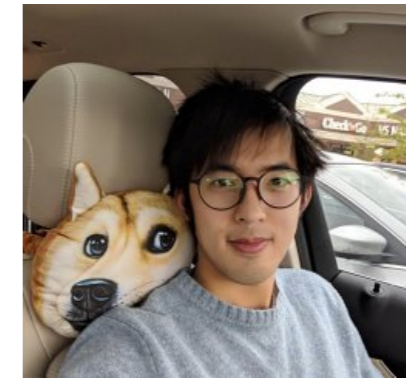
The heat capacity of cerium zirconate pyrochlore, $\text{Ce}_2\text{Zr}_2\text{O}_7$, was measured from 0.4 to 305 K by hybrid adiabatic relaxation method for various magnetic field strengths. Magnetisation measurements were performed on the sample also. The results revealed a low-temperature anomaly that showed Schottky-type characteristics with increasing magnetic field strength. The estimated entropy due to the magnetic ordering of the two Ce^{3+} moments is $1.37R$, close to the theoretical value for a doublet ground state ($1.39R$). The enthalpy increments relative to 298.15 K were measured by drop calorimetry from 531 to 1556 K. The obtained results significantly differ from those reported in the literature; the origin of the discrepancy is due to the probable oxidation of the pyrochlore structure into fluorite.

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Ce₂Zr₂O₇: a non-spin-ice pyrochlore U(1) spin liquid

Experimental signatures of a three-dimensional quantum spin liquid in effective spin-1/2 Ce₂Zr₂O₇ pyrochlore

Bin Gao^{1,11}, Tong Chen^{1,11}, David W. Tam¹, Chien-Lung Huang¹, Kalyan Sasmal², Devashibhai T. Adroja³, Feng Ye⁴, Huibo Cao⁴, Gabriele Sala⁴, Matthew B. Stone⁴, Christopher Baines⁵, Joel A. T. Barker⁵, Haoyu Hu¹, Jae-Ho Chung^{1,6}, Xianghan Xu⁷, Sang-Wook Cheong⁷, Manivannan Nallaiyan⁸, Stefano Spagna⁸, M. Brian Maple², Andriy H. Nevidomskyy¹, Emilia Morosan¹, Gang Chen^{9,10} and Pengcheng Dai^{1*}



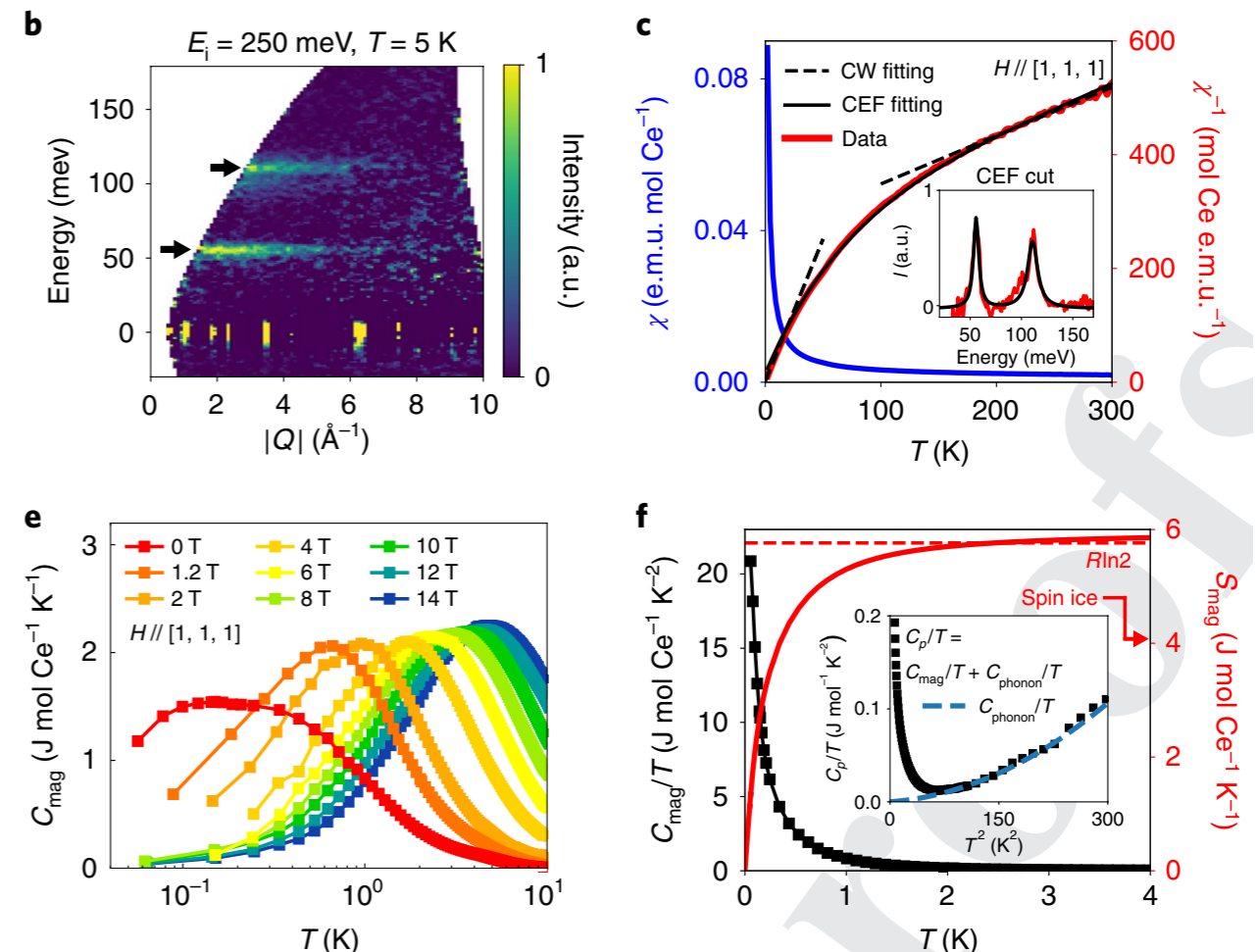
Tong Chen



Pengcheng Dai

Nature Physics, 2019

also Gaulin's group, PRL 2019



Our suggestion [[YD Li & GC, 1902.07075](#)]: this material is in U1B phase !

Summary-2

We have used some fundamental ideas to predict novel experiments:

- translation symmetry enrichment
 - electromagnetic duality
 - spin fractionalization and multipolariness
- > **spectral periodicity enhancement**
 - > **“magnetic monopole” continuum**
 - > **topological thermal Hall effect**
 - > **emergent Anderson Higgs transition**

The model, that we derive, can be understood by us theoretically and solved mostly by QMC.

“Real model, no sign problem and non-trivial phase”.

What does inelastic neutron scattering measure
in a pyrochlore U(1) quantum spin liquid in general ?

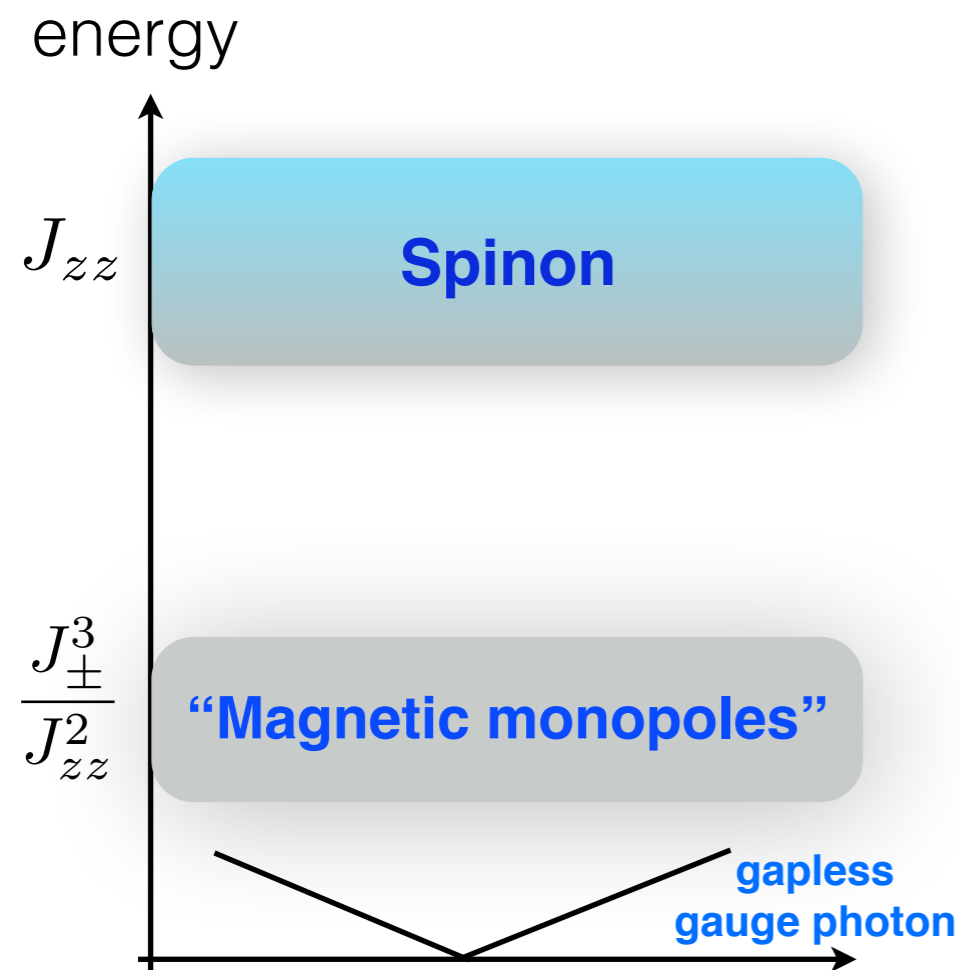
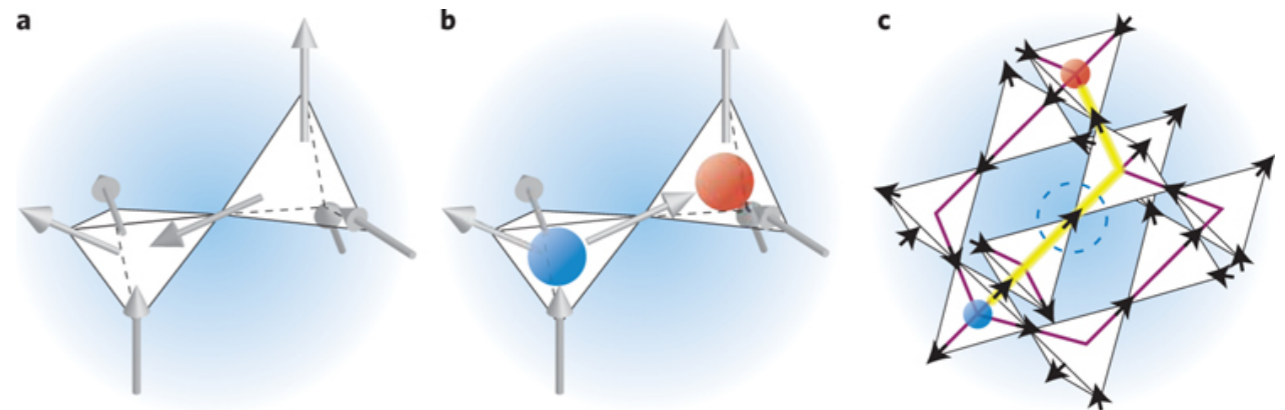


illustration for XXZ model



Figs from Moessner&Schiffer,2009

Spinon deconfinement

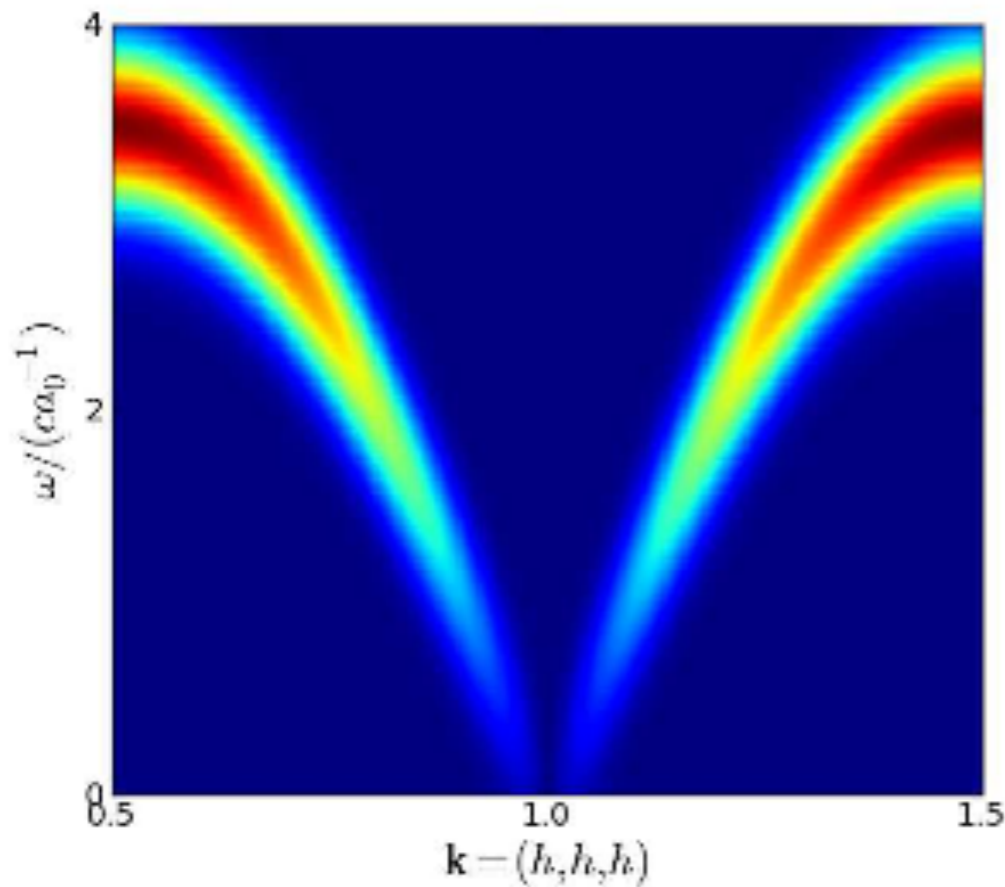
Hermele, Fisher, Balent 2004

Nic Shannon, etc 2012

Lucile Savary, Balents, 2012

Bruce Gaulin,

Emergent gauge photon



$$I(\omega) \sim \omega$$

$$S_z \sim E \text{ (emergent electric field)}$$

Low energy theory

$$\text{Im}[E_{-\mathbf{k},-\omega}^\alpha E_{\mathbf{k},\omega}^\beta] \propto [\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mathbf{k}^2}] \omega \delta(\omega - v|\mathbf{k}|),$$

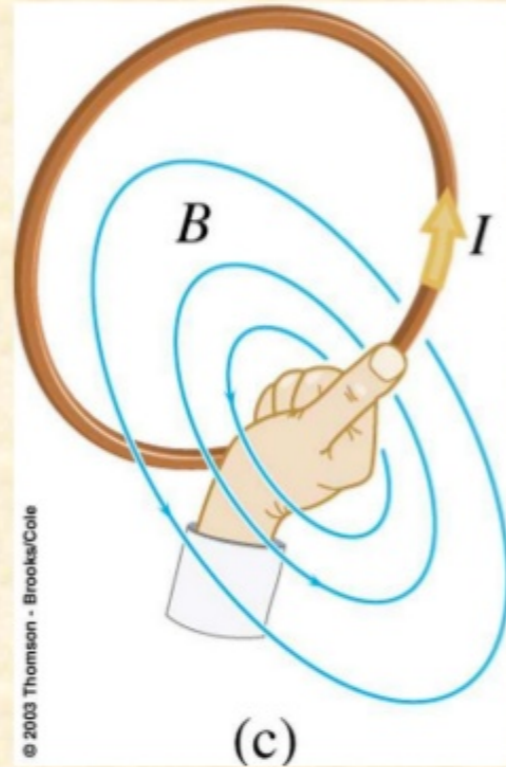
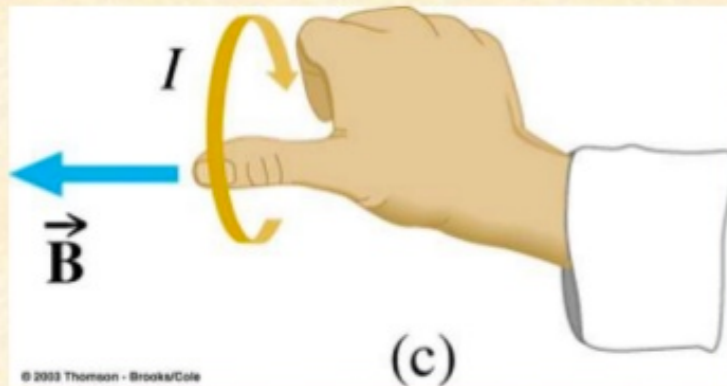
emergent U(1) photon in U(1) QSL

Hermele etc 2004
N Shannon etc 2012,
L Savary, Balents 2012

Electromagnetic duality

For loop or coil of wire, can still use 1st RHR, but direction of current constantly changes.

Easier to use 2nd Right Hand Rule. Fingers curl in direction of current, thumb points to direction of magnetic field.



Duality

Electric loop current \rightarrow Magnetic field
Magnetic loop current \rightarrow Electric field

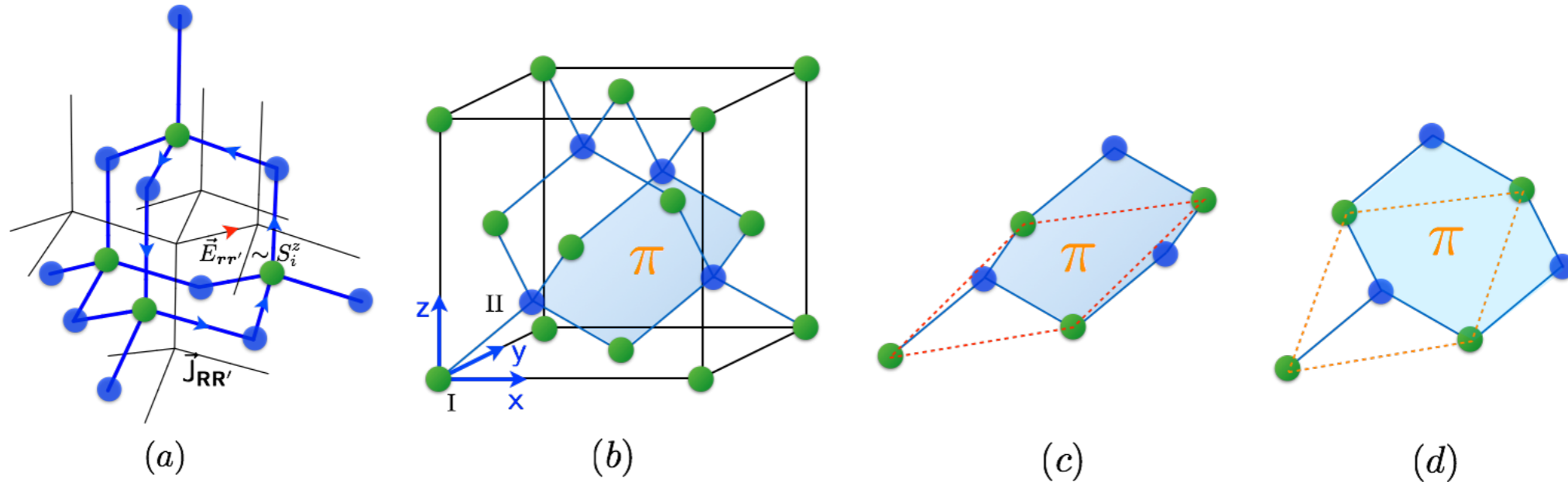
$$S_z \sim E \text{ (emergent electric field)}$$

GC, PRB 96, 195127 (2017)

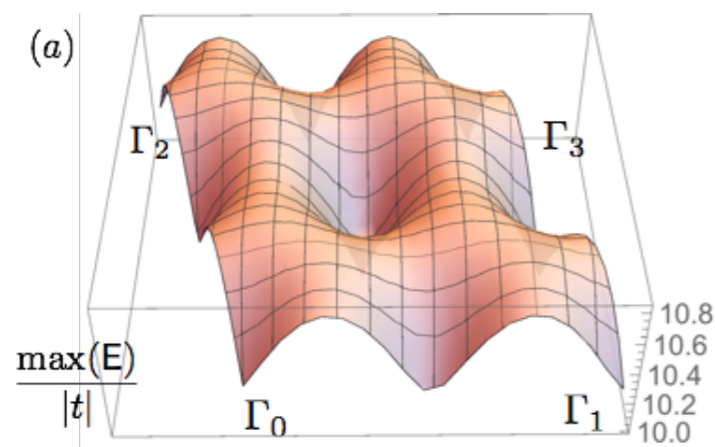
GC, PRB 96, 085136 (2017)

Motrunich & Senthil, 2004,
Bergman, Fiete, Balents, 2006

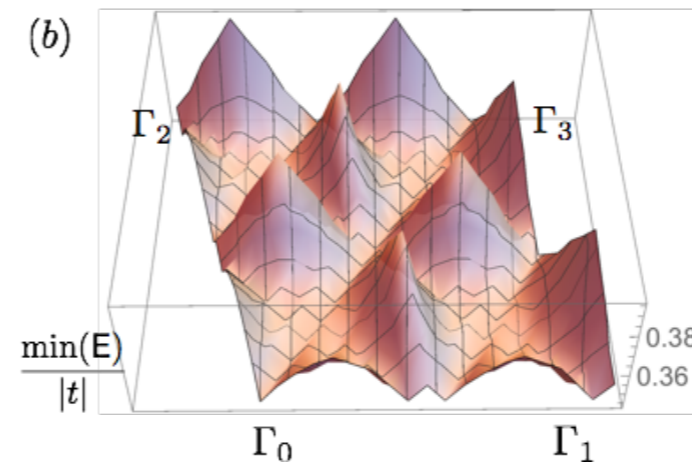
Sz correlation = monopole loop current correlation



$$H_{\text{dual}} = -t \sum_{\langle \mathbf{R}\mathbf{R}' \rangle} e^{-i2\pi\alpha_{\mathbf{R}\mathbf{R}'}} \Phi_{\mathbf{R}}^{\dagger} \Phi_{\mathbf{R}'} - \mu \sum_{\mathbf{R}} \Phi_{\mathbf{R}}^{\dagger} \Phi_{\mathbf{R}} + \frac{U}{2} \sum_{\square^*} (\text{curl} \alpha - \frac{\eta_{\mathbf{r}}}{2})^2 - K \sum_{\langle \mathbf{R}\mathbf{R}' \rangle} \cos B_{\mathbf{R}\mathbf{R}'} + \dots$$



the upper edge

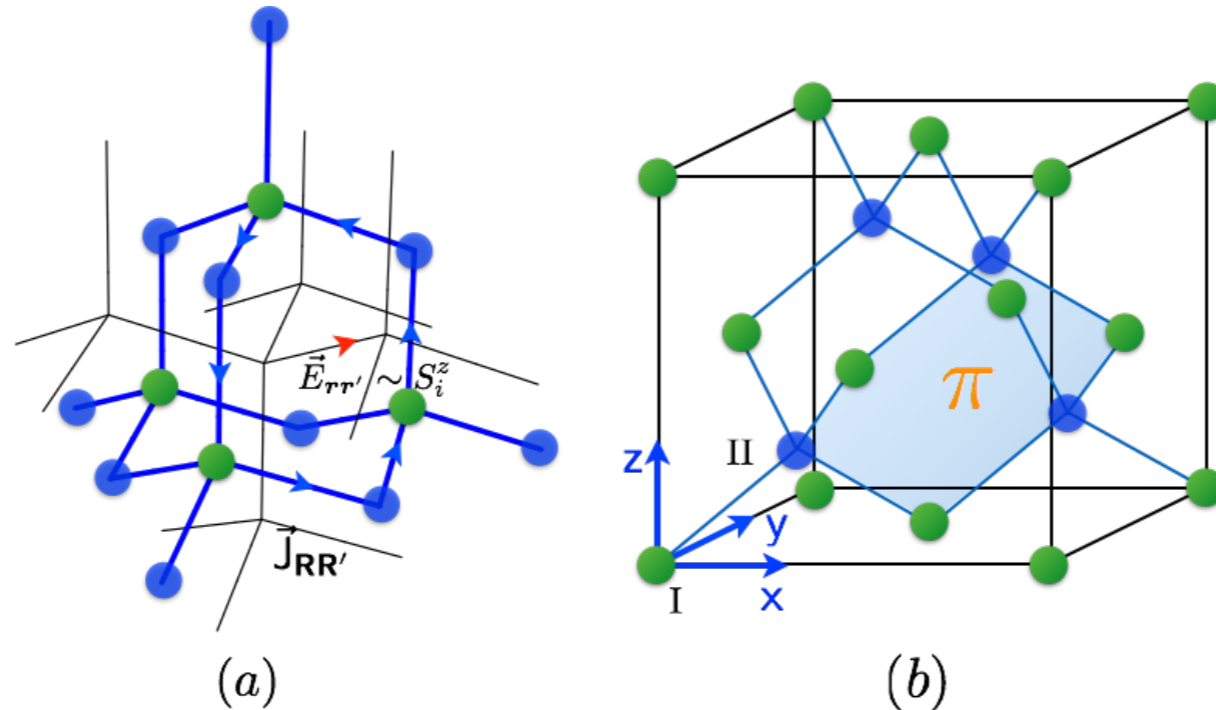


the lower edge

Monopole always experiences Pi flux

GC, PRB 96, 195127 (2017)
GC, PRB 96, 085136 (2017)
Xiao-Gang Wen, 2002
Essin, Hermele 2016

Suggestion 1: effect of the external magnetic field

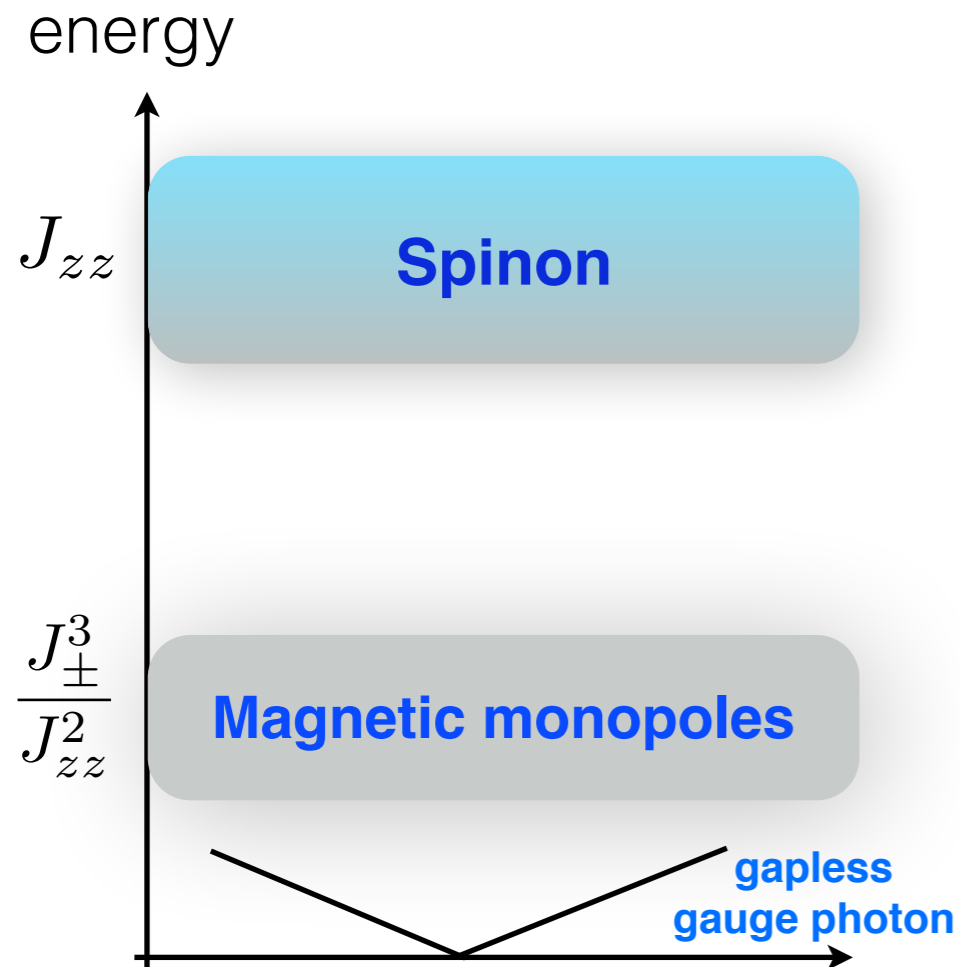


$$H_{Zeeman} = \vec{B} \cdot \sum_i S_i^z \hat{z}_i$$

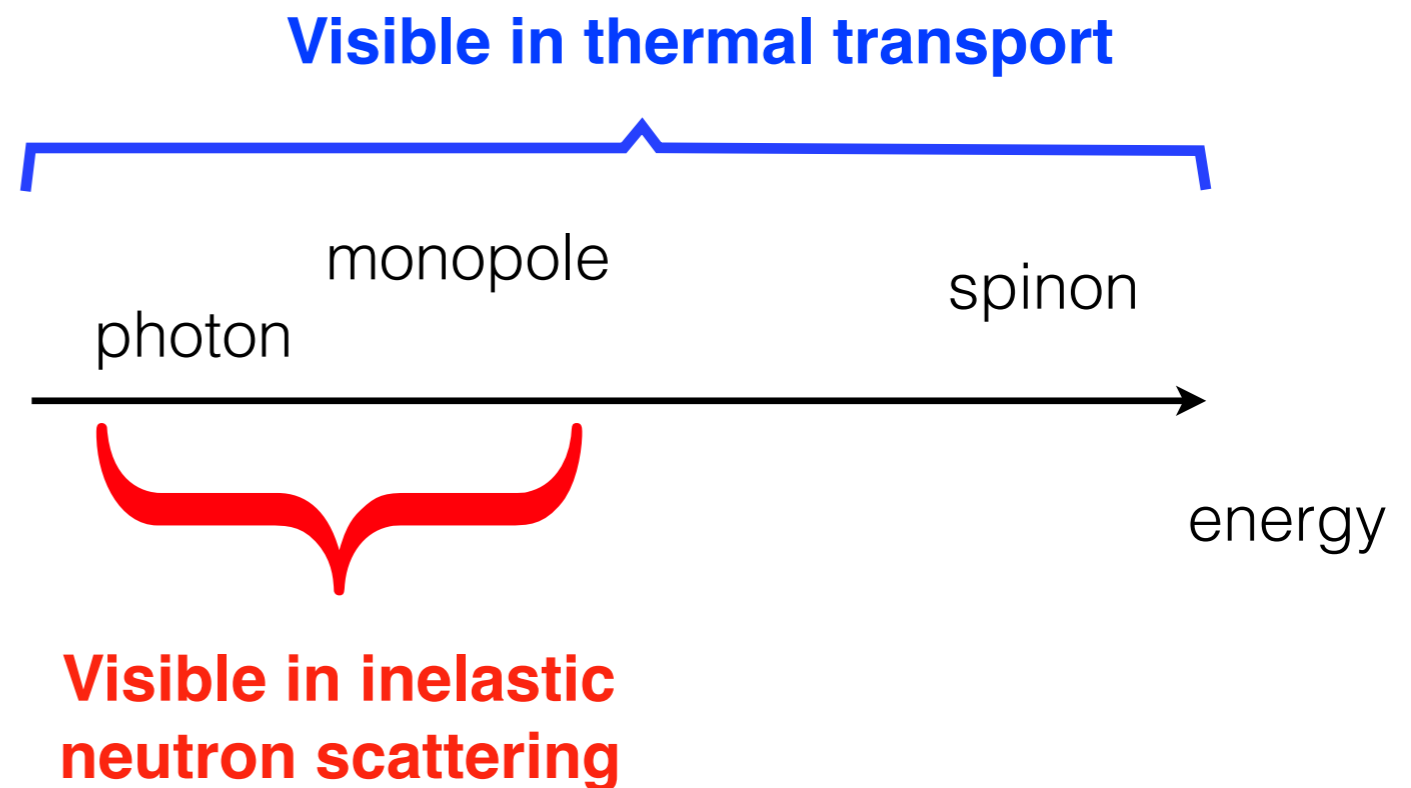
The weak magnetic field polarizes S_z slightly, and thus modifies the background electric field distribution. This further modulates monopole band structure, creating “**Hofstadter**” monopole band, which may be detectable in inelastic neutron.

This is also the origin of topological **thermal Hall effect** for “magnetic monopoles”.

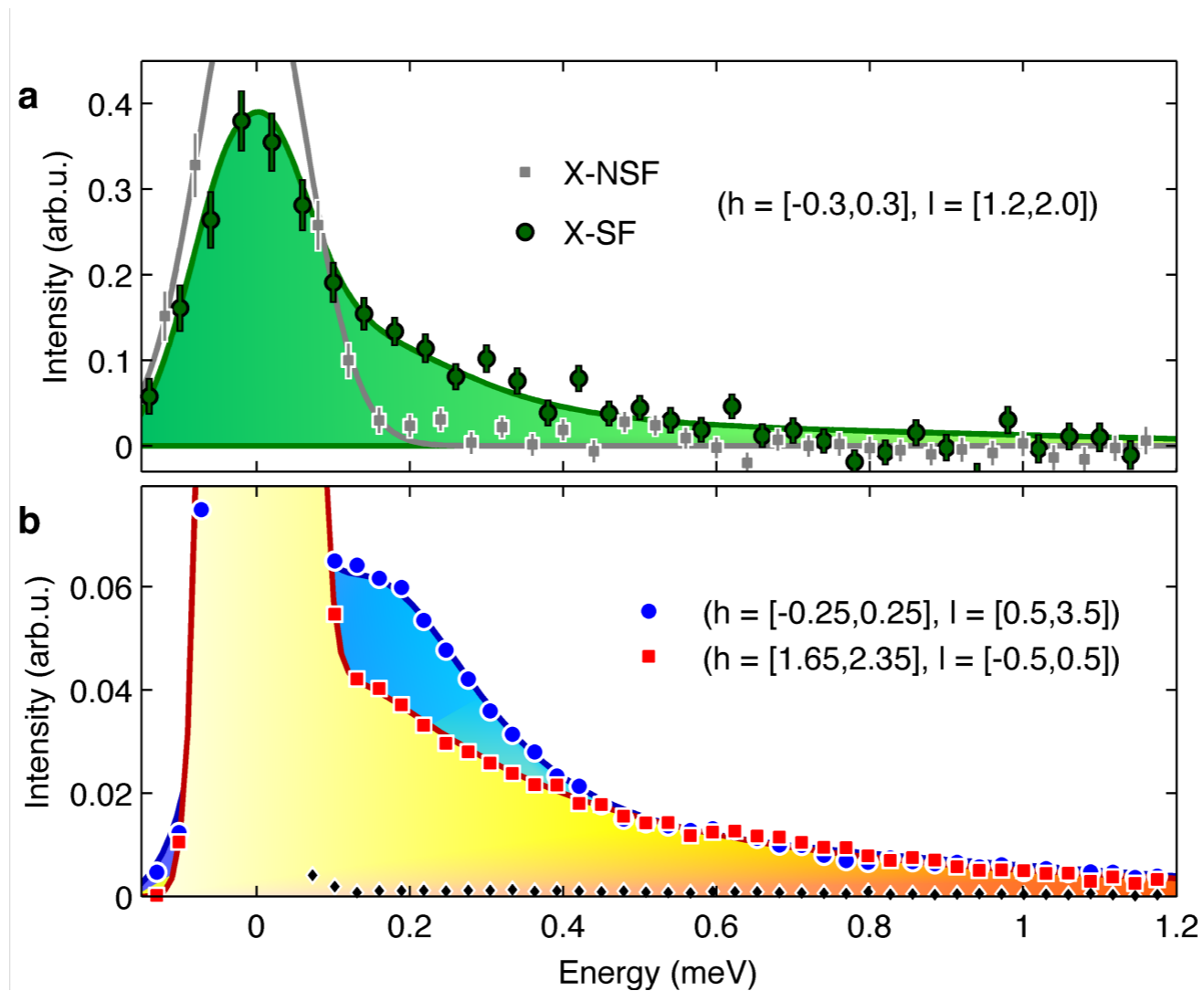
Suggestion 2: combine thermal transport with inelastic neutron



For **non-Kramers doublets** such as Pr ion in $\text{Pr}_2\text{Zr}_2\text{O}_7$ and Tb ion in $\text{Tb}_2\text{Ti}_2\text{O}_7$



In fact, continuum has been observed in $\text{Pr}_2\text{Hf}_2\text{O}_7$
(R. Sibille, et al, arXiv 1706.03604). Nature Physics



This is a non-Kramers doublet version of pyrochlore U(1) spin liquid candidate.