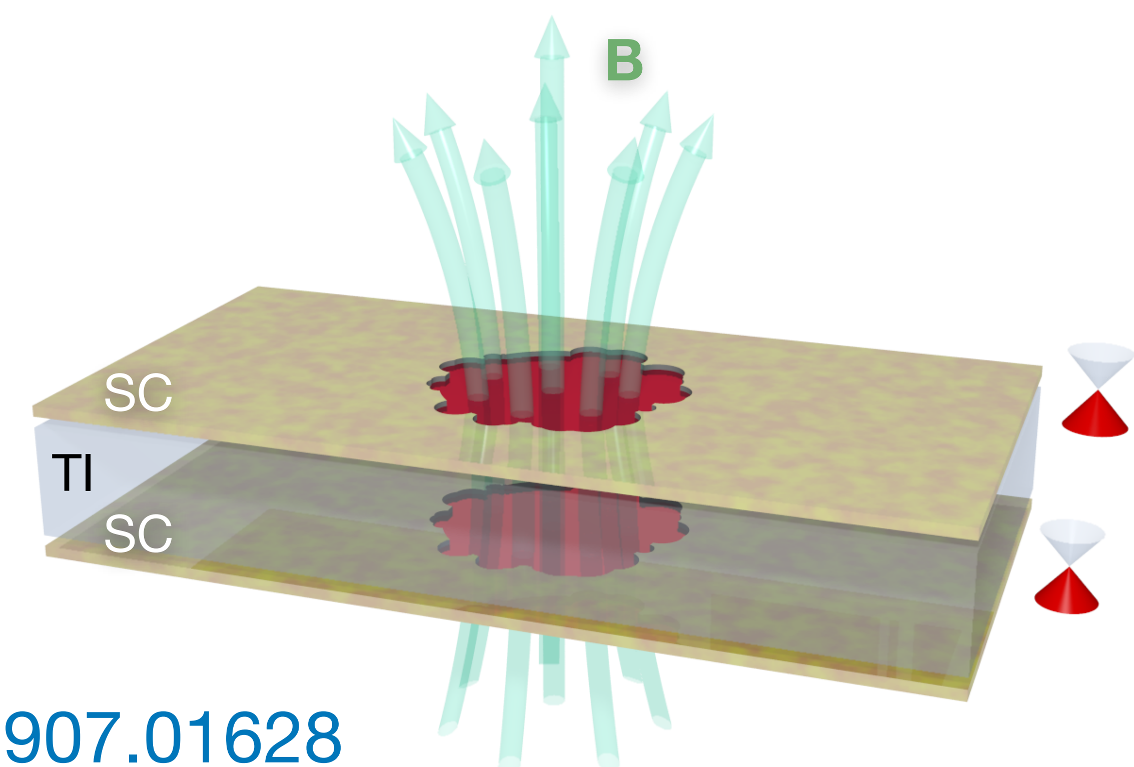
Diagnosing quantum chaos using entanglement

E´tienne Lantagne-Hurtubise, Stephan Plugge, Oguzhan Can, and Marcel Franz



arXiv:1907.01628

NSERC CRSNG
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UBC THE UNIVERSITY OF BRITISH COLUMBIA
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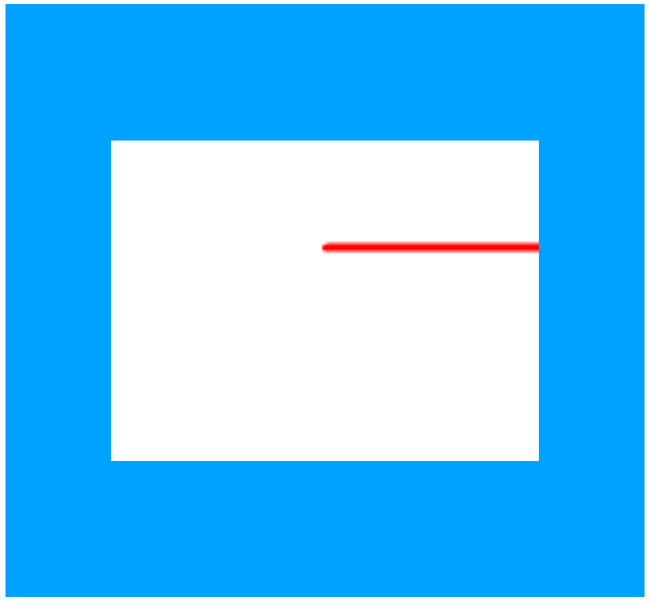
Classical chaos

- Exponential sensitivity of classical trajectories to small changes in initial conditions.
- Butterfly effect, breakdown of predictability in deterministic systems
- Many examples of classically chaotic systems exist in mathematics, physics and natural phenomena



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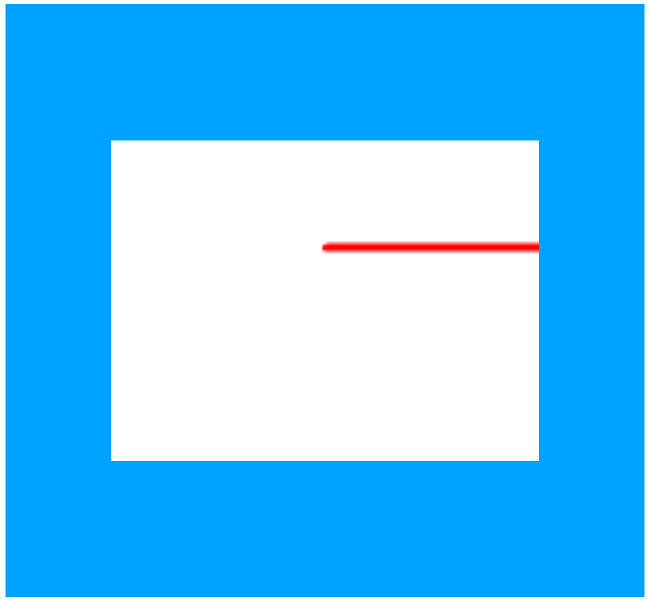

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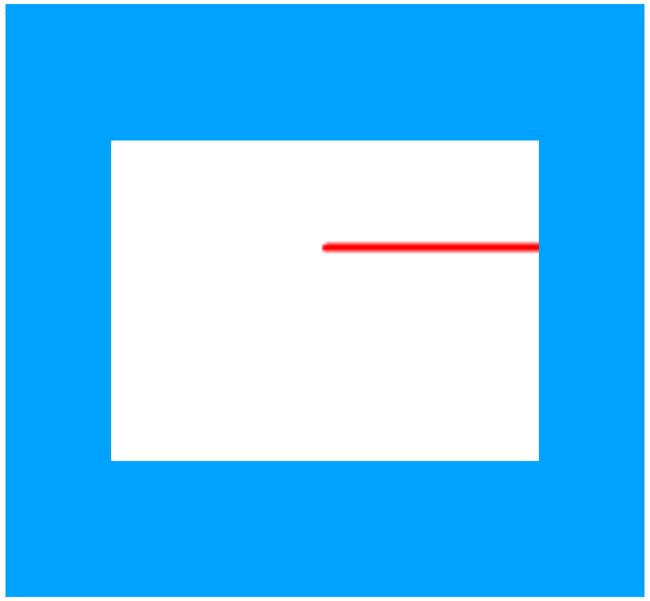

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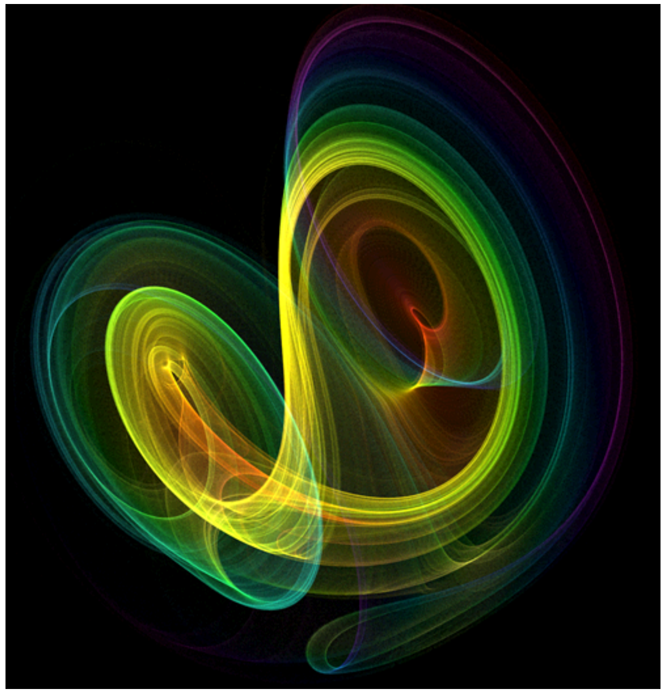

Lorenz attractor

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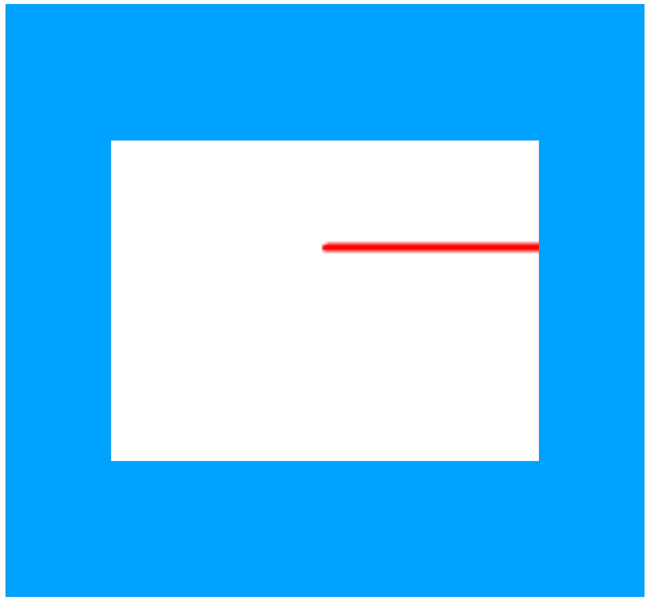

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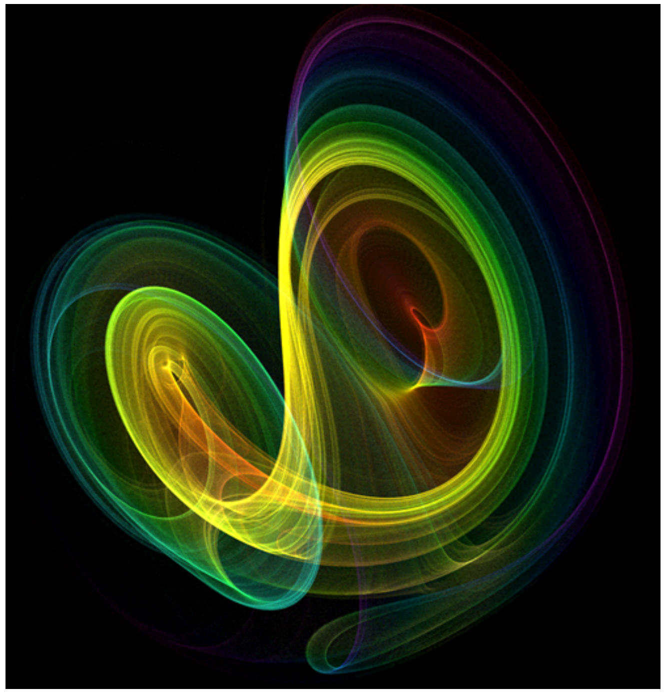

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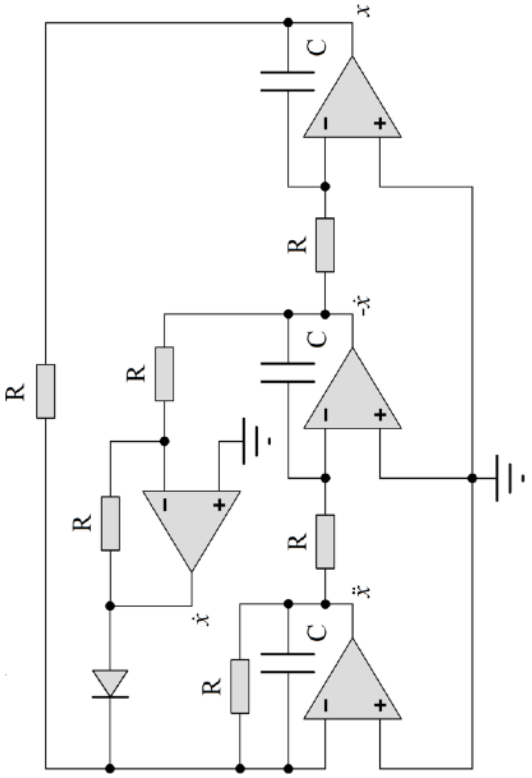

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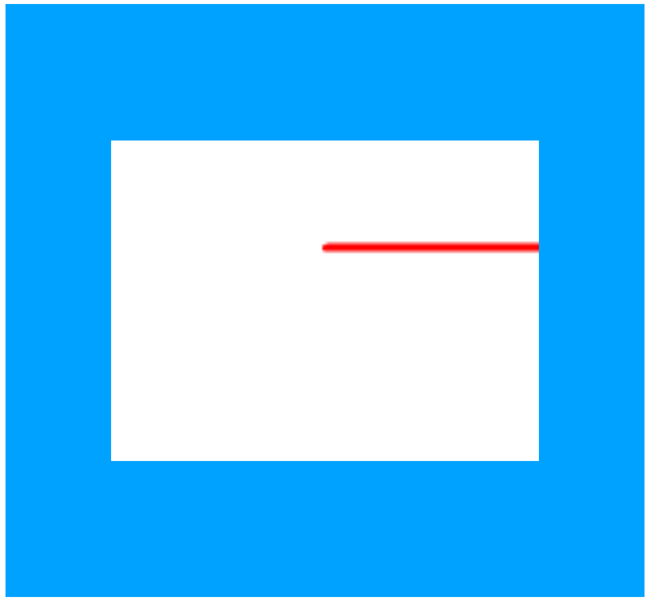

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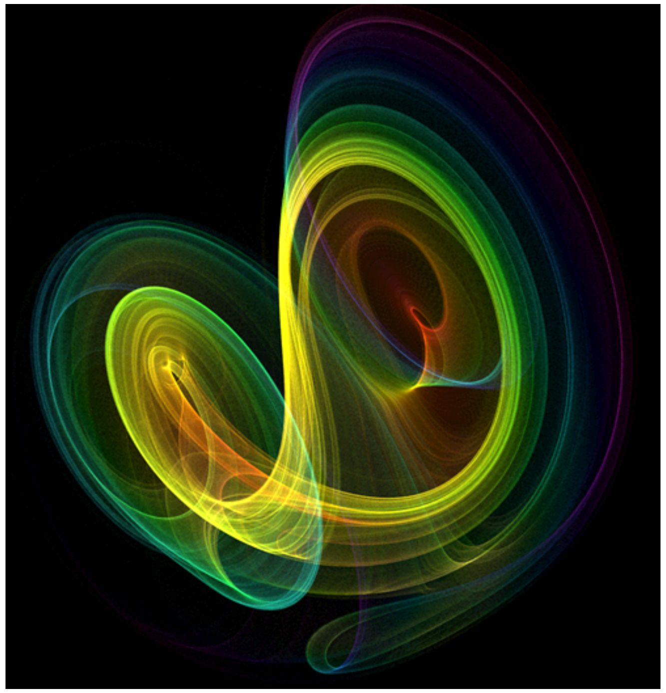

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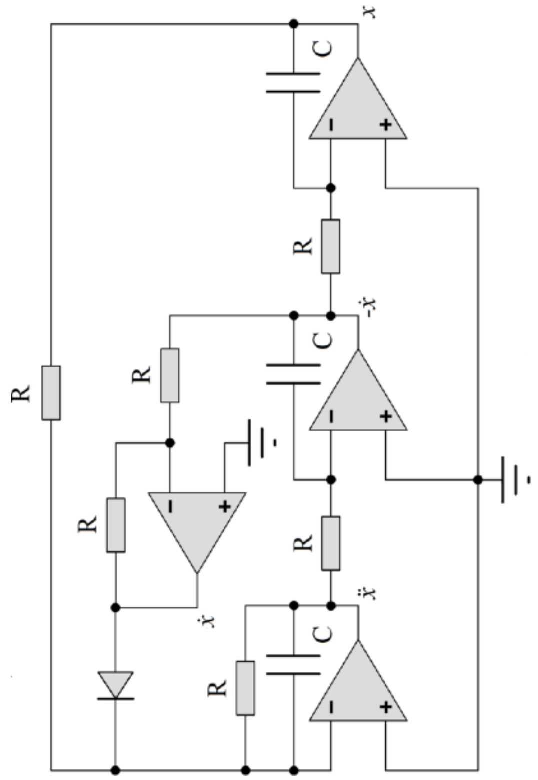

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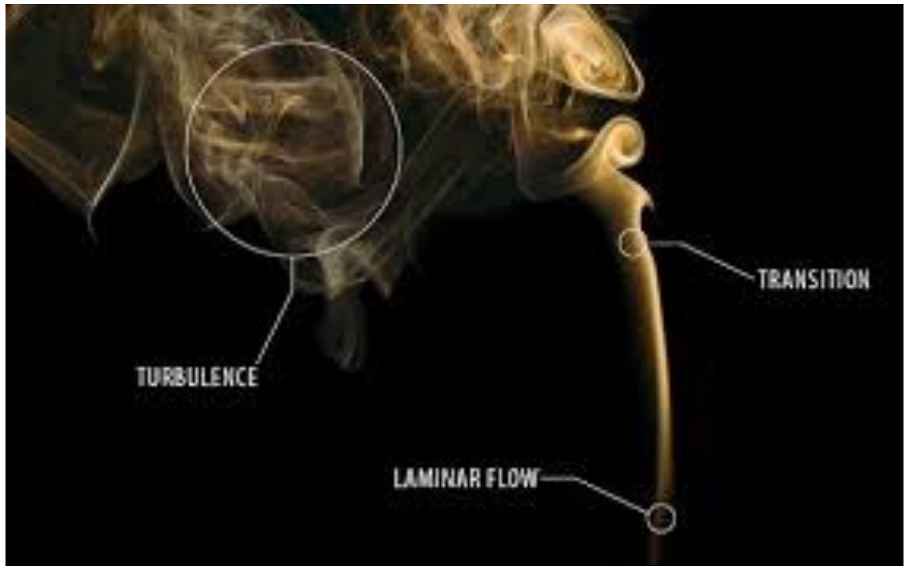

turbulent flow

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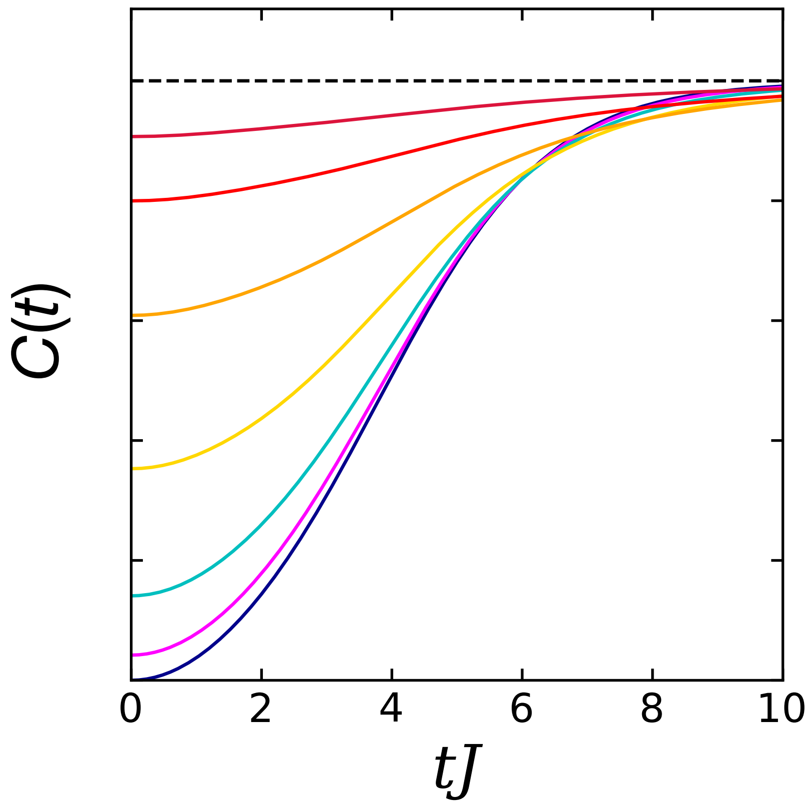

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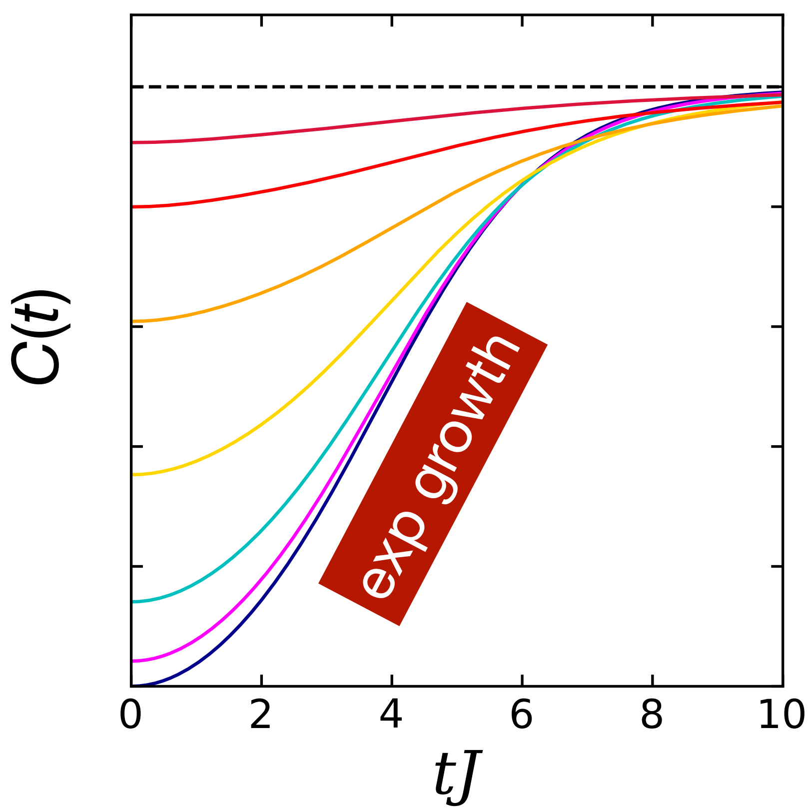

Operator growth intuitive picture:

- As the initially “simple” operator *W* becomes more complex due to the time evolution it eventually fails to commute with *V*.

$t = 0$ $\qquad$ $t > 0$

$[V(0), W(0)] = 0$ $\qquad$ $[V(0), W(t)] \neq 0$

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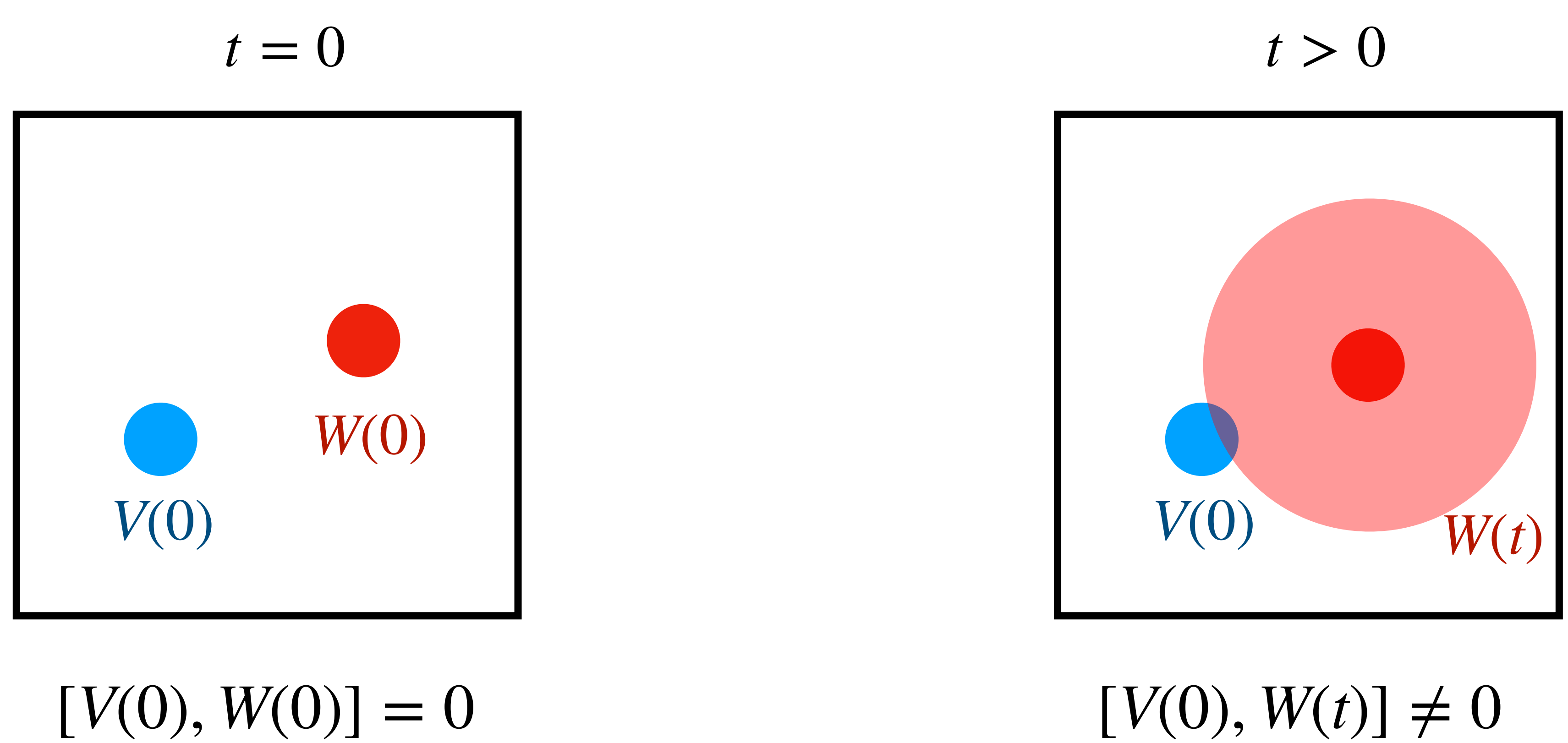

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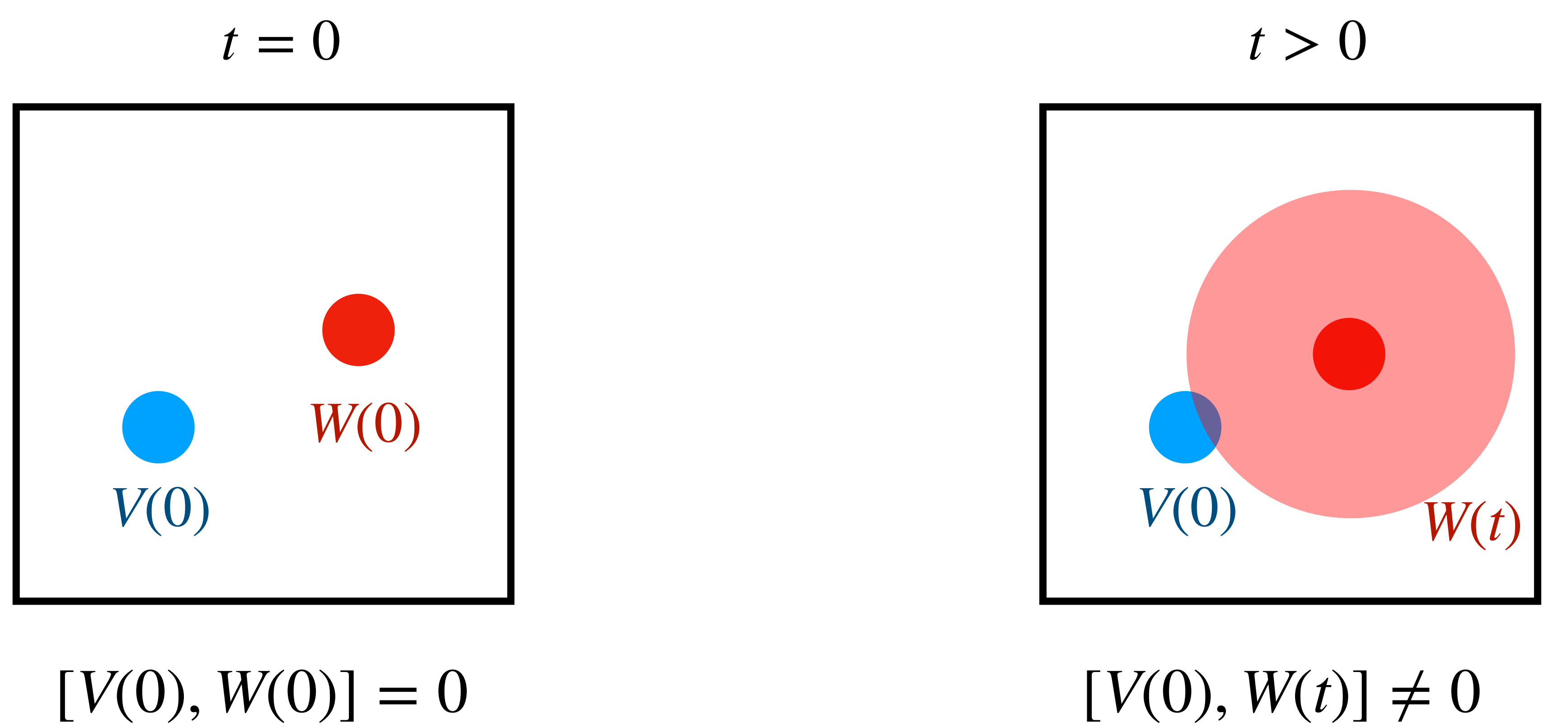

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- Here λ_L is the quantum Lyapunov exponent

The chaos bound

- The Lyapunov exponent λ_L characterizing the exponential growth of $C(t) = -\langle [W(t), V(0)]^2 \rangle$ at intermediate times is subject to a fundamental upper bound

$$\lambda_L \leq 2\pi T$$

[Maldacena, Stanford, Shenker 2017]

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- Such systems are usually holographically dual to a quantum gravity theory in a geometry with a black hole.
- *Black holes* are also maximally chaotic, i.e. they thermalize in shortest possible time consistent with causality and unitarity.

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- It underlies our understanding of important concepts including many-body localization and eigenstate thermalization hypothesis (ETH)
- Quantum chaos is important in attempts to reconcile quantum mechanics with general relativity: It points to a resolution of fundamental open questions such as the Hawking black hole information paradox

Out-of-time-order correlators

Expand the commutator squared:

$$C(t) = \langle W(t)VVW(t)\rangle + \langle VW(t)W(t)V\rangle$$
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(ii) evolve the perturbed state forward in time and
(iii) perform a measurement of the quantity represented by $W(t)^2$

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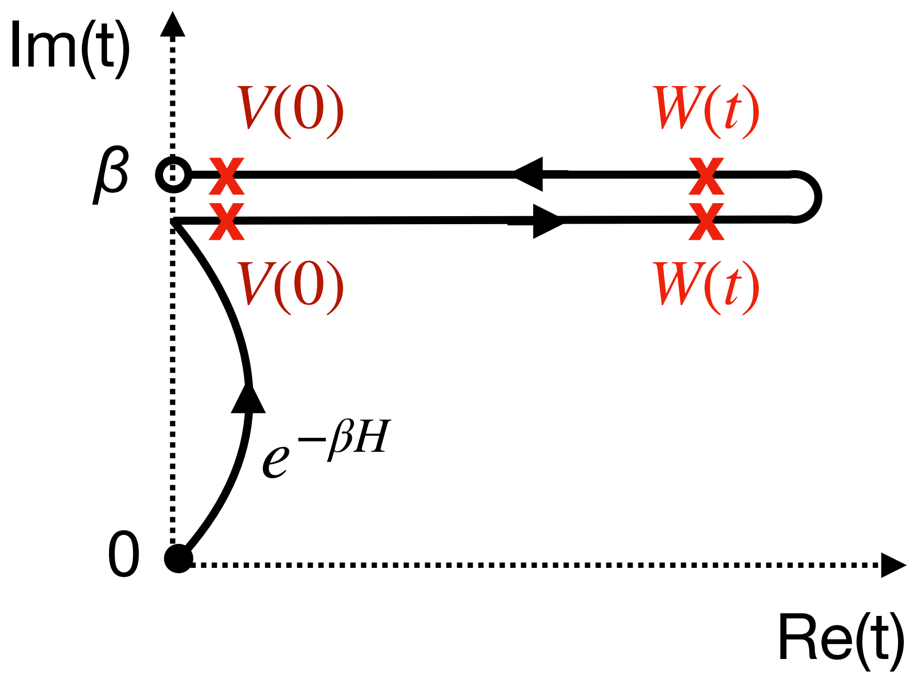
- Backward time evolution can be achieved by reversing the sign of the Hamiltonian

$$W(t) = e^{iHt} W e^{-iHt} \to e^{-iHt} W e^{iHt}$$

- In most systems however this is difficult or impossible to achieve

- The difference between NTOC and OTOC may be visualized by placing the operators on the Schwinger-Keldysh contour

a) NTOC $\langle V(0)W(t)W(t)V(0)\rangle$



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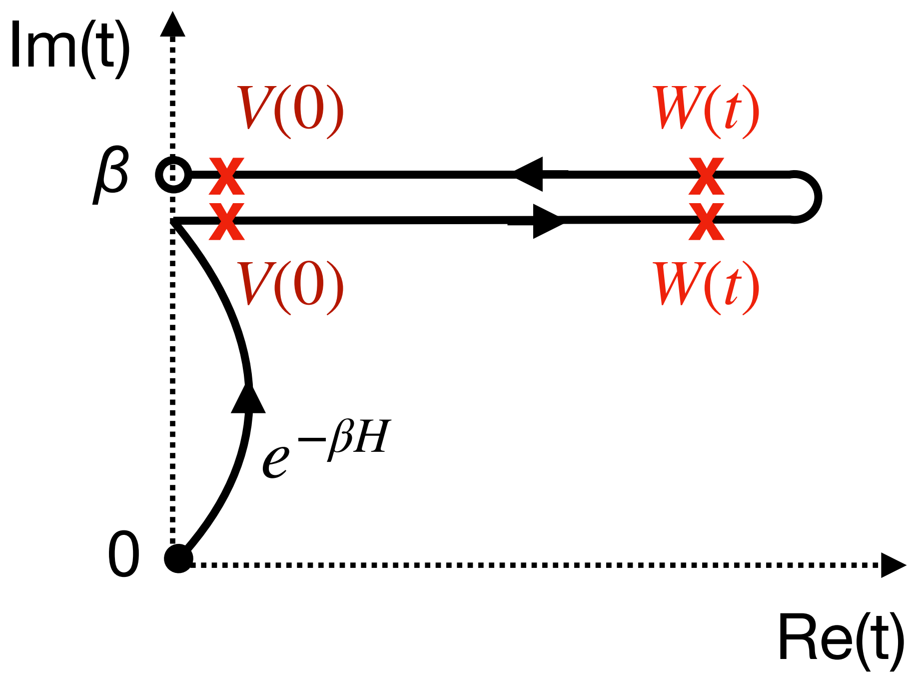
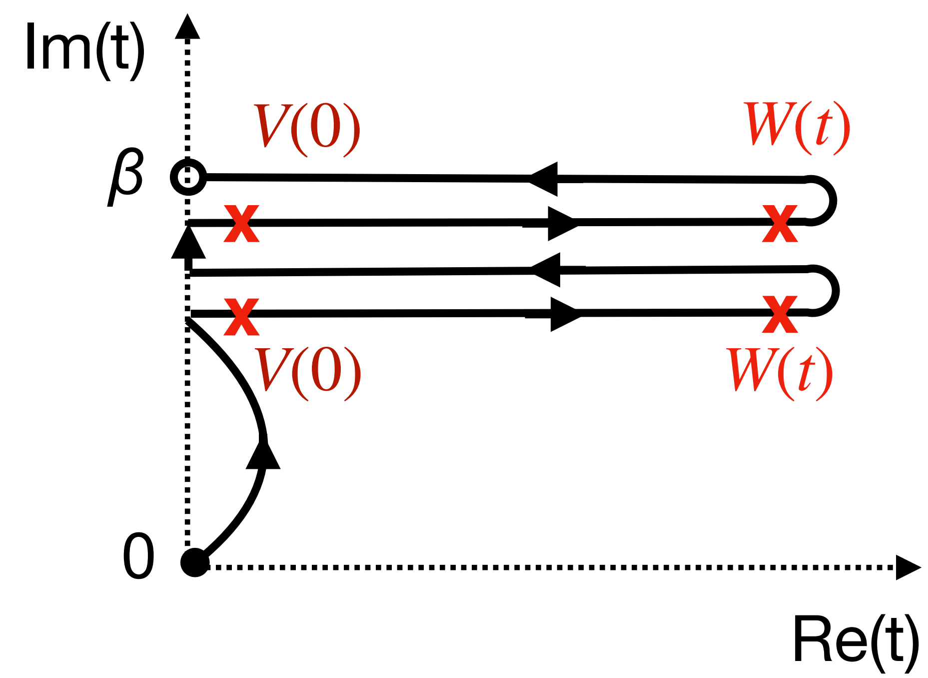


b) OTOC $\langle W(t)V(0)W(t)V(0)\rangle$



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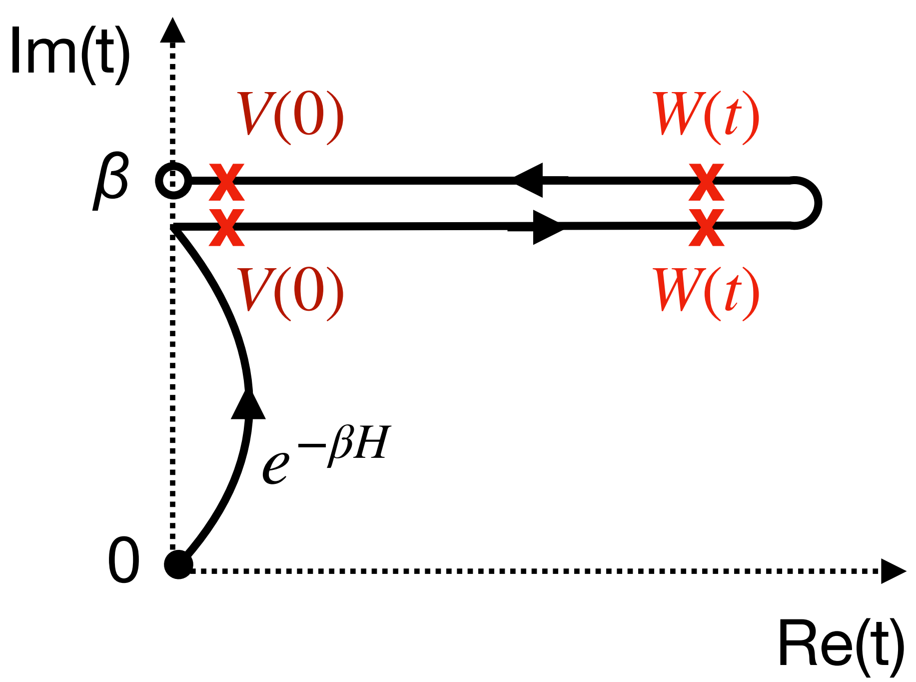

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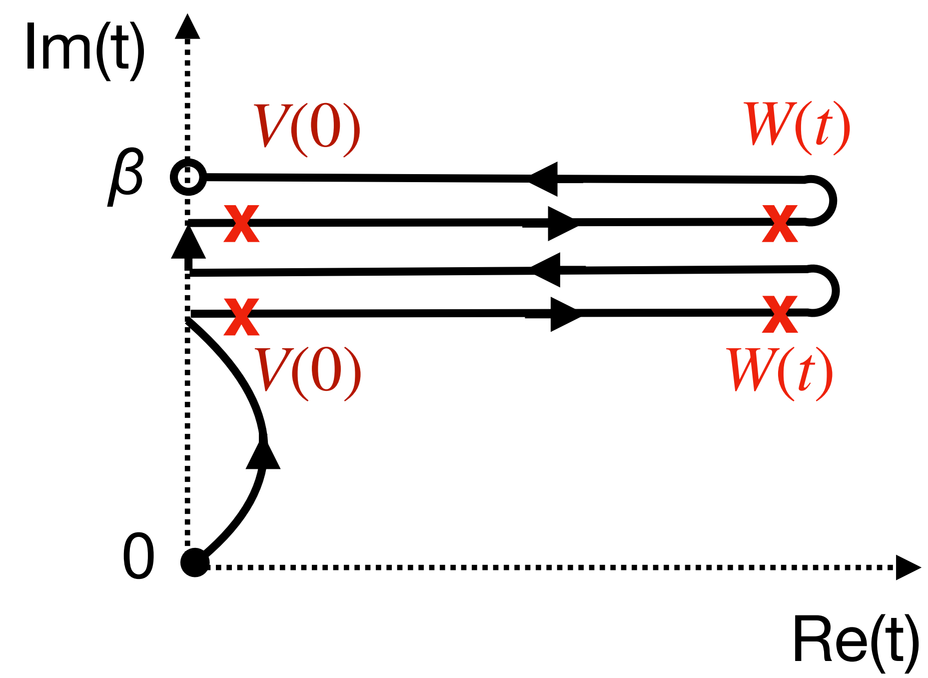

- NTOC can be placed on an ordinary S-K contour with one forward and one backward moving branch while
- OTOC require a **doubled contour**

nature physics

ARTICLES

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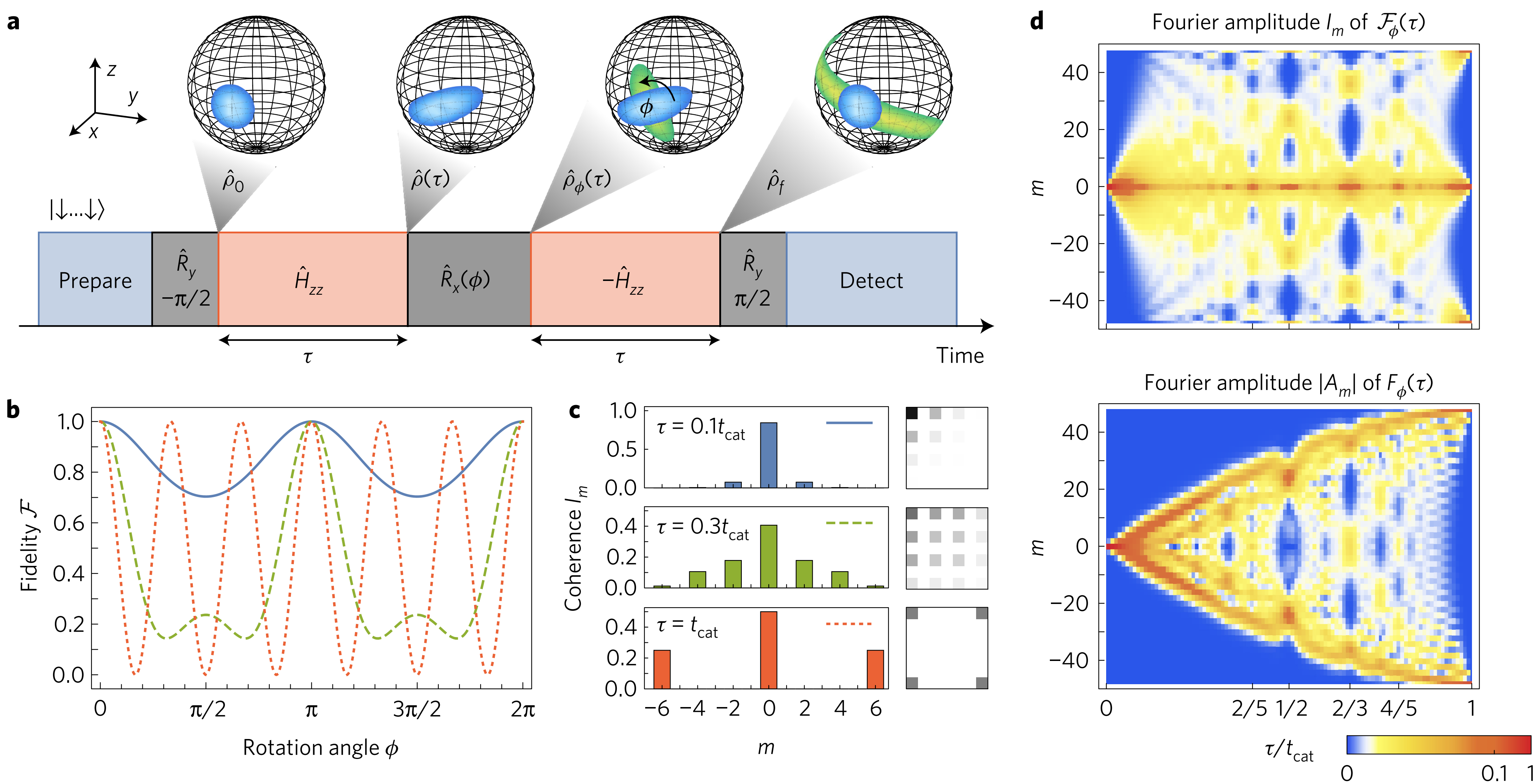
Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet

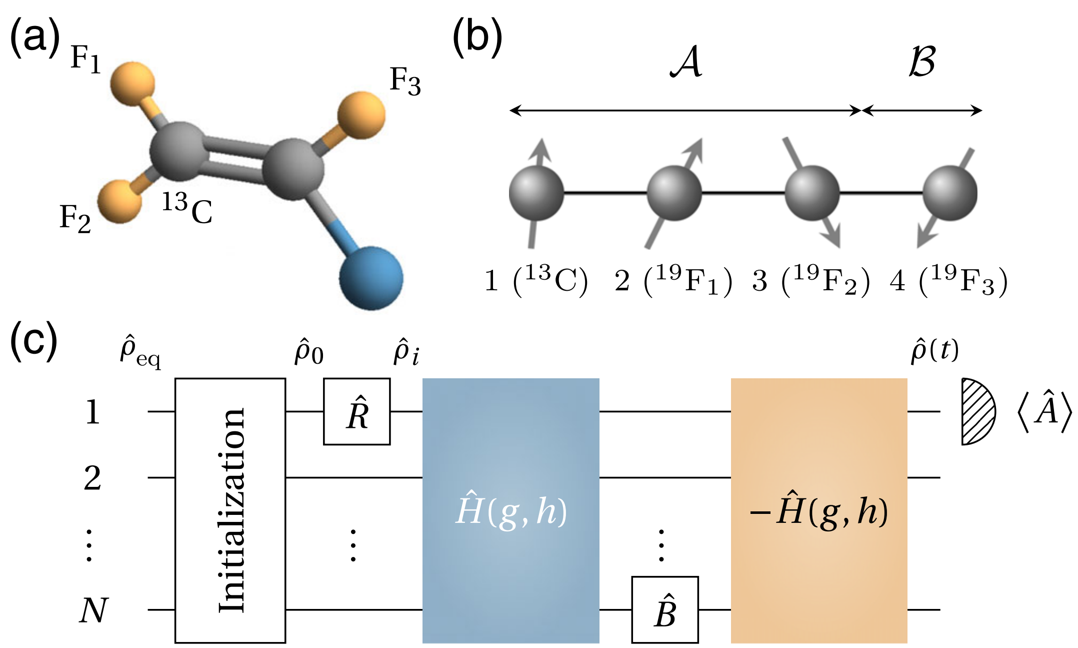
Martin Gärttner[1†], Justin G. Bohnet[2†], Arghavan Safavi-Naini[1], Michael L. Wall[1], John J. Bollinger[2] and Ana Maria Rey[1*]

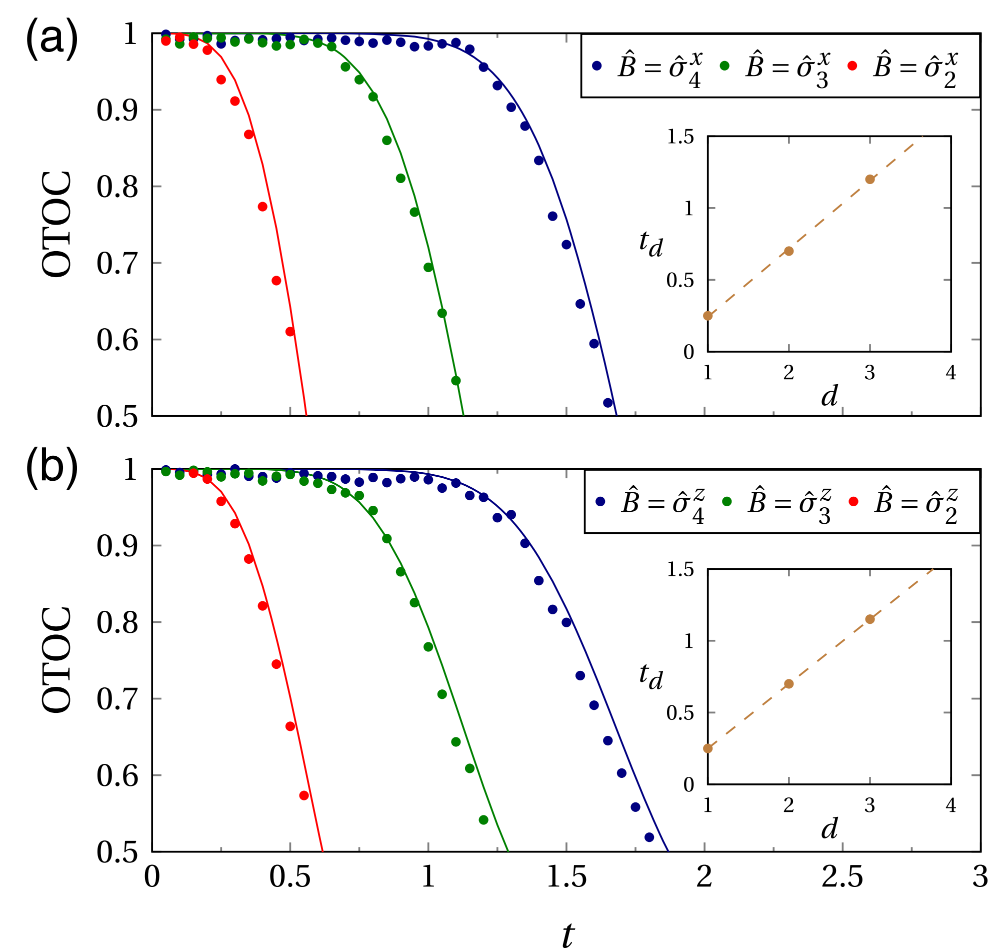


Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031011 (2017)

Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator

Jun Li,[1] Ruihua Fan,[2,3] Hengyan Wang,[3] Bingtian Ye,[3] Bei Zeng,[4,5,2,*] Hui Zhai,[2,6,†] Xinhua Peng,[7,8,9,‡] and Jiangfeng Du[7,8]
[1]*Beijing Computational Science Research Center, Beijing 100193, China*
[2]*Institute for Advanced Study, Tsinghua University, Beijing 100084, China*
[3]*Department of Physics, Peking University, Beijing 100871, China*





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Is there an alternative that could be used in complex many-body systems?

Quantum chaos diagnosis using entangled states

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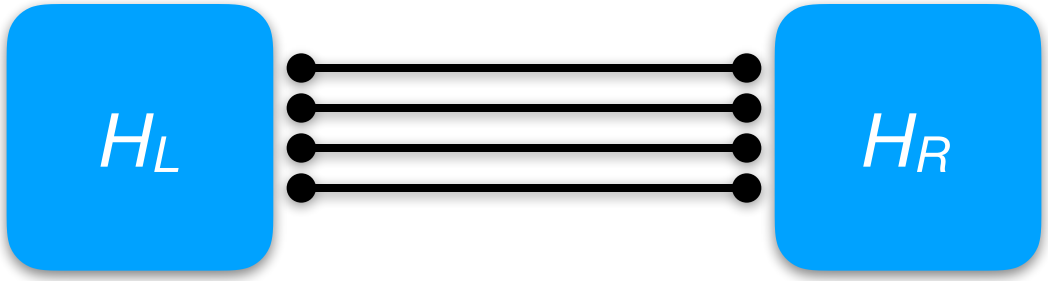

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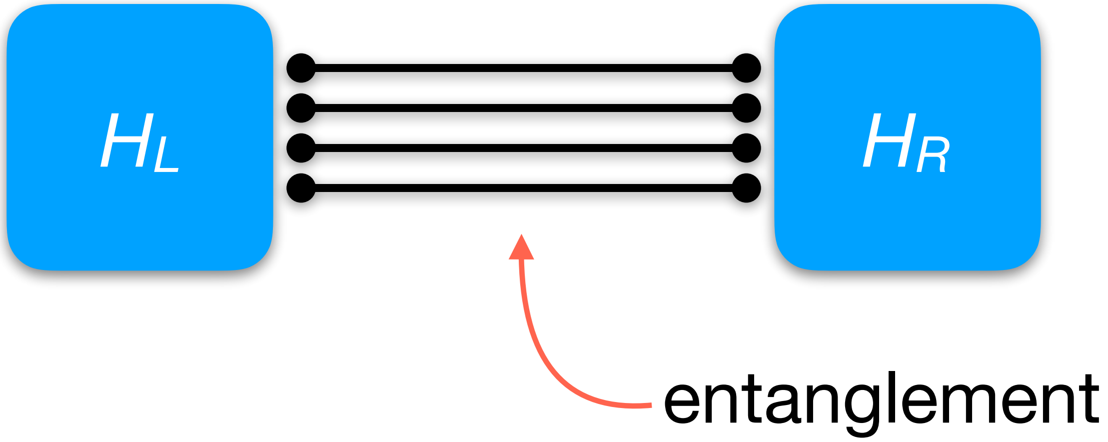

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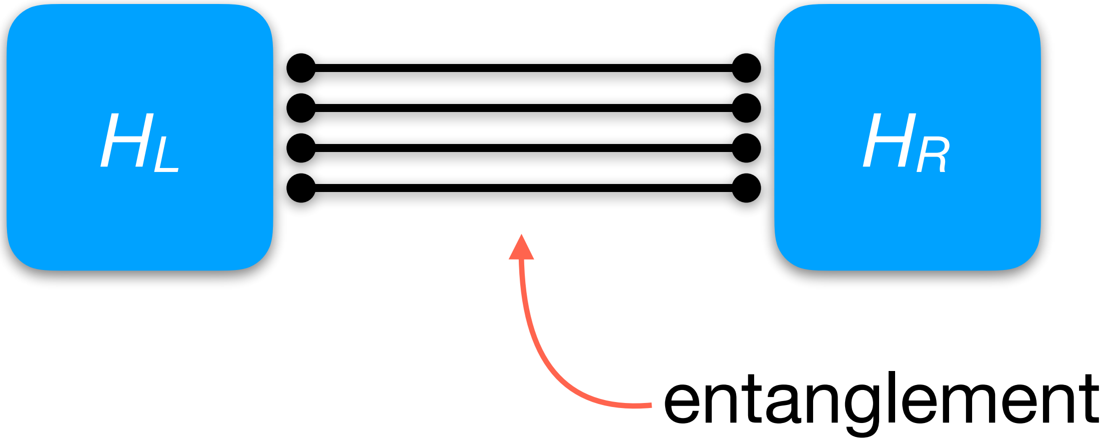

Specifically, we want the "thermofield double state" defined as

$$|\mathrm{TFD}_\beta\rangle = \frac{1}{\sqrt{Z_\beta}} \sum_n e^{-\beta E_n/2} |\bar{n}\rangle_L \otimes |n\rangle_R$$

where $|n\rangle_{L/R}$ is an eigenstate with energy E_n of $H_{L/R}$ and $|\bar{n}\rangle = \Theta|n\rangle$.

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Item 2 above has important consequence for the time-translation invariance in the TFD state:

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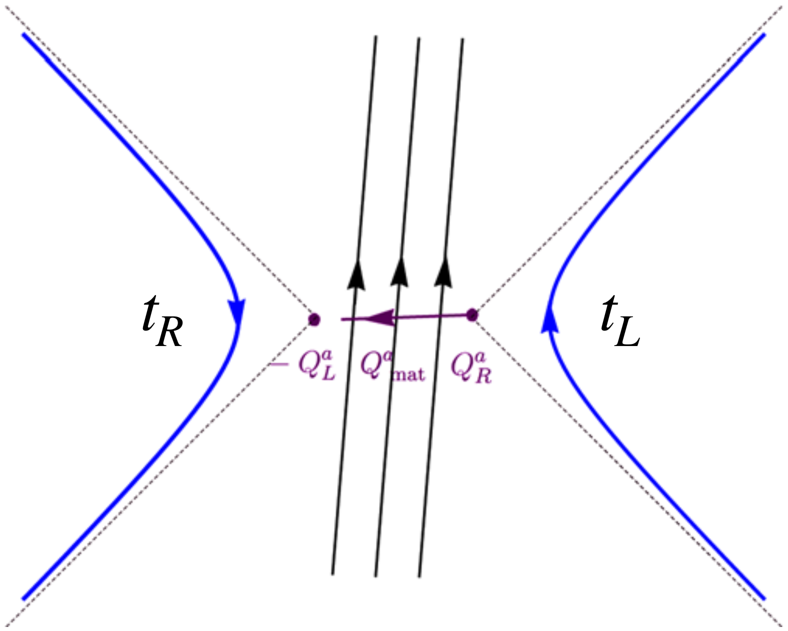

Time effectively flows in the opposite direction in two subsystems forming the TFD pair!

$$\mathcal{F}(t_1,t_2) = \langle \mathcal{O}_L(t_1)\mathcal{O}_R(t_2)\rangle_{\mathrm{TFD}} = \langle \mathcal{O}_L(t_1+t)\mathcal{O}_R(t_2-t)\rangle_{\mathrm{TFD}} = \mathcal{F}(t_1+t_2)$$

Probing OTOCs by means of conventional measurement in the TFD state

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The goal: Find a protocol to measure $F(t, t')$ without explicit backward time evolution.

Consider the following 4-point NTOC correlator evaluated in the TFD state:

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By using the definition of $|\mathrm{TFD}_\beta\rangle$ it is straightforward to show (about one page of calculation, details in arXiv:1907.01628) that

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The expression for $\tilde{F}(t,-t)$ above with density matrix insertions y^2 is called "thermally regularized OTOC" and has been argued in the literature to most directly diagnose quantum chaos.

Summary of the main result:

Certain naturally ordered 4-point correlators evaluated in the TFD entangled state map onto **regularized OTOCs**.

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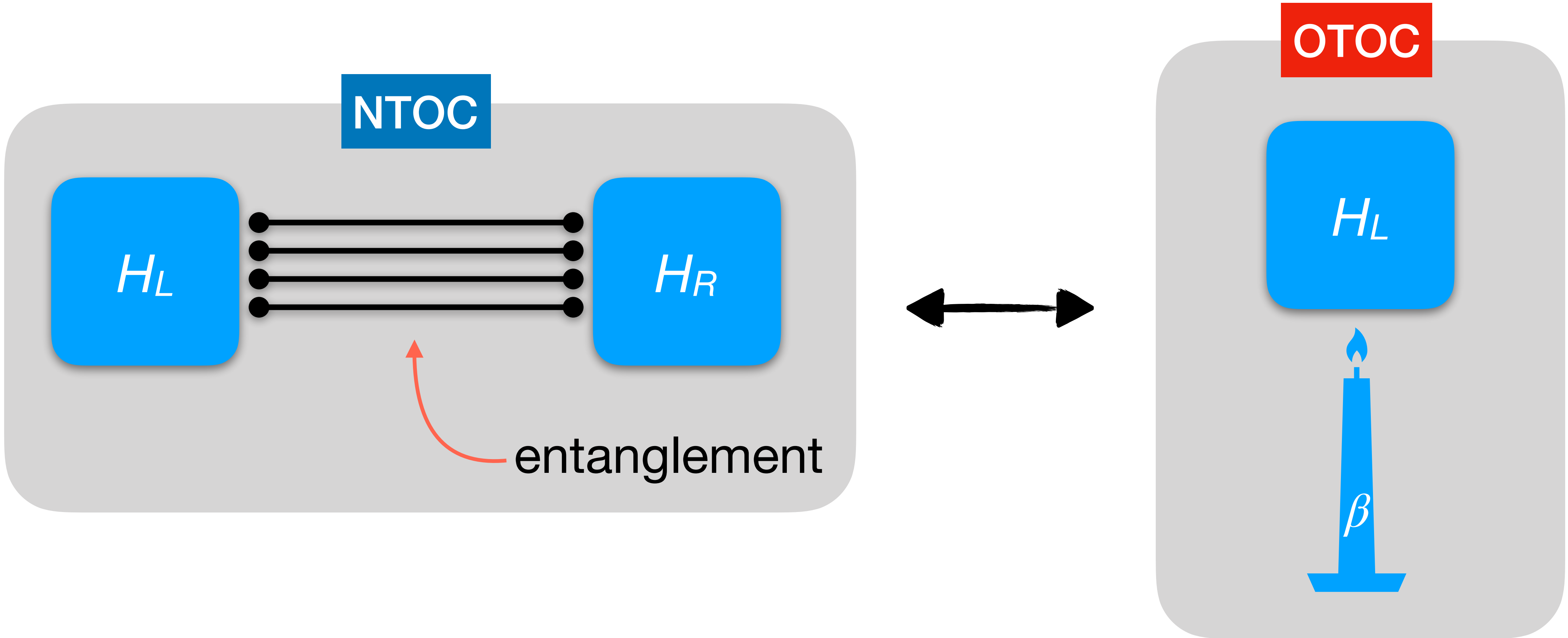

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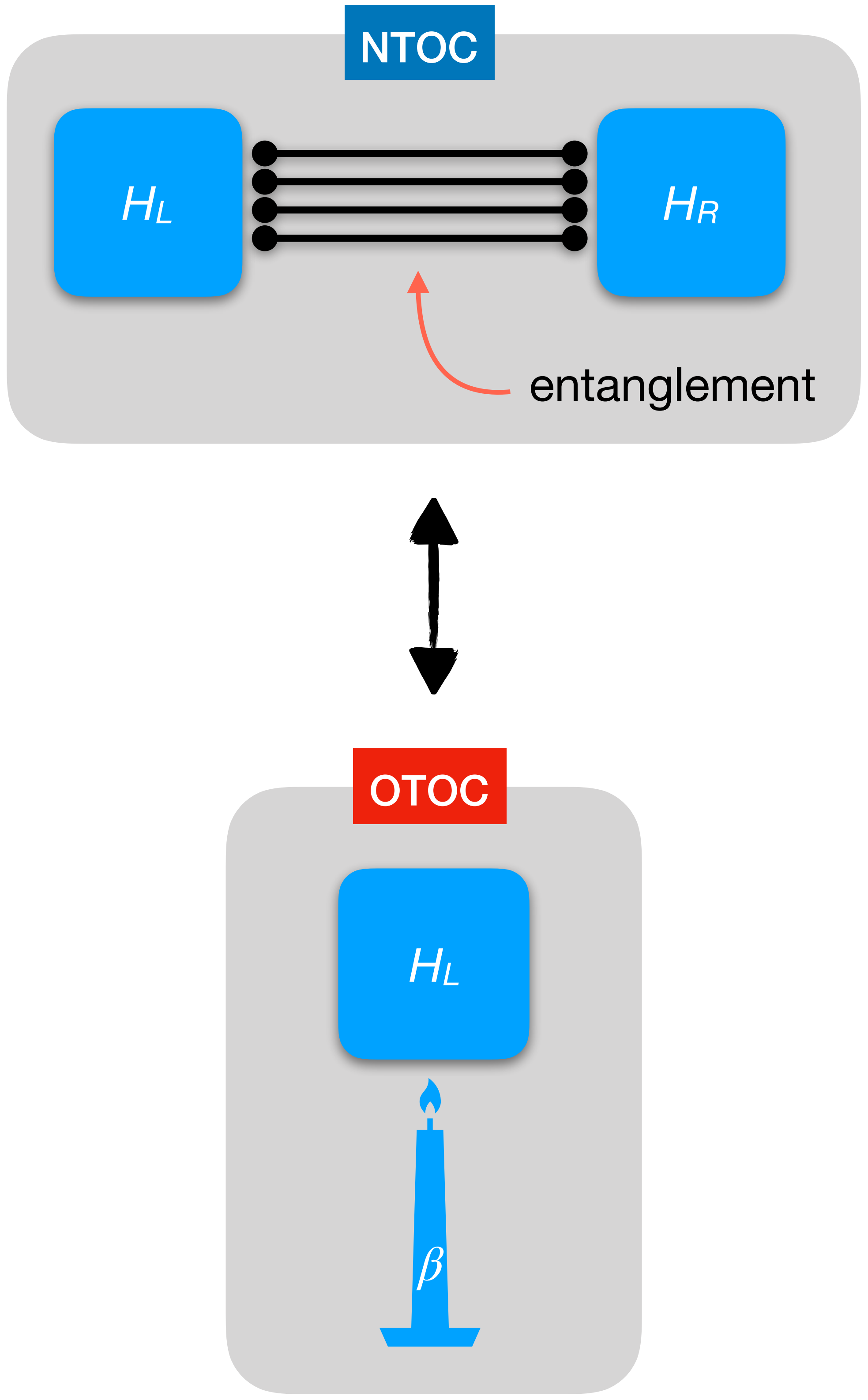

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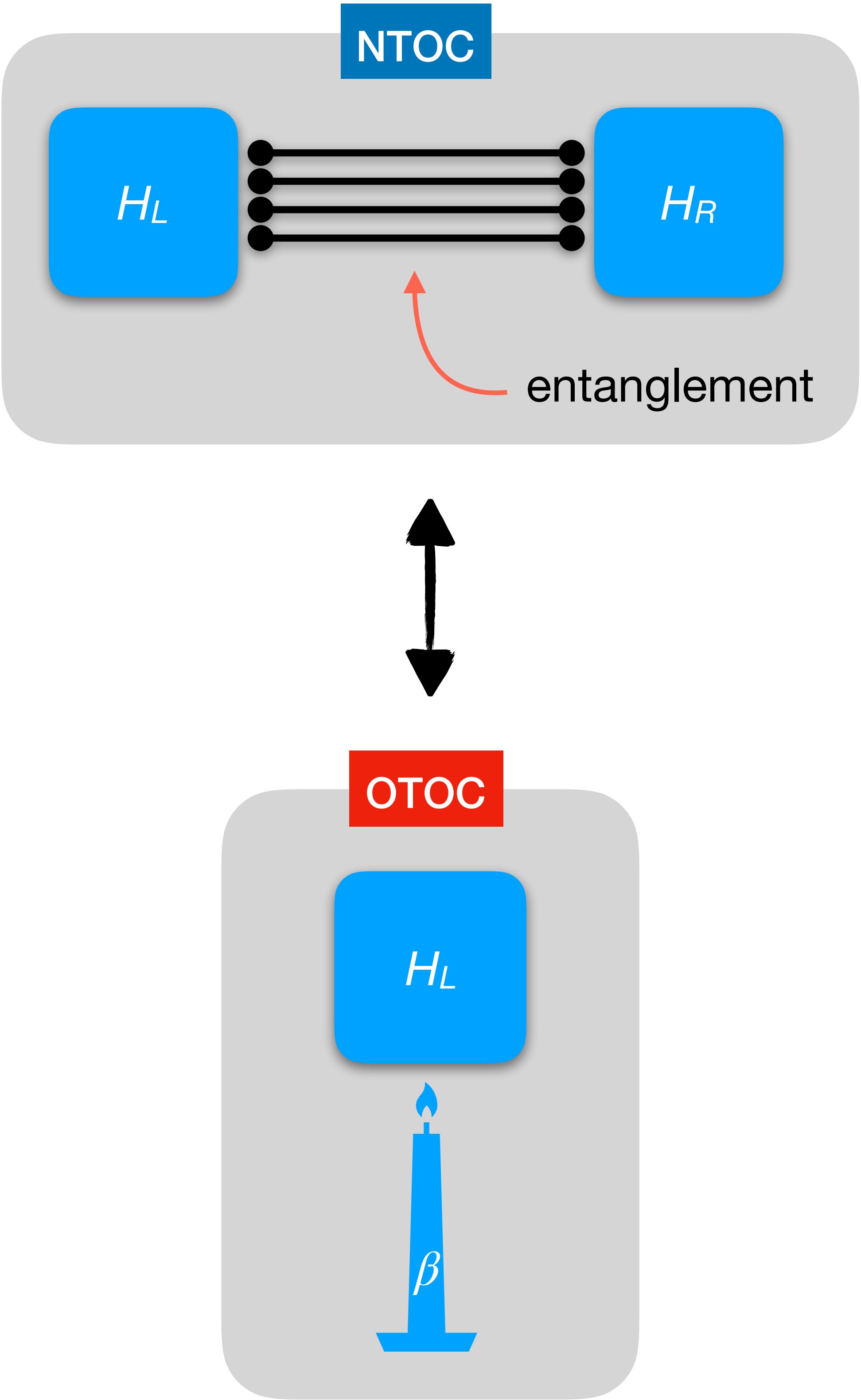

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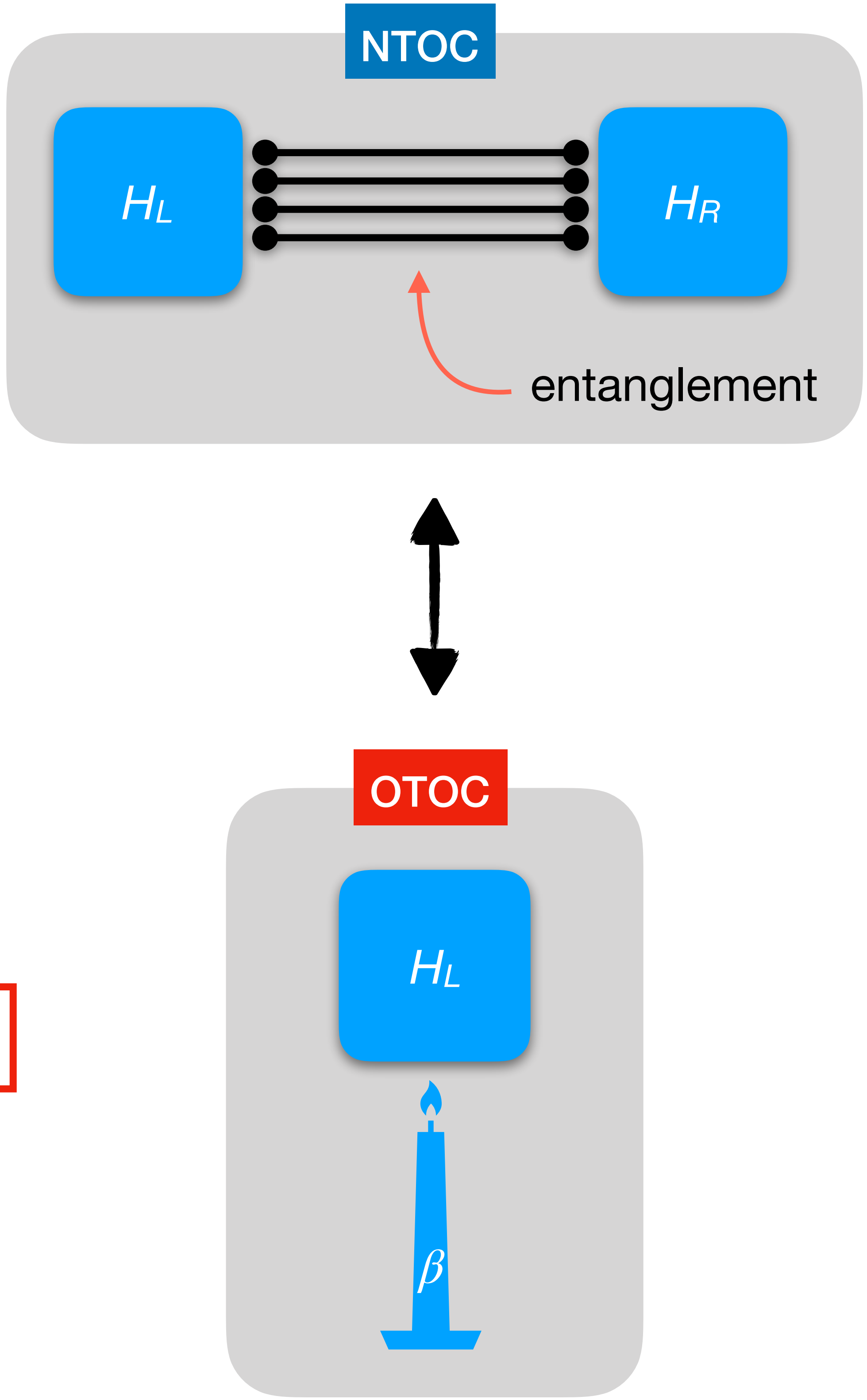

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Is it possible to efficiently prepare the TFD state?

TFD state preparation

Recent theoretical work showed how to construct a Hamiltonan H_S which admits $|\mathrm{TFD}_\beta\rangle$ as its ground state.

[W. Cottrell, B. Freivogel, D.M. Hofman, and S.F. Lokhande, J. High Energy Phys. 2019, 58 (2019)]

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$$H_S = H_L + H_R + H_I \quad \text{with} \quad H_I = i\mu \sum_j \mathcal{O}_L^j \mathcal{O}_R^j$$

has a ground state that is **very close** to $|\mathrm{TFD}_\beta\rangle$, that is $\langle \Psi_0 | \mathrm{TFD}_\beta \rangle \simeq 1$.

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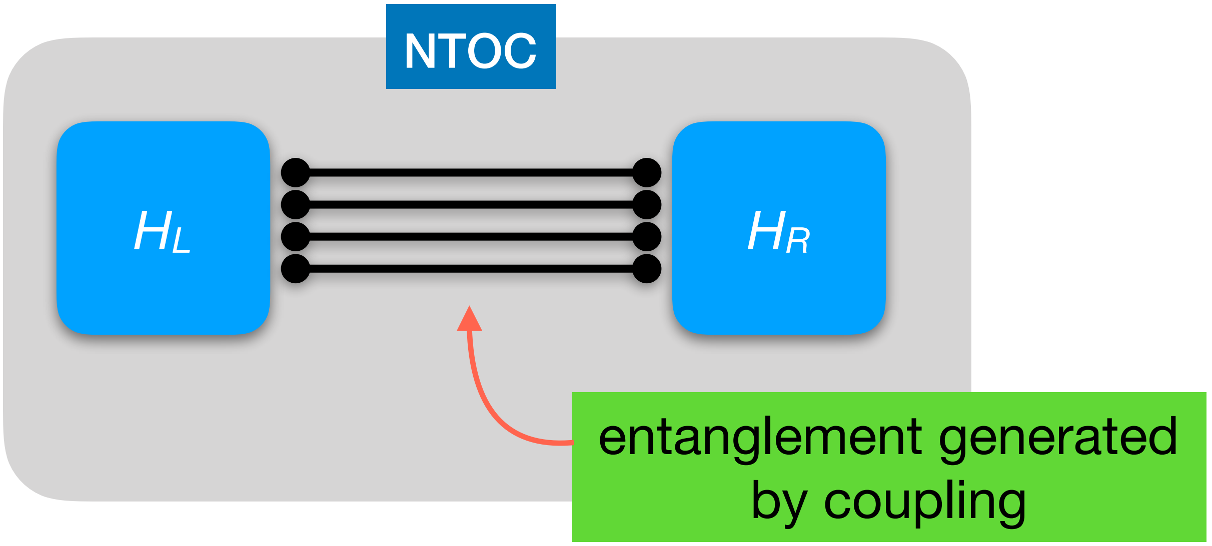

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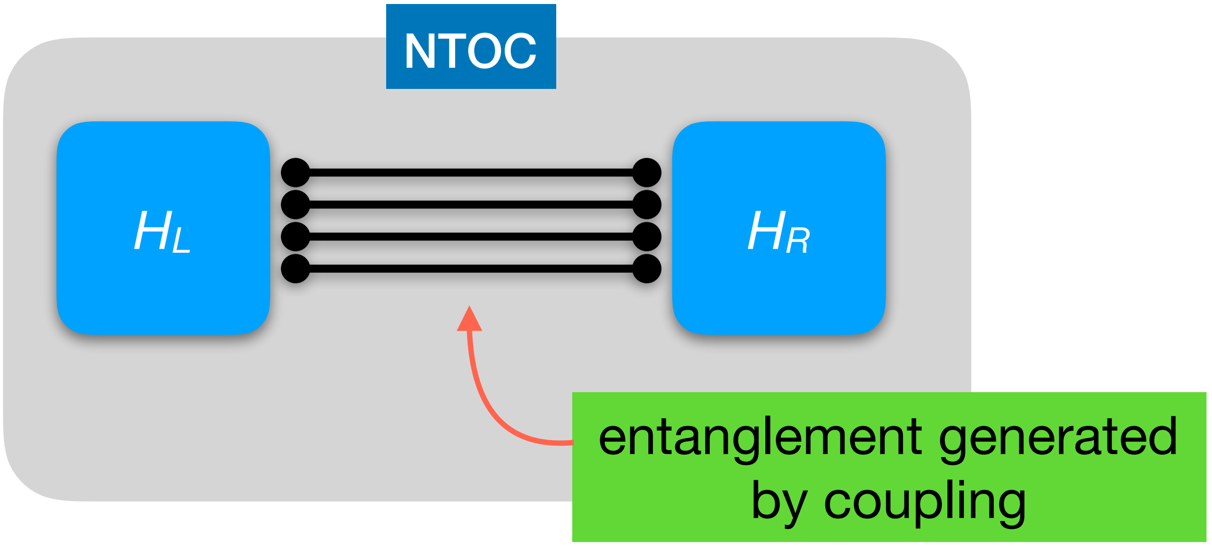
Strategy to prepare TFD state:

1. Engineer a system with Hamiltonian H_S
2. Cool the system to its ground state
   $|\Psi_0\rangle \simeq |\mathrm{TFD}_\beta\rangle$



Measurement strategy

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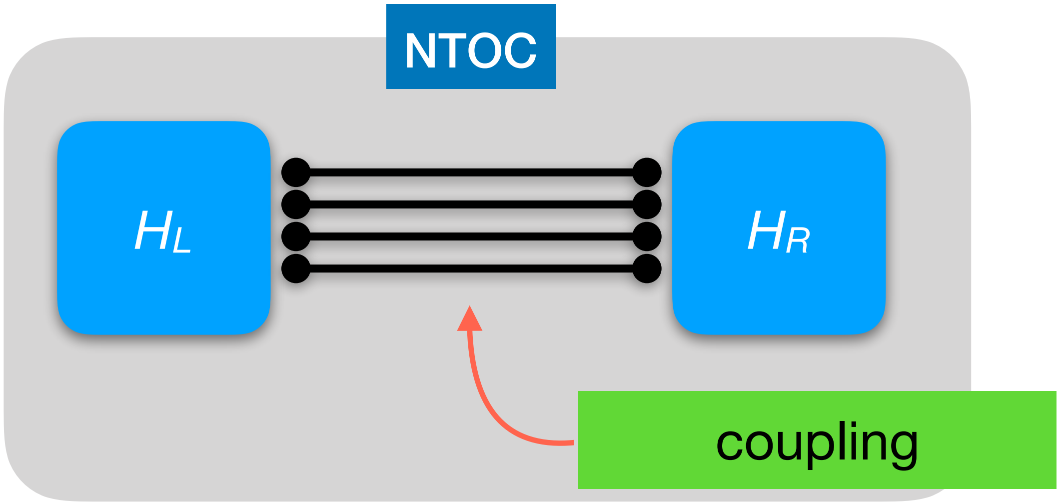

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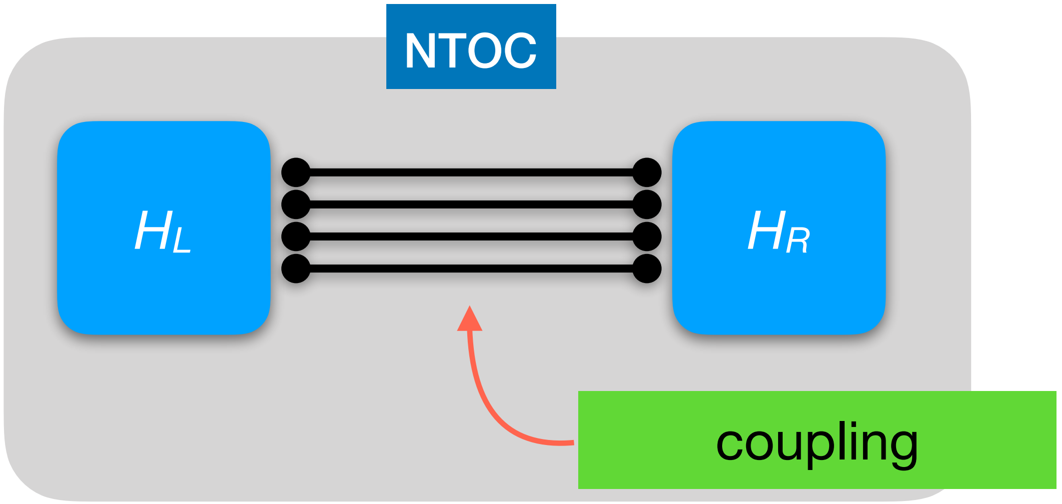

It turns out to be sufficient to measure an ordinary two-sided correlator $iG^{\text{ret}}_{LR}(t,t') = \theta(t-t')\langle\{V_L(t), V_R(t')\}\rangle_0$ in the TFD ground state.

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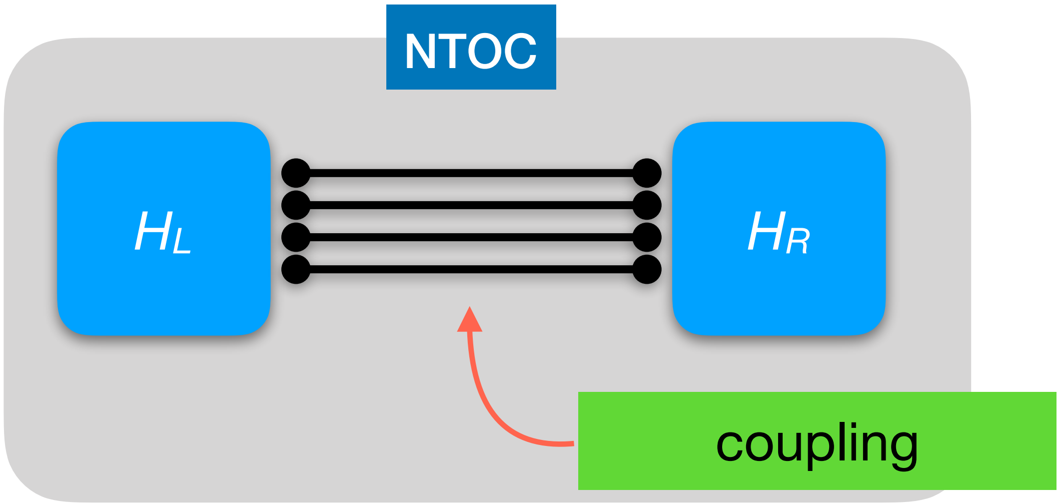

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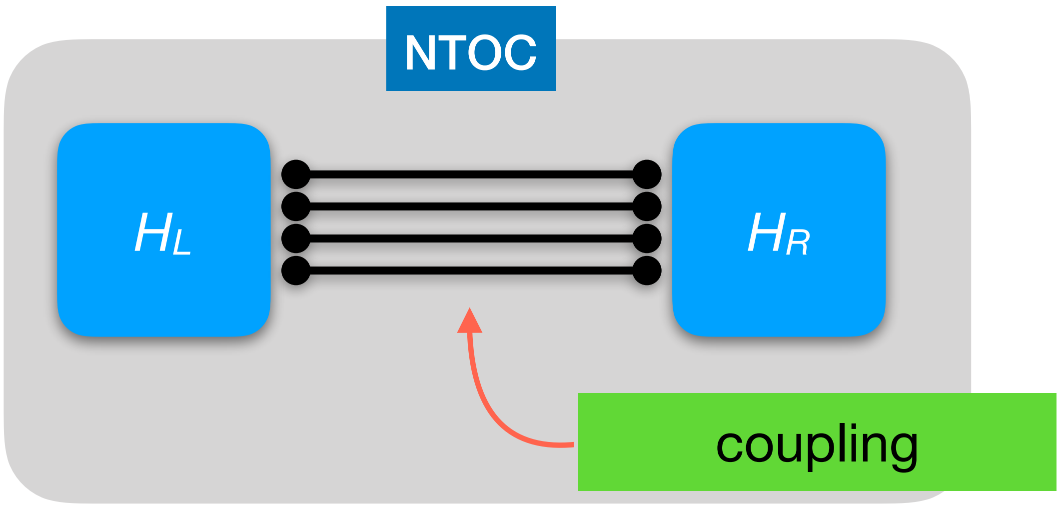

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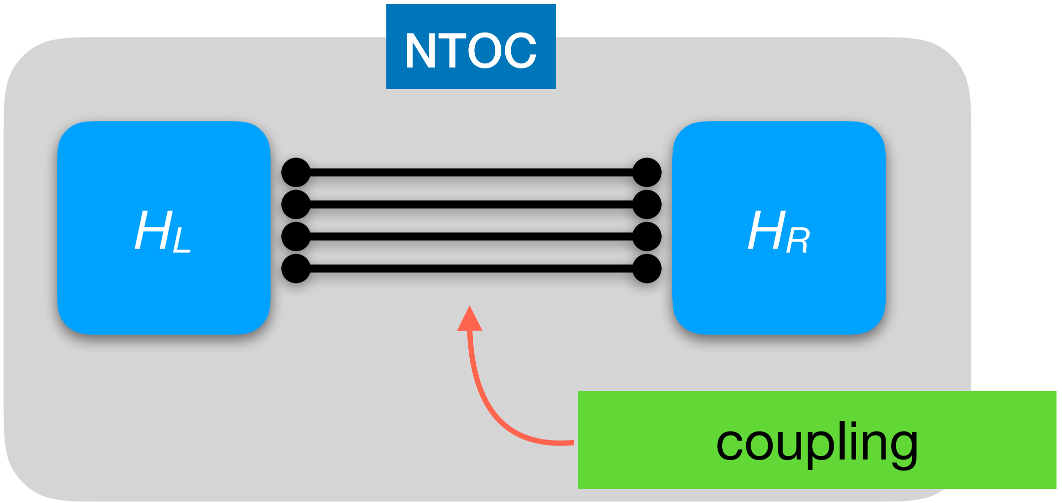

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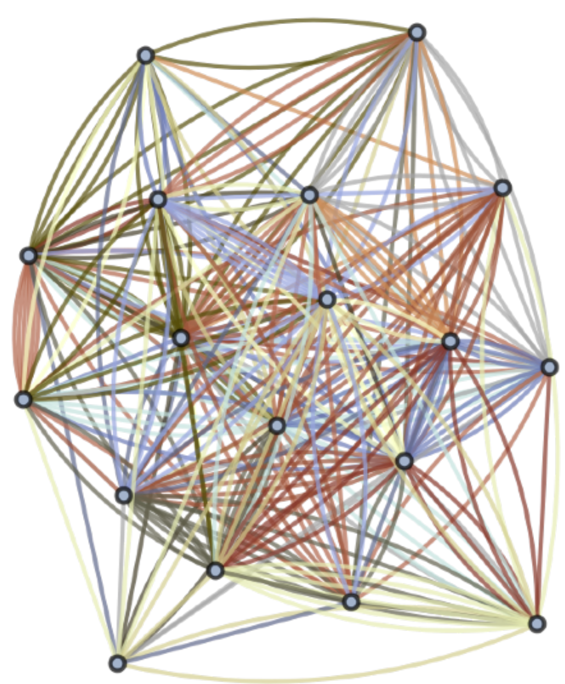
We thus expect $iG^{\text{ret}}_{LR}(t,-t) \simeq A + Be^{2\lambda_L t}$

Example: Black holes, wormholes and the Sachdev-Ye-Kitaev model



Sachdev-Ye-Kitaev (SYK) Model review:



A toy model that is both a black hole and a "strange metal."

$$\mathcal{H}_{\rm SYK} = \frac{1}{4!} \sum_{i,j,k,l} J_{ijkl} \chi_i \chi_j \chi_k \chi_l$$

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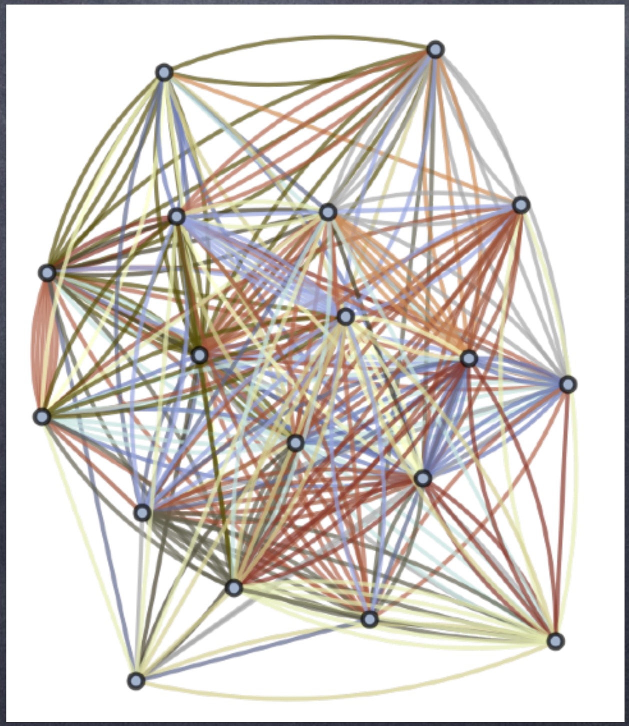
A system of N (Majorana) fermions with random all-to-all interactions



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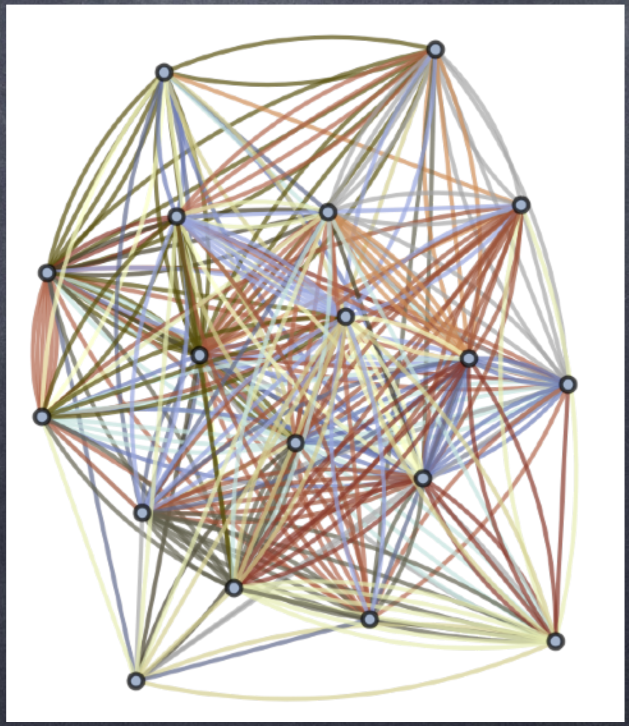

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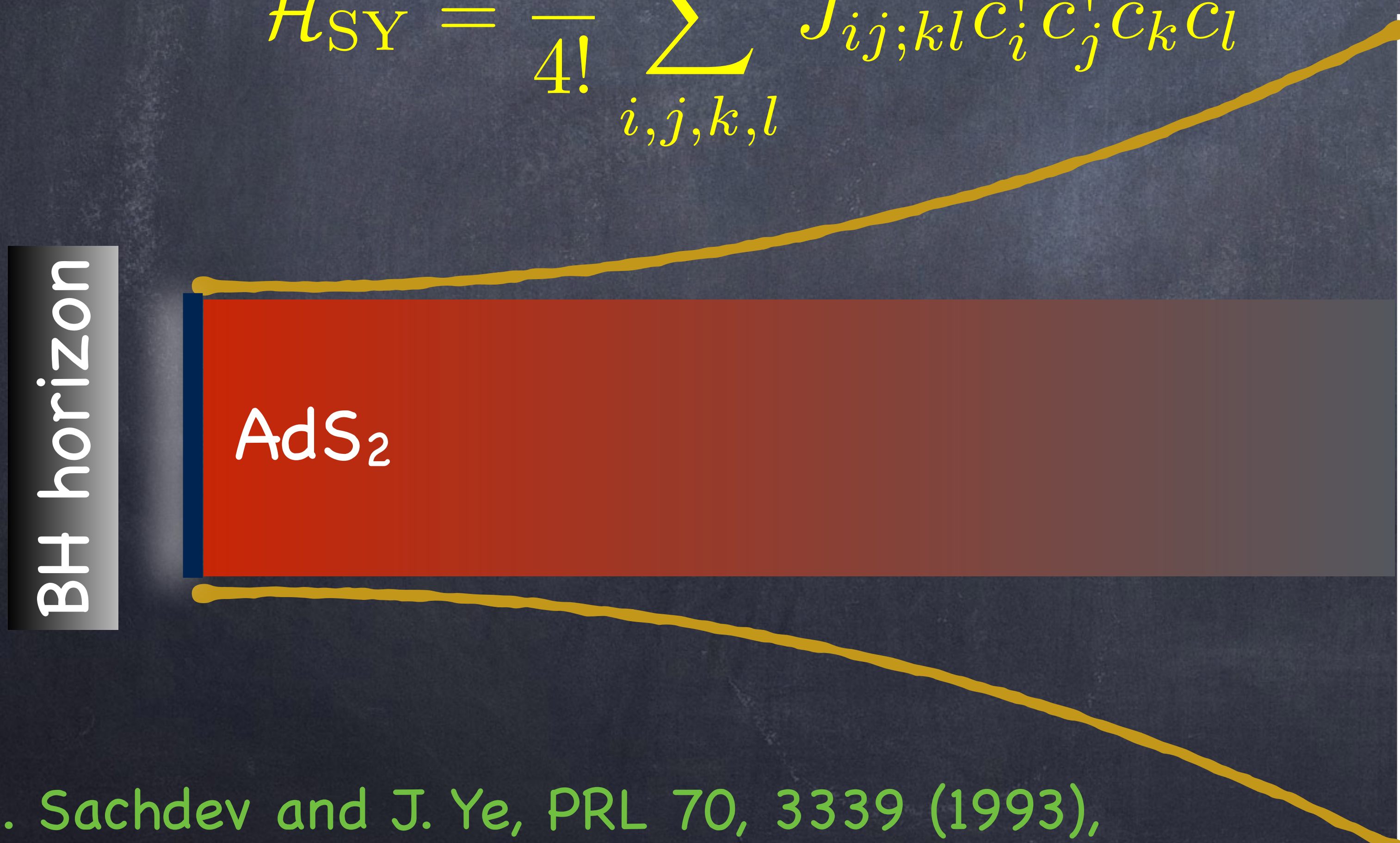

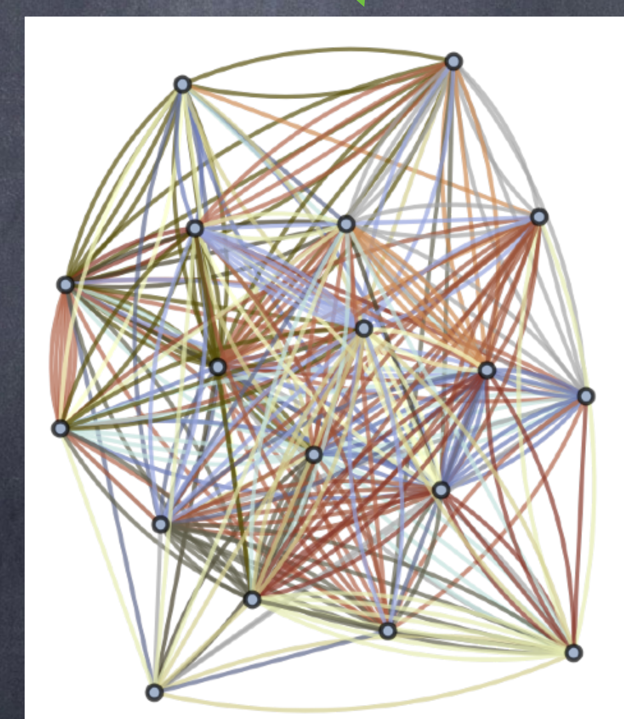

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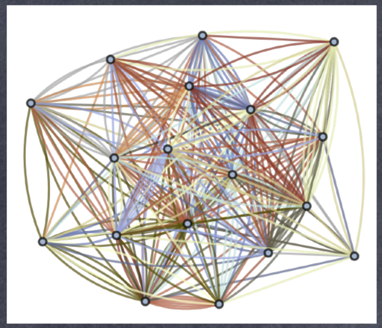

SYK versus black holes

- SYK model exhibits extensive T=0 entropy $S_{SYK} \sim N$.
- Emergent conformal invariance at low energies
- SYK model is maximally chaotic: its Lyapunov exponent saturates the Maldacena-Shenker-Stanford chaos bound $\lambda_L \leq 2\pi T$

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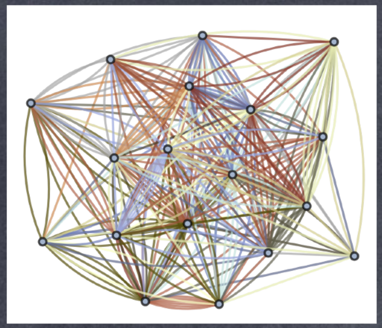

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Detailed considerations that involve effective action calculations, thermodynamics and scrambling properties confirm this expectation.
[J. Maldacena and D. Stanford, Phys. Rev. D 94, 106002 (2016),
S. Sachdev, PRX 5, 041025 (2015)]

Maldacena-Qi model and the TFD

$$H = H_L^{\text{SYK}} + H_R^{\text{SYK}} + i\mu \sum_j \chi_L^j \chi_R^j$$

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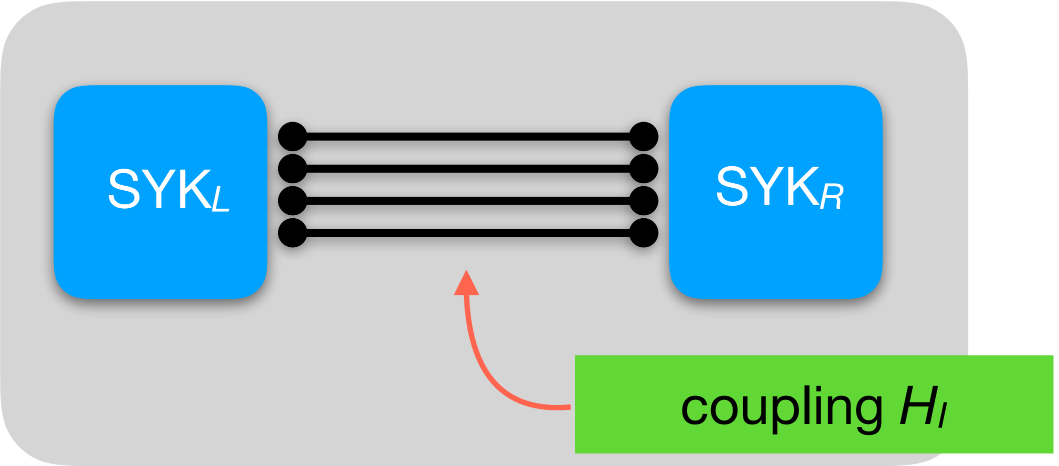

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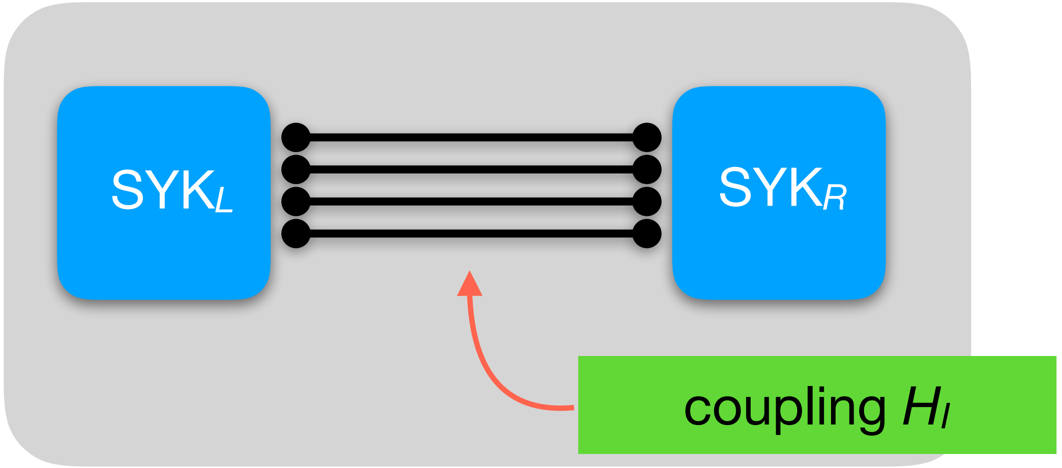

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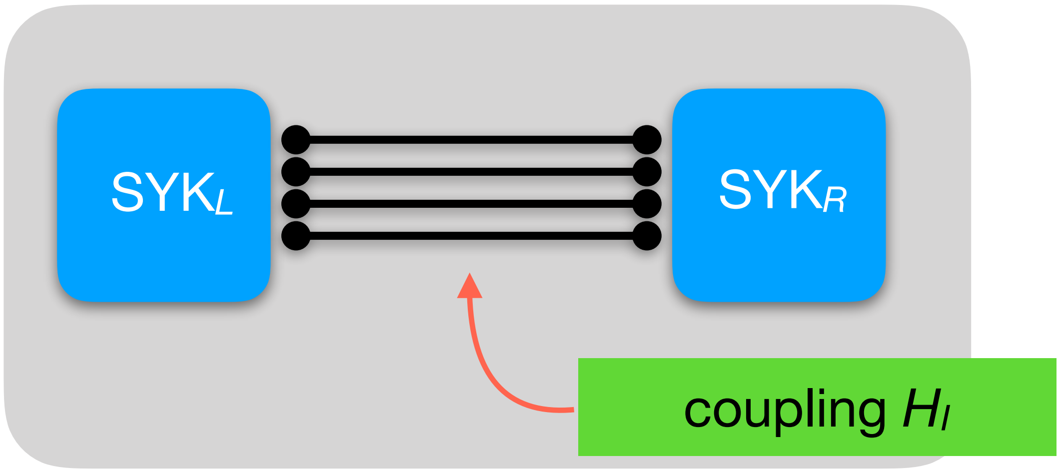

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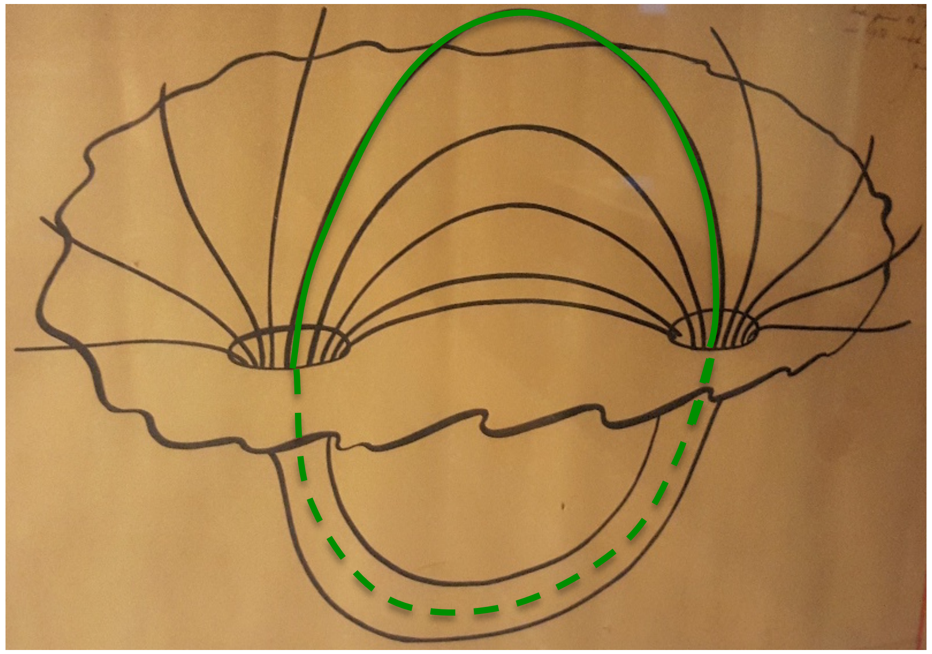

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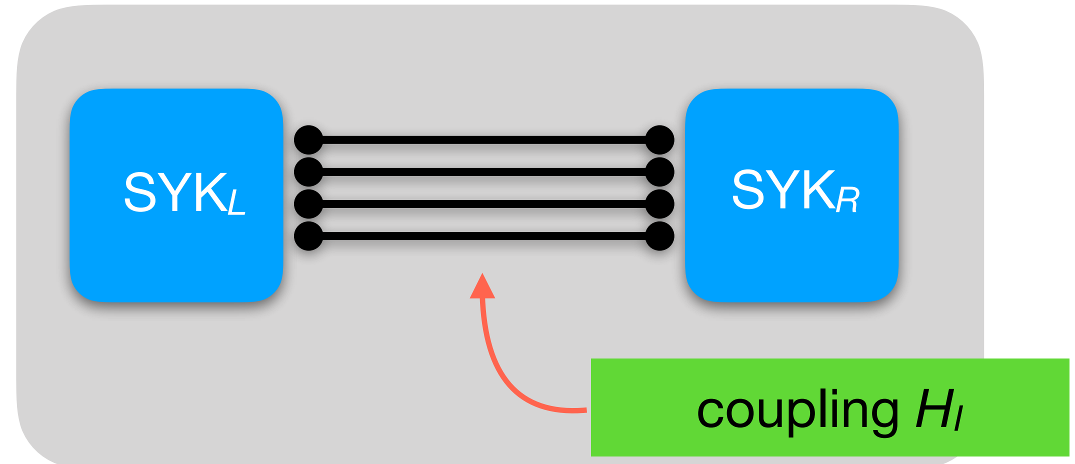

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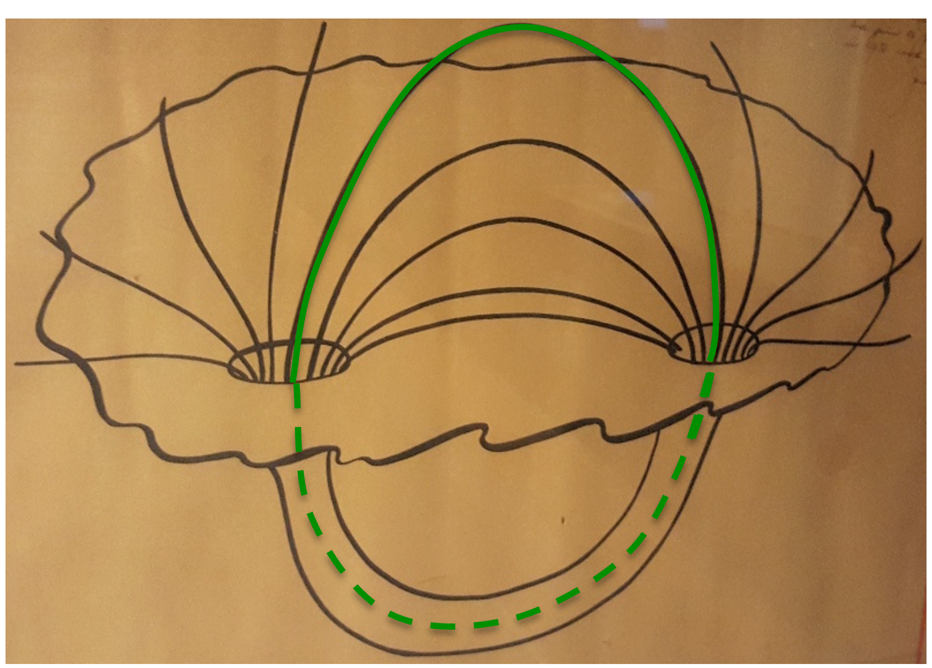

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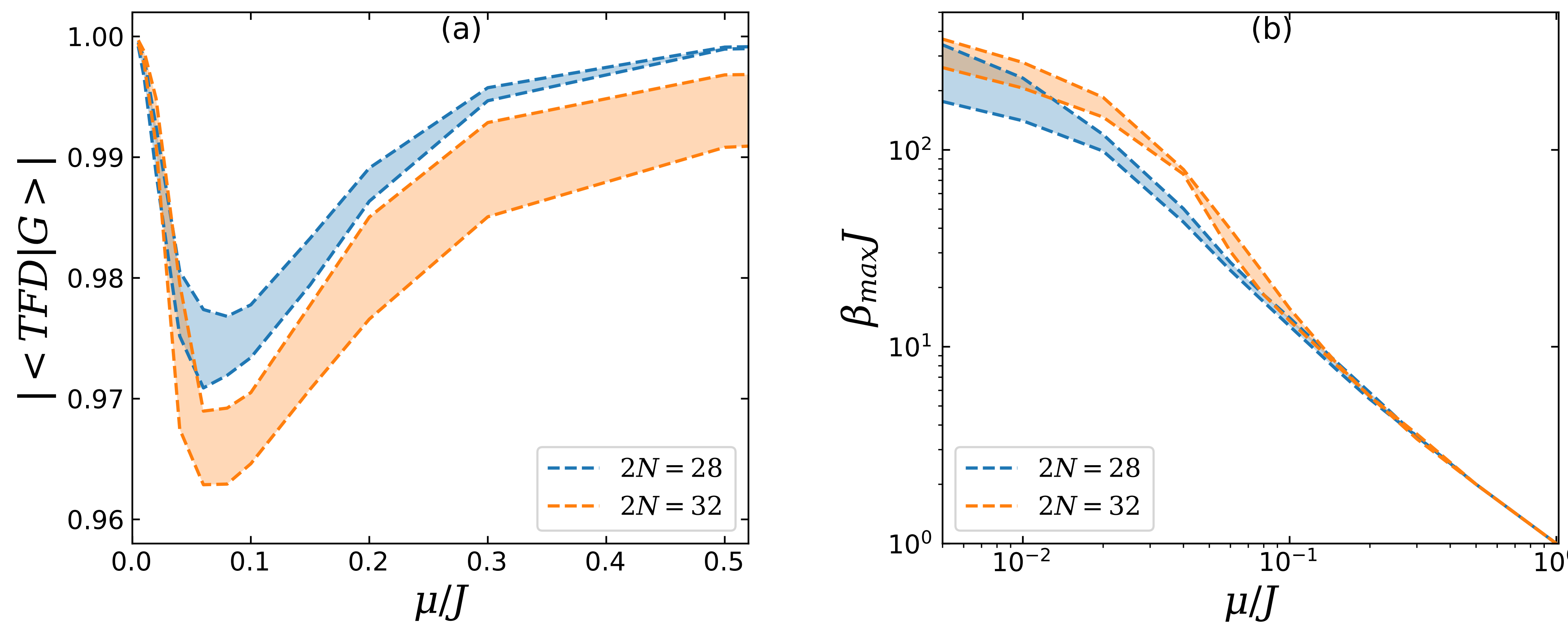

Results of numerical exact diagonalization arXiv:1907.01628

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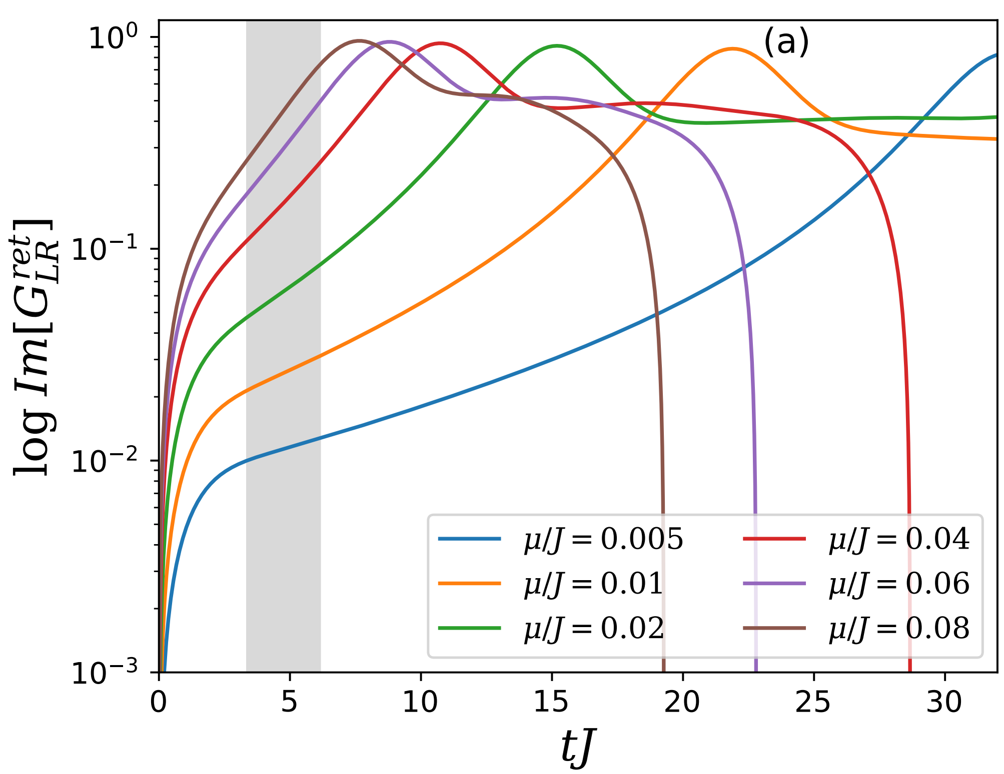

Results of large-*N* calculation arXiv:1907.01628

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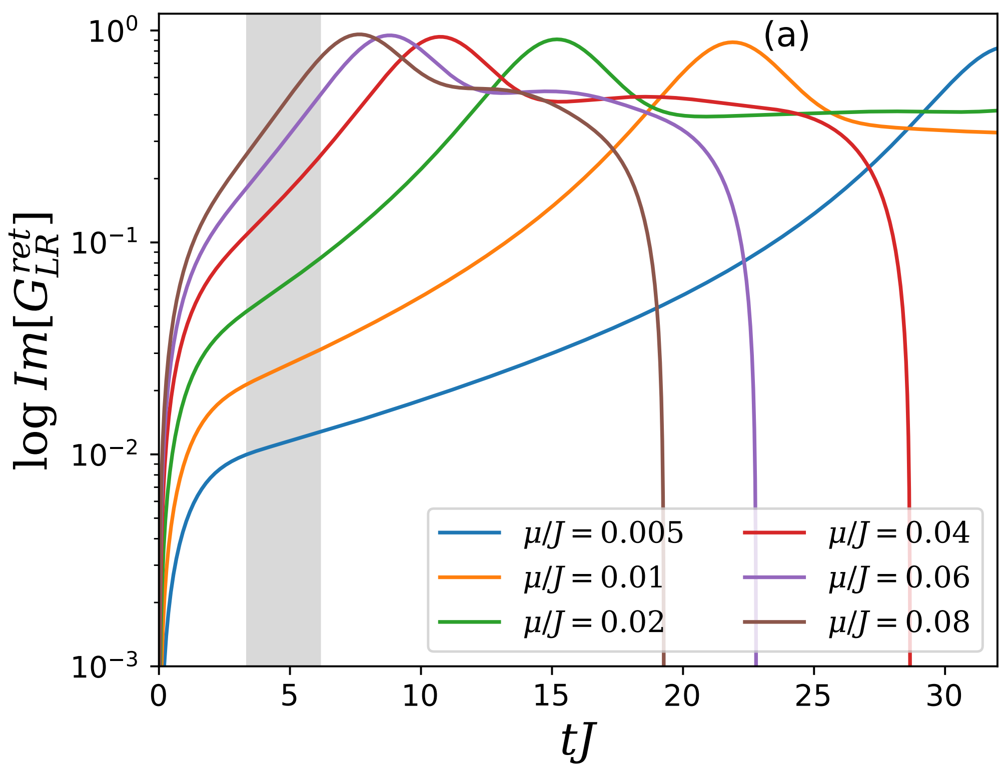
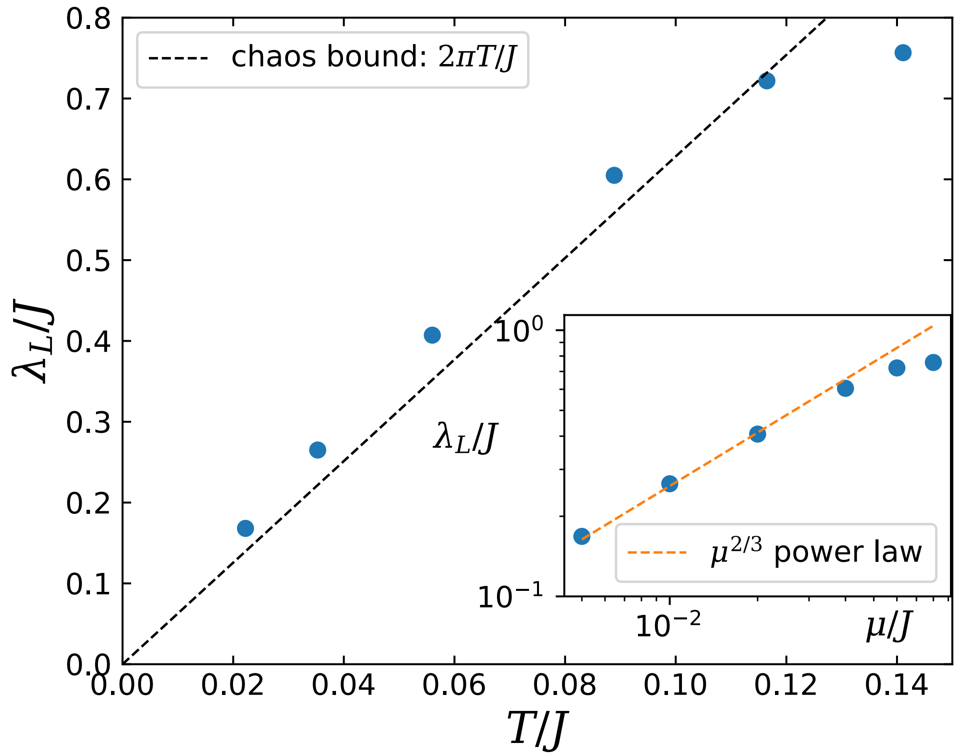


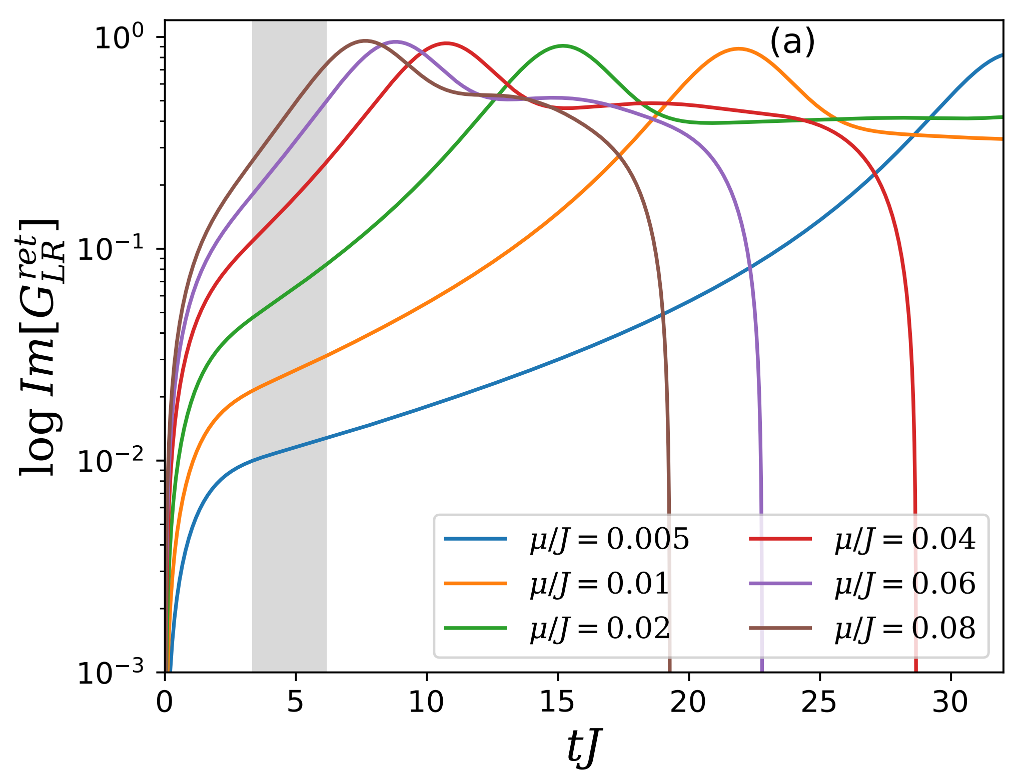


Results of large-*N* calculation arXiv:1907.01628

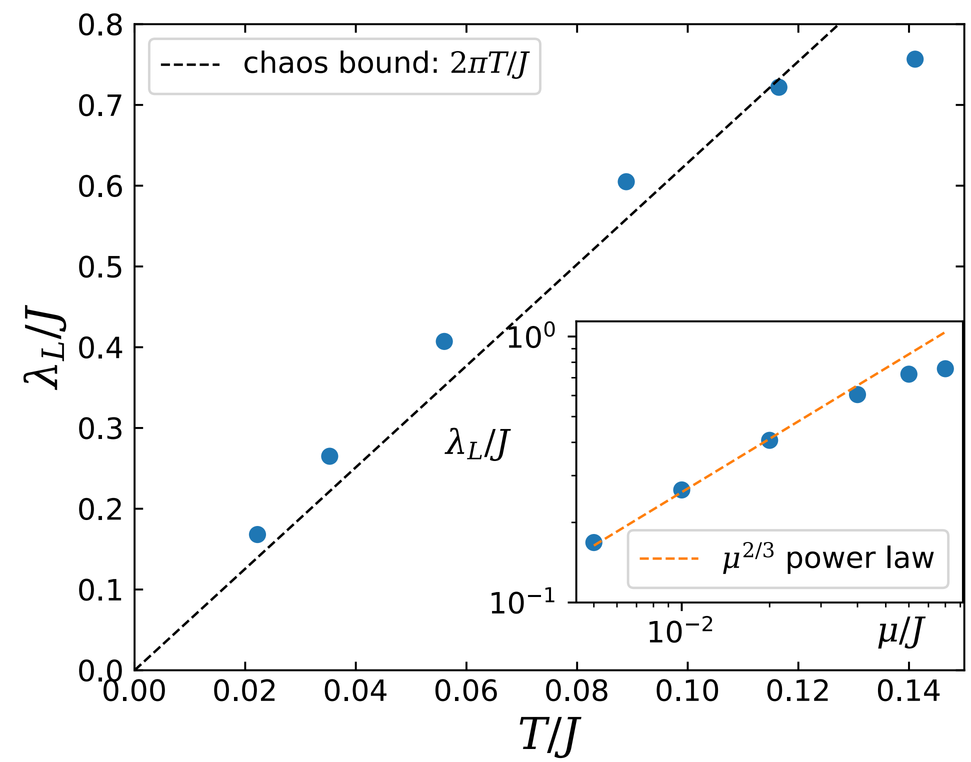
OTOC and Lyapunov exponent

We argued previously that LR correlator in TFD ground state contains OTOC contribution, i.e. $iG^{\text{ret}}_{LR}(t,-t) \simeq A + Be^{2\lambda_L t}$. For Maldacena-Qi we find:

$$iG^{\text{ret}}_{LR}(t,-t) = \frac{\theta(t)}{N}\sum_j \langle\{\chi^j_L(t), \chi^j_R(-t)\}\rangle$$
$$\simeq \frac{4\mu}{N}\sum_{j,k}\int_0^t ds \ \text{Re}\ \text{tr}[\chi^j(t+s)\chi^k(0)y^2\chi^j(t-s)\chi^k(0)y^2] \ + \ \text{NTOC}$$



Results of large-N calculation arXiv:1907.01628



Consistent with maximal chaos $\lambda_L = 2\pi T$

Possible experimental realizations

We want to realize the Maldacena-Qi model $H = H_L^{\mathrm{SYK}} + H_R^{\mathrm{SYK}} + i\mu \sum_j \chi_L^j \chi_R^j$

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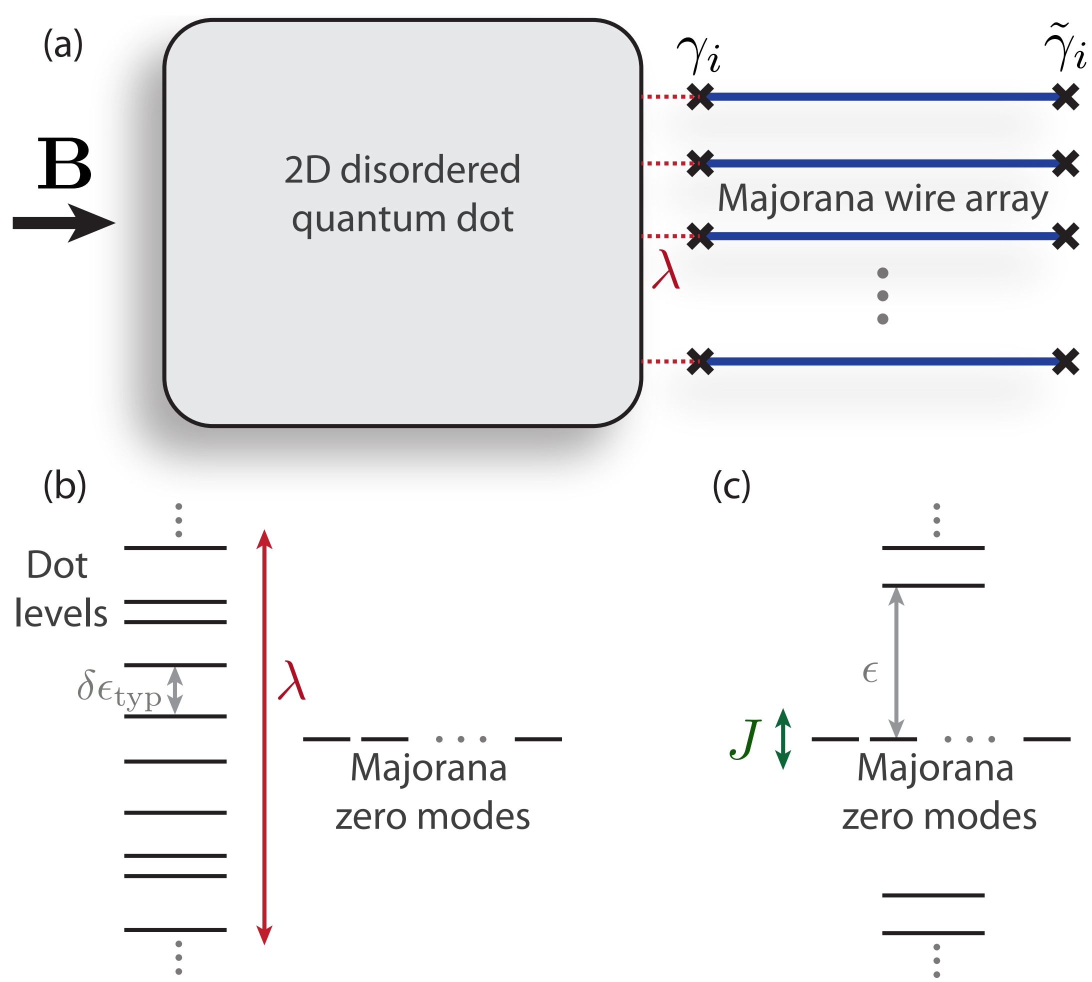
SYK model proposed realizations:



[Chew, Essin and Alicea, PRB 2017]

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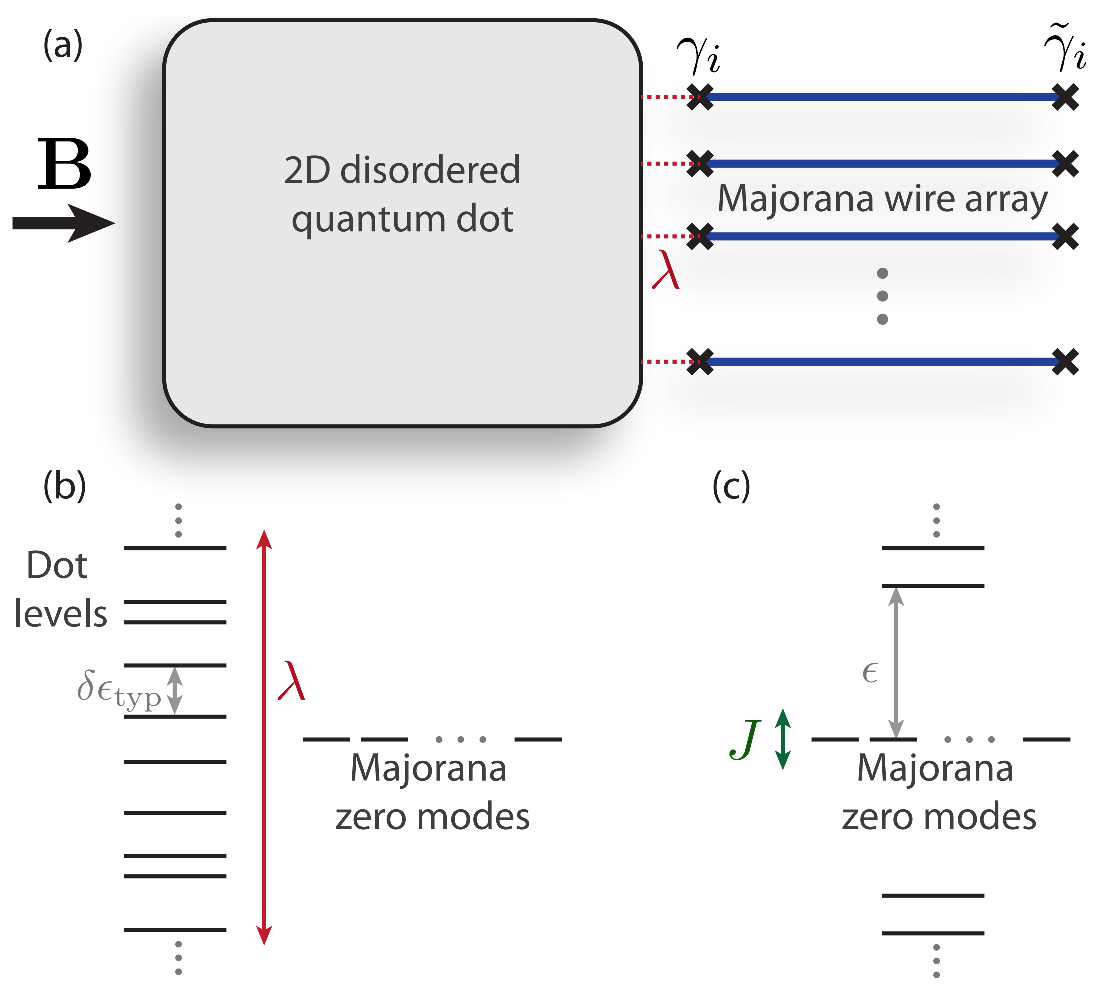

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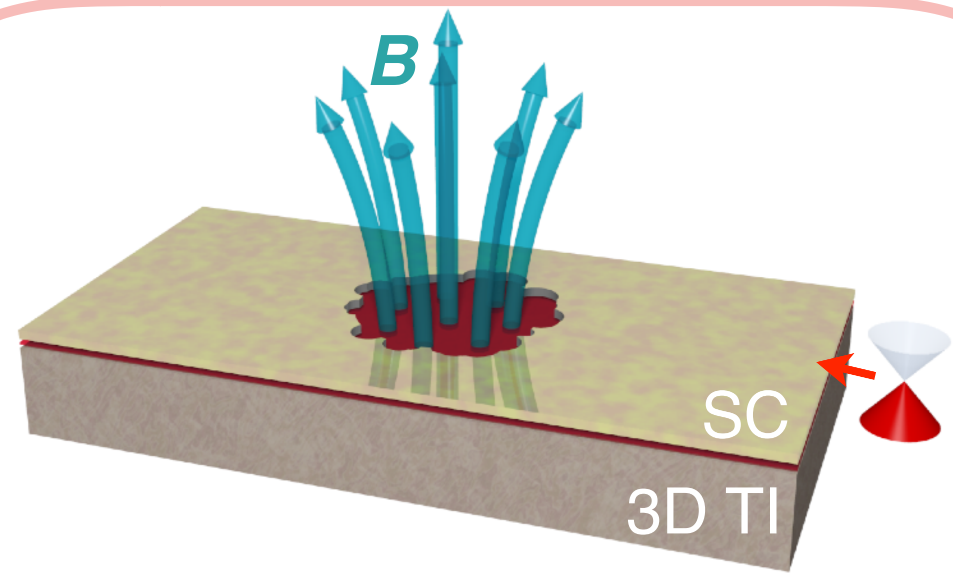

[Pikulin and Franz, PRX 2017]

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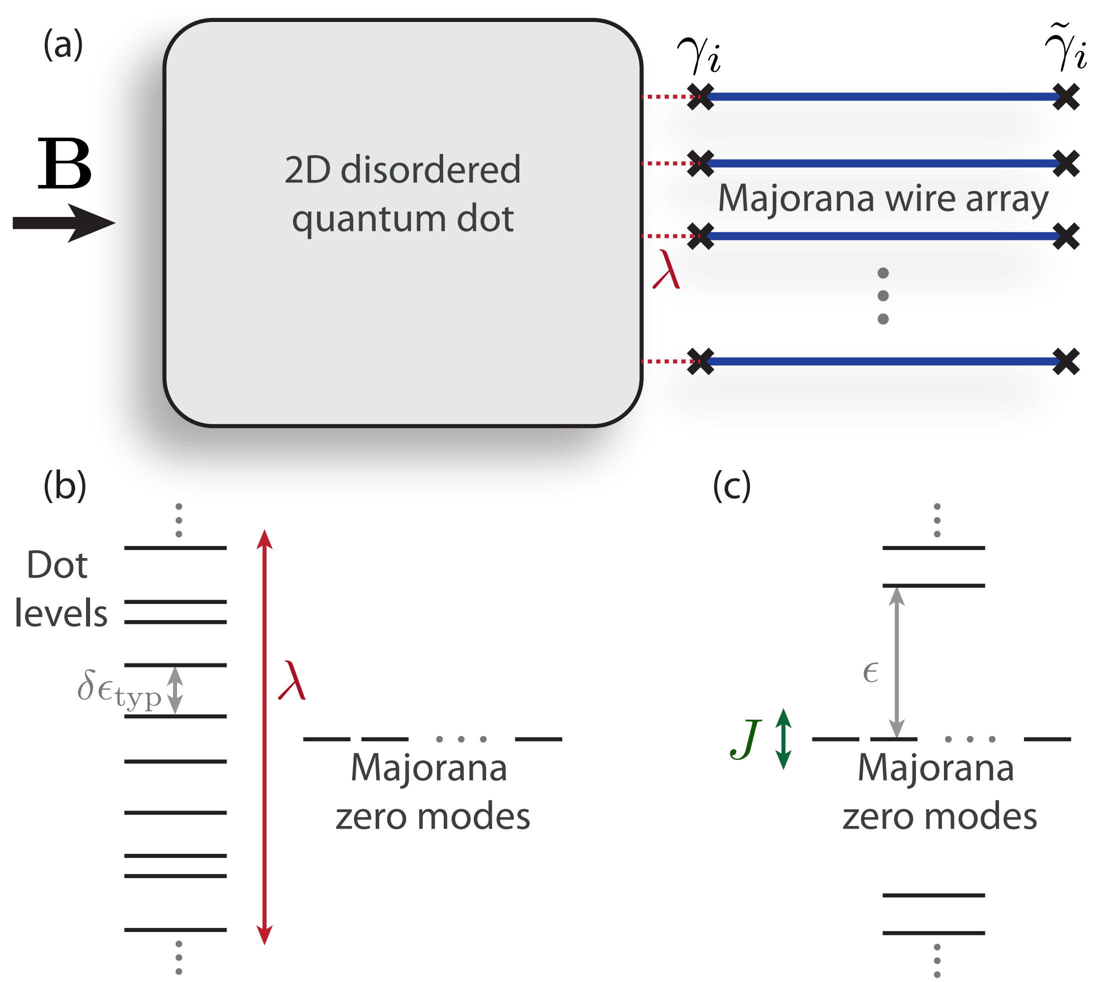

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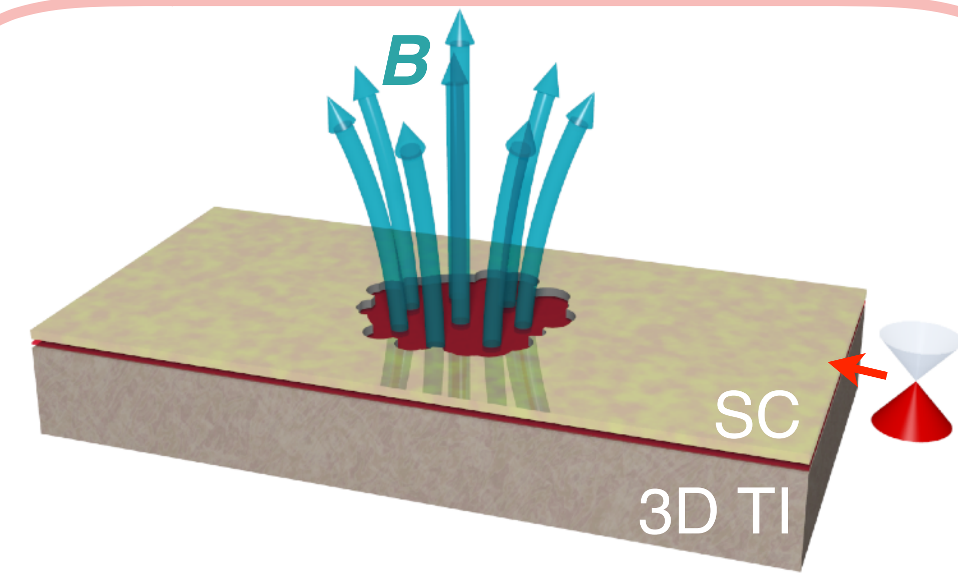
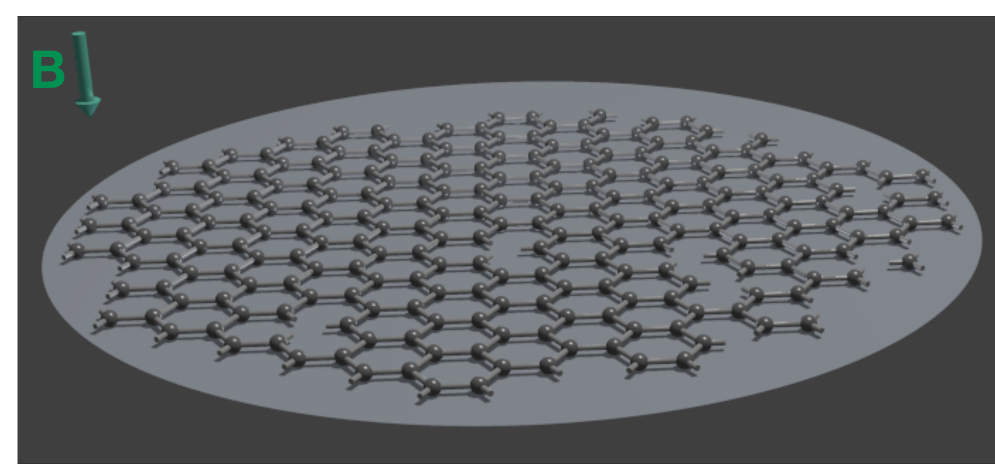


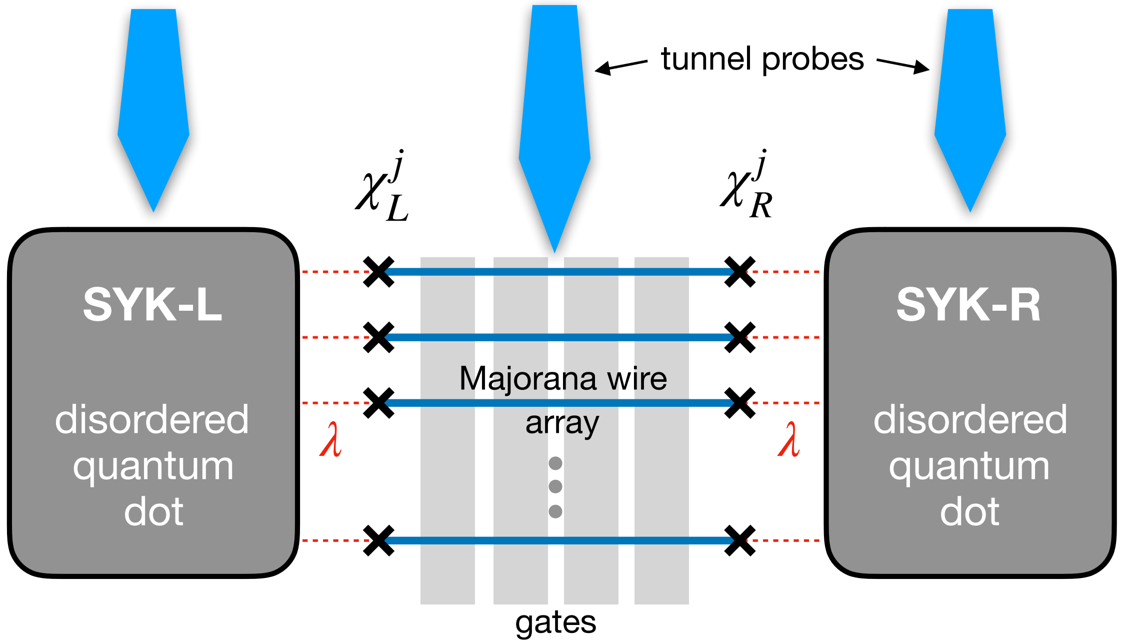
[Pikulin and Franz, PRX 2017]



[Chen, Ilan, de Juan, Pikulin and Franz, PRL 2018]

Two identical quantum dots bridged by Majorana wires could approximate the Maldacena-Qi model

$$H = H_L^{\mathrm{SYK}} + H_R^{\mathrm{SYK}} + i\mu \sum_j \chi_L^j \chi_R^j$$



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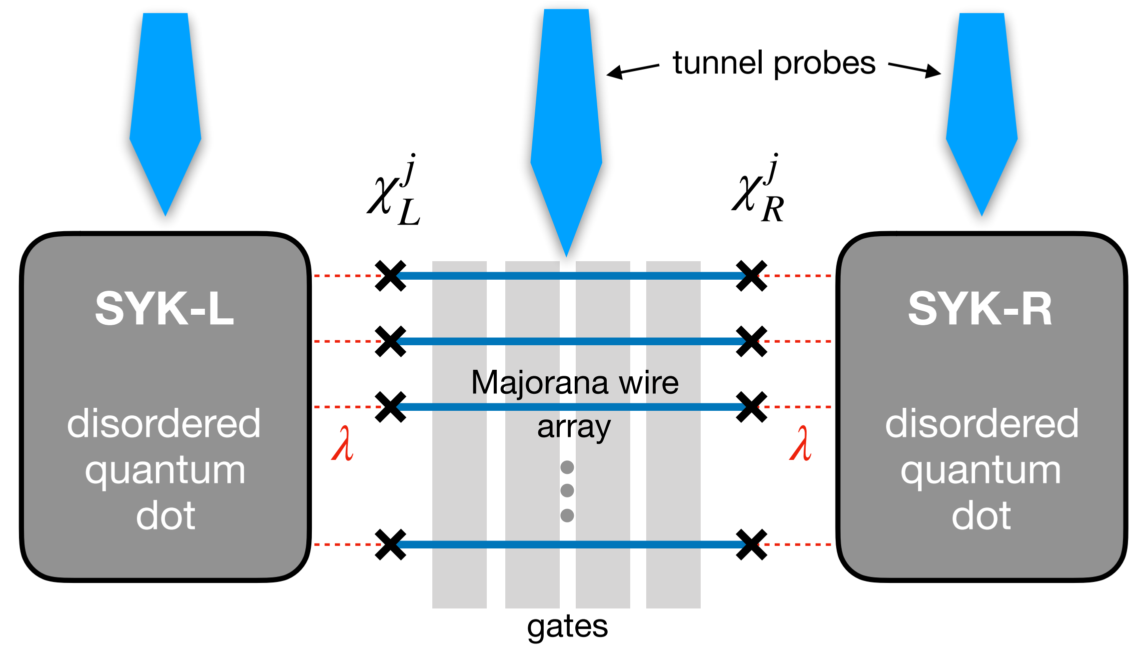

- Tunnel probes can be used to probe electron spectral function $\rho_x(\omega)$ in each wire which is related to the LR Majorana correlator. We find:

$$iG_{LR}^{\mathrm{ret}}(t) \simeq K_x \theta(t) \int_{-\infty}^{\infty} d\omega \; \rho_x(\omega) \sin \omega t \, .$$

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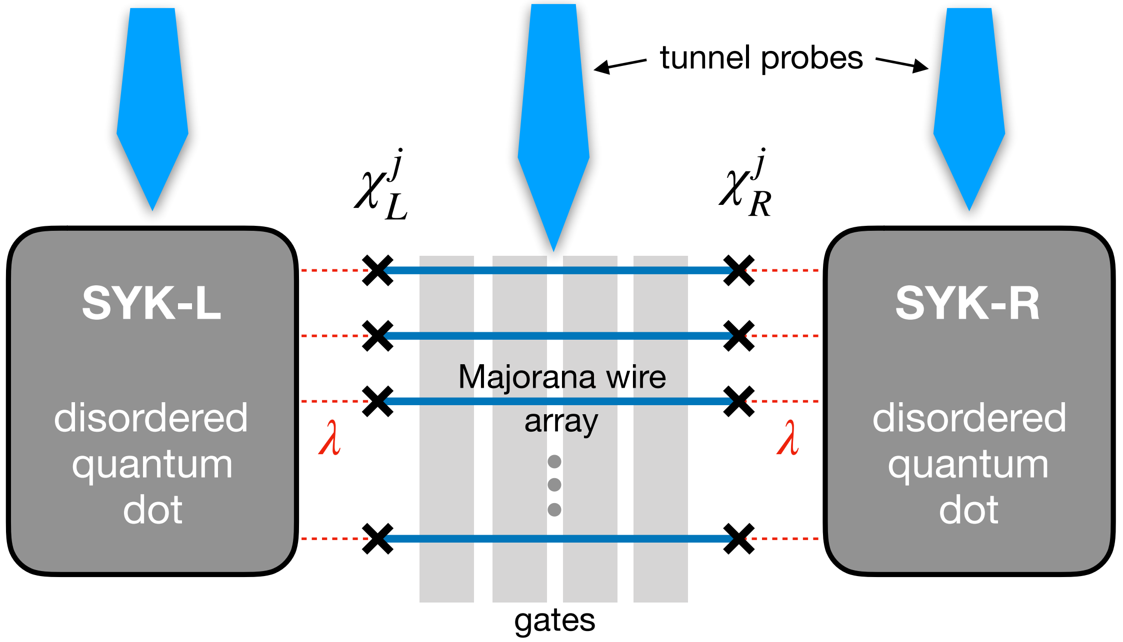

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Tunneling conductance experiment in this setup therefore gives access to the Lyapunov chaos exponent through $iG_{LR}^{\rm ret}(t) \simeq A + Be^{\lambda_L t}$

Conclusions

- Diagnosing quantum chaos traditionally requires backward time evolution which is hard or impossible in complex many-body systems
- We proposed a new protocol for chaos detection which replaces a complicated measurement scheme by a simple measurement on a specific entangled state
- The challenge now is to fabricate two identical copies of an interesting system that are weakly coupled to one another
- A simple spectroscopic measurement then yields the chaos exponent λ_L

arXiv:1907.01628