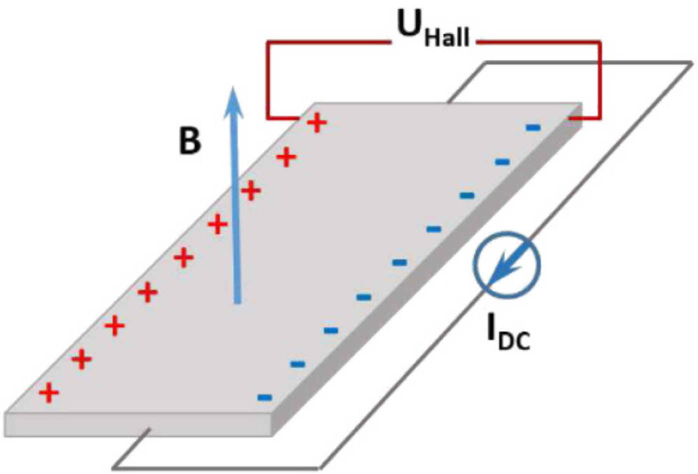
New Variations on Hall Effect

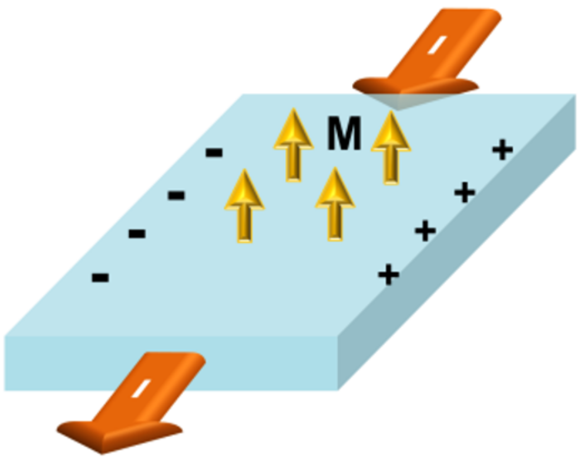
Liang Fu

Hall Effect
(1879)

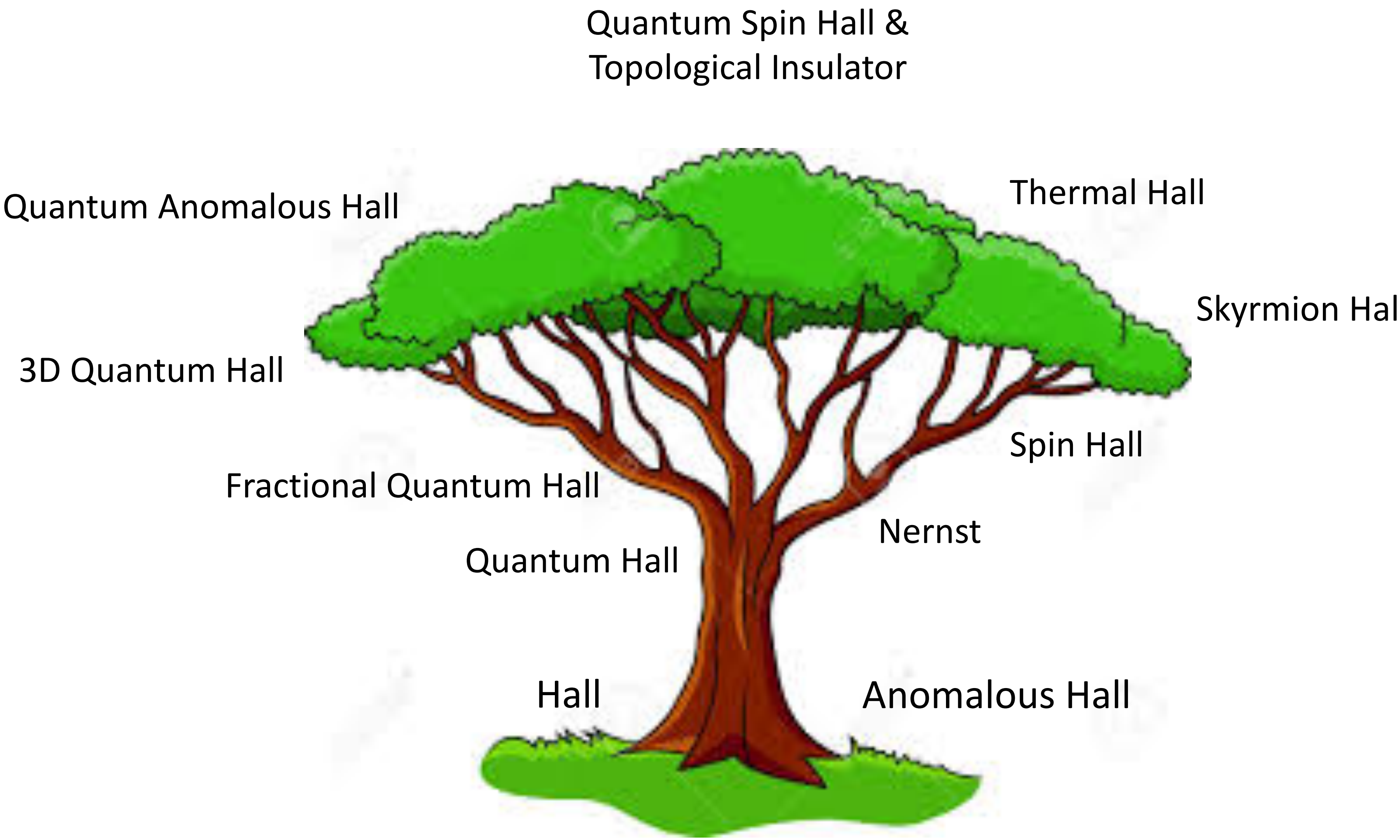


$$eE_x = ev_yB$$
$$\Downarrow$$
$$\rho_{xy} \equiv \frac{E_x}{J_x} = \frac{B}{ne}$$

Anomalous Hall Effect
(1881)



$$\rho_{xy} = R_0H_z + R_sM_z$$


Quantum Spin Hall &
Topological Insulator
Quantum Anomalous Hall
Thermal Hall
Skyrmion Hall
3D Quantum Hall
Spin Hall
Fractional Quantum Hall
Nernst
Quantum Hall
Hall
Anomalous Hall

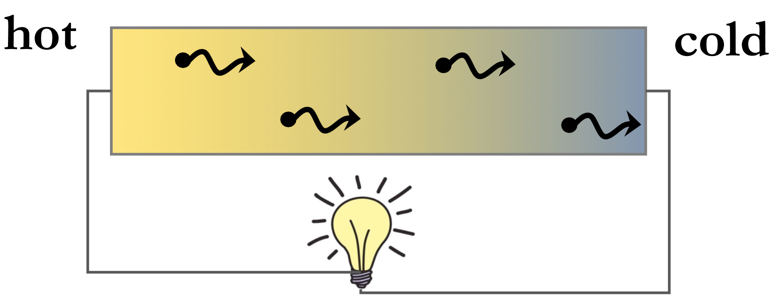
New Hall Phenomena

- "Quantized" thermoelectric Hall effect: $\boldsymbol{I_x = \alpha_{xy} \nabla_y \mathrm{T}}$
  (allowed at charge neutrality)

- Nonlinear Hall effect: $\boldsymbol{I_x = \chi_{xyy} E_y^2}$
  (allowed with time-reversal symmetry)

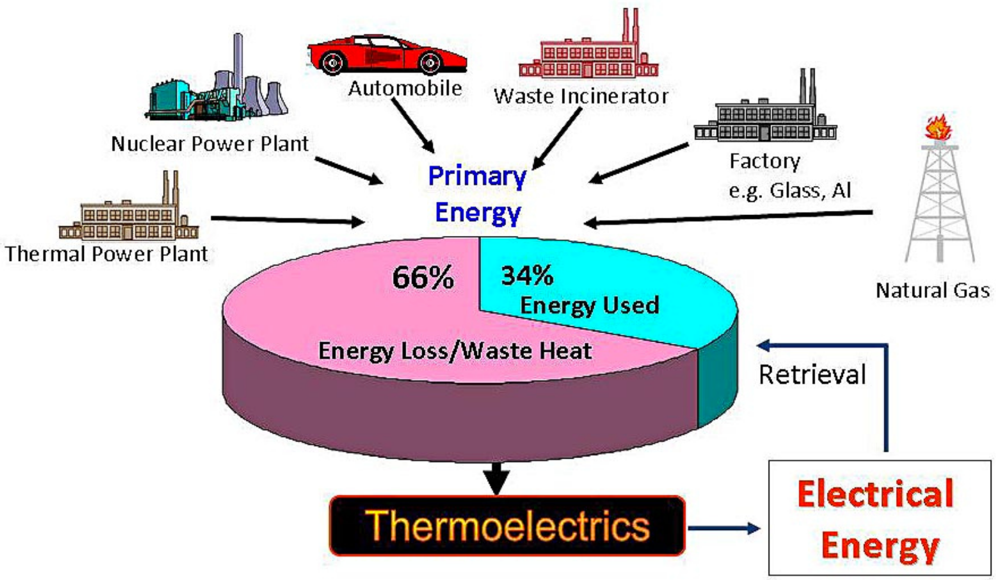
Thermoelectric Generator

Turn Heat into Electricity



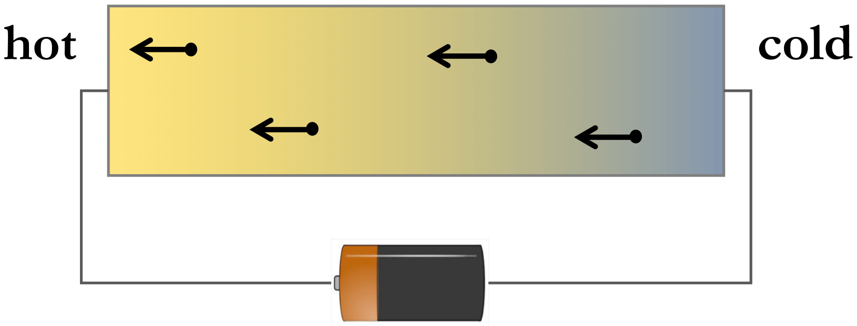
Thermopower (Seebeck)

$S = \Delta V / \Delta T$

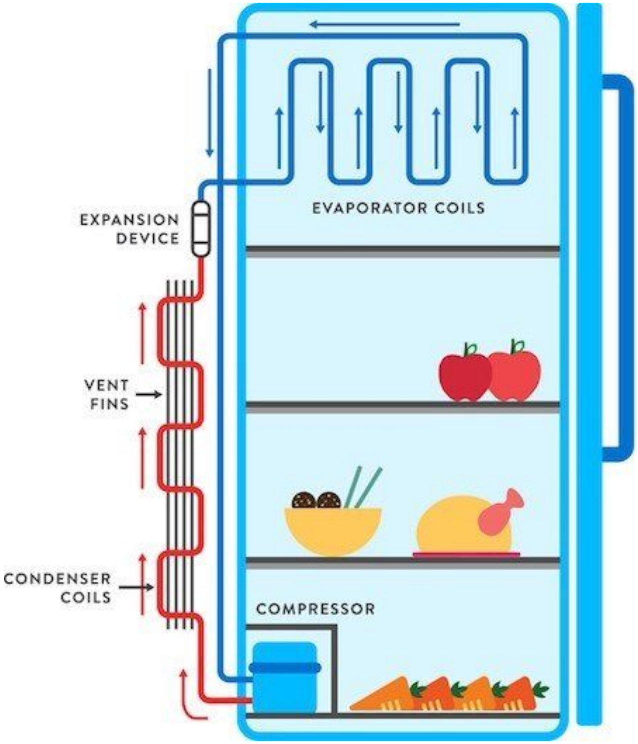


Thermoelectric Refrigerator

Solid-State Cooling



Peltier Coefficient: $\Pi = Q/I$



The HFCs widely used in air conditioning and refrigerator are thousands of times more potent than carbon dioxide. (climate.org)

Niche Applications

Mars 2020 Rover

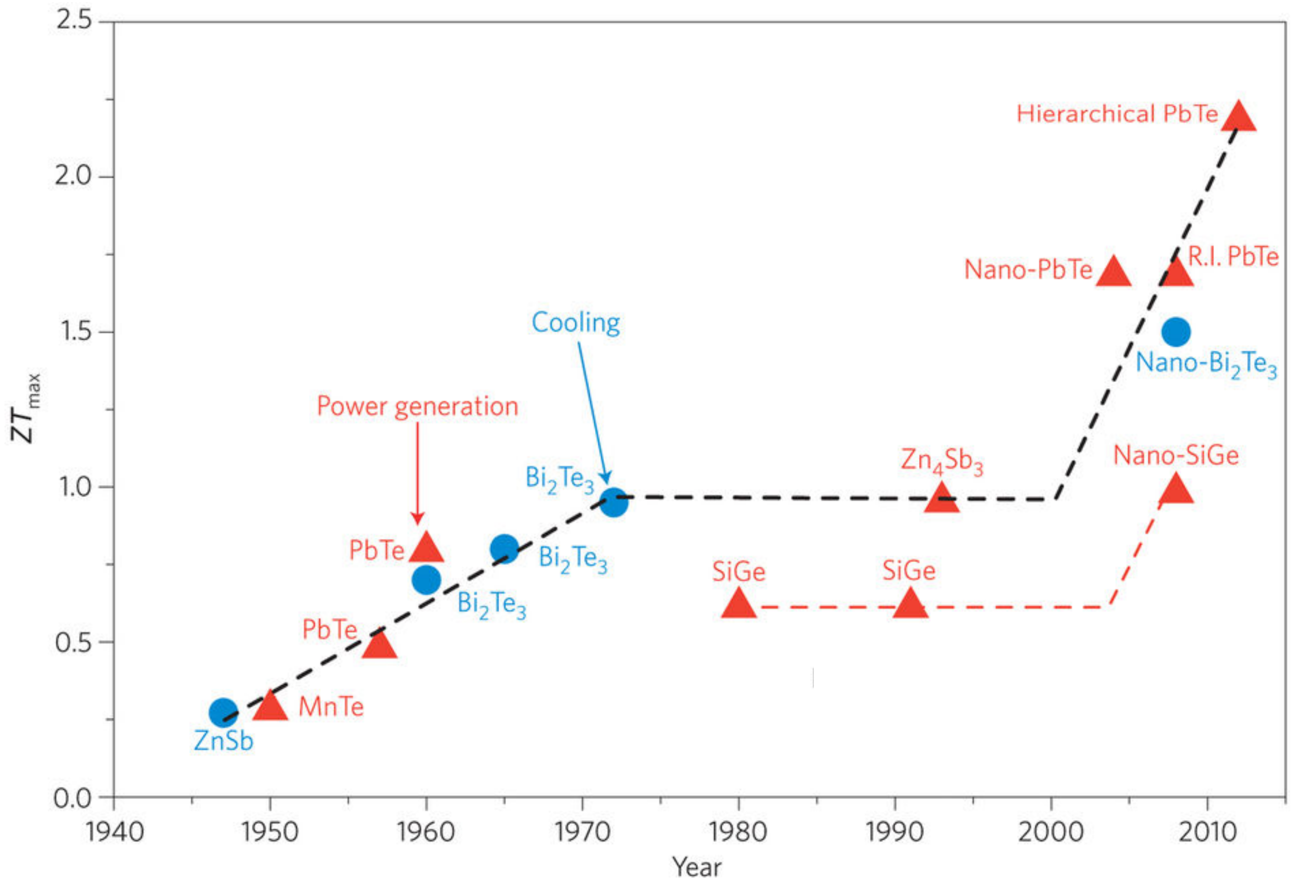
Radioisotope Thermoelectric Generator using PbSnTe

Portable fridge

Wine cooling using Bi_2Te_3

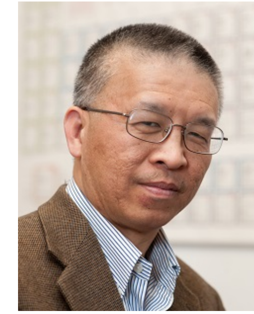
Both thermoelectrics are topological insulators!

Thermoelectric Figure of Merit



Knowledge is power

How I got started …

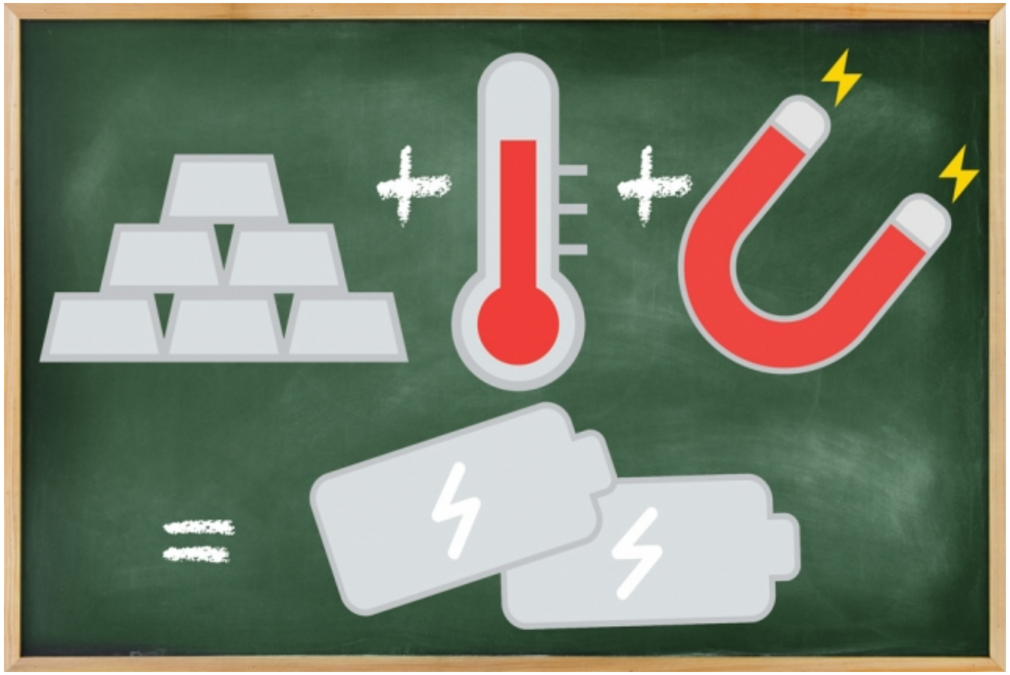


From: Gang Chen
Sent: Friday, September 12, 2014 1:57 PM

“I hope to find time getting together to explore whether topological insulator will be a good topic for a seed fund at our S3TEC center. “

S^3TEC | Solid-State Solar Thermal Energy Conversion Center

Thermoelectricity in Quantum Limit



Brian Skinner

Skinner & LF, Science Advance (2018)

(MIT News, May 2018)

How I got started …
… and got hooked

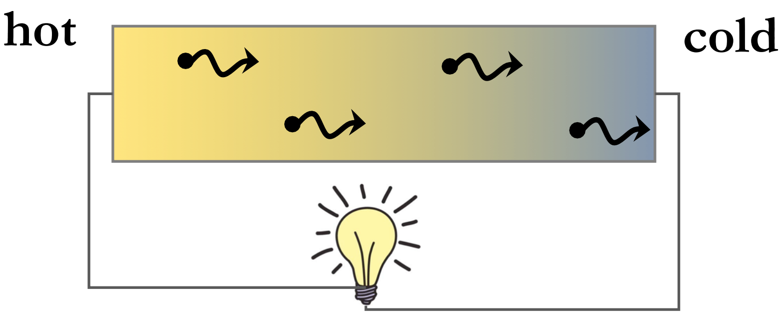
From: Gang Chen

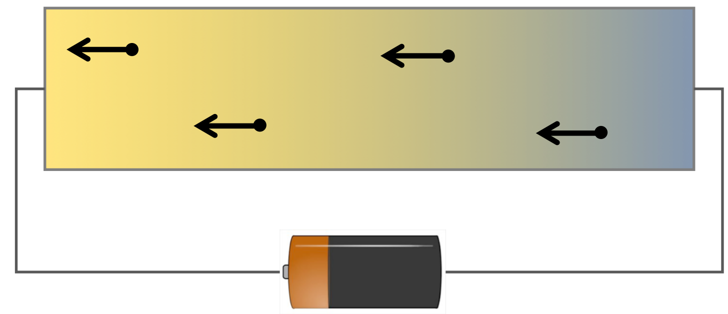
Sent: Tuesday, February 6, 2018 4:07 PM

"S3TEC has had a great run since its inception in 2009…
As the Center is drawing to a close this July …"

Skinner & LF, Science Advance (2018)
Kozii, Skinner & LF, PRB (2019)
LF, arXiv:1909.09506

Thermoelectric Transport Coefficients





Thermopower (Seebeck): $S = \Delta V / \Delta T$

(under open-circuit condition $I=0$)

Peltier Coefficient: $\boldsymbol{\Pi} = \boldsymbol{Q}/\boldsymbol{I}$

(at constant temperature $\nabla \boldsymbol{T} = \boldsymbol{0}$)

Onsager relation: $\Pi = TS$

Thermoelectric conductivity:

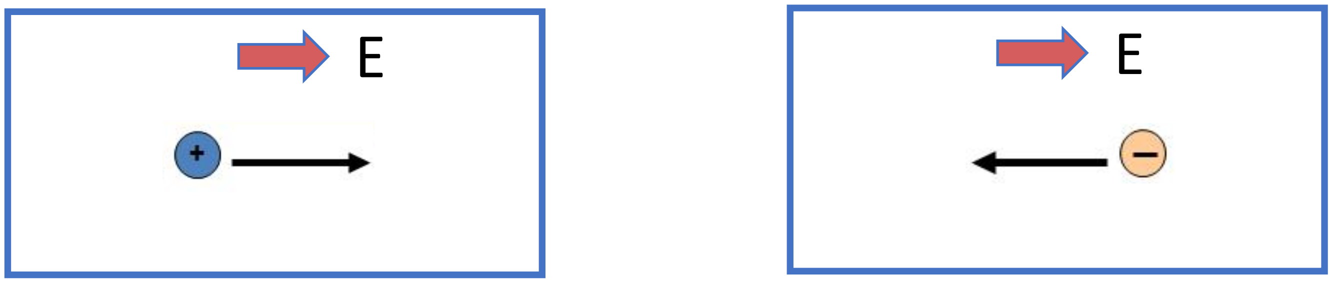
$\boldsymbol{\alpha} = \boldsymbol{Q}/(\boldsymbol{T} \cdot \boldsymbol{E})$ at constant temperature $\nabla \boldsymbol{T} = \boldsymbol{0}$

$\boldsymbol{\alpha} = -\mathbf{I}/\nabla \boldsymbol{T}$ under zero voltage $\boldsymbol{E} = \boldsymbol{0}$

$$S = \alpha \cdot \rho$$

Under magnetic field, all transport coefficients are tensors.

Thermoelectric Response at B=0

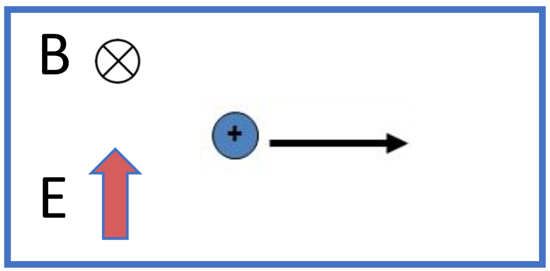


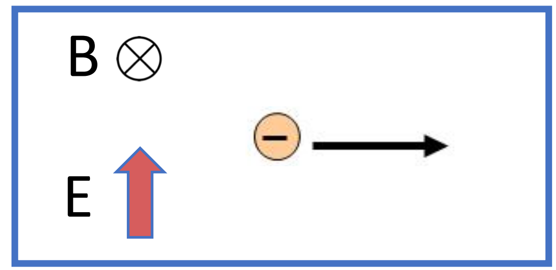
$I > 0, \quad Q > 0$ $\qquad$ $I > 0, \quad Q < 0$

At B=0, electron and hole move in opposite direction under E field, producing opposite heat current.

Hence $\alpha_{xx} = Q_x/(T \cdot E_x)$ is odd under charge conjugation.

Thermoelectric Hall Response of Electron and Hole





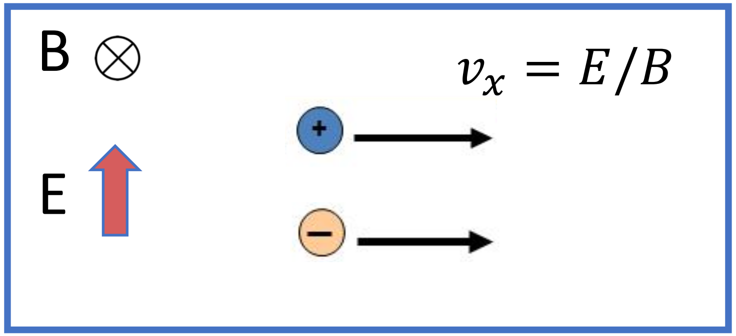
$I > 0, \qquad Q > 0$ $\qquad\qquad$ $I < 0, \qquad Q > 0$

$$eE_y = ev_xB \;\Rightarrow\; v_x = E/B$$

At $B \neq 0$, electron and hole drift in the same transverse direction, producing opposite electrical current but same heat current.

Hence $\alpha_{xy} = Q_y/(T \cdot E_x)$ is invariant under charge conjugation.

Thermoelectric Hall Response of Electron and Hole



In clean limit $\omega_c \tau \gg 1$:

$$\alpha_{xy} = \frac{Q_x}{T \cdot E_y} = \frac{T(\boldsymbol{s_e} + \boldsymbol{s_h})v_x}{T \cdot E_y} = \frac{s}{B}$$

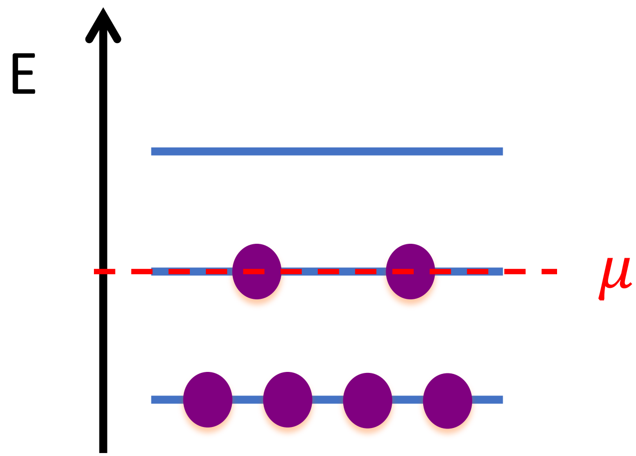
thermoelectric Hall conductivity is determined by **total entropy** density s

$$\sigma_{xy} = \frac{I_x}{E_y} = \frac{e(\boldsymbol{n_e} - \boldsymbol{n_h})v_x}{E_y} = \frac{en}{B}$$

Hall conductivity is determined by **net charge** density en

α_{xy} is Fermi surface property, σ_{xy} is not.

Maximize Entropy with Landau Level Degeneracy



Graphene at B=1T:
$E_1 - E_0 \sim 400\mathrm{K}, \quad \Gamma \sim 10\mathrm{K}$

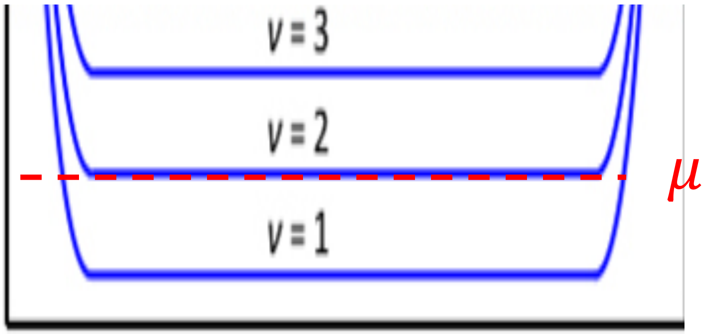
When $\hbar\omega_c \gg T \gg \Gamma$ (LL broadening)

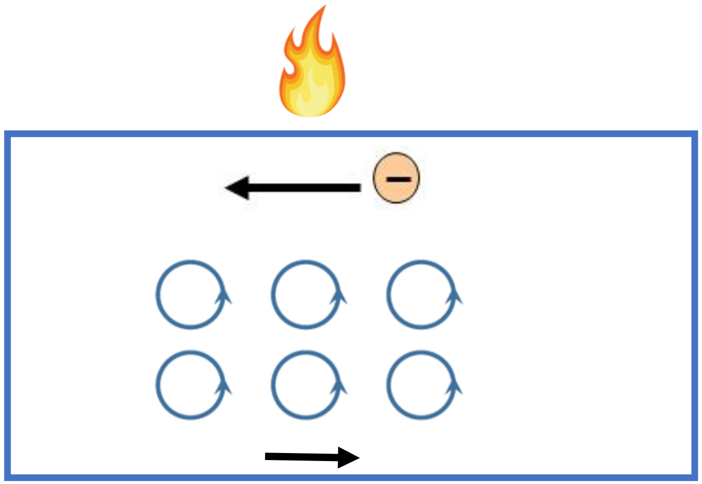
α_{xy} peaks at half-filling of every Landau level, with quantized peak value

$$\alpha_{xy} = \frac{g_L (\log 2)\, k_B \cdot \#\text{ of LL orbitals}}{\text{magnetic flux}} = \frac{g_L (\log 2)\, k_B e}{h}$$ (~2.3nA/K)

g_L: LL spin/valley degeneracy

Chiral Edge State Transport



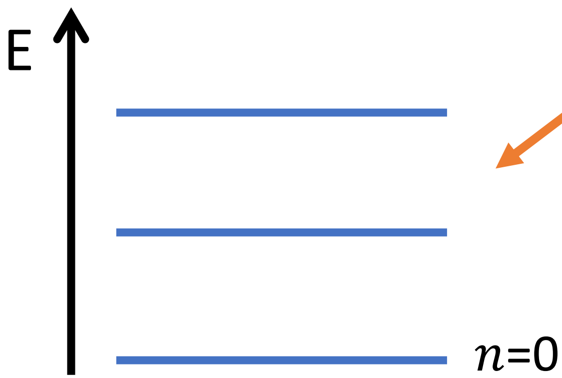


$$I_e = \frac{e}{h}\int_0^\infty dE\,\frac{\partial f}{\partial T}\Delta T = \frac{ek_B}{h}\int_0^\infty dE\,(\frac{E}{k_B T})\,(-\frac{\partial f}{\partial E})\Delta T \qquad f = 1/(e^{E/k_B T}+1)$$

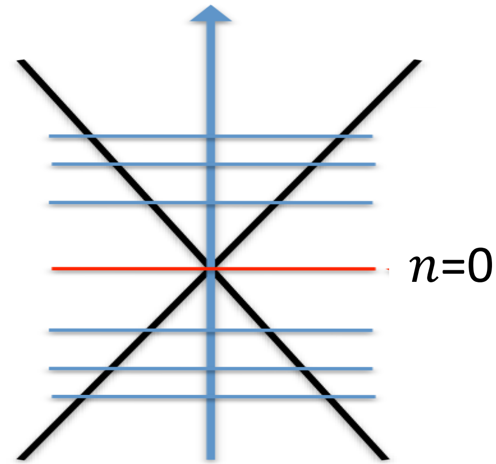
$$\alpha_{xy} = I_e/\Delta T = \log 2 \cdot ek_B/h$$

See for example Girvin & Johnson (1981), Bergman & Oganesyan (2009)

$$\alpha_{xy} = \frac{(\log 2)\, k_B e}{h}$$



GaAs



Graphene

When n-th Landau level is half-filled and $\sigma_{xy} \gg \sigma_{xx}$

$$\sigma_{xy} = \frac{(n+1/2)e^2}{h}$$

$$S_{xx} = \alpha_{xy}\rho_{yx} = \frac{\log 2\ \ k_B}{(n+1/2)e}$$

Girvin & Johnson (1981)

$$\sigma_{xy} = \frac{ne^2}{h}$$

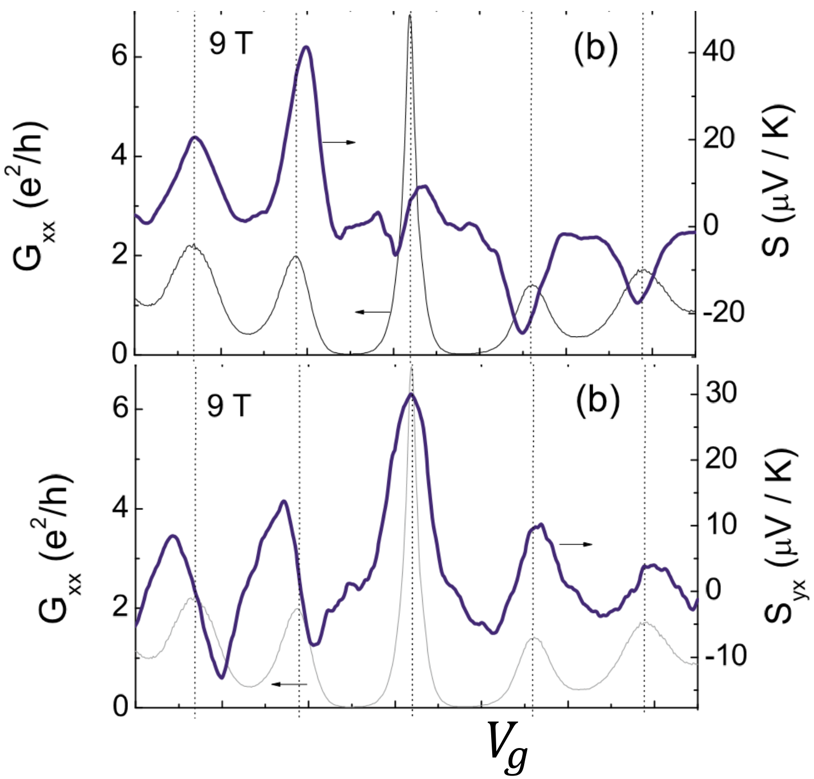
$$S_{xx} = \alpha_{xy}\rho_{yx} = \frac{\log 2\ \ k_B}{ne}$$

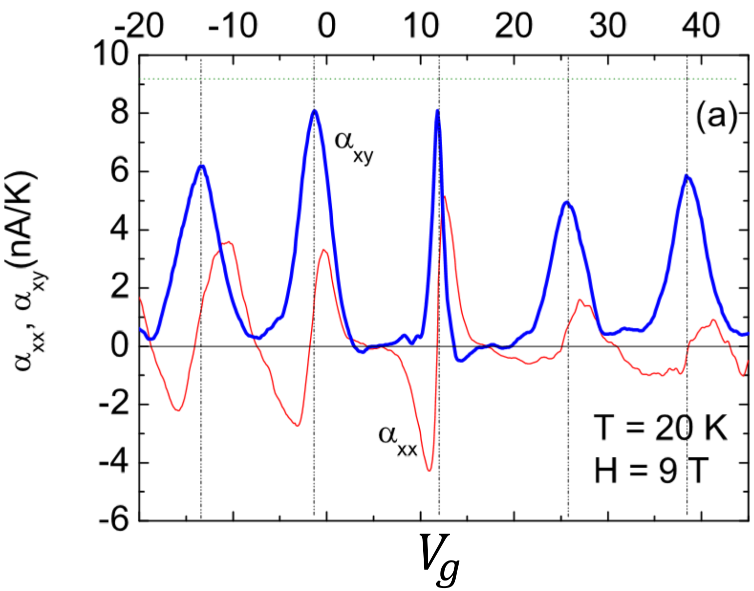
LF, arXiv:1909.09506

Thermopower is less universal than thermoelectric Hall conductivity.

Thermopower and Nernst effect in graphene in a magnetic field

Joseph G. Checkelsky and N. P. Ong

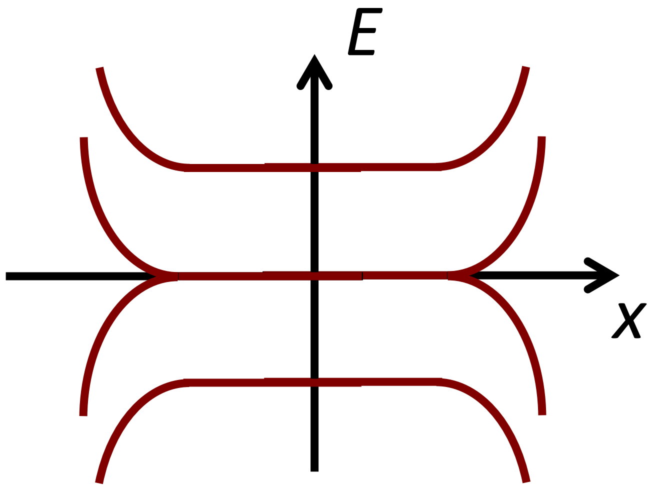


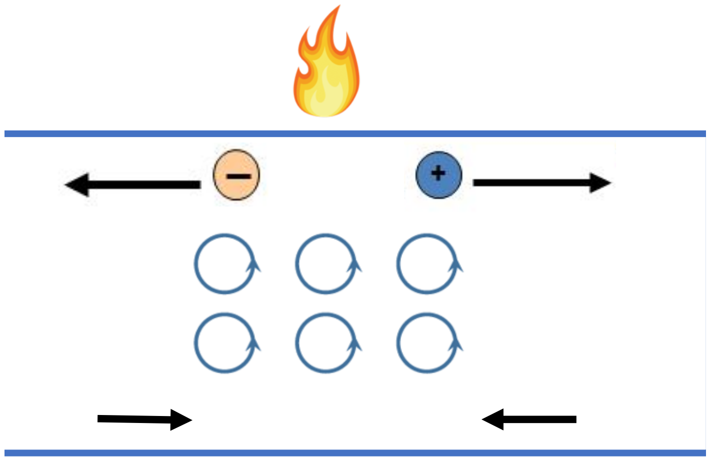


"it is not clear how the edge-current calculation of GJ is to be generalized to the n=0 LL, which is neither holelike nor electronlike."

See also Peng et al, Zuev et al (2009)

Thermoelectric Hall Effect & Edge States at $\nu = 0$





Ambipolar edge states: E>0 and E<0 modes have opposite chirality.

$$I_e = I_h = \frac{e}{h}\int_0^{\infty} dE\,\frac{\partial f}{\partial T}\Delta T = \frac{ek_B}{h}\int_0^{\infty} dE\,(\frac{E}{k_B T})\,(-\frac{\partial f}{\partial E})\Delta T$$

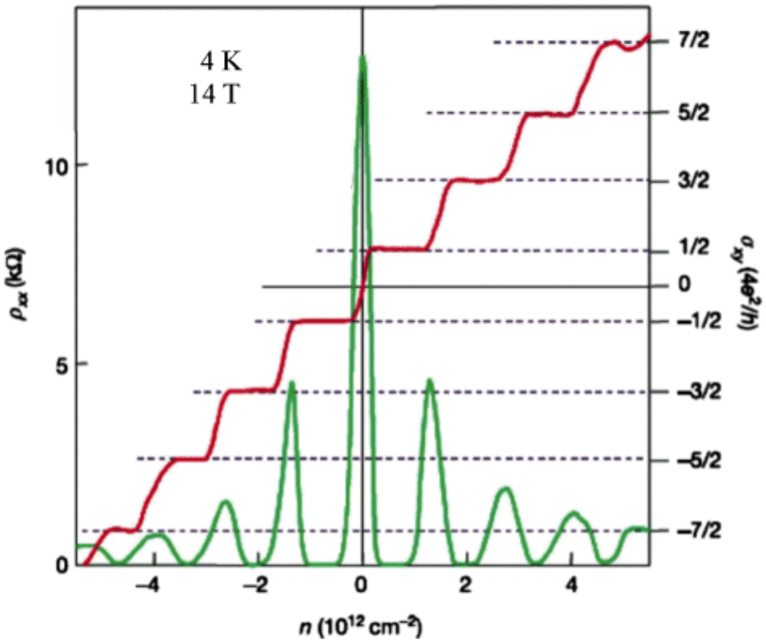
$$\alpha_{xy} = (I_e + I_h)/\Delta T = 2\log 2 \cdot ek_B/h$$

valley degeneracy

LF, arXiv:1909.09506

$\nu = 0$ State in Graphene

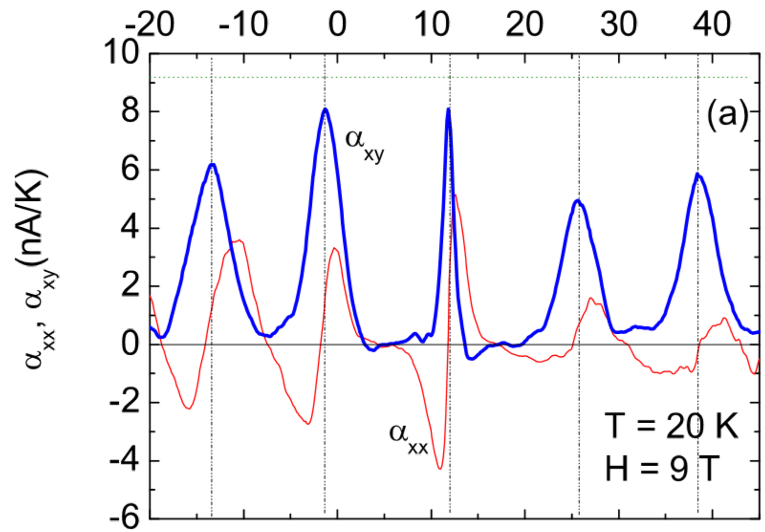
Geim & Kim (2005)



$\sigma_{xy} = \rho_{xy} = 0$ due to e-h symmetry

$\sigma_{xx} = 1/\rho_{xx}$ is finite

Checkelsky & Ong (2009)



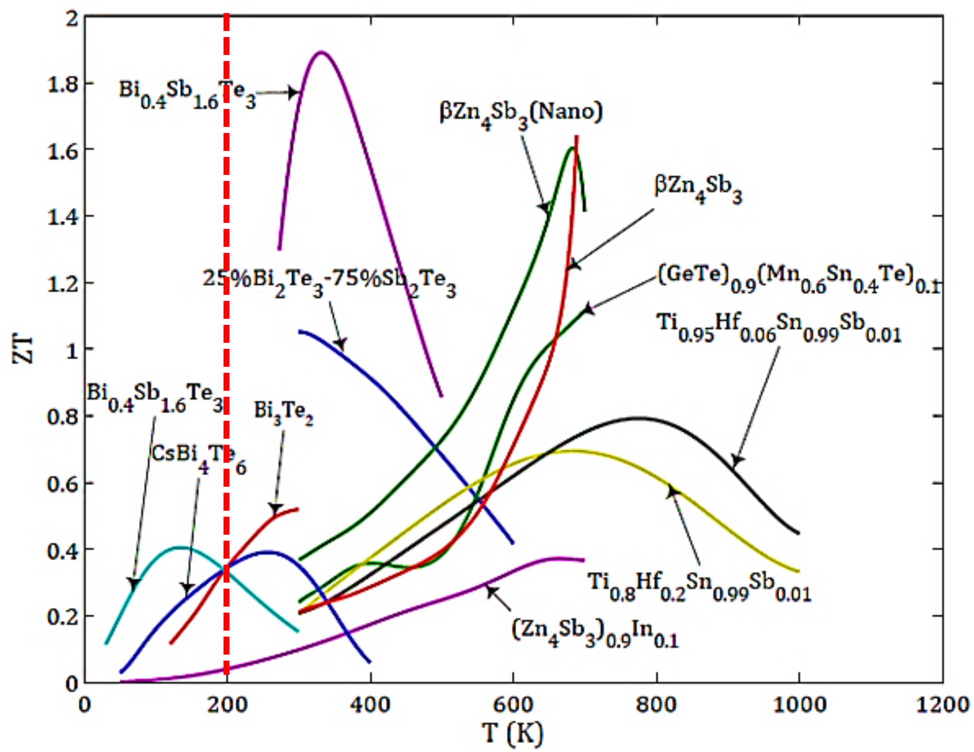
Thermoelectric Hall effect peaks at charge neutrality:

$$\alpha_{xy} = g_L (\log 2)\, k_B e/h$$

($S_{xy} = \alpha_{xy}\rho_{yy}$ is non-universal)

α_{xy} is the only Hall response at $\nu = 0$!

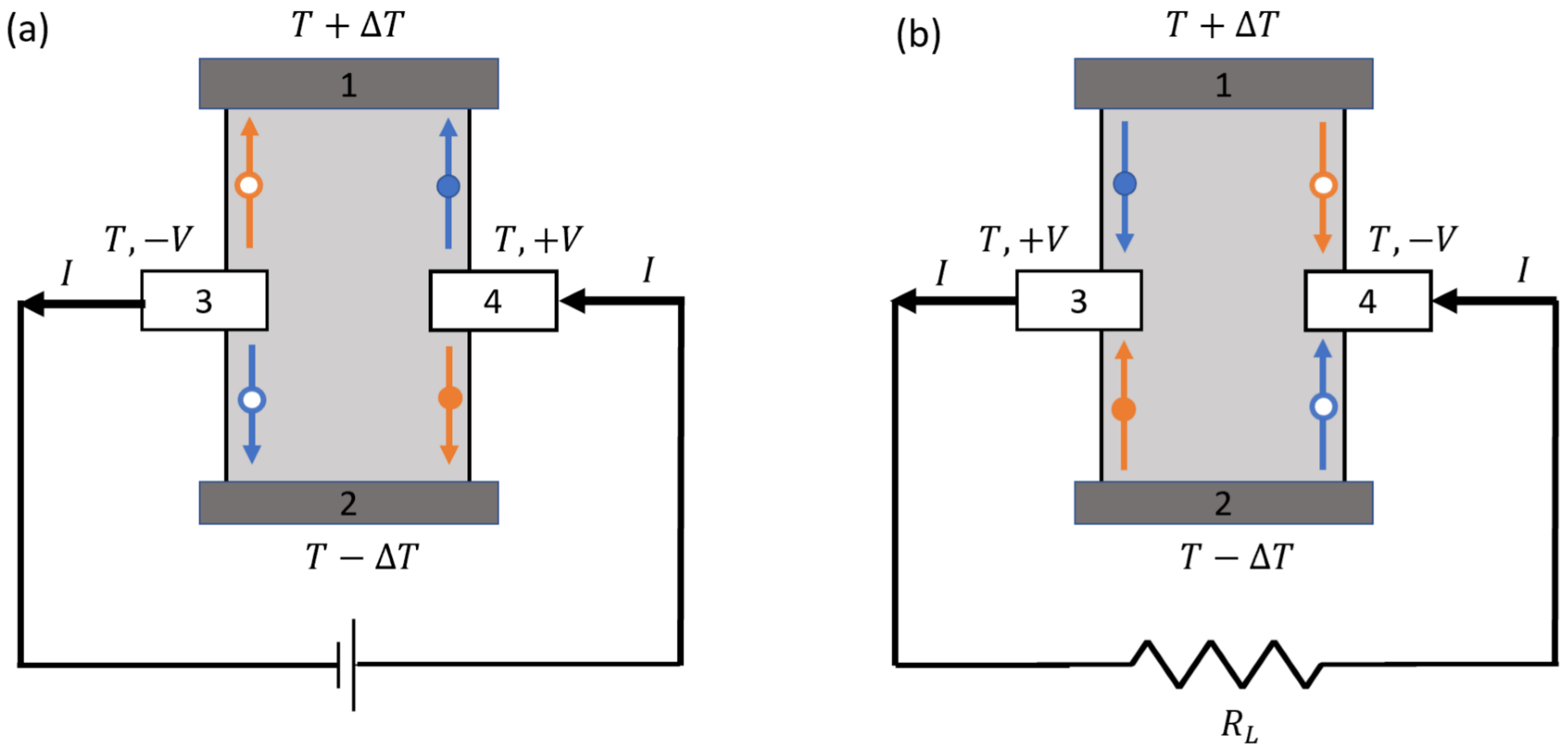
Cooling at Low Temperature (<200K)



- essential for quantum electronics, infrared detection, quantum computing
- low efficiency because thermal carriers freeze out at low T

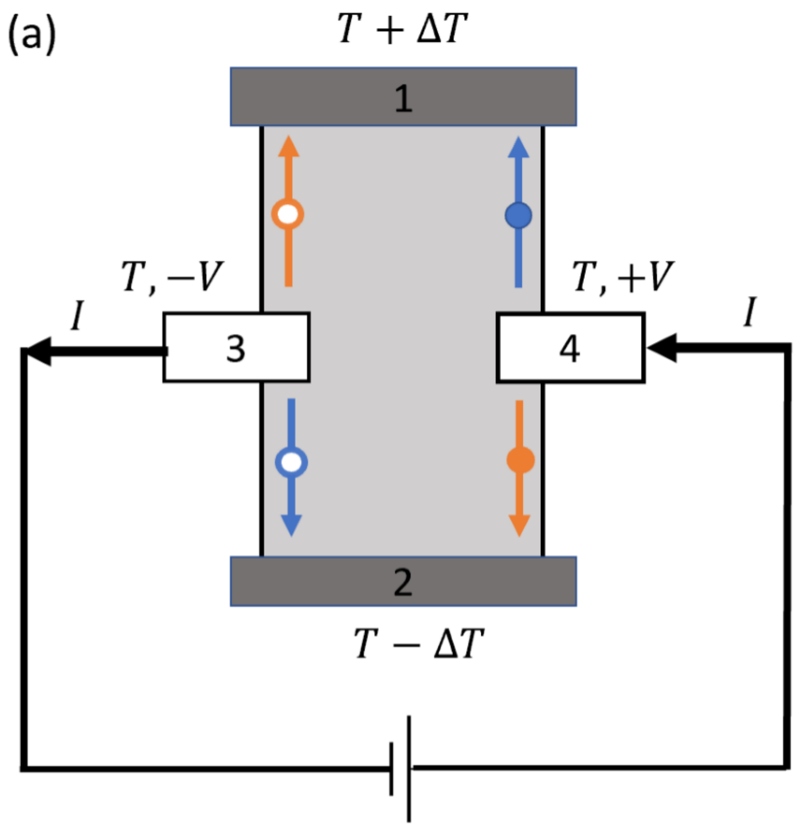
For $k_B T \ll E_F,\ \alpha_{xx} \propto k_B T \frac{d\sigma}{dE}$ (Mott formula)

Cryogenic Cooling and Power Generation using $\nu = 0$ State



LF, arXiv:1909.09506

Cryogenic Cooling and Power Generation using $\nu = 0$ State



$$I_x = GV_x + L^{eh}\Delta T_y$$
$$Q_y = -TL^{he}V_x + \tilde{K}\Delta T_y.$$

Cooling efficiency = heat taken out of cold bath / electrical power

$$\phi_c = (Q_y - I_x V_x/2)/(I_x V_x)$$

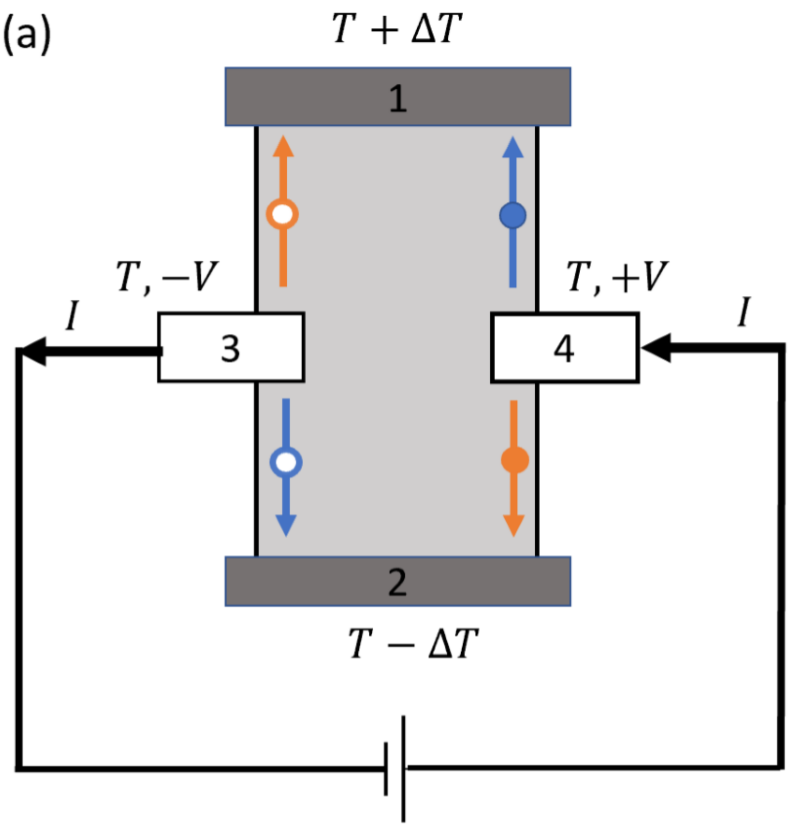
Max. efficiency only depends on transport coefficients:

$$G = \frac{e^2}{2h}, L^{eh} = L^{he} = \frac{\log 2 k_B e}{h}, \widetilde{K} = \frac{\pi^2 k_B^2 T}{6h}$$

LF, arXiv:1909.09506

Cryogenic Cooling and Power Generation using $\nu = 0$ State

(a)



$$I_x = GV_x + L^{eh}\Delta T_y$$
$$Q_y = -TL^{he}V_x + \tilde{K}\Delta T_y.$$

Cooling efficiency = heat taken out of cold bath / electrical power

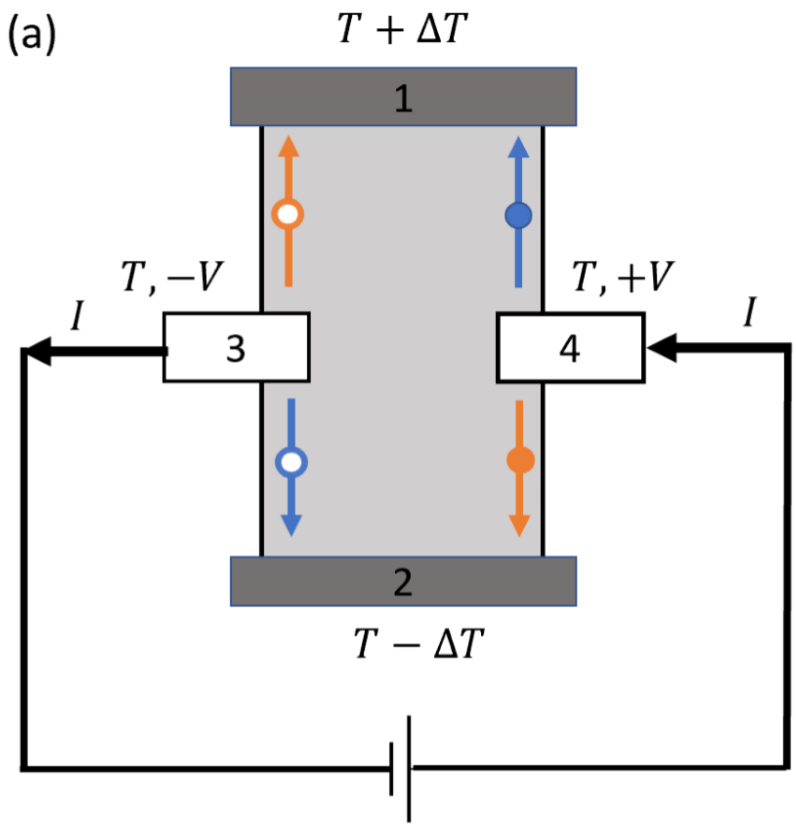
$$\phi_c = (Q_y - I_x V_x/2)/(I_x V_x)$$

Max. efficiency parametrized by ZT

$$ZT = \frac{L^{eh}L^{he}T}{GK} = \frac{S_{xy}^2\, G\, T}{K}$$
$$= \frac{\log^2(2)}{\frac{1}{2}(\frac{\pi^2}{6} + 2\log^2(2))} \approx 0.37.$$

LF, arXiv:1909.09506

Cryogenic Cooling and Power Generation using $\nu = 0$ State



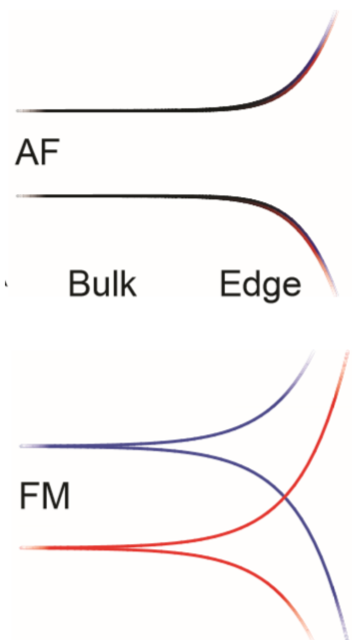
Candidate materials

- Graphene: high-mobility, small B field, but large lattice thermal conductivity
- Bi_2Se_3 thin film
- Multilayered Dirac system:
  topological insulator superlattice
  organic conductor α-(BEDT-TTF)$_2$I$_3$ …

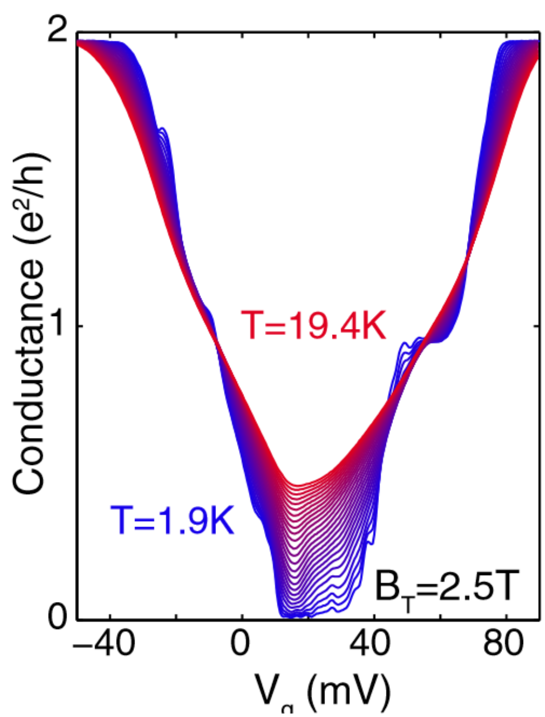
LF, arXiv:1909.09506

Effect of Electron Interaction

At sufficiently low T, LL splitting opens gap at $\nu = 0$ in graphene => $\alpha_{xy} \to 0$

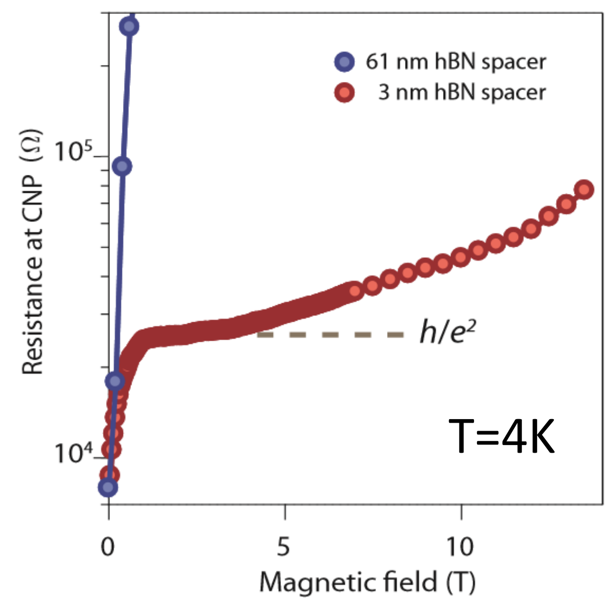


Abanin, Lee & Levitov (2006)
Kharitonov (2012)



Young, Hunt et al (2013)
Checkelsky, Li & Ong (2008)

Screening by $SrTiO_3$ substrate



Veyrat et al, arXiv (2019)

Thermoelectric Response and Entropy of Multi-Component & Fractional Quantum Hall States

$\hbar\omega_c \to \infty$

$\alpha_{xy} = (\log 2)\, k_B e/h$

T

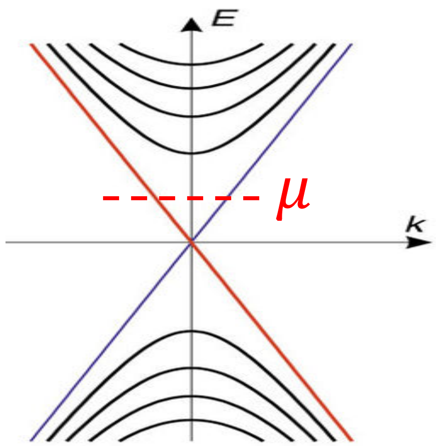
$e^2/\epsilon l_B$??

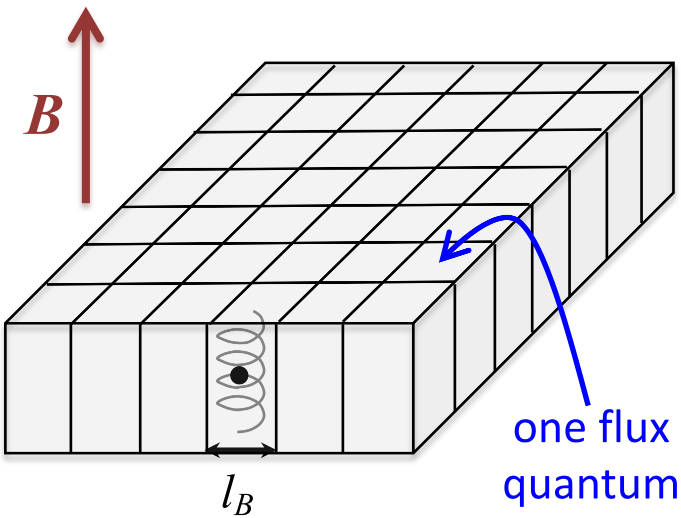
E_g ??

$T = 0$ $\alpha_{xy} = 0$

Work in progress with Donna Sheng

3D Topological Semimetal in Magnetic Field



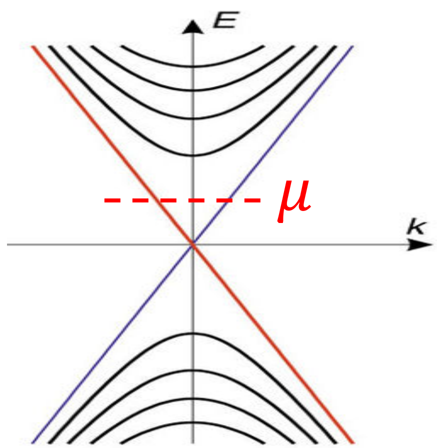


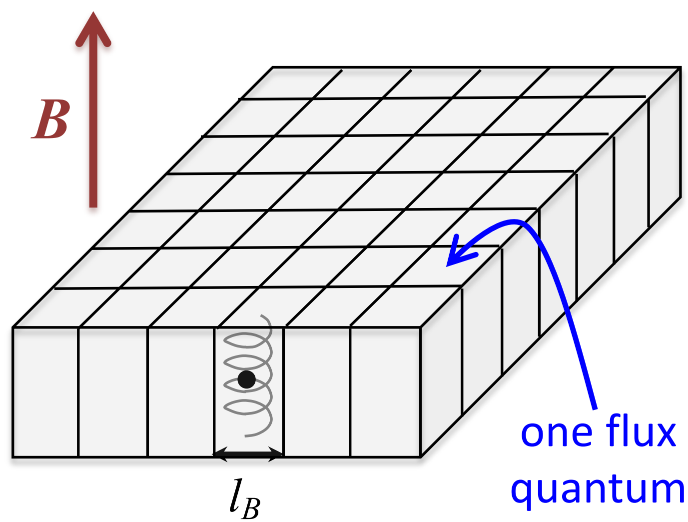
- 1D chiral Landau band: zero-gap state protected by topology/symmetry
- In extreme quantum limit, entropy grows with B field unlimited

$$s \sim k_B(k_B T \cdot DoS) = k_B^2\, T \cdot \left(\frac{eB}{h}\right) \cdot \left(\frac{1}{hv}\right) \qquad \text{for } k_B T \ll \hbar\omega_C$$

Kozii, Skinner & LF, PRB (2019)

3D Topological Semimetal in Magnetic Field





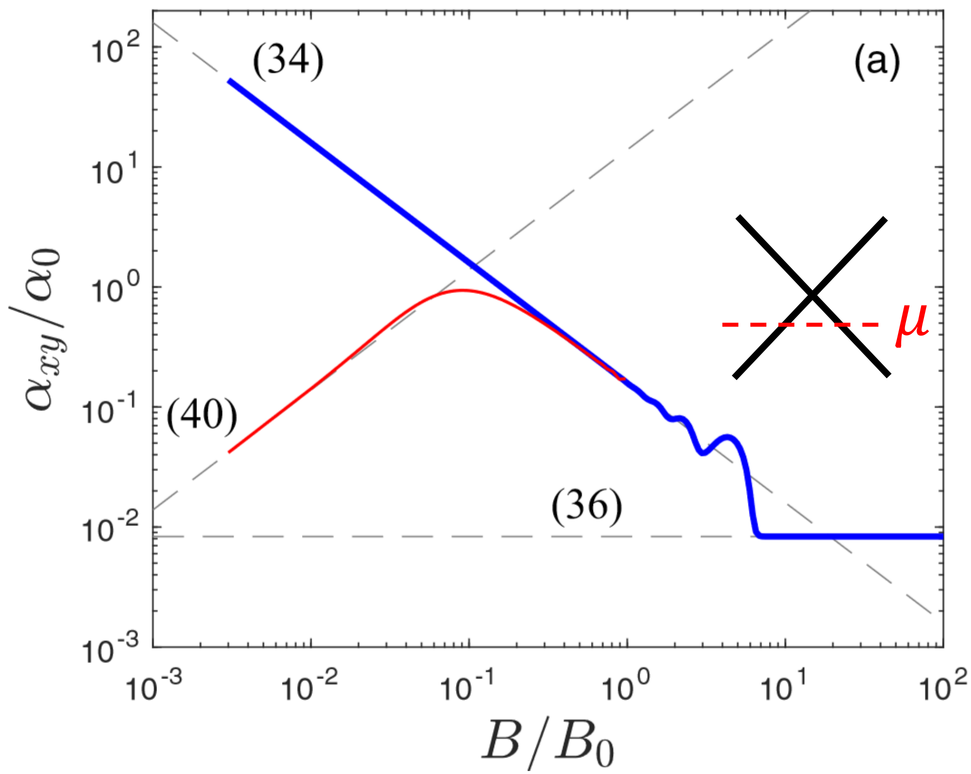
- 1D chiral Landau band: zero-gap state protected by topology/symmetry
- In extreme quantum limit, α_{xy}/T is a constant independent of *B* and n

$$\boldsymbol{\alpha_{xy}} = \frac{\boldsymbol{s}}{B} = \frac{\pi^2 e k_B^2 T}{3h^2 v_z}$$

for $k_B T \ll \hbar\omega_C$

Kozii, Skinner & LF, PRB (2019)

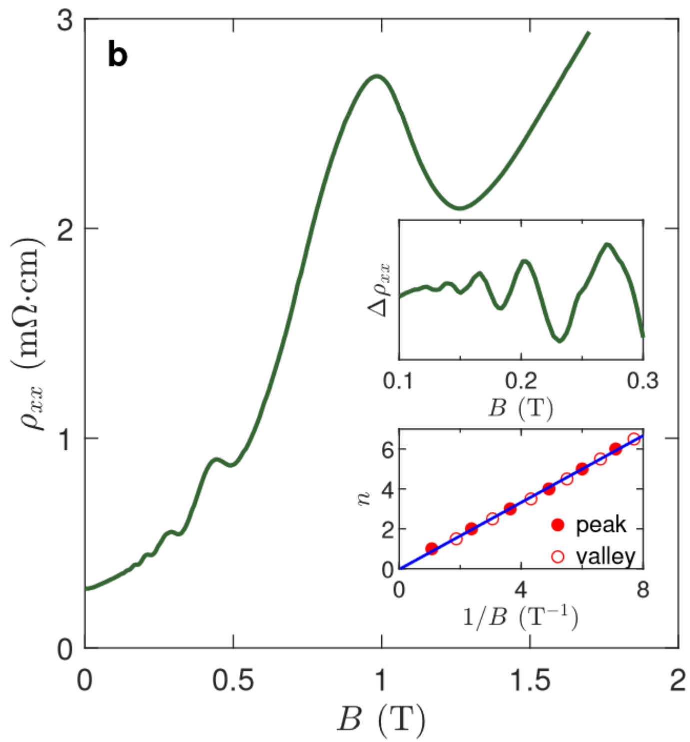
“Quantized” Thermoelectric Hall Effect in Dirac/Weyl Semimetal

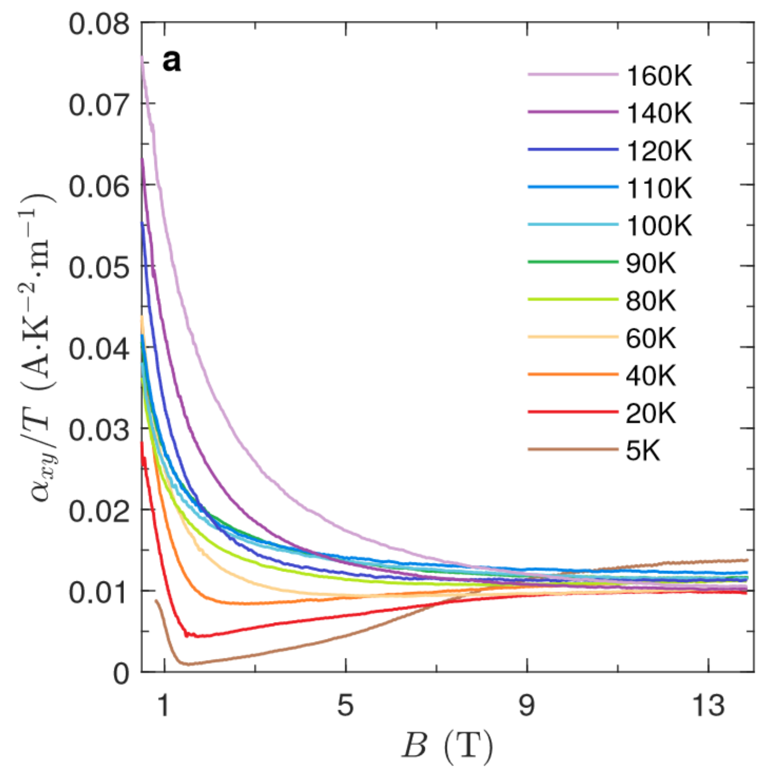


In contrast, at sufficiently large B semiconductors reach nondegenerate regime where entropy saturates and $\alpha_{xy} \propto 1/B$ (up to log correction)

$ZrTe_5$

with Liyuan Zhang, SUSTech
Xiaosong Wu, PKU
Gengda Gu, Brookhaven





- SdH oscillations onsets at 0.13 T, extreme quantum limit at ≈ 2T.

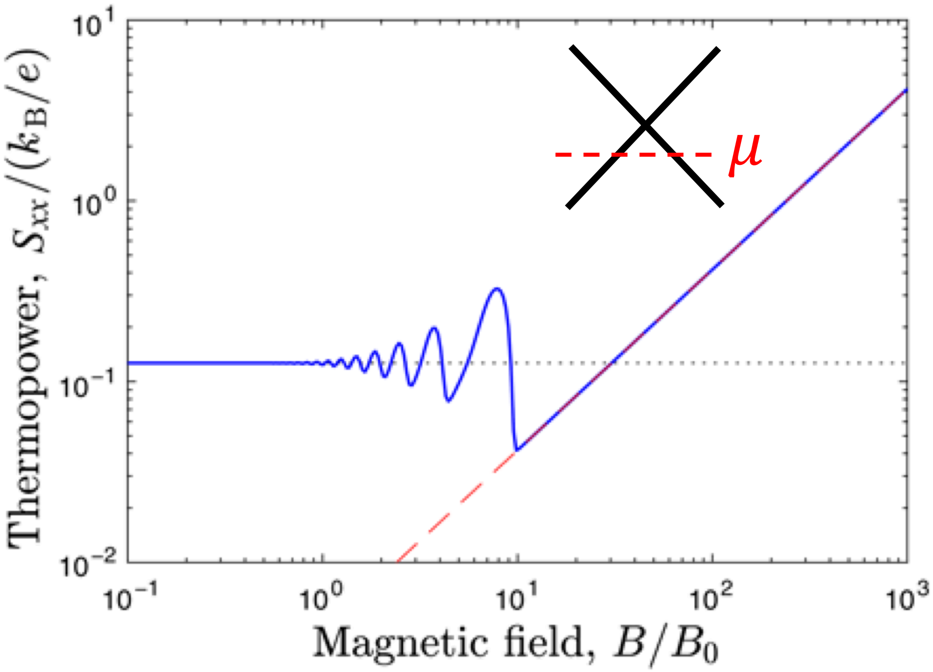
plateau $\alpha_{xy}/T \approx 0.01\ AK^{-2}m^{-2}$

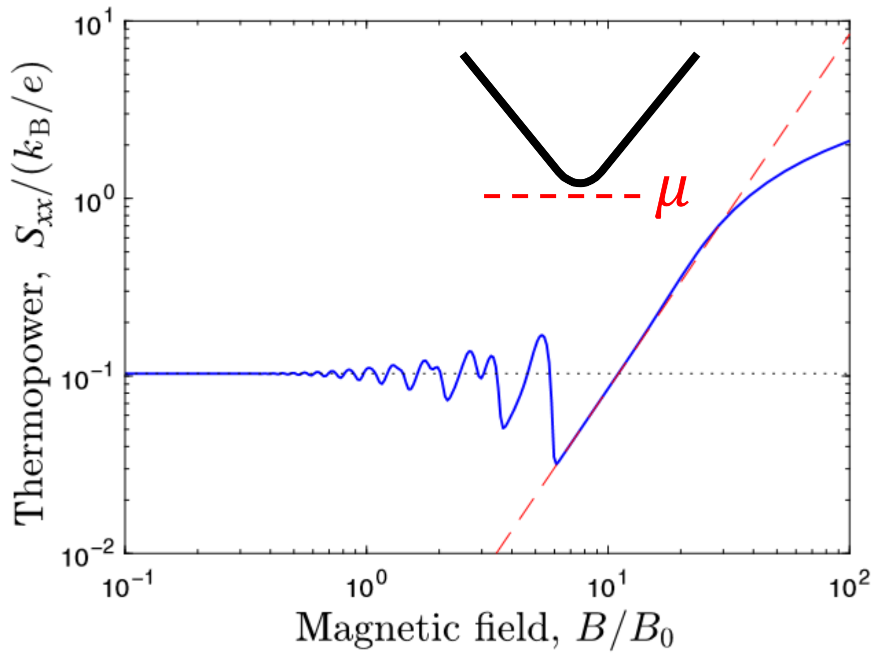
arXiv:1904.02157

Non-Saturating Thermopower of 3D Topological Semimetal

Skinner & LF (2018)

For $\rho_{xy} \gg \rho_{xx}$, $S_{xx} = \alpha_{xy}\rho_{yx} = \alpha_{xy}B/(ne)$



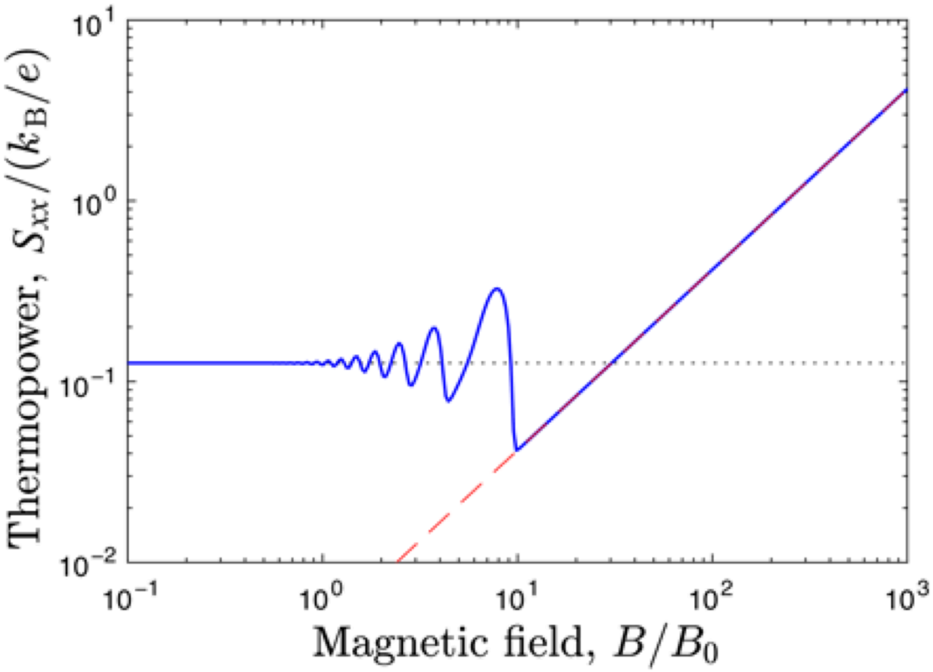


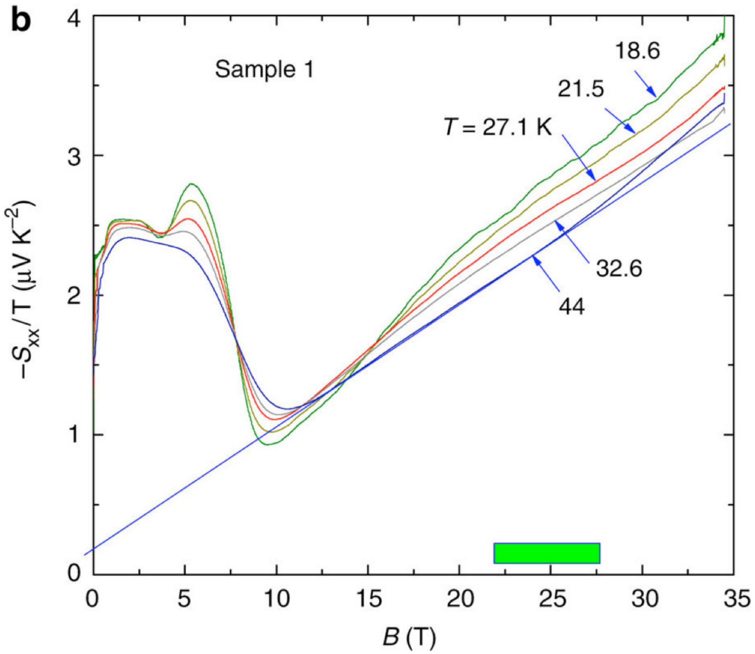
Non-Saturating Thermopower of 3D Topological Semimetal

For $\rho_{xy} \gg \rho_{xx}$,

$$S_{xx} = \alpha_{xy}\rho_{yx} = \alpha_0 B/(ne)$$

$Pb_{0.77}Sn_{0.23}Se$

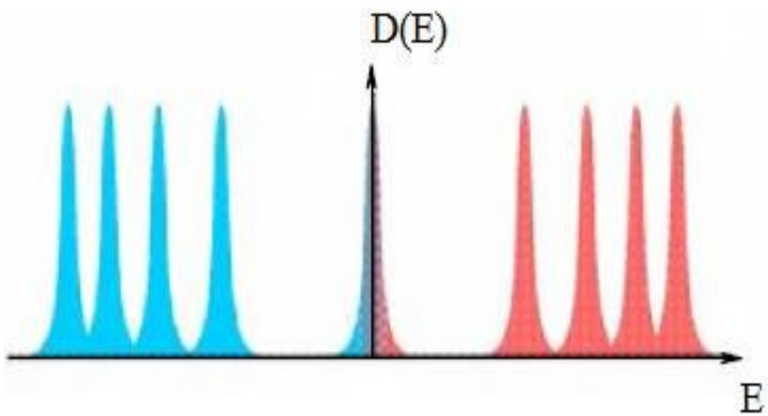




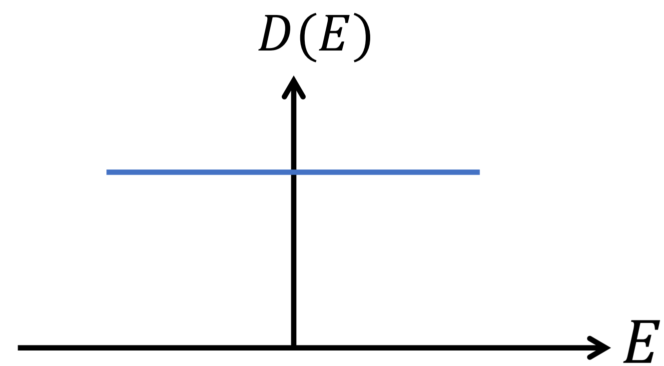
Skinner & LF, Science Advance (2018)

Tian et al, Nat. Commun. (2013)

Thermoelectric Hall Effect in 2D and 3D



2D Dirac Landau level spectrum



3D chiral Landau level: constant DOS unaffected by weak disorder

Peak value of $\alpha_{xy} = \log 2\, k_B e/h$
at high temperature $k_B T \gg \Gamma$;
reduced to $\alpha \propto T$ at low T

Plateau of $\frac{\alpha_{xy}}{T} = \frac{\pi^2 e k_B^2}{3h^2 v_z}$
independent of *B* and *n*

α_{xy} manifests itself in (1) Nernst signal $S_{xy} = \alpha_{xy}\rho_{yy}$ at charge neutrality;
(2) thermopower $S_{xx} = \alpha_{xy}\rho_{yx}$ at large Hall angle.

New Hall Phenomena

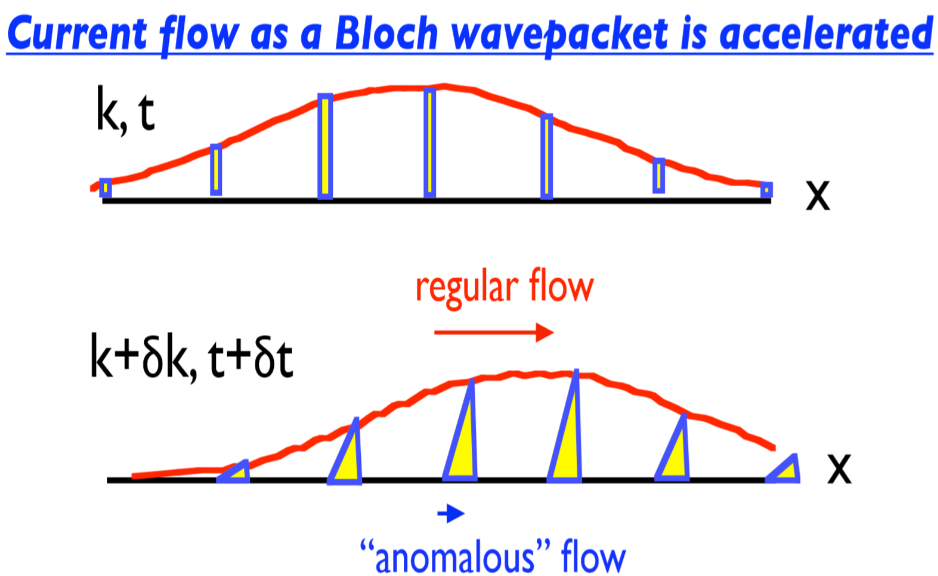
- "Quantized" thermoelectric Hall effect: $\boldsymbol{I_x = \alpha_{xy} \nabla_y \mathrm{T}}$
  (allowed at charge neutrality)

- Nonlinear Hall effect: $\boldsymbol{I_x = \chi_{xyy} E_y^2}$
  (allowed with time-reversal symmetry)

Anomalous Hall Effect

$$\rho_{xy} = R_0 H_z + R_s M_z,$$

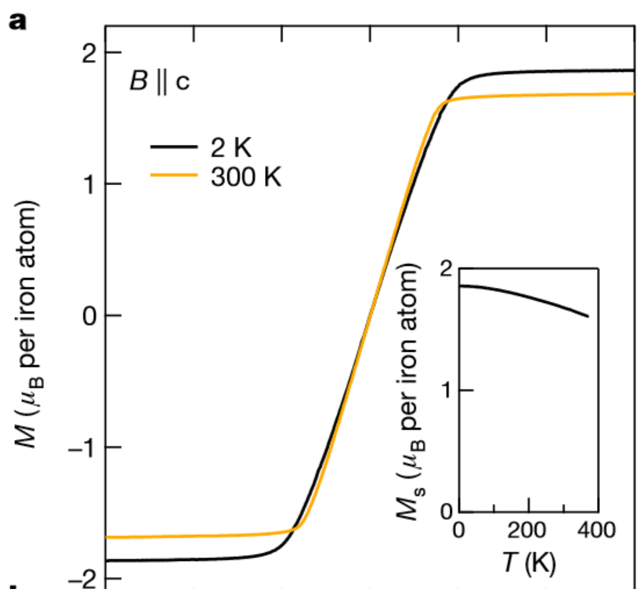
Intrinsic contribution from anomalous velocity of Bloch electron

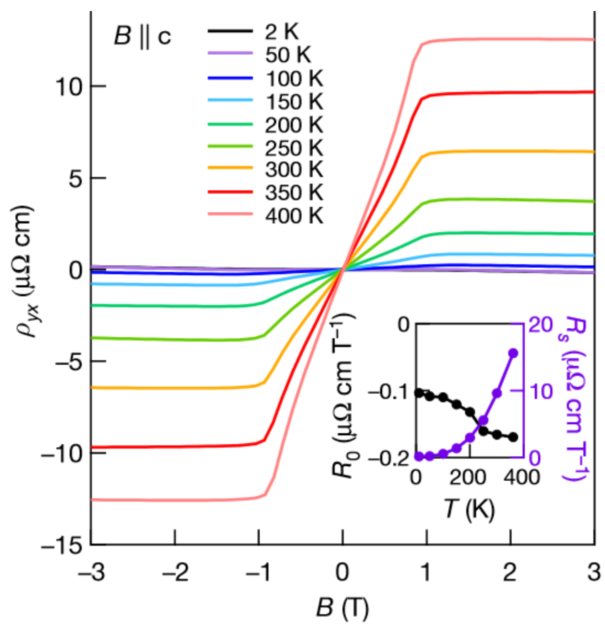


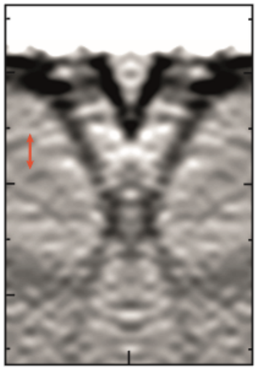
(from Haldane)

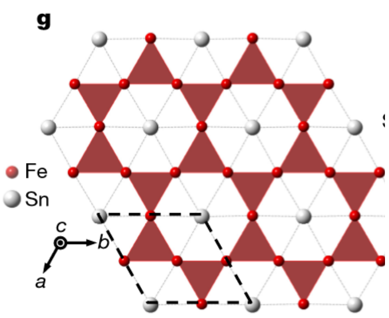
Anomalous Hall Effect in Fe_3Sn_2

$$\rho_{xy} = R_0 H_z + R_s M_z$$









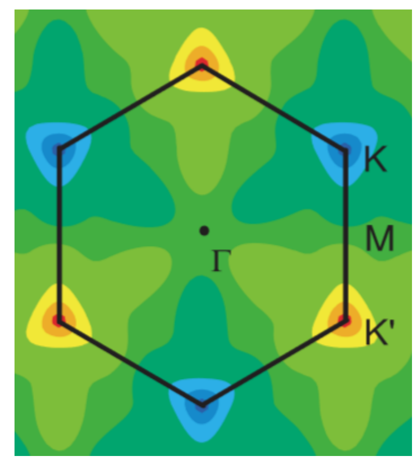
Checkelsky, Comin et al, Nature (2018)

Berry Curvature in T-Invariant & P-Breaking Systems

T-invariance: $\boldsymbol{\Omega}(\boldsymbol{k}) = -\boldsymbol{\Omega}(-\boldsymbol{k})$

~~P-invariance~~: $\boldsymbol{\Omega}(\boldsymbol{k}) = \boldsymbol{\Omega}(-\boldsymbol{k})$

- Biased bilayer graphene
- TMD MoS_2, WSe_2
- Weyl semimetal TaAs…



Equivalence of hundreds Tesla B field but hidden in dark

Semiclassical Transport with Berry Curvature

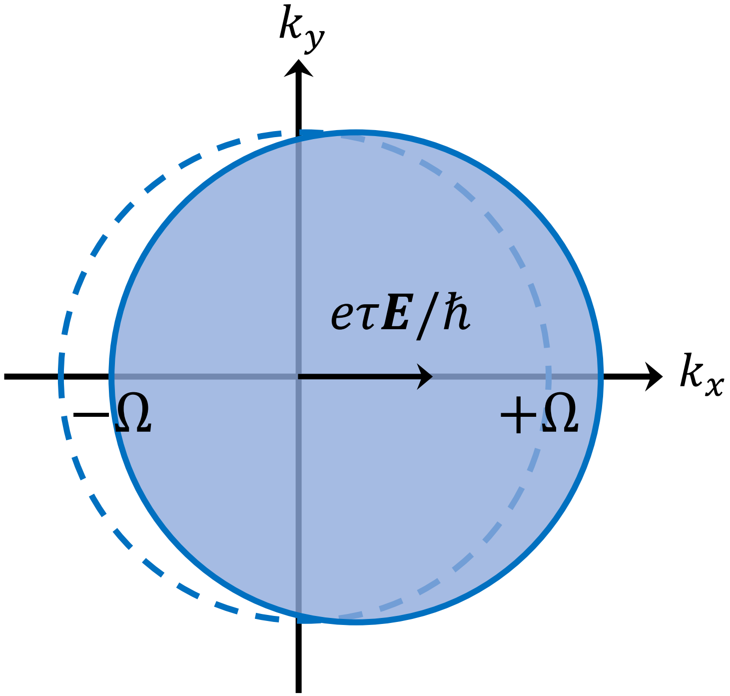
$$\boldsymbol{j_H} = \frac{e^2}{\hbar}\int d\boldsymbol{k} f(\epsilon_{\boldsymbol{k}})\boldsymbol{\Omega}(\boldsymbol{k})\times\boldsymbol{E}$$

In a current-carrying state, $f = f_0 + \delta f$ and $f(\epsilon_{\boldsymbol{k}}) \neq f(\epsilon_{-\boldsymbol{k}})$

$$\boldsymbol{j_H} = \frac{e^2}{\hbar}\int d\boldsymbol{k}\, \delta f(\epsilon_{\boldsymbol{k}})\boldsymbol{\Omega}(\boldsymbol{k})\times\boldsymbol{E}$$

$\delta f = \partial_{\boldsymbol{k}} f_0 \cdot e\boldsymbol{E}\tau/\hbar$ (Boltzmann theory)

$\Rightarrow j_H \propto E^2$



Inti Sodemann & LF, PRL (2015)

2^{nd} Order Response

$$(E\cos\omega t)^2 = E^2(1+\cos 2\omega t)/2$$

Photocurrent (rectification):

$$j_a^0 = \chi_{abc}\mathcal{E}_b\mathcal{E}_c^*$$

Second-harmonic generation:

$$j_a^{2\omega} = \chi_{abc}\mathcal{E}_b\mathcal{E}_c$$

$$\chi_{abc} = \varepsilon_{adc}\frac{e^3\tau}{2(1+i\omega\tau)}\int(\partial_b f_0)\Omega_d d\boldsymbol{k}$$

$$= -\varepsilon_{adc}\frac{e^3\tau}{2(1+i\omega\tau)}\boxed{\int f_0(\partial_b\Omega_d)d\boldsymbol{k}}$$

Berry curvature dipole

compare with

$$\sigma_{xy} = (e^2/h)\int\Omega_z d\boldsymbol{k}$$

Related works:

Moore & Orenstein, PRL (2010);

Deyo, Golub, Ivchenko & Spivak, arXiv (2009); Genkin & Mednis, JETP (1968)

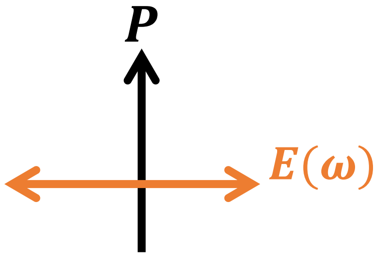
Berry Curvature Dipole

$$D_{ab} = \int f_0(\partial_a \Omega_b) d\boldsymbol{k}$$ In 2D: unit = Length

allowed in inversion breaking materials
same symmetry as current-induced magnetization

2D: must have a polar axis $\boldsymbol{P}$

$$D_a = \int f_0(\partial_a \Omega_z) d\boldsymbol{k}$$ in the direction $\boldsymbol{P} \times \hat{\boldsymbol{z}}$

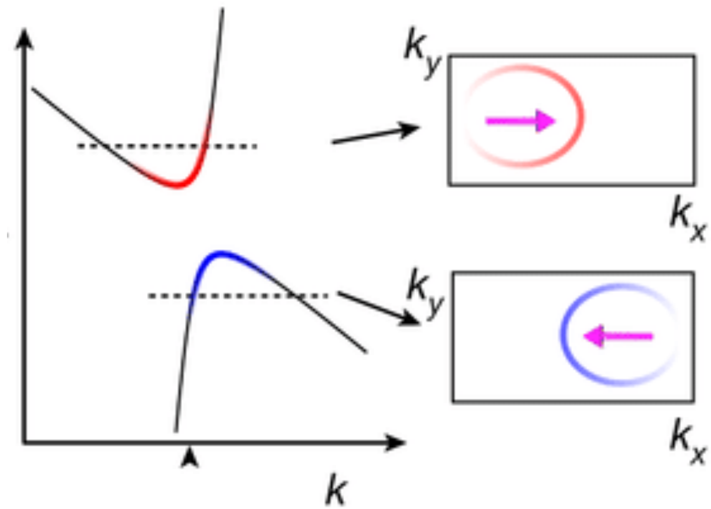


so that $\boldsymbol{J}_{\boldsymbol{0}}^{\parallel}, \boldsymbol{J}_{\boldsymbol{2\omega}}^{\parallel} \sim \boldsymbol{E}(\boldsymbol{\omega})_{\perp}^{\boldsymbol{2}}$

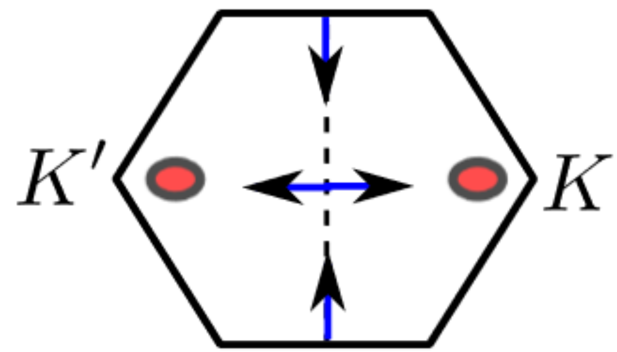
3D: $C_n, C_{nv}, n = 1,2,3,4,6$ and S_4 point group

Proposed Materials

2D systems with *titled* massive Dirac cone



e.g., TCI with ferroelectric distortion



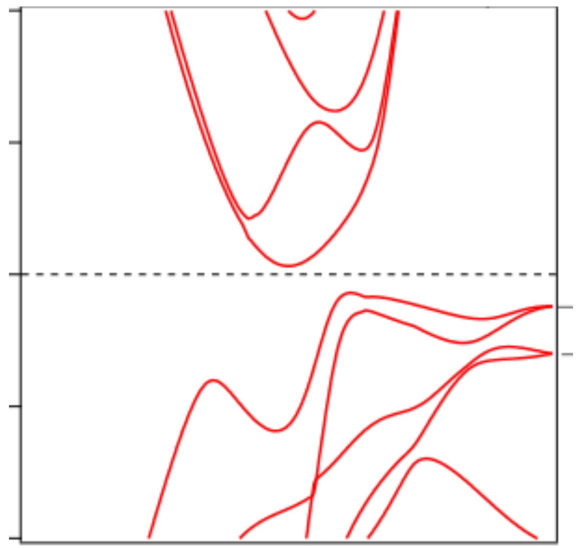
TMD under uniaxial strain

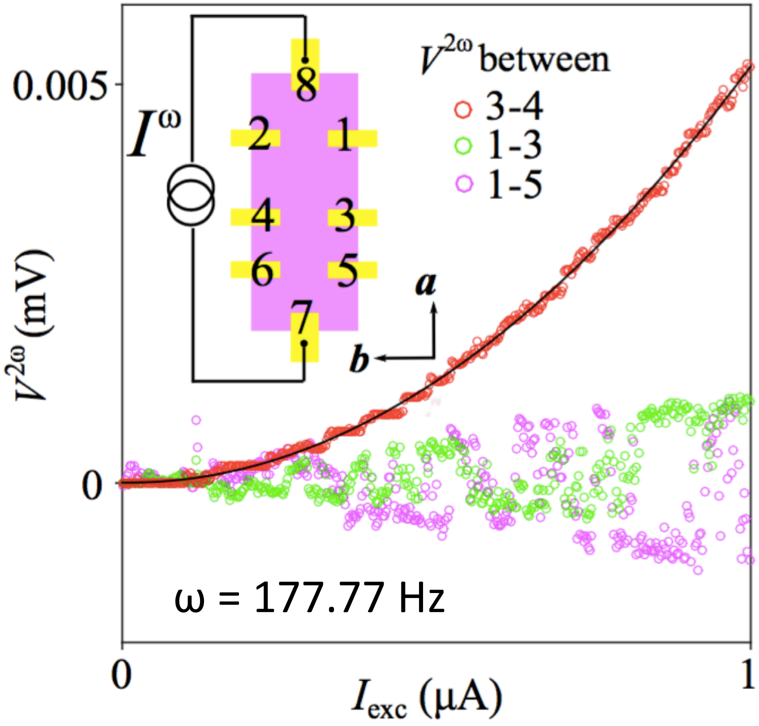
Related expt: Mak & Shan (2017)

3D Weyl semimetal with polar axis (TaAs ...)

Inti Sodemann & LF, PRL (2015)

Quantum Nonlinear Hall in Bilayer WTe_2





- Layer stacking breaks inversion
- Berry curvature dipole from tilted massive Dirac cone

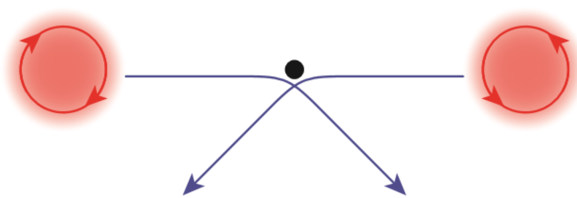
Ma, Xu, Shen et al, Nature (2018)

Mechanisms for Second-Order Electrical Response

Hiroki Isobe

- Berry curvature dipole $\chi \propto \tau$

- Skew scattering $\chi \propto \tau^2 \cdot \tau/\tau_s$



- T-breaking energy dispersion

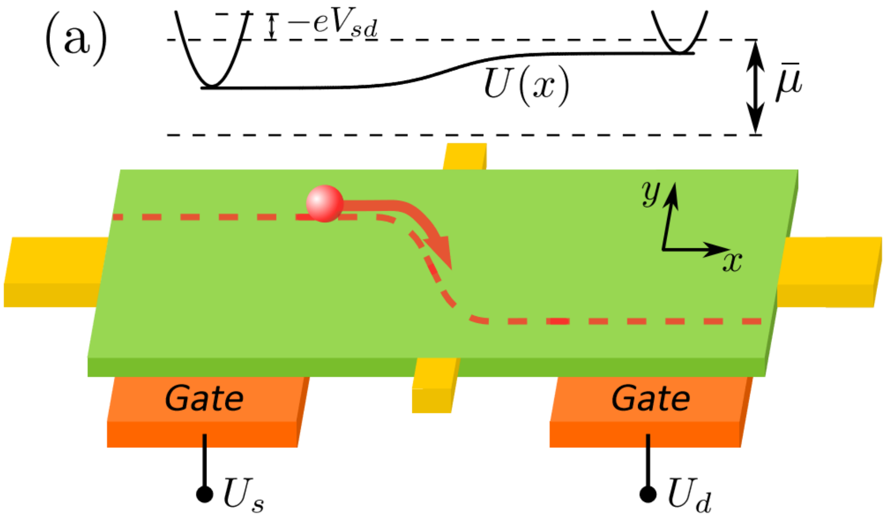
Second-order response is symmetry allowed in all crystals without inversion center, while Berry curvature dipole exists in a subset.

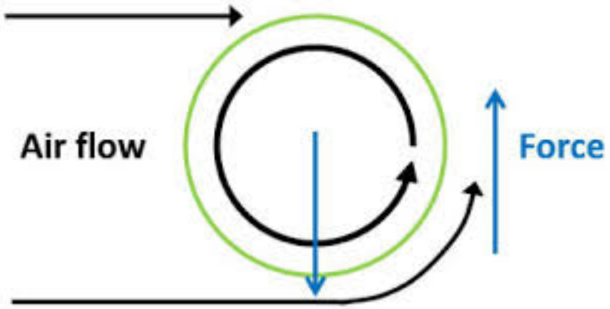
Isobe, Xu & LF, arXiv:1812.08162
Du et al, arXiv:1812.08377

Magnus Hall Effect

Papaj & LF, arXiv:1904.00013

A bias voltage between source and drain creates (1) a flow of electrons with net velocity; (2) an electrostatic potential difference





$$\Delta y_A = -\int_0^t \frac{\Omega(\mathbf{k})}{\hbar}\frac{\partial U}{\partial x}dt' = \frac{1}{\hbar v_x}\Omega(\mathbf{k}_0)\Delta U$$

Magnus Hall Effect

Papaj & LF, arXiv:1904.00013

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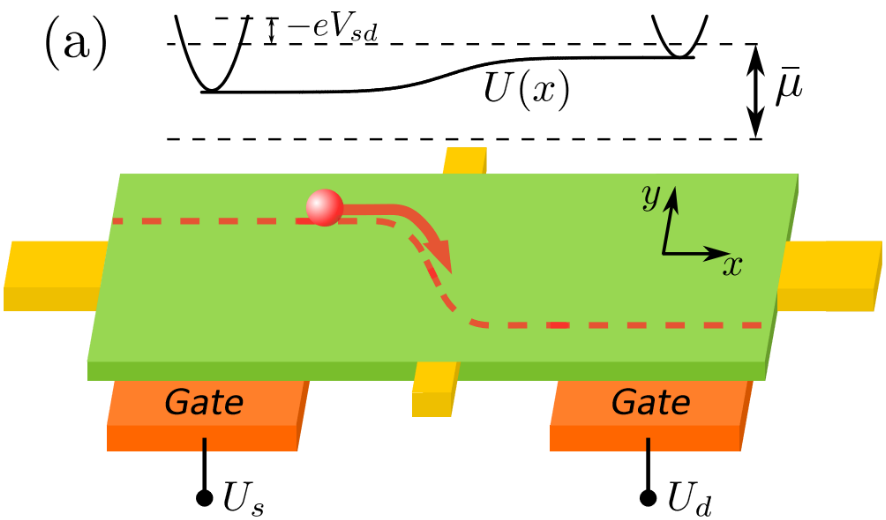
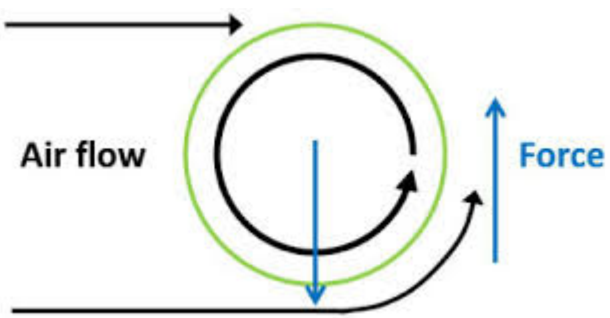


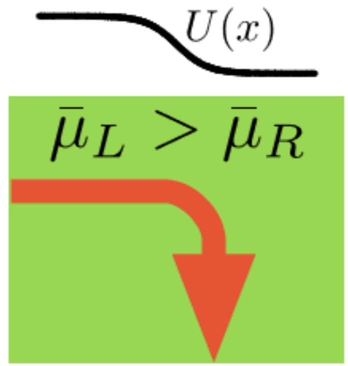


$$G_H = \frac{e^2}{h}\frac{\Delta U}{2\pi}\int_{v_x(\mathbf{k})>0} d^2k\,\Omega(\mathbf{k})\,\delta\left(\epsilon_{\mathbf{k}}-\mu\right)$$

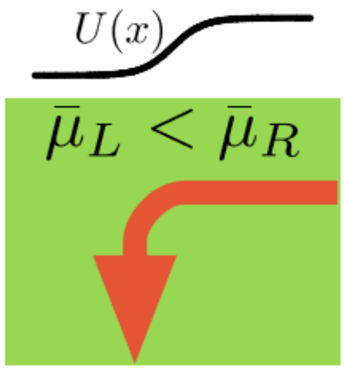
Magnus Hall Effect

Papaj & LF, arXiv:1904.00013

$n - 1/2 < t/T < n$



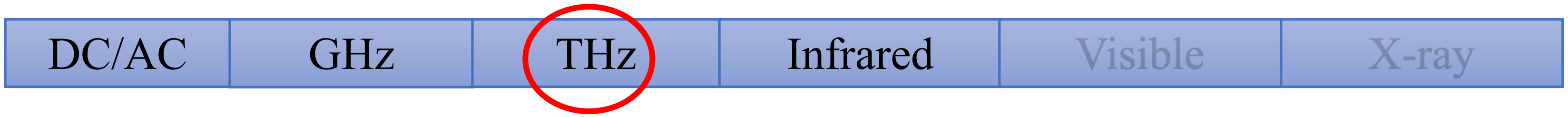
$n < t/T < n + 1/2$

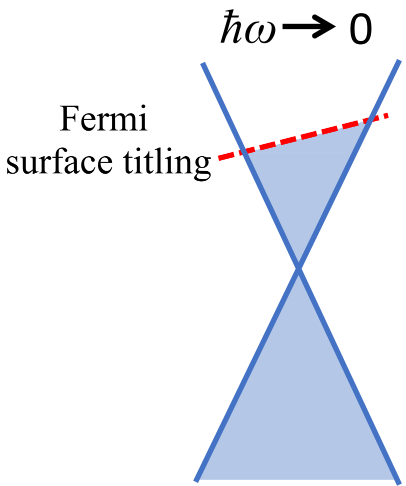


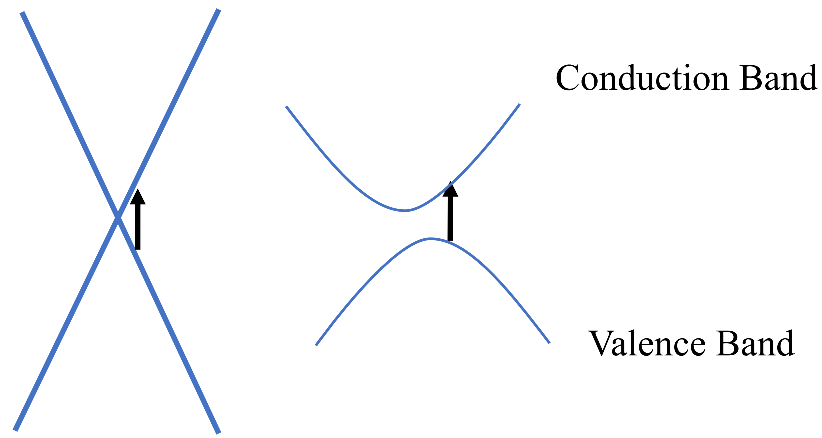
Reversing source-drain voltage flips both the direction of electric field and the net velocity of incident electrons, hence transverse current is preserved, leading to rectification of Hall current.

Nonlinear Response

Devices
Optics
Transport





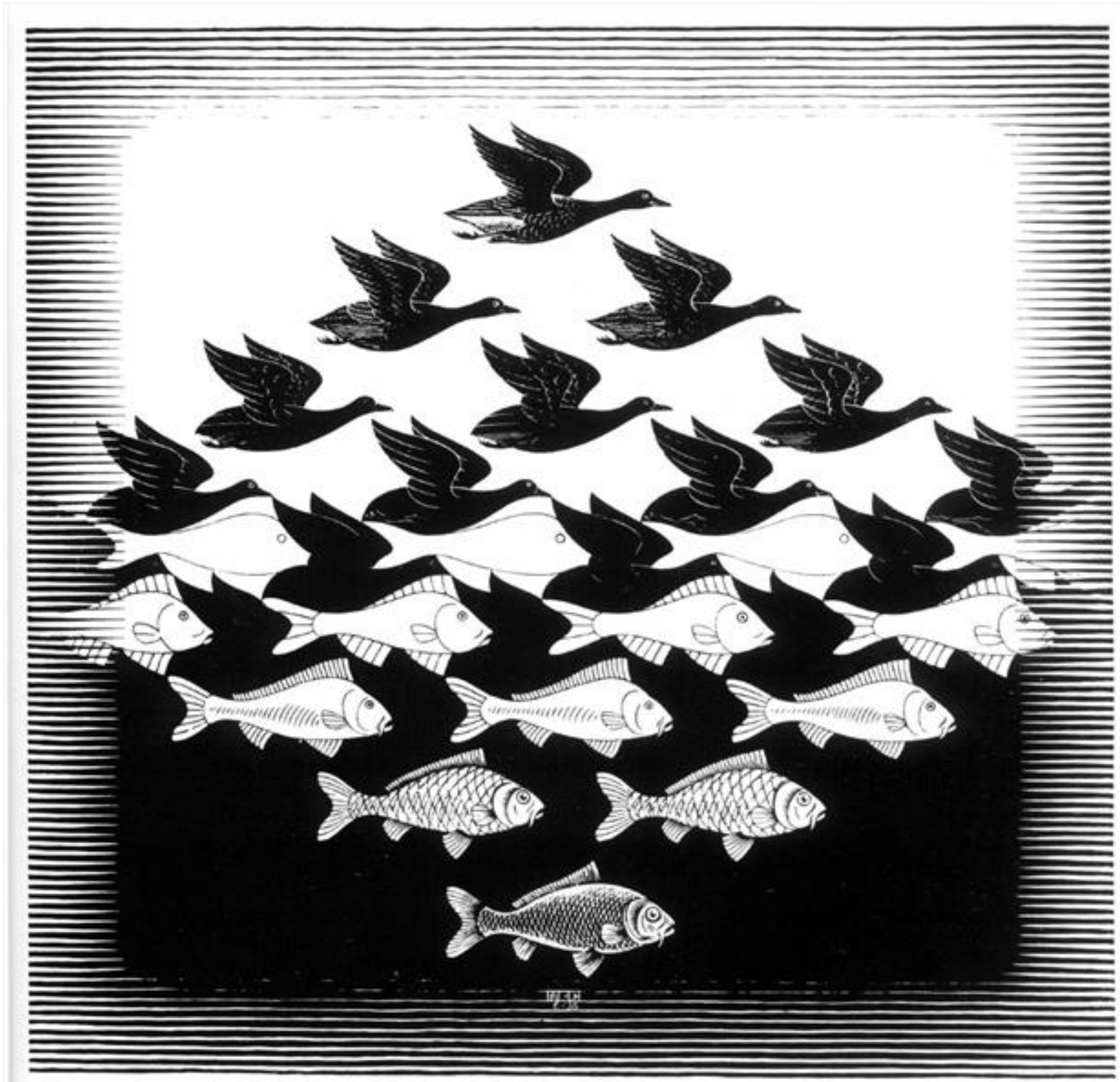


Berry curvature &
Intraband process

Interband transition

microwave and THz rectification and SHG:
application in wireless communication & charging etc

Topological Quantum Matter: From Fantasy to Reality



Escher: sky and water