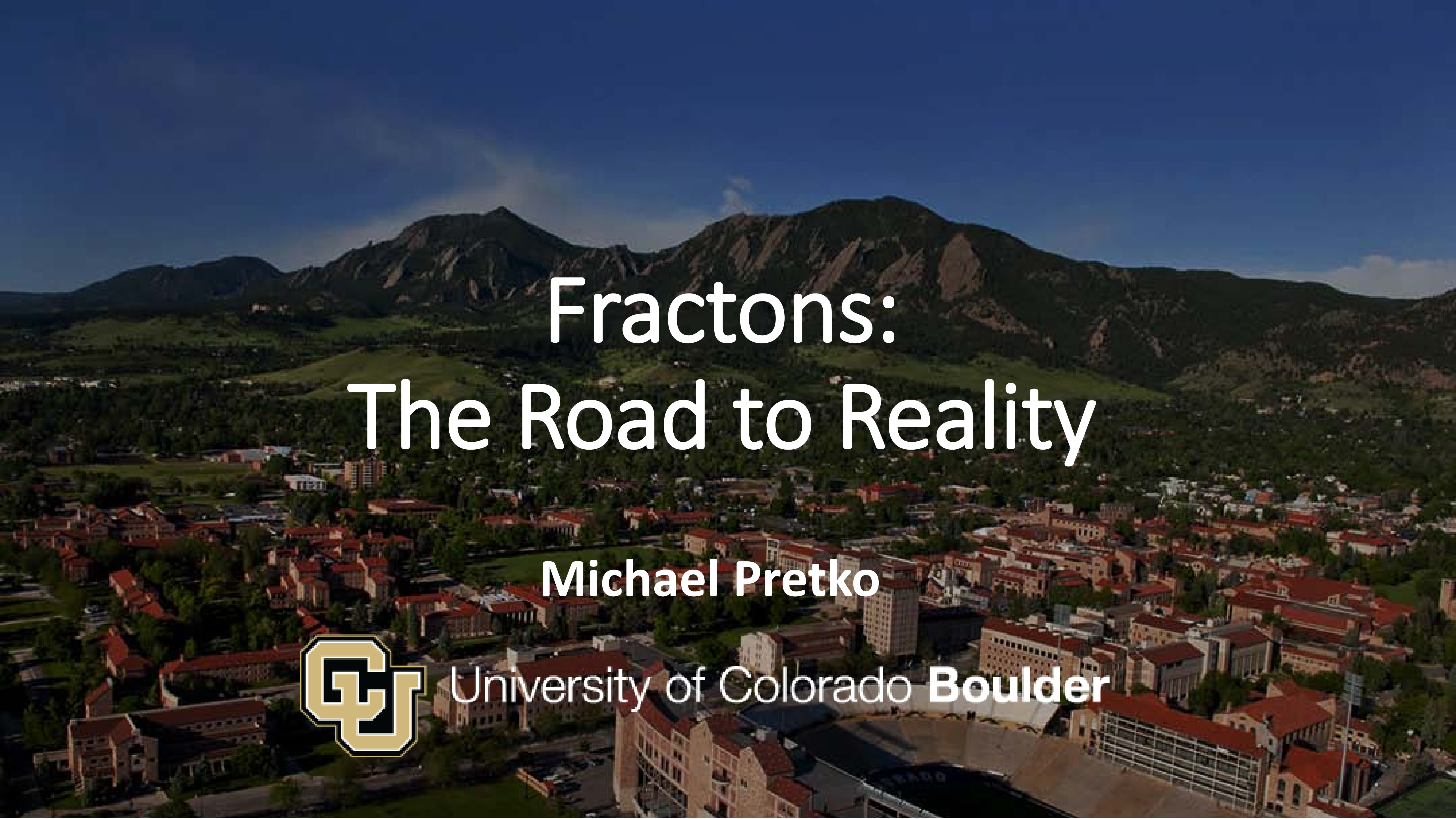


Fractons:
The Road to Reality

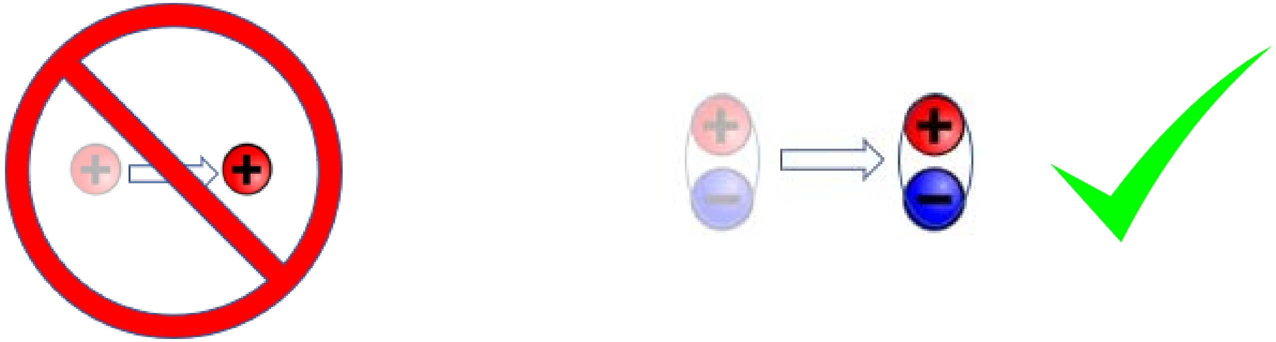
Michael Pretko

University of Colorado **Boulder**

Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

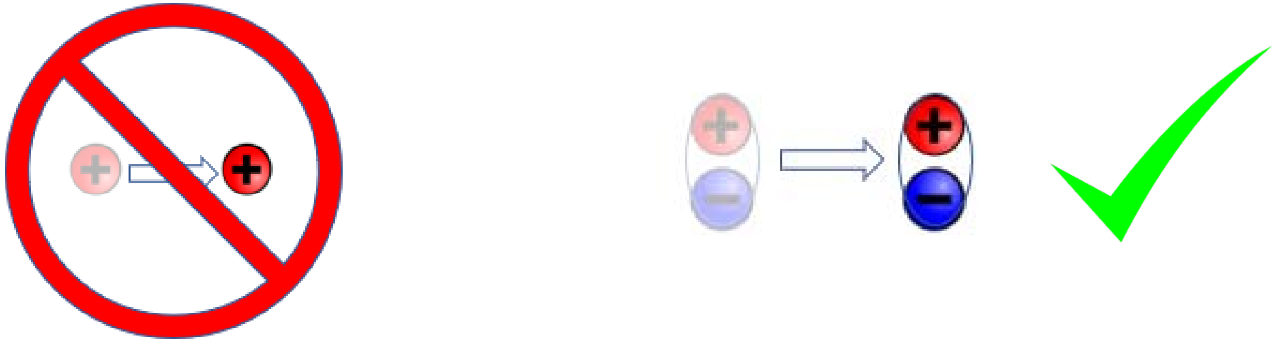
- Fractons are immobile in isolation, but can move collectively



Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

- Fractons are immobile in isolation, but can move collectively



- Mobility restrictions enforced by higher moment conservation laws

Ex: Dipole Conservation

$$\int d^d x \, (\rho \vec{x}) = \text{constant}$$

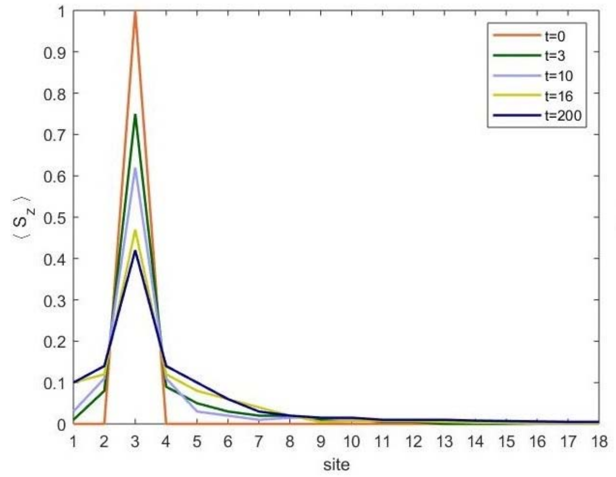
MP, PRB 95, 115139 (2017)

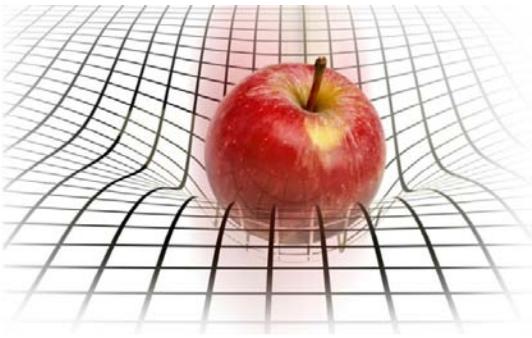
Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

- Exhibit a wide variety of unusual phenomenology
  - Slow thermalization / non-ergodicity
  - Gravitational behavior
  - Potential applications to quantum memory storage

Shriya Pai, **MP**, and
Rahul Nandkishore
PRX 9, 021003 (2019)





MP, PRD 96, 024051 (2017)

Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

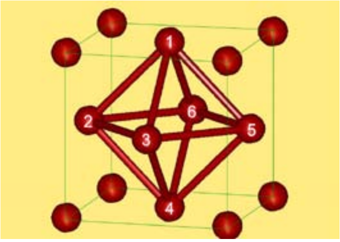
- Exhibit a wide variety of unusual phenomenology
  - Slow thermalization / non-ergodicity
  - Gravitational behavior
  - Potential applications to quantum memory storage

- Deep theoretical connections with various other fields
  - Elasticity theory
  - Higher order topological insulators
  - Holography
  - …

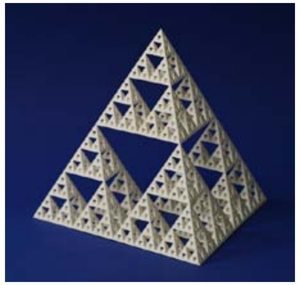
Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

- First encountered in certain exactly-solvable quantum spin models



Chamon Model
PRL 94, 040402 (2005)

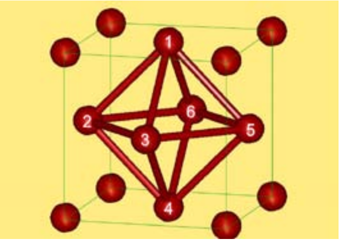


Haah's Code
PRA 83, 042330 (2011)

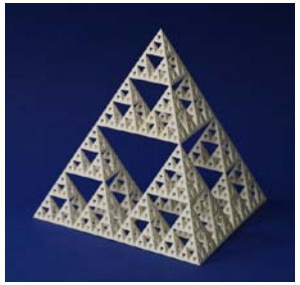
Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

- First encountered in certain exactly-solvable quantum spin models



Chamon Model
PRL 94, 040402 (2005)

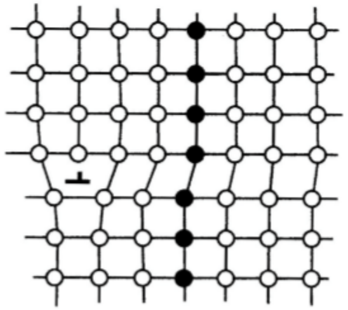


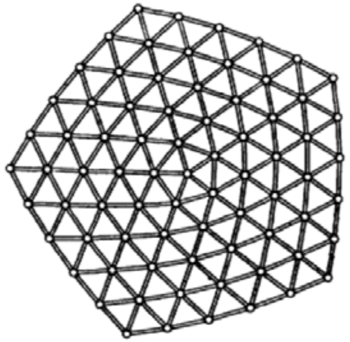
Haah’s Code
PRA 83, 042330 (2011)

- Very complicated! (e.g. 9-spin interactions in Haah’s code)

Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics
- First encountered in certain exactly-solvable quantum spin models
- Concrete realization as topological defects of ordinary crystals





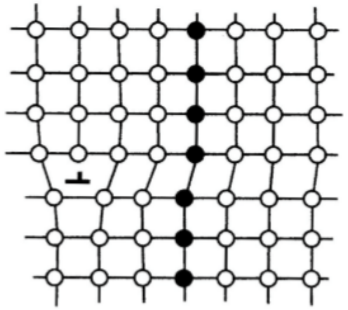
MP, Leo Radzihovsky
PRL 120, 195301

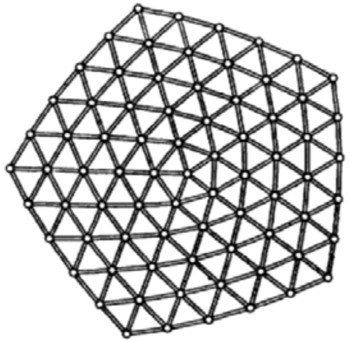
Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics

- First encountered in certain exactly-solvable quantum spin models

- Concrete realization as topological defects of ordinary crystals



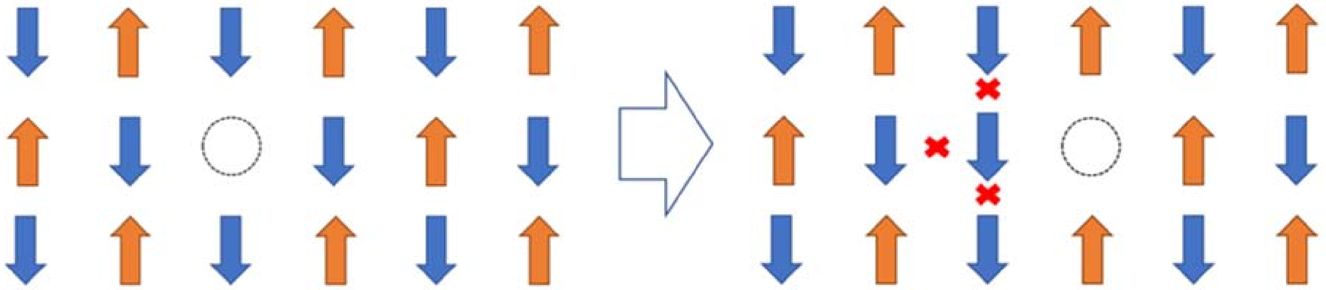


MP, Leo Radzihovsky
PRL 120, 195301

- Fractons are present, but very energetically costly

Fractons: A New Type of Particle

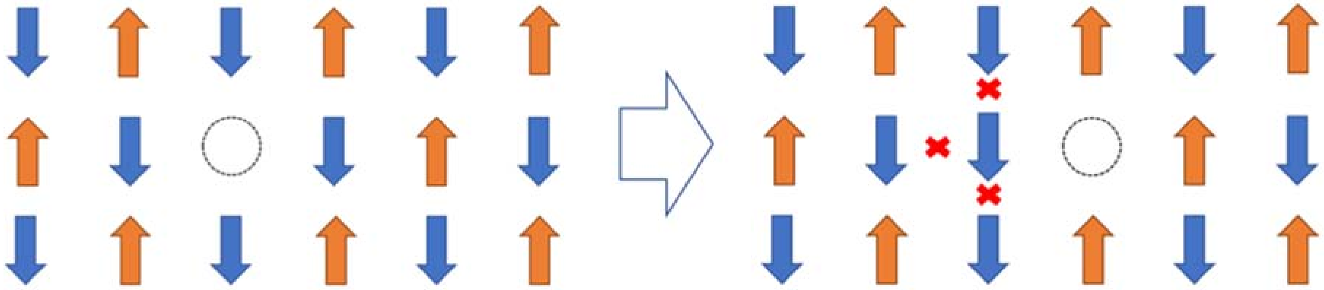
- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics
- First encountered in certain exactly-solvable quantum spin models
- Concrete realization as topological defects of ordinary crystals
- Approximate realization in hole-doped antiferromagnets



John Sous, **MP**
arXiv:1904.08424

Fractons: A New Type of Particle

- Fractons are an exotic type of emergent particle found in certain condensed matter systems, exhibiting severely restricted dynamics
- First encountered in certain exactly-solvable quantum spin models
- Concrete realization as topological defects of ordinary crystals
- Approximate realization in hole-doped antiferromagnets



John Sous, **MP**
arXiv:1904.08424

- Breaks down at 6th order in perturbation theory

Goal of this Conference

Understand “real-world realizations of interacting topological phases” by addressing the following questions:

1. Can putative spin liquids be realized in an experiment and what “smoking-gun” signatures can one expect?
2. What are the spectroscopic footprints of topological matter at finite temperatures?
3. To what extent can one mimic the topological aspects of topological quantum states in classical frameworks, such as mechanical systems and electrical circuits?

Goal of this talk

Understand "real-world realizations" of fracton physics by addressing the following questions:

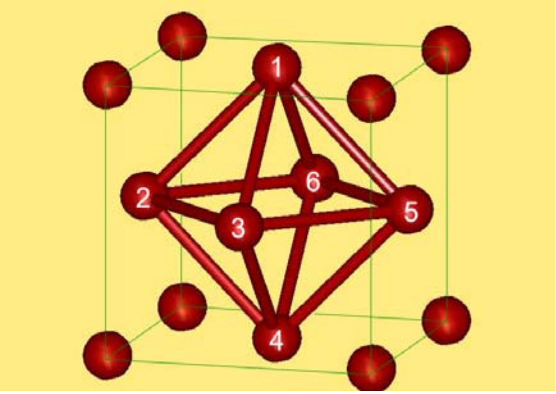
1. Can spin liquids with fracton excitations be realized in experiments and what "smoking-gun" signatures can one expect?
2. What are the signatures of fracton phases at finite temperatures?
3. To what extent can one mimic the mobility restrictions of fractons in classical frameworks, such as electrical circuits?

Part 1:

Towards Realization and Detection of Fractons in Spin Liquids

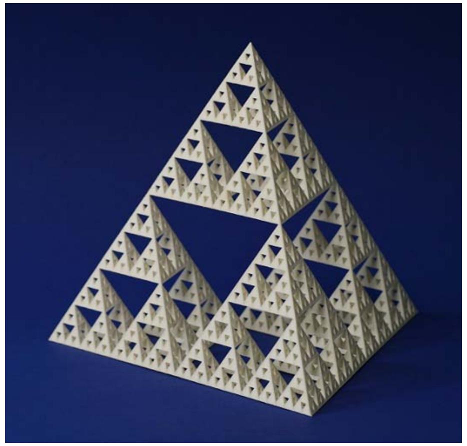
Spin Models

Fractons are realized in a wide variety of exactly-solvable spin models



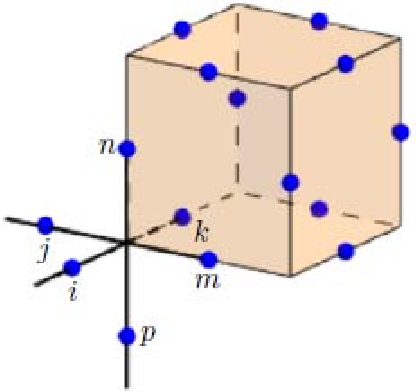
Chamon Model

PRL 94, 040402 (2005)



Haah's Code

PRA 83, 042330 (2011)



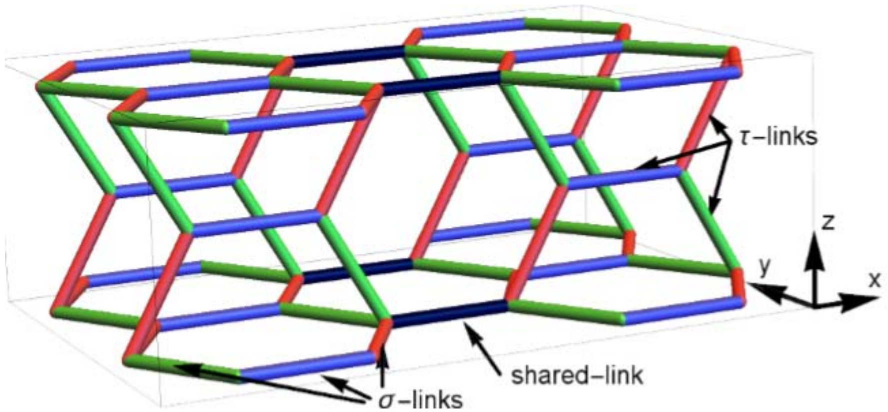
X-cube Model

PRB 94, 235157 (2016)

- Early models all featured complicated beyond-nearest-neighbor multi-spin interactions
- Little hope of realization in materials

Spin Models

Fractons are realized in a wide variety of exactly-solvable spin models



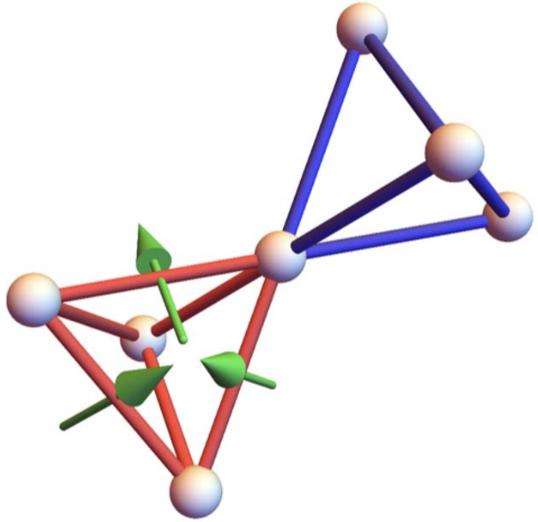
Slagle-Kim Model

PRB 96, 165106 (2017)

- Only features nearest-neighbor two-spin interactions
- No concrete material candidate, but much more realistic

Spin Models

Fractons are realized in a wide variety of exactly-solvable spin models



Yan, Benton, Jaubert, Shannon

arXiv:1902.10934

- “Spin-ice” model on the breathing pyrochlore lattice
- Two-spin nearest-neighbor interactions, including DM interactions
- Identifies certain Yb compounds as potential fracton candidates

Experimental Diagnostics of Fractonic Spin Liquids

- Today’s talk:
  - Pinch point singularities: smoking gun for gapless fracton phases
  - Glassy dynamics / localization without disorder (next section)

Experimental Diagnostics of Fractonic Spin Liquids

- Today’s talk:
  - Pinch point singularities: smoking gun for gapless fracton phases
  - Glassy dynamics / localization without disorder (next section)

- Other signatures for fracton phases:
  - Correlation function diagnostics (nonlocal)
    - Devakul, Parameswaran, Sondhi, PRB 97, 041110
  - Thermal Hall conductance (in certain special cases)
    - Prem, **MP**, Nandkishore, PRB 97, 085116
  - Dynamic spin structure factor?

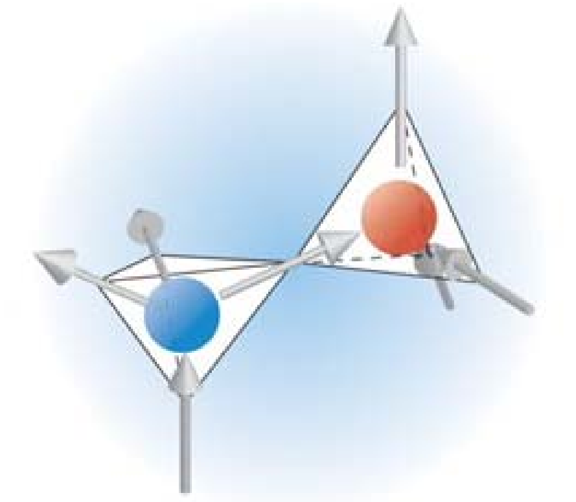
Experimental Diagnostics of Fractonic Spin Liquids

- Today's talk:
  - **Pinch point singularities: smoking gun for gapless fracton phases**
  - Glassy dynamics / localization without disorder (next section)

- Other signatures for fracton phases:
  - Correlation function diagnostics (nonlocal)
    - Devakul, Parameswaran, Sondhi, PRB 97, 041110
  - Thermal Hall conductance (in certain special cases)
    - Prem, **MP**, Nandkishore, PRB 97, 085116
  - Dynamic spin structure factor?

Pinch Point Singularities

Provide clear indication of an emergent gauge theory



Pyrochlore spin ice
(e.g. $Ho_2Ti_2O_7$)

- Example: Spin ice materials exhibit emergent U(1) gauge theory

- Characteristic singularities in spin-spin correlation functions

$$\langle S_z(q)S_z(-q)\rangle = \sum_{ij} C^{ij}\langle E_i(q)E_j(-q)\rangle$$

$$\langle E_x(q)E_y(-q)\rangle \propto q\sin(2\theta)$$

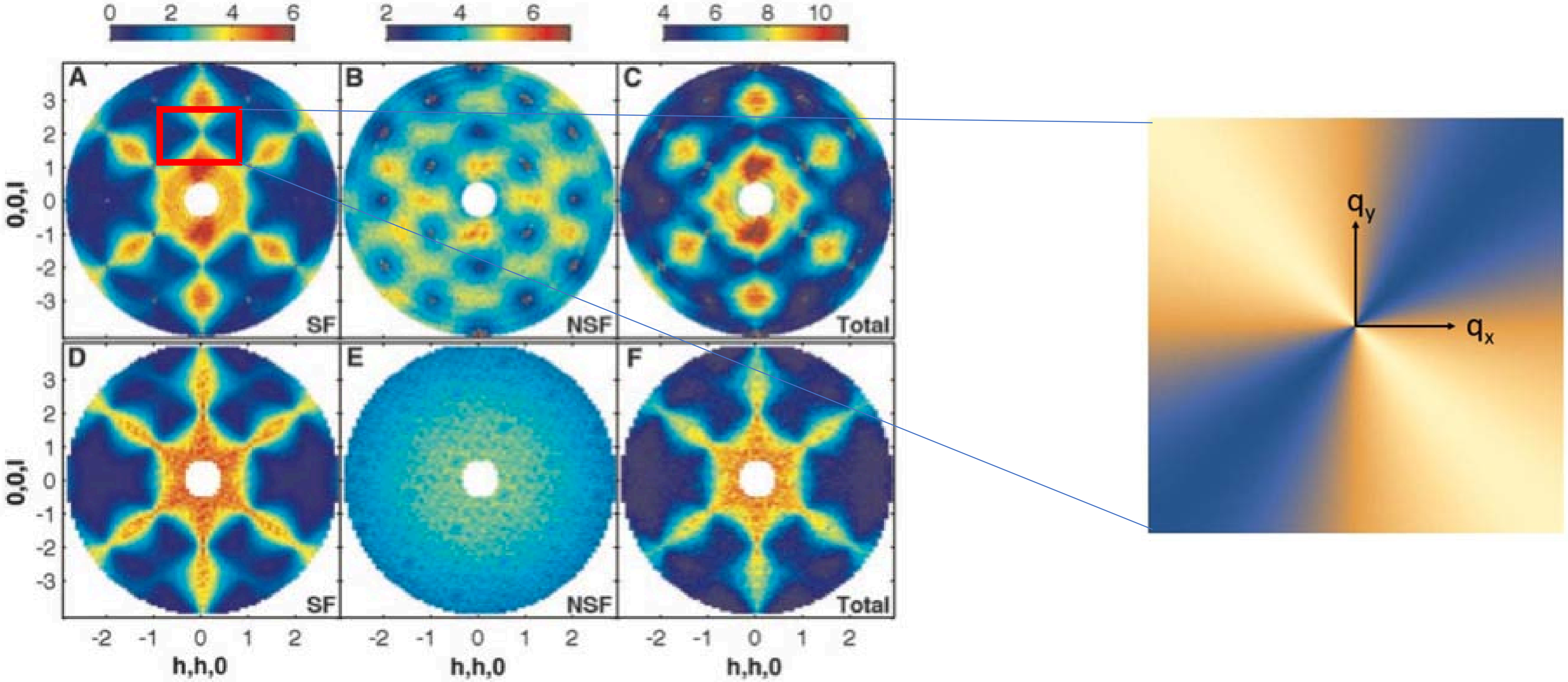
emergent electric field

azimuthal angle

Pinch Point Singularities

Pinch point singularities can be readily observed in polarized neutron scattering data

Fennell et al., Science 326, 5951 (2009)



MP, PRB 95, 115139

Pinch Point Singularities

Fractons are described by symmetric tensor gauge theories

Example: Scalar Charge Theory

- Tensor generalization of Maxwell theory:

$$A_{ij} \qquad E_{ij} \qquad B_{ij}$$

- Modified Gauss's law:

$$\partial_i \partial_j E^{ij} = \rho$$

MP, PRB 95, 115139

Pinch Point Singularities

Fractons are described by symmetric tensor gauge theories

Example: Scalar Charge Theory

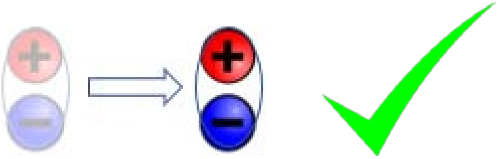
- Conservation laws:

$$Q = \int d^3x \; \rho = \text{ constant}$$

Conservation of charge

$$P^i = \int d^3x \; \rho x^i = \text{ constant}$$

Conservation of dipole moment



Prem, Vijay, Chou, **MP**, Nandkishore, PRB 98, 165140

Pinch Point Singularities

Fractons are described by symmetric tensor gauge theories

Example: Scalar Charge Theory

- Straightforward Calculation:

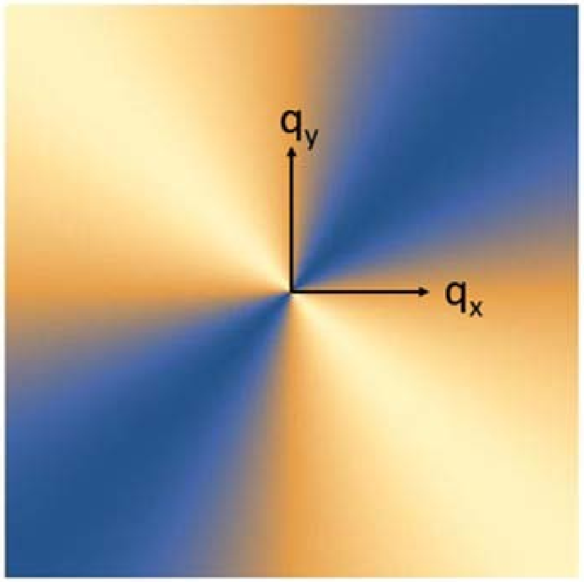
$$\langle E^{ij}(q)E^{k\ell}(-q)\rangle \propto q\left(\frac{1}{2}\left(\delta^{ik}\delta^{j\ell}+\delta^{i\ell}\delta^{jk}\right)-\frac{q^iq^jq^kq^\ell}{q^4}\right)$$

$$\langle E_{xx}(q)E_{yy}(-q)\rangle \propto q\sin(4\theta)$$

Prem, Vijay, Chou, **MP**, Nandkishore, PRB 98, 165140

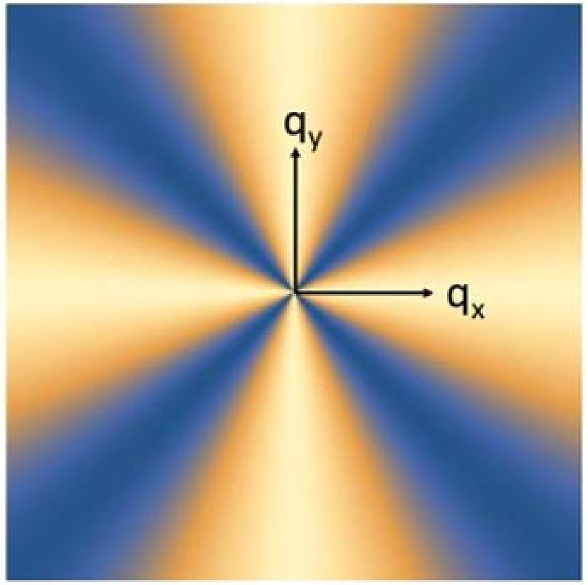
Pinch Point Singularities

Fractonic spin liquids have qualitatively different pinch points from conventional spin ice materials



Conventional U(1) Spin Liquid:

Two-fold symmetry



Rank-2 Tensor Spin Liquid:

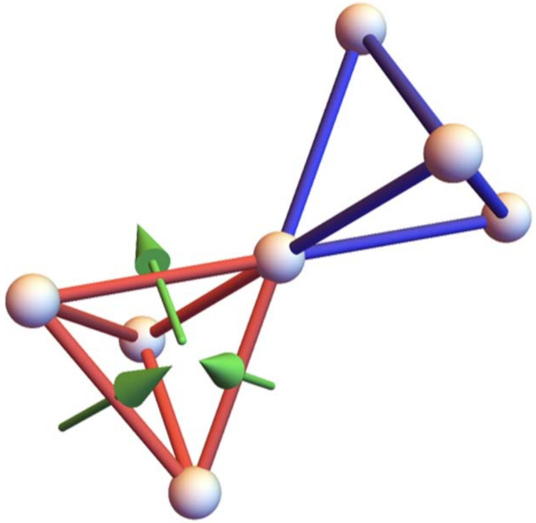
Four-fold symmetry

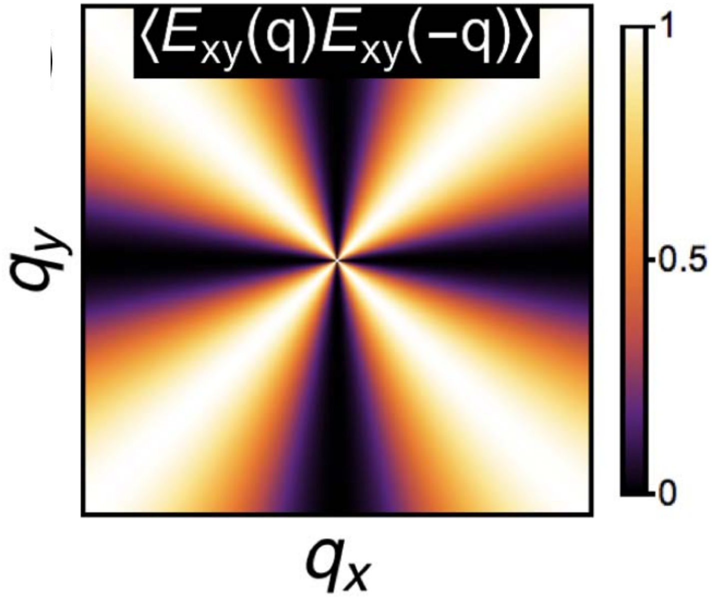
Pinch Point Singularities

Four-fold pinch points numerically observed in material-inspired model on breathing pyrochlore lattice

Yan, Benton, Jaubert, Shannon

arXiv:1902.10934

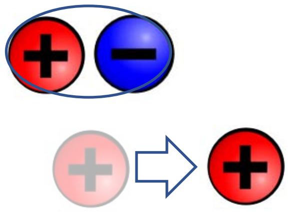




Part 2:
Finite-Temperature Behavior of Fractons

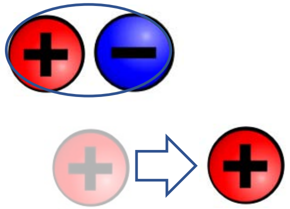
Glassy Dynamics

- Dipole conservation severely restricts motion of particles. However, fractons can move through interactions with thermal dipoles



Glassy Dynamics

- Dipole conservation severely restricts motion of particles. However, fractons can move through interactions with thermal dipoles

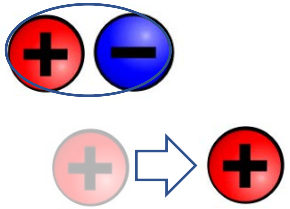


- In 3d, interactions eventually cause the system to thermalize, BUT:
  - Logarithmically slow relaxation to equilibrium
  - Glassy dynamics without disorder

Prem, Haah, Nandkishore,
PRB 95, 155133 (2017)

Glassy Dynamics

- Dipole conservation severely restricts motion of particles. However, fractons can move through interactions with thermal dipoles



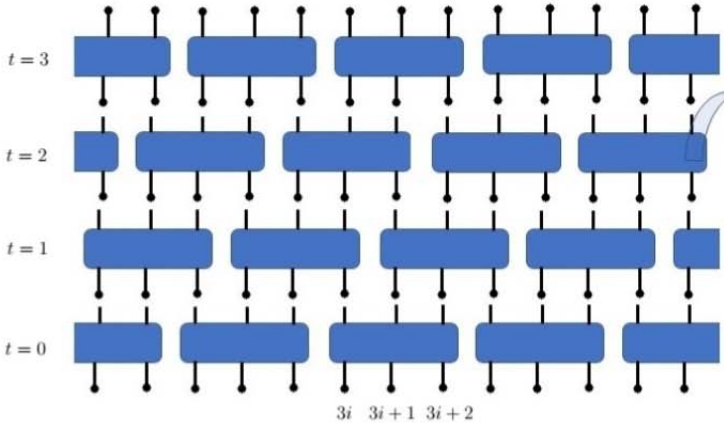
- In 3d, interactions eventually cause the system to thermalize, BUT:
  - Logarithmically slow relaxation to equilibrium
  - Glassy dynamics without disorder
- In certain systems (e.g. Haah’s code), relaxation time is superexponential in the inverse temperature
  - At low temperatures, can hold memory of initial conditions for longer than the age of the universe

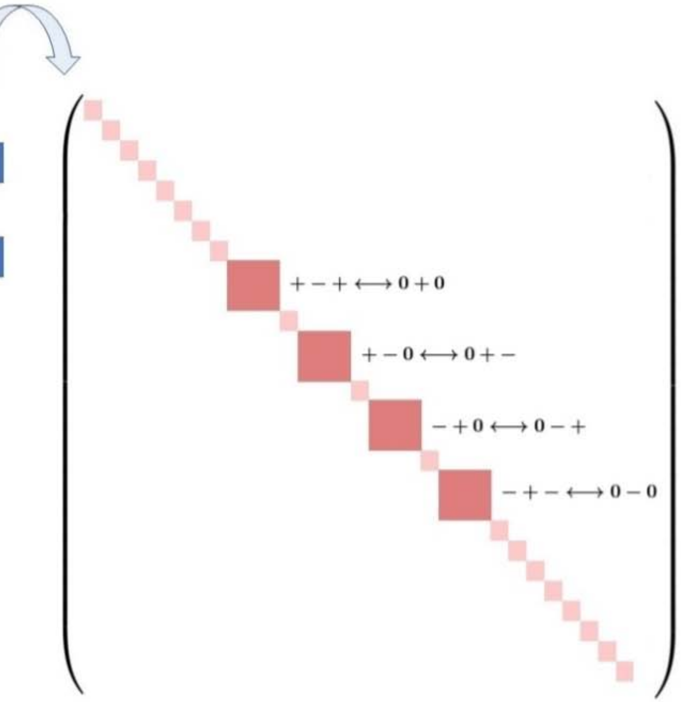
Prem, Haah, Nandkishore,
PRB 95, 155133 (2017)

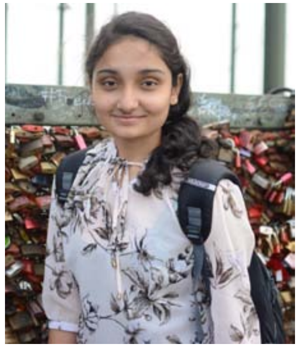
Non-Ergodic Dynamics

- In one dimension, certain fracton systems can maintain a permanent memory of their initial conditions

- Minimal model: random unitary circuit with 3-site gates exhibiting fracton conservation laws





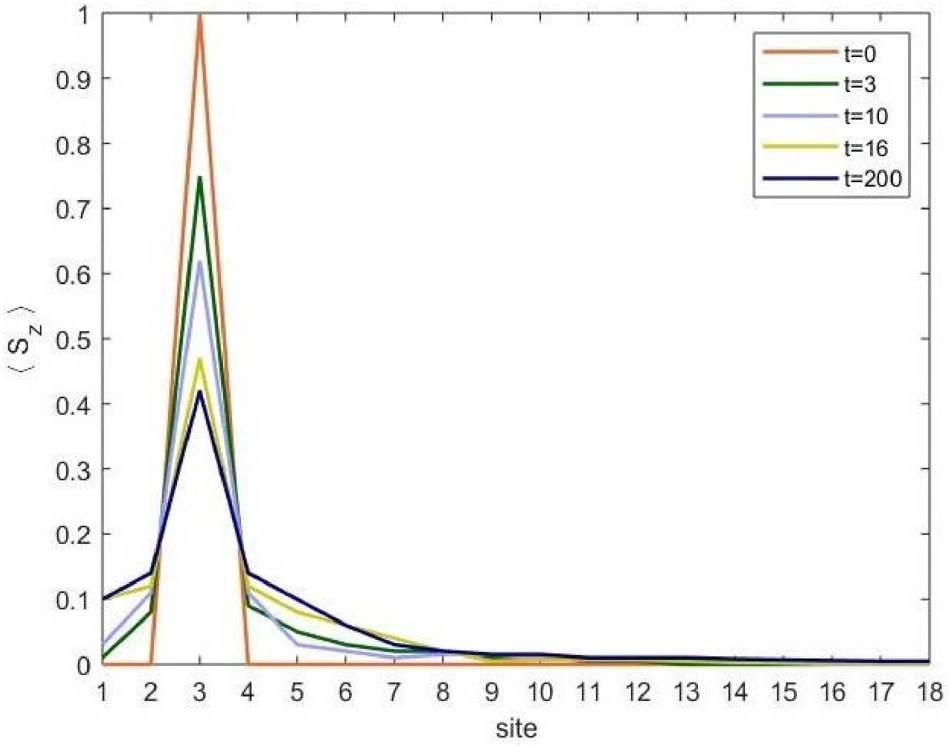


Shriya Pai, **MP**, and
Rahul Nandkishore
PRX 9, 021003 (2019)

- Spin-1 system
- Conserved “charge” (S_z) and dipole moment

Non-Ergodic Dynamics

- In one dimension, certain fracton systems can maintain a permanent memory of their initial conditions



- Minimal model features permanent peak in S_z expectation value at initial location of fracton

- Perfect localization can only be disrupted by third-nearest-neighbor interactions

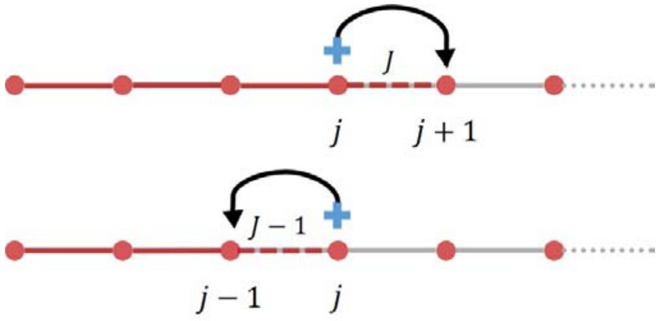
Platforms for Realization

- External linear potential, $V(x) \sim x$, can lead to conserved dipole moment
  - Theoretical support for non-ergodic behavior with strong potential
    - van Niewenburg, Baum, Refael (PNAS, 2019)
  - Experimental results indicate sub-diffusive behavior with weak coupling
    - Guardado-Sanchez et al. (arXiv:1909.05848)

Platforms for Realization

- External linear potential, $V(x) \sim x$, can lead to conserved dipole moment
    - Theoretical support for non-ergodic behavior with strong potential
        - van Niewenburg, Baum, Refael (PNAS, 2019)
    - Experimental results indicate sub-diffusive behavior with weak coupling
        - Guardado-Sanchez et al. (arXiv:1909.05848)

- Mapping between 1d confining models and fracton Hamiltonians
    - Shriya Pai and **MP** (arXiv:1909.12306)



- Known to exhibit non-ergodic behavior, such as many-body scars
    - e.g. James, Konik, Robinson (PRL 2019)

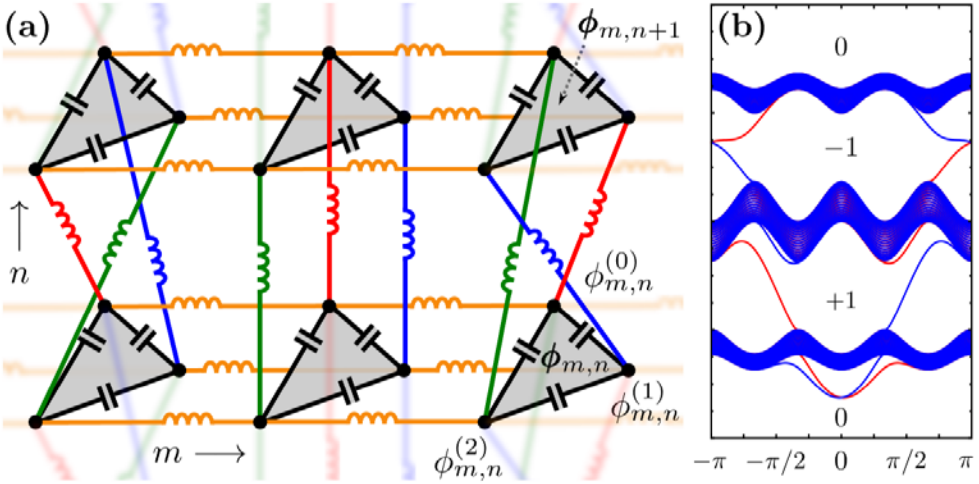
Part 3:

Fractons in Classical Electric Circuits


Michael Pretko
presents
FUN WITH
Fractons
REC

Topological Physics in Electric Circuits

- Physics of topological insulators can be mimicked in classical AC circuits

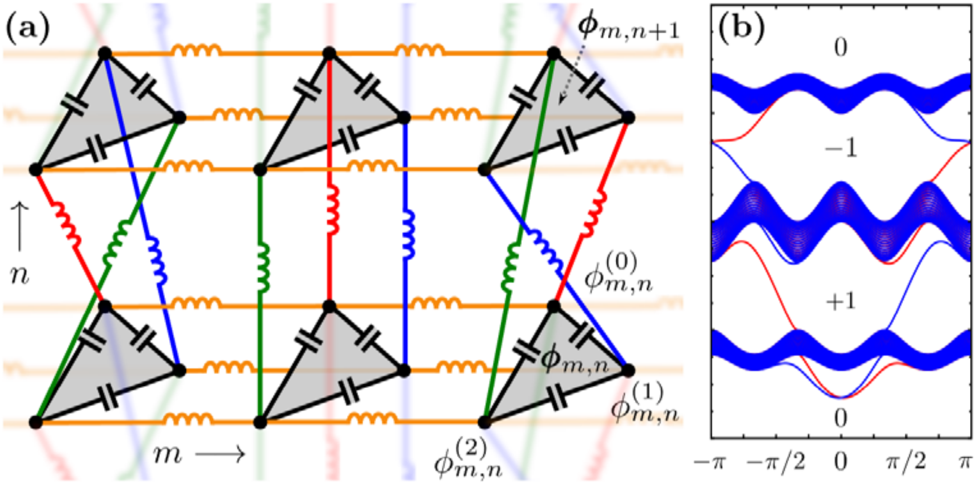


Albert, Glazman, Jiang, PRL (2015)
Jia et al., PRX (2015)

- Lattice of capacitors and inductors can give rise to topological admittance bands, hosting robust edge modes

Topological Physics in Electric Circuits

- Physics of topological insulators can be mimicked in classical AC circuits



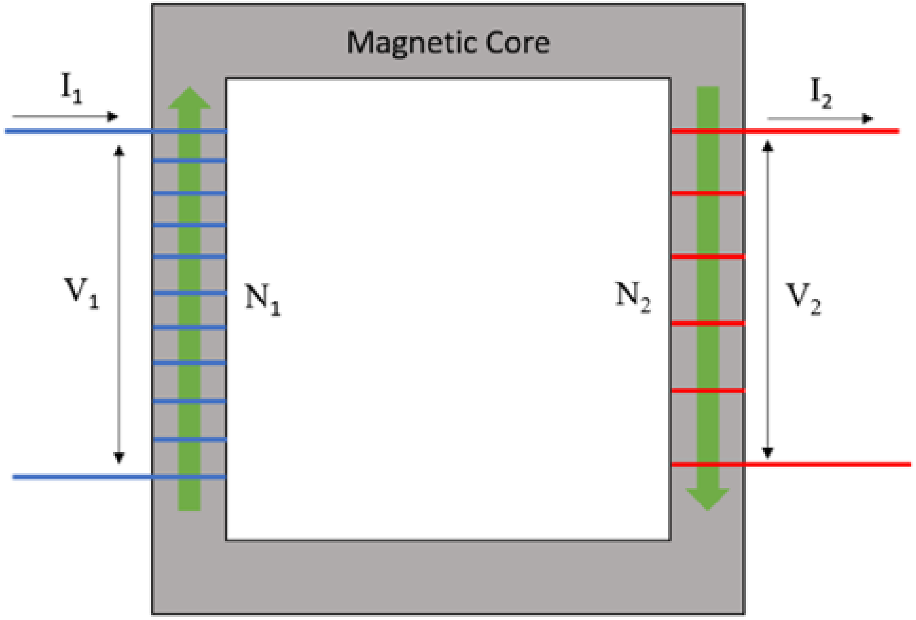
- Lattice of capacitors and inductors can give rise to topological admittance bands, hosting robust edge modes

- Can classical circuits also mimic the behavior of fractons?

MP, arXiv:1908.08536

Fracton Physics in Electric Circuits

- Conservation of dipole moment can be enforced by transformers



- Ideal transformer:

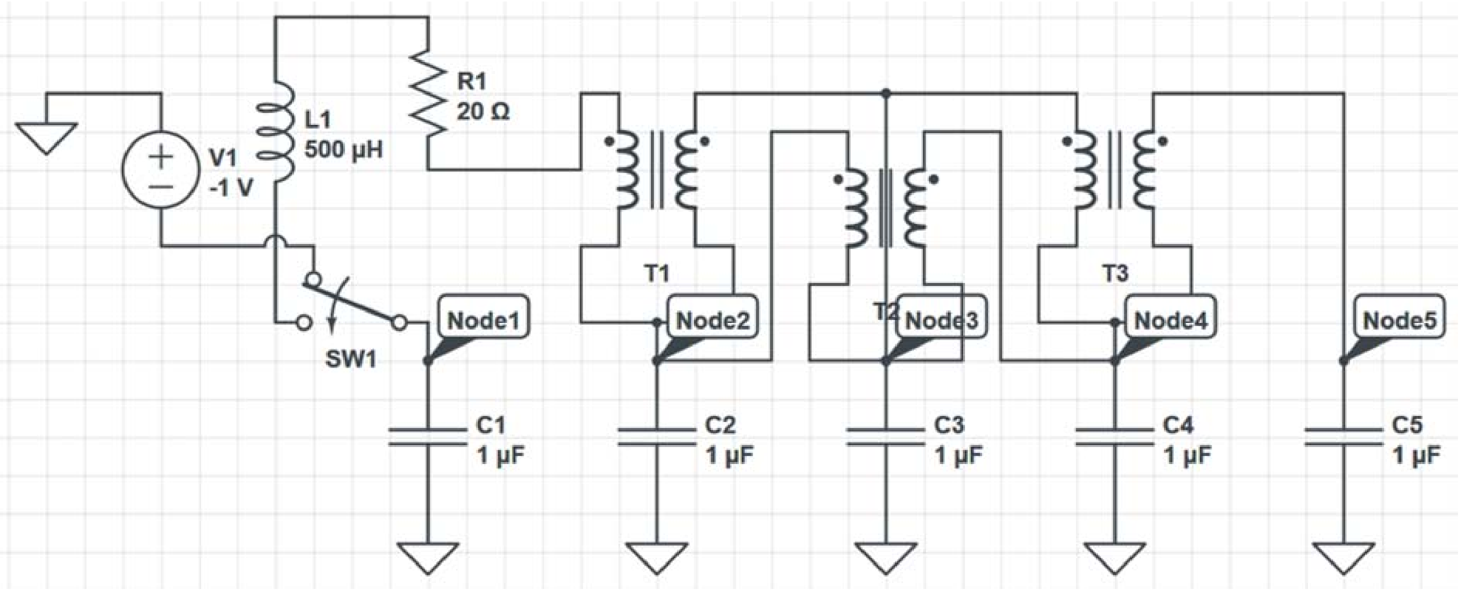
$$\frac{I_1}{I_2} = \frac{N_1}{N_2}$$

- Choosing $N_1 = -N_2$ results in perfect counterflow of current in the two wires

MP, arXiv:1908.08536

Fracton Physics in Electric Circuits

- Lattice of capacitors connected by transformers exhibits conserved dipole

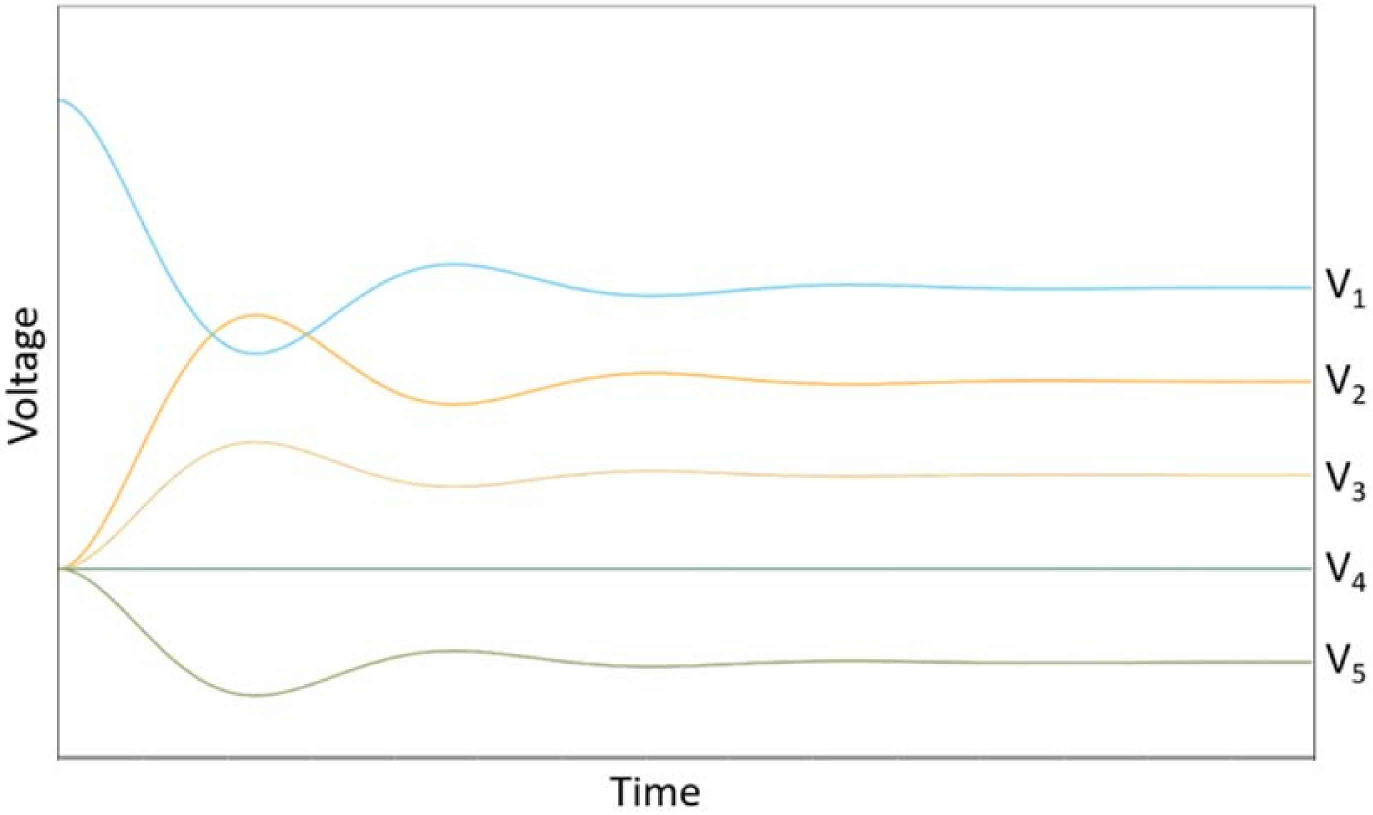


$$\sum_n Q_n x_n = \text{constant}$$

MP, arXiv:1908.08536

Fracton Physics in Electric Circuits

- Equilibrium voltage distribution takes characteristic linear form



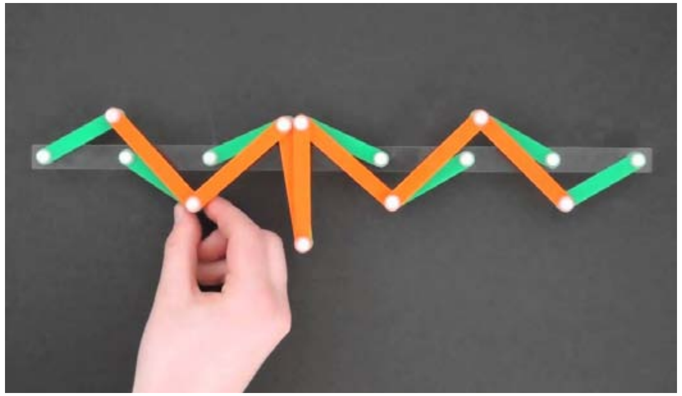
Direct signature of conservation of dipole moment

Fracton Physics in Electric Circuits

- To the future:
  - Can quantum fracton models be directly simulated in superconducting quantum circuits?

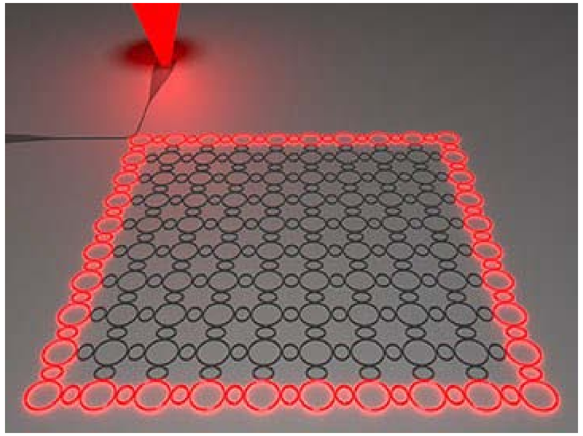
Fracton Physics in Electric Circuits

- To the future:
  - Can quantum fracton models be directly simulated in superconducting quantum circuits?
  - Can fractons be mimicked on other physical platforms?



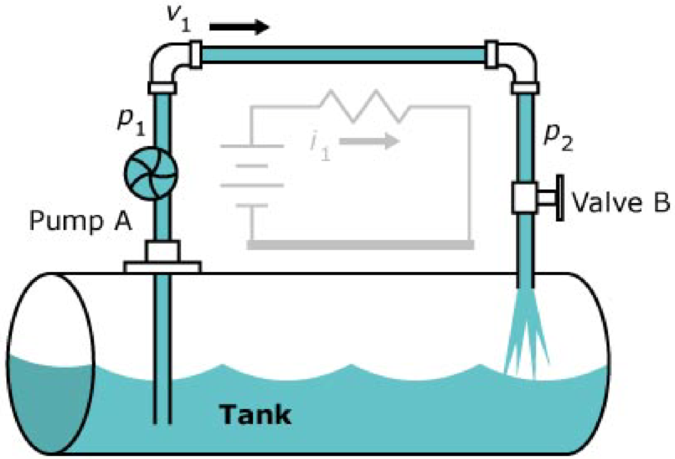
Mechanical systems?

(Vitelli)



Photonics?

(Wittek and Bandres)



Hydraulics?

(H. Johnson)

Summary

- Fractons are on the cusp of realization in material systems, but some final pushes are required
  - Earliest models are either cumbersome or unrealistic
  - Newer models are starting to connect to experiments

- Fractons can be detected by various diagnostics, such as pinch-point singularities and restricted thermalization

- Fracton physics can be engineered on various platforms
  - Electric circuits
  - Linear potentials (e.g. confinement)
  - …