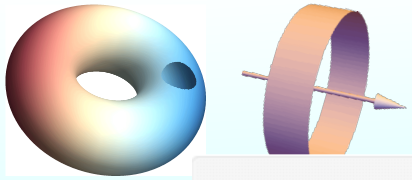
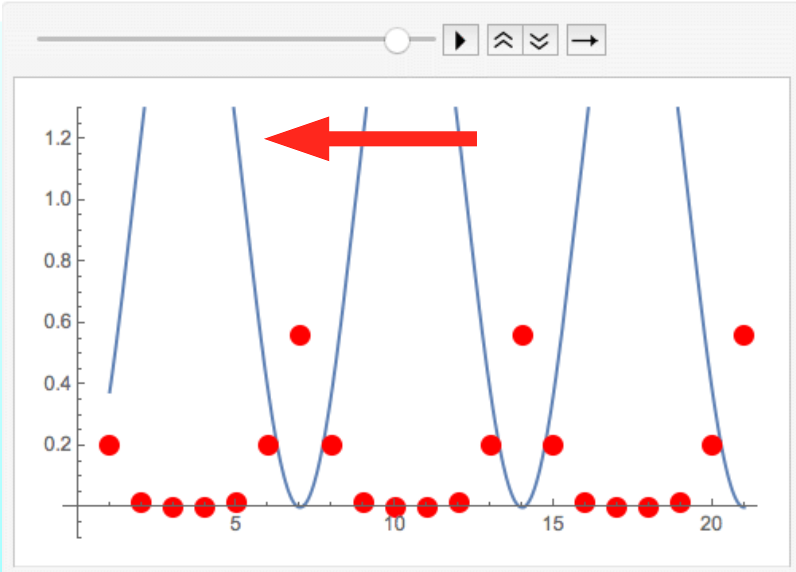
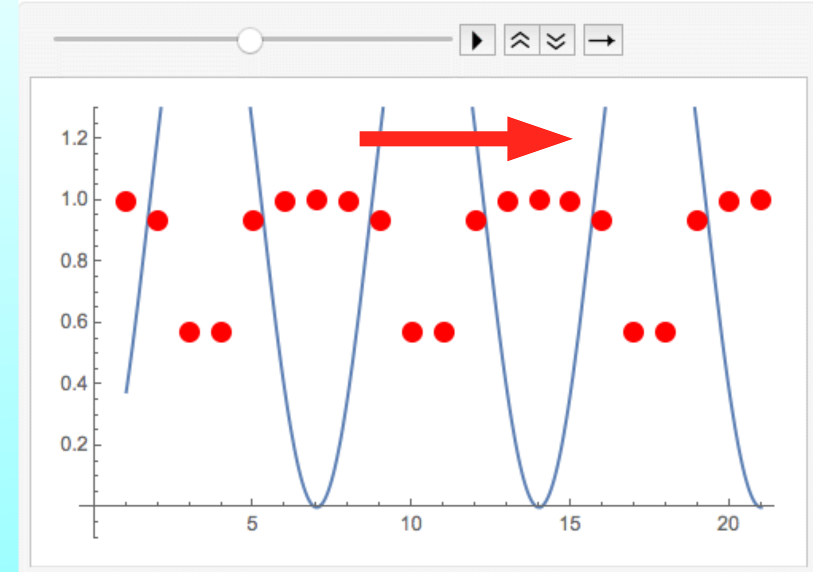
Topoquant, KITP, Oct. 14 (2016)



OSLO INTERNATIONAL SCHOOL





juggling at each site

Bulk-edge corresponding in topological pumping

A simple reason of quantization of pumped charge

Y. Hatsugai

Univ. Tsukuba

"Bulk-edge correspondence in a topological pumping", Y.Hatsugai & T. Fukui, Phys. Rev. B 94, 041102(R), (2016)

Plan

- **Topological pumping**
  - Back to Thouless
  - Time as a synthetic dimension of QHE
  - Experimental realizations after 30 years
  - Edge states ?
- **Pumped charge & Berry connection**
  - Pumped charge, Berry connection & Temporal gauge
- **Pumped charge & edge states**
  - Temporal gauge & center of mass (CM)
  - Singular motion of CM
  - The Chern number & BEC
- **Observations**
  - Adiabatic & non-adiabatic
  - Direct simulations

Adiabatic pump (Thouless '83)

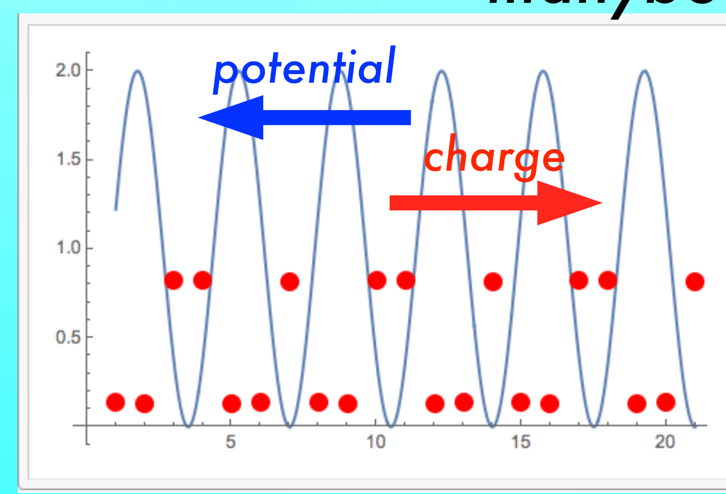
Periodically driven1D charge transport

Many-body but non-interacting as IQHE

$$i\hbar\partial_t|G(t)\rangle = H(t)|G(t)\rangle \quad |G(t)\rangle = Te^{-(i/\hbar)\int_{t_0}^{t} d\tau H(\tau)}|G(t_0)\rangle$$

$$H(t) = \sum_j^L \left[-t_x c_{j+1}^\dagger c_j + h.c. + v_j(t) c_j^\dagger c_j\right]$$

free fermion
manybody



$$v_j(t+T) = v_j(t) \quad \text{period } T$$

ex. $v_j(t) = -2t_y \cos(2\pi \frac{t}{T} - 2\pi\phi j)$

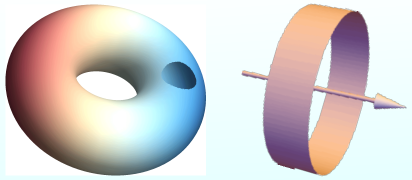
$$\phi = p/q$$

Adiabatic : ground state is gapped & slow pumping

$$\boxed{\Delta E \gg \hbar/T}$$

Topological !

Pumped charge is quantized as an integer



Topoquant, KITP, Oct. 14 (2016)

Back to Thouless '83

Time dependent 1D charge transport

PHYSICAL REVIEW B VOLUME 27, NUMBER 10 15 MAY 1983

1+1=2

Time as a synthetic dimension

Quantization of particle transport

D. J. Thouless

Department of Physics, FM-15, University of Washington, Seattle, Washington 98195

(Received 4 February 1983)

2D Integer quantum Hall effect

TKNN '82 Hall conductance by the Chern number

Brouwer '98 Wang-Troyer-Dai '13

… Marra-Citro-Ortix '15

Experimentally realized in cold atoms after 30+ years in '15

Y.Takahashi, Kyoto

Nakajima et al.,

Nature Phys. 12, 296 (2016)

Topological Thouless Pumping of Ultracold Fermions

Shuta Nakajima, Takafumi Tomita, Shintaro Taie, Tomohiro Ichinose, Hideki Ozawa, Lei Wang, Matthias Troyer, Yoshiro Takahashi

I. Bloch, Munich

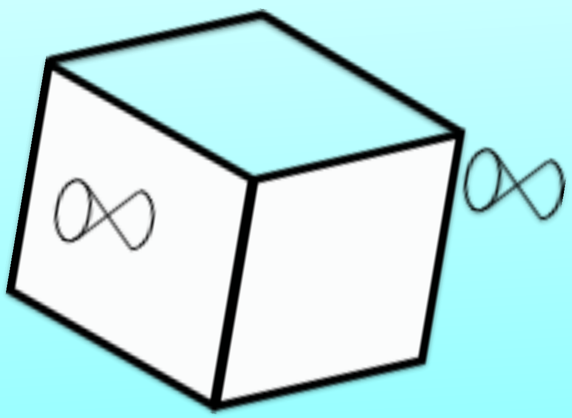
Lohse et al.,

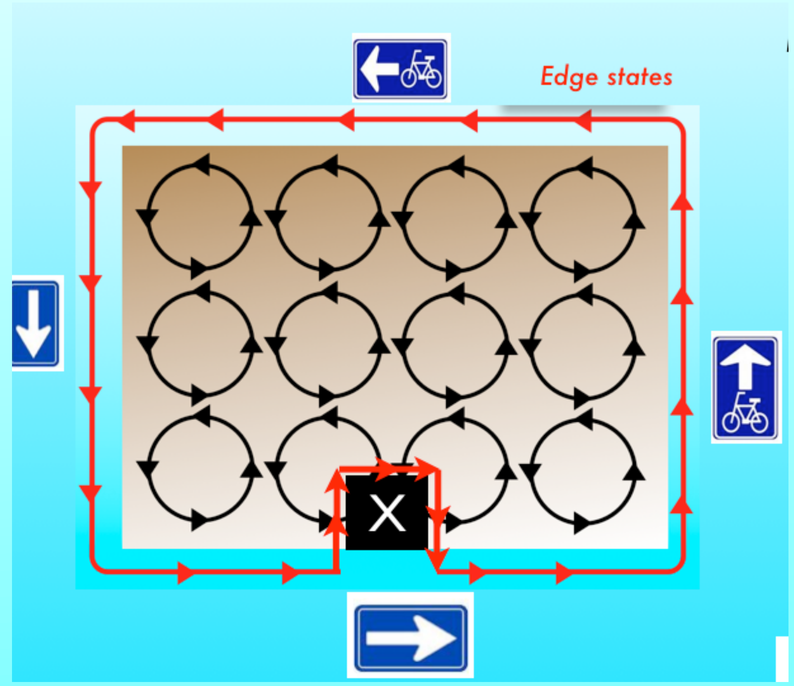
Nature Phys. 12, 350 (2016)

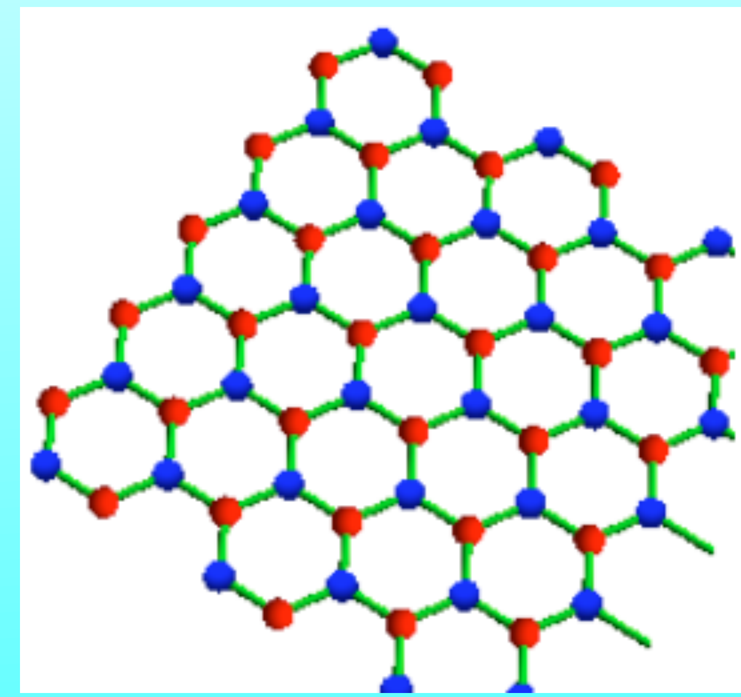
A Thouless Quantum Pump with Ultracold Bosonic Atoms in an Optical Superlattice

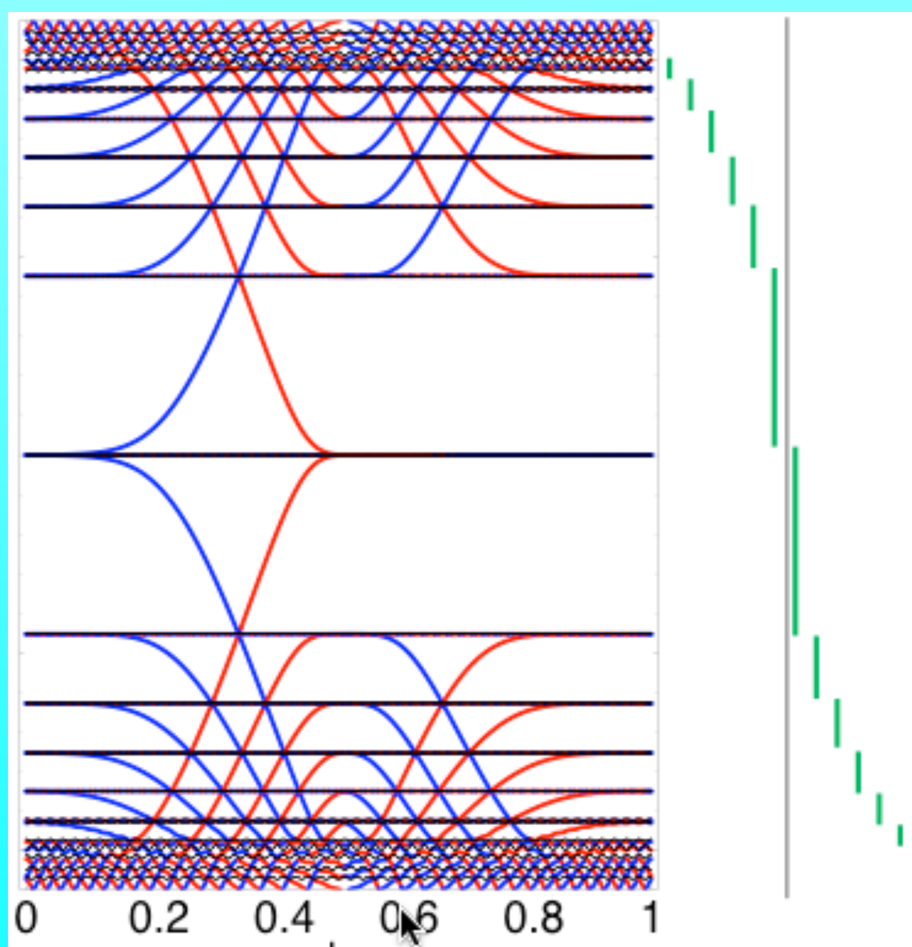
Michael Lohse, Christian Schweizer, Oded Zilberberg, Monika Aidelsburger, Immanuel Bloch

If topological, then edge states ?







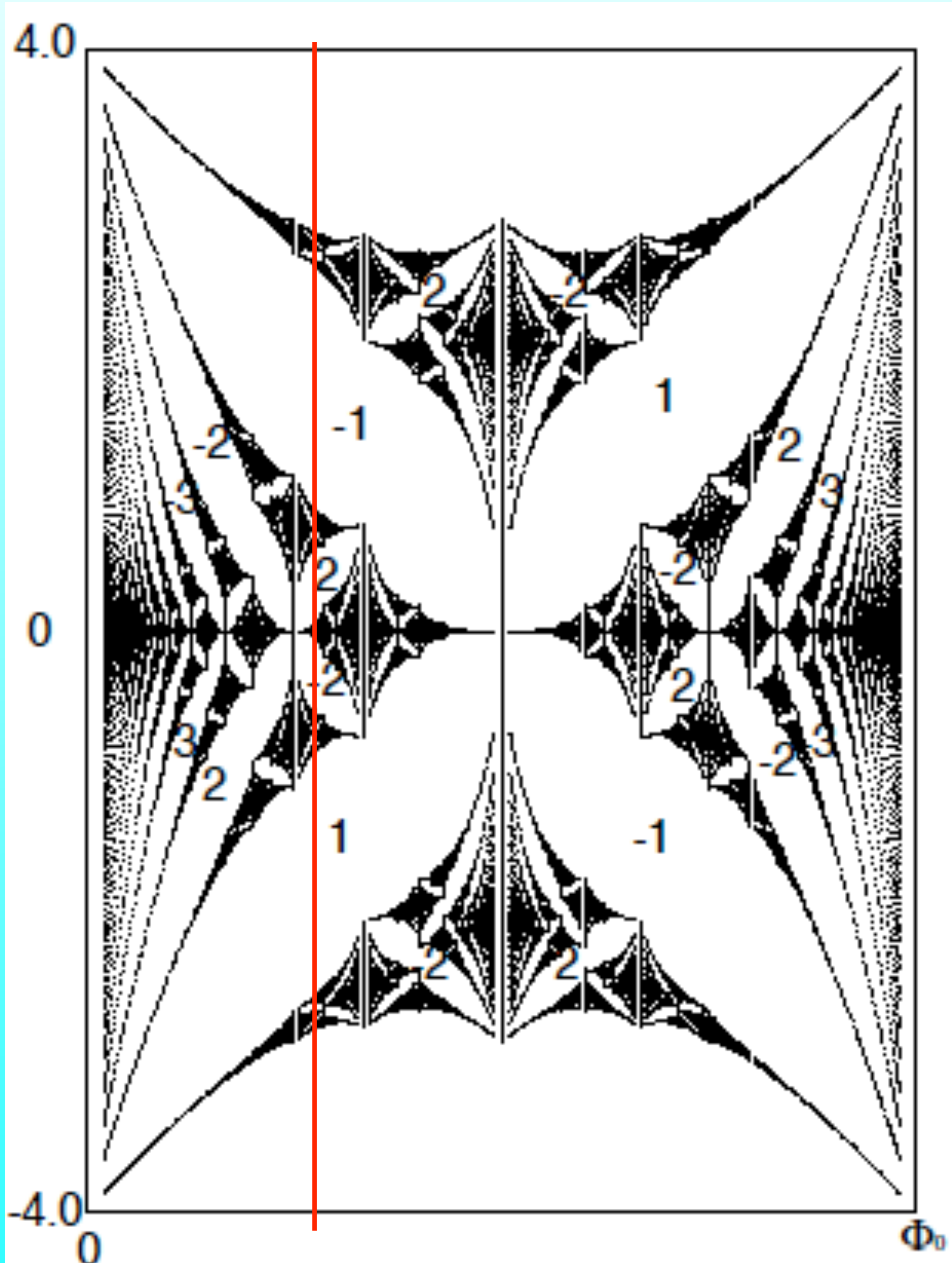


Not much for the topological pump

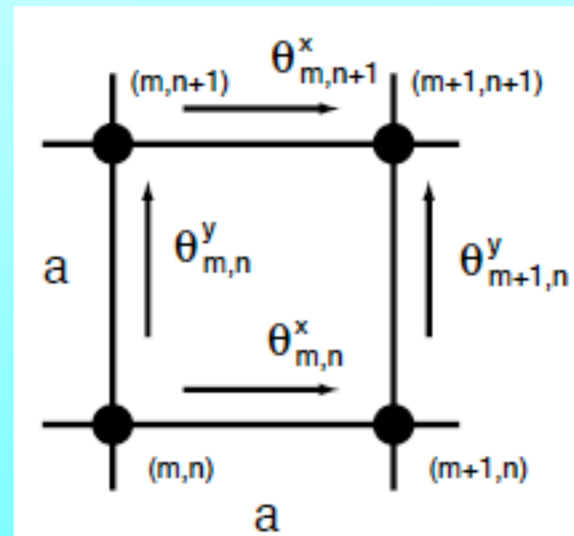
Try to revisit the old problem

- *More than reinterpretation*
- *New view points even technically*

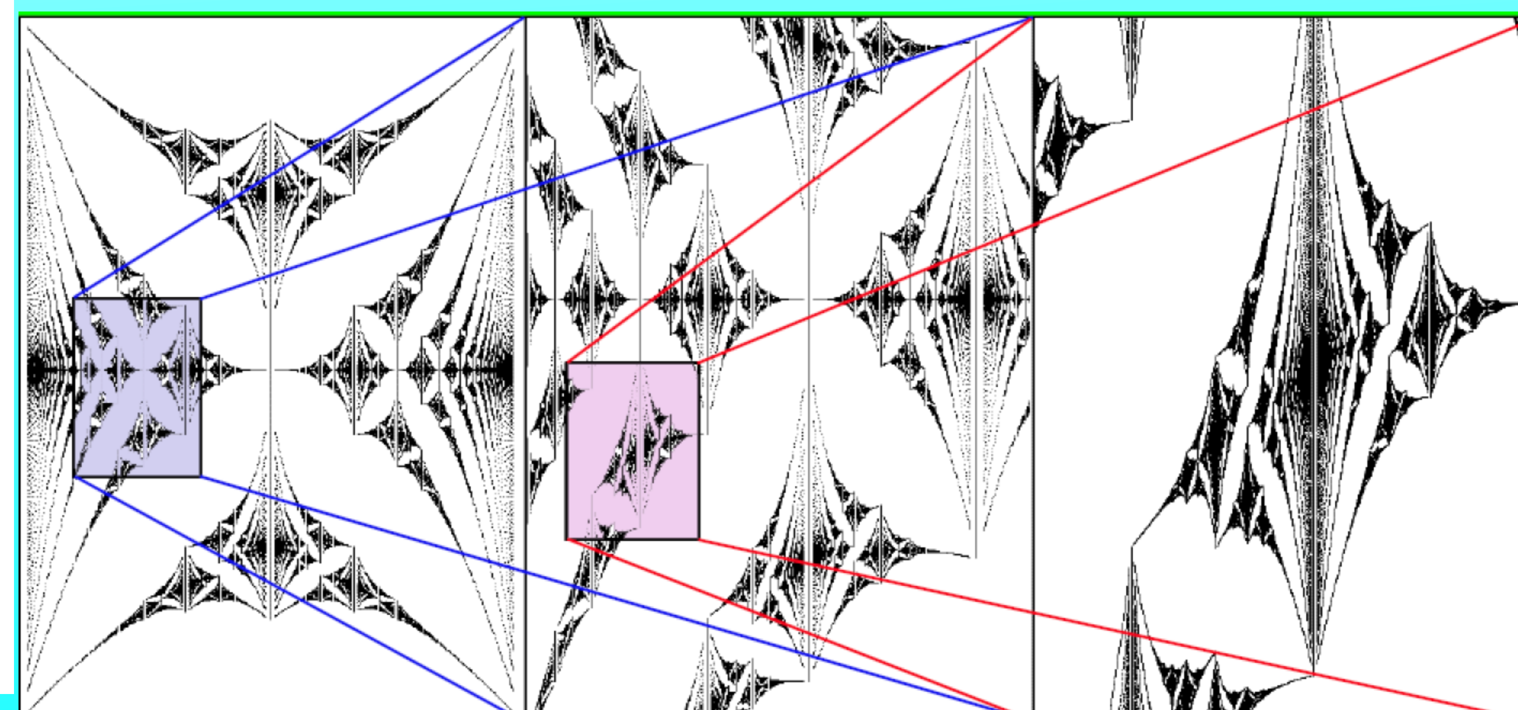
QHE (Hofstadter's)

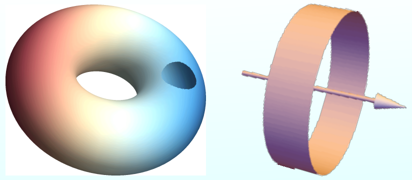


$\phi = 2/7$

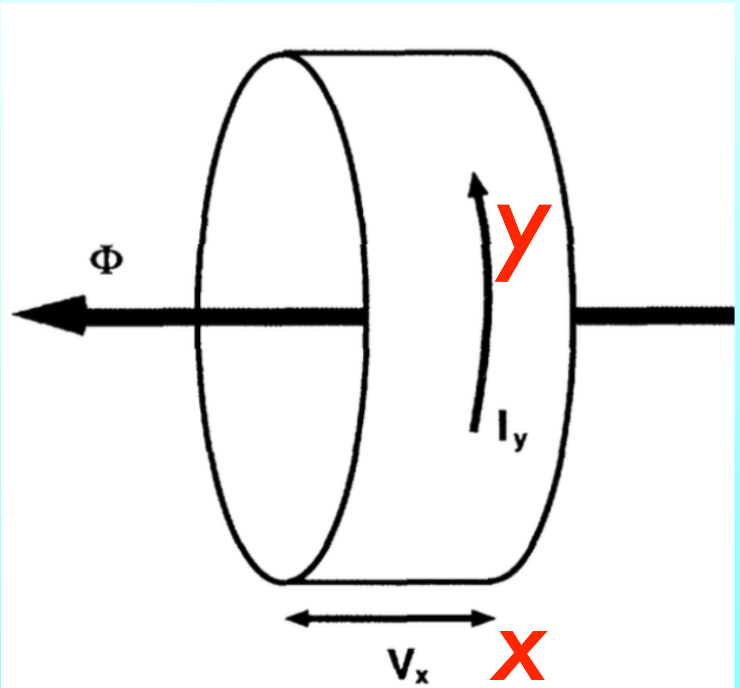


$$\sum_{j\in\langle ij\rangle} \theta_{ij} = 2\pi\phi_i = 2\pi\phi$$





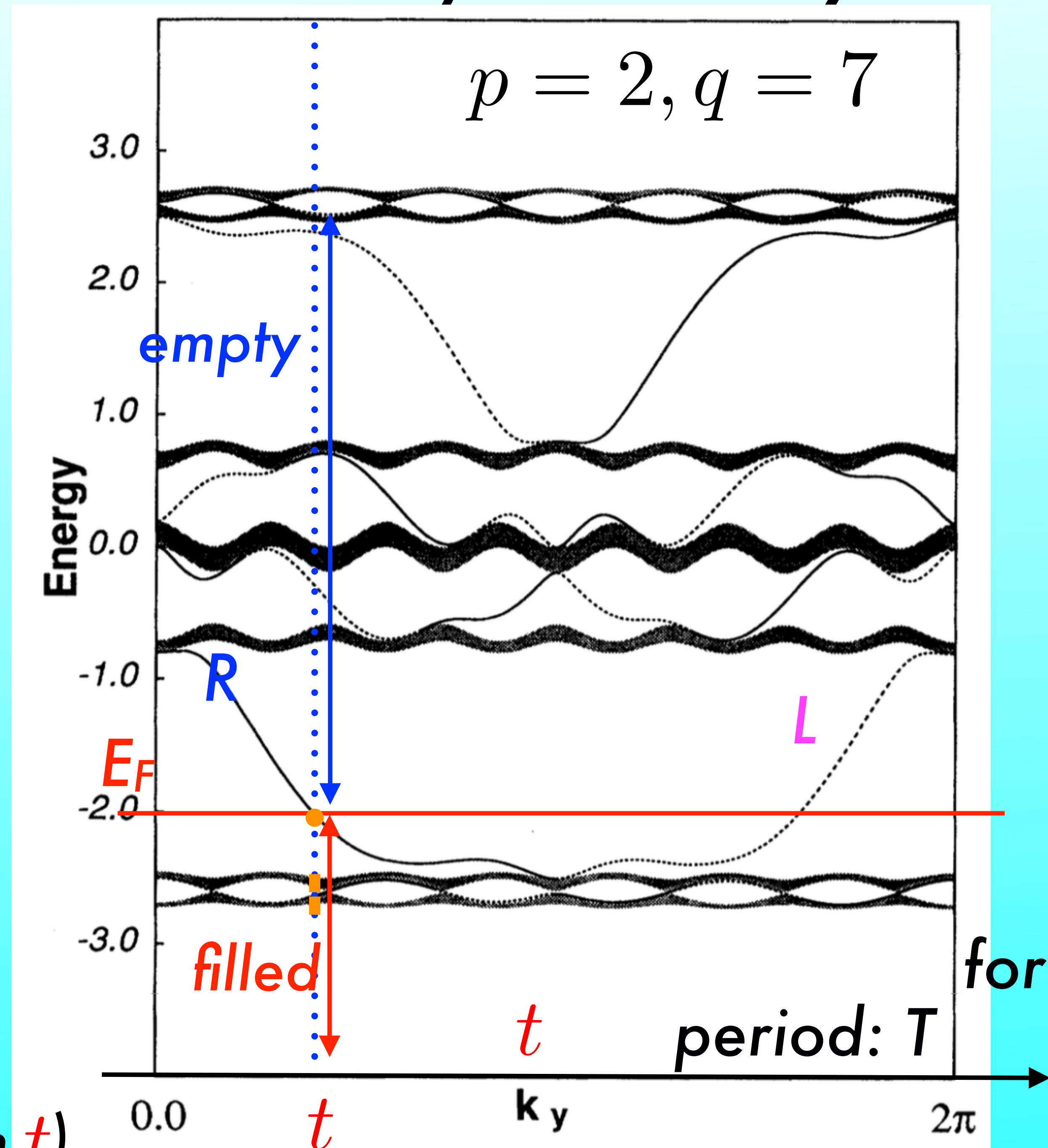
QHE (Hofstadter's) on 2D cylinder & TP



Periodic in

$$\boxed{k_y \to 2\pi\frac{t}{T}}$$

periodic pumping in 1D



Non interacting but many body

Adiabatic limit

$$\Delta E \gg \hbar/T$$

Filling states below E_F for the snapshot Ham.

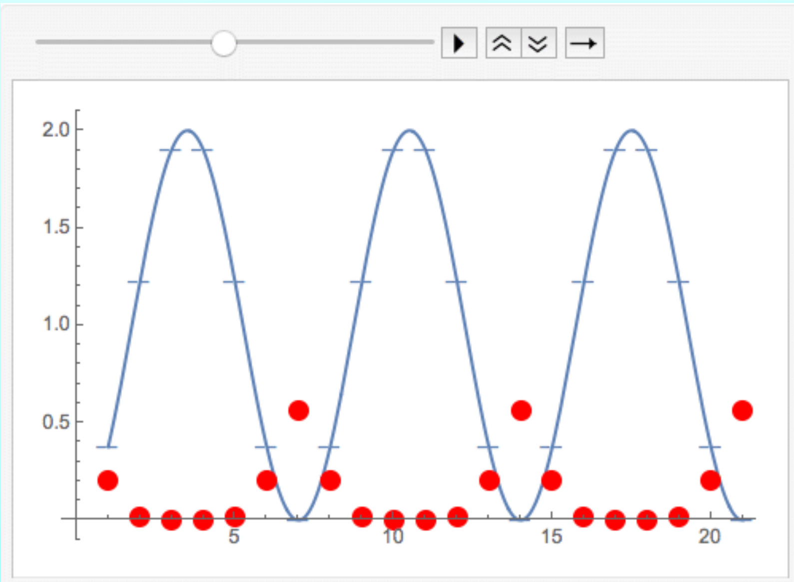
Harper eq. (1D for each t)

$$-t_x(\psi_{m+1}+\psi_{m-1}) - 2t_y\cos(k_y - 2\pi\frac{p}{q}m)\psi_m = E\psi_m$$

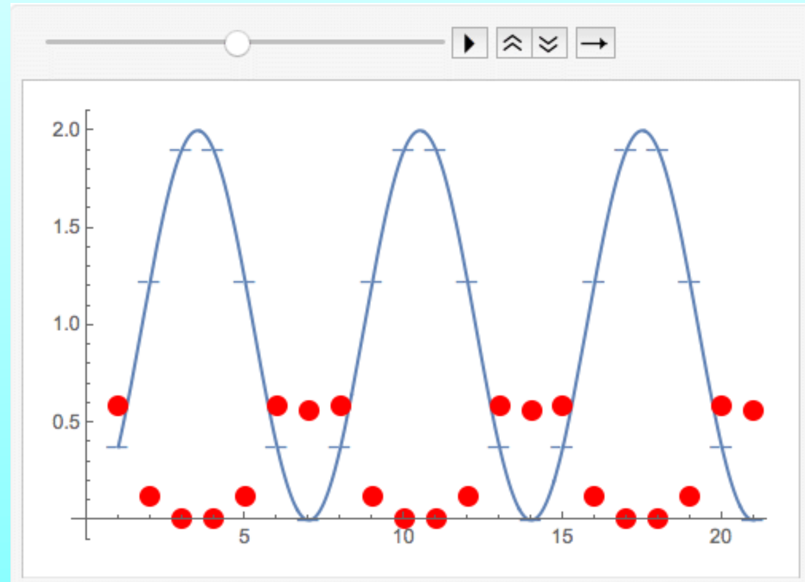
$$-t_x(\psi_{m+1}+\psi_{m-1}) - 2t_y\cos(2\pi\frac{t}{T} - 2\pi\frac{p}{q}m)\psi_m = E\psi_m$$

Examples 1 $\phi = -1/7 \quad t_x = 1,\ t_y = 1$

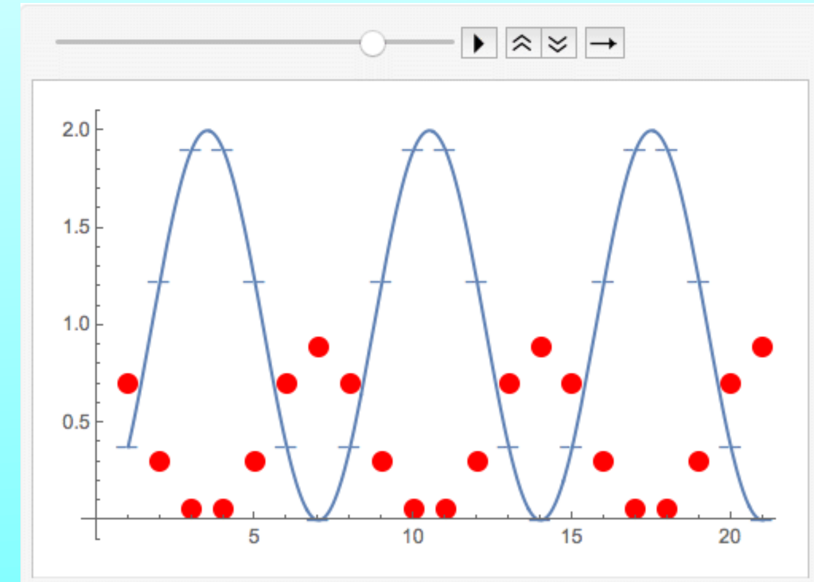
$\rho = 1/7$ C= -1



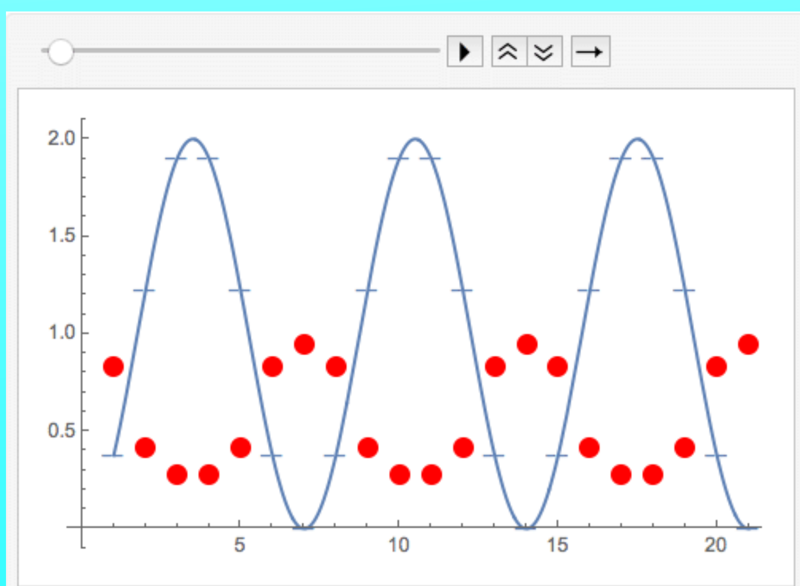
$\rho = 2/7$ C= -2



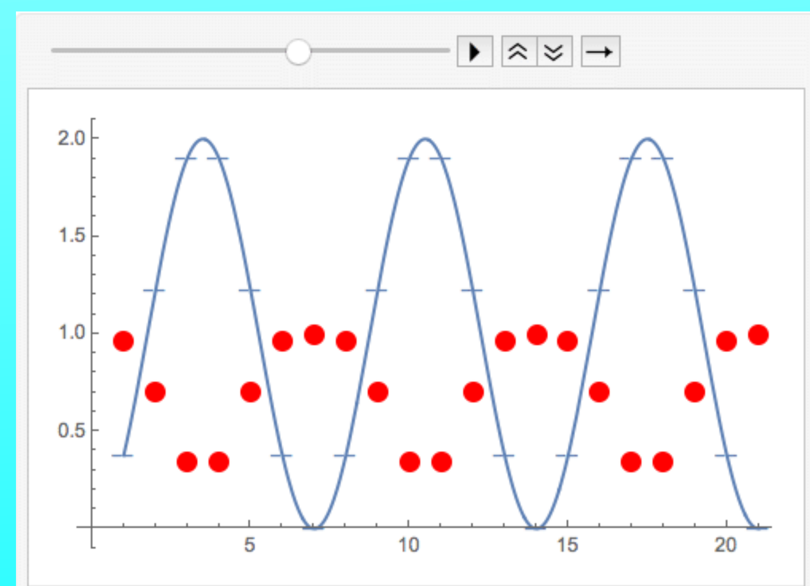
$\rho = 3/7$ C= -3



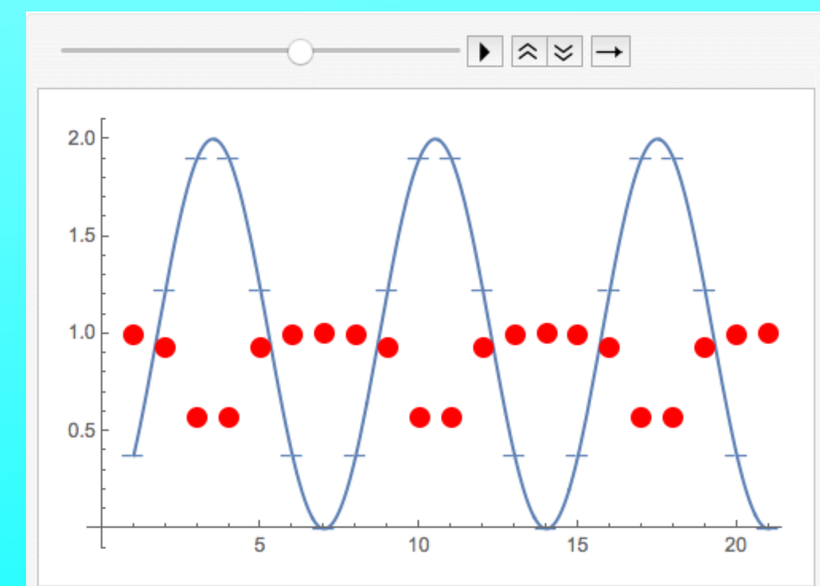
$\rho = 4/7$ C= +3



$\rho = 5/7$ C= +2



$\rho = 6/7$ C= +1



$$v_j(t) = -2t_y \cos(2\pi \frac{t}{T} - 2\pi\phi j) \qquad \phi = p/q$$

The pumping is topological ! (Thouless)

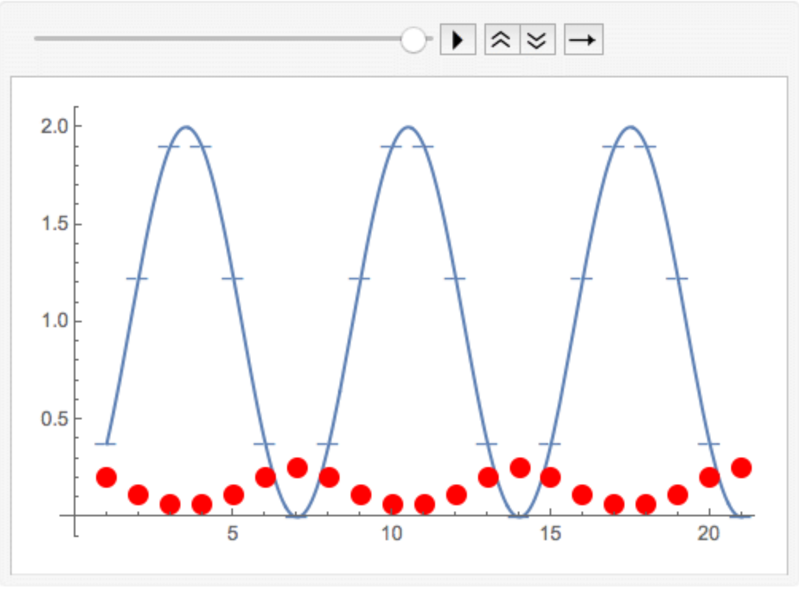
$\mathcal{O}(N^0)$ charge is pumped for an insulator with N particles

Pumped charge is quantized if gapped

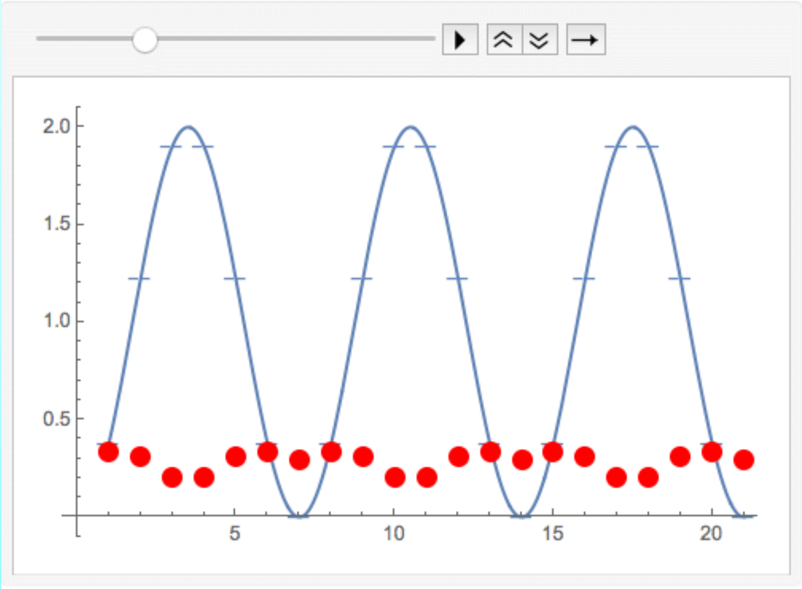
Independent of the parameters

Examples 2 $\phi = -1/7 \quad t_x = 1,\ t_y = 0.1$

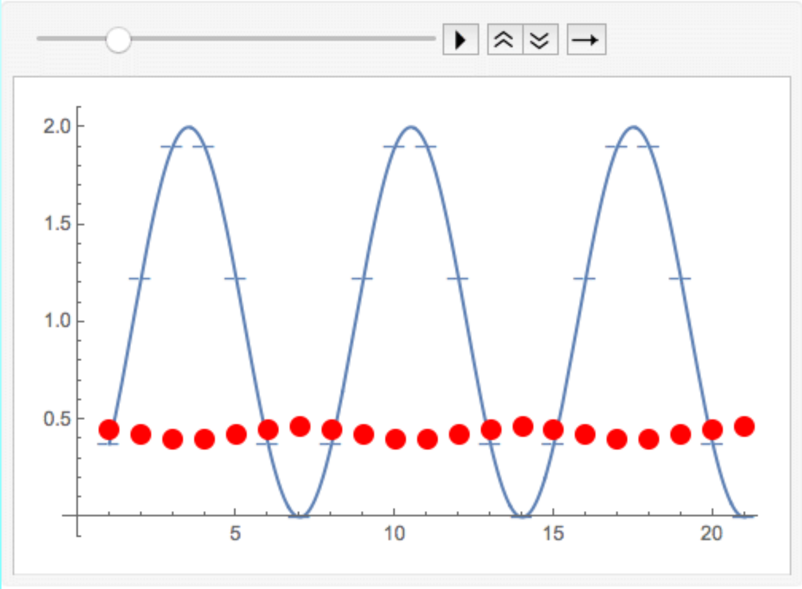
$\rho = 1/7$ C= -1



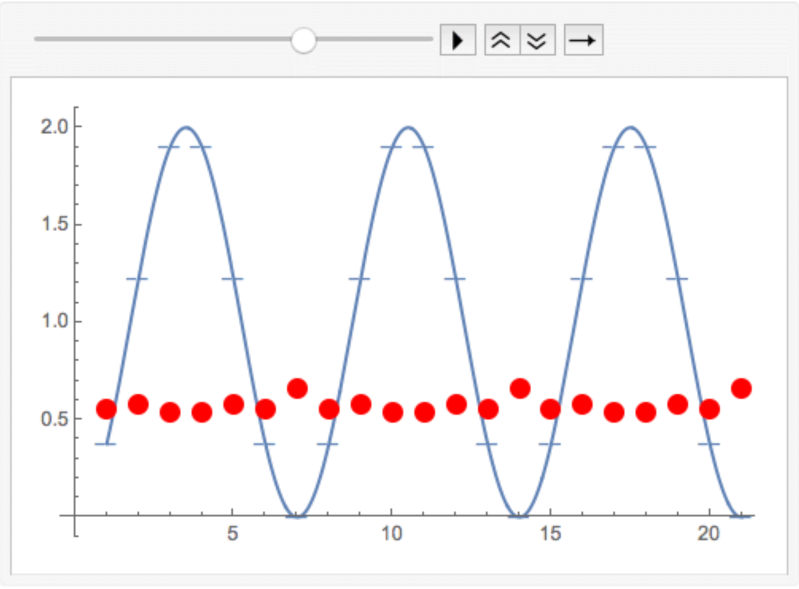
$\rho = 2/7$ C= -2



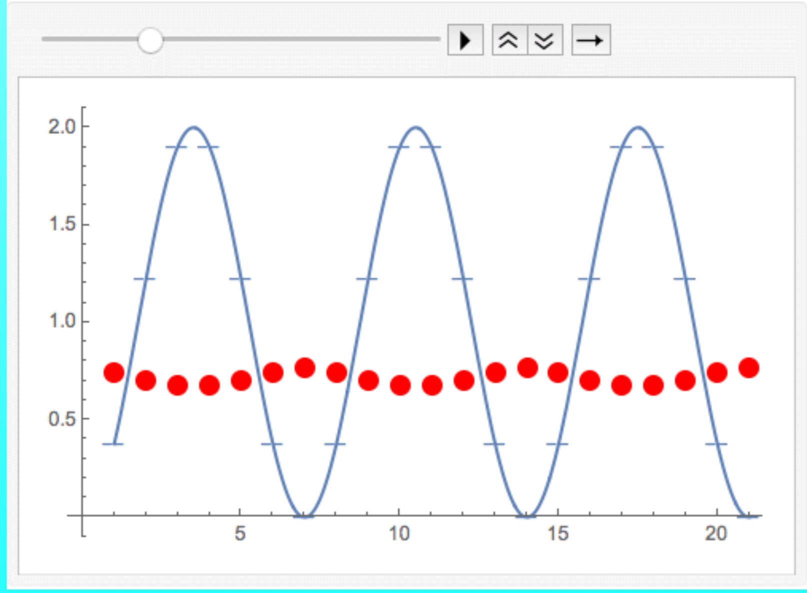
$\rho = 3/7$ C= -3



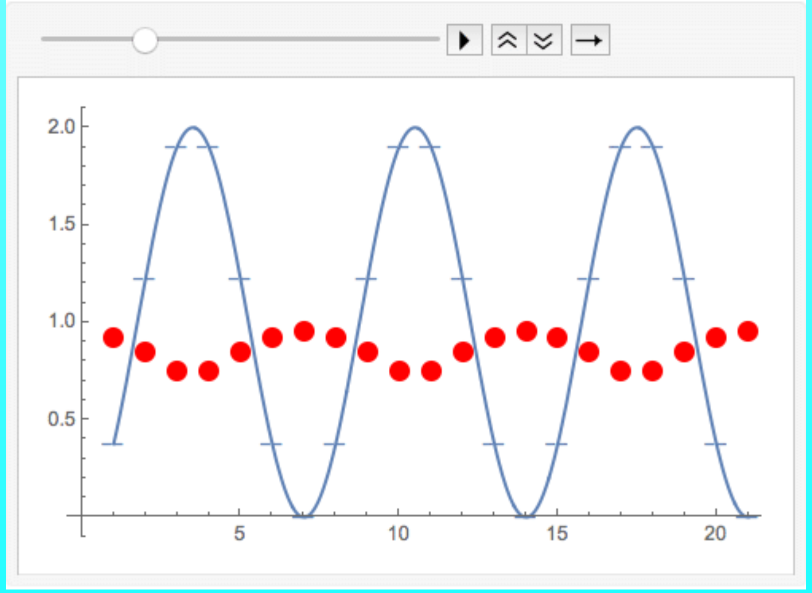
$\rho = 4/7$ C= +3



$\rho = 5/7$ C= +2



$\rho = 6/7$ C= +1



$t_x \gg t_y$

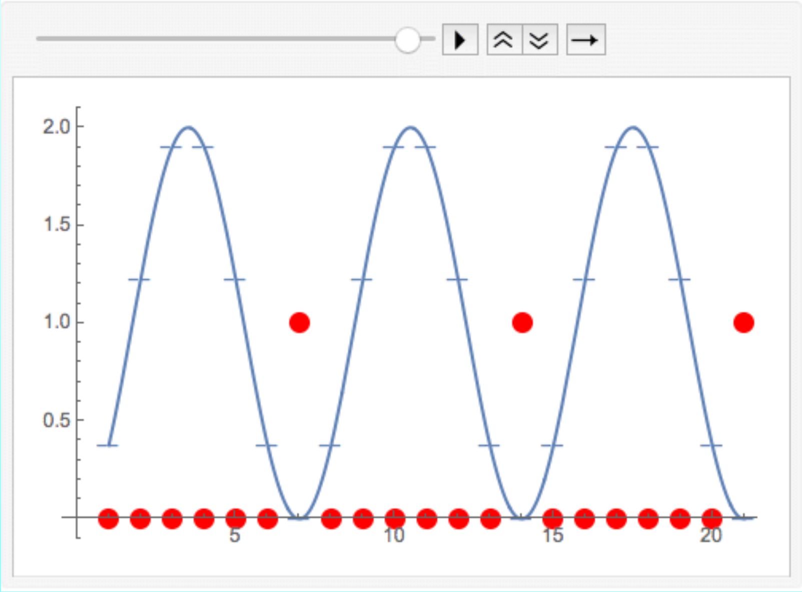
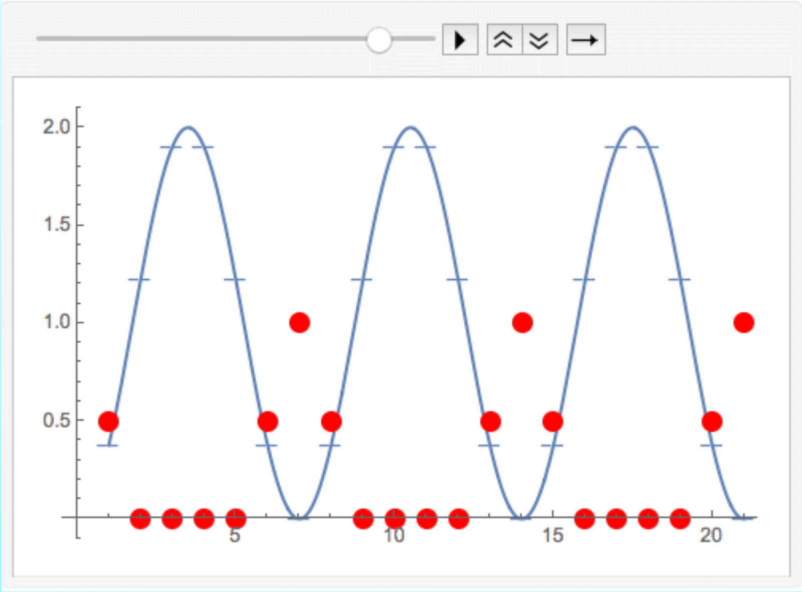
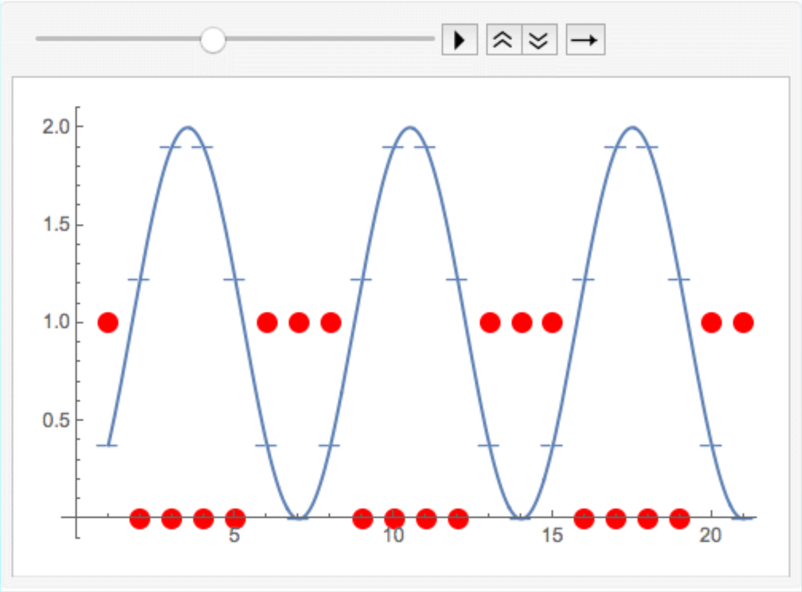
$t_x \gg |v_j|$

wave dynamics: quantum

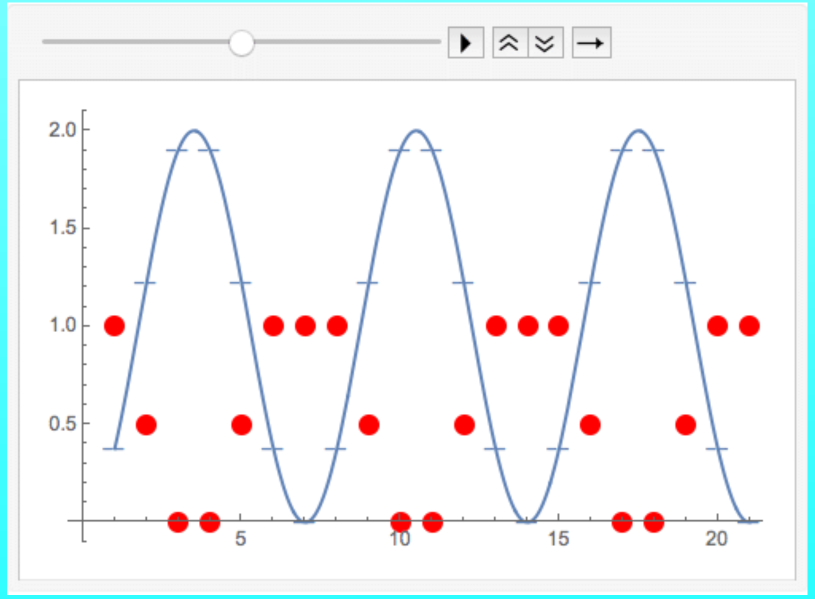
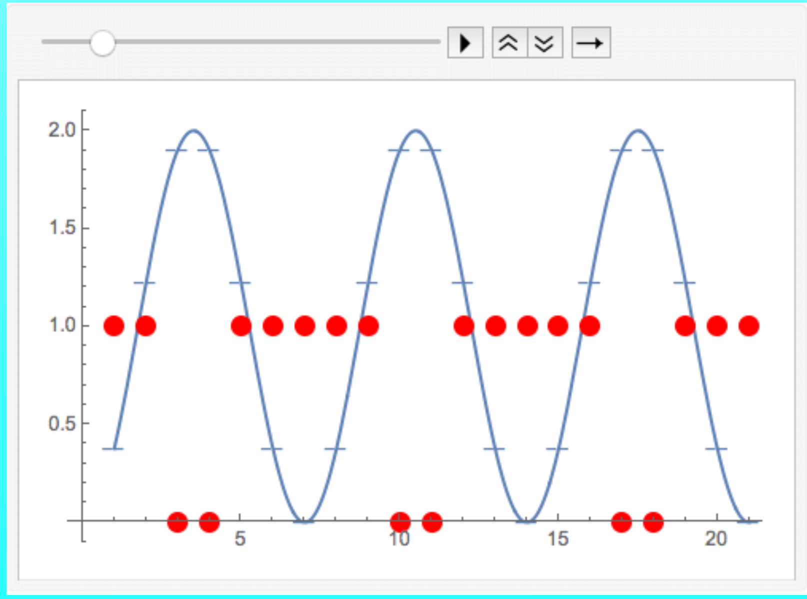
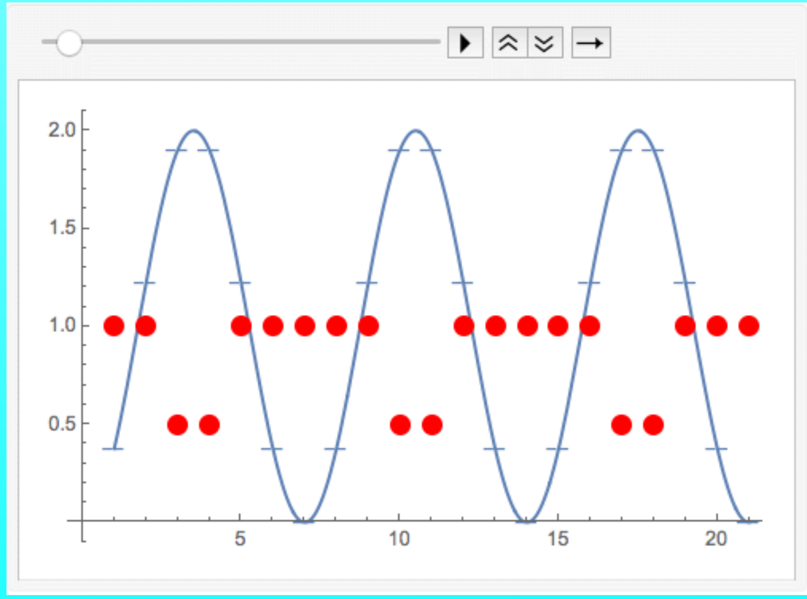
(weak potential)

Examples 3 $\phi = -1/7 \quad t_x = 0.01,\ t_y = 1$

$\rho = 1/7$ C= -1 $\rho = 2/7$ C= -2 $\rho = 3/7$ C= -3

$\rho = 4/7$ C= +3 $\rho = 5/7$ C= +2 $\rho = 6/7$ C= +1

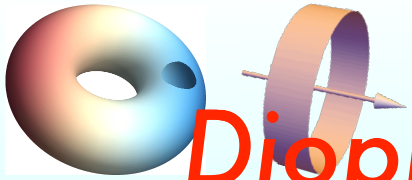
$t_x \ll t_y$

$t_x \ll |v_j|$

quantum tunneling: semi-classical

(deep potential)

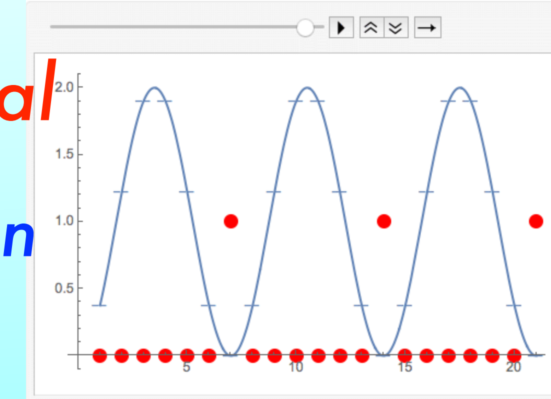
easy to count charge !



(quantum tunneling) $t_x/t_y \ll 1$

Topoquant, KITP, Oct. 14 (2016)

Diophantine eq. (TKNN) $t_x \ll |v_j|$



Count in the tunneling limit if topological

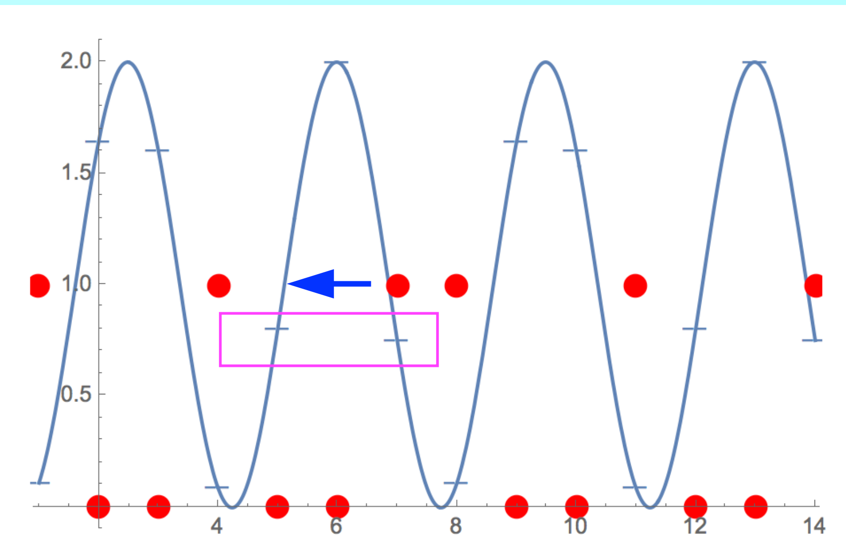
$$\phi = p/q = 2/7$$

$$\rho = r/q = 3/7$$

$$\boxed{\epsilon_j(t) = \epsilon_{j'}(t)}$$ Tunneling condition

$$\epsilon_j(t) = -2t_y \cos(2\pi\phi j - 2\pi\frac{t}{T})$$

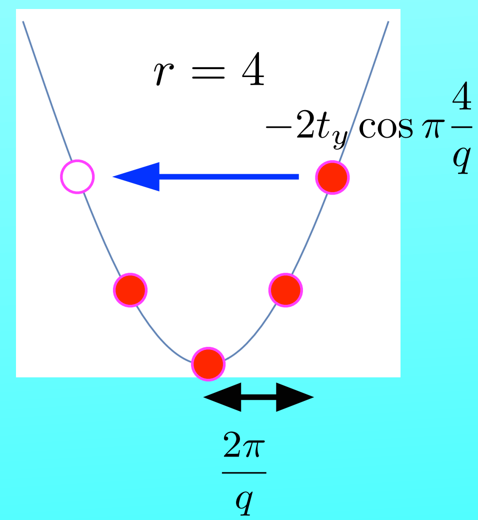
★ 1 $2\pi\phi j - 2\pi\frac{t}{T} = -(2\pi\phi j' - 2\pi\frac{t}{T}) + 2\pi s \quad s \in \mathbb{Z}$



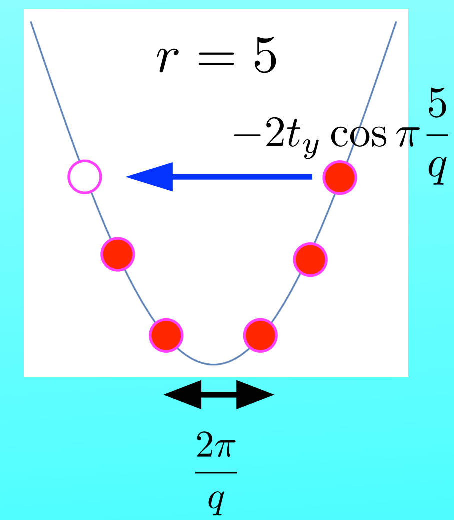
Tunneling at the filling r per unit

$$\boxed{\epsilon_j(t) = -2t_y \cos(\pi\frac{r}{q})}$$

r : even

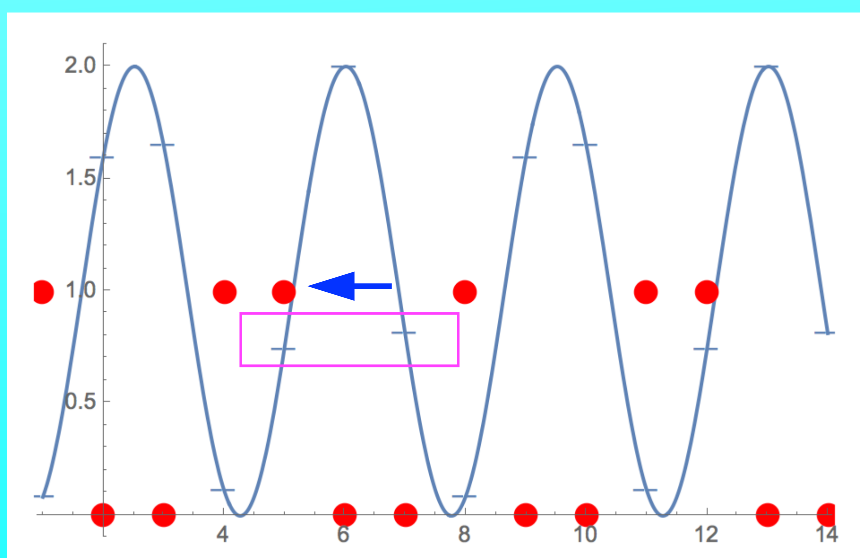


r : odd



★ 2 $\pi\frac{r}{q} = 2\pi\phi j - 2\pi\frac{t}{T}$

$$\Delta j = t = C = -2$$



★ 1& ★ 2

Diophantine eq. (TKNN)

$p = 2, q = 7$

$$\begin{aligned}(r,t,s) = &(1, -3, 1)\\ &(2, +1, 0)\\ &(3, -2, 1)\\ &(4, +2, 0)\\ &(5, -1, 1)\\ &(6, +3, 0)\end{aligned}$$

$$r = pt + qs \equiv pt \pmod q$$: algebraic

$$t = \Delta j = j - j' \quad |t| \le q/2$$

= Chern number C : analytic (integral)

Magic !

Streda formula: $C = \frac{dN}{dB} = \frac{d(r/q)}{d\phi} = \frac{r/q}{p/q} = t$

Diophantine eq. (TKNN) $t_x/t_y \ll 1$ (quantum tunneling)

$$\phi = p/q = 2/7$$

$$\rho = r/q = 3/7$$

★1 $$2\pi\phi j - 2\pi\frac{t}{T} = -(2\pi\phi j' - 2\pi\frac{t}{T}) + 2\pi s$$

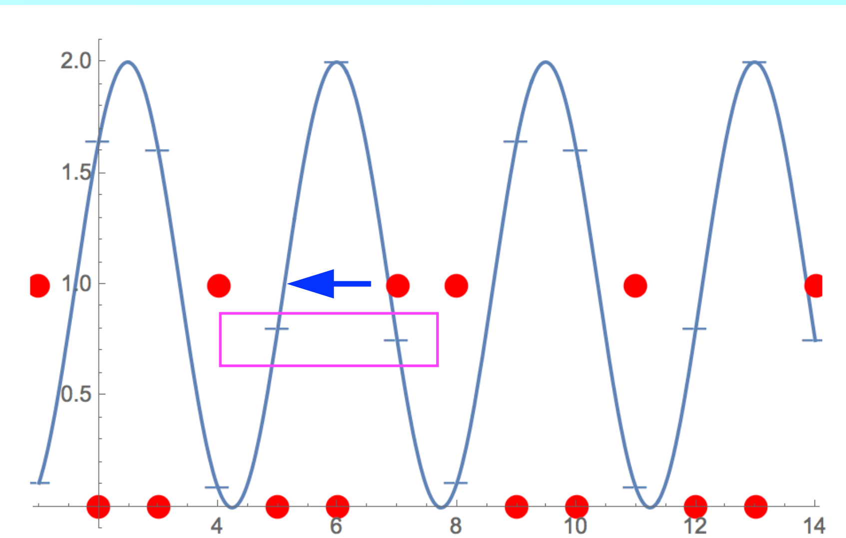
$$\frac{p}{q}j = -\frac{p}{q}j' + \frac{2t}{T} + s$$

★2 $$\pi\frac{r}{q} = 2\pi\phi j - 2\pi\frac{t}{T}$$ +

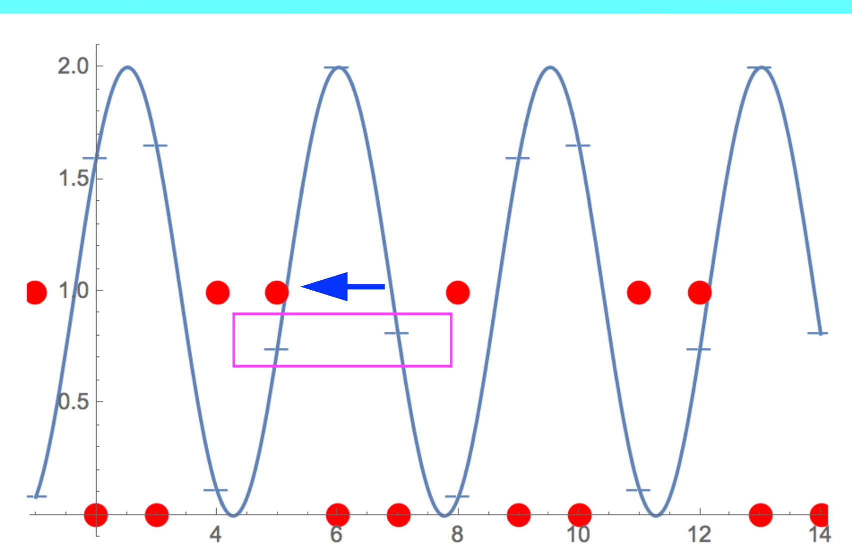
$$\frac{r}{q} = 2\frac{p}{q}j - \frac{2t}{T}$$

||

$$\frac{r}{q} = \frac{p}{q}(j - j') + s$$



$$\Delta j = t = C = -2$$



★1& ★2

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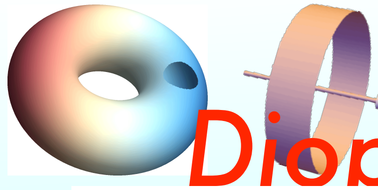

Diophantine eq. (TKNN) $t_x/t_y \ll 1$ *(quantum tunneling)*

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Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195

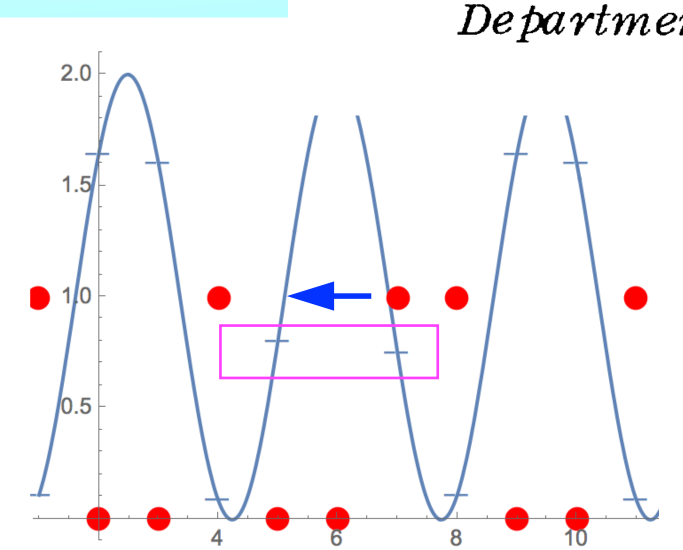
(Received 30 April 1982)

$s \in \mathbb{Z}$

per unit

en

r : odd



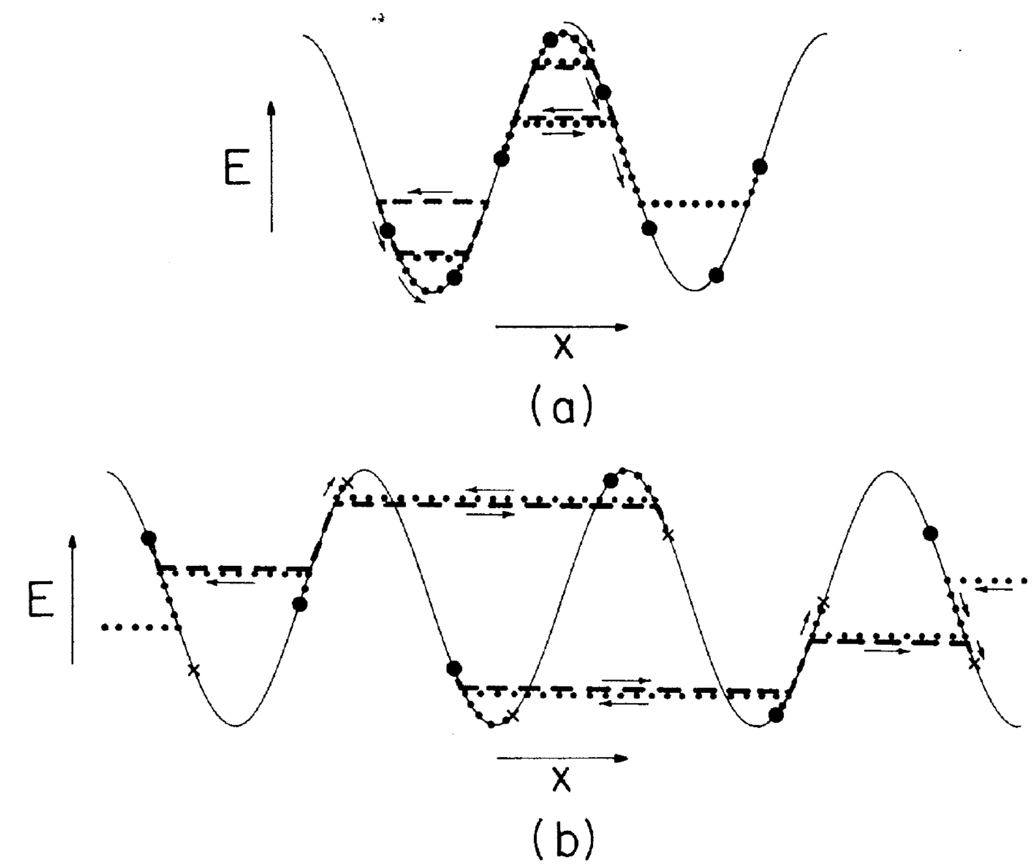
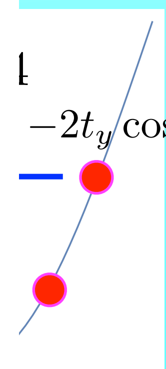
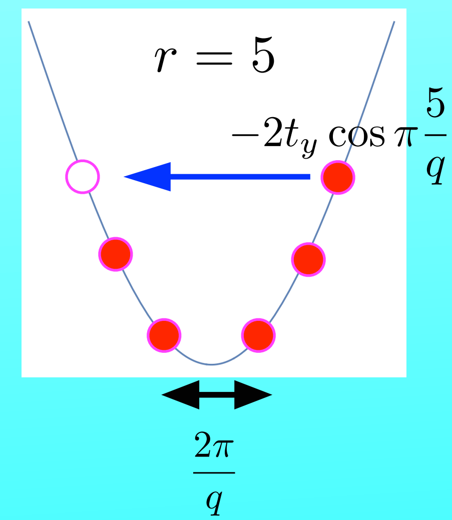
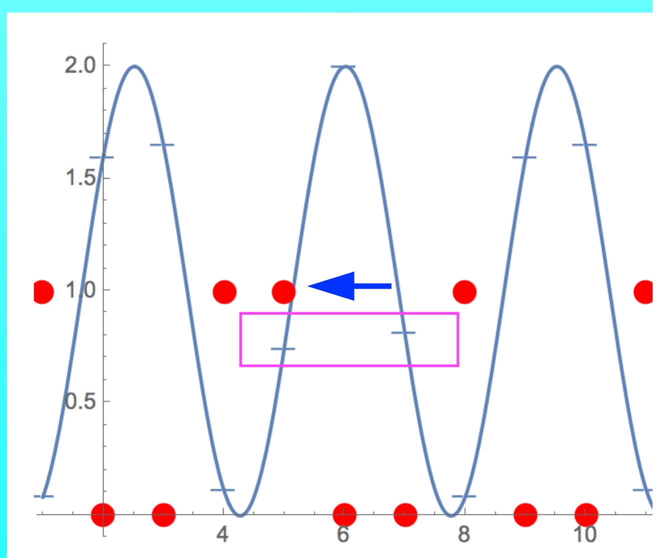


FIG. 1. Motion of electrons in the x direction under the influence of an electric field in the y direction for $V \ll V'$, for the two cases (a) $\varphi = 5$ and (b) $\varphi = \frac{5}{3}$.





$$\Delta j = t =$$



N)

$(\mathrm{mod}\, q)$ *: algebraic*

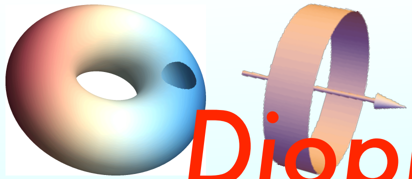
$|t| \le q/2$

: analytic (integral)

!

Streda formula: $C = \dfrac{dN}{dB} = \dfrac{d(r/q)}{d\phi} = \dfrac{r/q}{p/q} = t$

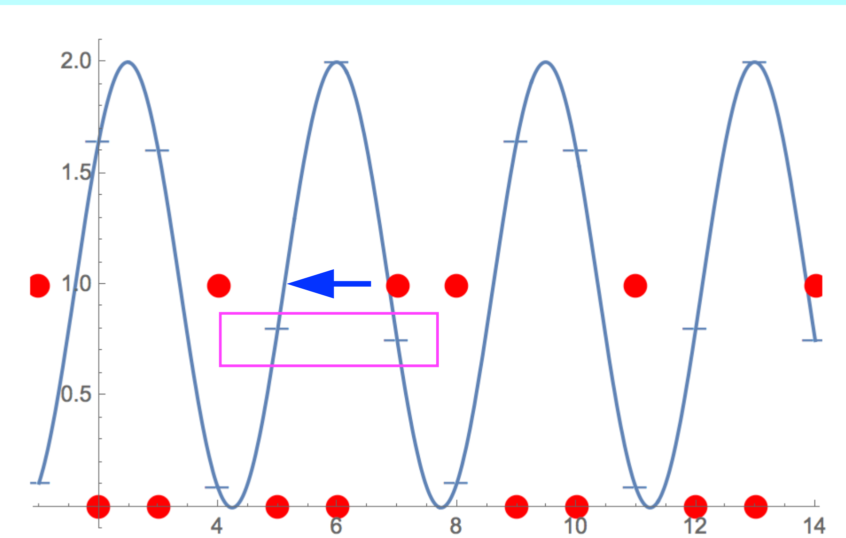
$(6, \boxed{+3,} 0)$



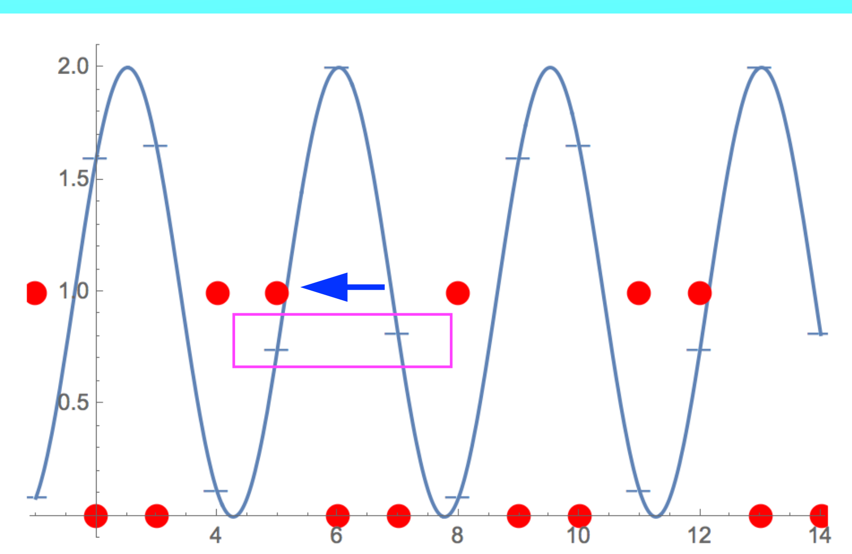
Diophantine eq. (TKNN) $t_x/t_y \ll 1$ (quantum tunneling)

$$\phi = p/q = 2/7$$

$$\rho = r/q = 3/7$$



$$\Delta j = t = C = -2$$



$\boxed{\epsilon_j(t) = \epsilon_{j'}(t)}$ **Tunneling condition**

$$\epsilon_j(t) = -2t_y \cos(2\pi\phi j - 2\pi \frac{t}{T})$$

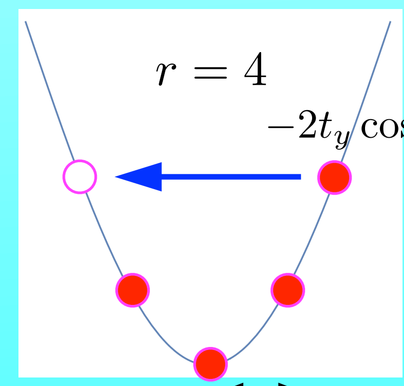
★ 1 $\quad 2\pi\phi j - 2\pi\frac{t}{T} = -(2\pi\phi j' - 2\pi\frac{t}{T}) + 2\pi s \quad s \in \mathbb{Z}$

Tunneling at the filling r per unit

$$\boxed{\epsilon_j(t) = -2t_y \cos(\pi \frac{r}{q})}$$

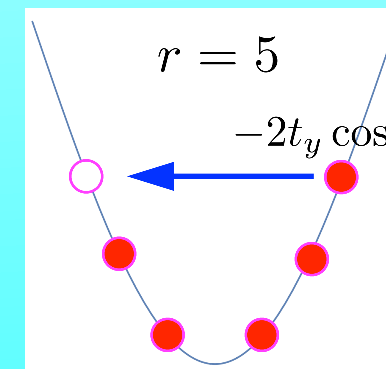
★ 2 $\quad \pi\frac{r}{q} = 2\pi\phi j - 2\pi\frac{t}{T}$

r : even



$\frac{2\pi}{q}$

r : odd



$\frac{2\pi}{q}$

★ 1& ★ 2

$p = 2, q = 7$

$$\begin{aligned}(r,t,s) = &(1, -3, 1)\\ &(2, +1, 0)\\ &(3, -2, 1)\\ &(4, +2, 0)\\ &(5, -1, 1)\\ &(6, +3, 0)\end{aligned}$$

Diophantine eq. (TKNN)

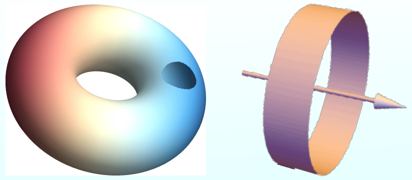
$$r = pt + qs \equiv pt \pmod q$$: algebraic

$$t = \Delta j = j - j' \quad |t| \le q/2$$

= Chern number C : analytic (integral)

Magic !

Streda formula: $C = \dfrac{dN}{dB} = \dfrac{d(r/q)}{d\phi} = \dfrac{r/q}{p/q} = t$



Adiabatic pump (Thouless '83)

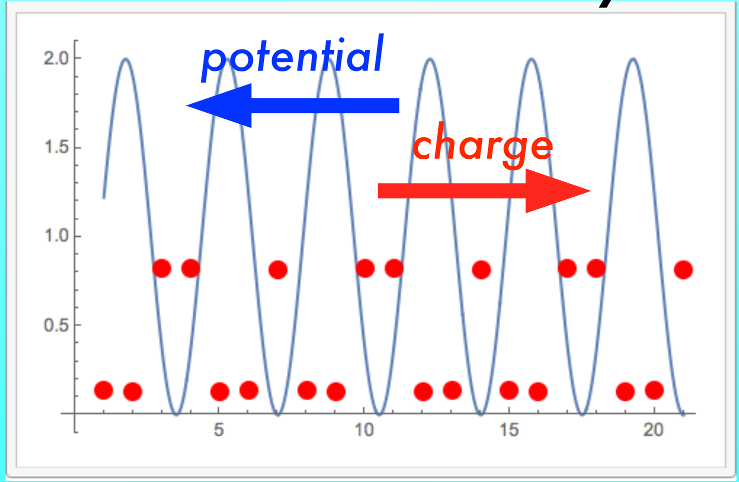
Periodically driven1D charge transport

$$\mathrm{i}\hbar\partial_t|G(t)\rangle = H(t)|G(t)\rangle \quad |G(t)\rangle = Te^{-(\mathrm{i}/\hbar)\int_{t_0}^{t} d\tau H(\tau)}|G(t_0)\rangle$$

$$H(t) = \sum_{j}^{L} [-t_x c_{j+1}^\dagger c_j + h.c. + \underline{v_j}(t) c_j^\dagger c_j]$$

free fermion

manybody



$$v_j(t+T) = v_j(t)$$ period T

ex. $v_j(t) = -2t_y \cos(\frac{t}{T} + 2\pi\phi j)$

Adiabatic : ground state is gapped & slow pumping

$$\boxed{\Delta E \gg \hbar/T}$$

Topological !

Pumped charge is quantized as an integer

Thouless '83

Pumped charge by adiabatic approximation

$$j = \langle G|J|G\rangle \qquad H(\theta,t) = \sum_j^L \left[-t_x e^{-\mathrm{i}\frac{\theta}{L_x}} c_{j+1}^\dagger c_j + h.c. + v_j(t) c_j^\dagger c_j\right]$$

twist

$$J = \frac{1}{L_x}(+\mathrm{i}\frac{t_x}{\hbar}e^{-\mathrm{i}\theta/L_x})\sum_j c_{j+1}^\dagger c_j + h.c$$

$$= +\hbar^{-1}\partial_\theta H(\theta)$$

$|\alpha(t)\rangle$: Snapshot eigen state

$$H(t)|\alpha(t)\rangle = E_\alpha(t)|\alpha(t)\rangle, \quad \langle\alpha|\beta\rangle = \delta_{\alpha\beta}.$$

$$|G\rangle = e^{-(\mathrm{i}/\hbar)\int_0^t dt' E_g(t')} e^{\mathrm{i}\gamma(t)}\left[|g\rangle + \mathrm{i}\hbar\sum_{\alpha\neq g}\frac{|\alpha\rangle\langle\alpha|\partial_t g\rangle}{E_\alpha - E_g}\right]$$

$$\delta j_x = \langle G|J|G\rangle - \langle g|J|g\rangle = -\mathrm{i}B$$

$$B = \partial_\theta A_t - \partial_t A_\theta,\ A_\mu = \langle g|\partial_\mu g\rangle, \quad \mu = \theta, t.$$

Pumped charge & Berry connection

Thouless '83

Pumped charge in T

$$\Delta Q = \int_0^T dt\, \delta j_x = -\mathrm{i} \int_0^T dt\, B$$

Adiabatic appr.

$$B = \partial_\theta A_t - \partial_t A_\theta$$

Berry connection $A_\mu = \langle g | \partial_\mu g \rangle, \quad \mu = t, \theta$

twist

$$t_x \to t_x e^{-\mathrm{i}\theta/L}$$

$$|g(t)\rangle : \quad H(t)|g(t)\rangle = E(t)|g(t)\rangle$$

snapshot ground state

B is invariant for the phase choice of $|g\rangle$: gauge freedom

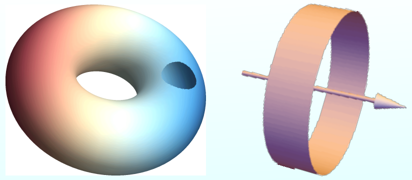
Temporal gauge: $A_t^{(t)} = 0$

$$B = \cancel{\partial_\theta A_t} - \partial_t A_\theta$$

$$\Delta Q = \mathrm{i} \int_0^T dt\, \partial_t A_\theta^{(t)} = \mathrm{i}\left[A_\theta^{(t)}(T) - A_\theta^{(t)}(0)\right]$$

Physical observable

Berry connection (gauge fixed)



Temporal gauge

$$\boxed{\text{Temporal gauge: } A_t^{(t)} = 0}$$

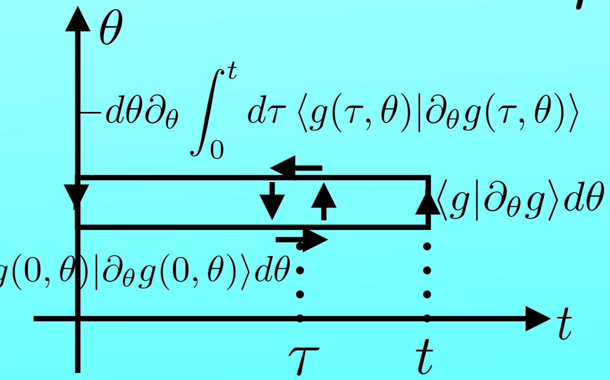
$$B = \cancel{\partial_\theta A_t} - \partial_t A_\theta$$

Gauge transformation $\langle g'|\partial_\mu g'\rangle = \langle g|\partial_\mu g\rangle + i\partial_\mu\chi, \;\; |g'\rangle = |g\rangle e^{i\chi}$

temporal general

$$A_\mu^{(t)}(t,\theta) = A_\mu(t,\theta) + \mathrm{i}\partial_\mu\chi(t,\theta)$$

$C : (0,0) \to (0,\theta) \to (t,\theta)$

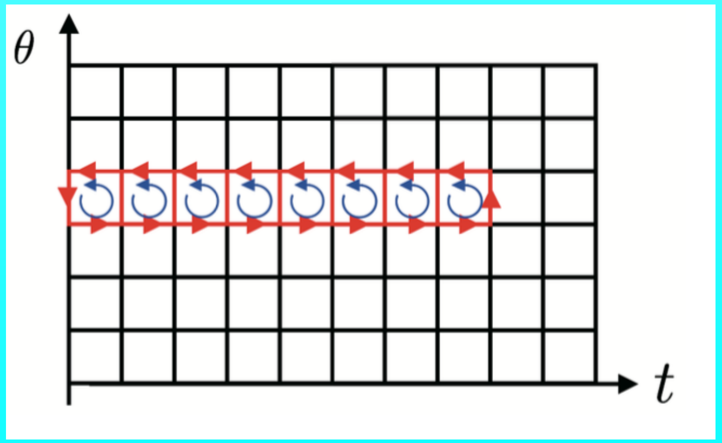


$$\chi(t,\theta) = \mathrm{i}\int_0^t d\tau\, A_t(\tau,\theta) + \mathrm{i}\int_0^\theta d\vartheta\, A_\theta(0,\vartheta)$$

$$A_t^{(t)}(t,\theta) = A_t(t,\theta) + \mathrm{i}\partial_t\chi(t,\theta) = 0$$

$$A_\theta^{(t)}(t,\theta) = A_\theta(t,\theta) + \mathrm{i}\partial_\theta\chi(t,\theta)$$

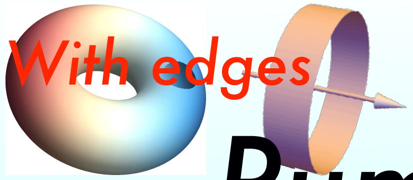
$$= A_\theta(t,\theta) - A_\theta(0,\theta) - \partial_\theta\int_0^t d\tau\, A_t(\tau,\theta)$$



$$A_\theta^{(t)}(t,\theta) \neq A_\theta^{(t)}(t+T,\theta)$$

non periodic gauge fixing

$$H(t,\theta) = H(t+T,\theta)$$



Pumped charge & Center of mass (CM)

$$H(\theta,t) = \sum_{j}^{L} \left[-t_x e^{-\mathrm{i}\frac{\theta}{L_x}} c^\dagger_{j+1} c_j + h.c. + v_j(t) c^\dagger_j c_j \right]$$

twist : gauged out for an open system (<u>with edges</u>)

$$\boxed{H(\theta,t) = \mathcal{U} H(0,t) \mathcal{U}^\dagger} \qquad \mathcal{U}(\theta) = \prod_{j=1} e^{-\mathrm{i}\theta n_j (j-j_0)/L_x}$$

$$j_0 = L/2$$

$$\mathcal{U} c_j \mathcal{U}^\dagger = e^{+\mathrm{i}\theta j/L_x} c_j$$
$$\mathcal{U} c_j^\dagger \mathcal{U}^\dagger = c_j^\dagger e^{-\mathrm{i}\theta j/L_x}$$

$$\boxed{|g(\theta)\rangle = \mathcal{U}(\theta)|g(0)\rangle}$$

large gauge tr.

θ independent

$$A_\theta = \langle g(\theta)|\partial_\theta g(\theta)\rangle = \langle g(0)|\underbrace{\mathcal{U}^\dagger \partial_\theta \mathcal{U}}_{-i\sum_j \frac{j-j_0}{L} n_j}|g(0)\rangle = -\mathrm{i}\boxed{P(t)}$$

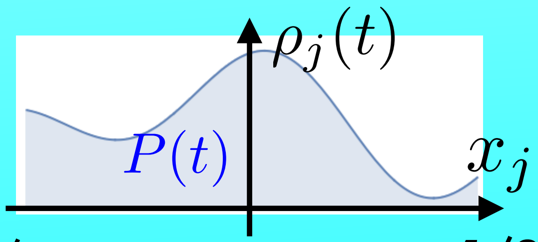
Center of mass (CM) $\quad P(t) = \sum_j x_j \rho_j$

$$x_j = (j-j_0)/L \in [-1/2, 1/2]$$

$$\sum_j \rho_j = N$$

number of particles

$$\rho_j = \rho(x_j) = \langle g(0)|n_j|g(0)\rangle$$



$$P(t) = \mathcal{O}(N^0)$$

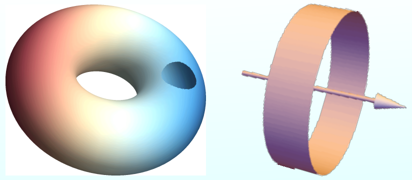
insulator

$$A_\theta^{(t)}(t,\theta) = A_\theta(t,\theta) - A_\theta(0,\theta) - \cancel{\partial_\theta \int_0^t d\tau A_t(\tau,\theta)}$$
$$= -\mathrm{i}[P(t) - P(0)]$$

$$\boxed{\Delta Q = P(T) - P(0)}$$

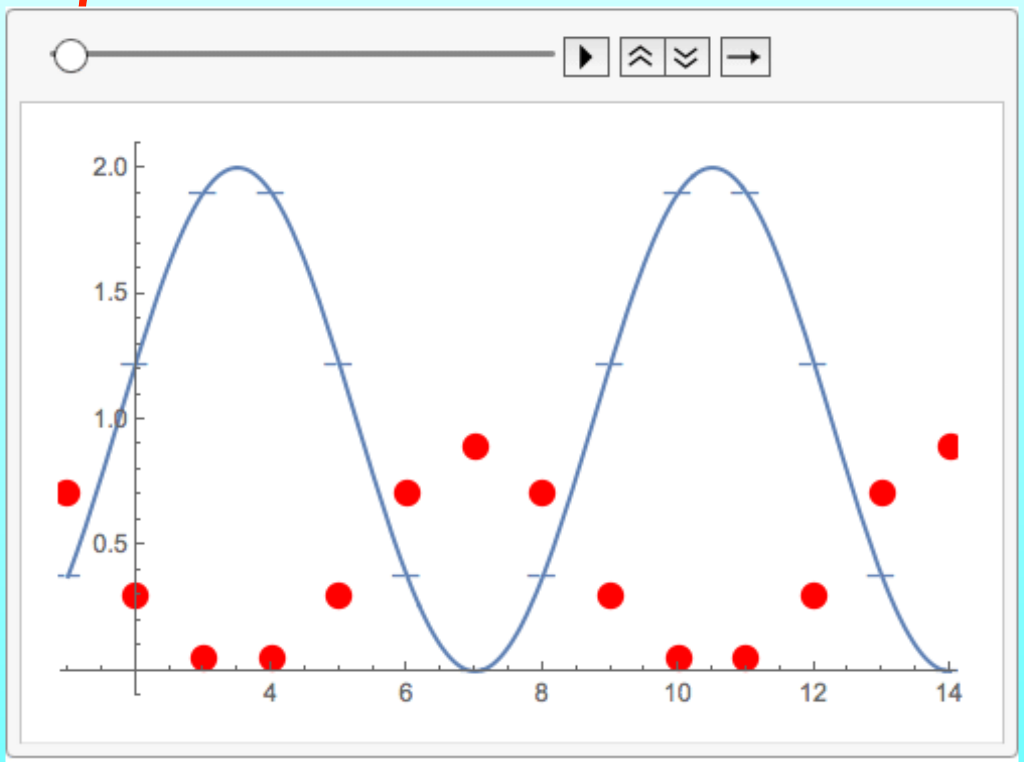
Shift of CM

ill-defined for periodic system

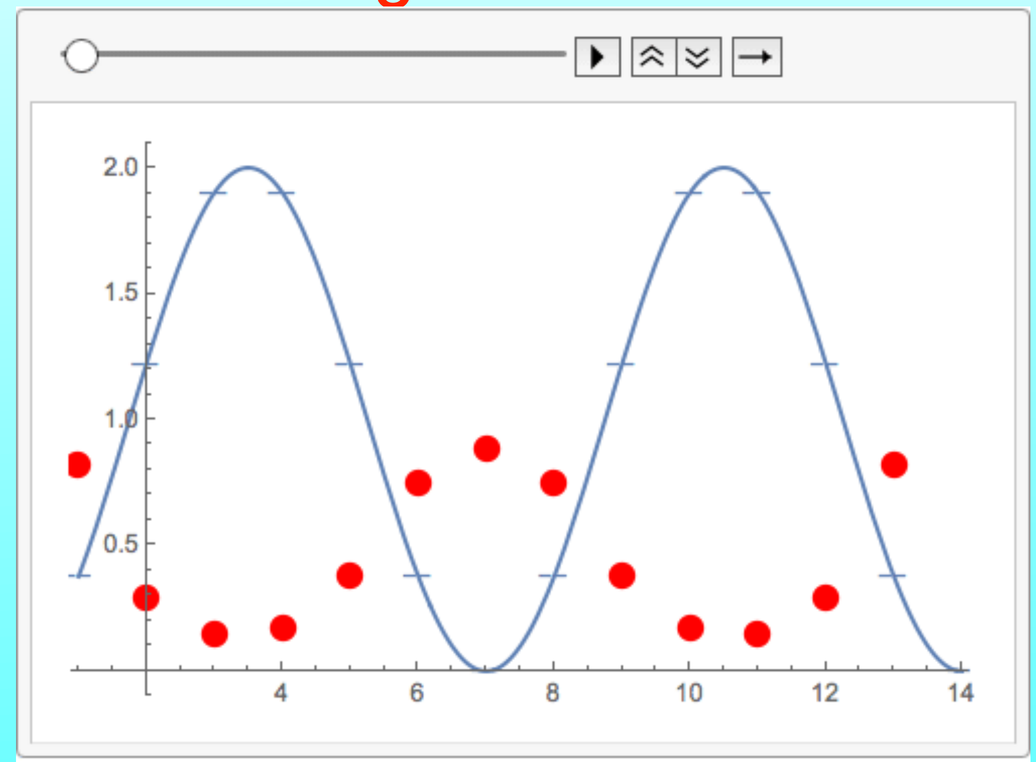


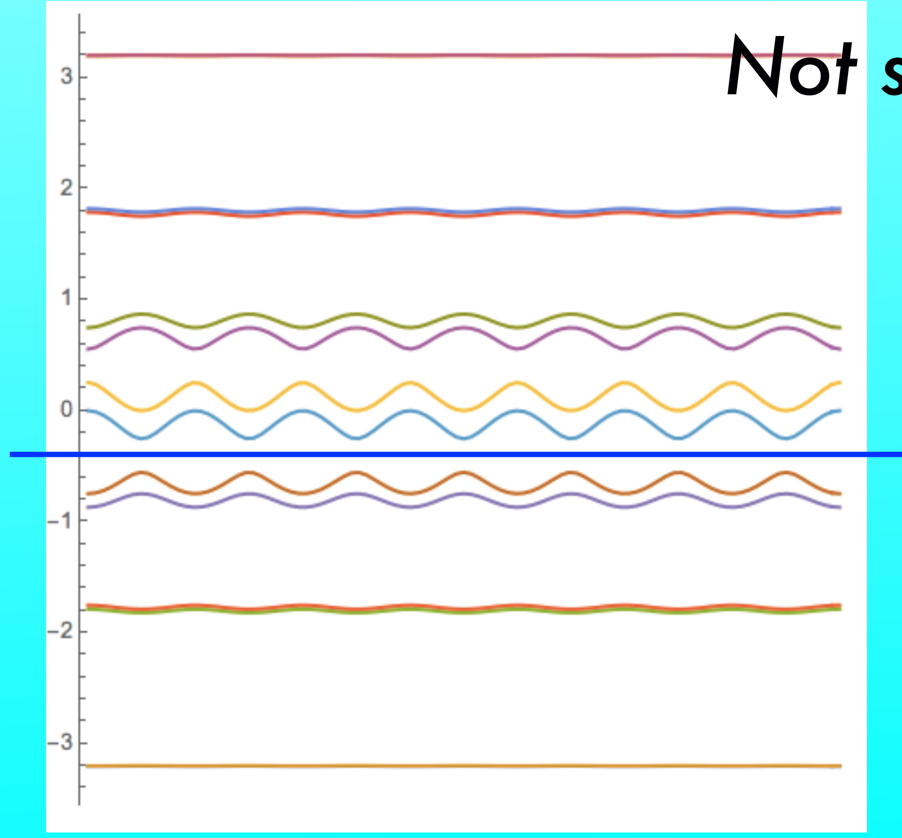
$\phi = 1/7, \rho = 3/7, C = 3$ With/without edges

periodic

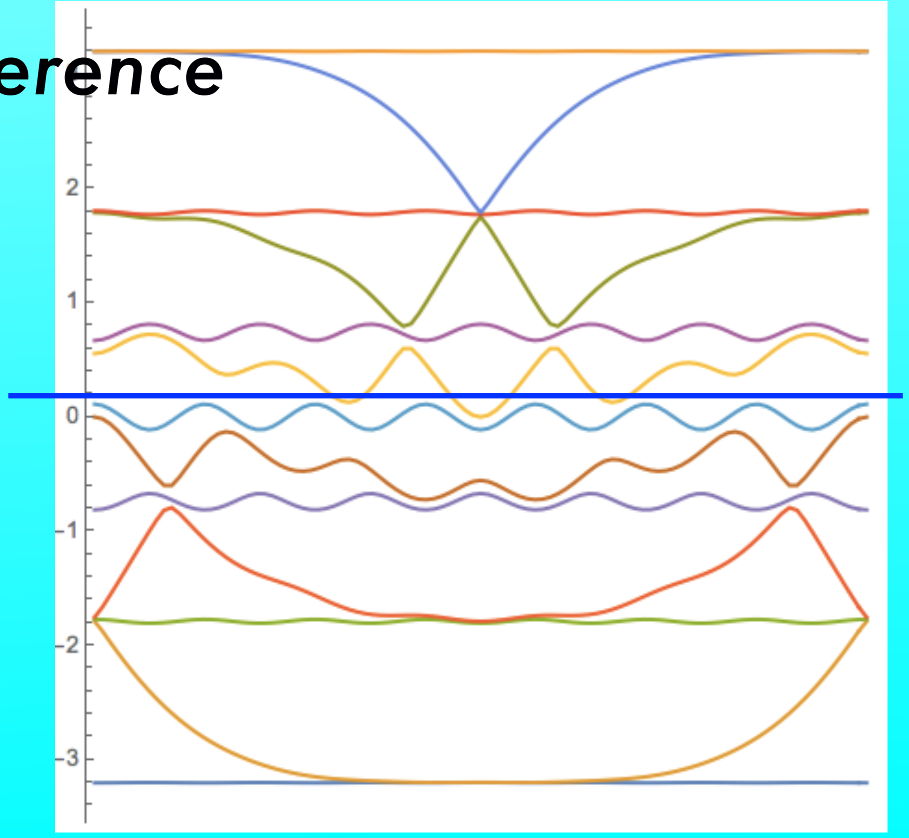


with edges





Not so much difference



Center of mass with edges

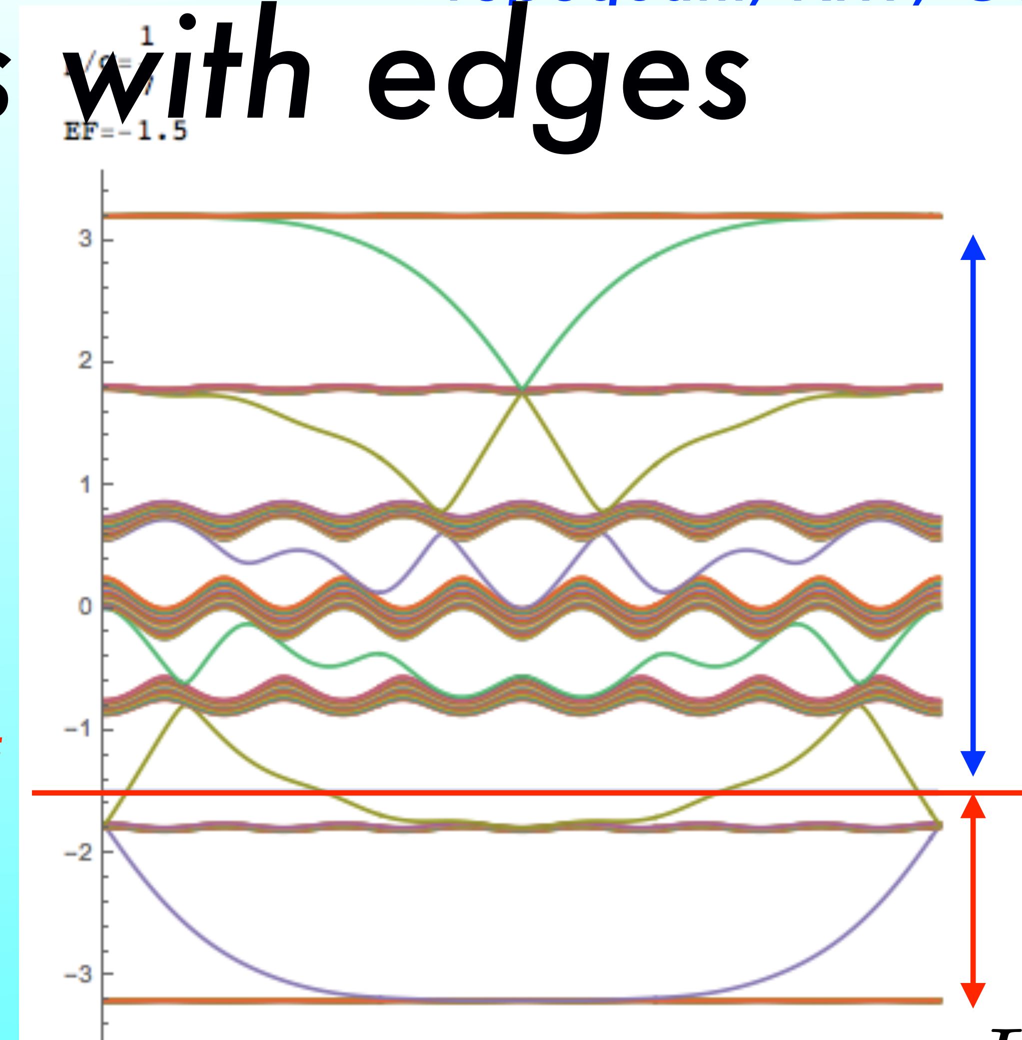
$\phi = 1/7, \qquad N \sim L \gg 1$

$E_F = -1.5, \rho \sim 2/7$

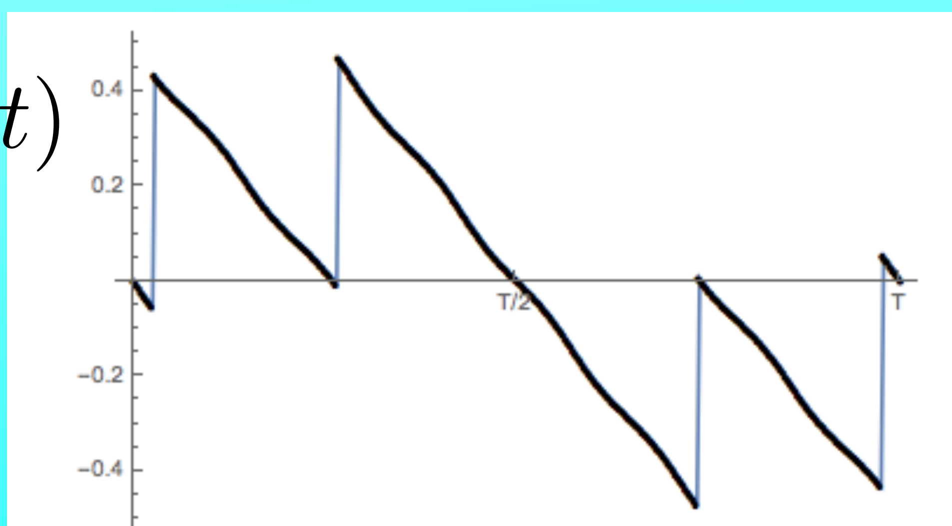
$P(t) = \sum_j x_j \rho_j \ \sim O(N^0)$

$x_j = (j - j_0)/L \in [-1/2, 1/2]$

$\rho_j = \rho(x_j) = \langle g(0) | n_j | g(0) \rangle$



$L = 139$



ΔP=0.483893 at time=0.0266667 T, Lx=139

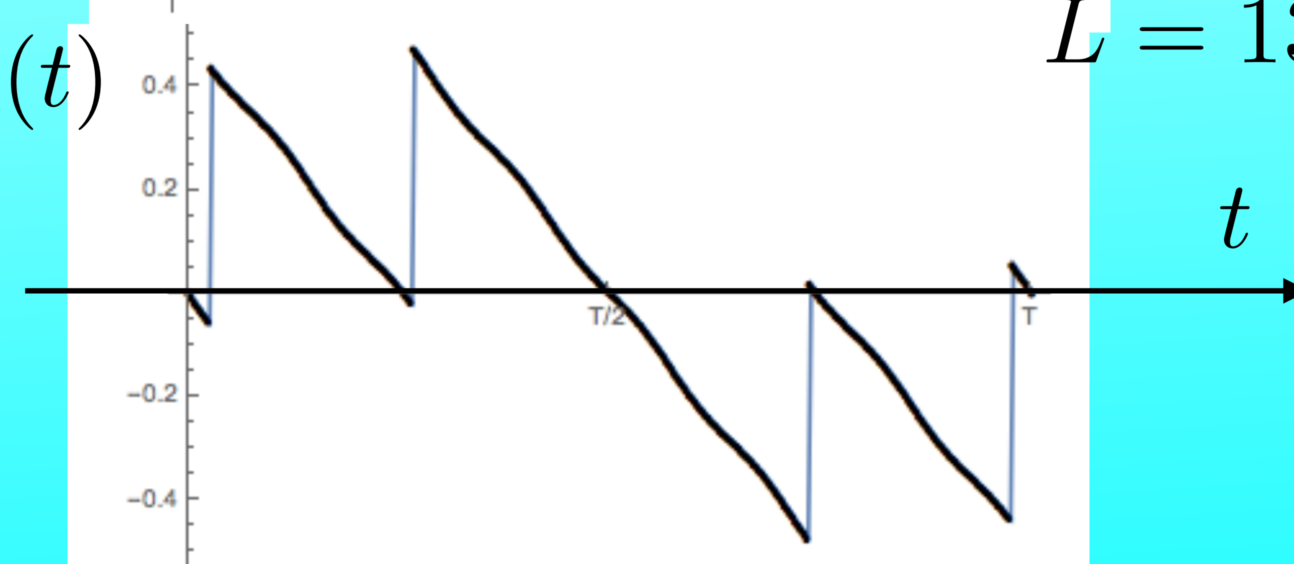
ΔP=0.475871 at time=0.266667 T, Lx=139

ΔP=0.475871 at time=0.736667 T, Lx=139

ΔP=0.483893 at time=0.976667 T, Lx=139

Total P=1.91953

$L = 1399$



ΔP=0.491839 at time=0.0266667 T, Lx=1399

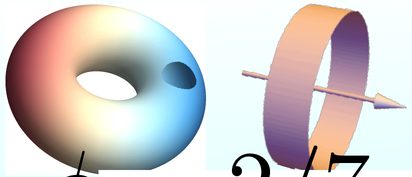
ΔP=0.490793 at time=0.266667 T, Lx=1399

ΔP=0.490793 at time=0.736667 T, Lx=1399

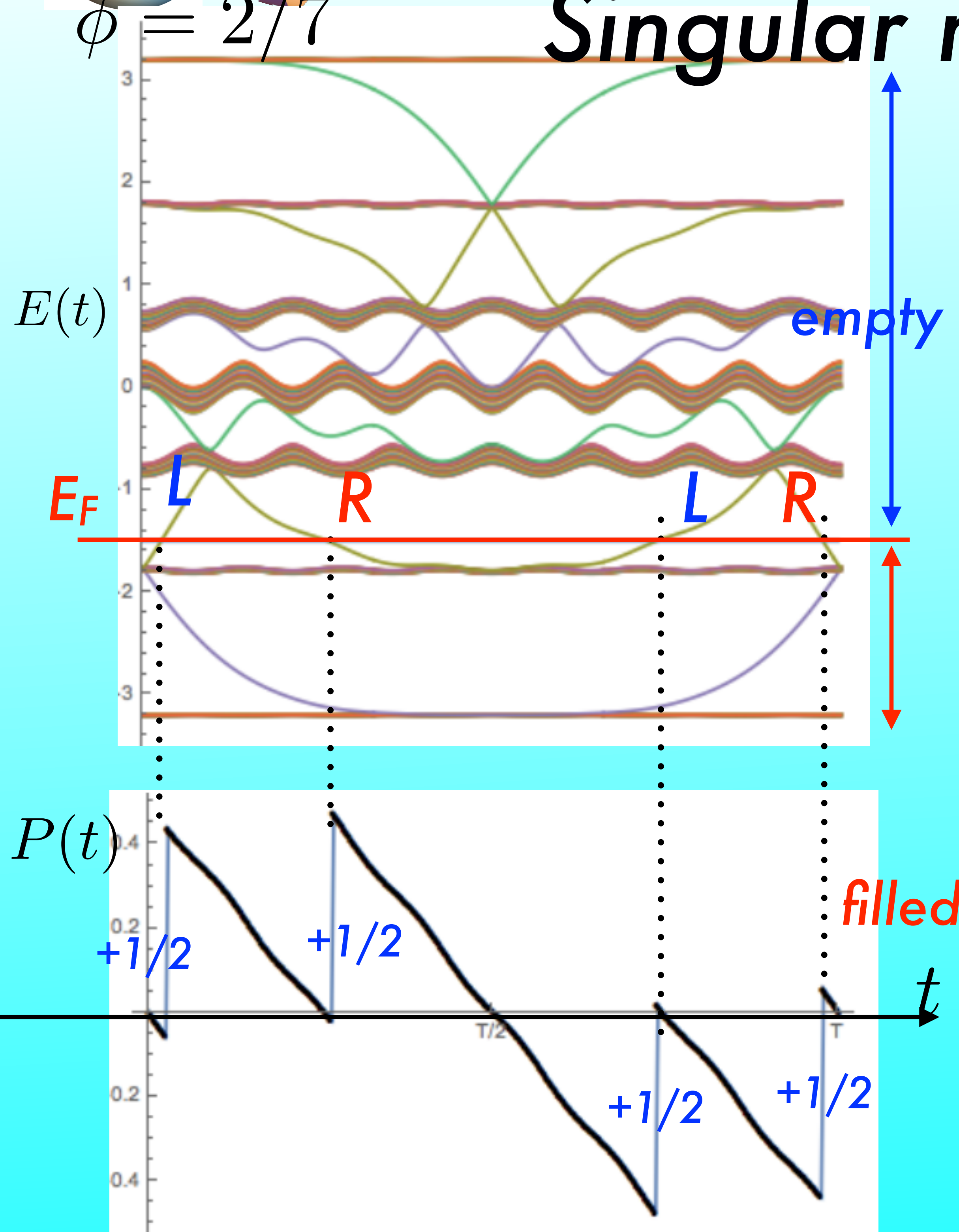
ΔP=0.491839 at time=0.976667 T, Lx=1399

Total P=1.96526

Many jump , discontinuities



Singular motion of CM
due to edge states



Contribution of the edge state for P is

$$\frac{j-j_0}{L} \to \pm\frac{1}{2}$$

exponentially localized

empty

$j \sim L$

$j \sim 1$

$j_0 = L/2 \quad (L \to \infty)$

filled

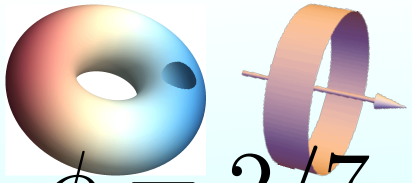
$$\Delta P(t_i) = P(t_i^+) - P(t_i^-)$$

$$= \begin{cases} -1/2\text{: becomes unoccupied at R} \\ +1/2\text{: becomes occupied at R} \\ +1/2\text{: becomes unoccupied at L} \\ -1/2\text{: becomes occupied at L} \end{cases}$$

t

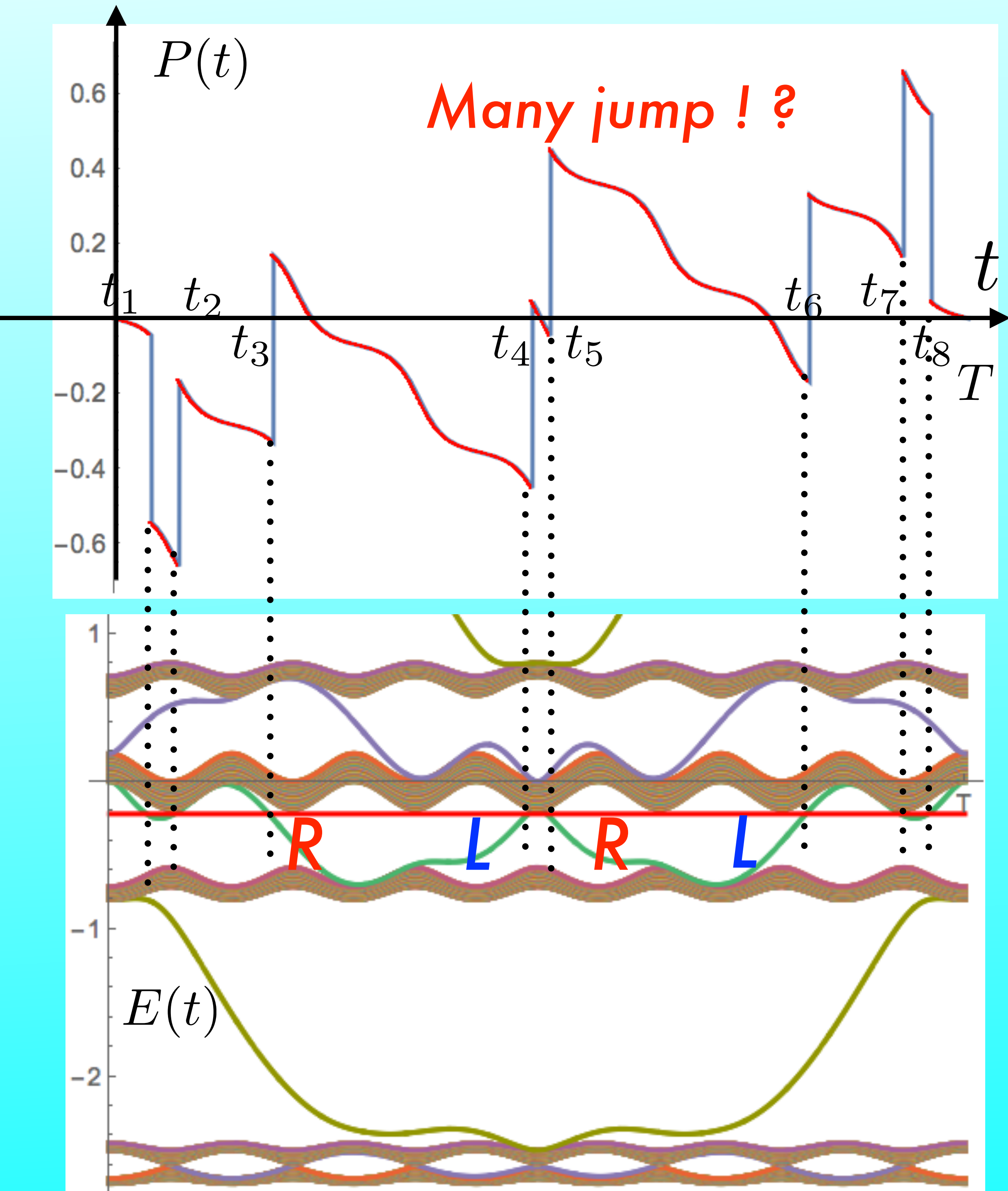
$$P(t) = \sum_j x_j \rho_j \qquad x_j = (j-j_0)/L \in [-1/2, 1/2]$$

$$\rho_j = \rho(x_j) = \langle g(0)|n_j|g(0)\rangle$$



$\phi = 2/7$

Singular motion of CM
due to edge states



Contribution of the edge state for P is

$$\frac{j-j_0}{L} \to \pm\frac{1}{2}$$

exponentially localized

$j \sim L$

$j \sim 1$

$j_0 = L/2 \quad (L \to \infty)$

$$\Delta P(t_i) = P(t_i^+) - P(t_i^-)$$

$=$
- -1/2: becomes unoccupied at R
- +1/2: becomes occupied at R
- +1/2: becomes unoccupied at L
- -1/2: becomes occupied at L

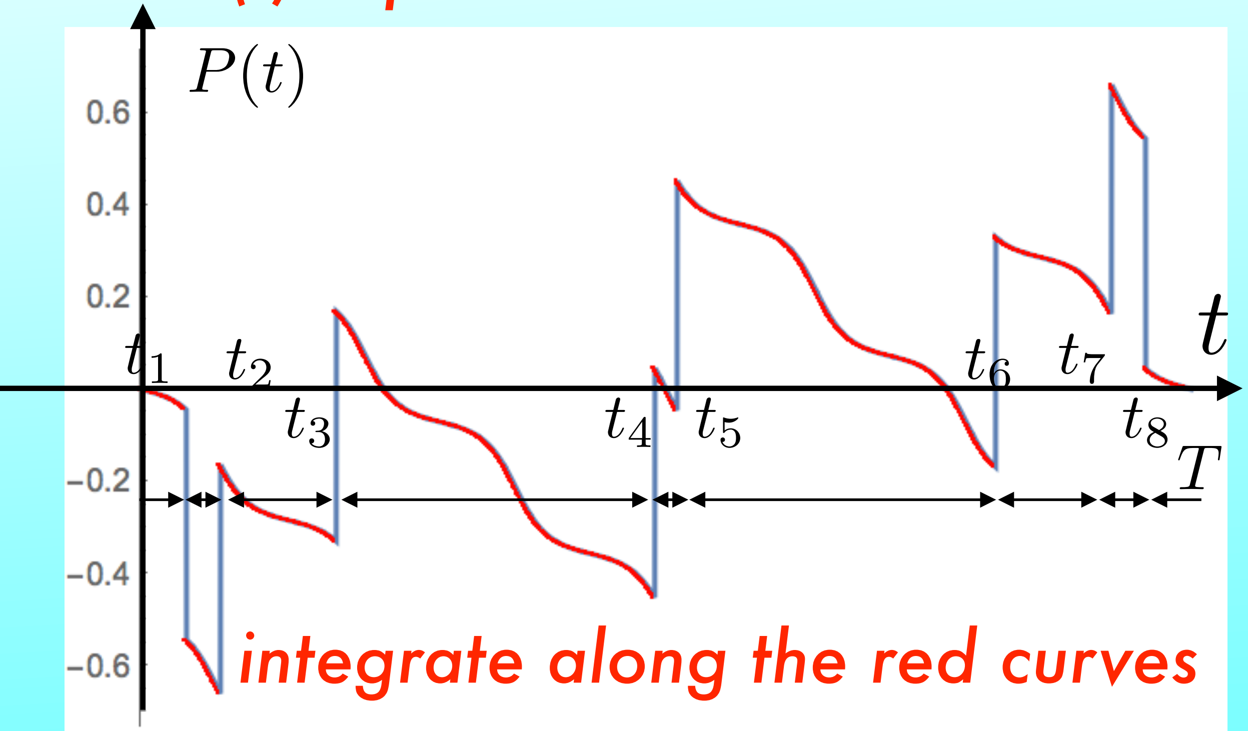
$$P(t) = \sum_j x_j \rho_j$$

$x_j = (j - j_0)/L \in [-1/2, 1/2]$

$\rho_j = \rho(x_j) = \langle g(0)|n_j|g(0)\rangle$

How much pumped?

P(t) is periodic function !



$$\Delta P(t_i) = P(t_i^+) - P(t_i^-)$$

$$= \begin{cases} \text{-1/2: become unoccupied at R} \\ \text{+1/2: become occupied at R} \\ \text{+1/2: become unoccupied at L} \\ \text{-1/2: become occupied at L} \end{cases}$$

patch work in time domain

Pump by bulk

$$\Delta Q = \sum_i \int_{t_i^+}^{t_{i+1}^-} dt\, \partial_t P(t) = \sum_i \left[P(t_{i+1}^-) - P(t_i^+)\right]$$

$$= -\sum_i \left[P(t_i^+) - P(t_i^-)\right] = -\boxed{\sum_i \Delta P(t_i)}$$

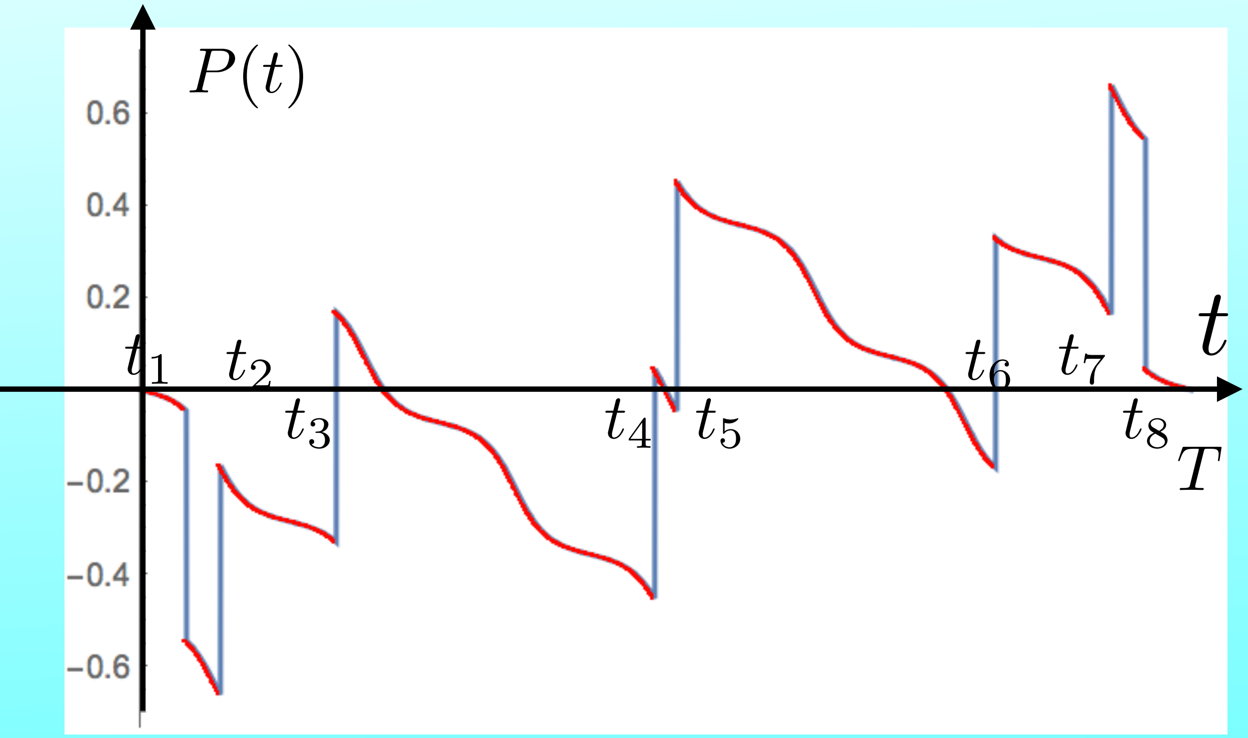
periodicity in time

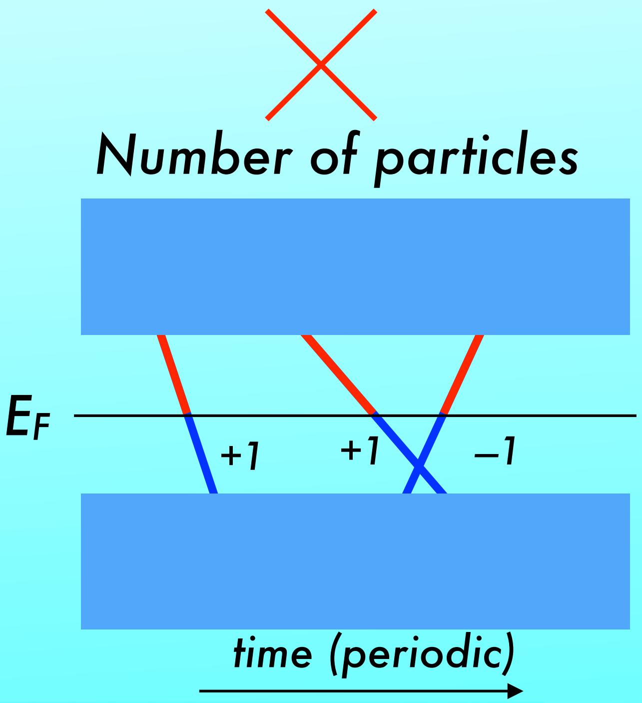
sum of the discontinuities

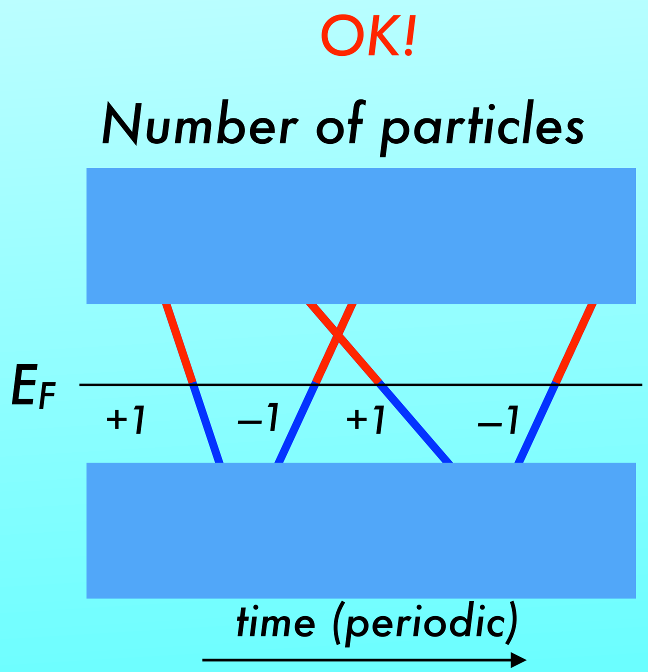
Bulk-edge correspondence in time domain → due to edge states

Topoquant, KITP, Oct. 14 (2016)

Quantization & conservation law







$$\boxed{\Delta Q = -\sum_i \Delta P(t_i)}$$

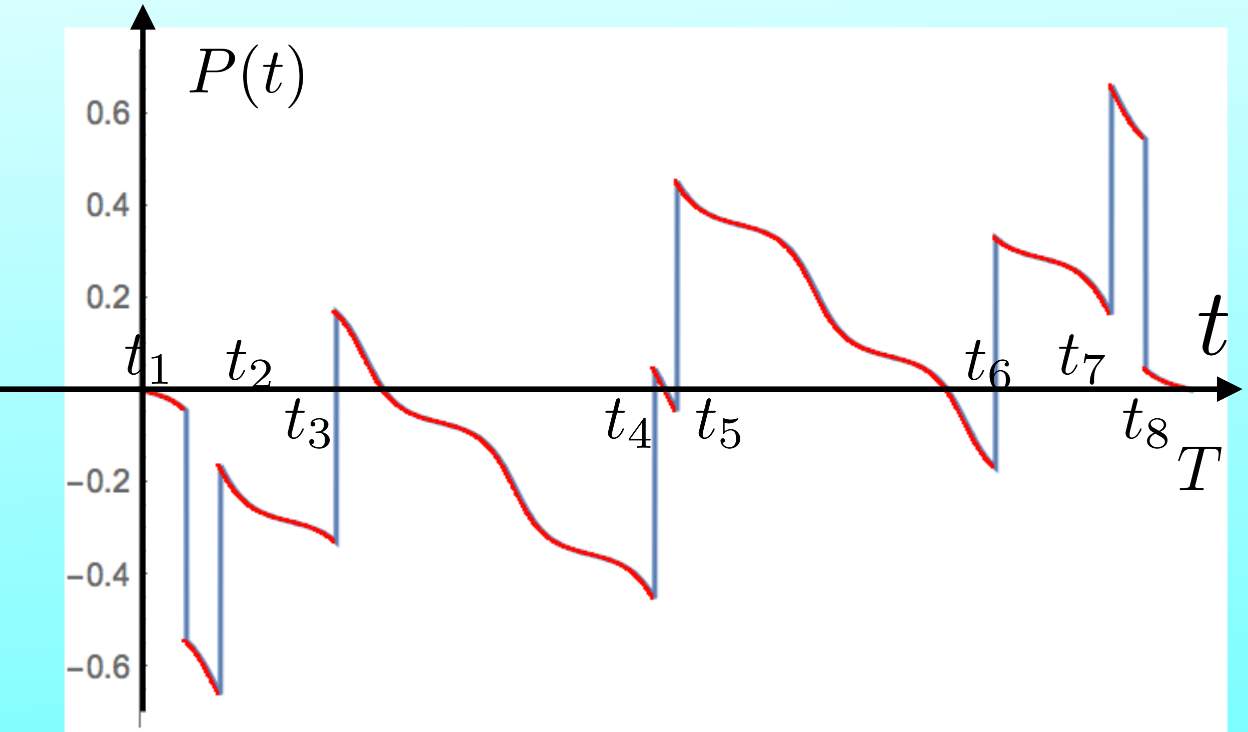
Number of the discontinuities (SUM) are EVEN !

Conservation of charge & periodicity in time

become occupied ⟷ become unoccupied

paired

Quantization & conservation law



$$\Delta P(t_i) = P(t_i^+) - P(t_i^-)$$

$= \left\{ \right.$
- -1/2: become unoccupied at R
- +1/2: become occupied at R
- +1/2: become unoccupied at L
- -1/2: become occupied at L

$$\boxed{\Delta Q = -\sum_i \Delta P(t_i) = -\sum_i \left(\pm \frac{1}{2} \right) = \text{integer} \quad I}$$

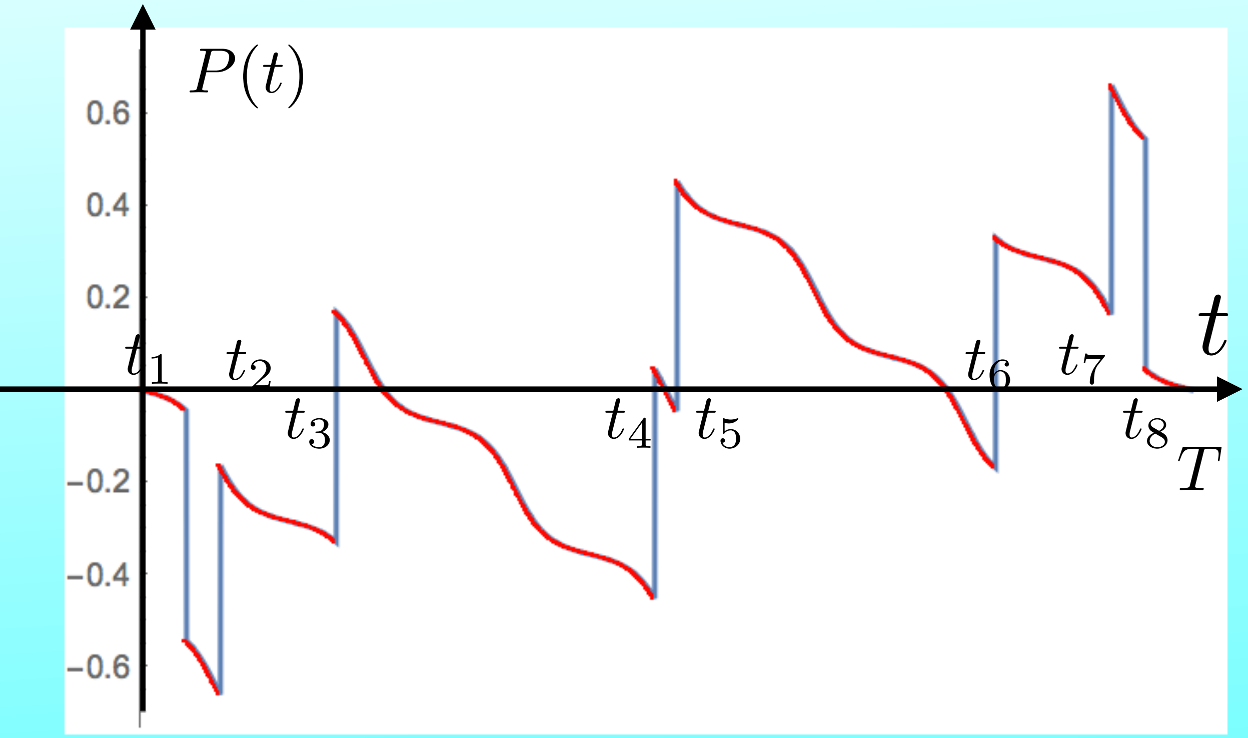
Number of the discontinuities (SUM) are EVEN !

Conservation of charge & periodicity in time

$\boxed{\text{become occupied}} \longleftrightarrow \boxed{\text{become unoccupied}}$

paired

Quantization & conservation law



$$\Delta P(t_i) = P(t_i^+) - P(t_i^-)$$

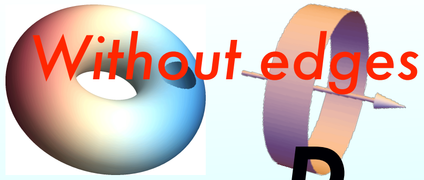
$=$

- -1/2: become unoccupied at R
- +1/2: become occupied at R
- +1/2: become unoccupied at L
- -1/2: become occupied at L

$$\boxed{\Delta Q = -\sum_i \Delta P(t_i) = -\sum_i \left(\pm \frac{1}{2} \right) = \text{integer } I}$$

Identify the discontinuities as massive Dirac fermions $\frac{1}{2}\operatorname{sgn} m$

Edge states correspond to massive Dirac fermions (fractionalized)

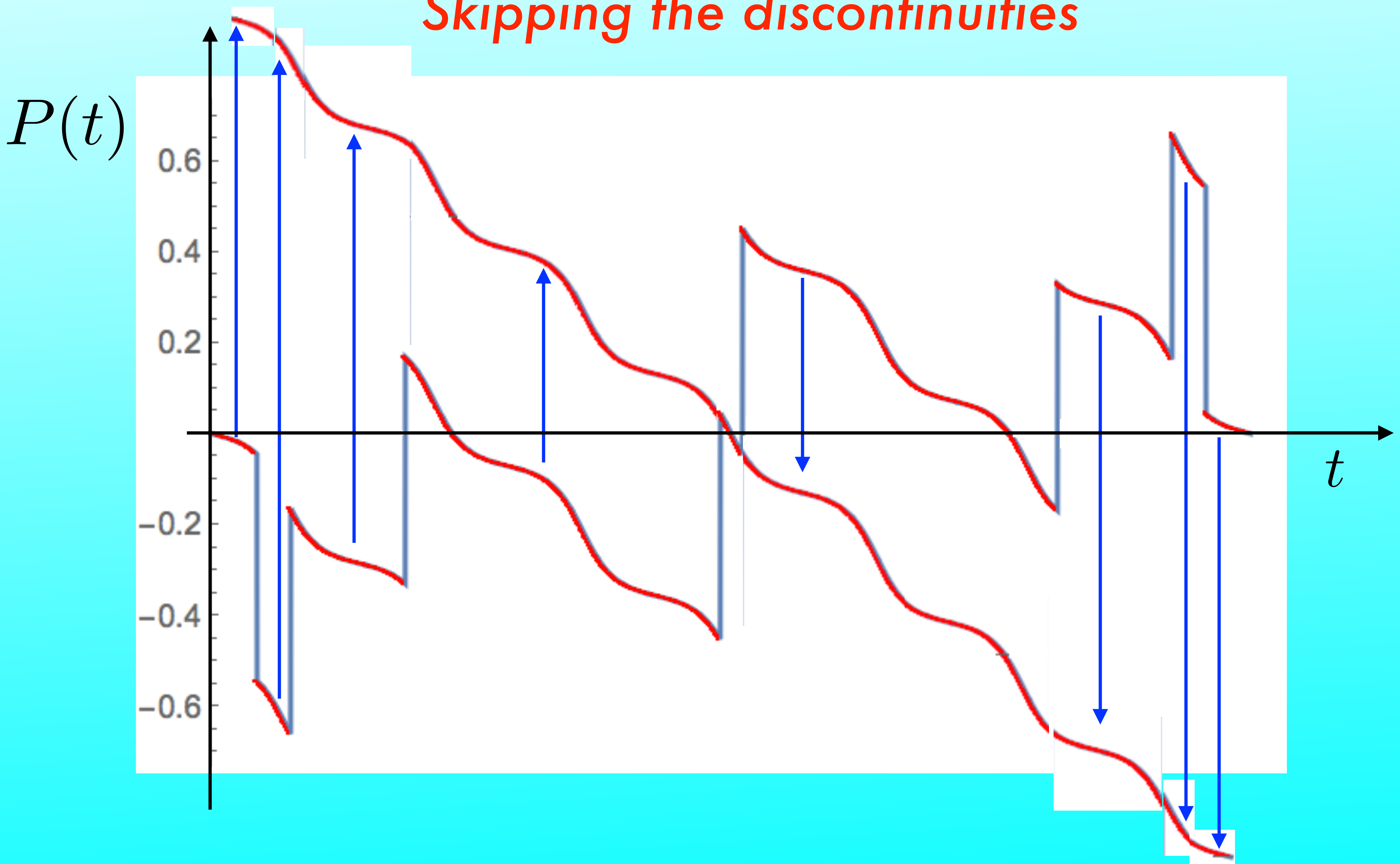


Pumped charge as a Chern number

Skipping the discontinuities

Pumped charge as a Chern number

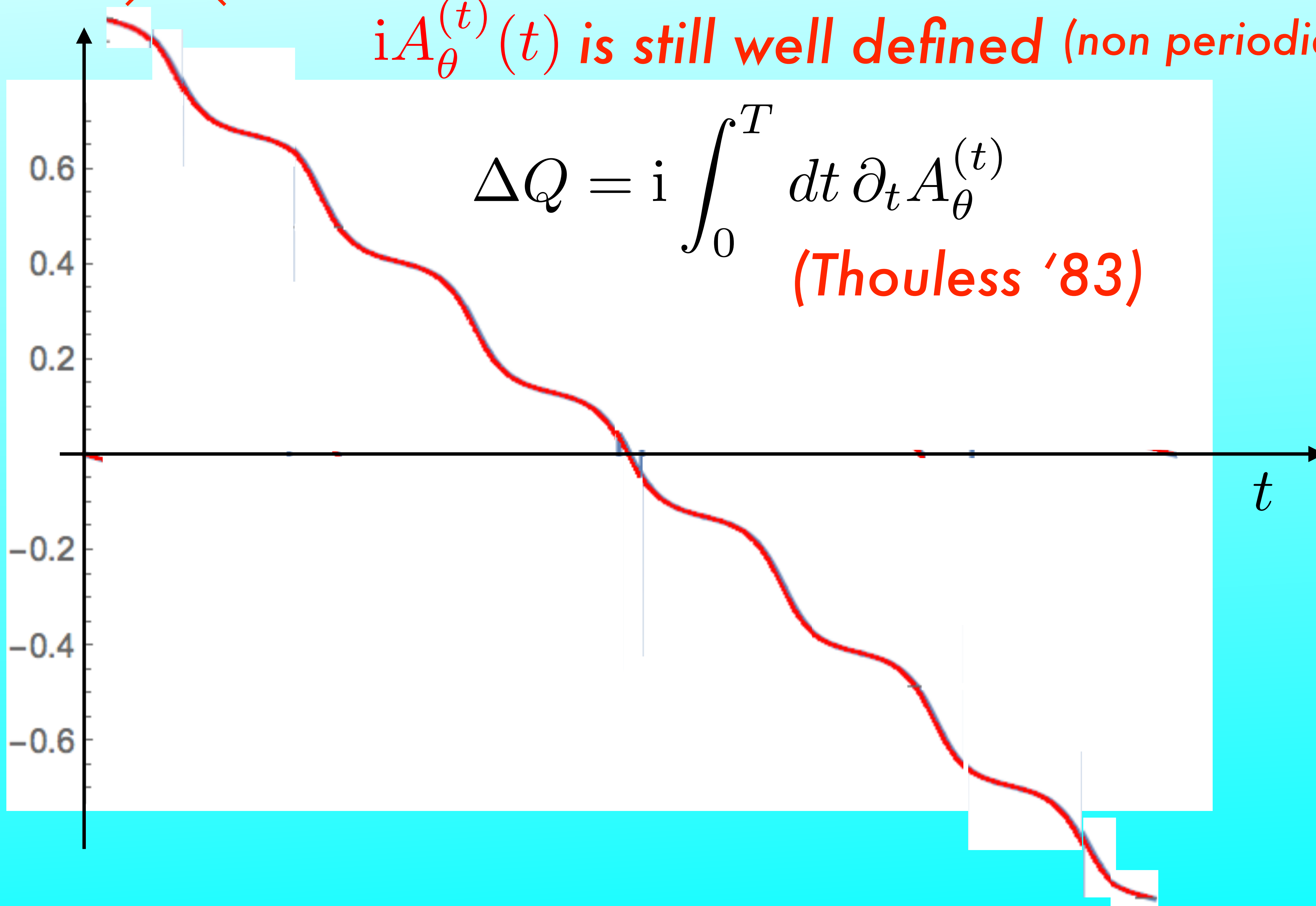
Skipping the discontinuities



Pumped charge as a Chern number

$\mathrm{i}A_\theta^{(t)}(t) = \cancel{P}(t)$ CM is not well defined for the bulk (Bloch state)

$\mathrm{i}A_\theta^{(t)}(t)$ is still well defined (non periodic in time)



$$\Delta Q = \mathrm{i}\int_0^T dt\,\partial_t A_\theta^{(t)}$$

(Thouless '83)

Pumped charge as a Chern number

$$\Delta Q = \mathrm{i}\int_0^T dt\,\partial_t A_\theta^{(t)} = \frac{1}{2\pi\mathrm{i}}\int_0^T dt\int_0^{\Delta k} dk_x\, b(k_x,t) \equiv C$$

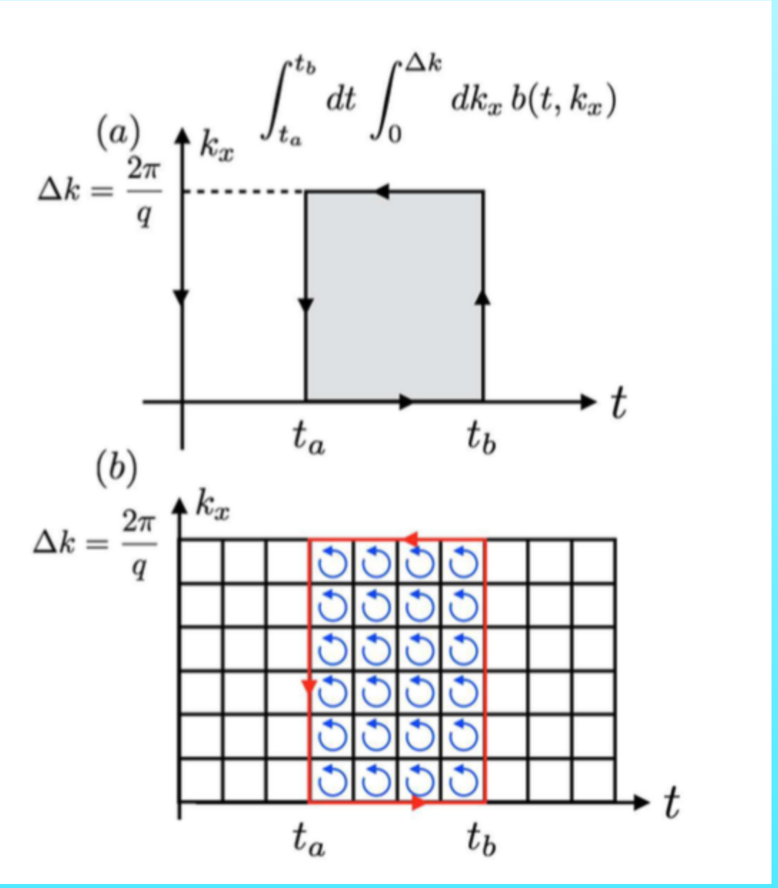
$$b = \partial_{k_x} a_t - \partial_t a_{k_x}$$

$$a_{k_x}^{(t)} = \mathrm{Tr}_M \mathcal{A}_{k_x}^{(t)}$$

$$\mathcal{A}_{k_x}^{(t)} = u^\dagger \partial_{k_x} u$$

$$u = (\boldsymbol{u}_1, \cdots, \boldsymbol{u}_M),$$

$\boldsymbol{u}_\ell(k_x, t)$ Bloch state of the ℓ-th band



non periodic gauge fixing

$$\boxed{I(\mathrm{edge}) = C(\mathrm{bulk})}$$

discontinuities Chern number

Bulk-edge correspondence between the topological numbers

Then the Chern number is integer as well ! (non trivial)

Note !

CM is only well-defined with edges
no way to define CM with periodic boundary condition

Pumped charge is carried by bulk
but is described
by the discontinuity due to edge states

This is the bulk-edge correspondence

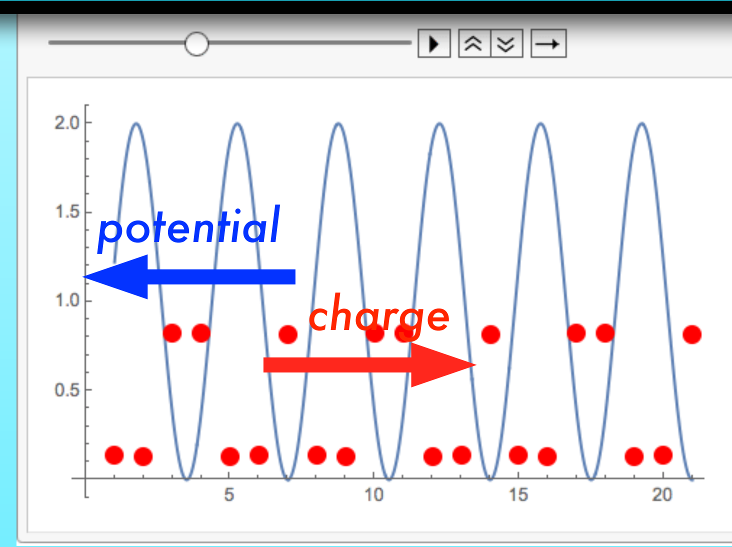
Discontinuity: breakdown of the adiabaticity
due to gapless edge states, then it is never observed in
real experiments of finite speed pump !

Edge states:

Do Not contribute the experiments

BUT still

protect quantization of the pumped charge



Note !

Discontinuity: breakdown of the adiabaticity
due to gapless edge states, then it cannot be observed
in real experiments of finite speed pump !
(if the system is large enough)

Thank you