Chiral topological spin liquids with PEPS

Didier Poilblanc

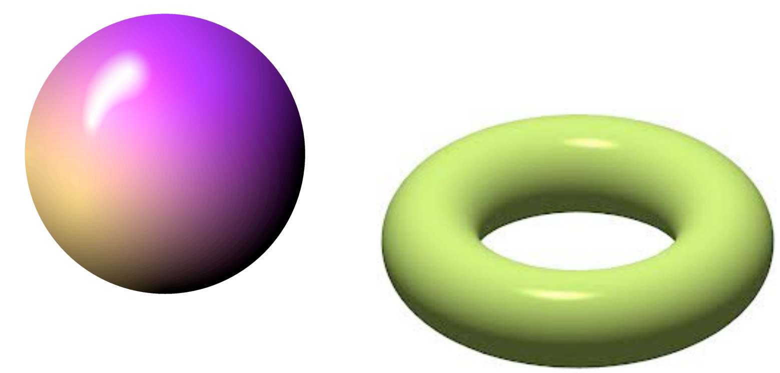
Laboratoire de Physique Théorique, Toulouse

OUTLINE

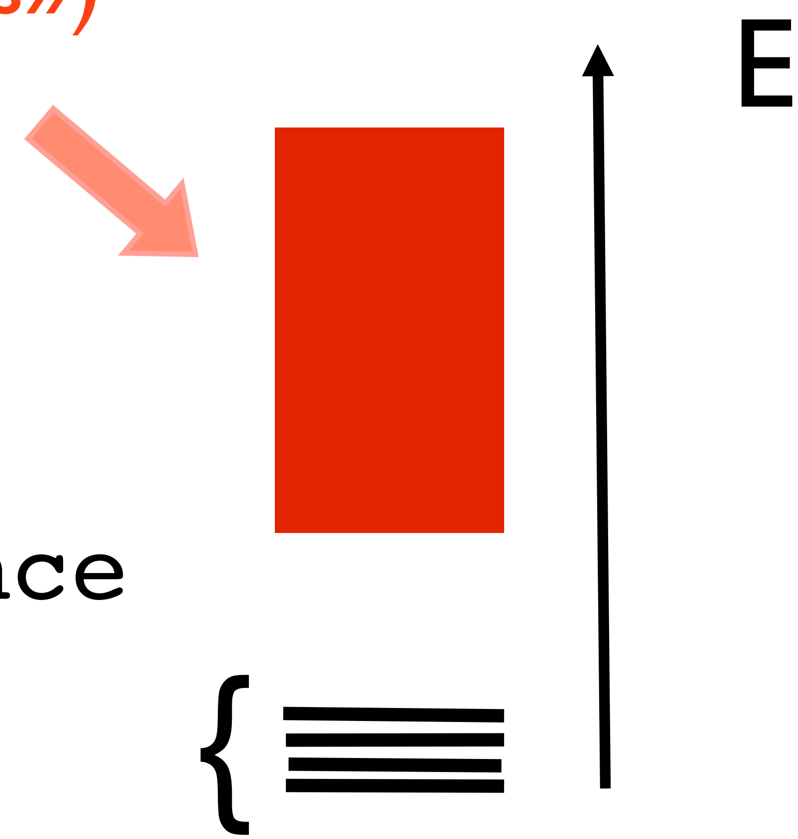
- Motivation: analogs of FQHS in the field of frustrated quantum magnetism ?
- «Projected Entangled Pair States» (PEPS) Ansätze and simple realizations:
    - The RVB Z_2 spin liquid
    - A d+id Abelian chiral spin liquid (CSL)
- «Holographic» framework : bulk-edge correspondence, «Entanglement Spectrum» and edge modes
- Towards constructing non-Abelian CSL

«Topological spin liquids» beyond the «order parameter» paradigm

* no spontaneous broken symmetry
* no local order but...
* Topological order



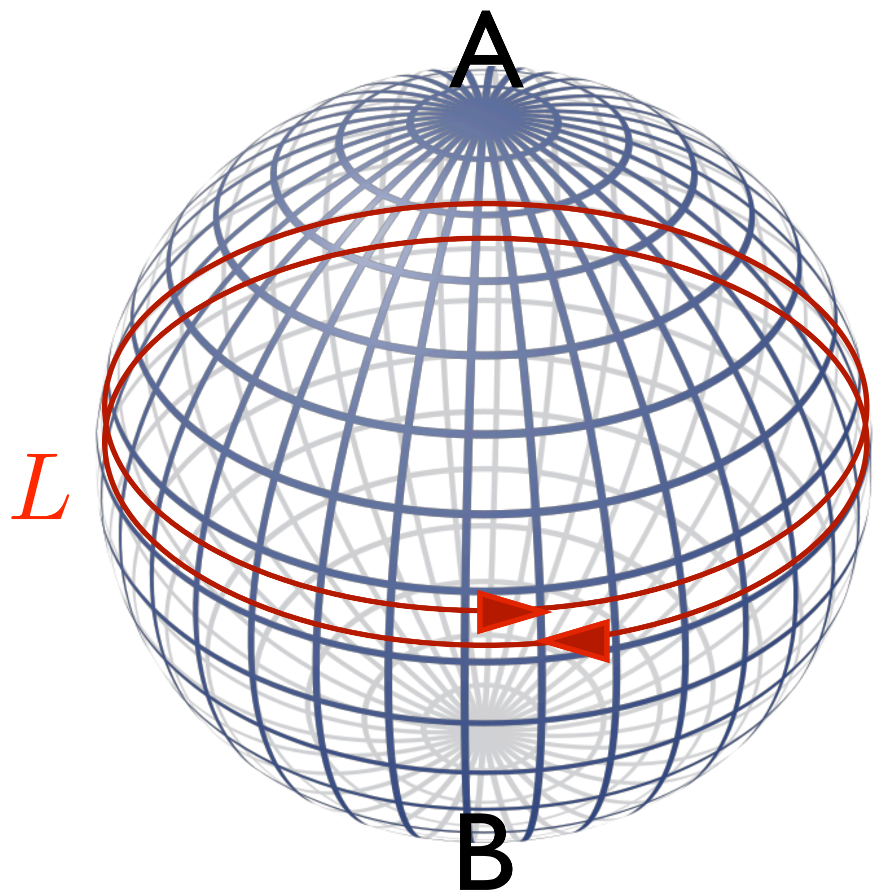
Excitations are fractional («anyons»)



GS degeneracy
depends on **topology** of space

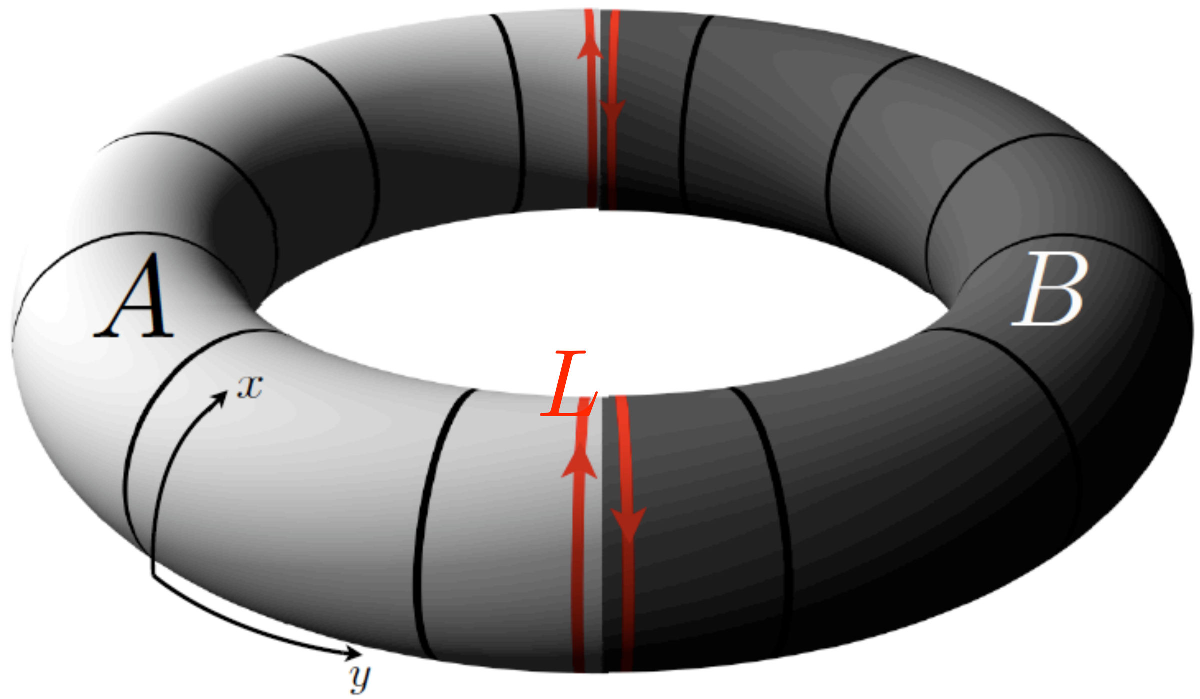
X. G. Wen

Edge states in topological chiral systems



Li & Haldane PRL 2008

Regnault, Bernevig & Haldane, 2009



Lauchli et al., 2009

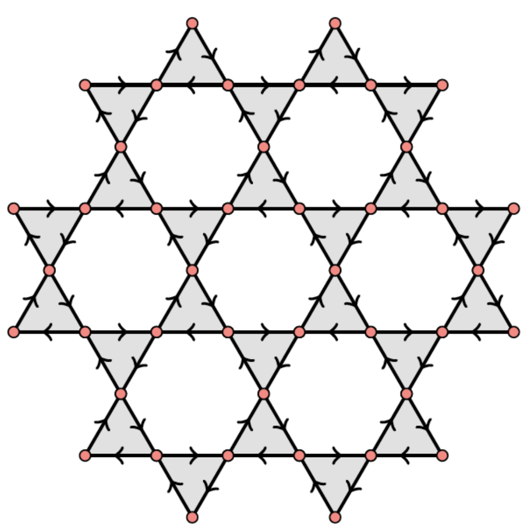
Long Range Entanglement → edge modes are protected !

Use new tools borrowed from quantum information
based on the concept of entanglement

Chiral SL in microscopic spin-1/2 models ?

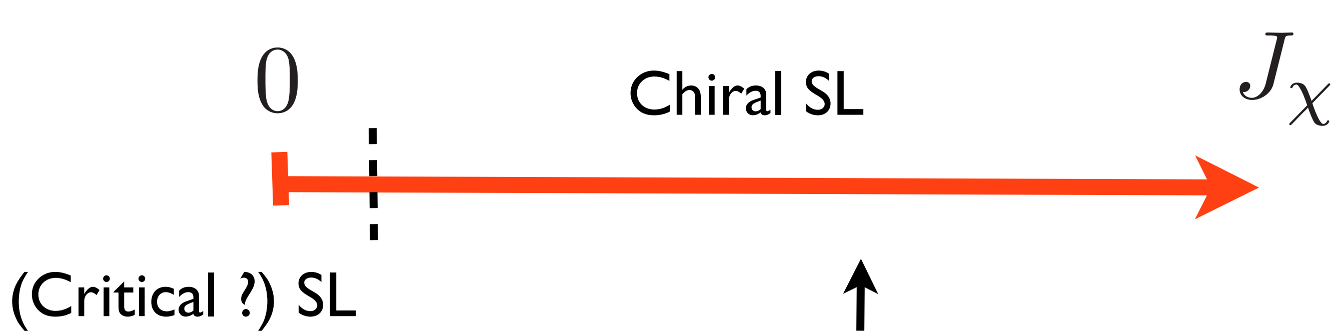
Paradigmatic chiral SL (Kalmeyer-Laughlin, 1987): $\nu = \frac{1}{2}$ FQHS on a lattice

B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig
Nature Communications 5, 5137 (2014)



Mott phase of a large-U Hubbard model

$$H = J_{\mathrm{HB}} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



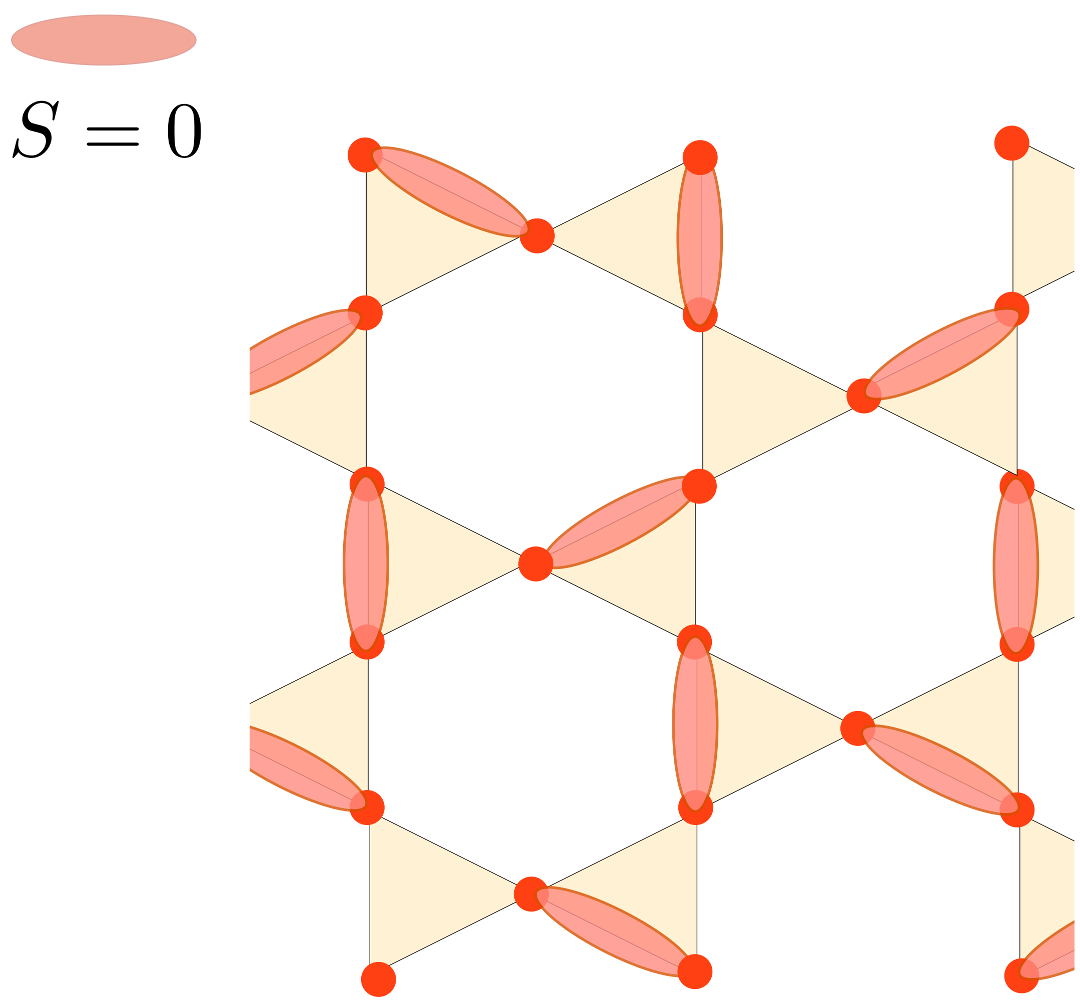
Edge modes : chiral c=1 CFT $(SU(2)_1$ WZW)

T-invariant H:

Yin-Chen He, D. N. Sheng, Yan Chen
Phys. Rev. Lett. 112, 137202 (2014)

Shou-Shu Gong, Wei Zhu, D. N. Sheng
Scientific Reports 4, 6317 (2014)

A «toy» gapped spin liquid: the RVB spin liquid



Equal-weight superposition
of NN singlet coverings

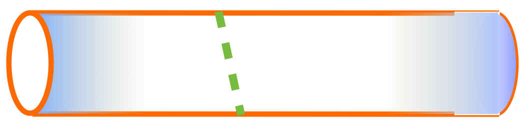
spin-1/2 RVB

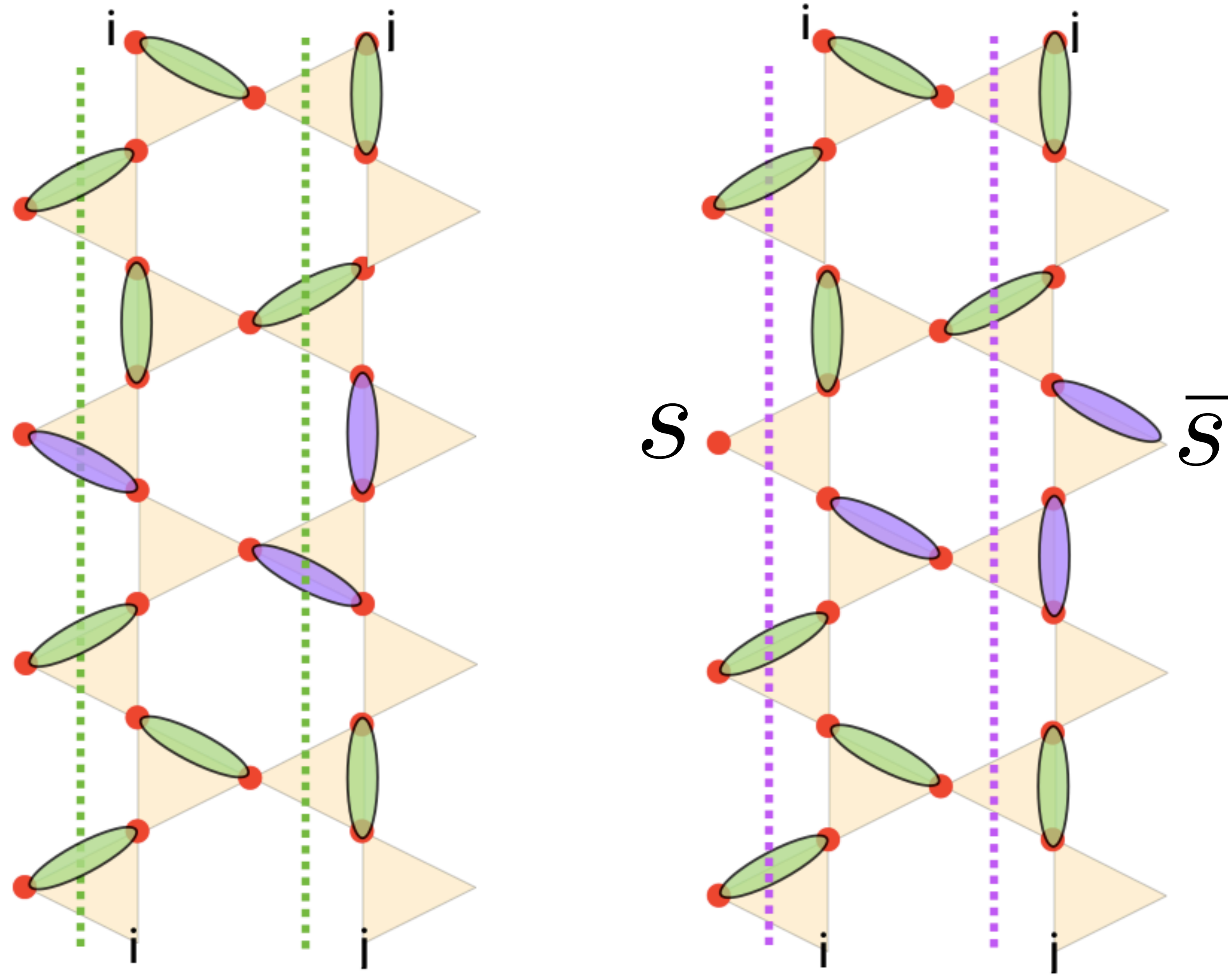
P. Fazekas and P.W. Anderson
Philosophical Magazine **30**, 423-440 (1974)

Topological liquid

Hasting-Oshikawa-Lieb-Schulz-Mattis theorem

Two «types» of cylinder boundaries





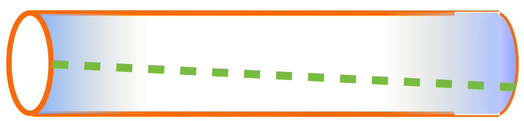
«even»

$$G_v = +1$$

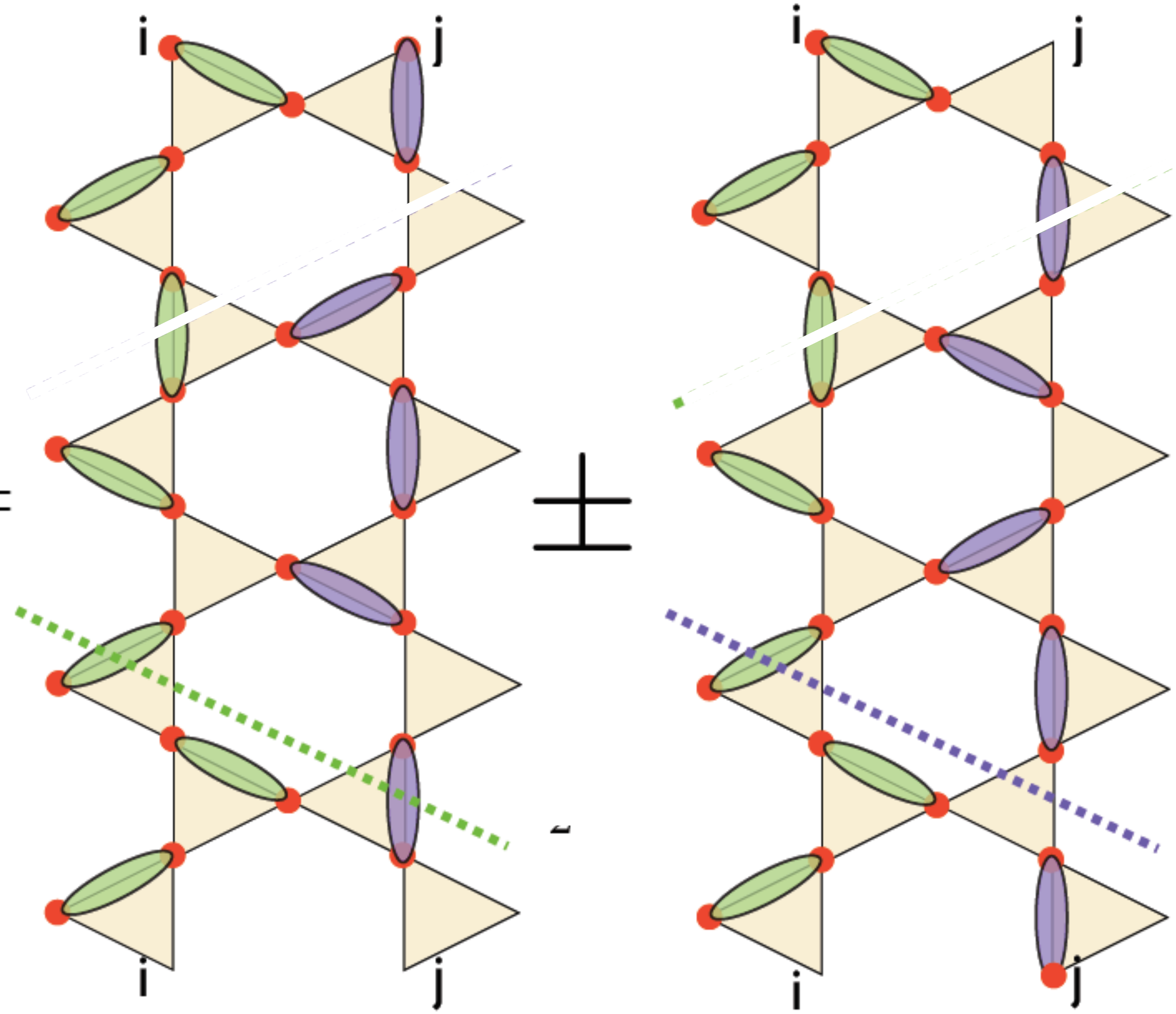
«odd»

$$G_v = -1$$

Eigenstates of a «Wilson loop» operator



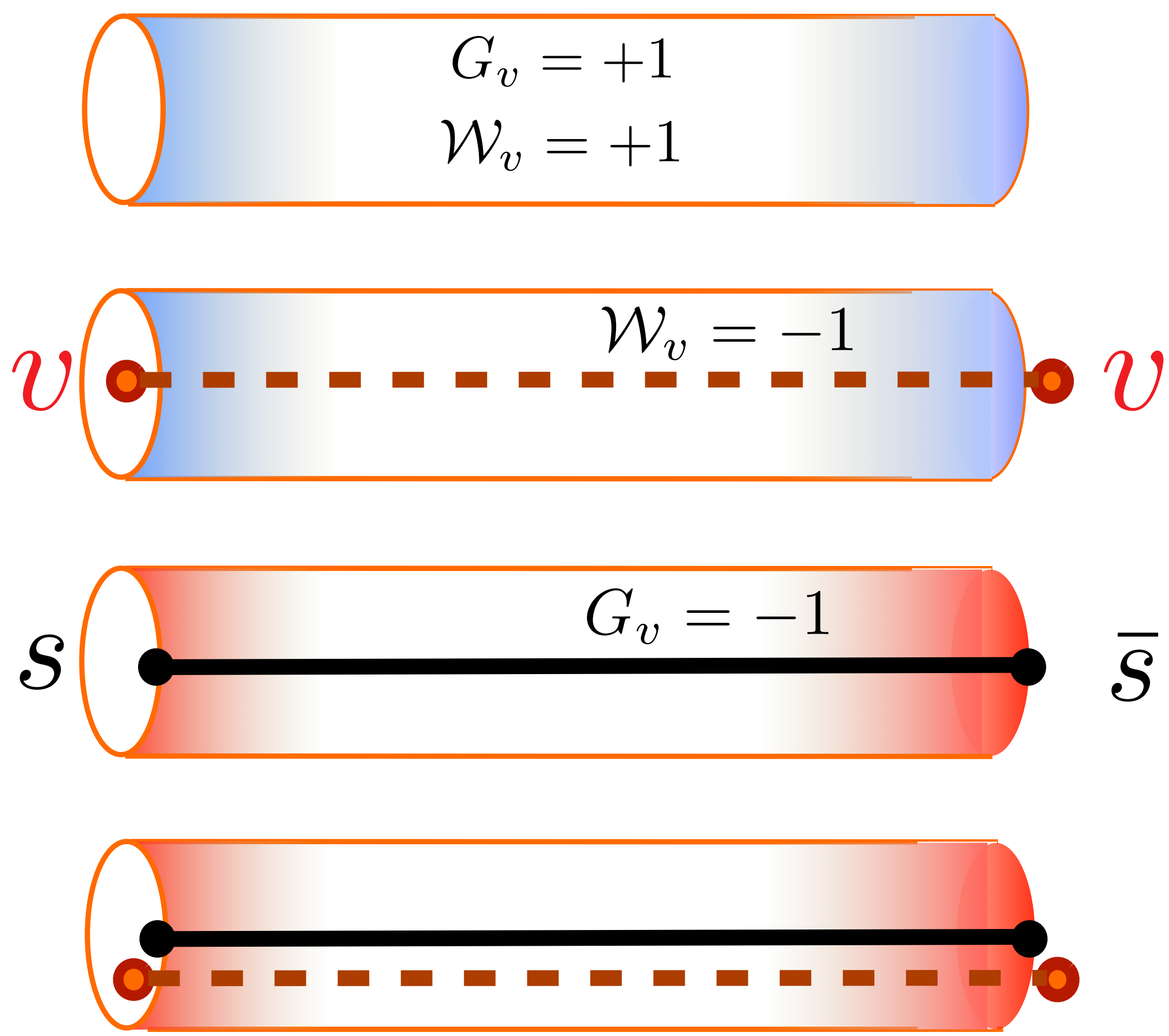
$$\Psi^{\pm}_{\mathrm{RVB}} =$$



$+$ no vison flux: $w_v = +1$

$-$ $\mathbb{Z}_2$ vison flux: $w_v = -1$

$\mathbb{Z}_2$ spin liquid : topological GS inserting «spinons» and «visons»

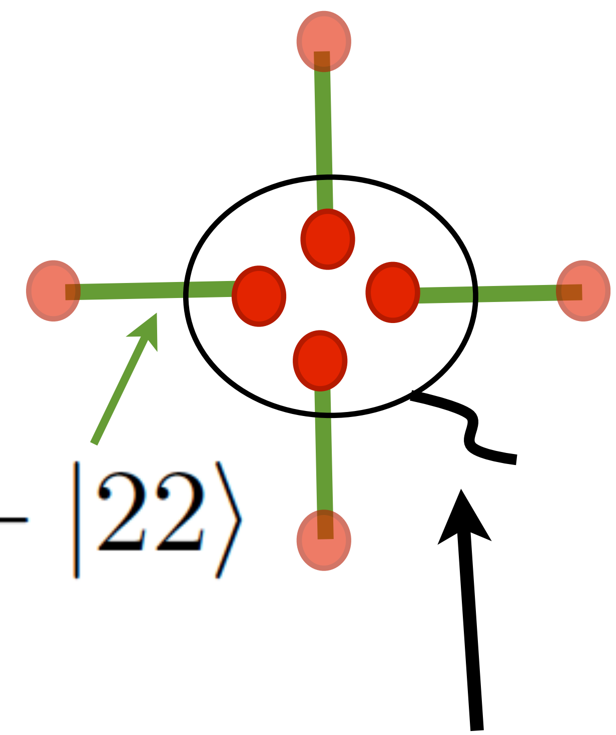


same class as
Kitaev's Toric Code
(fixed point $\xi = 0$)

The spin-1/2 RVB can be written as a PEPS !

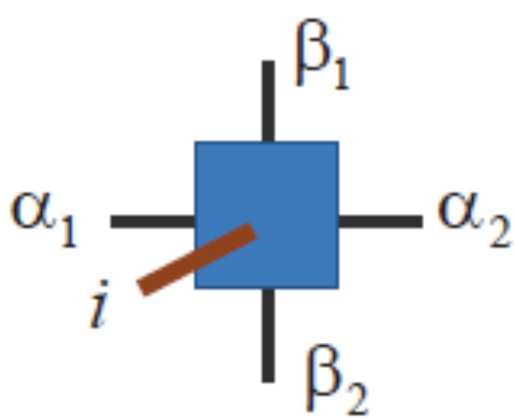
virtual states: $1/2 \oplus 0$

(D=3)



$$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$$

Project onto physical subspace S=1/2 (d=2)



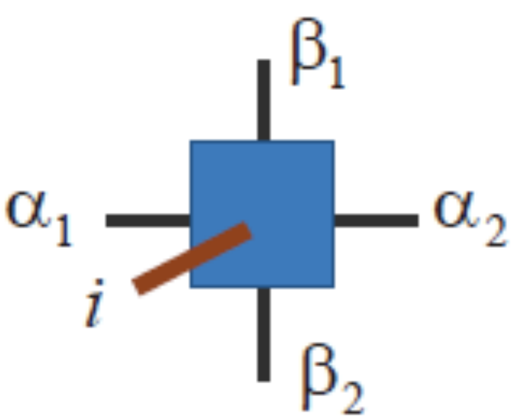
PEPS tensor

Gauge symmetry: topological order is encoded at the tensor level

The PEPS as a variational ansatz

$$|\Psi\rangle = \sum C_{i_1,i_2,\ldots,i_N} |i_1, i_2, \ldots, i_N\rangle$$

$$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$$



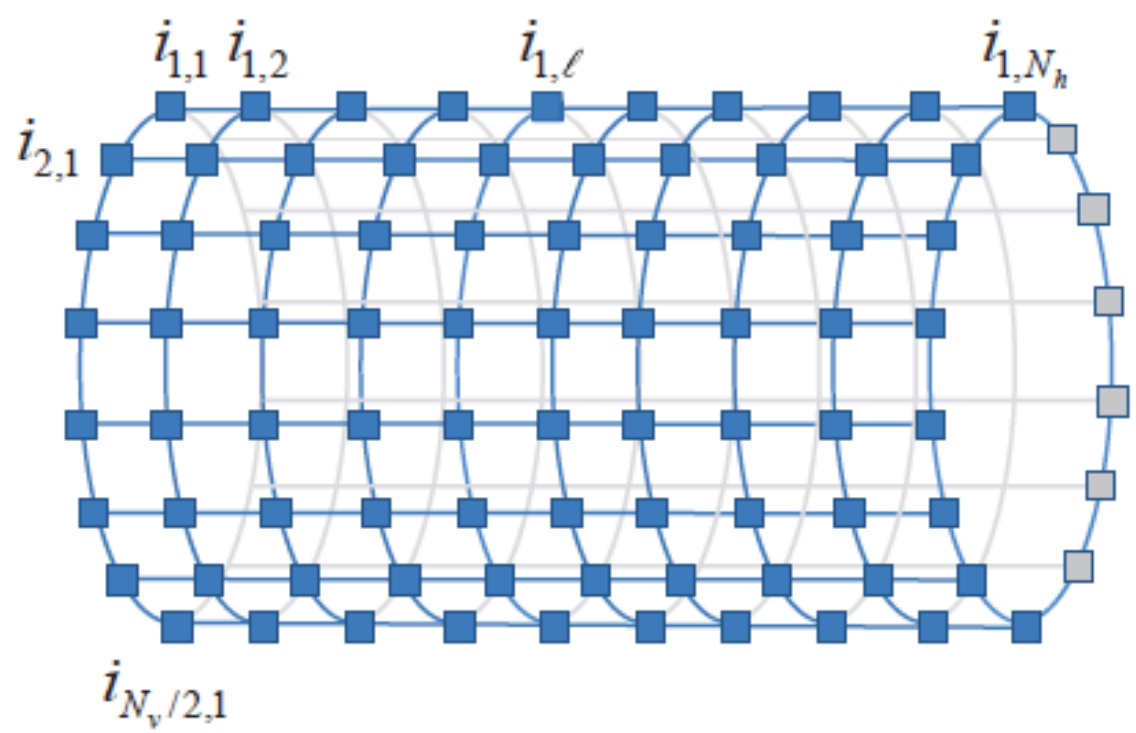
$$i = \{1, \cdots, d_{\text{phys}}\}$$

$$\alpha, \beta = \{1, \ldots, D\}$$

dimension of auxilary (or virtual) space

I. Cirac
F. Verstraete
G. Vidal

$$N = N_v N_h$$



Coefficients $C_{\{i_{1,1},\ldots,i_{N_v,N_h}\}}$ of the wavefunction

"contract" product of tensors

Two possible «routes» :

→ use simple PEPS ansatz to investigate physical (e.g. topological, etc...) properties

Conceptual understanding

→ optimize PEPS to construct competitive ansatz for microscopic models

G. Vidal, T. Xiang, R. Orus, P. Corboz,... & many more

Simulations of real materials

No-go theorem for chiral PEPS ?

J. Dubail, N. Read
Phys. Rev. B 92, 205307 (2015)

Chiral TNS of free fermions have
no gapped (local) parent Hamiltonians at E_F

Exemple by T.B. Wahl, H.-H. Tu, N. Schuch, J.I. Cirac
Phys. Rev. Lett. 111, 236805 (2013)

«They are ground states of two different kinds of free-fermion Hamiltonians: (i) local and gapless;
(ii) gapped, but with hopping amplitudes that decay according to a power law.»

For interacting spins is there any obstruction to construct chiral topological PEPS ?

not fully !

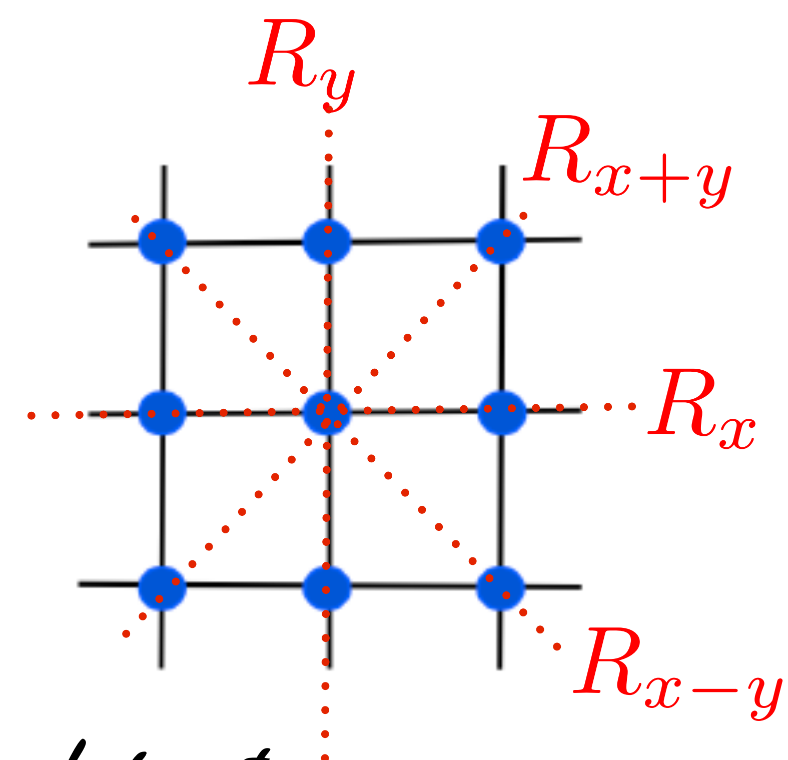
Exemple by Shuo Yang, Thorsten B. Wahl, Hong-Hao Tu, Norbert Schuch, J. Ignacio Cirac
Phys. Rev. Lett. 114, 106803 (2015)

but ... diverging correlation length

Constructing a chiral RVB PEPS

Necessary conditions:

$\forall \; G_i \in C_{4v}$ reflexion symmetries



$$G_i|\Psi\rangle = |\Psi^*\rangle$$ *Time-reversed partner*

Realized for a PEPS ansatz with the form:

$$\Psi = \Psi_s + i\,\Psi_g$$

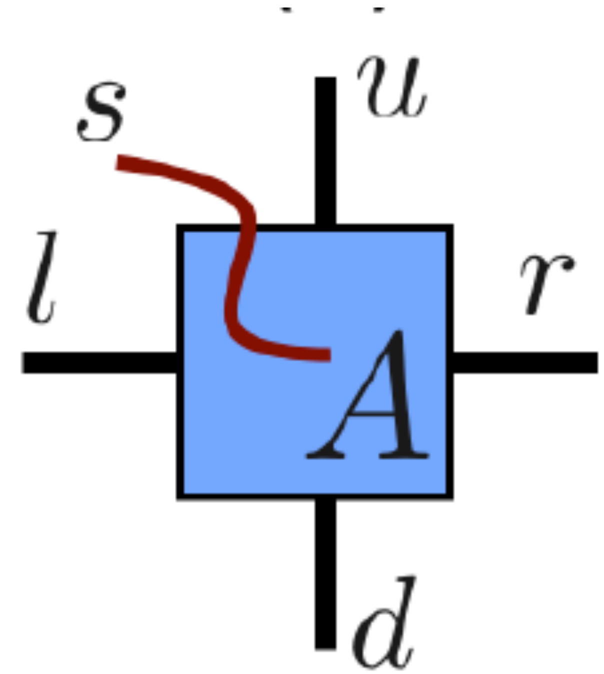
$s + ig$ symmetry

Implementing point-group symmetries

Can be achieved **at the level of the local PEPS tensor**:

$$A = R + iI$$

$$d_{x^2-y^2} + i\, d_{xy}$$



«Poor-man» approach for small D :

$$R^s_{lurd} = R^s_{ldru} = R^s_{ruld} = -R^s_{drul} = -R^s_{uldr},$$
$$I^s_{lurd} = -I^s_{ldru} = -I^s_{ruld} = I^s_{drul} = I^s_{uldr}.$$

Systematic classification of SU(2)-symmetric tensors

D=3 PEPS Ansatz

«Teleportation» tensors

L.Wang, DP, Z.-C. Gu, X.-G. Wen, F. Verstraete, PRL111, 037202 (2013)

RVB tensor

(NN singlets)

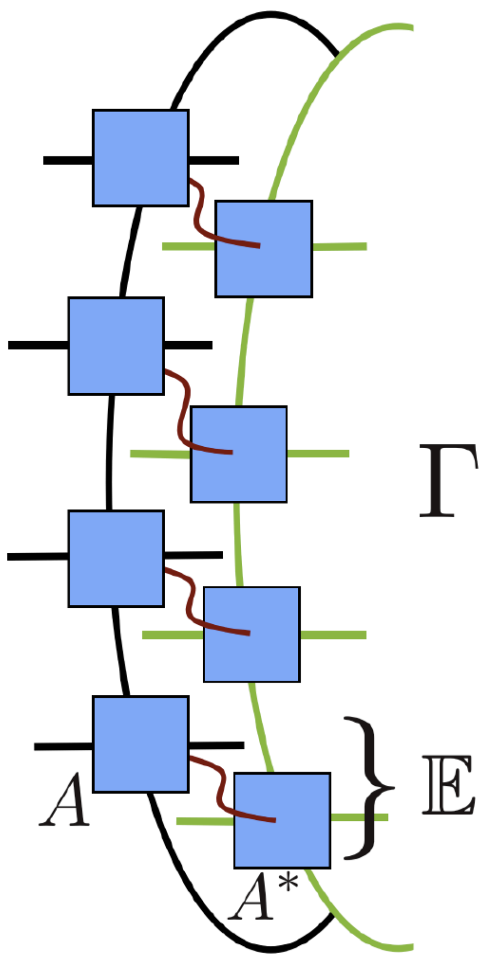
Generate NNN singlets

$$A = \lambda_1 R_1 + \lambda_2 R_2 + i\lambda_{\mathrm{chiral}} I$$

Point group sym.: $d_{x^2-y^2}$ d_{xy}

inherits $\mathbb{Z}_2$ gauge symmetry

Transfer Matrix



$$\langle \Psi | \Psi \rangle = V_{\text{left}} \Gamma^{N_h} V_{\text{right}}$$

$$D^{2N_v} \times D^{2N_v} \text{ matrix}$$

Build «double layer» tensor network
by contracting physical variables

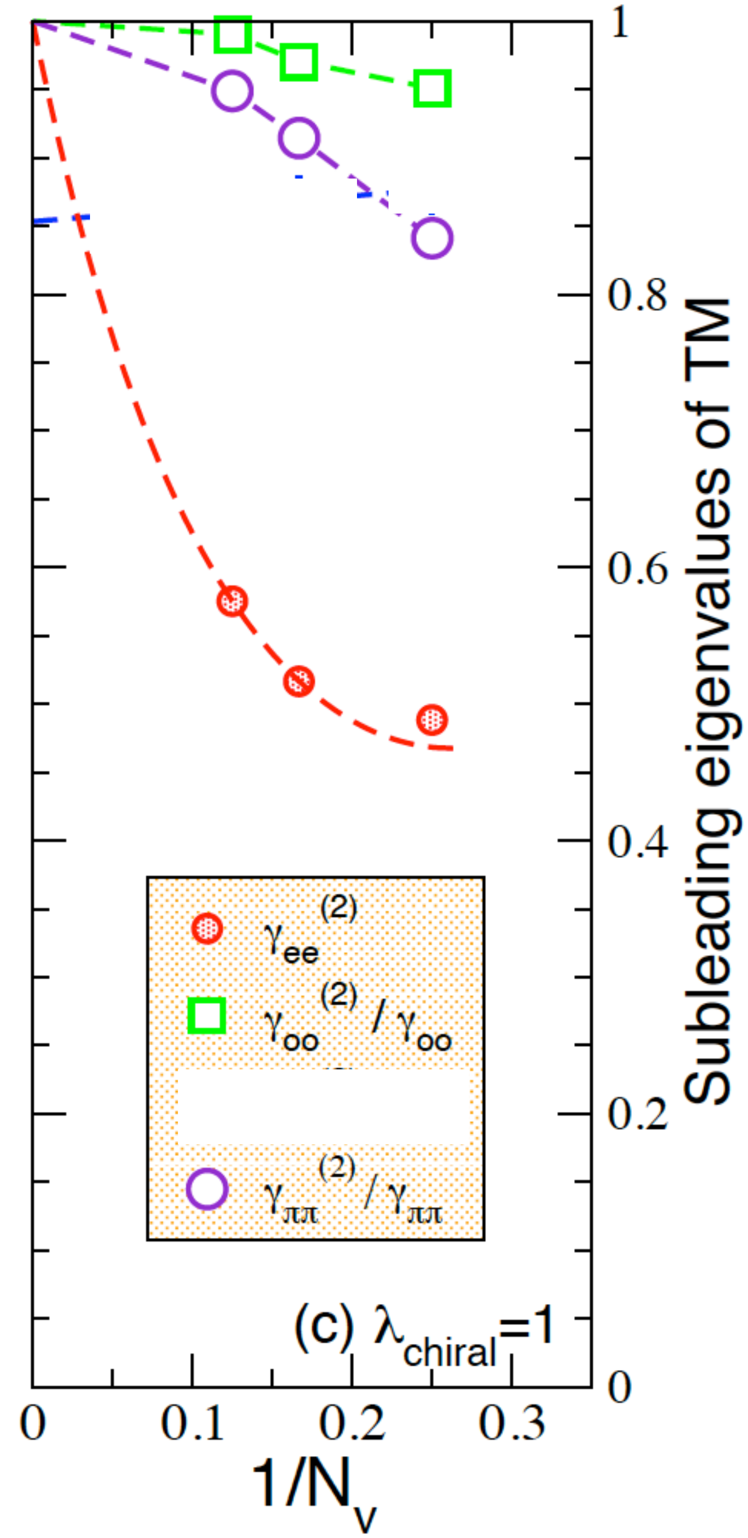
Spectrum of TM provides correlation lengths
Leading eigenvector («fixed point») gives Entanglement Spectrum

Gapless topological sectors

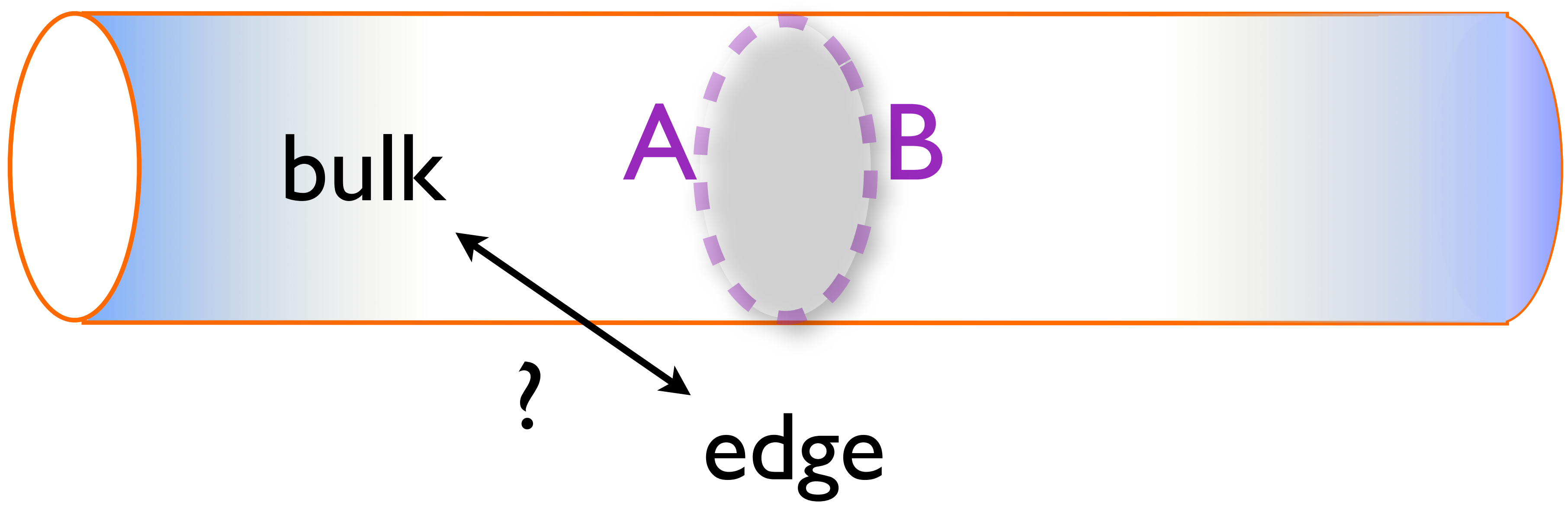
- 4 sectors can be constructed

- TM gapless: compatible with algebraic SL

retain critical character of Rohksar-Kivelson dimer liquid on a bipartite lattice

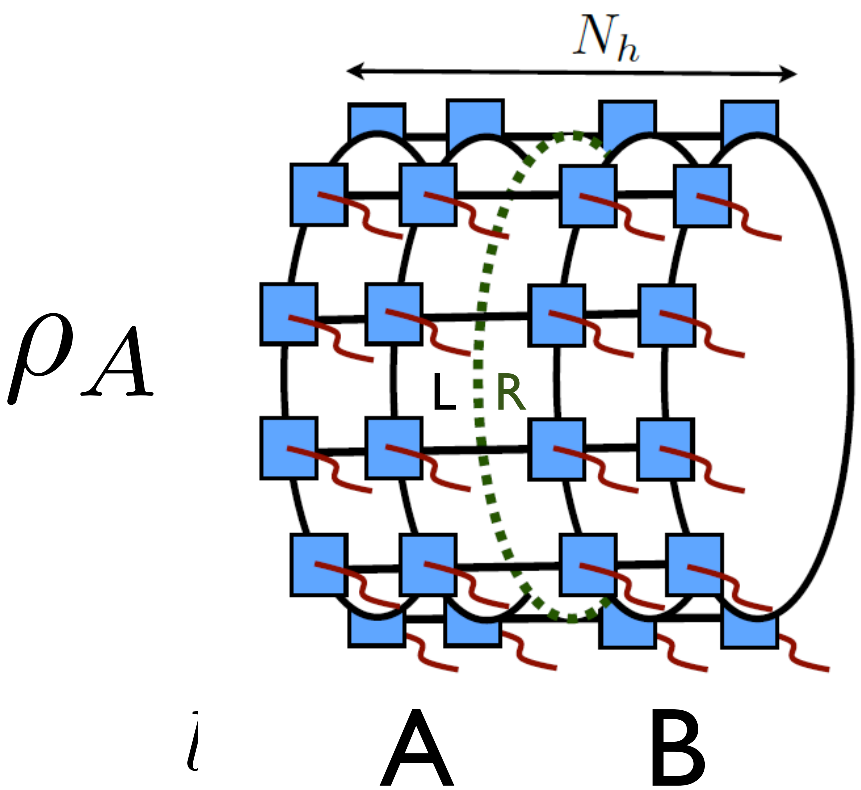


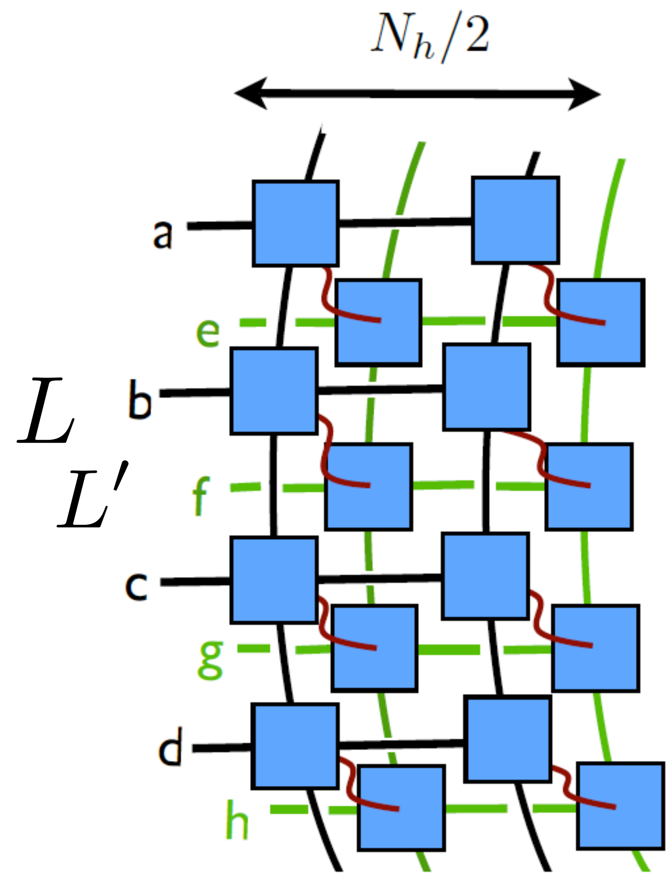
«Holographic» framework



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

Reduced density matrix





$$\sigma_b^2$$

lives” on the boundary

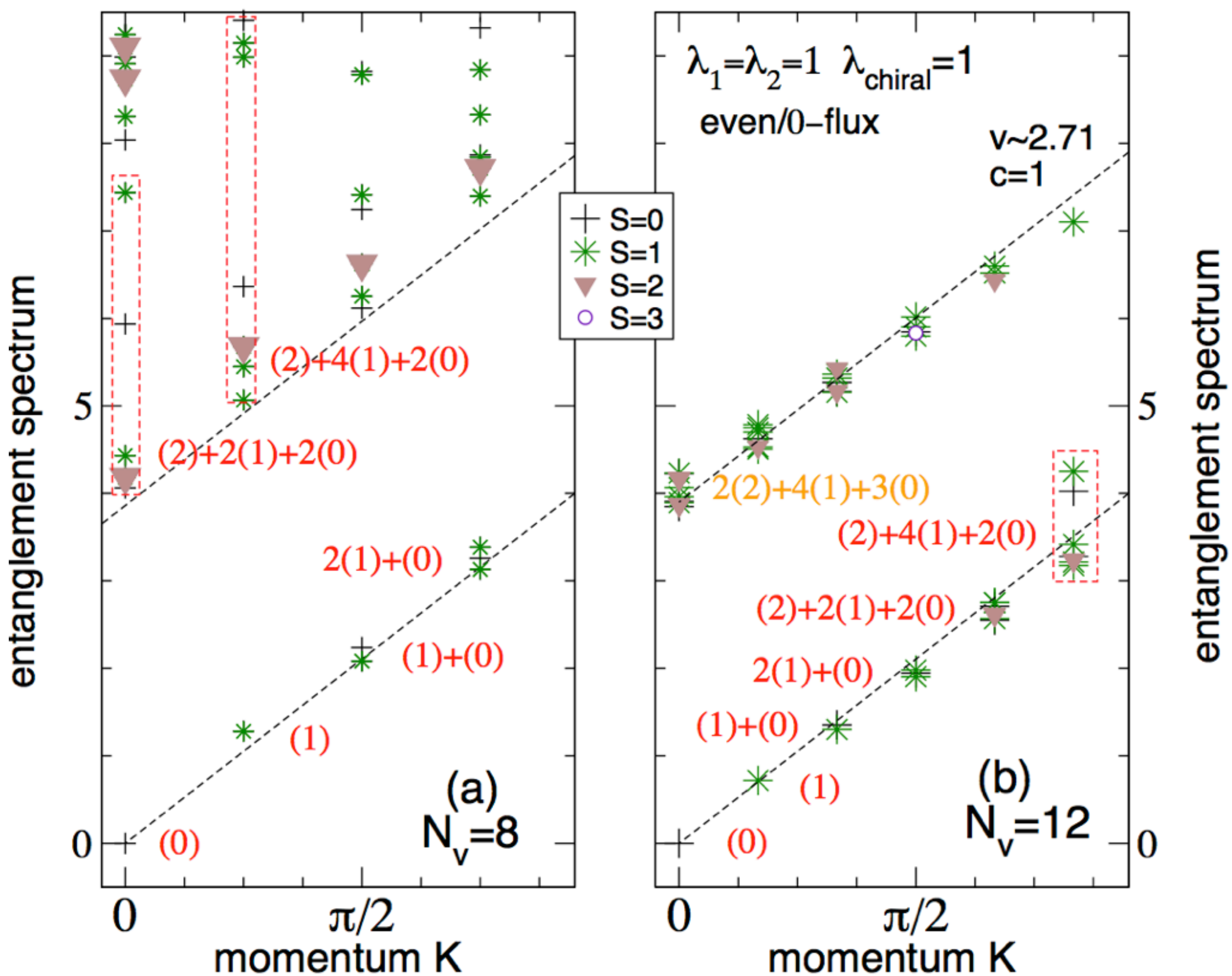
Basic formula: $\rho_A = U \sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

J. Ignacio Cirac, DP, Norbert Schuch, Frank Verstraete, Phys. Rev. B 83, 245134 (2011)

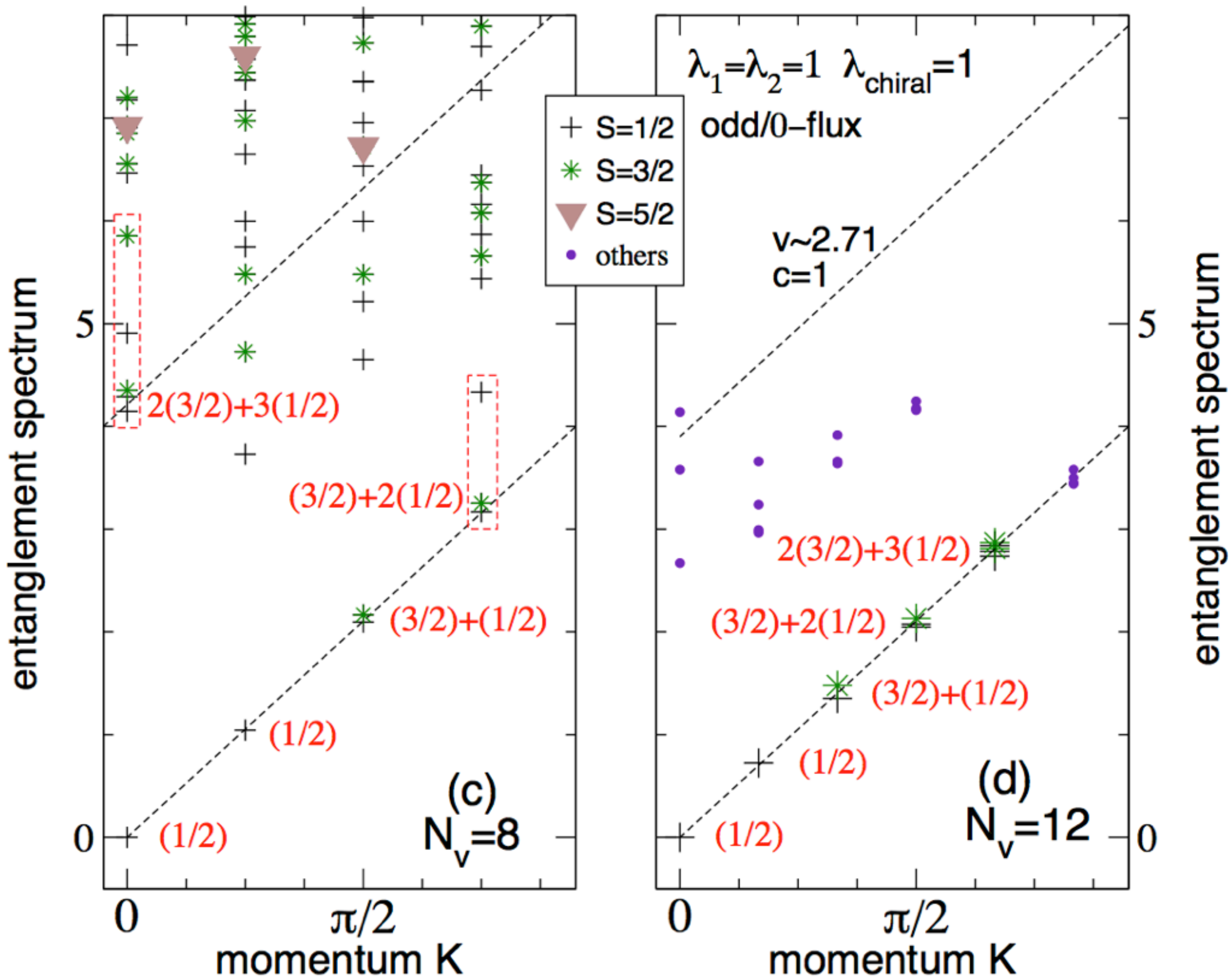
Entanglement spectrum of chiral RVB (I)

Even sector «tower of states»



Entanglement spectrum of chiral RVB (II)

Odd sector «tower of states»



SU(2)$_1$ Entanglement spectrum

WZW SU(2)$_1$ CFT

$$E_{\mathrm{CFT}}(S_z, m_n, n) = \frac{\pi u}{N_v}\left(-\frac{c}{24} + S_z^2 + m_n n\right)$$

= Luttinger Liquid
at SU(2)-symmetric point

Even sector

Table 15.1. States in the lowest grades of the $\widehat{su}(2)_1$ module $L_{[1,0]}$.

L_0	S_z: −2	−1	0	1	2	$su(2)$ decomposition
0			1			(0)
1		1	1	1		(2)
2		1	2	1		(2)+(0)
3		2	3	2		2(2)+(0)
4	1	3	5	3	1	(4)+2(2)+2(0)
5	1	5	7	5	1	(4)+4(2)+2(0)
6	2	7	11	7	2	2(4)+5(2)+4(0)

Odd sector

Table 15.2. States in the lowest grades of the $\widehat{su}(2)_1$ module $L_{[0,1]}$.

L_0	$2S_z$: −2	−1	0	1	2	3	$su(2)$ decomposition
$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$		1	2	2	1		(3)+(1)
$\frac{13}{4}$		1	3	3	1		(3)+2(1)
$\frac{17}{4}$		2	5	5	2		2(3)+3(1)
$\frac{21}{4}$		3	7	7	3		3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)

Philippe Di Francesco
Pierre Mathieu
David Sénéchal

Conformal Field Theory

$$E \equiv N_v e_0 + e_{\mathrm{topo}} + E_{\mathrm{CFT}}$$

$$e_0 = \frac{\pi c}{6u}$$

$$e_{\mathrm{topo}} = -\ln 2/2$$

Entanglement entropy

$$S_n = -\frac{1}{n-1}\mathrm{Ln}\{\mathrm{Tr}(\rho_{\mathrm{A}})^{\mathrm{n}}\}$$

(Renyi)

$$S_{\mathrm{VN}} = -\mathrm{Tr}\{\rho_A \ln \rho_A\}$$

(Von Neumann)

"area" law

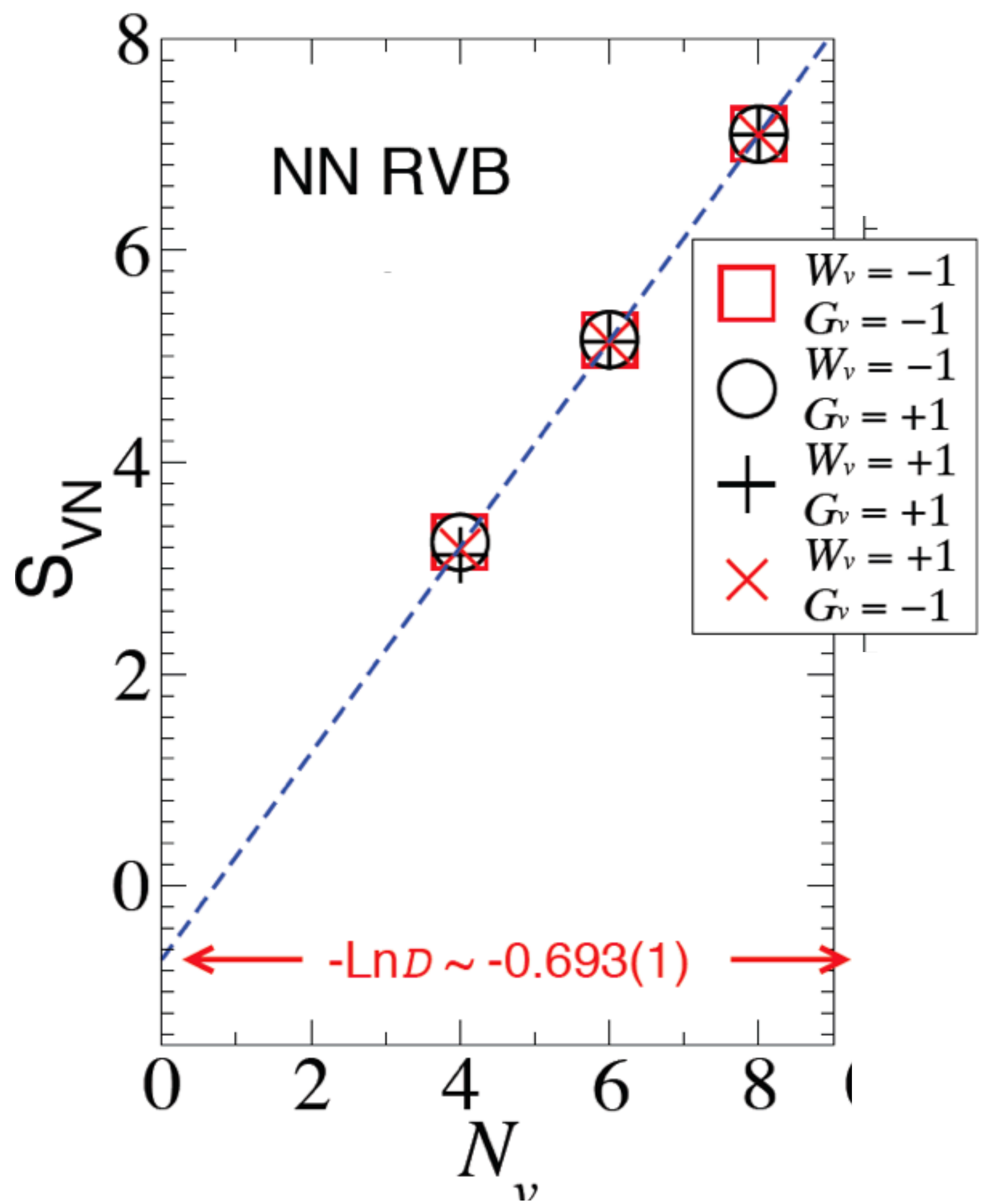
$$S_{\mathrm{VN}} \sim C N_v - S_{\mathrm{TE}}$$

subleading correction to area law: topological entropy

Kitaev & Preskill, 2006

Levin & Wen, 2006

EE of non-chiral RVB (kagome)



$S_{\mathrm{TE}} \simeq -\ln 2$ → $\mathbb{Z}_2$ spin liquid

EE of chiral liquid (CFT predictions)

Kitaev & Preskill, 2006

Levin & Wen, 2006

Renyi: $$S_q \sim \frac{q+1}{q} e_0 N_v + e_{\text{topo}}$$

$$e_{\text{topo}} = -\ln 2/2$$

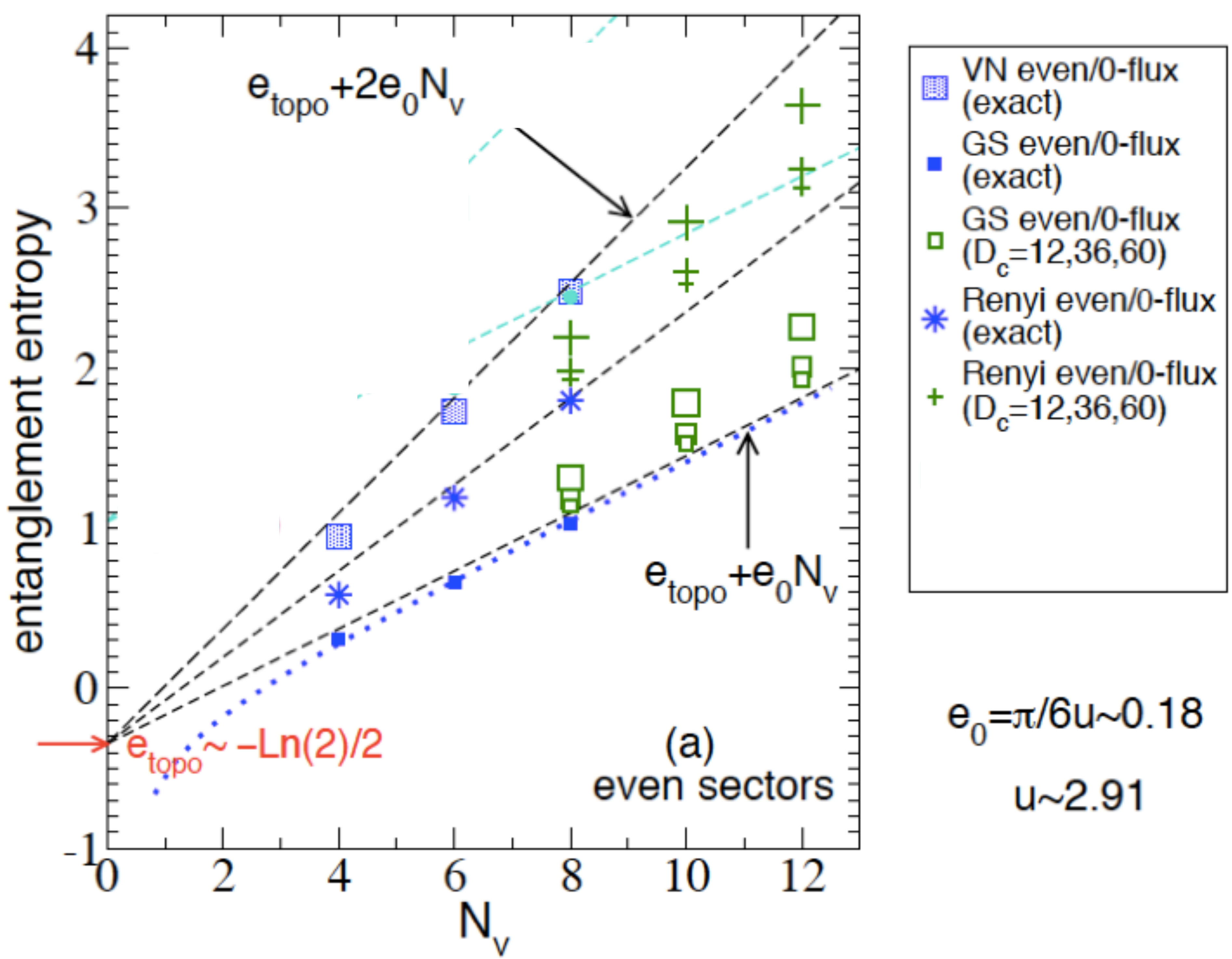
Von Neumann $$S_{VN} \sim (2e_0) N_v + e_{\text{topo}}$$

$$S_\infty \sim e_0 N_v + e_{\text{topo}} + \frac{\pi u}{N_v} \alpha^{(p)}$$

$$\alpha^{(e)} \sim -\frac{c}{24}$$

$$\alpha^{(o)} \sim \frac{1}{4} - \frac{c}{24}$$

EE of square chiral RVB liquid (PEPS)



$e_0=\pi/6u\sim 0.18$

$u\sim 2.91$

Non-Abelian CSL ?

Would host non-Abelian fractional excitations (anyons) !

Several exemples in the FQHE world:
- Moore-Read FQHS with «Ising anyons»
- Read-Rezayi FQHS with «Fibonacci anyons»

New developments:

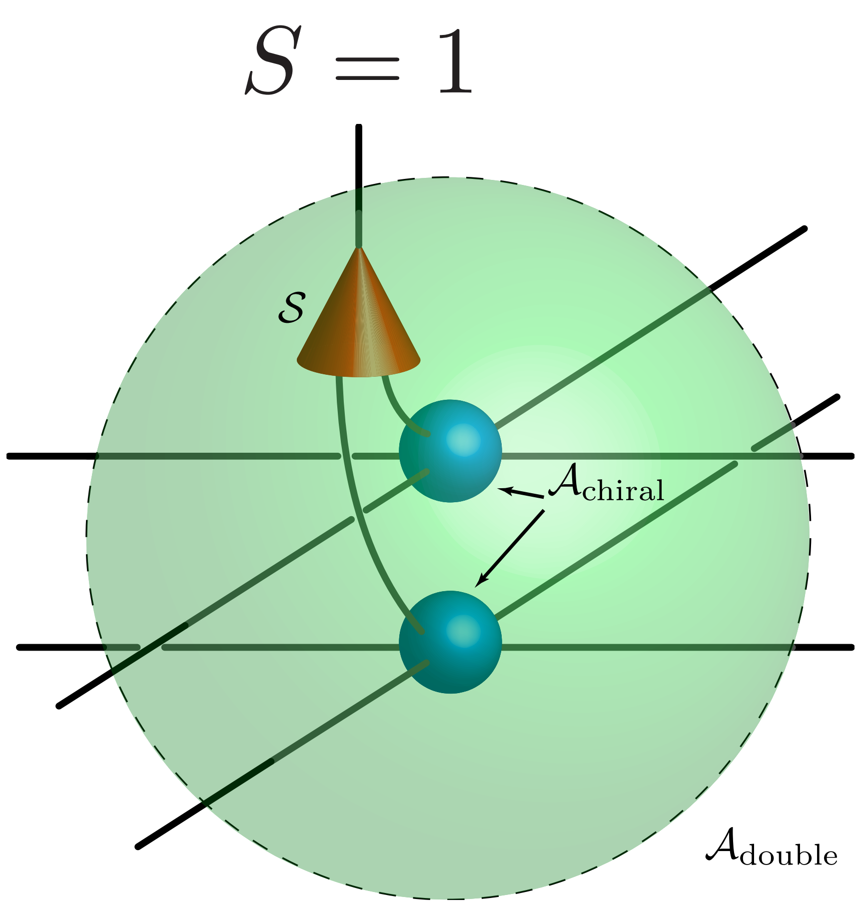
B. Estienne, Z. Papic, N. Regnault, and B. A. Bernevig
«Matrix product states for trial quantum hall states», Phys. Rev. B 87, 161112 (2013)

Symmetrization of multi-layers system:

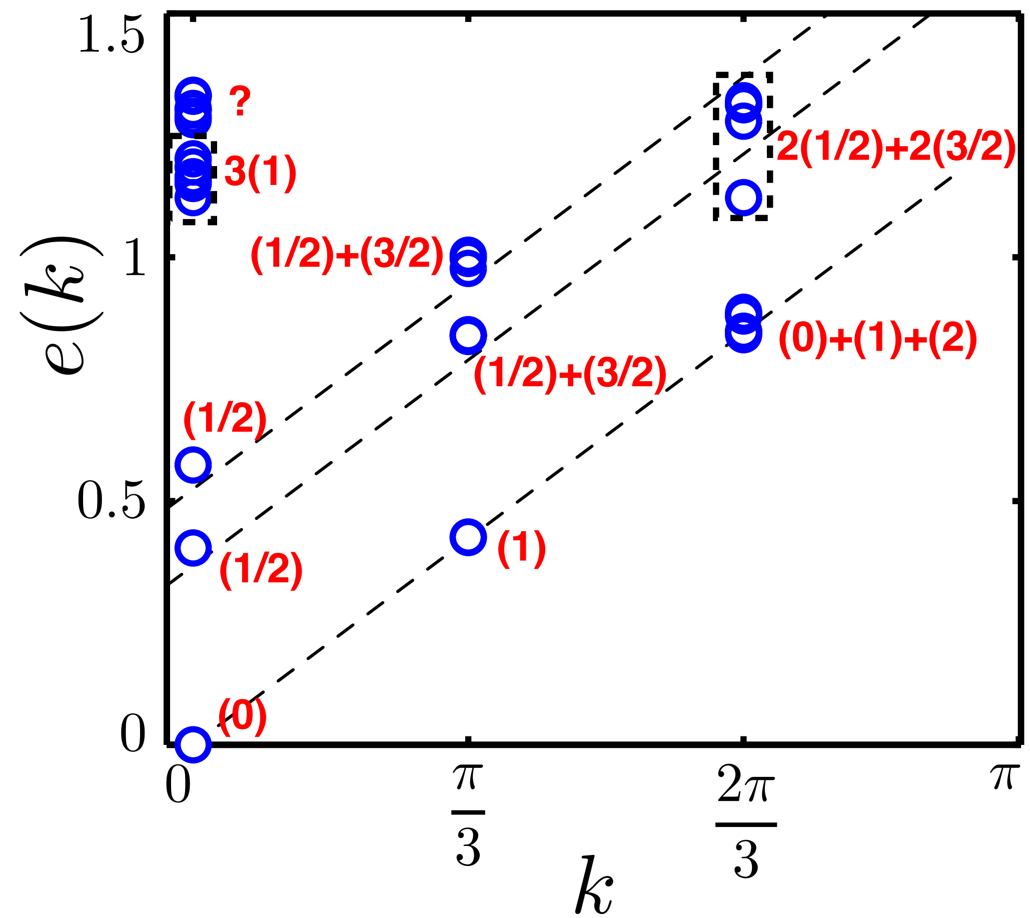
Cécile Repellin et al.
Projective construction of the Zk Read-Rezayi fractional quantum hall states and their excitations on the torus geometry," Phys. Rev. B 92, 115128 (2015)

Carlos Fernandez-Gonzalez, Roger S. K. Mong, Olivier Landon-Cardinal, David Perez-Garcia, and Norbert Schuch
Constructing topological models by symmetrization: A peps study," arXiv:1608.00594

Double-layer tensor



Entanglement spectrum



$n\backslash j$	0	$\frac{1}{2}$	1
0	(0) [1]	$(\frac{1}{2})$ [2]	(1) [3]
1	(1) [3]	$(\frac{1}{2})+(\frac{3}{2})$ [6]	(0)+(1) [4]
2	(0)+(1)+(2) [9]	$2(\frac{1}{2})+2(\frac{3}{2})$ [12]	(0)+2(1)+(2) [12]
3	(0)+3(1)+(2) [15]	$4(\frac{1}{2})+3(\frac{3}{2})+(\frac{5}{2})$ [26]	2(0)+3(1)+2(2) [21]

3 sectors of the $SU(2)_2$ WZW model

Summary

- Conceptual understanding of important correlated phases in quantum magnetism :
  - Non-chiral & chiral topological SLs
- Virtual degrees of freedom play «physical role» at boundary : edge states
- Some issues:
  - Are PEPS chiral SLs always gapless ?
    (see Dubail & Read)
  - Could they be considered as critical lines between gapped topological phases ?

PEPS framework for spin liquids

Norbert Schuch, DP, **J. Ignacio Cirac**, and **David Pérez-García**, Phys. Rev. B **86**, 115108 (2012)

DP, Norbert Schuch, David Pérez-García, and J. Ignacio Cirac, Phys. Rev. B **86**, 014404 (2012)

Norbert Schuch, DP, J. Ignacio Cirac, and David Perez-Garcia Phys. Rev. Lett. 111, 090501 (2013)

DP and Norbert Schuch, Phys. Rev. B 87, 140407 (2013)

Chiral spin liquids

DP, J. Ignacio Cirac and Norbert Schuch, Phys. Rev. B 91, 224431 (2015)

DP, Norbert Schuch and **Ian Affleck**, Phys. Rev. B 93, 174414 (2016) (Editor's suggestion)

Classification of spin liquids

Matthieu Mambrini, **Roman Orus** & DP, arXiv:1608.06003