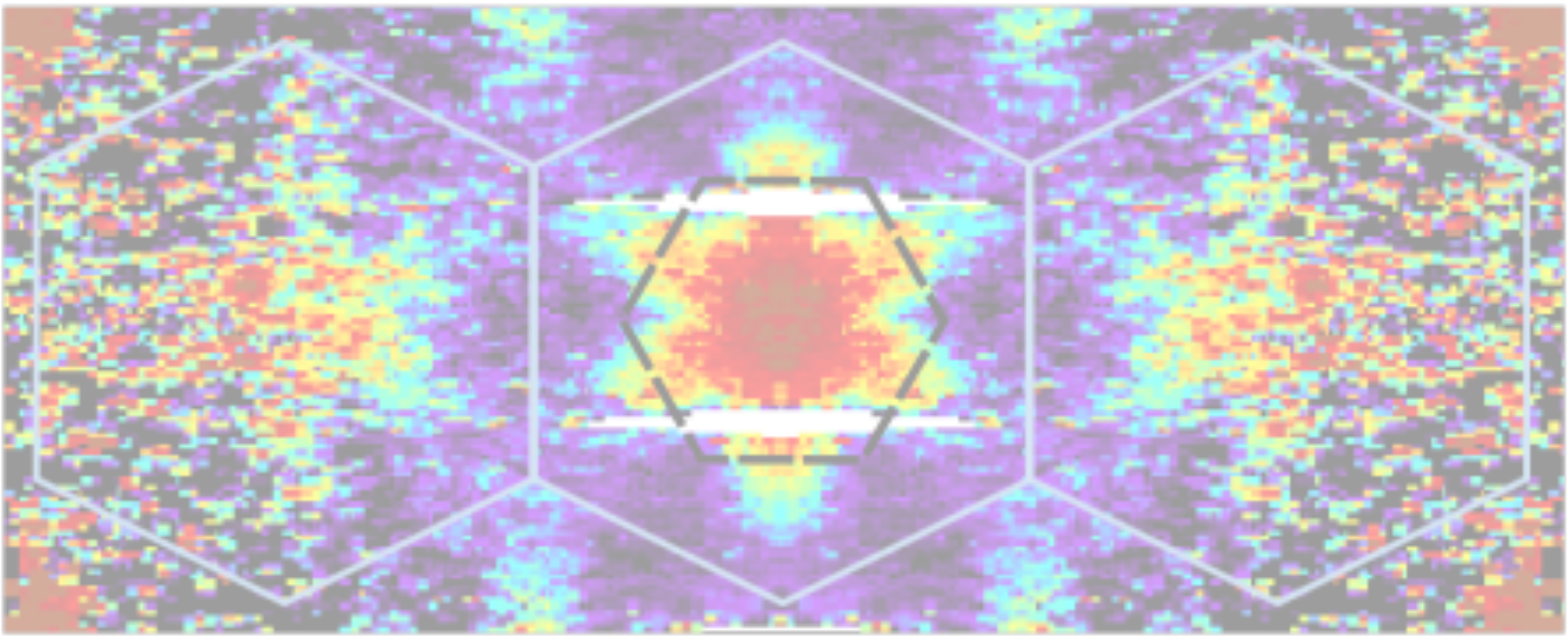
Dynamical signatures of topological spin liquids

Frank Pollmann,
MPI for the Physics of Complex Systems

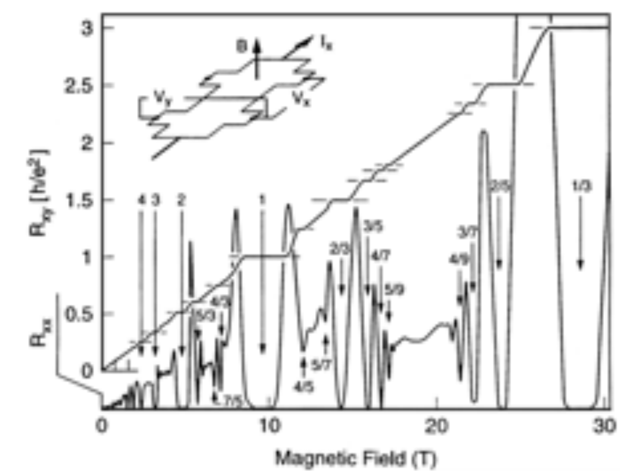


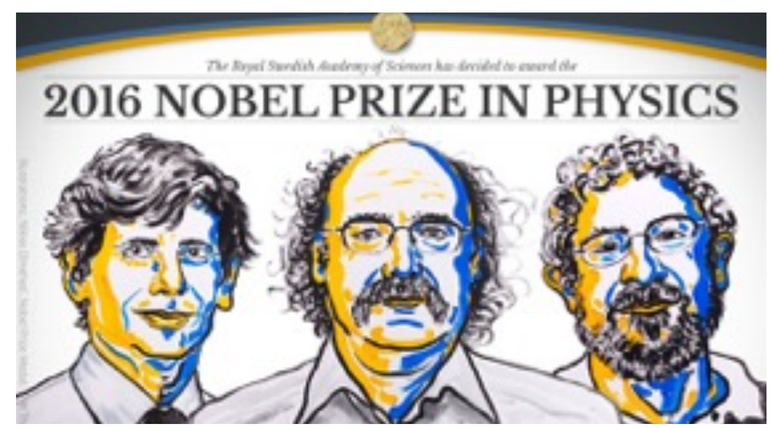
DFG SFB 1143

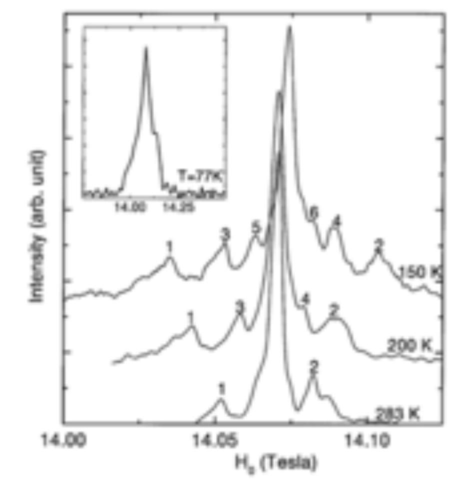
topoquant16
KITP, Santa Barbara
Oct. 4, 2016

Topological phases of matter

- Distinct phases of matter that cannot be characterized by symmetry breaking
  - Quantum Hall effects [Klitzing '80, Tsui '82, Laughlin '83]
  - Topological insulators [Kane & Mele '05]
  - Haldane spin chain [Haldane '83]
  - Spin-liquids [Anderson '73]







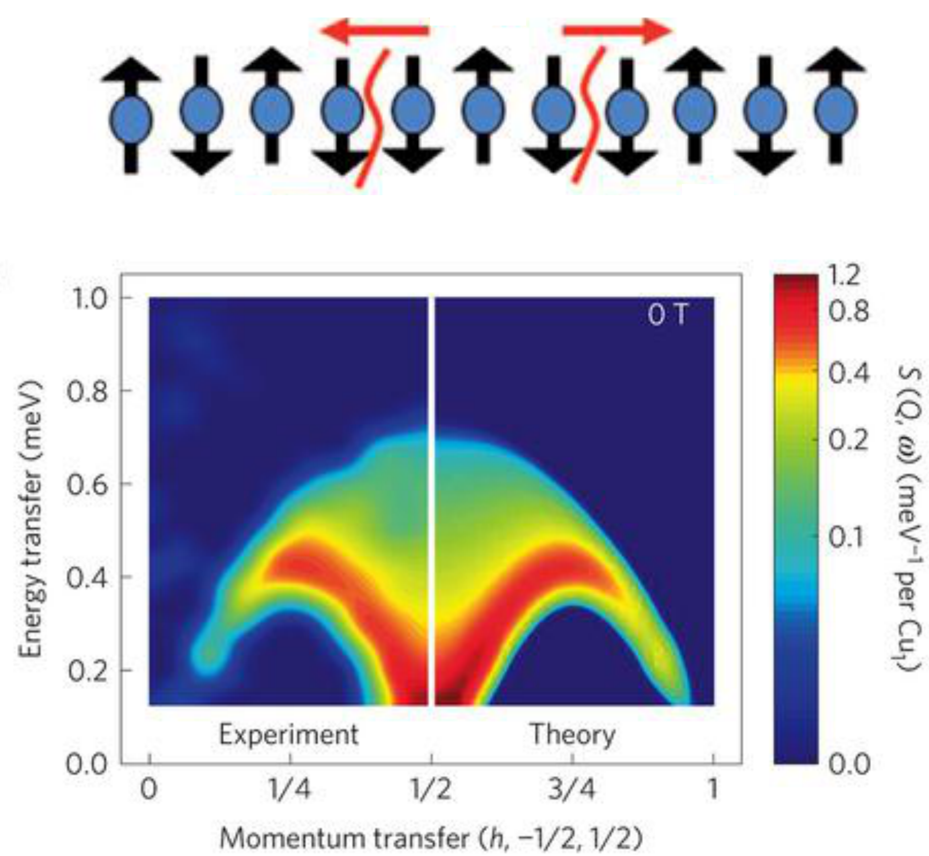
- Characterizing features: quantized conductance, fractionalization, anyonic quasiparticles, …

Dynamical signatures of quantum spin liquids

- Dynamical structure factor:

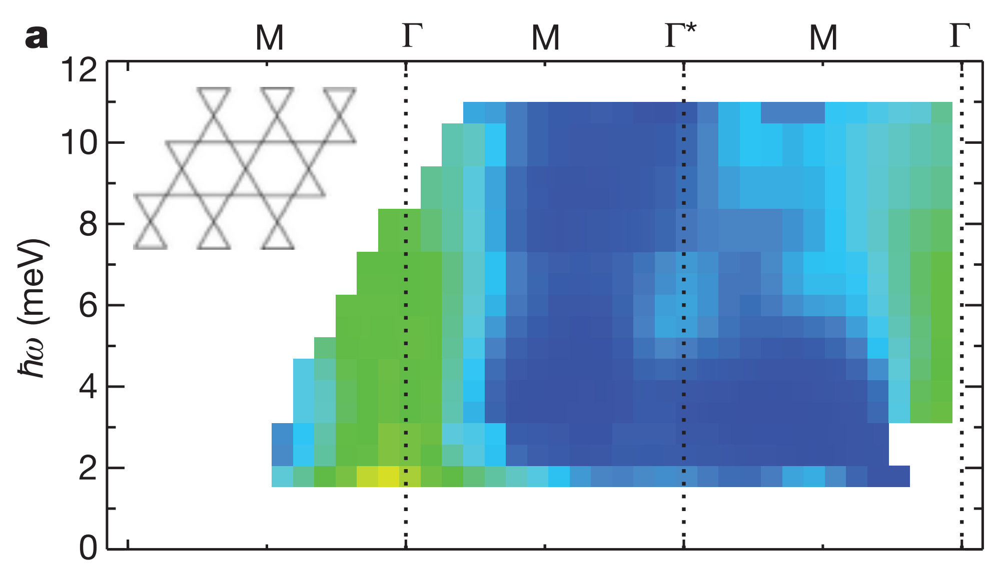
$$S(\vec{q},\omega) = \sum_n \left| \langle \psi_n | S^+_{\vec{q}} | \psi_0 \rangle \right|^2 \delta(\omega + \omega_0 - \omega_n)$$

- Fractional **spinon excitations** in the quantum Heisenberg antiferromagnetic chain ($CuSO_4 \cdot 5D_2O$)



[Mourigal et al, '13]

Kagome antiferromagnetic



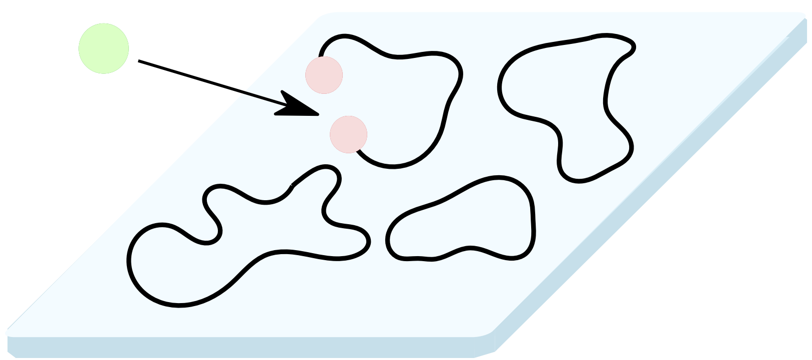
➡ featureless

[Han et al '12]

Dynamical signatures of topological spin liquids

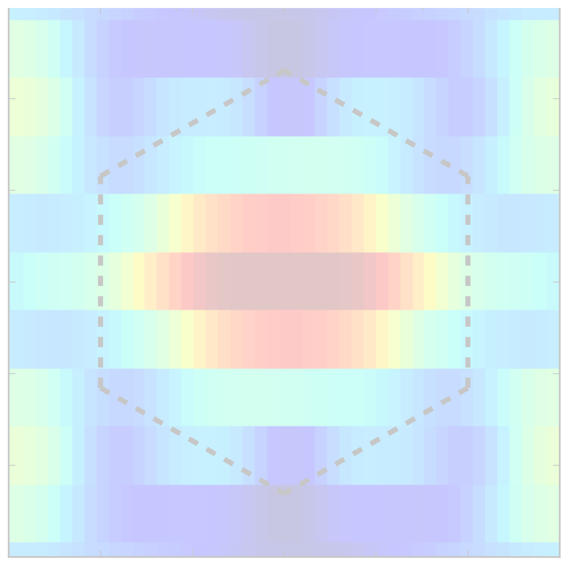
(1) Universal signatures of topological spin liquids
in threshold spectroscopic measurements

with **Siddhardh Morampudi,** Ari Turner, Frank Wilczek
[arXiv:1608.05700]



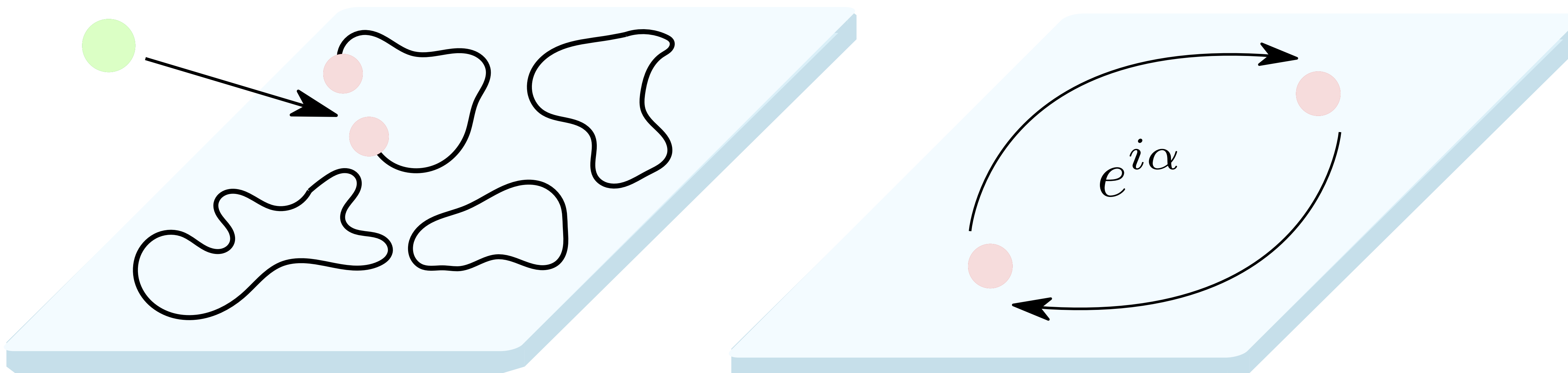
(2) Dynamical structure factor of the Kitaev-Heisenberg model
using large scale DMRG simulations

with **Matthias Gohlke, Ruben Verresen,** Roderich Moessner
[in preparation]



Topologically ordered spin liquids

- Local operators operators create **pairs of gapped anyonic excitations** (e.g., two spinons)



Excitations above a gap Δ

Anyons characterize topological order
(e.g., chiral spin liquid: "semions" with $\alpha = \pm\frac{\pi}{2}$)

[Morampudi, Turner, FP, Wilczek, arXiv:1608.05700]

Topologically ordered spin liquids

PHYSICAL REVIEW VOLUME 73, NUMBER 9 MAY 1, 1948

On the Behavior of Cross Sections Near Thresholds

EUGENE P. WIGNER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received January 16, 1948)

The energy dependence of the cross section for the formation of a product, near the threshold energy for that formation, is considered. It is shown that the cross section is, apart from a constant, in the neighborhood of the threshold the same function of energy, no matter what the reaction mechanism is, as long as the long-range interaction of the product particles is the same. The same must hold, because of the principle of detailed balance, for the back reaction, i.e., the reaction between particles with very low relative velocities. In this case, the cross section, as function of the energy, depends only on the long-range interaction of the reacting particles. The energy dependence of the cross section is determined for three types of interactions, *viz.* no interaction, Coulomb repulsion and Coulomb attraction. The rule for a $1/r^2$ interaction can be obtained from the first case. Reasons are adduced to show that two interactions, the difference of which goes to zero at least as fast as $r^{-2-\epsilon}$ with ($\epsilon>0$), give the same energy dependence of the cross section. Hence, long-range interaction in the above connection should mean an interaction which, at large distances of the particles, does not go to zero faster than r^{-2}. The effect of small perturbations in the long-range interaction is discussed in general.

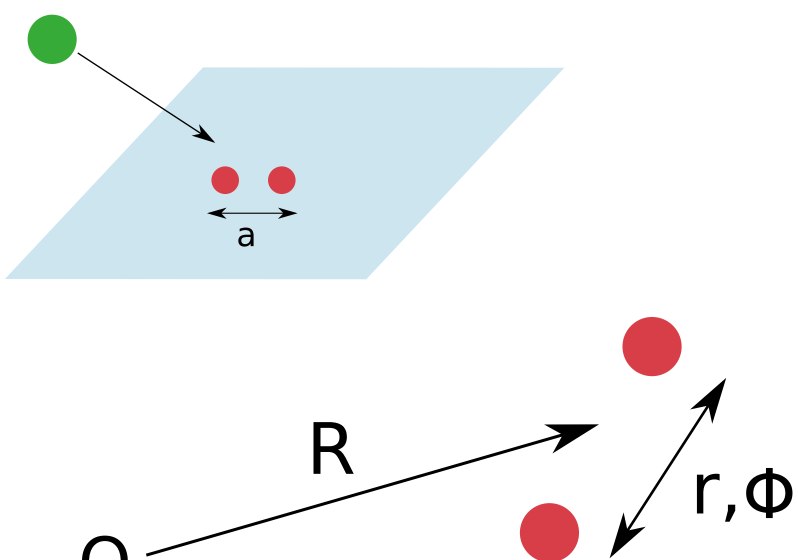
[Morampudi, Turner, FP, Wilczek, arXiv:1608.05700]

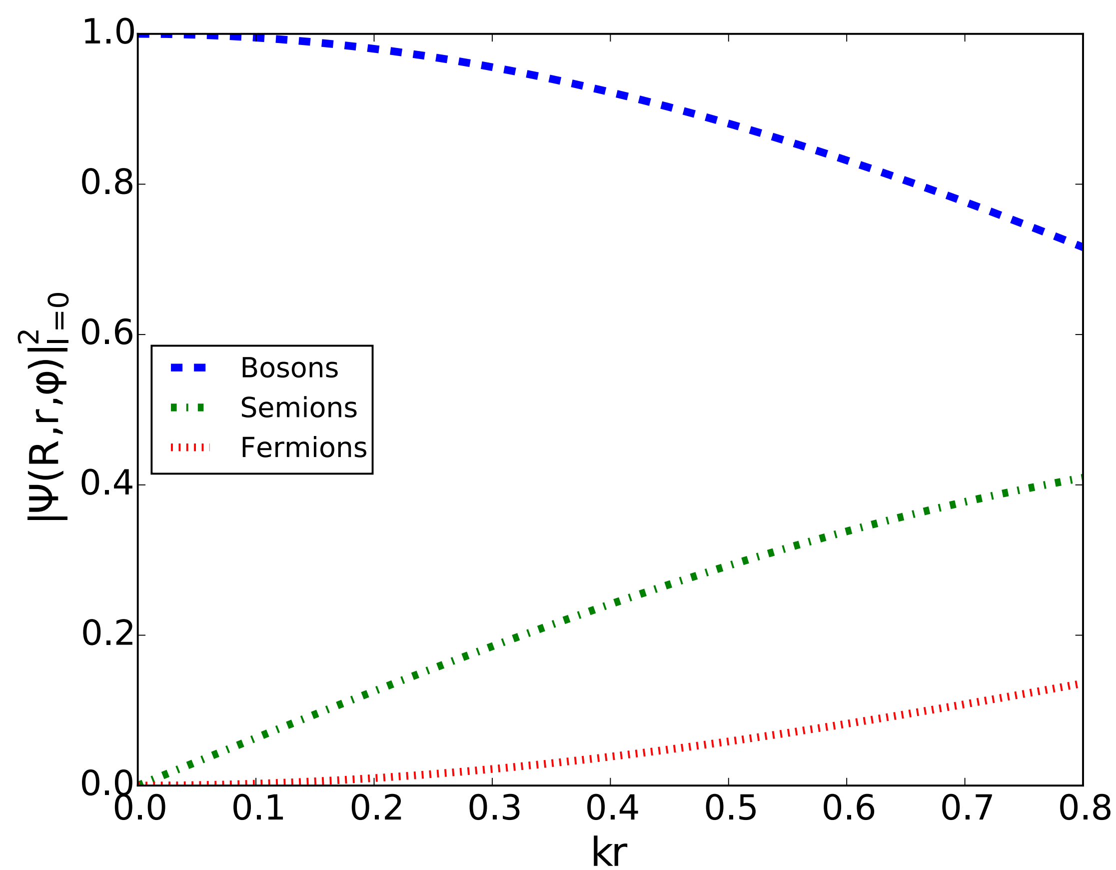
Topologically ordered spin liquids

- Solving the **two-anyon problem**

$$\Psi_{k,\vec{Q},l}(R,r,\phi) = \sqrt{\frac{k}{2L}} J_{|l-\alpha|}(kr)\exp(il\phi)\exp(i\vec{Q}\cdot\vec{R})$$

$$\underset{\text{(bosons)}}{0} \leq \alpha \leq \underset{\text{(fermions)}}{1}$$





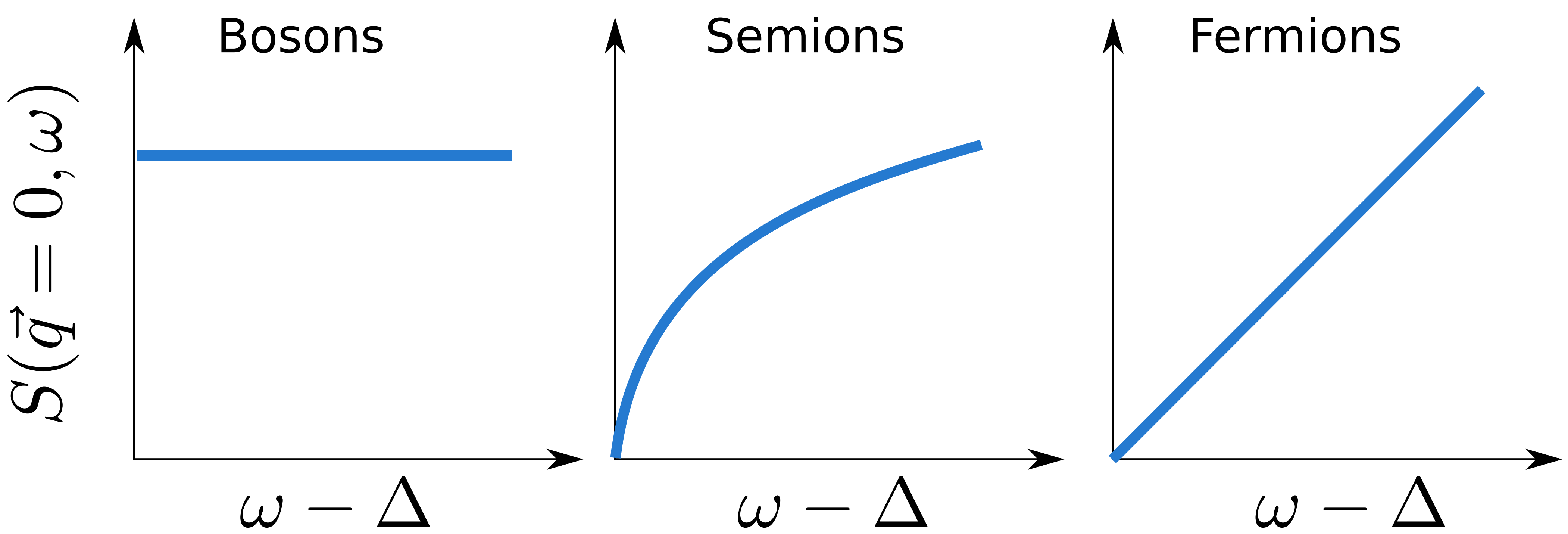
[Morampudi, Turner, FP, Wilczek, arXiv:1608.05700]

Topologically ordered spin liquids

- Spectral functions for **fractionalization into two anyons**:

$$S(\vec{q},\omega) = \sum_n \left| \langle \psi_n | S^+_{\vec{q}} | \psi_0 \rangle \right|^2 \delta(\omega + \omega_0 - \omega_n) \;= m J^2_\alpha(\sqrt{m\omega})$$

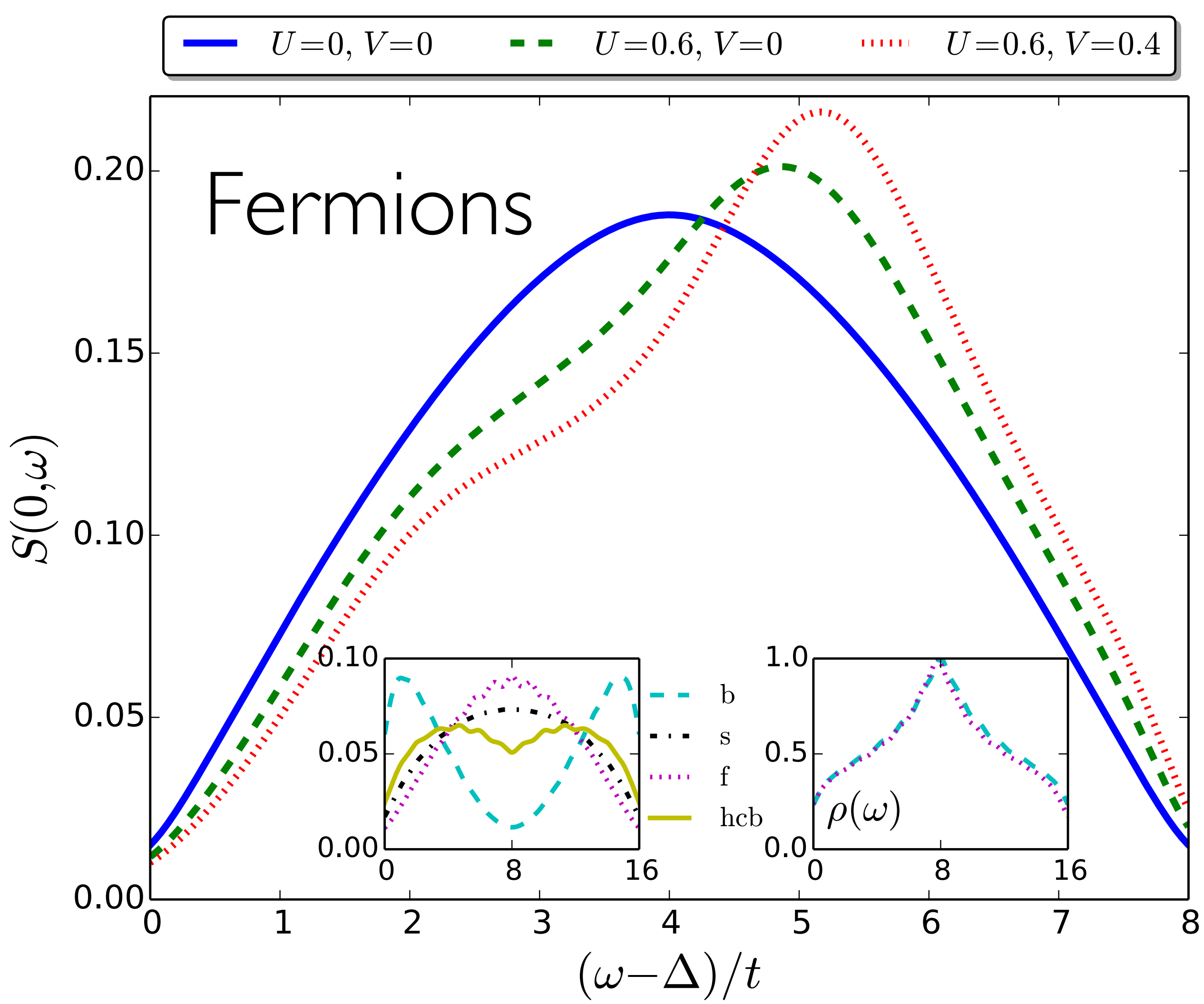
- At low energies $S(\vec{q},\omega) \approx m^{\alpha+1}\omega^\alpha$

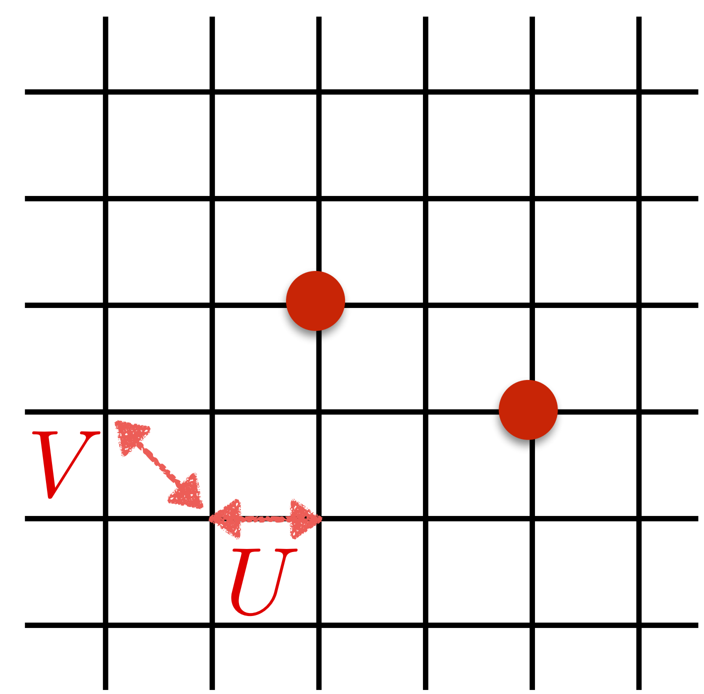


[Morampudi, Turner, FP, Wilczek, arXiv:1608.05700]

Topologically ordered spin liquids

- $\alpha > 0$: **short-range repulsive interactions do not affect the low-energy answer**
- Bosons do get a correction: $S(\vec{q},\omega) \approx \ln(m\omega^2 b^2)^{-2}$

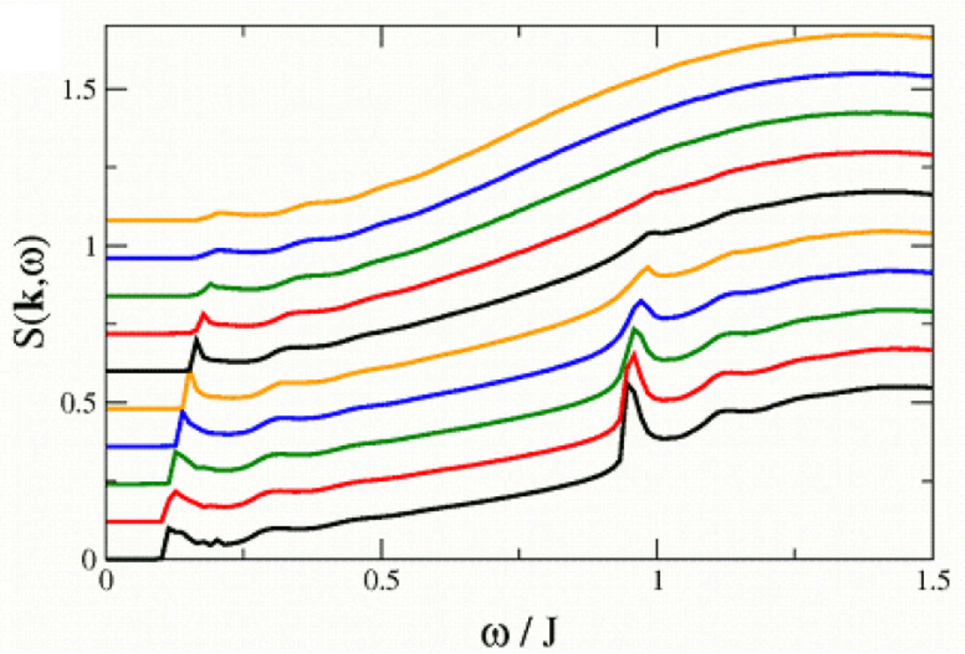




[Morampudi, Turner, FP, Wilczek, arXiv:1608.05700]

Topologically ordered spin liquids

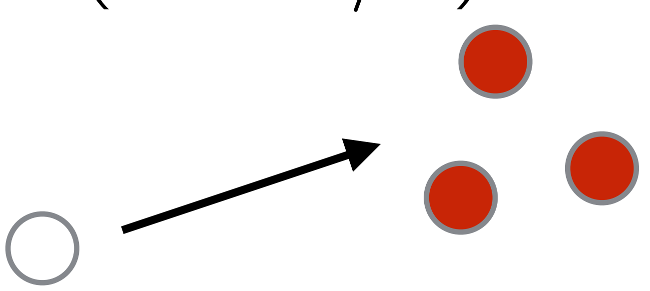
- Bosonic spin liquids on kagome lattice / triangular lattice
  [Punk, Chowdhury, Sachdev '14] [Qi, Xu, Sachdev '08]



$$\mathrm{Im}\chi(k,\omega) = \frac{\mathcal{A}\,\mathcal{C}\,\mathrm{sgn}(\omega)}{\Delta_z^{2-\bar{\eta}}}\,\frac{\theta\left(|\omega| - \sqrt{k^2+4\Delta_z^2}\right)}{\ln^2\left(\frac{|\omega^2-k^2-4\Delta_z^2|}{16\Delta_z^2}\right)}$$

- Generalization to three identical anyons ($\alpha = 1/3$)

$$S(\vec{q},\omega) \approx (\omega - \Delta)^2$$



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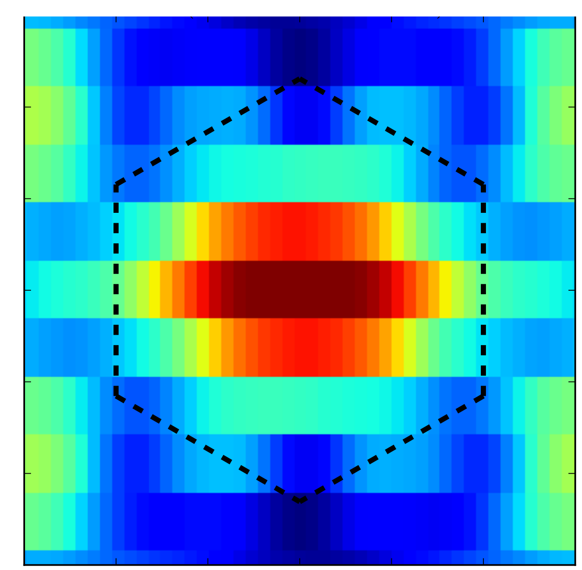
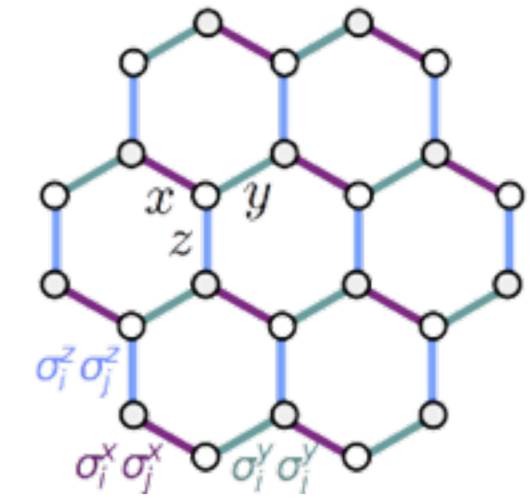


Kitaev-Heisenberg model

- **Kitaev model** on the honeycomb lattice is an exactly solvable quantum spin liquid
  [Kitaev '06]



- Fractionalized excitations:
  gapless majoranas and **static gapped fluxes**

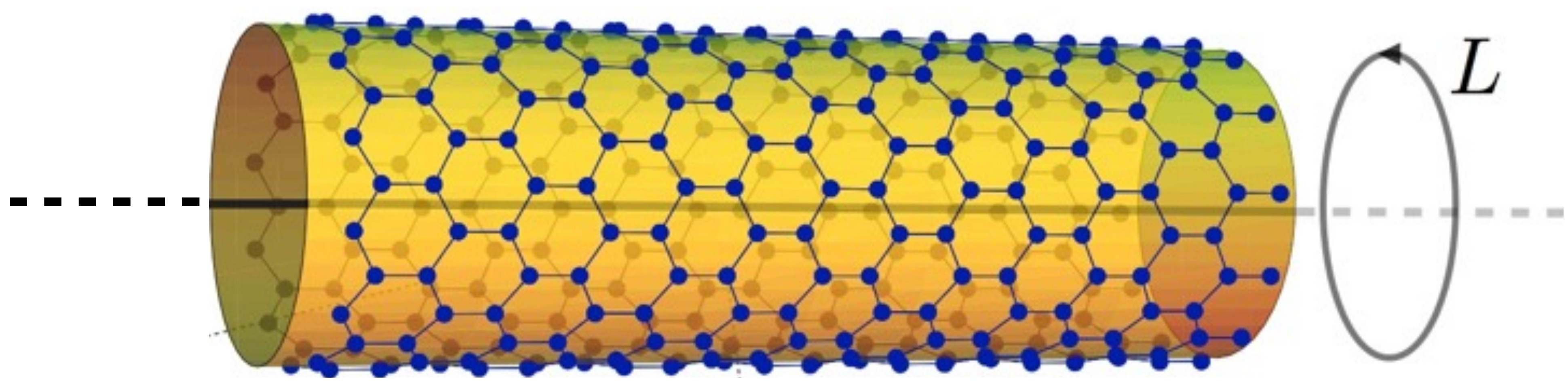
- **Kitaev-Heisenberg model**
  [Jackeli and Khaliullin '09]

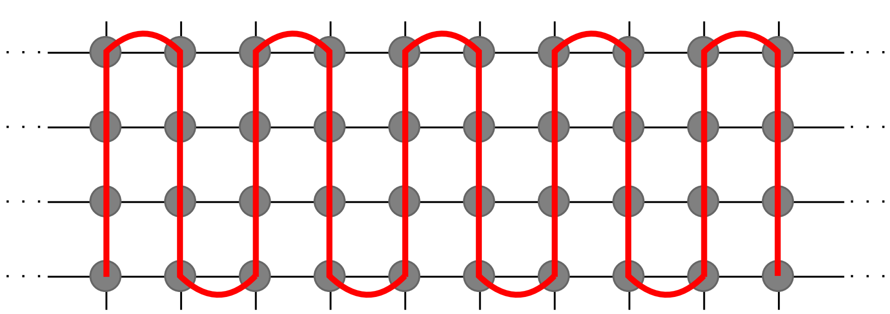
$$H = \sum_{\langle i,j \rangle} \left(2 \sin\alpha \; S_i^\gamma S_j^\gamma + \cos\alpha \; \boldsymbol{S_i} \cdot \boldsymbol{S_j} \right)$$

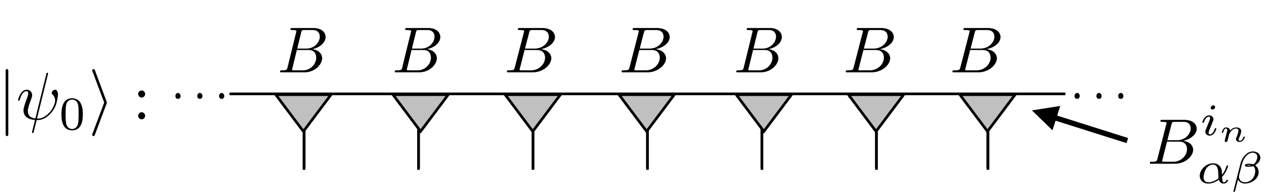
Experimental relevance: Iridates, alpha-$RuCl_3$,…?

Kitaev-Heisenberg model

- Density-Matrix Renormalization Group (DMRG) [White '92]
  - Infinite cylinders with circumference up to $L = 12$

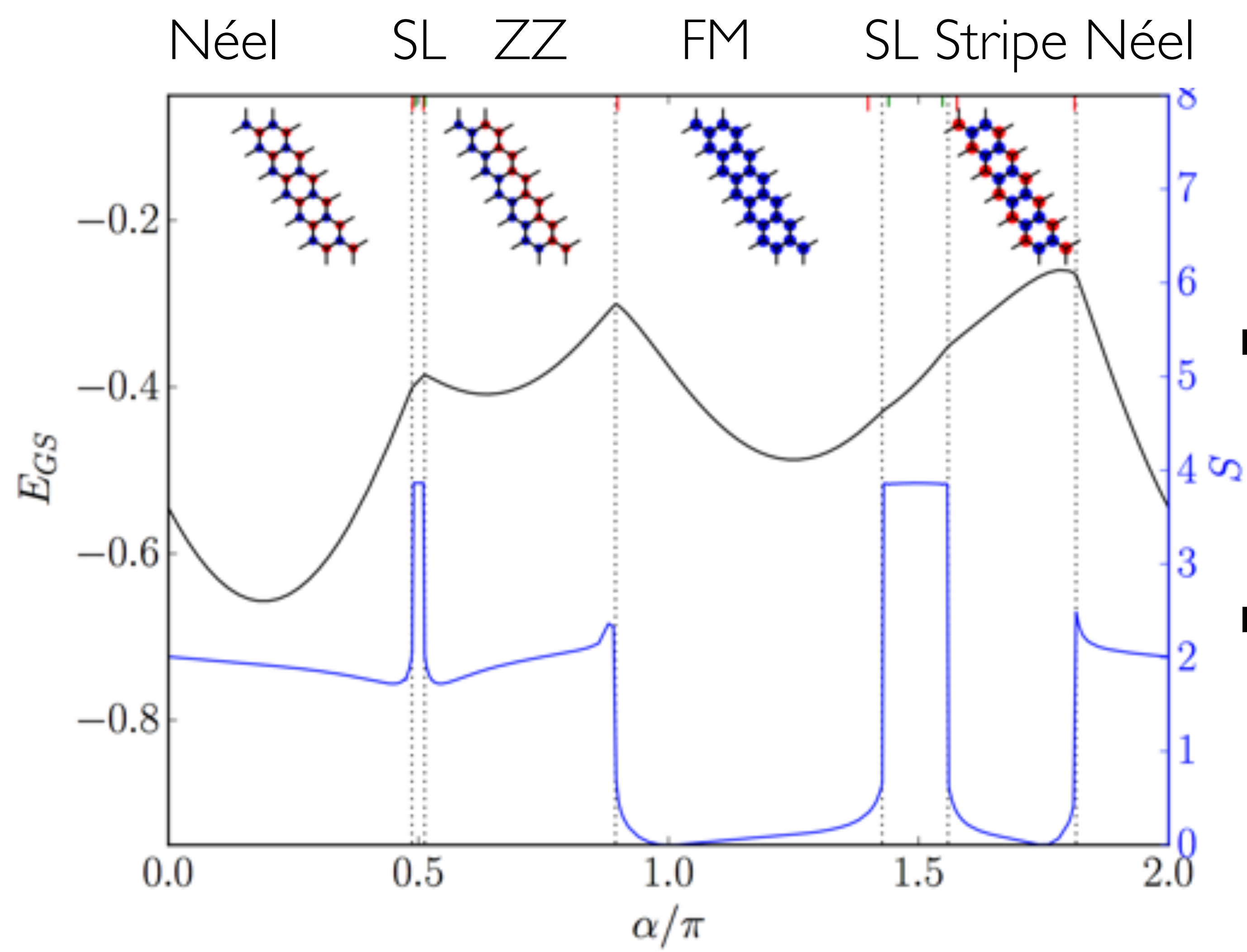


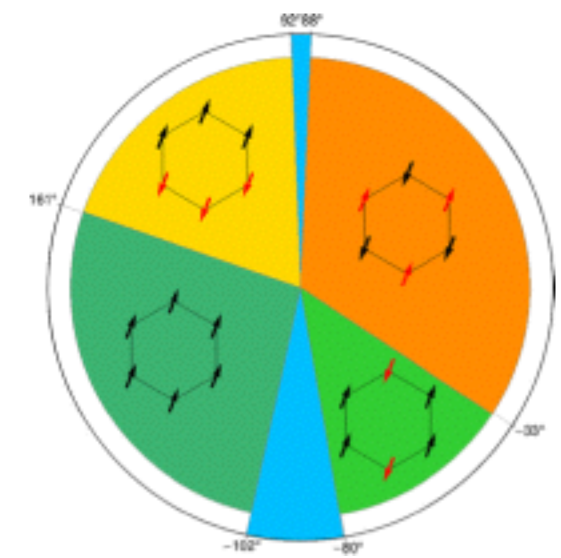




Kitaev-Heisenberg model

- DMRG study ($L = 12$) of the **Kitaev-Heisenberg model**





➡ DMRG results indicate first order transitions (?)

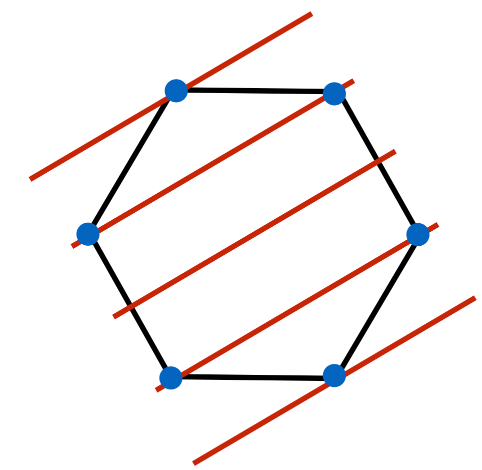
➡ Critical properties in SL robust (boundary conditions important)

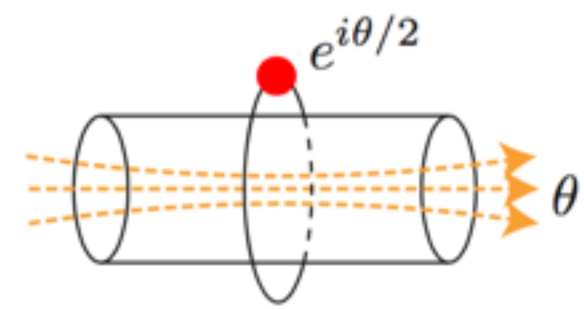
[Matthias Gohlke, Ruben Verresen, Roderich Moessner, FP]

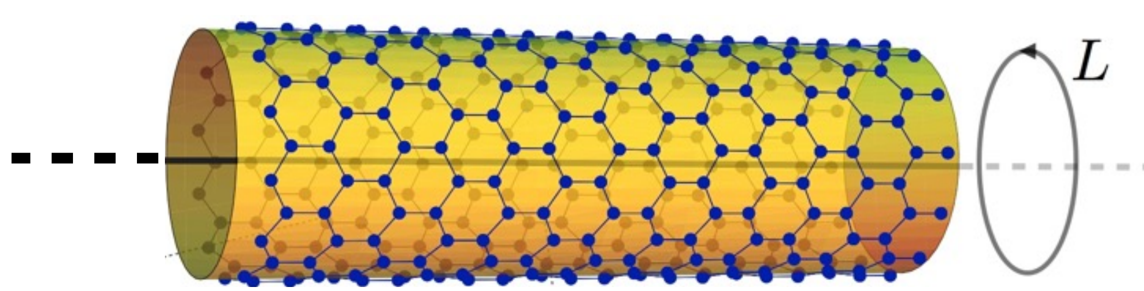
Kitaev-Heisenberg model

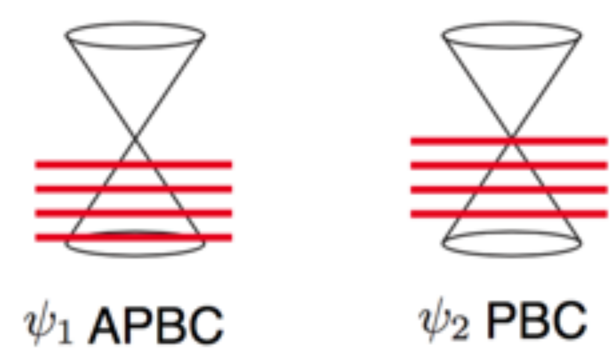
- Finite circumference yields L cuts through Brillouin zone
  - ➡ Dirac cones can be gapped out due to cylinder geometry
  - ➡ Topological sectors with flux have shifted momenta: Generically only one sector can be gapless

    ☞ Yin-Chen He





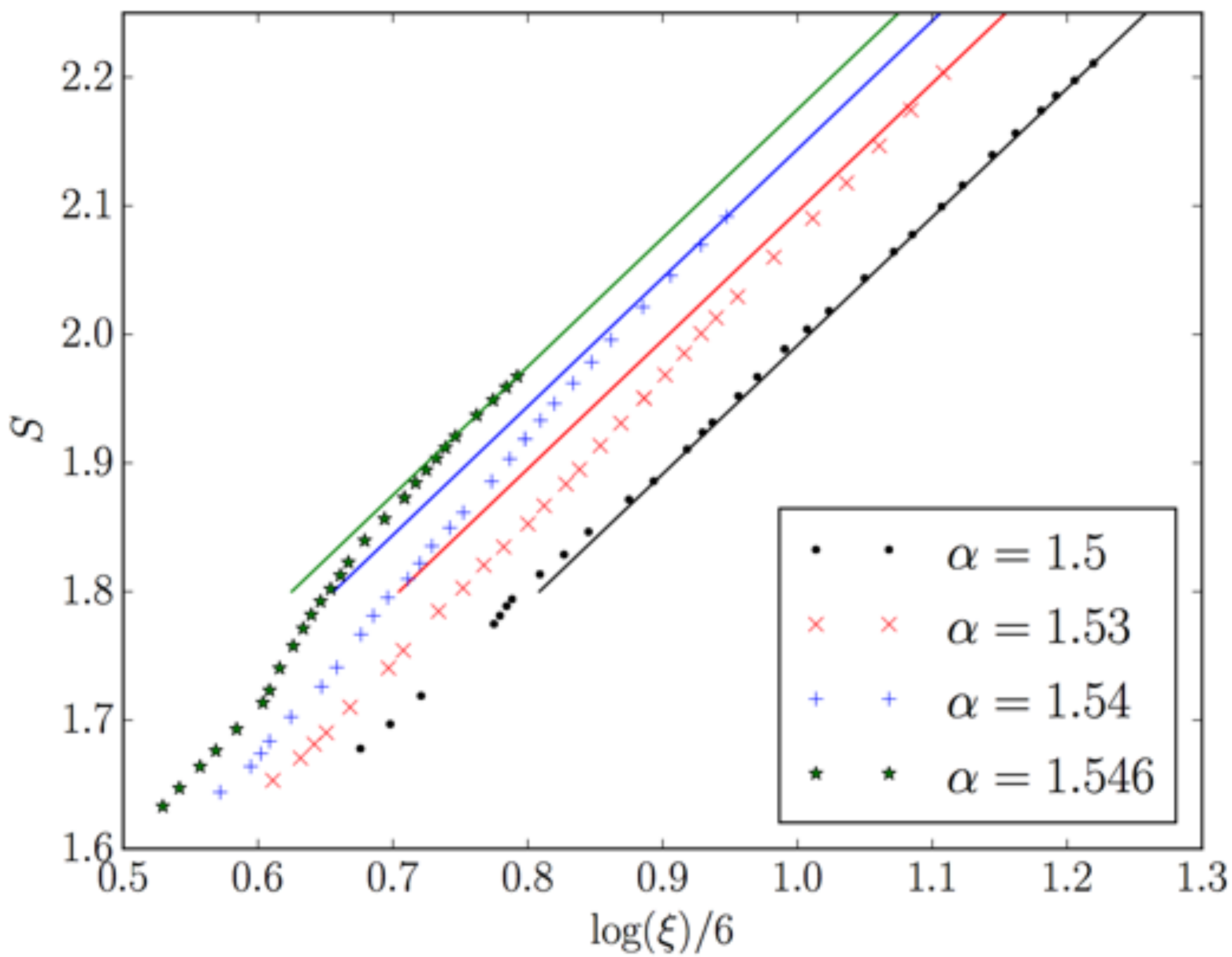


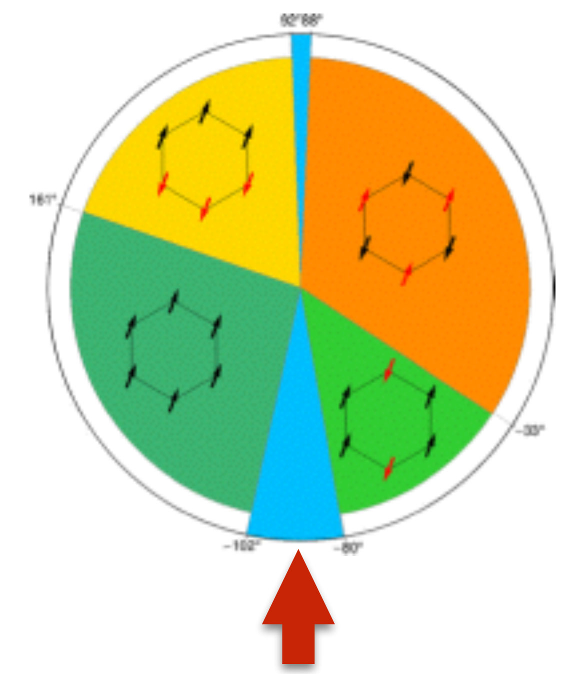


[Matthias Gohlke, Ruben Verresen, Roderich Moessner, FP]

Kitaev-Heisenberg model

- Critical phase robust to Heisenberg perturbations





➡ Finite entanglement scaling $S = \frac{c}{6} \log \xi$

➡ Central charge of $c = 1$ throughout the SL phase

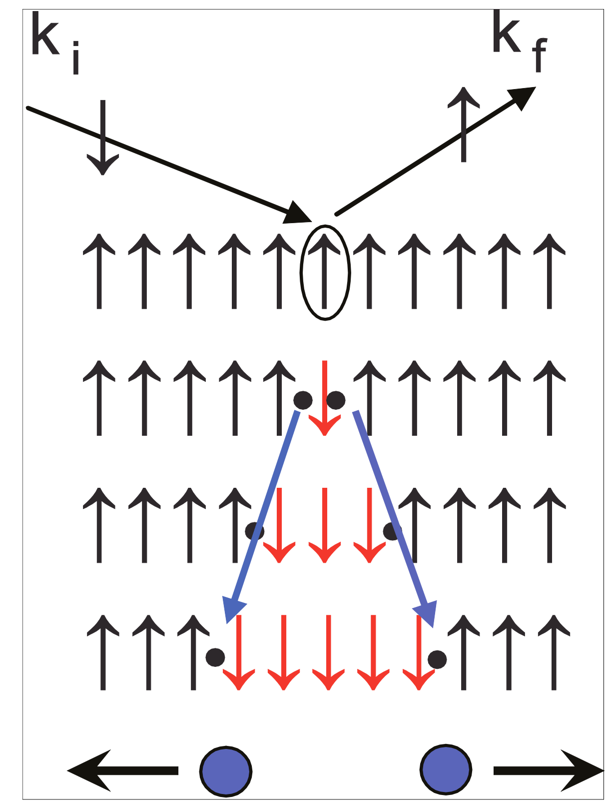
[Matthias Gohlke, Ruben Verresen, Roderich Moessner, FP]

Dynamics of the Kitaev-Heisenberg model

- Numerical calculation of the **dynamical structure factor**

$$S(k,\omega) = \sum_x \int_{-\infty}^{\infty} dt e^{-i(kx+i\omega t)} C(x,t)$$

$$C(x,t) = \langle \psi_0 | S_x(t)^- S_0^+(0) | \psi_0 \rangle$$

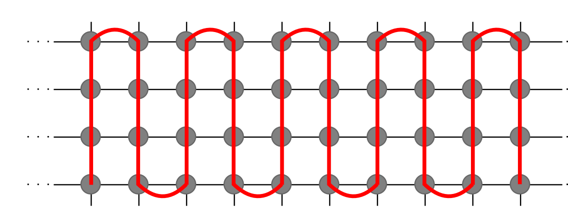


(1) Find the ground state $|\psi_0\rangle$: DMRG

(2) Time evolve $S_0^+|\psi_0\rangle$: tDMRG

$$C(x,t) = e^{itE_0} \langle \psi_0 | S_x^- e^{-itH} S_0^+ | \psi_0 \rangle$$

using new **"matrix-product operator"** based method [Zaletel et al '15]



Dynamics of the Kitaev-Heisenberg model

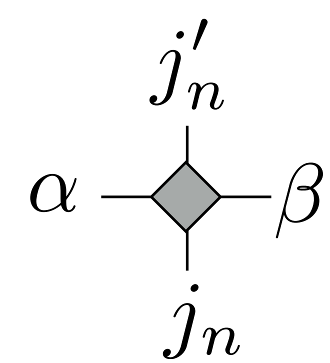
- Hamiltonian expressed as a sum of terms $H = \sum_x H_x$
  Expand $U = \exp(-itH)$ for $t \ll 1$: [Zaletel et al '15]

$$\underbrace{1 + t \sum_x H_x}_{\epsilon \sim L^2 t^2} \rightarrow \underbrace{\prod_x (1 + tH_x)}_{\epsilon \sim L t^2}$$

Neglect overlapping terms in expansion

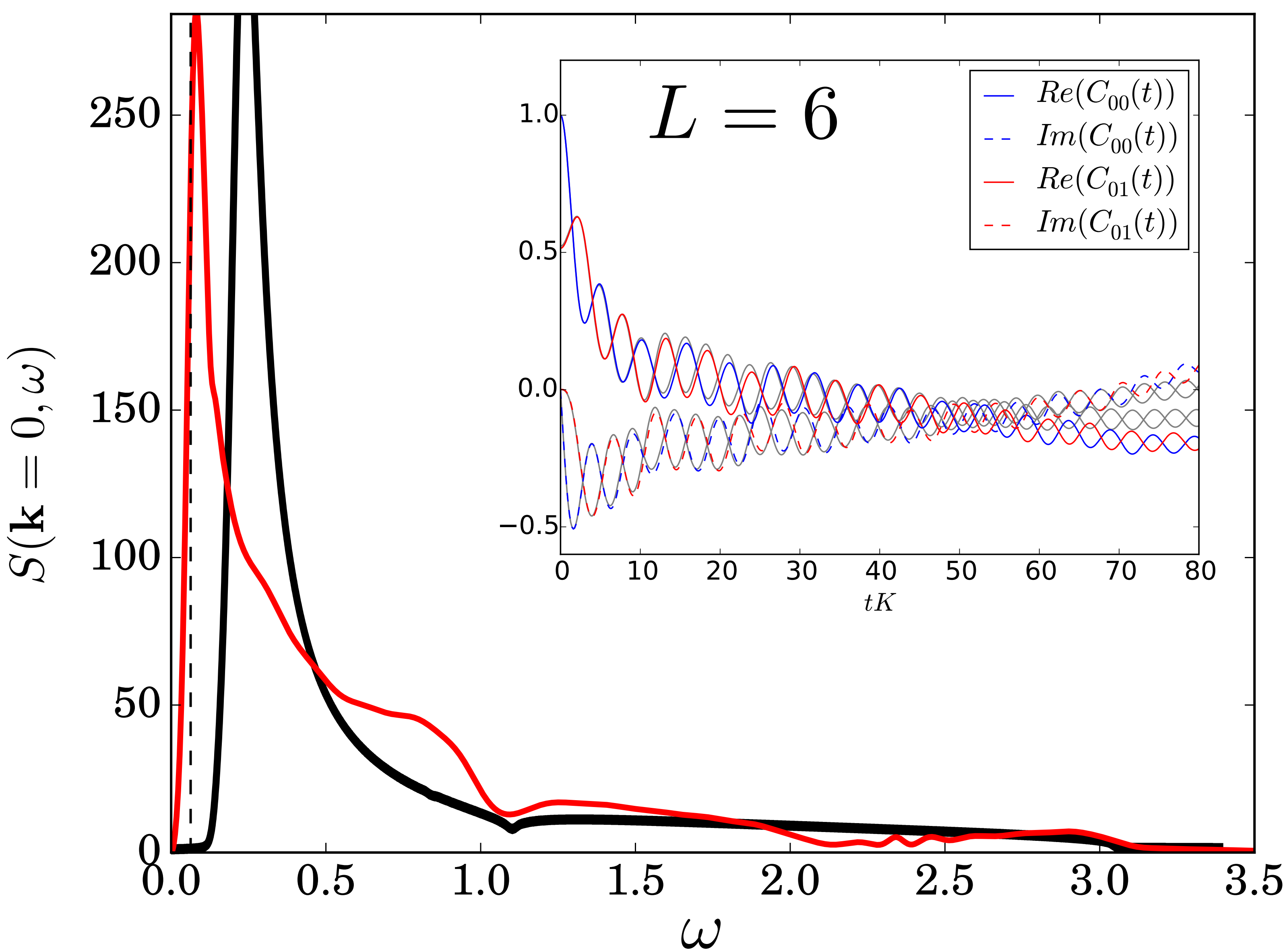
$$\approx 1 + t \sum_x H_x + t^2 \sum_{x<y} H_x H_y + t^3 \sum_{x<y<z} H_x H_y H_z + \dots$$

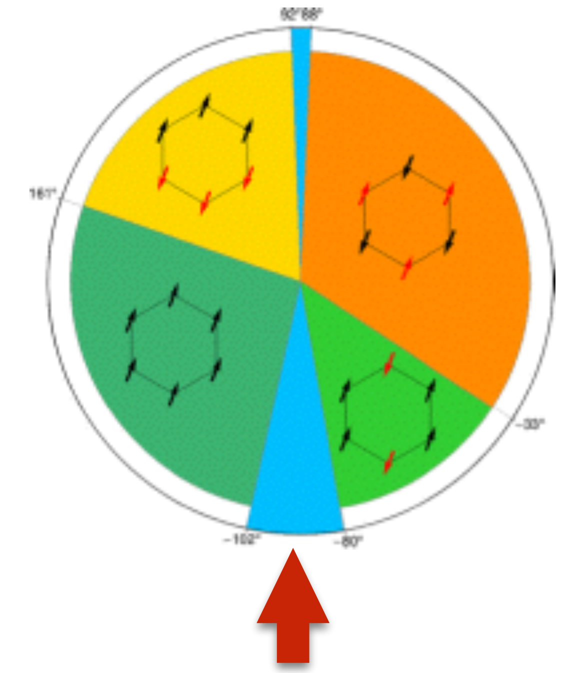
Compact matrix product operator representation

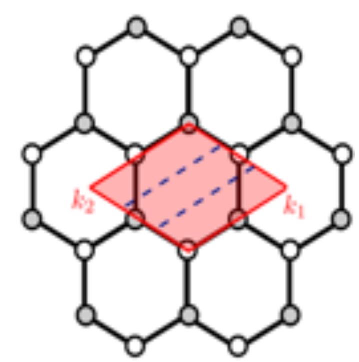
$W^{[n] j_n j'_n}_{\alpha\beta} =$ 

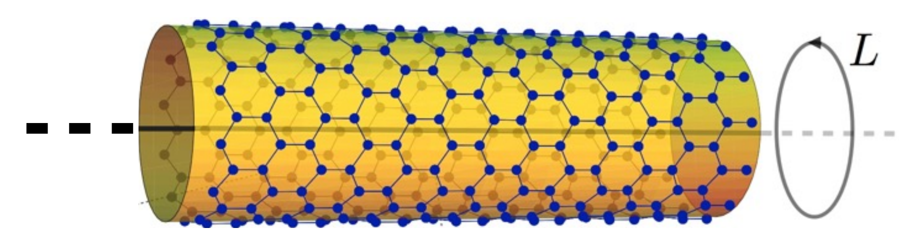
Dynamics of the Kitaev-Heisenberg model

Benchmark: **Pure Kitaev model**
(no Heisenberg term) [Knolle et al. 13]





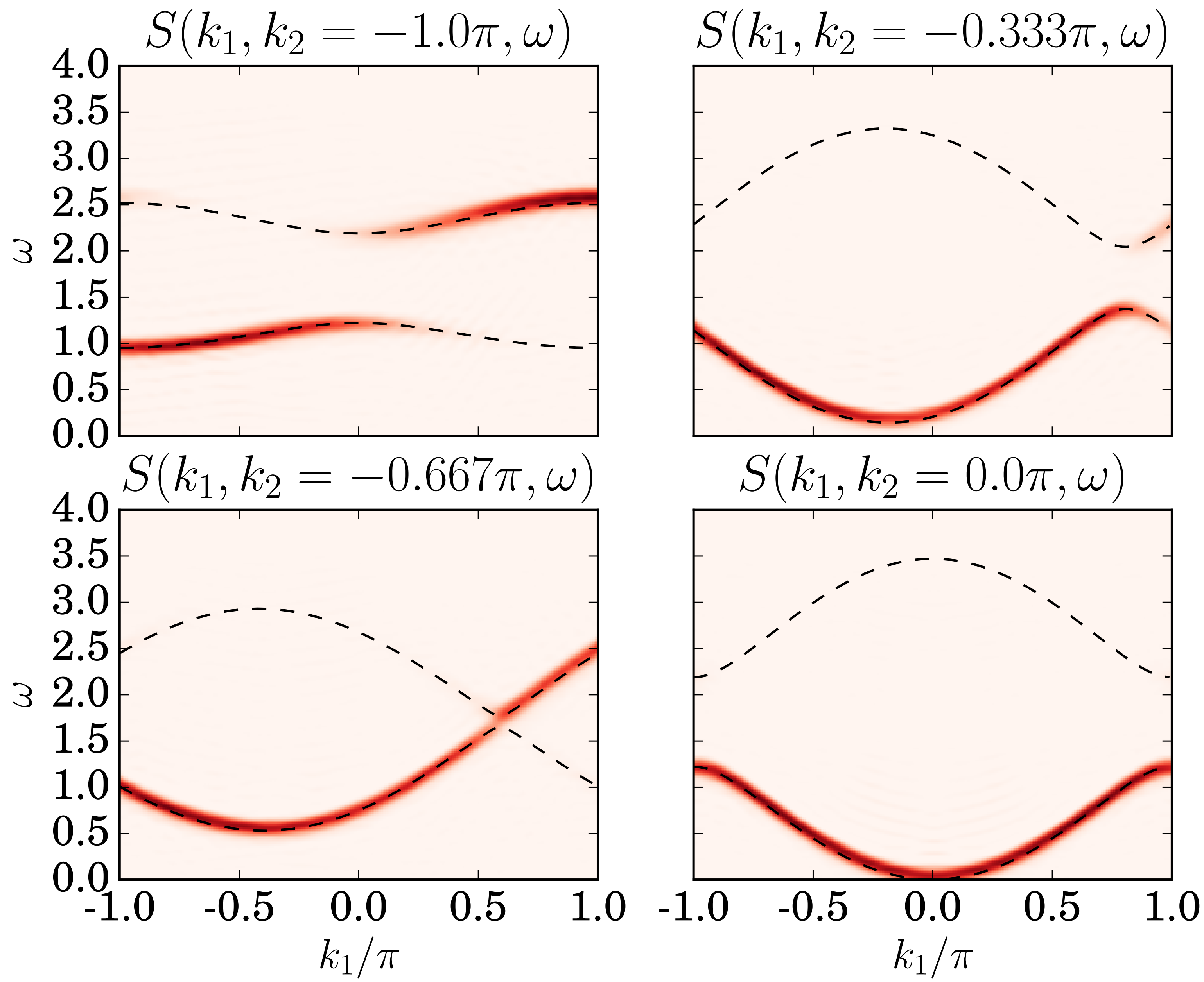


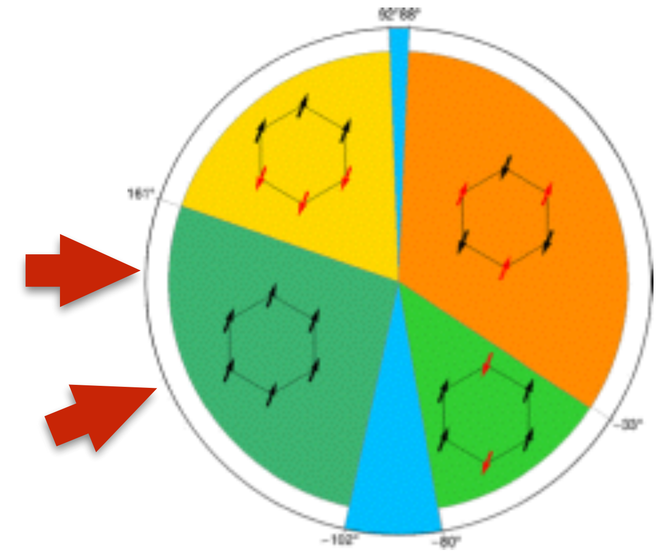


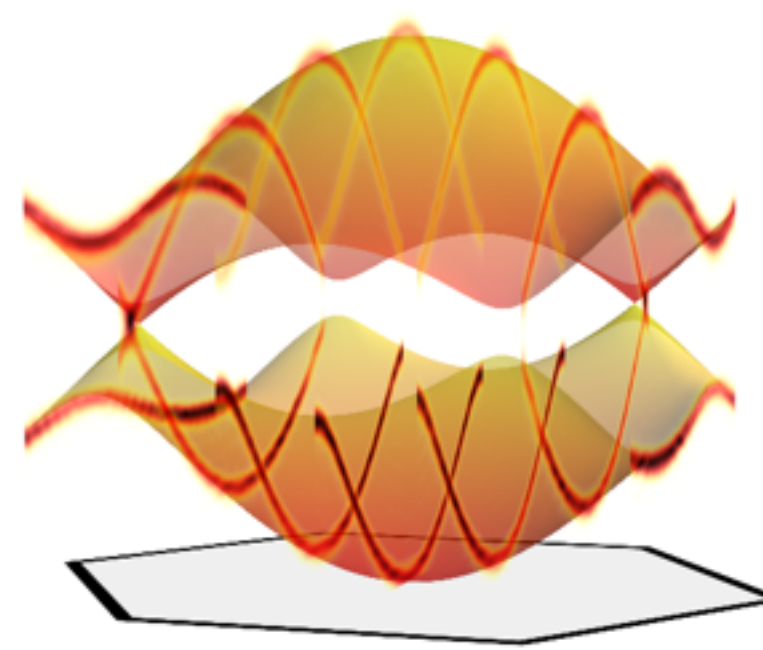
[Matthias Gohlke, Ruben Verresen, Roderich Moessner, FP]

Dynamics of the Kitaev-Heisenberg model

Ferromagnetic phase





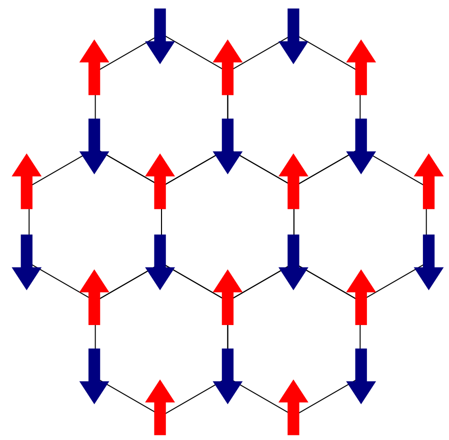


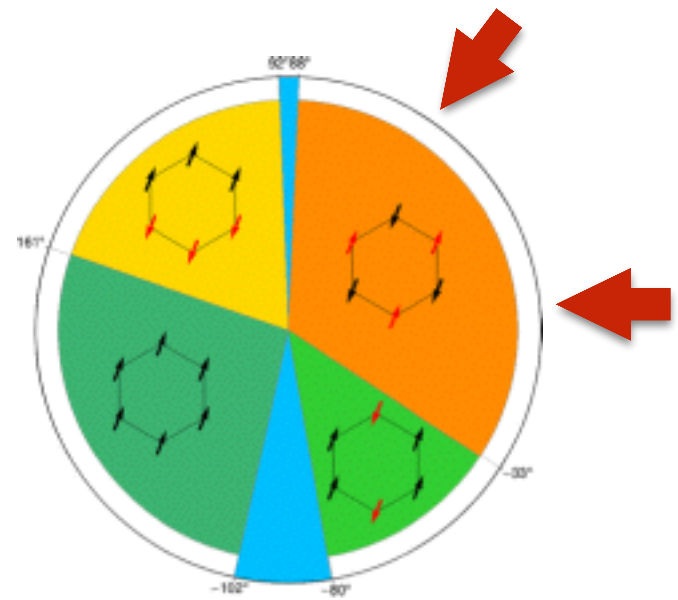
[Matthias Gohlke, Ruben Verresen, Roderich Moessner, FP]

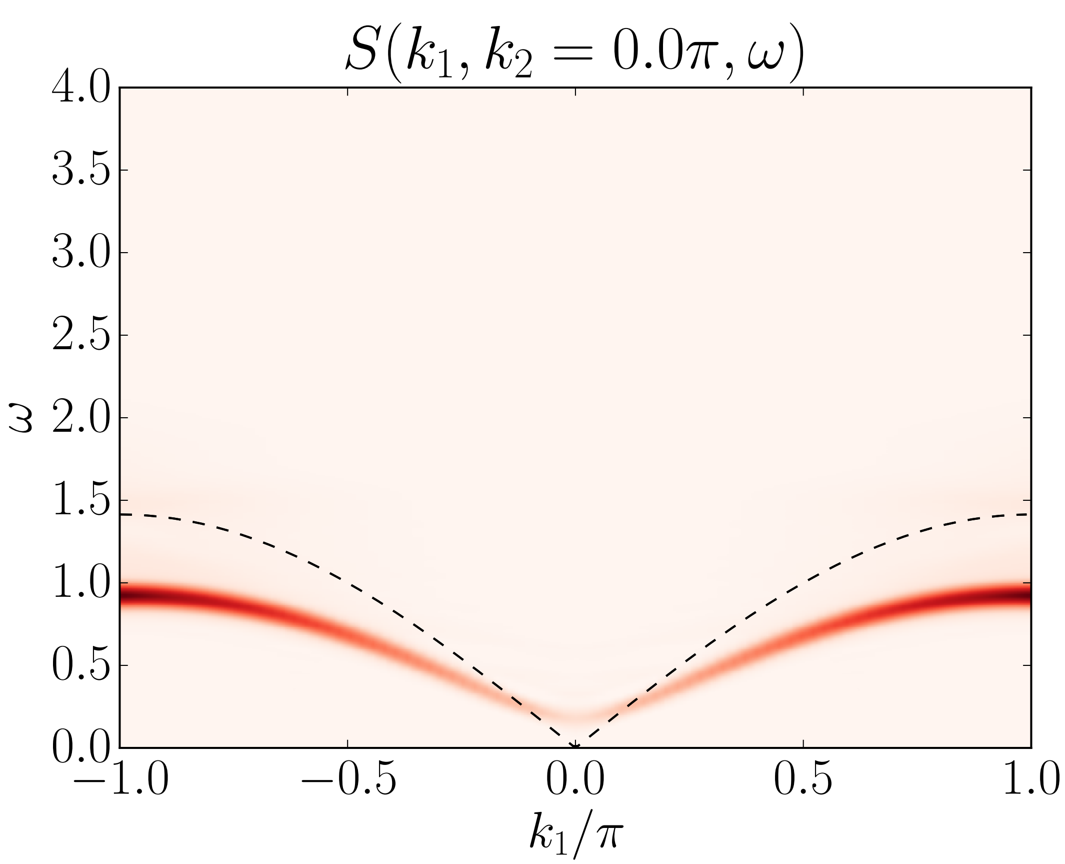
Dynamics of the Kitaev-Heisenberg model

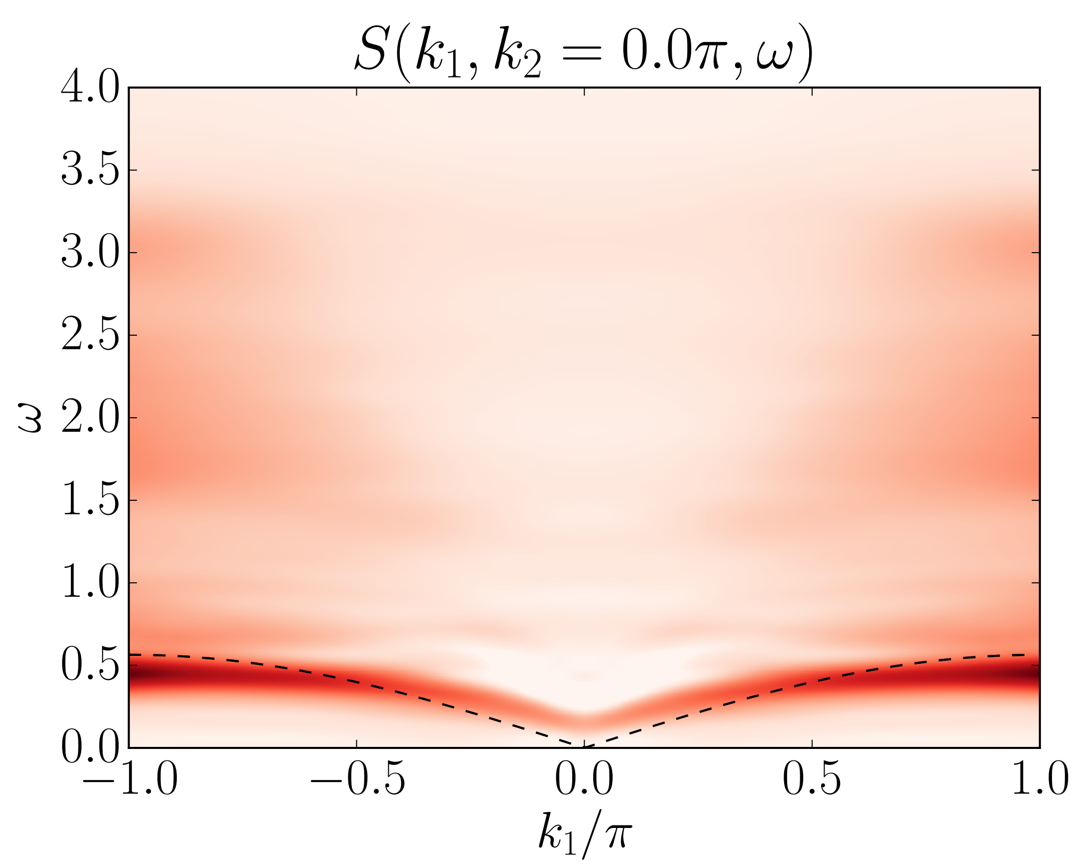
Néel phase

➡ Broad high energy features near transition to SL







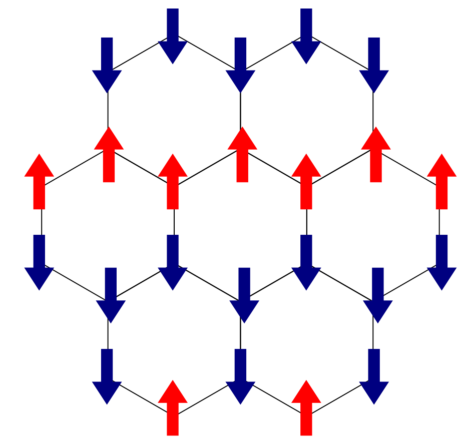


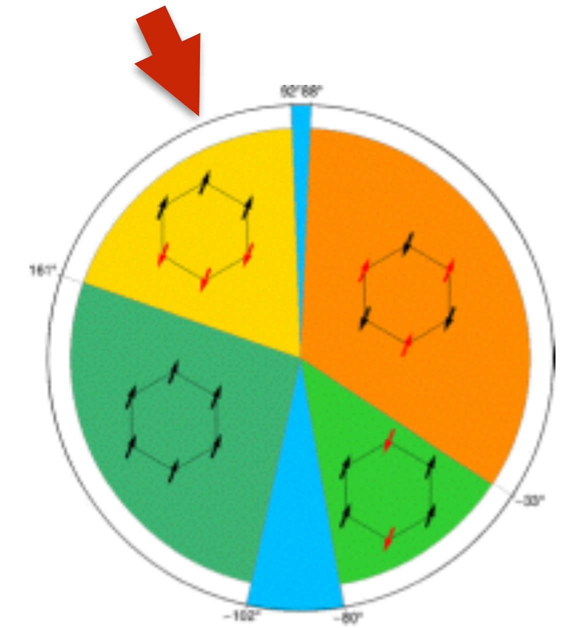
[Matthias Gohlke, Ruben Verresen, Roderich Moessner, FP]

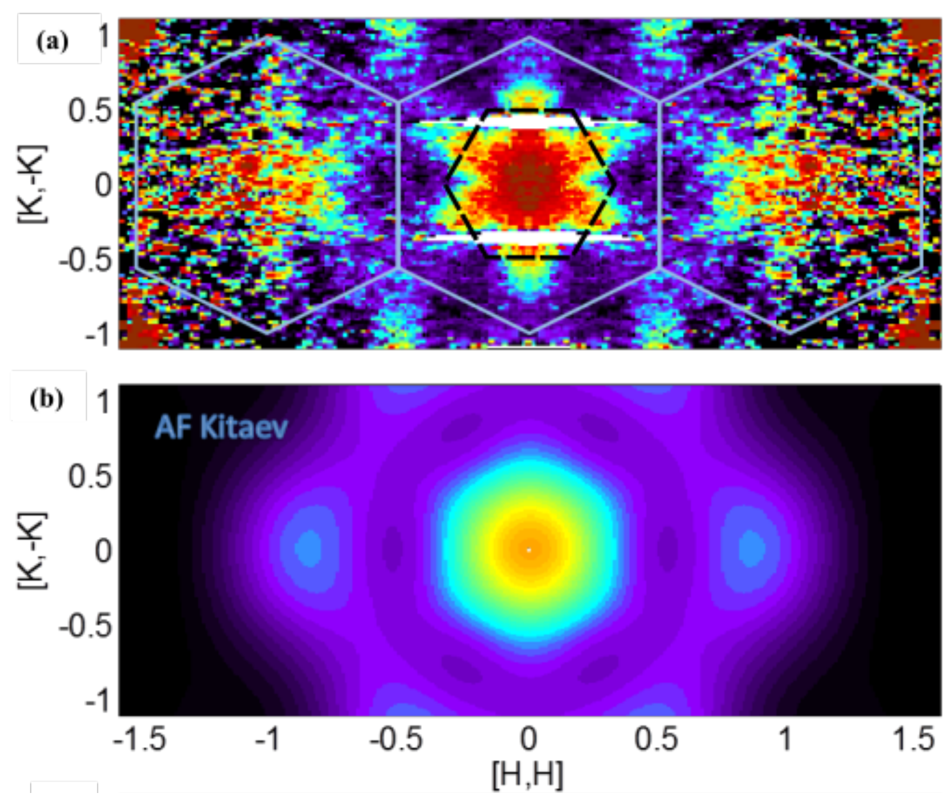
Dynamics of the Kitaev-Heisenberg model

Zig-Zag phase

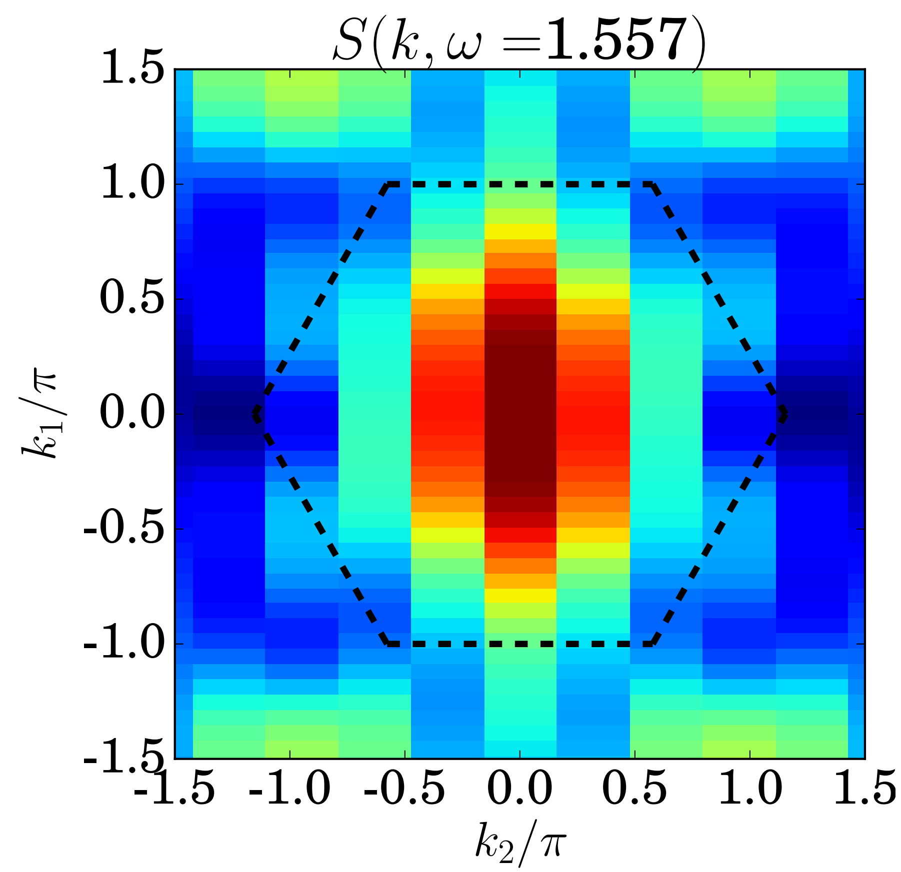
➡ Six fold symmetric
high energy feature (?)







[Banerjee et al.'16]



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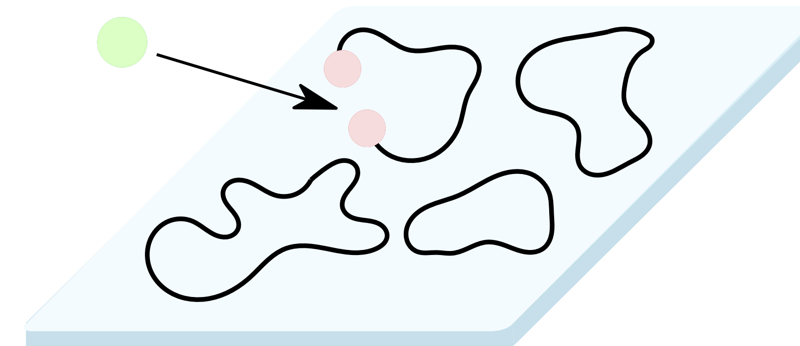

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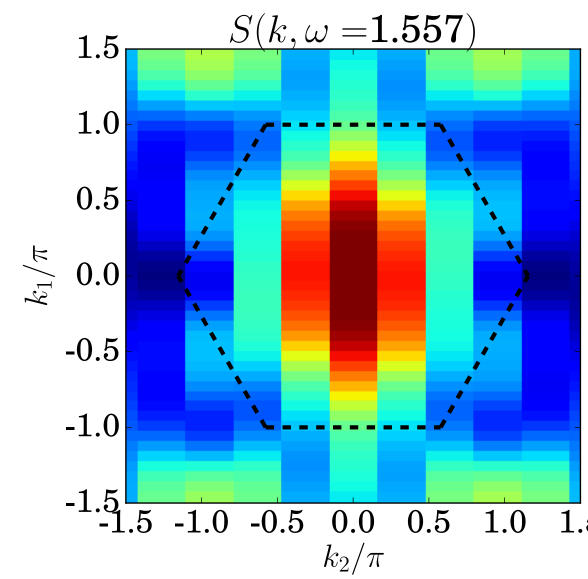


Siddhardh


Matthias


Ruben

Thank You!