Stable Self-Dual Interacting conformal field theories in 2+1d, theory and possible experiments

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Stable Self-Dual Interacting conformal field theories in 2+1d

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References:
arXiv:1510.06032, Cenke Xu, Yi-Zhuang You
arXiv:1609.02560, Meng Cheng, Cenke Xu
arXiv:1611.xxxxx, Wang, Nahum, Metlitski, Xu, Senthil

Example of stable 2+1d CFT:

QED3 with large-*N* flavors of Dirac fermions:

$$\mathcal{L} = \sum_{j=1}^{N} \bar{\psi}_j \gamma_\mu (\partial_\mu - i a_\mu) \psi_j$$

A standard theory of algebraic spin liquid, a stable CFT with large enough *N*, all relevant perturbations (Dirac fermion mass terms) forbidden by symmetry, can be studied quantitatively using a 1/*N* expansion.

Introduction

Questions:

1, can we find more examples of stable 2+1d CFTs, namely all symmetry allowed perturbations are irrelevant?

2, what can we compute about a 2+1d CFT without large-*N* flavors of matter fields?

A powerful nonperturbative method: duality

Example 1: duality between 2+1d photon and superfluid:

$$\mathcal{L} = \frac{1}{2e^2} f_{\mu\nu}^2 \quad \leftrightarrow \quad \mathcal{L} = \frac{e^2}{2}(\partial_\mu \theta)^2$$

Gapless photon is dual to the Goldstone mode. 2+1d photon phase can be viewed as the Goldstone mode of the condensate of gauge flux.

Introduction

Example 2: 3d O(2) Wilson-Fisher and 3d Higgs transition of 3d bosonic QED (Dastupta, Halperin, 1981, Fisher, Lee, 1989):

$$\mathcal{L} = |(\partial_\mu - iA_\mu)\Psi|^2 + r|\Psi|^2 + g|\Psi|^4$$

$r < 0$ SF, ordered — disordered $r > 0$

$\tilde{r} > 0$ Photon — Higgs $\tilde{r} < 0$

$$\mathcal{L} = |(\partial_\mu - ia_\mu)\Phi|^2 + \tilde{r}|\Phi|^2 + \tilde{g}|\Phi|^4 + \frac{i}{2\pi} a \wedge dA$$

Introduction

How do we know two seemingly different non-SUSY CFTs are secretly dual to each other?

Necessary conditions (consistency check):
The two theories have the same set of operators, and can be driven into the same phases (fixed points) when deformed by these operators;

Conversely, if two CFTs have the same excitations, and are always deformed into the same phases, they could be (though not proven to be) dual to each other.

Other logics for these dualities: (1) deformation from known SUSY dualities; (2) Extending from the known dualities in the large-N and large-k limit.

Assuming a small number of basic dualities are correct, many more dualities can be derived.

Example 3: 3d Dirac fermion and bosonic QED with Chern-Simons term at level-1 (Chen, Fisher, Wu 1993):

$$\mathcal{L} = |(\partial_\mu - ia_\mu)\Phi|^2 + r|\Phi|^2 + g|\Phi|^4 + \frac{i}{4\pi} a \wedge da - \frac{i}{2\pi} a \wedge dA$$

$r < 0$ Trivial insu. Integer QH $r > 0$

$m < 0$ $m > 0$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi + m\bar{\psi}\psi + \bar{\Psi}\gamma_\mu(\partial_\mu - iA_\mu)\Psi + M\bar{\Psi}\Psi$$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi + m\bar{\psi}\psi - \frac{i}{8\pi} A \wedge dA$$

A finite *N* version of this duality was found (Hsin, Seiberg 2016), using other logic (large-*N*, deformation from SUSY theories, etc.)

Introduction

Example 3*: 3d Dirac fermion and bosonic QED with Chern-Simons term at level-1 (Chen, Fisher, Wu 1993):

$$\mathcal{L} = |(\partial_\mu - ib_\mu)\tilde{\Phi}|^2 - r|\tilde{\Phi}|^2 + g|\tilde{\Phi}|^2 + \frac{i}{4\pi}(b - A) \wedge d(b - A)$$

$r < 0$ Trivial insu. | Integer QH $r > 0$

$m < 0$ | $m > 0$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi + m\bar{\psi}\psi + \bar{\Psi}\gamma_\mu(\partial_\mu - iA_\mu)\Psi + M\bar{\Psi}\Psi$$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi + m\bar{\psi}\psi - \frac{i}{8\pi}A \wedge dA$$

A finite *N* version of this duality was found by Hsin, Seiberg, using other logic (large-*N*, deformation from SUSY theories, etc.)

Example 4: 3d QED with “level-1/2” CS and 3d O(2) Wilson-Fisher (Chen, Fisher, Wu 1993, Barkeshli, McGreevy 2014):

$$\mathcal{L} = |(\partial_\mu - iA_\mu)\Psi|^2 + r|\Psi|^2 + g|\Psi|^4$$

$r < 0$ SF, ordered disordered $r > 0$

$m < 0$ $m > 0$

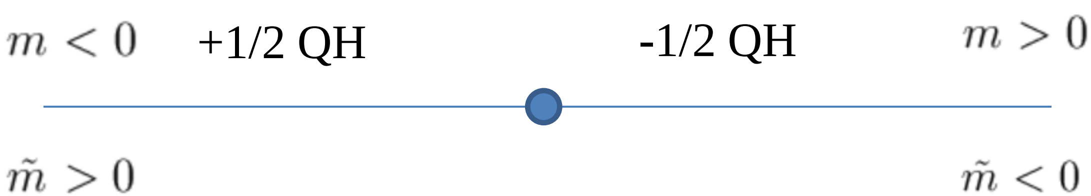
$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + m\bar{\psi}\psi + \bar{\Psi}\gamma_\mu(\partial_\mu - ia_\mu)\Psi + M\bar{\Psi}\Psi$$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + m\bar{\psi}\psi - \frac{i}{8\pi}a \wedge da$$

Introduction

Example 5: 3d Dirac fermion and QED with *N*=1 (Son, 2015, Metlitski, Vishwanath, 2015, Wang, Senthil, 2015)

$$\mathcal{L} = \bar{\chi}\gamma_\mu(\partial_\mu - iA_\mu)\chi + m\bar{\chi}\chi$$



$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + \frac{i}{4\pi} a \wedge dA + \tilde{m}\bar{\psi}\psi$$

1, the +-1/2 QH state is the anomalous effect at the boundary of the 3d TI;
2, *a* only allows $4\pi n$ flux;
3, another more complete version of the dual theory which does not require special flux quantization of *a* is given by Seiberg, etc, 2016.

Self-dual N=2 QED3

Restating the conjecture: the *N*=1 QED3 flows to an IR fixed point, which is equivalent to a noninteracting Dirac fermion.

$$\mathcal{L} = \bar{\chi}\gamma_\mu(\partial_\mu - iA_\mu)\chi$$

$$\leftrightarrow\ \mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + \frac{i}{4\pi}a \wedge dA$$

Assuming this is true, we can derive the following conclusion:
The *N*=2 QED3, if it is a CFT, is self-dual, Xu, You, arXiv:1510.06032

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j\gamma_\mu(\partial_\mu - ia_\mu)\chi_j + iA_\mu\bar{\chi}\gamma_\mu\tau^z\chi + \frac{i}{2\pi}a \wedge dB$$

$$\leftrightarrow\ \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j\gamma_\mu(\partial_\mu - ib_\mu)\psi_j + iB_\mu\bar{\psi}\gamma_\mu\tau^z\psi + \frac{i}{2\pi}b \wedge dA$$

Self-dual N=2 QED3

Deriving the self-duality (Xu, You, arXiv:1510.06032):

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

Step 1: run the fermion-fermion duality for each flavor:

$$\leftrightarrow \ \mathcal{L} = \bar{\psi}_1 \gamma_\mu (\partial_\mu - i b_\mu) \psi_1 + \bar{\psi}_2 \gamma_\mu (\partial_\mu - i c_\mu) \psi_2 - \frac{i}{4\pi} a \wedge d(b + c - 2B) - \frac{i}{4\pi} A \wedge d(b - c)$$

Step 2: Integrating out dynamical gauge field *a*:

$$\leftrightarrow \ \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i b_\mu) \psi_j + i B_\mu \bar{\psi} \gamma_\mu \tau^z \psi + \frac{i}{2\pi} b \wedge dA$$

Other Derivation of this duality: Mross, et.al. arXiv:1605.03582
Karch, Tong, arXiv:1606.01893, Hsin, Seiberg, arXiv:1607.07457

Self-dual N=2 QED3

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

$$\leftrightarrow \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i b_\mu) \psi_j + i B_\mu \bar{\psi} \gamma_\mu \tau^z \psi + \frac{i}{2\pi} b \wedge dA$$

Comments:

1, the *N*=2 QED3 has O(4) symmetry. The SO(4) ~ SU(2) x SU(2) is the flavor symmetry of both ψ and χ. The Z_2 subgroup of O(4) is the self-duality transformation.
There is another way to see the O(4) symmetry, by mapping this model to a low-energy effective field theory in terms of gauge invariant O(4) vector boson (Senthil, Fisher 2005).

Self-dual N=2 QED3

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

$$\leftrightarrow \quad \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i b_\mu) \psi_j + i B_\mu \bar{\psi} \gamma_\mu \tau^z \psi + \frac{i}{2\pi} b \wedge dA$$

Comments:

2, there is an O(4) breaking but SO(4) invariant relevant perturbation:

$$m\bar{\psi}\psi \sim -m\bar{\chi}\chi$$

Tuning *m* drives a transition from a bosonic SPT state to a trivial state (Grover, Vishwanath, 2012, Lu, Lee, 2012):
the Chern-Simons level of *A* and *B* changes by +2 and -2 respectively.

Self-dual N=2 QED3

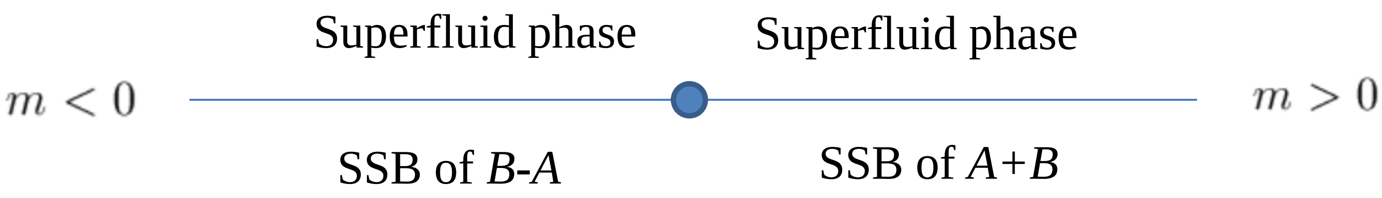
$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

$$\leftrightarrow \ \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i b_\mu) \psi_j + i B_\mu \bar{\psi} \gamma_\mu \tau^z \psi + \frac{i}{2\pi} b \wedge dA$$

Comments:

3, there is an O(4) breaking fermion mass term, that drives the system into a superfluid phase:

$$m \bar{\psi} \sigma^z \psi \sim -m \bar{\chi} \sigma^z \chi$$



Self-dual N=2 QED3

4, evidences for the *N*=2 QED3 to be a CFT
4.1 Direct numerical evidence: Karthik, Narayanan arXiv:1606.04109

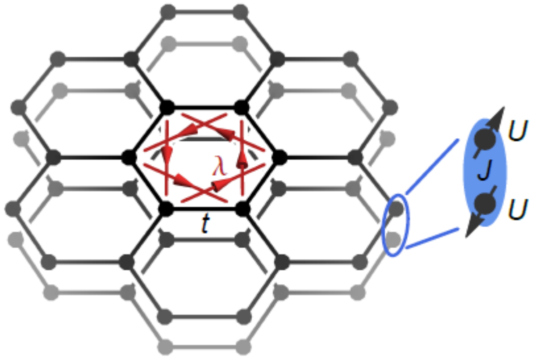
4.2 the tuning parameter $m\bar{\psi}\psi \sim -m\bar{\chi}\chi$ drives a topological phase transition, and the Chern-Simons level of the back ground field *A* and *B* change by +2 and -2 respectively. If we enhance *A* and *B* to SU(2) background gauge fields, their levels change by +1 and -1 respectively.

Simulation on a lattice model with the same transition (Slagle, You, Xu, arXiv:1409.7401, He, etc. arXiv:1508.06389):

$$H = H_{\text{band}} + H_{\text{int}},$$

$$H_{\text{band}} = -t \sum_{\langle ij\rangle,\ell} c^\dagger_{i\ell} c_{j\ell} + \sum_{\langle\langle ij\rangle\rangle,\ell} \mathrm{i}\lambda_{ij} c^\dagger_{i\ell}\sigma^z c_{j\ell} + H.c.,$$

$$H_{\text{int}} = \quad + J \sum_i \left[\boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \frac{1}{4}(n_{i1}-1)(n_{i2}-1) - \frac{1}{4} \right]$$

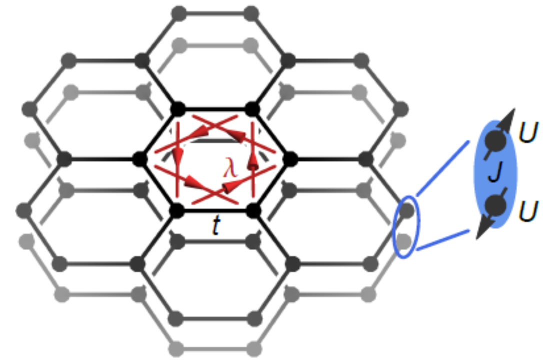


Self-dual N=2 QED3

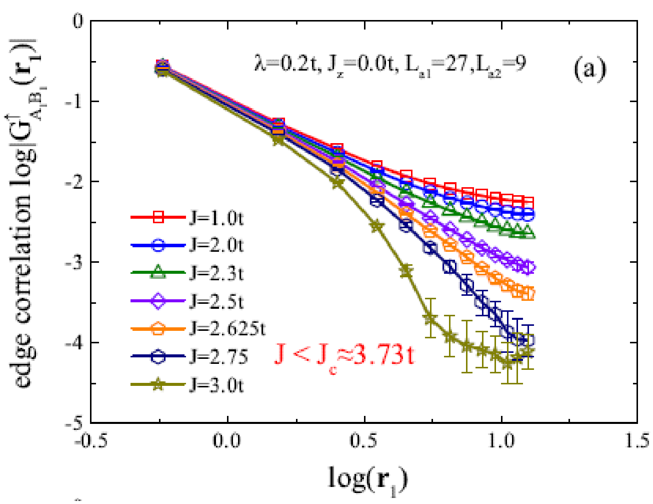
$$H = H_{\text{band}} + H_{\text{int}},$$

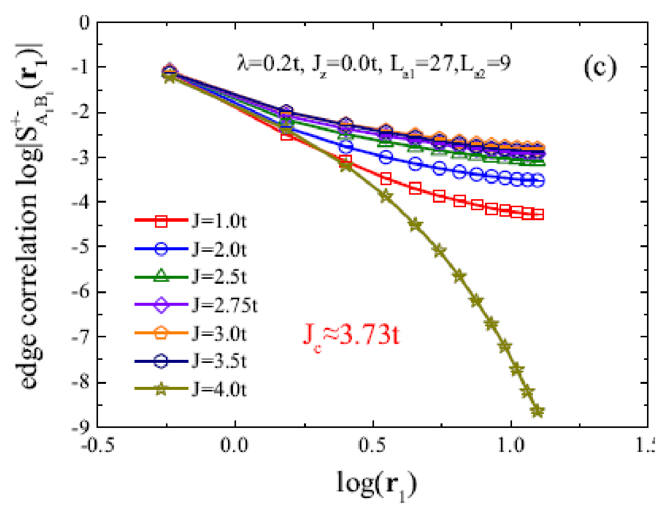
$$H_{\text{band}} = -t \sum_{\langle ij \rangle, \ell} c_{i\ell}^{\dagger} c_{j\ell} + \sum_{\langle\langle ij \rangle\rangle, \ell} \mathrm{i}\lambda_{ij} c_{i\ell}^{\dagger} \sigma^z c_{j\ell} + H.c.,$$

$$H_{\text{int}} = \quad + J \sum_i \left[\boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \frac{1}{4}(n_{i1} - 1)(n_{i2} - 1) - \frac{1}{4} \right]$$



When J is finite but not too large, the system has gapless bosonic edge states, but no gapless fermion edge states. Detailed calculation is consistent with the CS term of the back ground gauge field.

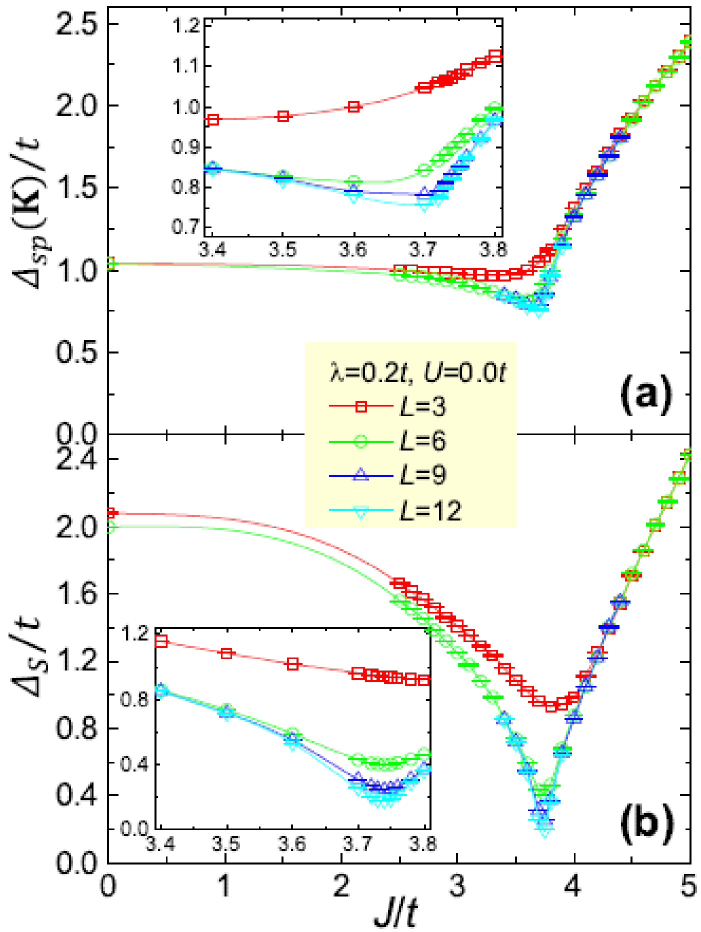




Self-dual N=2 QED3

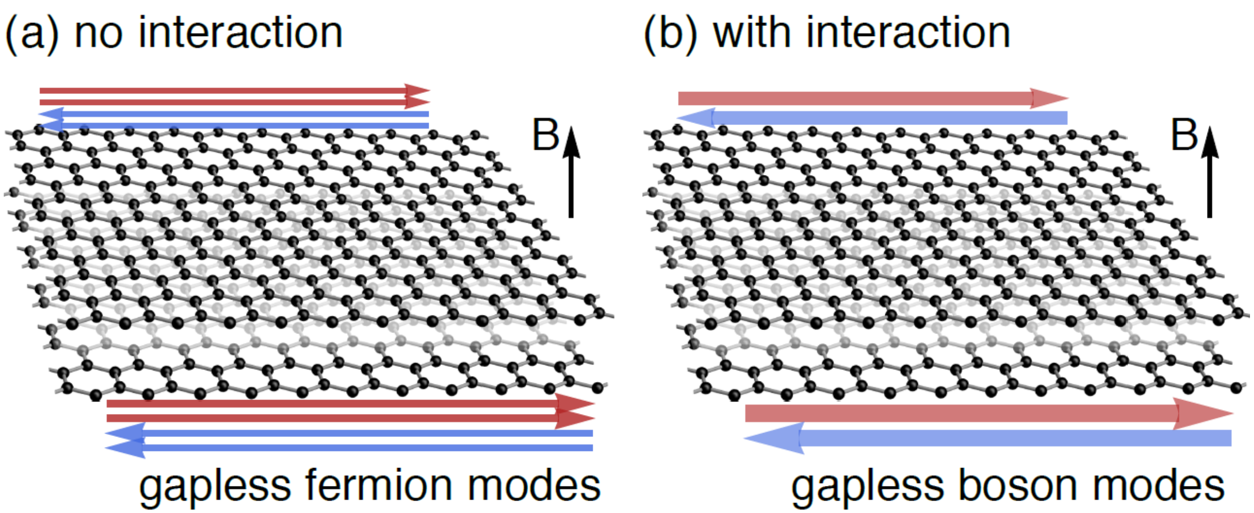
Increasing *J/t* drives a bulk topological phase transition, where the bosonic edge states disappear.
local fermions are gapped by interaction at the bulk transition, i.e. no gauge invariant gapless fermions at the transition.

But bosonic degrees of freedom close gap at the transition.



Possible Experimental realization

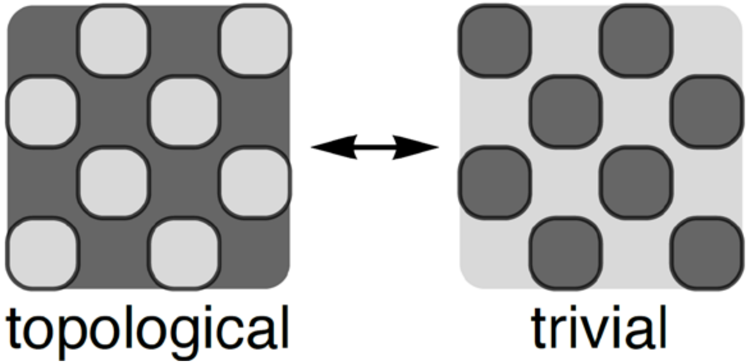
Possible realization of the physics: bilayer graphene with Coulomb interaction (Bi etc. arXiv:1602.03190):



The Coulomb interaction plays a key role of gapping out all the electrons, while leaving the bosonic degrees of freedom (charge and spin) gapless at the edge, i.e. this system is effectively a bosonic SPT state.

Possible Experimental realization

Possible realization of the physics: bilayer graphene with Coulomb interaction (Bi etc. arXiv:1602.03190):



The competition between the out-of-plane magnetic and electric field may realize the desired topological transition described by N=2 QED.

A series of self-dual QED3

Using the same logic, we can derive the following duality:

$$\mathcal{L} = \bar{\psi}_1\gamma_\mu(\partial_\mu - ika_\mu - i2n_B B_\mu)\psi_1 + \bar{\psi}_2\gamma_\mu(\partial_\mu - ia_\mu)\psi_2 + \frac{in_A}{2\pi}\epsilon_{\mu\nu\rho}a_\mu\partial_\nu A_\rho$$

$$\updownarrow$$

$$\mathcal{L} = \bar{\chi}_1\gamma_\mu(\partial_\mu - ib_\mu)\chi_1 + \bar{\chi}_2\gamma_\mu(\partial_\mu - ikb_\mu - i2n_A A_\mu)\chi_2 + \frac{in_B}{2\pi}\epsilon_{\mu\nu\rho}b_\mu\partial_\nu B_\rho$$

In the large-*k* limit, ψ_2 and χ_2 effectively decouple from the gauge fields. Thus in this limit we can compute the gauge field propagator exactly:

$$G^a_{\mu\nu}(\vec{p}) = \frac{\pi^2}{k^2|p|}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

A series of self-dual QED3

$$G^{a}_{\mu\nu}(\vec{p}) = \frac{\pi^2}{k^2|p|}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

Using this gauge field propagator, we can perform a 1/*k* expansion for various quantities, like the scaling dimension of gauge invariant operators:

$$\Delta[\bar{\psi}_j\psi_j] = 2 - \frac{4}{3k^2}$$

We can also compute the DC conductivity of *A* and *B* in the large-k limit:

$$\langle J^A_\mu(p)\, J^A_\nu(-p)\rangle = \tilde{\sigma}_A|p|\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$J^A_\mu = \tfrac{n_A}{2\pi}\epsilon_{\mu\nu\rho}\partial_\nu a_\rho \qquad \tilde{\sigma}_A = \frac{n_A^2}{4k^2}$$

A series of self-dual QED3

With odd integer k, by turning on Dirac fermion mass operators, this CFT can be driven into a topological order with $(k^2 + 1)/2$ different Abelian anyons. This implies that the entanglement entropy of this topological order is

$$S = \alpha \frac{R}{\epsilon} - F, \qquad F = \frac{1}{2} \log \left(\frac{k^2 + 1}{2} \right)$$

The F-theorem states that the topological F is the lower bound for the F of the CFT.

For $k = 1$, a different bound of F can be found, because this theory can be driven into the 3d O(2) Wilson-Fisher fixed point by turning on one mass term, F must be larger than that of the 3d O(2) WF fixed point.

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - i a_\mu)\psi + m\bar{\psi}\psi + \bar{\Psi}\gamma_\mu(\partial_\mu - i a_\mu)\Psi + M\bar{\Psi}\Psi$$

Possible Numerical Test

To get our (k, 1) theory, and compare with our $1/k$ expansion, we can design two flavors of fermions f_1, f_2 on a lattice model, which couple to a lattice gauge field with charge k and 1.

Then we can tune f_1 to the critical point between Chern number 0 and -1, and f_2 to the critical point between Chern number $(k^2 - 1)/2$ and $(k^2 + 1)/2$. This multi critical point is the (k, 1) theory.
We have given up time-reversal in this construction.

A standard way of engineering dynamical U(1) gauge field coupled to matter field, is by using the following Gutzwiller projected wave function:

$$\prod_J P(kn_1 = n_2)_J |f_1\rangle \otimes |f_2\rangle$$

Another "trick" is needed to make the gauge field noncompact.

More duality of N=2 QED3

Another duality of the same theory (Wang, ctc. To appear)

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

Step 1: run the fermion-boson duality for each flavor:

$$\leftrightarrow \ \mathcal{L} = |(\partial_\mu - i b_\mu) z_1|^2 + g|z_1|^4 + |(\partial_\mu - i c_\mu) z_2|^2 + g|z_2|^4 + \frac{i}{2\pi} a \wedge (b - c) + \frac{i}{4\pi} b \wedge db - \frac{i}{4\pi} c \wedge dc$$

Step 2: Integrating out dynamical gauge field *a*:

$$\mathcal{L} = \sum_j |(\partial_\mu - i b_\mu) z_j|^2 + g|z_j|^4$$

More duality of N=2 QED3

Phase diagram matching:

$$\mathcal{L} = \sum_j |(\partial_\mu - ib_\mu)z_j|^2 + r|z_j|^2 + g|z_j|^4$$

$r < 0$ SF ordered SF order $r > 0$

$m < 0$ $m > 0$

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - ia_\mu)\psi_j + m\bar{\psi}\sigma^z\psi$$

The easy plan NCCP1 model is also self-dual. It is the theory for the deconfined quantum phase transition between easy plane Neel order and VBS order.

More duality of N=2 QED3

An UV fixed point for both (dual) CFTs:

$$\mathcal{L} = \sum_j |(\partial_\mu - ib_\mu)z_j|^2 + g(\sum_j |z_j|^2)^2 \qquad \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - ia_\mu)\psi_j + h\phi\bar{\psi}\psi + g'\phi^4$$

Easy plane anisotropy ↓

Gaping out Ising field ↓

$$\mathcal{L} = \sum_j |(\partial_\mu - ib_\mu)z_j|^2 + g|z_j|^4 \qquad \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - ia_\mu)\psi_j$$

Summary

We propose a series of self-dual stable CFTs in (2+1)d, whose infrared properties can be computed in a controlled method.

The $1/k$ calculation can be tested numerically, which would provide quantitative evidence for the originally proposed duality between Dirac fermion and N=1 QED.

We also propose an experimental realization of one of the cases under study.